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LAYER FLOW OVER INDENTED SURFACES

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Highlights

- Stability of compressible boundary layer flow over indented surfaces is considered.
- Small surface indentations enhance certain flow instabilities.
- An increase in Mach number enhances further this behaviour.
- Amplification for deepest case are locally up to 20 times larger than in flat plate.

A STABILITY ANALYSIS OF THE COMPRESSIBLE BOUNDARY LAYER FLOW
OVER INDENTED SURFACES

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Abstract

This contribution presents a stability analysis for compressible boundary layer flows over indented surfaces. Specifically, the effects of increasing depth D/δ^* and Ma_∞ number on perturbation time-decay rates and spatial amplification factors are quantified and compared with those of an unindented configuration. The indented surfaces represent aeronautical lifting surfaces endowed with the smooth gap resulting when a filler material applied at the junction of leading-edge and wing-box components retracts upon its curing process. Since the configuration considered is such that the parallel/weakly-parallel assumptions are necessarily compromised, a global temporal stability analysis is considered in this study. Our analysis does not require a parallel flow constrain, and hence it is believed to be valid when two dimensional effects are relevant.

We find that small surface modifications enhance certain flow instabilities. An increase in Ma_∞ enhances further this behaviour: for the $D/\delta^* = 1.5$, $Ma_\infty = 0.5$ case, amplification factors at a given location can be up to 20 times larger than those corresponding to the unindented case.

Keywords: stability analysis, compressible boundary layer flow, flat plate, indentation, high-order numerical method

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1. Introduction

The aeronautics industry has shown an increased interest on natural laminar flow (*NLF*) wings. These wings are carefully designed to maintain the flow under laminar conditions over a relatively large extent of the wing area. The advantage is a lower skin friction and the consequent reduction in fuel consumption; this is achieved at the price of more stringent manufacturing tolerances.

From the manufacturing viewpoint, wings are assembled by joining several components, e.g. the main central wing box and the leading and trailing edges. The fitting between these elements is never perfectly tight: small grooves are always left at the wing-box/leading edge and trailing edge junctions. *Filler* materials, of resinous nature, are applied at these locations to alleviate the misfitting problem. However, since filler materials retract during its curing process, a small, possibly smooth indentation remains. The question arises then, whether this smaller but somewhat unavoidable groove in the wing box/leading edge junction can enhance the growth of boundary layer instabilities. In such case, a significant forward movement of transition location would spoil the effort invested in the design of the natural laminar flow region.

It is well known that in real swept wings, transition to turbulence is mainly driven by cross-flow and Tollmien-Schlichting instability mechanisms [1]. However, and contrarily to more established wing concepts, *NLF* wings operate at comparatively lower sweep angles [2]; this in turn translates in an increased relevance of Tollmien-Schlichting over cross-flow dominated transition mechanisms.

Spatial growth of Tollmien-Schlichting (or **TS**) structures is one of the avenues explaining laminar to turbulent transition. Through this mechanism, *i.e.* the natural transition scenario, the **TS** waves grow exponentially over a finite length, to then saturate and interact in a non-linear fashion, leading eventually to transition to turbulence. Alternatively, non-linear interactions may appear without

a definite preliminary exponential growth phase [3] (hence the term *bypass* transition). These alternative mechanisms are not covered in this work.

In the aircraft industry, it is common practice to employ semi-empirical, but
 35 extensively validated methods, to predict natural transition location for flows
 over wings and fuselages at flight conditions. The most common tools either
 perform a local stability analysis [4, 1] or solve the Parabolized Stability Equa-
 tions [or *PSE*, 5] upon a base state obtained numerically; application of the e^N
 criteria [6, 7] allows then to predict approximate transition locations for nat-
 40 ural scenarios, as long as the parallel or weakly parallel assumptions are fulfilled.

Whenever small surface imperfections are present, the preliminary exponential
 growth phase (natural transition) may be compromised, resulting possibly into
 a different transition location. In this case, the parallel hypothesis is not valid
 45 anymore, and classical methods may have difficulties in predicting the modified
 transition location [2]. Alternative methods are needed to handle these situa-
 tions.

Zahn & Rist report in [8] a detailed analysis of the effect of deep gaps in laminar-
 50 to-turbulent transition for a $Ma_\infty = 0.6$ flow, employing direct numerical sim-
 ulation. They succeed in identifying an acoustic feedback mechanism between
 standing waves at the gap and the boundary layer, and derived a model that suc-
 cessfully accounts for amplification factor modifications. They also investigate
 the transition delay effect induced by a deep cavity placed before a forward-
 55 facing step.

