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SYNCHRONIZATION AND RESONANCE EFFECTS IN NONLINEAR ELECTRONIC CIRCUITS

WITH NEUROSCIENCE APPLICATIONS

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Mariano Alberto García Vellisca

Prólogo

Entre Molinos de Viento

En un lugar de Madrid y en una familia humilde nació una pequeña persona que no imaginaba que “x” años después iba a hacer grandes cosas. Todo empezó, sin darse cuenta, destripando los juguetes de su hermano mayor, pasando por desmontar piezas de un ciclomotor y cacharreando aquí y allá. Era un niño inquieto que pasó a tener inquietudes pero algunos no saben qué pasó por su cabeza para llegar a convertirse en lo que es hoy día, un magnífico ingeniero (persona que discurre con ingenio las trazas y modos de conseguir o ejecutar algo) y un profesor en potencia, porque sabe transmitir sus conocimientos a todos los niveles, porque le gusta lo que hace.

En esto, antes de ni siquiera empezar, con buen ojo y criterio empezaste a traducir el lenguaje de las cosas lejanas a los pobres ojos de los que solo veíamos gigantes, cuyos nombres para muchos seguirán siendo dragones, y no, unos simples molinos de ocho años de acometida.

Hay que estar loco para hablar del futuro, pero la diferencia entre tú y los locos es que tú no estás loco, así que sigue pintando sueños y no dejes de vencer dragones.

Inspirado en la obra:
El Ingenioso Hidalgo Don Quijote De La Mancha.

Pues bien, a través de esta tesis doctoral el autor acerca aquellos gigantes a una distancia donde más bien parecen de nuestra estatura. En particular, mostrando un pequeño pedazo de un gigantesco campo como es la neurociencia, y cómo éste puede ser modificado simplemente con un “clic” que, como en la vida, marca la diferencia.

Velli y Maca

Abril 2017.

“Where there is a will there is a way”.

Old English Proverb

“Shoot for the moon. Even if you miss, you’ll land among the stars.”

Norman Vincent Peale

“Life is like riding a bicycle. To keep your balance, you must keep moving.”

Albert Einstein

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Abstract

This thesis analyzes the dynamics of three nonlinear systems through their theoretical and experimental models. Two of them are used in neuroscience studies *Chua's circuit* and *Hindmarsh-Rose model*.

In particular, the main contribution is based on figuring out how this type of systems get synchronized and the resonant effects that appear as a consequence of the interaction among them. We use *master-slave* and *ring* topology to couple these oscillatory systems in unidirectional (or bidirectional) diffusive manner. On one hand, we numerically simulate a general system compound of oscillators. We extract time series, for their statistical analyses, in order to figure out not only what kind of synchronization exists among them but also if any of them exhibits some resonance effect. On the other hand, we design the electronic circuit associated with the general system, mentioned above, to check how robust the results are. Following this research line, we find the experimental evidence of the appearance of some order in a chaotic system under the influence of a chaotic signal. Moreover, we also run into bistability in a master-slave neural system.

Finally, we developed a visual application to show the dynamics of two electronic neurons connected by chemical and electrical synapse.

Keywords

dynamics; nonlinear systems; Chua's circuit; Hindmarsh-Rose model; synchronization; time series; unidirectional and bidirectional diffusive coupling; resonance effect; chaotic system; bistability; electronic neurons.

Resumen

En esta tesis se analiza la dinámica de tres sistemas no lineales a través de sus respectivos modelos teóricos y experimentales. Dos de ellos, el conocido como circuito Chua y el modelo de Hindmarsh-Rose, se usan en estudios de neurociencia.

En particular, la principal contribución se basa en averiguar cómo se sincronizan este tipo de sistemas oscilatorios y los efectos resonantes que aparecen como consecuencia de la interacción entre ellos. Para acoplar osciladores de una forma difusiva y unidireccional (o bidireccional) utilizamos básicamente las configuraciones *maestro-esclavo* y en anillo. Por un lado, simulamos computacionalmente el sistema general compuesto de osciladores; extraemos los datos en forma de series temporales aplicándoles un análisis estadístico para averiguar tanto el tipo de sincronización como si aparece algún efecto resonante. Por otro lado, diseñamos los circuitos electrónicos para reproducir el sistema general con el objetivo de contrastar cuán robustos son nuestros resultados. Siguiendo esta línea de investigación, mostramos la evidencia experimental de la aparición de cierto orden en un sistema caótico bajo la influencia de una señal caótica. Además, también mostramos biestabilidad en un sistema neuronal tipo maestro-esclavo.

Finalmente, desarrollamos una aplicación visual que muestra la dinámica de dos neuronas electrónicas conectadas a través de sinápsis química y eléctrica.

Chapter 1

INTRODUCTION AND MOTIVATION

One of the first and last necessities for the human being is eating. It is something known that hunters handle accurately the motion technique, to get food. As an example, spheroids found in African archaeological sites suggest hunters used that stones as a projectile weapons ¹. Even though the mechanism of throwing it against the target seems easy it requires a high precision for hunting the prey. Particularly, to be successful in hunting it is necessary to know how the prey moves. Hence, the hunter makes some calculus, using the brain, to figure out where the prey will be at each moment. Then, he may choose a right moment to throw a spheroid against the prey ². To calculate the future prey location, the brain has to manage a huge number of variables, to process information received through visual perception. This

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¹Wilson, A. D., Zhu, Q., Barham, L., Stanistreet, I., & Bingham, G. P. (2016). A dynamical analysis of the suitability of prehistoric spheroids from the cave of hearths as thrown projectiles. *Scientific Reports*, 6, 30614.

²The world famous Spanish researcher Santiago Ramón y Cajal (1852-1934) was one of the pioneers of science who studied the brain neurons, the ancestor of neuroscience. For his achievements in neuroscience, in 1906 he was awarded a Nobel Prize in Physiology and Medicine together with Camillo Golgi. Cajal was convinced by the idea that a nervous system is made up of billions of nerve cells. The Cajal's work led to the conclusion that a basic unit of the nervous system is represented by an individual cellular element, later called "neuron". This conclusion is the basis of modern neuroscience [21].

1. INTRODUCTION AND MOTIVATION

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process is very important to reach the goal, i.e. to get the food. In particular, the eyes' reaction related with the prey movement arises from the sensitivity of ocular neurons to visual perception [59, 135]. Even, it has been proved that "...whilst the sensitivity to movement decreases as one goes towards the periphery of the visual field, this decrease is more rapid in the vertical than in the horizontal..." [102].

Going deeper into the brain studies, we know that the brain belongs to the Central Nervous System (CNS) and it is involved in performing information processing. The neural system is the principal component of any nervous system. The CNS consists of sensory systems which monitor the organism's state under environment conditions, motor systems which organize and generate actions, and the associative systems linked to other systems, providing the basis for high-level functions such as perception, attention, emotions, etc. The understanding of human beings turns around these three principal systems, although many questions of how they work remain open [122].

Nowadays, the scientific community makes a great effort in the study of the brain functionality with the objective to understand the biological mechanisms inherent to the human mental activity. L'Ecole Polytechnique Fédérale de Lausanne (Switzerland) together with IBM started an ambitious European project with the aim to create a functional brain model by means of reverse engineering of the mammalian brain [97], called *Blue Brain Project*, where several Spanish institutions are involved. The Cajal Blue Brain project as a part of the main Blue Brain Project, combines efforts from the Universidad Politécnica de Madrid (UPM) and the Consejo Superior de Investigaciones Científicas (CSIC) through the Cajal Cortical Circuits Laboratory located at the Center for Biomedical Technology (CTB).

Such an interest from the scientific community oriented my research towards neuroscience applications. Taking into account my background as an electronic engineer, electronic circuits have been used to support my research. The design of these circuits are based on theoretical neural models. The comparison of numerical and experimental results allowed to estimate the robustness of the circuits to parameter mismatch, as well as to take advan-

1. INTRODUCTION AND MOTIVATION

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tage in computation time . The implementation of neurons into electronic circuits is a challenge in the study of neural dynamics in real time during every stage, including design, simulation, manufacturing, soldering, testing, and commercialization. These neural electronic circuits have been used for experimental study of neural dynamics and synchronization in a neural network.

1. INTRODUCTION AND MOTIVATION

Chapter 2

METHODS AND MODELS

2.1 Introduction to dynamical systems

First of all, I would like to introduce a basic concept of motion. It is a matter proved that an object that moves with a constant velocity, keeps moving in the same direction unless an external force or perturbation changes its state. Let me illustrate this with a simple example. Imagine a human being cycling with a constant pedaling cadence ¹ (a dynamical system), in a calm windless day. Suddenly, a hard wind starts to blow in the cyclist's opposite direction (external perturbation). Accordingly, the cyclist changes the pedaling cadence (velocity) to keep going.

We could synthesize such example from a physical point of view, saying that the cyclist changed his momentum. In fact, physicists use to talk about the momentum of motion which is equal to a velocity multiplied by a mass. In addition, the rate of change of the momentum is directly related to the force of Newton's second law; one of the fundamental physical laws which govern dynamics [19].

Generally speaking, a system state can change with time. Consequently, a set

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¹Kautz, S. A., Feltner, M. E., Coyle, E. F., & Baylor, A. M. (1991). *The pedaling technique of elite endurance cyclists: Changes with increasing workload at constant cadence. International Journal of Sport Biomechanics*, 7(1), 29-53.

2.1. INTRODUCTION TO DYNAMICAL SYSTEMS

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of all possible states is called *phase space* [3]. When we observe an evolution of the system state, we actually refer to a *dynamical system*. Moreover, dynamical systems can be either continuous or discrete, depending on whether they are described by differential or difference equations [142]. Ordinarily, a system state is described by *state variables*. A dynamical system can be described by differential equations as follows:

$$\dot{\mathbf{x}} = \frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}; \mathbf{p}), \quad (2.1)$$

where $\mathbf{x}$ is a vector of state variables and $\mathbf{p}$ is a set of parameters, $\mathbf{F}$ is a vector function that can be either linear or nonlinear and determines the system evolution. Equation (2.1) is *autonomous* because on its right-hand side time variable does not explicitly appear. Systems, whose derivative of the state variables depends explicitly on time, are called *non-autonomous* or *driven* [73].

My main interest is focused on *dissipative systems* [10, 36] i.e., where “... the energy or phase volume varies...”² in time. Among a wide class of dynamical systems, we are interested in nonlinear oscillators which are the main subject of this thesis. The oscillators can be either autonomous or driven by an external force, respectively called *self-sustained* and *non-self-sustained* oscillators [3]. The dynamical systems properties can be analyzed through graphical representation of the system evolution in the phase space, i.e. the orbits in the space of the system variables. A projection of the phase space on the plane of two variables (2D) is called *phase portrait* [17].

In continuation, let me go deeper inside some mathematical details. Nonlinear differential equations can have stable and unstable solutions. To analyze the system stability, first, we need to solve Eq. (2.1) and find equilibrium or *fixed* points $\mathbf{x}^*$ where the system variables are constant over time. Then, we take $\mathbf{X}(t) = \mathbf{x}(t) - \mathbf{x}^*$ as a linear function. The linear approximation of Eq.

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²Anishchenko, V. S., Astakhov, V. V., Neiman, A. B., Vadivasova, T. E., & Schimansky-Geier, L. (2007). *Nonlinear dynamics of chaotic and stochastic systems*. Springer-Verlag Berlin Heidelberg, , XIII, 446

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(2.1) near the fixed point $\mathbf{x}^*$ yields

$$\dot{\mathbf{X}} = \frac{d\mathbf{X}}{dt}[\mathbf{F}'(\mathbf{x}^*)] \cdot \mathbf{X} = \mathbf{a} \cdot \mathbf{X}, \quad (2.2)$$

where $\mathbf{a} = \mathbf{F}'(\mathbf{x}^*) = \frac{\partial \mathbf{F}}{\partial \mathbf{x}}|_{\mathbf{x}^*}$. The solution for this sort of equations obey an exponential function like $C \cdot e^{\mathbf{a}t}$. The constant C can be found if we know the system state at a given time t , for instance, $\mathbf{x}(t = 0) = \mathbf{x}_0$, where x_0 is the initial condition. Since C is a positive constant, the absolute value of this function decreases to zero when $|\mathbf{a}| < 0$, and goes to infinity when $|\mathbf{a}| > 0$. The fixed point $\mathbf{x}^*$ is locally stable when $|\mathbf{F}'(\mathbf{x}^*)| < 0$, and locally unstable when $|\mathbf{F}'(\mathbf{x}^*)| > 0$, [69].

Now, we apply the linear stability analysis to a two-dimensional dynamical system in the explicit form:

$$\begin{aligned} \frac{dx}{dt} &= f(x, y), \\ \frac{dy}{dt} &= g(x, y), \end{aligned} \quad (2.3)$$

where $f(x, y)$ and $g(x, y)$ are nonlinear functions of x and y . Let us carry out the linear stability analysis of Eq. (2.3), as we explained above. In particular, we take $f'(x, y) = g'(x, y) = 0$ in the neighborhood of the fixed point (x^*, y^*) and look for linear approximation $X = x - x^*$, $Y = y - y^*$ [145], to denote the deviations of the solutions from the steady state. Thus, we can expand (2.3) to obtain:

$$\begin{aligned} \frac{dX}{dt} &= A \cdot X + B \cdot Y, \\ \frac{dY}{dt} &= C \cdot X + D \cdot Y, \end{aligned} \quad (2.4)$$

where

$$A = \left. \frac{\partial f}{\partial x} \right|_{x^*, y^*}, B = \left. \frac{\partial f}{\partial y} \right|_{x^*, y^*}, C = \left. \frac{\partial g}{\partial x} \right|_{x^*, y^*}, D = \left. \frac{\partial g}{\partial y} \right|_{x^*, y^*}. \quad (2.5)$$

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These terms are the components of the Jacobian matrix

$$\mathbf{J} = \begin{pmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix} \quad (2.6)$$

that determines the system behavior near equilibrium points and their stability.

To calculate the eigenvalues at the fixed points we must solve the following characteristic equation:

$$\det[\mathbf{J}(x^*, y^*) - \lambda I] = 0. \quad (2.7)$$

An equilibrium point is *hyperbolic* if none of the real parts of all eigenvalues are zero [69]. Instead, if every eigenvalues real part is zero, the equilibrium point is *non-hyperbolic*. In this thesis we focus on hyperbolic attractors only. Depending on its stability, this point can be one of the following types [17]:

- Focus. The eigenvalues are complex-conjugated. The focus is unstable for cases with positive real part and stable for cases with negative real part.
- Node. The eigenvalues are both real and of the same sign. The node is unstable when the eigenvalue is positive and stable when the eigenvalue is negative.
- Saddle point. The eigenvalues are both real, but with different signs. The saddle point represents instability.

In dynamical systems with more than one variable, periodic-cycled attractors or limit-cycles also can exist. Furthermore, in high- (more than two-) dimensional systems chaotic attractors³ can arise. The chaotic systems are characterized by strong dependence on initial conditions [145] e.g., Chua attractor, Rössler attractor. Consequently, other techniques for the stability analysis of periodic and chaotic orbits were developed.

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³Nusse, Helena Engelina, Yorke, James A., (1998). *Dynamics: Numerical explorations*. New York: Springer

2.2 Nonlinear systems

Recovering the example of the previous section, in this case, we must focus on how fast the cyclist changes the rate of pedaling cadence when the wind starts to blow. The amplitude of that magnitude could change linearly over time, or not. Ordinarily, real systems modeled with mathematical equations include not only linear terms but also, at least, one term that contains the square or higher power, a product of system variables, or more complicated functions from them, or some sort of threshold functions. Therefore, the addition of two solutions does not yield a valid new solution [142].

For most dynamical systems modeled through nonlinear differential equations, a general solution in the analytical sense does not exist; nevertheless, there are some paths to achieve a useful knowledge of the possible solutions and to figure out the dynamical properties.

Nowadays, the way of computing a numerical solution, supply a set of data called *time series*, which contributes to have an sufficiently accurate idea of the system behavior through a reconstruction of the dynamics. From this quantitative point of view, there are several algorithms, e.g., Euler, Runge-Kutta, to reach an approximate solution that needs the initial conditions to start the numerical computation [69]. Moreover, the set of all initial condition whose solutions tend to the equilibrium point is called the *basin of attraction*. In fact, initial conditions play an important role when we want to know whether the dynamical system is *deterministic*; due to, two identical dynamical systems describing the same orbit if, and only if, they have exactly the same initial conditions in the absence of external perturbations. However, a tiny difference in the initial condition can generate substantial differences over time; then, we refer to it as deterministic chaos. On the contrary, for the *stochastic systems*, not even the first values of the solution (time series) are predictable.[55, 145] .

Time series data are able to reveal a huge amount of information; for instance, if we want to know how the distance between adjacent orbits or trajectories

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changes in time, we must have calculated the time series to draw the trajectories. *Lyapunov exponents* represent an average of these rate changes and also let characterize stability properties of dynamical systems, e.g. with only one positive Lyapunov exponent the system will be *chaotic*.

Another useful statistical tool, applicable to time series data, comes from correlation concept. Once time series have been computed, we can obtain the measure of how different is a signal with itself, i.e. *autocorrelation*. In fact, we may use it to distinguish between deterministic and stochastic systems [55, 64].

For non-linear systems, particularity in chaotic oscillator analysis, *power spectra* places a good position to show the frequency range of such systems. Power spectrum (based on fast Fourier transform applied to the time series) gives information about how strong each frequency ω_i is represented in the oscillating signal [147]. Besides, a strategy for analyzing the chaotic oscillator frequency consists of counting peaks (for example maximums) in the time series [23]. The strongest frequency is the dominant ⁴ or mean ⁵ frequency. This is another characteristic for chaotic oscillator along with the amplitude and the phase of the oscillation.

On other side, from a mathematical modeling point of view, the equations which describe a dynamical system usually depend on several parameters. Varying a certain system parameter, qualitative changes are sometimes observed in the time series, known as bifurcations . Obviously, the parameter value, at which point bifurcation takes place, is called bifurcation point and the parameter space where we are able to plot such bifurcation point is known as bifurcation diagram. Moreover, a cloud of data points (maximums which come from a time series) in the same parameter range represents one typical feature for systems with complex dynamics [3].

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⁴Bolger, C., Bojanic, S., Sheahan, N. F., Coakley, D., & Malone, J. F. (1999). *Dominant frequency content of ocular microtremor from normal subjects*. *Vision Research*, 39(11), 1911-1915.

⁵Bolger, C., Sheahan, N., Coakley, D., & Malone, J. (1992). *High frequency eye tremor: Reliability of measurement*. *Clinical Physics and Physiological Measurement*, 13(2), 151.

2.2.1 Chua's Circuit

During a four-month visiting from October 1983-January 1984 at Waseda University, Japan, Leon Chua of the University of California, Berkeley, was working with autonomous circuits that show chaotic attractors [99]. A simplified version of the circuit suggested by Leon Chua is compounded by two sections: on one hand, the non-linear resistor R which has a characteristic shown in Figure 2.1 in parallel with a capacitor $C1$. On the other hand, another capacitor $C2$, connected in parallel with an inductance $L1$; both are connected by means of a resistor $R = 1/G$ (see Figure 2.2). The differential equations which describe the circuit dynamics are:

$$C_1 \frac{dv(C1)}{dt} = G(v(C2) - v(C1)) - g(v(C1)) \quad (2.8)$$

$$C_2 \frac{dv(C2)}{dt} = G(v(C1) - v(C2)) + i_L \quad (2.9)$$

$$L \frac{dv(i_L)}{dt} = -v(C2) \quad (2.10)$$

with $g(v_{C1})$:

$$g(v_{C1}) = m_0 v_{C1} + \frac{1}{2}(m_1 - m_0)[|v_{C1} + B_p| - |v_{C1} - B_p|] \quad (2.11)$$

Where, $v(C_1)$, $v(C_2)$, and i_L denote voltage across $C1$, the voltage across $C2$, and current through $L1$, respectively. Accordingly, D in Figure 2.2 represents the famous Chua diode. The slopes in the inner and outer regions are m_0 and m_1 , respectively; B_p denote the breakpoints[81].

Notice that, the system described by equation(2.8) has three equilibria, one of them at the origin, another in the half space $V_{C1} > 0$ and the last one in the half space $V_{C1} < 0$. The equilibrium at the origin has one positive real eigenvalue and a pair of complex-conjugate eigenvalues with negative

Figure 2.1: Piecewise-linear function.

Figure 2.2: Chua circuit

real part. Other equilibria have one negative real eigenvalue and a pair of complex-conjugate eigenvalues with positive real part. If we take a look at the time series, for chaotic behavior, we will see that each state variable has the following aspect (see Figure 2.3).

2.2.2 Rössler oscillator

Rössler oscillator was proposed for the first time by the prestigious researcher O. E. Rössler in 1976. It was for throwing out some light into the qualitative behavior of other dynamical equations widely known as the equations of the Lorenz model [133]. Consequently, the system proposed was described by

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Figure 2.3: x (magenta), y (black) and z (green) state variables time series. Chua oscillator attractor (blue).

the ordinary differential equations:

$$\dot{x} = -(y + z) \quad (2.12)$$

$$\dot{y} = x + a \cdot y \quad (2.13)$$

$$\dot{z} = b + z \cdot (x - c) \quad (2.14)$$

Rössler studied the chaotic attractor fixing the standard values $a = b = 0.2$ and $c = 5.7$ [55]. This attractor has two stationary points, which can be found by solving the system for the xy plane, i.e. $z = 0$. Hence, we consider the equations shown in (2.15) and (2.16).

$$\dot{x} = -y \quad (2.15)$$

$$\dot{y} = x + a \cdot y \quad (2.16)$$

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The stability in the xy plane can be found by calculating the eigenvalues of the Jacobian shown in (2.17).

$$\mathbf{J} = \begin{pmatrix} 0 & -1 \\ 1 & a \end{pmatrix} \quad (2.17)$$

Solving the characteristic equation shown in (2.18), we obtain (2.19).

$$\lambda^2 - a\lambda + 1 = 0 \quad (2.18)$$

$$\lambda = \frac{a \pm \sqrt{a^2 - 4}}{2} \quad (2.19)$$

We may observe that for $0 \leq a \leq 2$, the eigenvalues are complex and at least one has a positive real component, making the origin unstable (see 2.1). Moreover, to find the fixed points, we set Rössler equations in (2.12), (2.13) and (2.14) to zero and the (x, y, z) coordinates of each fixed point were determined by solving the resulting equations.

$$x = \frac{c \pm \sqrt{c^2 - 4ab}}{2} \quad (2.20)$$

$$y = -\left(\frac{c \pm \sqrt{c^2 - 4ab}}{2a}\right) \quad (2.21)$$

$$z = \frac{c \pm \sqrt{c^2 - 4ab}}{2a} \quad (2.22)$$

The fixed point coordinates are shown in (2.20), (2.21) and (2.22), fixing parameter values this yield two fixed points: one of them located at the center of the attractor loop and the other one away from the attractor. To visualize the dynamical behavior for standard parameter values, we show each state variable, its time series and the attractor in Figure 2.4.