An alternative approach, based in the definition of a Local Scattering Problem,
 has been proposed recently in [9, 10, 11]. This method leads to an eigenvalue
 problem whose solution bridges the spatial behaviour much before and after the
 60 scatter location (indentation, bump, different materials junction, ...)

In this context, we propose to quantify the effect of small indentations by using global stability analysis techniques [12], since these do not rely on the parallel or weakly-parallel flow assumptions. Indeed, many contributions describe the application of global techniques -both in its modal and non-modal variants- to study laminar separation bubbles on flat plate (*FP*) configurations, be they generated by a convex bump [13, 14, 15], by a concave indentation [16] or by an adverse pressure gradient [17]. Alternatively, direct numerical simulation followed by solution of the linearised Navier-Stokes equations may be employed: e.g. [18] investigates the effect of very small-scale, localised bumps and indentations on the Tollmien-Schlichting waves appearing on a *FP* configuration.

In this work, we aim at studying how the presence of an indentation modifies the stability characteristics of a canonical zero pressure gradient boundary layer (or *BL*) over a flat plate. We specifically seek to quantify the effects of increasing indentation depth and flow compressibility (*i.e.* Mach number) on the linear stability (*i.e.* the spectrum and amplification factors) by means of global stability tools. In line with most of the studies mentioned above, the flow is considered bidimensional.

The rest of the document is structured as follows: next section describes the flow configurations considered and presents the tool chain employed in our study. Section 3 gathers the results and discussions. Finally, section 4 summarises our conclusions.

2. Flow configuration and numerical methods

We study a zero pressure gradient boundary layer flow over a flat plate geometry that includes a smooth groove or indentation. The indentation, of infinite spanwise extent, sits at a certain distance downstream of its leading edge (see section 2.1 for the problem description). We proceed -as in a classical stability analysis- by obtaining first a steady (numerical) solution to the flow governing

equations: the *base* flow (*cf.* sections 2.2 and 2.3); a linear perturbation of this *basic* flow solution and its subsequent expansion in terms of Fourier modes allows then to assemble a discrete eigenvalue problem, (or *EVP*, *cf.* section 2.4).

95 The spectral information (eigenvalues and eigenfunctions) retrieved is analysed along two dimensions: on the one hand, the eigenvalue locations in the complex plane; on the other hand, the spatial evolution of individual components along the streamwise direction, as given by their amplification factors (section 2.5).

2.1. Problem description

100 We consider compressible boundary layer flows with $Re_{\delta^*(x)} \in [610, 1050]$ at upstream Mach numbers $Ma_\infty = 0.1$ and 0.5 . The incompressible boundary layer flow over a flat plate configuration is, in the range of $Re_{\delta^*(x)}$ considered, convectively unstable [4], and has been addressed in [19, 20].

105 The Reynolds number is based on a displacement thickness $\delta^*(x)$:

$$Re_{\delta^*} = \frac{\rho_\infty U_\infty \delta^*(x)}{\mu_\infty}, \quad (1)$$

where ρ_∞ , U_∞ and μ_∞ are the density, speed and dynamic viscosity upstream.

Figure 1 shows the computational domain studied: it is rectangular in shape, of length L_x and height L_z , and the air flows from left to right. The leading edge of the flat plate is not simulated, instead a solution to the compressible boundary layer equations at the corresponding Ma_∞ is imposed at the leftmost edge of the computational domain, *i.e.* the inlet. The choice of the domain extent is partly guided by previous results on incompressible *BL* flows at the same Re_{δ^*} [19, 20]. Specifically, L_z/δ^* needs to be chosen large enough so neither the *BL* growth nor
115 the global eigenfunctions are artificially constrained, see [12]; this consideration becomes more and more restrictive as both Ma_∞ and D/δ^* increase.

The isolated indentation, when present, is located at a distance x_c from the left edge and is characterised by its breadth L and depth D . The notch considered

Figure 1: Computational domain definition: wall profile (a); and part of the Spectral Element mesh employed (b, every other element shown) for $D/\delta^* = 1.5$. Axes are not to scale.

Re_{δ^*}	Ma_{∞}	L_x/δ^*	L_z/δ^*	x_c/δ^*	L/δ^*	D/δ^*	L/D
610	0.1 and 0.5	400	40	100	50	0, 1, 1.5	$\infty, 50, 33.3$

Table 1: Configurations considered.

120 presents a smooth, Gaussian-like profile given as $z = -D \exp\left(\frac{x-x_c}{L/2}\right)^2$.

All the geometrical parameters defining the problem are non-dimensionalised with the mass displacement thickness δ^* at the leftmost edge of the domain. In this study we fix the groove extent L/δ^* and location x_c/δ^* and vary the
 125 groove depth D/δ^* and the upstream Ma_{∞} number [21]; table 1 summarises the different configurations considered. Notice that the range for the ratio L/D here included is essentially different from previous studies on rectangular low aspect ratio cavities - $L/D \in (1/5, 4)$ - at high Reynolds and Mach numbers described e.g. in [22, 23, 8].