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Figure 2.4: x (magenta), y (black) and z (green) state variables time series. Rössler oscillator attractor (blue)

2.2.3 Hindmarsh-Rose oscillator

There are many kinds of biological oscillators in both vertebrate-like human and invertebrate like insects, mollusks etc. In this section, I focus on the pond snail *Lymnaea stagnalis* which belongs to the mollusc family [51]. It has a Central Nervous System or brain (see Figure 2.5(a) with several ganglia involved in different tasks as respiration, control of the heartbeat, control locomotion through muscle movements involving the shell, sensory processing of information from the eyes, tentacles and lips and some others.

Each ganglion contains a number of neurons ⁶ to carry out its own tasks. Focusing on the structure, neurons have four distinct regions ⁷ (see Figure 2.5(b)):

- The cell body (soma) contains the nucleus and synthesizes neuronal proteins and membranes.
- The dendrites are cellular extensions with branches which receive sig-

⁶Fodstad, H. (2001). *The neuron theory. Stereotactic and Functional Neurosurgery*, 77(1-4), 20-24.

⁷Lodish, H. F., Berk, A., & Zipursky, S. L. (2016). *Molecular cell biology*

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nals at synapses with other neurons.

- The axon is a cable-like projection from the cell body that is capable of conducting an electric impulse.
- The axon terminals are small branches of the axon that form the connections, or synapses, with other cells (through the membranes).

Neurons generate electrical signals called action potentials⁸ with an elaborate mechanisms based on the flow of ions Na^+ , K^+ , Cl^- and Ca^{2+} across their membranes. This flow of ions generates a potential difference across the membrane, also known like membrane potential [122]. If the value of membrane potential becomes more positive, overcoming a threshold potential (depolarization), the neuron is activated firing an action potential.

Figure 2.5: *Lymnaea* CNS⁹. A set of ganglia: cerebral ganglion, parietal ganglion, pedal ganglion, pleural ganglion and visceral ganglion. (Right) Structural schematic of one individual neuron. Notice that a ganglion is a group of nerve cell bodies¹⁰.

On other side, small assemblies of neurons, such as central pattern generators (CPG) [137], are widely known to express regular oscillatory firing patterns carrying bursts of action potentials; the electrical event that carries signals among neurons is called action potential (also called “spikes” or “impulses”

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⁸In Neuroscience is also common “spikes” or “impulses”.(Purves, 2012)

⁹Menzel, R., & Benjamin, P. R. (2013).

¹⁰Sadava, D. E., Hillis, D. M., Heller, H. C., & Hacker, S. D. (2017). *Life: The science of biology* W. H. Freeman.

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[122]). In fact, some neurons exhibit much more complicated firing patterns than simple repetitive firing, therefore, nonlinear systems theory plays an important role. A common mode of firing in many neurons and other excitable cell is bursting oscillation. This is characterized by a silent phase of near-steady-state resting behavior alternating with an active phase of rapid, spike-like oscillations [14].

The most famous are the Hodgkin-Huxley model of the nerve impulse, but its four coupled nonlinear differential equation, six functions, and seven constants make this model of a high complexity [76]. Another one is the Fitzhugh-Nagumo model, but the equations predict an action potential duration which is similar to the inter-spike interval [50]. However, Hindmarsh-Rose model generalizes the second-order Fitzhugh-Nagumo equations, moreover, is based on the form of the data obtained from the visceral ganglion of the pond snail [67] and even occupies an interesting position from implementation efficiency point of view among spiking models [74]. Afterward, in 1984, two years after publishing Hindmarsh-rose neuron model, the authors added to their model a third equation related to an adaptation current [68]. The equations are as follows:

$$\dot{x} = y - ax^3 + bx^2 + I_{ext} - z \quad (2.23)$$

$$\dot{y} = c - dx^2 - y \quad (2.24)$$

$$\dot{z} = r(s(x - x_0) - z) \quad (2.25)$$

where x is the membrane potential, y is the recovery variable associated with the fast current of Na⁺ or K⁺ ions, z is the adaptation current associated with the slow current of Ca₂⁺ ions, I_{ext} is the external current input and a, b, c, r and s are the constants. Ordinarily, the state variables show the most important firing/bursting modes of real central pattern generators (CPGs).

2.3. CHAOS: SYNCHRONIZATION AND RESONANCE EFFECTS

For having a general idea of its behavior when chaotic, e.g., $a = 1, b = 3, c = 1, d = 5, r = 5 \cdot 10^{-3}, s = 4, x_0 = 1.6$ and $I_{ext} = 3.28$. We will see that each state variable has the aspect shown in 2.6.

Figure 2.6: x (magenta), y (black) and z (green) state variables time series. Hindmarsh-Rose oscillator attractor (blue).

2.3 Chaos: synchronization and resonance effects

Chaos is an important form of dynamical movement in nature. In the late 20th century, when the computational techniques became an important scientific tool, many scientists focused their efforts on developing deterministic methods to stabilize chaos. Roughly speaking, we can say that in a deterministic method, there is no space for random events; i.e., such methods always produces the same output for a given system with the same state and initial conditions. Those requirements are fulfilled in deterministic chaotic systems.

Since a chaotic attractor is composed of an infinite number of unstable periodic orbits, several researchers proposed to stabilize an unstable periodic orbit embedded within the chaotic attractor using feedback and non-feedback

2.3. CHAOS: SYNCHRONIZATION AND RESONANCE EFFECTS

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control methods. In fact, the most recognized techniques are based on an adjustment of a system parameter or a variable. These methods require a very small change in the parameter or variable, therefore the control is assumed to be small. Instead, non-feedback control requires an external modulation to induce a new stable orbit [27, 89, 106, 119, 120] and therefore cannot be considered small, because the external forcing should be strong enough to modify the system dynamics.

Recently, chaos suppression in coupled chaotic oscillators was found in two cases, first, in the presence of asymmetry in coupling and, second, when there is a small mismatch between natural frequencies of the coupled oscillators. While the former was observed in bidirectionally coupled identical systems [26], the latter was theoretically predicted in unidirectionally coupled oscillators [118].

2.3.1 Synchronization

Certainly, Huygens is very famous by his observation in the 17th century, when he realized how two pendulum clocks, hanging on a wall, became synchronized between them. Synchronization is widely studied even in other fields, not only in physics but also in music [141], transport [129], communication [114], physiology [62], computation science [39], neuroscience [48], etc., .

Going back to the dynamical systems, there are many researchers focusing on synchronization in chaotic systems where a tiny difference in the initial conditions shows different trajectories [92]. In this framework, we can distinguish some kinds of synchronization regime. Complete synchronization (CS) occurs when two chaotic trajectories of two systems become completely identical, then both trajectories have the same amplitude at the same time [113]. Generalized synchronization (GS) means that using two systems, even completely different, both outputs are associated with a given function [2, 84]. Phase synchronization (PS) is observed when the phases of two dynamical systems become locked, whereas amplitudes remain uncorrelated [20, 115, 130]. Phase synchronization is abundant in science and plays a crucial role in many weakly interacting natural systems, including electronic circuits [35, 84, 132], cardiorespiratory rhythm [136] and neurons [94, 148].

Dynamics of a chaotic system can be regularized due to its interaction with other systems in order to reach a synchronous state. In fact, synchronization

2.3. CHAOS: SYNCHRONIZATION AND RESONANCE EFFECTS

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is an example of self-organization in nature [34, 146] and it is usually assumed that the interaction between oscillators enhances their synchronization. However, this is not always true. Indeed, the increasing coupling between chaotic systems may result in unexpected behaviors, such as, e.g., oscillation death [8, 101] and the coherence enhancement [26, 118]. The latter was predicted theoretically in two coupled oscillators. It was surprisingly found that adequate coupling can force a chaotic oscillator towards more regular oscillations, so although coupled oscillators have the same dominant frequency in their power spectra, they follow different phase trajectories. In terms of synchronization theory, this means that the oscillators are phase synchronized, i.e., they develop a perfect phase-locking relation for relatively weak coupling, although their amplitudes remain almost uncorrelated [20, 86, 130].

Although synchronization of unidirectionally coupled chaotic oscillators has been extensively investigated [115], some features are not yet well understood, in particular, in the presence of a small detuning between natural frequencies of the coupled oscillators. Recently, it was shown that a chaotic slave oscillator coupled with a chaotic master oscillator becomes more regular when the oscillators are in phase synchronization. Such coherence enhancement has a resonant character with respect to both the coupling strength and frequency mismatch [118].

2.3.2 Resonance effects

The term resonance comes from Latin *resonantia*, “echo” from *resonare* “re-sound”. One of the meanings in the Physics field is “The reinforcement or prolongation of sound by reflection from a surface or by the synchronous vibration of a neighboring object”¹¹. As a matter of fact, resonance effects are everywhere, if we look back to the early 40s, we find the famous event related to the Tacoma Narrows Bridge. This event was presented moreover, as an example of elementary forced resonance in physics textbooks [77]; it was one of the longest suspension bridges at that moment. However, Tacoma Bridge unfortunately collapsed into Puget Sound¹² due to self-oscillations. Generally speaking, in physics, resonance is applied to a dynamical system having periodic oscillations at some frequencies, when subject to a periodic forcing of frequencies near one of the former frequencies; shows a marked response [16].

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¹¹ Oxford, E. D. (2016). *Resonance*, noun.

¹² Billah, K., and Scanlan, R. (1991). *Resonance, tacoma narrows bridge failure, and undergraduate physics textbooks. American Journal of Physics*, 59(2), 118-124.

The regularity or coherence of a chaotic system can also be improved by noise, as we mentioned in the previous section. Sometimes, the influence of noise has a resonance character referred to as noise-induced coherence resonance. This effect was detected in both excitable [57, 60, 90, 108, 116, 121] and bistable systems [6, 90, 111, 151]. Later, a similar behavior was discovered in completely deterministic systems without any noise. For example, in a bistable system chaos plays a role similar to noise by inducing switches between coexisting states; the switches become more regular at certain amplitude of the chaotic signal [4, 53, 117]. Such an effect, known as deterministic coherence resonance, was also observed in monostable chaotic systems subject to time-delayed feedback [31, 71, 79, 98], where the increasing feedback signal induced optimal regularity in the chaotic system.