130 2.2. Governing equations

We are interested in flows governed by the compressible Navier-Stokes equations, that once expressed in terms of non-dimensional, conserved variables $\mathbf{U} = [\rho, \rho \vec{v}^t, \rho E]^t$ (mass, momentum and total energy per unit volume), can

be written in compact vector form as:

$$\frac{\partial \mathbf{U}}{\partial t} + \mathcal{N}(\mathbf{U}) = \vec{0} \quad (2)$$

135 where $\mathcal{N}$ is the divergence of the flux tensor:

$$\mathcal{N} = \nabla \cdot \bar{\bar{F}}(\mathbf{U}), \quad (3)$$

and $\bar{\bar{F}}$ gathers convective and diffusive effects:

$$\bar{\bar{F}}(\mathbf{U}) = \begin{pmatrix} \rho \vec{v} \\ \rho \vec{v} \cdot \vec{v}^t + p \bar{I} - \bar{T} \\ \rho H \vec{v} - \bar{T} \cdot \vec{v} + \vec{q} \end{pmatrix}. \quad (4)$$

In equation (2) above, p and ρH stand for thermodynamic pressure and total enthalpy per unit volume, respectively; $\bar{I}$ is the identity matrix.

A Newtonian behaviour is assumed, the viscous stress tensor $\bar{T}$ is therefore given
140 by the expression:

$$T_{i,j} = \frac{1}{Re} \left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{i,j} \right], \quad (5)$$

while the heat flux vector $\vec{q}$ is given by Fourier law:

$$q_i = -\frac{1}{Re \, Pr} \frac{\partial T}{\partial x_i}. \quad (6)$$

2.3. Base flow computation

In order to solve system (2), we employ a high-order, staggered multi-domain Spectral Element Method (*SEM*) technique, described in detail in [24, 25]. The
145 numerical tool solves the strong form of the compressible Navier-Stokes equations using collocated values for $\mathbf{U}$ at Gauss-Chebyshev nodes; the flux tensor $\bar{\bar{F}}$ is discretised at Gauss-Chebyshev-Lobatto nodes to ensure inter-element continuity. The solution and the flux tensor are expressed as:

$$\mathbf{U}^h = \sum_{e=1}^{N_{Dom}} \sum_{i,k=1}^{p_x+1, p_z+1} \mathbf{U}_{i,k}^e \Phi_{i,k}^e(x, z), \quad \bar{\bar{F}}^h = \sum_{e=1}^{N_{Dom}} \sum_{i,k=1}^{p_x+1, p_z+1} \bar{\bar{F}}_{i,k}^e \Psi_{i,k}^e(x, z), \quad (7)$$

150 where $\Phi_{i,k}^e(x, z)$ and $\Psi_{i,k}^e(x, z)$ are the basis functions obtained as a tensor product of one-dimensional Legendre polynomials (orders p_x and p_z). Note that the flux tensor discretisation employs a higher-order polynomial, so that its divergence has the same order as the one used for the solution.

The spatially discretised time varying and non-dimensional form of system (2) is:

$$\mathcal{M} \frac{\partial \mathbf{U}^h}{\partial t} = \mathcal{N}(\mathbf{U}^h). \quad (8)$$

The non-linear discrete operator $\mathcal{N}(\mathbf{U}^h)$ above gathers the convective and diffusive contributions, while $\mathcal{M}$ is the mass matrix resulting from the *SEM* spatial discretisation. It is convenient to proceed by redefining

$$\frac{\partial \mathbf{U}^h}{\partial t} = \mathbf{G}(\mathbf{U}^h) ; \text{ where } \mathbf{G}(\mathbf{U}^h) = \mathcal{M}^{-1} \mathcal{N}(\mathbf{U}^h). \quad (9)$$

155 We use a marching procedure in pseudo-time [26] to converge system (9) towards a steady state solution, reached whenever the pointwise maximum of $\frac{\partial \mathbf{U}}{\partial t}$ term falls below 1×10^{-9} . This methodology enables stability studies upon steady base flows and instabilities of convective nature. Additional details on the base
160 flow solver can be found in [24, 25].

At the interfaces between adjacent domains, the advective contributions are computed with the Roe approximated Riemann solver, while viscous terms are simply averaged. High-order representation of curved boundaries is achieved
165 by means of a conformal mapping between the physical and computational domains: this succeeds at representing accurately the wall surface, while avoiding spurious perturbations in the solution.

At the left, top and right boundaries the compressible *BL* profile at the corresponding $Re_{\delta^*(x)}$ is imposed through a Riemann flux solver. Additionally,
170 the viscous fluxes are extrapolated from the interior domain. For the wall, an

adiabatic, no-slip boundary condition is enforced.

2.4. Eigenvalue problem formulation and stability analysis

Let $\mathbf{U}_b^h$ be a (discrete) steady solution to system (9), obtained with the *SEM* solver described above¹. Consider next a linearly perturbed base flow, say:

$$\mathbf{U}_* = \mathbf{U}_b^h + \mathbf{U}', \text{ with } \|\mathbf{U}'\| \ll \|\mathbf{U}_b^h\|. \quad (10)$$

Aiming at a Global (modal) temporal linear stability analysis [12], the perturbation $\mathbf{U}'$ is expanded as:

$$\mathbf{U}' = \hat{\mathbf{U}}(x, z) e^{-i\omega t} + \text{c.c.}, \quad (11)$$

where $\omega \in \mathbb{C}$ and *c.c.* stands for complex conjugate. Substitution of equation (11) into (9) and subsequent linearisation (*i.e.* using a first order Taylor series approximation around the base flow) lead to a (discrete) linearised system of equations which, when rearranged as the block-vector $\mathcal{U}$, can be written as the eigenvalue problem:

$$\mathcal{A}\mathcal{U} = -i\omega\mathcal{U}, \quad (12)$$

Matrix $\mathcal{A}$ is the Jacobian $\frac{\partial \mathbf{G}^h}{\partial \mathbf{U}_b^h}$. In this work, the Jacobian matrix $\mathcal{A}$ has been computed numerically using a complex-step approximation [27], which has already been used for stability analysis by the authors in [28].

The complex-step derivative approximation is accomplished by approximating a nonlinear function g (*i.e.* any of the components of $\mathbf{G}$) with a complex variable using a Taylor series expansion:

$$g(x + i\epsilon) = g(x) + \frac{dg}{dx}i\epsilon - \frac{d^2g}{dx^2}\frac{\epsilon^2}{2} - \dots \quad (13)$$

¹In this work, the stability analysis is performed on the same domain where the base flow has been computed: contrarily to other approaches [19, 16], no interpolation is applied between base flow and stability domains.

taking the imaginary part and neglecting terms of order ϵ^2 and higher:

$$\frac{dg}{dx} = \frac{\text{Im}[g(x + i\epsilon)]}{\epsilon}. \quad (14)$$

The advantages of the complex-step approximation approach over a standard finite differencing include: 1) it provides second-order accuracy with a single function evaluation, 2) the Jacobian approximation is not subject to subtractive cancellations inherent in roundoff errors, and 3) it is easy to implement in a black-box manner, thereby making applicable to general nonlinear functions. On the downside part, this approach requires to implement a complex version of the numerical fluxes $\bar{\bar{F}}^h$.

Assembling the eigenvalue problem in equation (12) through the discrete approach [29, 28] implies that the Jacobian matrix $\mathcal{A}$ is built by repeated application of the base flow solver. In this sense, boundary conditions (BCs) for the eigenfunctions $\mathcal{U}$ are defined by the base flow solver and the complex-step derivative approximation just described. Namely, BCs for the EVP are inherited from the base flow solver. Thus, in view of how BCs are enforced in the base flow problem, BCs for the eigenfunctions $\mathcal{U}$ are equivalent to Robin conditions.

A Shift and Invert methodology [30], relying on an Arnoldi iteration technique [31] implemented by means of MUMPS library [32], allows to obtain approximations to the spectrum of $\mathcal{A}$. The number of eigenvalues/eigenvectors retrieved is precisely the dimension of the Krylov space employed, N_K .

Complex eigenvalues $\omega = \omega_R + i\omega_I$ obtained from equation (12) characterise the corresponding eigenfunctions $\mathcal{U}$: the real part ω_R is the mode angular pulsation (related to its temporal frequency) while the imaginary part ω_I establishes how fast the mode grows/decays exponentially in time.

215 2.5. Amplification factors

In order to quantify in a consistent manner the effect of the indentation on the spatial growth of the TS-like eigenmodes, we follow [19] and define the spatial amplification factor of the eigenfunctions as:

$$A(x) = \sqrt{\int_{z_{min}(x)}^{L_z} (\delta u^\dagger \delta u + \delta w^\dagger \delta w) dz}, \quad (15)$$

where δu and δw are the velocity components of the 2D global perturbation; superscript $\dagger$ indicates complex conjugation.