At the beginning of the 21st century, dynamical systems like communication electronic devices had yet low sound quality because showed some background noise. However, some noise may improve instead of hindering the performance of electronic devices. If noise stems from an external frequency, the enhancement phenomenon is called stochastic resonance. It has been found not only in electronic devices but also in physics, chemistry and biomedical sciences [56]. Over time, this concept has grown to include different mechanisms. Nowadays, from a general point of view, we consider it as the increased sensitivity to small perturbations at an optimal noise level.

One of the main reasons for most of the stochastic resonance studies is its application in biology, concretely in excitable neuronal systems. In fact, there are studies showing the ability of sensory neurons to process weak input signals can be enhanced by adding noise to the system [28, 155]. Furthermore, nonlinear systems with noise can display similar behavior to the one reported in stochastic resonance, even without external signal [108]. This phenomenon has been called coherence resonance [116] and we have observed this phenomenon in section 4.2.

2.4 Neuroscience and complex systems

From a functional point of view, neurons communicate with each other through synapses. Moreover, they are connected by both chemical and electrical synapses. The former can be excitatory (inhibitory), if they increase

2.4. NEUROSCIENCE AND COMPLEX SYSTEMS

(decrease) the likelihood of a postsynaptic action potential occurring. The latter represent a minority but are found in all nervous systems. They allow synchronization among populations of neurons through an electrical current flow from one neuron to another, in most cases with a bidirectional transmission, although there are special cases with unidirectional transmission [122]. In fact, “the membranes of the two communicating neurons come extremely close at synapse and are actually linked together by an intercellular specialization called a gap junction” ¹³.

It is a matter proved that neurons are implicated in several physiological aspects of the brain function and in anomalous population activities, such as epilepsy [44, 61]. Synchronous firing neurons can produce either regular or irregular oscillations of the brain electrical activity in different brain areas, including the neocortex, thalamus, and hippocampus [11, 33]. To understand how interaction among neural oscillators promotes synchrony, different neural models were developed and investigated (for comprehensive review see, e.g., [76] and references therein).

For the last three decades, several studies in neuroscience have revealed that a healthy brain works as a coordinated system where many oscillations are involved, with different spatiotemporal scales [30, 33, 54, 140]. Particularly, previous studies show how networks of reciprocally coupled spiking neurons become synchronized in fast transitions from uncorrelated states [140], in spite of each oscillatory rhythm following a balance between excitatory and inhibitory neurons in a network [29, 30].

Brain signals such as local field potentials (LFP), electroencephalograms (EEG) and magnetoencephalograms (MEG) show oscillatory activity coming from the synchronized activity of neuronal groups. In fact, synchronization plays a main part in coordinating and processing information at different spatiotemporal scales [96], without interfering with them [9]. The interaction of the different synchronized group of neurons arises in tasks as learning about item-context associations [150], selective attention [25], or even in con-

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¹³*Purves, D. (2012). Neuroscience. Sunderland, Mass.: Sinauer Associates.*

2.4. NEUROSCIENCE AND COMPLEX SYSTEMS

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scious perception [87]. In addition to the brain operating at multiple scales, it can also be described in terms of logic calculus when facing information processing tasks [103, 159].

From the mesoscopic scale point of view, the structures of oscillatory networks such as Cellular Neural Networks (CNNs) [40–43], can be taken like a computational model; where the dynamics of coupled oscillators, or nodes ¹⁴, is related to the logical states [158]. Therefore, we can also consider CNNs dynamics as that of limit-cycle oscillator submitted to weak forcing and coupling. Taking into account coupled oscillator theory [40], the network’s node can work in one or several synchronization regimes [20], in fact, in mesoscopic brain oscillators different forms of synchronization might coexist [96].

Another interesting research line studies the question of how regularity can be broken and how chaos can be induced in coupled neurons. Specific motivation for such a general problem is the search for a way to destroy an organism with a stable dynamics by destabilizing its metabolism.

To address this issue, we focus on the model of a pair of neuron cells unidirectionally coupled via an electrical synapse. In particular, we consider the Hindmarsh-Rose (HR) model [68] which provides a simple description of the patterned activity observed in molluscan neurons as shown in section 2.2.3. Even though this model is not wholly based on physiology as the accurate Hodgkin-Huxley model [70] it allows basic phenomenological description of neuron dynamics, such as quiescence, spiking, irregular spiking and chaotic bursting [13, 125], and reveals nonlinear dynamical mechanisms underlying many biological processes.

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¹⁴*In complex networks field, oscillators and nodes are interchangeable.*[112]

2.4. NEUROSCIENCE AND COMPLEX SYSTEMS

Chapter 3

ELECTRONIC IMPLEMENTATION

This chapter treats with three electronic circuits, used during this Thesis period, where the difficulty increases one by one. One of the first steps to face big challenges, from a prudent development line, starts with the well-known activities. Consequently, we started with an electronic circuit whose behavior shows chaotic oscillations being one of most famous electronic oscillators used in neural networks ¹. After that, we went further with another chaotic oscillator that uses more electronic components, and finally, we implemented the most difficult of those three, based on a neural model.

3.1 Chua electronic oscillator

For the Chua electronic oscillator, there are different implementation options in the literature, e.g. with diodes [99], operational amplifiers [160], with transistors [100] among others . In fact, there is an accurate study, which synthesizes the circuit with the lower number of components: two

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¹*Madan, R. N. (1993). Chua's circuit: A paradigm for chaos. Singapore .etc..: World scientific.*

3.1. CHUA ELECTRONIC OSCILLATOR

operational amplifiers, six resistors, two capacitors, an inductor and a potentiometer [81]; our implementation is based on the latter study. Nowadays, the most expensive component is the potentiometer; however, for building up a Chua low-cost circuit, the amount of money necessary is around five euros, according to the web quotation of the largest distributor of electronic components ².

Leaving by side, at the moment, the market for electronic components let me turn into the Chua's circuit equations, already seen in (2.8), (2.9) and (2.10).

Figure 3.1: Electronic Schematic (left) and electronic implementation (right) for Chua oscillator. We use two coils of 10 mH in a series configuration to obtain the value of $L1$.

For this circuit, we handle milliamperes in electronic circuits instead of amperes, hence we may rescale all currents by a factor of 10^3 ; consequently, reduce the capacitances by the same factor, and at the same time increase resistances and the inductance, again, by a factor of 10^3 . Thereby, the equations not only remain apparently the same, but also we may use electronic component values available inside the market [81].

²P. S. (2008). *An interview with Steve Phillips, chief information officer avnet, inc. Journal of Global Information Technology Management*, 11(2), 80-83.

³Texas Instruments datasheet. <http://www.ti.com/lit/ds/symlink/tl082.pdf> [on line] [query: 15-1-2017].

3.2. RÖSSLER ELECTRONIC OSCILLATOR

Component	Value
$R1$	222Ω
$R2$	$22k\Omega$
$R3$	$2.2k\Omega$
$R4$	$3.3k\Omega$
$R5$	$2k\Omega$
$C1$	$10nF$
$C2$	$100nF$
$L1$	$20mH$
$OpAmp$	$TL082$

Table 3.1: Values for the Chua electronic components. For further information about $TL082$ see the datasheet ³.

3.2 Rössler electronic oscillator

First of all, we rewrite the equations (2.12), (2.13) and 2.14) by explicitly showing the natural frequency ω :

$$\dot{x} = -\omega \cdot y - z \quad (3.1)$$

$$\dot{y} = \omega \cdot x + a \cdot y \quad (3.2)$$

$$\dot{z} = b + z \cdot (x - c) \quad (3.3)$$

Then, we use the Kirchhoff's law, Ohm's law [149], the Laplace transform [110], the operational amplifier building blocks (see chapter ??) to obtain the equivalent electronic circuit. Notice all values for the passive components are included in Table 3.2.

To analyze the circuit in Figure 3.2, we start applying the Kirchhoff's law. We need to know which currents go inside and which ones go out from the

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⁴*Analog Devices datasheet.* <http://www.analog.com/en/products/linear-products/analog-multipliers-dividers/ad633.html#product-overview> [on line] [query: 11-2-2017].

3.2. RÖSSLER ELECTRONIC OSCILLATOR

Component	Value
$R1, R2, R3, R4, R5, R6$	$10k\Omega$
$R7$	$50k\Omega$
$R8$	$9.3k\Omega$
$R9$	$5.72k\Omega$
$R10$	$14.98k\Omega$
$C1, C2, C3$	$100nF$
OpAmp	TL082
Multiplier	AD633

Table 3.2: Values for the Rössler's passive electronic components. For further information about TL082 see the datasheet ⁴.

Figure 3.2: Rössler electronic circuit.

3.2. RÖSSLER ELECTRONIC OSCILLATOR

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green circle show in 3.3 (see [149] for signs criteria). For this purpose, we draw upon the famous Ohm's law. Consequently, we are able to find which one is the current I_1 across R_4 and I_2 across R_1 ; the equations are:

$$I_1 = \frac{V_z}{R_4} \quad (3.4)$$

$$I_2 = \frac{V_y}{R_1} \quad (3.5)$$

Figure 3.3: We analyze this mesh, extracted from 3.2, to show the relation between the x state variable and the V_x experimental state variable.

Besides, there is another current which come from the capacitor C_1 ; the current across a capacitor is defined in ??, therefore, I_3 is given by:

$$I_3 = C_1 \frac{V_x}{dt} \quad (3.6)$$

Afterward, the current outgoing, from the green circle in 3.3 and the current going into the operational amplifier (taking into account the ideal features of the operational amplifier), are considered the same. That current is neglected [149]. Going back to the Kirchhoff's current law, we have the equations:

$$\frac{V_z}{R_4} + \frac{V_y}{R_1} + C_1 \frac{V_x}{dt} = 0 \quad (3.7)$$

$$\frac{V_z}{R_4} + \frac{V_y}{R_1} = -C_1 \frac{V_x}{dt} \quad (3.8)$$

3.2. RÖSSLER ELECTRONIC OSCILLATOR

Taking the Laplace transform in (3.5), we get:

$$\frac{V_z(s)}{R4} + \frac{V_y(s)}{R1} = -C1 \cdot s \cdot V_x(s) \quad (3.9)$$

$$C1 \cdot s \cdot V_x(s) = - \left(\frac{V_z(s)}{R4} + \frac{V_y(s)}{R1} \right) \quad (3.10)$$

And making the Laplace transform inverse, we finally obtain (3.11).

$$\frac{V_x}{dt} = -\frac{1}{C1} \left(\frac{V_y}{R1} + \frac{V_z}{R4} \right) \quad (3.11)$$

Particularly, in this case $R1 = R4$, and the equation 3.11 converts to:

$$\frac{V_x}{dt} = -\frac{1}{C1 \cdot R1} \cdot (V_y + V_z) \quad (3.12)$$

Defining, the temporal scale factor as:

$$\alpha_1 = \frac{1}{C1 \cdot R1} \quad (3.13)$$

We rewrite (3.12) as:

$$\frac{V_x}{dt} = -\alpha_1 \cdot (V_y + V_z) \quad (3.14)$$

We already have the equivalent equation for the first Rössler equation shown in (3.1). Let me analyze the mesh associated to the V_y state variable shown in Figure 3.4.

If we enforce Kirchhoff's law into the point out circumference in Figure 3.4, we figure out which currents go inside and which ones go outside. For this purpose, we draw upon the famous Ohm's law. Consequently, we are able to find which one is the current across $R5$, $R6$ and $R7$; the equations are shown

3.2. RÖSSLER ELECTRONIC OSCILLATOR

Figure 3.4: We analyze this mesh, extracted from 3.2, to show the relation between the y state variable and the V_y experimental state variable.

(3.15) in and (3.16).

$$\frac{V_x}{R6} + \frac{V_y}{R7} + \frac{V_{U3}}{R5} = 0 \quad (3.15)$$

$$V_{U3} = -\frac{R5}{R6} \cdot V_x - \frac{R5}{R7} \cdot V_y \quad (3.16)$$

Analogously, for the sub mesh of the right hand side in Figure 3.4.

$$\frac{V_{U3}}{R2} + C2 \frac{dV_y}{dt} = 0 \quad (3.17)$$

$$V_{U3} = -R2 \cdot C2 \cdot \frac{dV_y}{dt} = 0 \quad (3.18)$$

Taking the Laplace transform in (3.18) and (3.16), moreover, equating the equations:

$$-\frac{R5}{R6} \cdot V_x(s) - \frac{R5}{R7} \cdot V_y(s) = -R2 \cdot C2 \cdot V_y(s) \quad (3.19)$$

$$-\frac{R5}{R2 \cdot C2 \cdot R6} \cdot V_x(s) + \frac{R5}{R2 \cdot C2 \cdot R6} \cdot V_y(s) = V_y(s) \quad (3.20)$$

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And making the Laplace transform inverse, we finally obtain:

$$\frac{dV_y}{dt} = \frac{R5}{R2 \cdot C2} \cdot \left[\frac{V_x}{R6} + \frac{V_y}{R7} \right] \quad (3.21)$$

We rewrite 3.21:

$$\frac{dV_y}{dt} = \frac{R5}{R2 \cdot C2 \cdot R6} \cdot \left[V_x + \frac{R6}{R7} V_y \right] \quad (3.22)$$

Then, we define the temporal scale factor α_2 :

$$\alpha_2 = \frac{R5}{C2 \cdot R2 \cdot R6} \quad (3.23)$$

and the parameter a :

$$a = \frac{R6}{R7} \quad (3.24)$$

Finally, the equivalent equation for the second Rössler equation (3.2), is represented as follows:

$$\frac{dV_y}{dt} = \alpha_2 \cdot (V_x + a \cdot V_y) \quad (3.25)$$

Let me analyze the mesh associated to the V_z state variable shown in Figure 3.5.

Figure 3.5: We analyze this mesh, extracted from 3.2, to show the relation between the z state variable and the V_z experimental state variable. First, we pay attention to the mesh associated to $R8$, $R9$ and $R10$.

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As we have done until now, we enforce Kirchhoff's law into the point out circumferences in Figure 3.5. Keeping the same signs criteria for currents [149], we need to know which currents go inside and which ones go outside. For this purpose, we draw upon the famous Ohm's law. Consequently, we are able to find i_1, i_2 and i_3 .

$$V_{CC} - V_p = i_1 \cdot R8 \Rightarrow i_1 = \frac{15 - V_p}{R8} \quad (3.26)$$

$$V_{CC} + V_q = i_2 \cdot R10 \Rightarrow i_2 = \frac{15 + V_q}{R10} \quad (3.27)$$

$$V_p - V_q = i_3 \cdot R9 \Rightarrow i_3 = \frac{V_p - V_q}{R9} \quad (3.28)$$

Moreover, in Figure 3.5 is satisfied (3.29) because of the signs criteria.

$$i_1 = i_2 \quad (3.29)$$

On other hand, the DC power source, V_{cc} , supply $15V$. Consequently, the voltage is given by (3.30).

$$\Delta V = 2 \cdot V_{CC} = 30V \quad (3.30)$$

Conveniently, we define a new variable R :

$$R = R8 + R9 + R10 \quad (3.31)$$

Applying Ohm's law:

$$\Delta V = i \cdot R = i \cdot 30k\Omega \quad (3.32)$$

3.2. RÖSSLER ELECTRONIC OSCILLATOR

We can obtain an approach for the current:

$$i = \frac{\Delta V}{R} = \frac{30V}{30k\Omega} = 1mA = 10^{-3}A \quad (3.33)$$

For this moment, we are able to figure out the values for i_1 and i_2 . Recovering (3.26) and (3.27), and then replacing values, we obtain:

$$10^{-3} = \frac{15 - V_p}{9.3k\Omega} \quad (3.34)$$

$$V_p = 15 - 9.3 = 5.7V \quad (3.35)$$

$$10^{-3} = \frac{-15 - V_q}{14.98k\Omega} \quad (3.36)$$

$$V_q = 14.98 - 15 = -0.02V \quad (3.37)$$

Once, we know the value of V_p and V_q shown in (3.35) and (3.37), we use them to calculate i_3 . Replacing in (3.28), we get i_3 :

$$i_3 = \frac{5.7 + 0.02}{5.72k} = 1mA \quad (3.38)$$

For analyzing the sub mesh from Figure 3.5, we take into account the AD633 multiplier information. Hence, we know how to calculate W :

$$W = \frac{(0 - V_z) \cdot (V_x - 5.7)}{10} + V_q = \frac{-V_z \cdot (V_x - 5.7)}{10} - 0.02 \quad (3.39)$$

Once again, we enforce Kirchhoff's law into the point out circumference in Figure 3.6. Through an analogous analyses, that we already have done, we obtain:

$$i_4 + i_5 = 0 \quad (3.40)$$

3.2. RÖSSLER ELECTRONIC OSCILLATOR

Figure 3.6: To complete the analysis, we use the submesh associated to the operational amplifier from Figure 3.5.

$$\frac{W}{R3} + C3 \cdot \frac{dV_z}{dt} = 0 \quad (3.41)$$

$$\frac{V_z}{dt} = -\frac{W}{C3 \cdot R3} = \frac{V_z \cdot (V_x - 5.7)}{10 \cdot C3 \cdot R3} + \frac{0.02}{C3 \cdot R3} \quad (3.42)$$

$$\frac{V_z}{dt} = \frac{1}{10 \cdot C3 \cdot R3} [(V_z \cdot (V_x - V_p)) + 0.2] \quad (3.43)$$

Defining, the temporal scale factor α_3 as:

$$\alpha_3 = \frac{1}{10 \cdot R3 \cdot C3} \quad (3.44)$$

the parameter b by:

$$\begin{aligned} b = V_1 = V_q = i \cdot R10 - V_{CC} &= \frac{\Delta V}{R} \cdot R10 - V_{CC} \\ &= \frac{2V_{CC}}{R8 + R9 + R10} \cdot R10 - V_{CC} = \\ &= V_{CC} \cdot \left[\left(\frac{2 \cdot R10}{R8 + R9 + R10} \right) - 1 \right] \end{aligned} \quad (3.45)$$

3.2. RÖSSLER ELECTRONIC OSCILLATOR

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and the parameter c by:

$$\begin{aligned}
 c = V_2 = V_p = V_{CC} - i \cdot R8 &= V_{CC} - \frac{\Delta V}{R} \cdot R8 \\
 &= V_{CC} - \frac{2V_{CC}}{R8 + R9 + R10} \cdot R8 = \\
 &V_{CC} \cdot \left[1 - \left(\frac{2 \cdot R18}{R8 + R9 + R10} \right) \right]
 \end{aligned} \tag{3.46}$$

Finally, the equivalent equation for the third Rössler equation (3.3) is represented as follows:

$$\frac{V_z}{dt} = \frac{1}{10 \cdot C3 \cdot R3} [(b + (V_z \cdot (V_x - c)))] \tag{3.47}$$

As a summary, we rewrite the Rössler equations (3.1), (3.2) and (3.3) as:

$$\frac{V_x}{dt} = A \cdot V_y - B \cdot V_z \tag{3.48}$$

$$\frac{V_y}{dt} = C \cdot V_x - D \cdot V_y \tag{3.49}$$

$$\frac{V_z}{dt} = E + F \cdot V_z \cdot (V_y - G) \tag{3.50}$$

Where, A , B , C , D and E are given by:

$$A = \frac{\omega}{\alpha_1} \tag{3.51}$$

$$B = \frac{\omega}{\alpha_1} \tag{3.52}$$

$$C = \omega \cdot \alpha_2 \tag{3.53}$$

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$$D = \omega \cdot \alpha_2 \cdot \frac{R6}{R7} \quad (3.54)$$

$$E = \omega \cdot \alpha_3 \cdot V_1 \quad (3.55)$$

$$F = \frac{15}{V_{CC}} \quad (3.56)$$

$$G = \frac{\alpha_3}{V_2} \quad (3.57)$$

With V_2 given by:

$$V_2 = V_{CC} \cdot \left(1 - \frac{2 \cdot R8}{R}\right) \quad (3.58)$$

Consequently, we have the necessary information to implement the electronic circuit over a protoboard just to make a proof of concept (see Figure 3.7). Notice, there is a $1k\Omega$ potentiometer compensating the component tolerances.

Figure 3.7: (Left) virtual image, (right) real image from Rössler implementation.

3.3 Hindmarsh Rose

As a personal challenge from previous experiences, I carried out a professional electronic circuit of Hindmarsh-Rose oscillator, for being a biological model and having more complexity instead of any Chua and Rössler oscillators.

3.3.1 Electronic analysis & schematic

There are different electronic implementations of the Hindmarsh-Rose model [105, 144], however, in this Ph.D. Thesis essentially, the electronic design of the Hindmarsh-Rose neuron is based on the work described in the latter reference. Consequently, there are three integrator circuits, which integrate the three state variables of the Hindmarsh-Rose model. Besides, for generating the squared and cubic terms that appear in the equations, I have used two AD633 multipliers due to its useful features as well as its low price.

In order to obtain the equivalent electronic circuit from the mathematical model in equations (2.20), (2.21) and (2.22), we proceed in the same way as we did in the previous section using the Kirchoff's law, Ohm's law [149] and operational amplifier building blocks [32], (see Appendix C).

Figure 3.8: Hindmarsh-Rose electronic circuit.

Consequently, we achieve the circuit shown in Figure 3.8. There, we distinguish one mesh by each state variable. For the first place, we deal with the mesh associated with the V_x state variable (see Figure 3.9). Notice, all values for the passive components are included in Table 3.3.

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Component	Value
$R1, R2, R3, R4, R5, R6, R7, R8, R10, R14, R15$	$10k\Omega$
$R4$	333Ω
$R5$	100Ω
$R9$	$0.2k\Omega$
$R11$	$2M\Omega$
$R12$	$500k\Omega$
$R13$	$100k\Omega$
$C1, C2, C3$	$20mH$
$C4, C5$	$TL082$

Table 3.3: Values for the Chua electronic components.

Figure 3.9: We analyze this mesh, extracted from 3.8, to show the relation between the x state variable and the V_x experimental estate variable.