Since the eigenvalue problem in equation (12) is formulated in terms of the conserved variables, δu and δw are not directly available; but can be retrieved straightforwardly as:

$$\delta u = \frac{\delta(\rho u) - u_b \delta \rho}{\rho_b}, \quad \delta w = \frac{\delta(\rho w) - w_b \delta \rho}{\rho_b}, \quad (16)$$

where $u_b = \rho^{u_b}/\rho_b$ and $w_b = \rho^{w_b}/\rho_b$ are computed from the base flow $\vec{U}_b$.

3. Results and discussion

3.1. Base flows

Solutions for the unindented flat plate and two different depths for different upstream Ma_∞ numbers (see Table 1) have been computed. In every case, the domain has been divided in $N_{Dom} = 48$ spectral elements distributed along the streamwise direction, with refinement near the indentation region (figure 1b). Different *intra-element* resolutions $p_x \times p_z$ have been considered; p_x and p_z are the polynomial orders of the underlying basis functions along the x and z directions respectively. For all the configurations discussed in this work, the residuals decreased by at least nine orders of magnitude.

In figure 2, we address the accuracy and the sensitivity of the numerical tool employed. Figure 2a compares the skin friction coefficient C_f for the unindented low $Ma_\infty = 0.1$ case (intra-element resolution $p_x \times p_z = 10 \times 60$) against

that corresponding to a incompressible Blasius profile: these curves overlap. Figure 2b presents the sensitivity of the C_f coefficient to intra-element grid resolution for the most challenging $D/\delta^* = 1.5$, $Ma_\infty = 0.5$ case: there is barely any appreciable difference for the different resolutions considered. For reasons explained in section 3.2, we retain results computed with the $p_x \times p_z = 10 \times 60$ resolution.

The effect of the notch is to deflect the incoming BL flow; of course, the deeper the indentation the larger the flow deflection. For the case $D/\delta^* = 1$ the flow remains attached, as revealed by skin friction $D1$ in figures 3c-3d; for the deepest case the flow separates from the wall and a steady-state recirculation zone, confined in the groove, is established (see figures 3a-3b), thus invalidating the parallel flow assumption. This fact is confirmed by curves $D1.5$ in figures 3c-3d.

Figure 2: (a) Skin friction for the $Ma_\infty = 0.1$ FP ($p_x \times p_z = 10 \times 60$) case against that corresponding to a Blasius profile (curves overlap); (b) Effect of numerical resolution on skin friction for $Ma_\infty = 0.5$, $D/\delta^* = 1.5$.

Figure 3: Horizontal velocity component for $D/\delta^* = 1.5$ case with $Ma_\infty = 0.1$ (a) and $Ma_\infty = 0.5$ (b) conditions. Skin friction for FP (—), and depths $D/\delta^* = 1$ (---) and 1.5 (-.-) for both $Ma_\infty = 0.1$ (c) and $Ma_\infty = 0.5$ (d).

3.2. Temporal modes

Figures 4a-4b show the temporal spectra for the different configurations described in Table 1. The first observation is that all the modes retrieved are temporally stable, *i.e.* $\omega_I < 0$. All spectra show two distinct branches: on the left, an almost straight branch runs diagonally and downwards, that is identifiable to the Orr branch of a parallel boundary layer flow [33]. The second branch contains modes that resemble Tollmien-Schlichting -or TS- perturbations (altered of course by the progressive *BL* thickness growth), see e.g. figure 7a. As reported by [20], no continuous branch is retrieved since the usage of a Riemann solver is equivalent to employing extrapolation *BCs*.

Prior to describing the effect of D/δ^* and Ma_∞ parameters on the spectrum, the sensitivity to both domain size and to the intra-element resolution is analysed in figure 5 for the $D/\delta^* = 1.5$, $Ma_\infty = 0.5$ case. Specifically, figure 5a shows the influence of domain extent (*i.e.* L_x , L_z) for a fixed polynomial resolution of $p_x \times p_z = 10 \times 60$. That L_x , L_z affect eigenvalue locations is expected; it has indeed been described already e.g. in [19, 15]). The spatial amplification behaviour of those modes according to equation (15) is nevertheless equivalent. Figure 5b considers in turn $L_x = 400$, $L_z = 40$ and polynomial resolutions of $p_x \times p_z = 10 \times 60$, 12×60 and 10×72 (*i.e.* the same used for the base flow computation, see section 3.1). Again, eigenmodes for the different resolutions considered behave in an equivalent manner.

Concerning the accuracy of the eigenmodes retrieved, all of them fulfil that $\|\mathcal{A}\mathcal{U} + i\omega\mathcal{U}\|_\infty \leq 1 \times 10^{-11}$ with $\|\mathcal{U}\|_2 = 1$. In both cases a Krylov space dimension of $N_K = 1000$ has been considered, as increasing N_K to 1500 leads to no change in the spectra over the region of interest.

Figure 4: Indentation depth effect on global temporal spectra for both upstream Mach numbers $Ma_\infty = 0.1$ (a) and 0.5 (b).

Figure 5: Grid study for $Ma_\infty = 0.5, D/\delta^* = 1.5$ case: sensitivity to boundary size location (a) and polynomial order (b).

		FP			$D/\delta^* = 1.5$			Change with D/δ^*
		ω_R	ω_I	$A/A_0 _{x/\delta^*=120}$	ω_R	ω_I	$A/A_0 _{x/\delta^*=120}$	$\frac{\Delta A/A_0 _{x/\delta^*=120}}{A/A_0 _{x/\delta^*=120}}$
Ma_∞	0.1	0.0529	-0.0117	1.86	0.0530	-0.0102	3.58	0.924
	0.5	0.0837	-0.0081	1.75	0.0802	-0.0067	6.60	2.77
Change with Ma_∞		$\frac{\Delta A/A_0 _{x/\delta^*=120}}{A/A_0 _{x/\delta^*=120}}$			$\frac{\Delta A/A_0 _{x/\delta^*=120}}{A/A_0 _{x/\delta^*=120}}$			
		-0.059			0.84			

Table 2: Angular pulsation ω_R , temporal amplification ω_I and spatial amplification A/A_0 at $x/\delta^* = 120$, and their relative changes with D/δ^* and Ma_∞ for eigenfunction M1.

		FP			$D/\delta^* = 1.5$			Change with D/δ^*
		ω_R	ω_I	$A/A_0 _{x/\delta^*=120}$	ω_R	ω_I	$A/A_0 _{x/\delta^*=120}$	$\frac{\Delta A/A_0 _{x/\delta^*=120}}{A/A_0 _{x/\delta^*=120}}$
Ma_∞	0.1	0.1280	-0.0310	35.48	0.1296	-0.0278	340.81	8.60
	0.5	0.1211	-0.0283	61.12	0.1217	-0.0285	1298.27	20.28
Change with Ma_∞		$\frac{\Delta A/A_0 _{x/\delta^*=120}}{A/A_0 _{x/\delta^*=120}}$			$\frac{\Delta A/A_0 _{x/\delta^*=120}}{A/A_0 _{x/\delta^*=120}}$			
		0.72			2.81			

Table 3: Angular pulsation ω_R , temporal amplification ω_I and spatial amplification A/A_0 at $x/\delta^* = 120$, and their relative changes with D/δ^* and Ma_∞ for eigenfunction M2.

		FP			$D/\delta^* = 1.5$			Change with D/δ^*
		ω_R	ω_I	$A/A_0 _{x/\delta^*=120}$	ω_R	ω_I	$A/A_0 _{x/\delta^*=120}$	$\frac{\Delta A/A_0 _{x/\delta^*=120}}{A/A_0 _{x/\delta^*=120}}$
Ma_∞	0.1	0.1291	-0.0215	3.41	0.1288	-0.0215	59.84	16.55
	0.5	0.1291	-0.0094	1.33	0.1292	-0.094	8.91	5.70
Change with Ma_∞		$\frac{\Delta A/A_0 _{x/\delta^*=120}}{A/A_0 _{x/\delta^*=120}}$			$\frac{\Delta A/A_0 _{x/\delta^*=120}}{A/A_0 _{x/\delta^*=120}}$			
		-0.61			-0.85			

Table 4: Angular pulsation ω_R , temporal amplification ω_I and spatial amplification A/A_0 at $x/\delta^* = 120$, and their relative changes with D/δ^* and Ma_∞ for eigenfunction M3.

At the sight of figures 4a-4b (see also figure 6), we conclude that the Orr-like branch is practically insensitive to increasing both depth D/δ^* and Ma_∞ . Since our objective is to study TS based transition, we focus on TS-like structures.

As shown in figures 4a-4b, the effect of the progressive increase of groove depth D/δ^* is, irrespective of Ma_∞ , to reduce the temporal decay rate ω_I for those

modes with angular pulsation $\omega_R \in [0.02, 0.12]$. This increase in ω_I does not destabilises the modes, however. Observe also how eigenvalues with pulsation lower than 0.02 and higher than 0.12 over the quasi-horizontal branch piece are
 290 insensitive to D/δ^* when Ma_∞ is fixed.

The effect of Ma_∞ is evident in figures 6a, 6b and 6c. At each depth considered, eigenvalues for the high $Ma_\infty = 0.5$ case are systematically less temporally stable (*i.e.* ω_I are comparatively larger). The effect of Ma_∞ is specially significant for the higher ω_R end of the TS branch, where $Ma_\infty = 0.5$ modes are
 295 much less temporally stable than their $Ma_\infty = 0.1$ counterparts.