The relationship among state variables is shown in:

$$\frac{dV_x}{dt} = -\frac{\left(\frac{V_x^3}{100}\right)}{C1 \cdot R5} + \frac{\left(\frac{V_x^2}{10}\right)}{C1 \cdot R4} + \frac{V_y}{C1 \cdot R2} - \frac{V_z}{C1 \cdot R3} + \frac{V_{ext}}{C1 \cdot R1} \quad (3.59)$$

Grouping terms, we reach the expression as:

$$\frac{dV_x}{dt} = \frac{1}{T_s} [-AV_x^3 + BV_x^2 + FV_y - GV_z + HV_{ext}] \quad (3.60)$$

Where, $T_s = 10^4 s$, is the time scale factor⁵. The relation between (3.60) and

⁵Buscarino, A., Fortuna, L., Frasca, M., & Sciuto, G. (2014). *A concise guide to chaotic electronic circuits* - Springer International Publishing.

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the first equation of Hindmarsh-Rose model (2.23), arise easily through:

$$a = \frac{A}{T_s} \quad (3.61)$$

$$b = \frac{B}{T_s} \quad (3.62)$$

$$F = T_s \quad (3.63)$$

$$G = T_s \quad (3.64)$$

$$H = T_s \quad (3.65)$$

For the second place, the mesh associated to the V_y state variable in Figure 3.10.

Figure 3.10: We analyze this mesh, extracted from 3.8, to show the relation between the y state variable and the V_y experimental state variable.

The relationship among state variables is:

$$\frac{dV_y}{dt} = -\frac{1}{C2 \cdot R10} + \frac{\left(\frac{V_x^2}{10}\right)}{C2 \cdot R9} + \frac{V_y}{C2 \cdot R8} \quad (3.66)$$

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Grouping terms, we reach an expression:

$$\frac{dV_y}{dt} = \frac{1}{T_s} [C - DV_x^2 - EV_y] \quad (3.67)$$

The relation between (3.67) and the second equation of Hindmarsh-Rose model (2.24), arise easily through:

$$c = \frac{C}{T_s} \quad (3.68)$$

$$d = \frac{D}{T_s} \quad (3.69)$$

$$E = T_s \quad (3.70)$$

For the third place, the mesh associated to the V_z state variable in (2.25).

Figure 3.11: We analyze this mesh, extracted from 3.8, to show the relation between the z state variable and the V_z experimental state variable

The relationship among state variables is:

$$\frac{dV_z}{dt} = -\frac{V_z}{C3 \cdot R11} + \frac{V_x}{C3 \cdot R12} + \frac{0.32}{C3 \cdot R13} \quad (3.71)$$

Grouping terms, we reach the expression:

$$\frac{dV_z}{dt} = R[S(V_x - X0) - V_z] \quad (3.72)$$

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The relation between (3.72) and the third equation of Hindmarsh-Rose model (2.25), arise easily through:

$$r = \frac{R}{T_s} \quad (3.73)$$

$$s = S \quad (3.74)$$

$$x_0 = X_0 \quad (3.75)$$

3.3.2 Simulation

Once we have designed the schematic, we must simulate it in order to analyze the system behavior. In fact, you also can compare the data obtained from the solution of the mathematical model, with the data yielded by the electronic circuit simulation, see 3.12 and 3.13.

Figure 3.12: Hindmarsh-Rose simulation run with Matlab.

There are a lot of software to develop this activity, in this case, we used Multisim. It offers a relatively new technique to deal with electronic simulations, i.e. co-simulation [37, 38]. This application basically consists of transforming

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Figure 3.13: Hindmarsh-Rose simulation run with Multisim. We present the electronic circuit design (left) and the time series plot for the V_x state variable (right).

the circuit into differential equations and solving it by numerical methods ⁶ (Euler, or Runge-Kutta).

Ordinarily, you use to design electronic circuits with ideal components supplied by the software. In addition, some of them offer you different options for changing the component properties becoming them more real, Multisim too. However, it is quite unusual to find software which allows combining analog design with control design; co-simulation joints them giving a reliable idea of how the real circuit works. I have found this technique useful for synchronization experiments among electronic circuits because it allows a secure path in the electronic circuit development for saving efforts and money during the manufacturing process.

3.3.3 PCB design

Accordingly, to the electronic analysis and the simulation, we already have the system specification. Hence, next step is to transform the schematic into printed circuit board (PCB). This type of circuit will improve its performance due to enhanced properties related with the noise level, short circuits, isolated lines, as well as written tags on the silkscreen layer to quickly locate and recognize the circuit. The Institute for Interconnecting and Packaging Electronic Circuits (IPC) publishes a series of standards that list in detail all the necessary information about how to design them [45].

⁶Hirsch, M. W., Smale, S., & Devaney, R. L. (2003). *Differential equations, dynamical systems & an introduction to chaos*.(p. 425)

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Following the suggestions from IPC, we chose a double side PCB's and through hole components which are the lowest-cost choice for consumer electronic products. Then, we export the bill of material (BOM), shown in Table 3.4, for making an idea about the size and the shape of the board where the electronic components will be placed. At this point, it is highly recommendable to check the electronic component market for coordinating your component choice with the market offers.

At that time, we placed the electronic components [5] and started the PCB routing process, adapting the spacing and length rules based on the suggestions of IPC and IEEE ⁷. The sequence of the latter is shown Figure 3.14 and 3.15, moreover, the final design and the real PCB are shown in 3.16. At the end of the routing process, we must review and inspect the layout before sending the Gerber files ⁸ to the factory.

Figure 3.14: Hindmarsh-Rose PCB Silkscreen layer (left). Hindmarsh-Rose PCB Top layer (right).

The manufacturing process has boundaries; even though you have followed a standard you should design the PCB satisfies the factory requirements like the minimum diameter for the holes, the pad size among others, otherwise, the manufacturing process won't walk in the right direction. In order to reduce the invoice, we decided to mount the components on the PCB by ourselves.

Finally, after receiving the PCB's from the factory, in our case, the last step consists in soldering the components and powering the PCB to test the

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⁷*Oxford, E. D. (2016). IEEE, abbreviation.*

⁸*Ucamco. (2016). The gerber file format specification.*

Quantity	Description	Description	Package
9	<i>RES, 10kΩ</i>	<i>R1, R2, R3, R6, R7, R8, R10</i>	<i>IPC – 2221A/22221300 – 700X250</i>
1	<i>RES, 333Ω</i>	<i>R4</i>	<i>IPC – 2221A/22221300 – 700X250</i>
1	<i>RES, 100Ω</i>	<i>R5</i>	<i>IPC – 2221A/22221300 – 700X250</i>
1	<i>RES, 0.2kΩ</i>	<i>R9</i>	<i>IPC – 2221A/22221300 – 700X250</i>
1	<i>RES, 2MΩ</i>	<i>R11</i>	<i>IPC – 2221A/22221300 – 700X250</i>
1	<i>RES, 500kΩ</i>	<i>R12</i>	<i>IPC – 2221A/22221300 – 700X250</i>
1	<i>RES, 100kΩ</i>	<i>R13</i>	<i>IPC – 2221A/22221300 – 700X250</i>
2	<i>OPAMP, LM324AN</i>	<i>U1, U2</i>	<i>IPC – 2221A/2222 – 14(N14A)</i>
2	<i>MULT, AD633AN</i>	<i>U3, U4</i>	<i>IPC – 2221A/2222 – N – 8</i>
2	<i>DC_POWER, 15V</i>	<i>V1, V2</i>	<i>Generic</i>
1	<i>GROUND</i>	<i>GND</i>	<i>Generic</i>
3	<i>CAP, 0.01</i>	<i>C1, C2, C3</i>	<i>Generic</i>
1	<i>DC_POWER, –1V</i>	<i>V3</i>	<i>Generic</i>
1	<i>DC_POWER, 0.32V</i>	<i>v4</i>	<i>Generic</i>
2	<i>CAP, 1</i>	<i>C4, C5</i>	<i>Generic</i>
1	<i>PULSE_VOLTAGE</i>	<i>V5</i>	<i>Generic</i>

Table 3.4: Hindmarsh-Rose BOM.

Chapter 4

RESULTS

4.1 Synchronization-based computation through coupled Chua oscillators

In this chapter, we present an experimental approach to suggest one mechanism of brain computation based on synchronized oscillations, upholding the idea of neural mass oscillators may process inputs in a complex Boolean-like manner [95]. Therefore, we have chosen a physical implementation of Chua electronic oscillator driven by oscillatory input signals. Of course, the processing possibilities of larger networks implemented by this type of system go further from placing logic gates.

The experimental set up is based on a network of electronic oscillators and the dynamics for every node i is determined by the classical Chua circuit [93]. Adapting the equations in (2.6), (2.7) and (2.8) to the systems for this particular case we obtain the equations shown in (4.1), (4.2) and (4.3), respectively.

$$\frac{dv_A^i}{dt} = \left(\frac{1}{C_1} \right) \cdot \left(\frac{v_B^i - v_A^i}{R_5} - f(v_A^i) + \frac{v_A^i - s(t)}{R_{in}} \right) \quad (4.1)$$

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$$\frac{dv_B^i}{dt} = \left(\frac{1}{C_2} \right) \cdot \left(\frac{v_B^i - v_A^i}{R_5} - i_L + \frac{v_B^i - v_B^j}{R_c} \right) \quad (4.2)$$

$$\frac{di_L^i}{dt} = -\frac{v_B^i}{L_1} \quad (4.3)$$

Where v_A^i and v_B^i are the voltages of the two capacitors (C_1 and C_2 and i_L^i is the intensity through the coil of the circuit L_1 . $s(t)$ is an external oscillatory input signal that we can (“1”) or can not (“0”) activate, controlling the dynamical state at which the Chua operates. The strength of the input signal is proportional to the inverse of R_{in} . We will tune its value depending on the type of logic gate we will be considering. $f(v_A^i)$ is a piece-wise (non-linear) function given by (4.4).

$$f(v_A^i) = G_1 v_A^i + \frac{1}{2}(G_1 - G_0) [|v_A^i + B_p| - |v_A^i| - B_p] \quad (4.4)$$

With $B_p = 1.7V$ being the breaking point of the piece-wise function. The values of G_1 and G_2 are obtained from different resistances of the electronic circuit shown in Figure 4.1.

$$G_1 = \frac{-R_1}{R_1 + R_3} + \frac{1}{R_2} \quad (4.5)$$

$$G_2 = \frac{-R_1}{R_1 + R_3} - \frac{R_2}{R_2 R_4} \quad (4.6)$$

And the rest of the parameters are $V_{CC} = 15$ V, $V_{EE} = -15$ V, $R_1 = 222$ Ω , $R_2 = 22$ k Ω , $R_3 = 2.2$ k Ω , $R_4 = 3.3$ k Ω , $C_1 = 10$ nF, $C_2 = 100$ nF, $L_1 = 20$ mH, and $R_5 = 1.88$ k Ω .