For the remainder of the discussion we will focus on three eigenpairs, shown in figure 6c corresponding to $Ma_\infty = 0.5$ and D/δ^* : M1 are the modes with largest
 300 temporal amplification; the choice of M2 is justified in section 3.3; M3 is chosen as representative of the modes in the large pulsation range. Tables 2-4 gather additional information on these modes for the *FP* and the $D/\delta^* = 1.5$ cases.

Let us describe now the shape of modes M1, M2 and M3 highlighted in figure 6c.
 305 Figure 7a shows M1: it presents the characteristic TS structure, with a relatively long wavelength. Higher frequency modes M2 and M3 display also a TS character, with progressively shorter wavelengths than M1, figures 7b and 7c. Notice also how M2 and M3 differ specially in the groove region, as M3 is (visually) appreciable already before the leading edge of the indentation and its magnitude is
 310 specially intense in the trailing edge of the indentation, near the reattachment region of the flow. And yet another difference, M1 and M2 grow monotonically over the region of interest while M3 decays after an initial growth behaviour (see also figure 8).

(a) FP (b) $D/\delta^* = 1$ (c) $D/\delta^* = 1.5$ Figure 6: Influence of Ma_∞ on spectra for FP (a), $D/\delta^* = 1$ (b) and $D/\delta^* = 1.5$ (c).

(a) M1 for $Ma_\infty = 0.5$ and $D/\delta^* = 1.5$

(b) M2 for $Ma_\infty = 0.5$ and $D/\delta^* = 1.5$

(c) M3 for $Ma_\infty = 0.5$ and $D/\delta^* = 1.5$

Figure 7: Real part of the x -momentum perturbation for eigenfunctions M1 (a), M2 (b), and M3 (c) for $D/\delta^* = 1.5$, $Ma_\infty = 0.5$.

3.3. Amplification factors

In this section we analyse the amplification factors -equation (15)- to quantify the spatial growth of the different eigenmodes considered. Amplification factors are non-dimensionalised with their value $A_0 = A(x/\delta^* = 50)$, well before the indentation: subsequent figures actually display the ratio $A(x)/A_0$ over the range $x/\delta^* \in [50, 300]$.

In order to measure the relative growth of the spatial modes, we have recourse to the e^N semi-empirical criteria. Engineering practice assumes laminar to turbulent transition [6, 7] whenever a linear perturbation has grown e^8 to e^{10} times. Figure 8 shows amplification factors for the deepest case $D/\delta^* = 1.5$ at both Mach numbers for modes M1, M2 and M3 in previous section. Eigenfunctions with pulsation $\omega_R \approx 0.121$ for $Ma_\infty = 0.5$ ($\omega_R \approx 0.130$ for $Ma_\infty = 0.1$) are those with the fastest and largest spatial growth, for both the Ma_∞ considered. This is the reason supporting the choice of M2 in figure 6c.

Figure 8: Amplification factors, for $D/\delta^* = 1.5$ configuration at $Ma_\infty = 0.1, 0.5$. Most unstable (M1), most spatially amplified (M2) and high frequency (M3) modes considered.

Observe how the associated amplification factors for both Ma_∞ numbers in the *FP* case do not differ significantly, figure 9a and table 3. However, as depth is increased, an increased Ma_∞ results in faster spatial growth, figures 9b-9c.

335 Figure 10 brings another perspective of M2 behaviour, complemented with M1 and M3 sensitivities to D/δ^* and Ma_∞ .

Modes M1 and M2 -both in the pulsation range where spectra is sensitive to D/δ^* - share a similar behaviour (figures 10a-10d), where the streamline deflection induced by the indentation alters the slope of ratio $A(x)/A_0$; this modification
340 increases with depth (and base flow separation), resulting in an increase of up to hundred times over the extent of the indentation width. By the end of the indentation, the slope of $A(x)/A_0$ decreases again, whereas the amplification factor continues growing, in accordance to the convectively unstable nature of the
345 *BL* flow. Regarding the effect of increasing D/δ^* on $A(x)/A_0$ at a given location -e.g. at $x/\delta^* = 120$, near the end of the groove- $A/A_0|_{x/\delta^*=120}$ is up to 20 times larger than that corresponding to the unindented case, tables 2-3. Finally, observe how the $A(x)/A_0$ reaches the level e^8 up to 60 δ^* units before the *FP* for the higher Ma_∞ .

350 Modes M3 (figures 10e-10f) behave slightly differently, as they show a differentiated behaviour with respect to Ma_∞ : whereas in both cases eigenmodes experience spatial amplification, an increase in indentation depth D/δ^* has a lasting effect only for $Ma_\infty = 0.1$ eigenmodes. $Ma_\infty = 0.5$ case modes see their
355 amplification factor affected by D/δ^* , but this effect disappears as x/δ^* increases. The $A(x)/A_0$ curves even collapse into each other beyond $x/\delta^* \approx 300$. Notice as well, how despite its relatively high temporal amplification factor ω_I (table 4), this mode does not grow significantly in space (note the scale range in figure 10f).