The coupling circuit between the different Chua circuits, represented as the last term of equation (4.1), consists on a voltage follower placed at the output of v_B^j combined with a coupling resistance R_c , whose value controls the amount of unidirectional coupling from v_B^j and v_B^i . The coupling from unit i

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Figure 4.1: Electronic implementation of the Chua circuit. Two *TL082* operational amplifiers are the core of the non-linear part of the circuit which follows the function given in (4.4). The input signal (0/1) is introduced through the capacitor C_1 , while the output of the circuit is the voltage V_B of both C_2 and L_1 .

to unit j is introduced in a similar way, thus leading to an effective bidirectional coupling. Values of the coupling and input resistances of the different gates implemented in the experiments are summarized in Table 4.1. Notice, the coupling strength is proportional to the inverse of the resistances. R_{in} accounts for the resistance of the input signal of each circuit. R_c is the coupling resistance of the output signal of each unit to its neighbors. R_{int} represents the integration resistance that quantifies the amount of coupling from each unit to the integrator. Parenthesis indicate the direction of the coupling between units i and j , which can be either unidirectional ($\rightarrow$) or bidirectional ($\leftrightarrow$).

The input signal $s(t)$ used as the “1” input for the logic gates is introduced through the variable V_A by means of a voltage follower and a coupling resistance R_{in} . The signal $s(t)$ was recorded previously from a Chua circuit with the same parameters as the ones used in the experiments. Both, the input and the output signals, are characterized by a chaotic behavior shown

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Node	$R_{in}[k\Omega]$	$R_c[k\Omega]$	$R_{int}[k\Omega]$
XNOR GATE			
A_1	$0.6(s(t) \rightarrow A_1)$	$3.13(A_1 \rightarrow A_2)$	$100(A_1 \leftrightarrow C)$
A_2	$1.2(s(t) \rightarrow A_2)$	$100(A_2 \rightarrow A_1)$	$100(A_1 \leftrightarrow C)$
AND GATE			
B_1	$32(s(t) \rightarrow B_1)$	$7.22(B_1 \rightarrow B_2)$	$100(B_1 \leftrightarrow C)$
B_2	$61(s(t) \rightarrow B_2)$	$96(B_2 \rightarrow B_1)$	$100(B_2 \leftrightarrow C)$

Table 4.1: Coupling R_c and input R_{int} resistances of the Chua circuits.

in Figure 4.2.

Figure 4.2: Bifurcation diagram for Chua electronic oscillator.

Finally, outputs of both gates are sent to an integrator system (C), by means of a bidirectional coupling, adjusted by an integrating resistance R_{int} (see Figure 4.3). Concretely, nodes A_1 and A_2 form a XNOR gate (in a CS regime) while nodes B_1 and B_2 implement an AND gate (in a PS regime). The output of the two sync-based gates is integrated through node C .

In order to determine the state of our logic gates, we need to quantify the synchronization between pairs of nodes. Each synchronized state is defined by a characteristic functional relationship between the dynamics of the interacting elements [20]. Therefore, we can distinguish several synchronizations regimes. For our purpose, I mainly use phase synchronization and complete synchronization although the latter is not likely to be achieved in a neural context. From a quantitative point of view, we use the synchronization error and the phase synchronization index between the output signals, as the

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Figure 4.3: Bifurcation diagram for Chua electronic oscillator.

reference indicators. For the former indicator, we evaluate it as the average of the difference between the outputs of two systems, e.g., units A_1 and A_2 in Figure 4.3. The mathematical definition is given by

$$\epsilon = \frac{1}{T} \sum_{t=1}^T |v_{A_1} - v_{A_2}| \quad (4.7)$$

Being T the total number of time steps. For the latter indicator, firstly we must know the phase $\phi^m(t)$, through calculating the Hilbert transform [107, 130] from the output signal. Then, we can figure out the phase synchronization index γ of two oscillators, given by equation (4.8).

$$\gamma \equiv \langle e^{i\Delta^{12}(t)} \rangle_t = \sqrt{\langle \cos \Delta\phi^{12}(t) \rangle_t^2 + \langle \sin \Delta\phi^{12}(t) \rangle_t^2} \quad (4.8)$$

Where, phase difference, $\Delta\phi$ [107] shown in equation(4.9).

$$\Delta\phi^{12}(t) = \phi^1(t) - \phi^2(t) \quad (4.9)$$

From our criteria, two oscillators are complete synchronized when the values of the synchronization error are lower than a certain threshold, $\epsilon_{th} = 0.10V$ and phase synchronization arises for high values of the phase synchronization index ($\gamma_{th} = 0.85$) and, simultaneously, high values of the synchronization

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error ($\epsilon > \epsilon_{th} = 0.10V$). Let take the synchronization state of two oscillators, assigning arbitrary “0” and “1” values to the absence of synchronization and its presence, respectively. At this point, we focus on the description and integration of a XNOR and AND gates in a network shown in Figure 4.3.

Inputs		XNOR			AND		
A_1	A_2	γ	$\epsilon(V)$	Out	γ	$\epsilon(V)$	Out
0	0	0.991	0.085	1	0.734	0.357	0
0	1	0.750	0.641	0	0.760	0.294	0
1	0	0.885	0.449	0	0.692	0.285	0
1	1	0.994	0.039	1	0.920	0.288	1

Table 4.2: Synchronization errors and truth tables of the XNOR (with complete synchronization) and the AND gate (with phase synchronization) embedded in a network of coupled Chua oscillators.

That network consists of an integrated circuit formed by five Chua oscillators, two of them implementing a XNOR gate (nodes A_1 and A_2), other two forming an AND gate (nodes B_1 and B_2) and the fifth Chua circuit (node C) integrating the output of both gates. The input of each dynamical unit can be either “1” (when complex signal is injected into the node) or “0” (in the absence of an input signal). In order to assess the emergence of the synchronize time evolutions we have computed the synchronization error (4.7) of the two pairs of nodes that implement the logic gates. Such error allow us to determine a proper threshold to define each synchronized state, and thus each output state, in the truth tables of the logic gates. Therefore, we have constructed truth tables based on synchronization states as shown in Table 4.2. Time traces and the corresponding truth table for the XNOR gate are shown in Table 4.4.

The AND gate is based on phase synchronization, and it is obtained by a fine tuning of the input and coupling resistances. In this case, we must reach a high phase synchronization value by preventing complete synchronization, the latter leading to unavoidable matching of the phases of the oscillators. To guarantee that we have phase, and not complete, synchronization we checked at the same time the synchronization error ϵ and the phase synchronization

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Figure 4.4: Time series of the XNOR gate. Functioning of the gate relies on the complete synchronization of units A_1 and A_2 . The upper time trace, obtained for low values of the coupling with node C ($R_{in} = 100k\Omega$), shows the different outputs of the truth table of the XNOR gate (see Table 4.2). In the bottom signal, coupling with node C is increased ($R_{in} = 25k\Omega$), and the gate begins to fail.

index γ given by equation (4.8). A combination between both indicators builds up the gate. Time series and the corresponding truth table for the AND gate are shown in Table 4.5.

It is worth mentioning that a tuning of the integrator resistances (see Table 4.1) allows the correct functioning of the ensemble of gates. When the integrator resistances are decreased (i.e., the coupling with the central node C is increased) to values close to $R_{in} \leq 25k\Omega$ the functioning of the gates begins to fail due to interferences between them (see bottom panels of Table 4.4 and Table 4.5).

Such an experimental implementation is a proof of the fact that system, considered for small networks of neural mass oscillators shown by [95], are robust against the parameter mismatch of electronic circuits and the intrinsic noise of real systems. Hence, we do not claim that the brain is a circuit of Boolean gates but a highly complex dynamical system able to process information in a very sophisticated manner at the mesoscale.

4.2. DETERMINISTIC COHERENCE RESONANCE IN COUPLED CHAOTIC OSCILLATORS WITH FREQUENCY MISMATCH

Figure 4.5: Time series of the AND gate. Functioning of the gate relies on the complete synchronization of units B_1 and B_2 . The upper time trace, obtained for low values of the coupling with node C ($R_{in} = 100k\Omega$), shows the different outputs of the truth table of the XNOR gate (see Table 4.2). In the bottom signal, coupling with node C is increased ($R_{in} = 25k\Omega$), and the gate begins to fail.

4.2 Deterministic coherence resonance in coupled chaotic oscillators with frequency mismatch

The main aim of this item is to provide the experimental evidence for the resonance phenomenon [118]. Since the experimental parameters do not coincide with the numerical parameters used in their studies, we have performed numerical simulations of the same equations, but with parameters exactly matched the experimental ones, and compared the obtained experimental and numerical results. Furthermore, to demonstrate the generality of the deterministic coherence resonance, we also considered another coupling scheme, namely a ring of unidirectionally coupled chaotic oscillators.

4.2. DETERMINISTIC COHERENCE RESONANCE IN COUPLED CHAOTIC OSCILLATORS WITH FREQUENCY MISMATCH

A system of coupled oscillator can be described as shown in equation (4.10).

$$\dot{\mathbf{x}}_j = \mathbf{F}(\mathbf{x}_j, \omega_j) + \sigma_{ji}(\mathbf{x}_i - \mathbf{x}_j) \quad (4.10)$$

Where $\mathbf{x}_{j,i}$ are vectors of state variables of j th and i th oscillators, $\mathbf{F}$ is a vector function, and σ_{ji} is a coupling strength. The oscillators are only distinct by their natural frequencies ($\omega_j \neq \omega_i$). Due to nonlinearity, the dominant frequency Ω_j in the chaotic power spectrum of the uncoupled j th oscillator usually does not coincide with its natural frequency ($\Omega_j \neq \omega_j$). When the oscillator drives the j th oscillator. Thus, the former acts as a master, while the latter acts as a slave. For sufficiently strong coupling, the master oscillator i entrains the dominant frequency Ω_j of the slave oscillator j , which results in phase synchronization. The time-average difference between the oscillators' phases $\delta_{ji} = \langle \phi_j - \phi_i \rangle$ is negative ($\delta_{ji} < 0$) if the frequency mismatch $\Delta_{ji} = \omega_j - \omega_i < 0$ and positive ($\delta_{ji} > 0$) if $\Delta_{ji} > 0$. In the former case, the phase of the slave oscillator is locked by the master oscillator with lag, whereas in the latter case with anticipation [123].

Deterministic coherence resonance in two unidirectionally coupled Rössler oscillators with small mismatch between their natural frequencies has been theoretically described in [118]. In the regime of phase synchronization, the coherence of the slave oscillator was shown to reach the maximum with respect to both the frequency mismatch Δ_{ji} and the coupling strength σ_{ji} . This coherence enhancement was attributed to the third and fourth Lyapunov exponents which took negative values in the parameter range where the resonance of this surprising phenomenon in two and three unidirectionally coupled oscillators. This effect resembles “stabilization of chaos by chaos” i.e., the chaotic system at a certain coupling strength and a frequency mismatch behaves more regularly in two coupled oscillators and completely periodically in a ring of three oscillators.

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Figure 4.6: Coupling circuit.