(a) *FP*

(b) $D/\delta^* = 1$

(c) $D/\delta^* = 1.5$

Figure 9: Amplification factors for $Ma_\infty = 0.1$ (—) and $Ma_\infty = 0.5$ (---) for most spatially amplified mode ($\omega_R \approx 0.12 - 0.13$). Horizontal lines run at levels e^8 (---) and e^{10} (- · - · -).

Figure 10: Amplification factors for FP (—), and depths $D/\delta^* = 1$ (---) and 1.5 (---). Horizontal lines run at levels e^8 (---) and e^{10} (- · - · -).

360 A complementary vision of the effect of the indentation on the eigenfunctions
can be gained by exploiting the concept of transmission coefficient, discussed
in [9, 10]. Consider the least temporally stable (M1) and the most spatially
amplified (M2) modes for the $Ma_\infty = 0.5$, $D/\delta^* = 1.5$ case. Curves (dashed-
blue ones in figure 11) fitting the high x/δ^* behaviour of those eigenfunctions
365 to the spatial growth of the corresponding flat plate eigenmode have been ob-
tained. These curves can be then *traced back* to the centre of the indentation
 $x/\delta^* = 100$. This allows to estimate transmission coefficients of $A_T \approx 3$ and 10
for M1 and M2 modes, respectively. These values are consistent with those re-
ported [9, 10]. It should be noted that $\omega_i < 0$ in our work, whereas ω is real
370 (*i.e.* $\omega_i = 0$) in the local scattering approach [9, 10]. However, since ω_i is
usually rather small, the transmission coefficient extracted here is likely to ap-
proximate that in [9, 10], and provides a measure of the destabilizing effect of
the indentation.

Figure 11: Transmission coefficient estimation for M1 and M2 for the $Ma_\infty = 0.5$, $D/\delta^* = 1.5$ case.

375 4. Conclusions

Current wing manufacturing processes allow two-dimensional surface imperfections during the assembly phase. These imperfections may compromise the efficiency of natural laminar flow designs: since *NLF* wings operate at Reynolds numbers in the unstable regime, the question arises whether these surface imperfections may advance laminar-to-turbulent transition via TS amplification. 380 If that were effectively the case, the *NLF* wing would be then operating sub-optimally (increasing drag and fuel consumption).

In this contribution the influence of a groove on the stability characteristics of a flat plate boundary layer flow has been investigated. The indentation considered is representative of the smooth gap left by the retraction of a filler material applied at the junction of leading-edge and wing-box components. 385

Since the configuration considered is such that the parallel/weakly-parallel assumptions are necessarily compromised, we have performed a global temporal stability analysis. Specifically, our investigation has addressed the effect of increasing depth D/δ^* and Mach number Ma_∞ on perturbation time-decay rates and spatial amplification factors. 390

We have shown that temporal spectra are sensitive to both groove depth D/δ^* and upstream Mach number Ma_∞ . While the Mach number effect is significant over the whole angular pulsation range, that of D/δ^* is relevant only in a part of that range. 395

At a given Ma_∞ number, an increase in groove depth translates into less temporally stable modes; for a given groove depth D/δ^* , an increase in Mach number also renders modes less temporally stable. However, no matter the combination of parameters considered, the eigenvalues retrieved remain confined in the temporally stable region (*i.e.* $\omega_I < 0$). 400

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We have also studied the impact of Ma_∞ and D/δ^* on the spatial amplification factors of the individual global eigenfunctions retrieved. As expected, not all the eigenmodes are equally processed by the indentation: while certain modes are slightly altered, others experience substantial modifications: the amplification at a given location can be up to 20 times larger than that corresponding to the unindented case. All in all, for a given angular pulsation² an increase in D/δ^* results in a larger spatial amplification. An increase in Ma_∞ may enhance further this behaviour.

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In this context, one can find combinations of ω_R , D/δ^* and Ma_∞ that, according to an e^N method, would lead to early transition soon after the indentation. This is more likely as D/δ^* and Ma_∞ increase, and poses concerns along two directions. On the one hand, it sets a limitation on the tolerances allowed when applying filler materials; on the other hand, an undesired premature transition might happen as flight Ma_∞ numbers are approached, invalidating the design effort invested in the *NLF* wing.

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Finally, care should be exerted when employing classical design tools relying on parallel or weakly non-parallel assumptions since indentations seem to enhance spatial growth of TS structures.

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²Included in the preferential angular pulsation range described already.

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