4.2.1 Experimental setup

The experimental setup is constructed on the base of the electronic circuits shown in Figure 3.2. These circuits are analog implementation of the Rössler oscillator [133] and the diffusive coupling. The equations, given by (4.11), (4.12) and (4.13), are based on the Kirchhoff's mesh analysis previously realized in 3.2.

$$\dot{V}_{xj} = \alpha A(V_{yj} + V_{zj}) \quad (4.11)$$

$$\dot{V}_{yj} = -\alpha[BV_{xj} + CV_{yj} + \sigma_{ji}(V_{yi} - V_{yj})] \quad (4.12)$$

$$\dot{V}_{zj} = \alpha[D + EV_{zj}(V_{xj} - F)] \quad (4.13)$$

Where V_{xj} , V_{yj} and V_{zj} are the output voltages of the three meshes, $\alpha = 10^3 s^{-1}$ is the time scale coefficient, σ_{ji} is the coupling strength between the oscillators j and i defined by the parameters of the coupler shown in 4.6, and A , B , C , D , E and F are the parameters expressed in terms of electronic components, showed in:

$$A = C_1 R_1 \quad (4.14)$$

$$B = \frac{R_5}{C_1 R_2 R_6} \quad (4.15)$$

4.2. DETERMINISTIC COHERENCE RESONANCE IN COUPLED CHAOTIC OSCILLATORS WITH FREQUENCY MISMATCH

$$C = \frac{R_5}{C_1 R_2 R_7} \quad (4.16)$$

$$D = \frac{V_0}{10C_3 R_3} \left(\frac{1 - 2R_{10}}{R_8 R_9 R_{10}} \right) \quad (4.17)$$

$$E = \frac{15}{V_0} = 1 \quad (4.18)$$

$$F = \frac{V_0}{10C_3 R_3} \left(1 - \frac{2R_8}{R_8 R_9 R_{10}} \right) \quad (4.19)$$

Where $V_0 = 15V$, is the power voltage of each mesh. Although in experimental equations shown in (4.11), (4.12) and (4.10) the coupling is realized through variable V_y , our experiments show that there is no principal difference if the oscillators are coupled through variable V_x . Since we observe coherence resonance in both cases, here we only present the results for the coupling given by the equation (4.12).

We carry out two kinds of experiments. First, we study two unidirectionally coupled Rössler oscillators and, second, three oscillators unidirectionally coupled in a ring. The natural frequencies of the oscillators are determined by resistors R_1 and R_6 , different for every oscillator. Since these resistors are not variable, in the experiments we do not use the natural frequencies as control parameters because variable resistors with the required variation step are not available. The full experimental process is controlled with a virtual interface developed in *LABVIEW*2014 (mostrar el diagram de flujo y el panel frontal), which can be considered as a state machine. The coupling strength is regulated by a variable resistor $R_{50} = 3/300k\Omega$ with step of $3k\Omega$. Since, $\sigma_{ji} \sim 1/R_{50}$, the oscillators can be considered to be uncoupled when R_{50} is high ($R_{50} = 300k\Omega$). This weak coupling does not act as a residual noise source able to synchronize the phase oscillators. The experimental procedure is realized as follows. First, the coupling is set to the minimum value ($R_{50} = 300k\Omega$). After a waiting time of $500ms$, (roughly corresponding to 60 cycles of the autonomous system), the output signals from all circuits are acquired by analog ports. Once the dynamics of the whole system is recorded, the value of R_{50} is decreased by one step ($3k\Omega$), and the signals are again stored in the PC for further analysis. This process is repeated until the maximum coupling ($R_{50} = 3k\Omega$) is reached.

4.2. DETERMINISTIC COHERENCE RESONANCE IN COUPLED CHAOTIC OSCILLATORS WITH FREQUENCY MISMATCH

4.2.2 Experiments with two coupled oscillators

The parameters of two Rössler oscillators in the master-slave configuration (master $i = 1$, slave $j = 2$) are chosen so when uncoupled both oscillators are in a chaotic regime. Since the coupling is unidirectional, we set $\sigma_{12} = 0$ and $\sigma_{21} = \sigma$. We observe that a certain coupling the slave oscillator becomes more coherent. A glimpse of the results are present in 4.7 with the time series, power spectra and phase trajectories of the uncoupled and coupled oscillators shrinks, meaning that the oscillations become more regular.

Figure 4.7: Experimental [(a) and (b)] time series, [(c) and (d)] power spectra, and [(e) and (f)] attractors of uncoupled (left-hand column) and coupled oscillators with coupling $1/R_{50} = 2 \times 10^{-5}$ (right-hand column). f_1 and f_2 are the dominant frequencies in the power spectra. The coherence enhancement in the slave oscillator is characterized by the shrinking of the attractor (dark blue line) in the phase space.

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The resistors $R1$ and $R6$ in the master and slave electronic circuits are chosen so the dominant frequencies of the uncoupled oscillators are, respectively, $f_1 = 162\text{Hz}$ and $f_2 = 191\text{Hz}$, 4.7. With sufficiently strong coupling the oscillators' phases become synchronized 4.7 because the master oscillator entrains the dominant frequency of the slave oscillator, so the oscillators have the same dominant frequency in their power spectra 4.7(c,d). This coupling leads to chaos suppression in the slave oscillator clearly seen in the phase-space plot in 4.7.

Figure 4.8: Experimental (a) peak value of V_{y2} , (b) interpeak intervals (IPI), (c) normalized standard deviation (NSD) of peak value, and (d) NSD of IPI versus coupling strength.

For quantitative description of coherence, we use (i) the peak value of the variable V_{y2} , (ii) interpeak intervals (IPI), (iii) normalized standard deviation (NSD) of the peak value of V_{y2} , and (iv) NSD of IPI. In Figure 4.8 we plot all these characteristics for variable V_{y2} as a function of coupling strength.

The minimum of NSD, in Figure 4.8(c), is the signature of amplitude coherence, while the minimum of NSD, in Figure 4.8(d) indicates time coherence. Interestingly, the coherence in time and amplitude occur for distinct coupling strengths.

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4.2.3 Experiments with three coupled oscillators

Now we consider a ring of three unidirectionally coupled chaotic Rössler oscillators. In this configuration, each oscillator acts simultaneously as a master for one oscillator and a slave for another. All oscillators are coupled with the same coupling strength $\sigma_{21} = \sigma_{32} = \sigma_{13} = \sigma$. Since the coupling is unidirectional, $\sigma_{21} = \sigma_{32} = \sigma_{13} = 0$. The resistor $R1$ and $R6$ in each of the three circuits are chosen so the dominant frequencies in the chaotic power spectra of the uncoupled oscillators are in an ascending order, i.e., $f_1 < f_2 < f_3$, where $f_1 = 152Hz$, $f_2 = 168Hz$, and $f_3 = 177Hz$. The time series, phase portraits, and power spectra of the uncoupled and coupled oscillators are shown in

Figure 4.9: Experimental [(a) and (b)] time series, phase portraits, and [(c) and (d)] power spectra of three [(a) and (c)] uncoupled and [(b) and (d)] coupled Rössler oscillators demonstrating coherence enhancement for $(1/R_{50} = 2 \times 10^{-5} \Omega^{-1})$.

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Figure 4.10: Experimental (a) peak output voltages of three oscillators and (b) normalized standard deviations of peak voltage as function of the coupling strength demonstrating coherence resonance for $(1/R_{50} = 2 \times 10^{-5} \Omega^{-1})$.

4.2.4 Numerical simulations

We perform numerical simulations of the system of equations (4.20), (4.21) and (4.22).

$$\dot{x}_j = -\omega_j \cdot y_j - z_j \quad (4.20)$$

$$\dot{y}_j = -\omega_j \cdot x_j + a \cdot y_j + \sigma_{ji}(y_i - y_j) \quad (4.21)$$

$$\dot{z}_j = b + z_j \cdot (x_j - c) \quad (4.22)$$

Where x_j , y_j , and z_j are the state variables of the j th oscillator. The system of equations (4.20), (4.21) and (4.22) is the dimensionless model of the experimental equations (4.11), (4.12) and (4.13). To simulate our experiments numerically, we consider the same configurations as in the experiments, i.e., two and three coupled oscillators. While in the experiments the oscillators' natural frequencies ω_j were not varied due to technical difficulties, in the numerical simulations we used frequency mismatch $\Delta = \omega_j - \omega_i$ as a control parameter, in addition to the coupling strength $\sigma_{ji} = \sigma$.

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4.2.5 Simulations of two couple oscillators

In the numerical simulations we use the same parameters as in the experiment, namely $a = \alpha C = 0.2$, $b = \alpha D = 0.2$, and $c = \alpha F = 5.7$. The natural frequency of the master oscillator is fixed to $\omega_1 = 1$ to provide a chaotic regime.

Figure 4.11 illustrates how mismatch $\Delta \equiv \Delta_{21} = \omega_2 - \omega_1 = 0.1$ between the natural frequencies of the slave and master oscillators enhances coherence of the slave oscillator dynamics. The uncoupled oscillators are chaotic [Figure 4.11(a, c and e)] with different dominant frequencies in their power spectra, $\omega_1 = 1.07$ and $\omega_2 = 1.16$ [Figure 4.11(c)]. Due to nonlinearity, they are shifted with respect to the natural frequencies $\omega_1 = 1$ and $\omega_2 = 1.1$. When the oscillators are coupled with $\omega = 0.2$, the dominant frequency of the slave oscillator ω_2 is entrained by the master oscillator [Figure 4.11(d)], resulting in phase synchronization [Figure 4.11(b)]. One can see in Figure 4.11(f), that the chaotic attractor of the slave oscillator shrinks, meaning that its dynamics becomes more regular.

Figure 4.12 demonstrates coherence resonance in the slave oscillator with respect to its natural frequency, while the natural frequency of the master oscillator remains fixed to $\omega_1 = 1$. The bifurcation diagrams of the peak value of the variable y_2 [Figure 4.12(a)] and the interpeak interval (IPI) [Figure 4.12(b)] display strong resonant suppression of the amplitude variation at $\omega_2 = 1.14$. The coherence resonances in amplitude and time are clearly seen in Figure 4.12(c) and (d), respectively.

In Figure 32 we plot NSD of peak y_2 [Figure 4.12(a)] and IPI [Figure 4.12(b)] in the space of two control parameters, the natural frequency ω_2 of the slave oscillator and the coupling strength ω . These diagrams have a structure of Arnold tongues centered at $\omega_2 = 1$. Within these tongues, the dominant frequency of the slave oscillator is entrained by the master oscillator, so the oscillators are in phase synchronization. The dark blue bands inside these tongues indicate the region of higher coherence.

The interesting question may arise of whether the coherence resonance is related to synchronization. To measure synchronization we use similarity function S_{ij} between the oscillators i and j introduced by Rosenblum et al. in [131] to describe phase and lag synchronization of non-identical chaotic

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Figure 4.11: Numerical [(a) and (b)] time series, [(c) and (d)] power spectra, and [(e) and (f)] chaotic attractors of two [(a), (c), and (e)] uncoupled and [(b), (d) and (f)] coupled Rössler oscillators [Eq.(4.20), (4.21) and (4.22)]. Coherence enhancement of the slave oscillator observed for, frequency mismatch $\Delta = 0.1$ and coupling $\sigma = 0.2$, when its dominant frequency is entrained by the master oscillator.

oscillators.

$$S_{ij}^2(\tau) = \frac{\langle [x_j(t) - x_i(t + \tau)]^2 \rangle}{\sqrt{\langle x_j(t)^2 \rangle \langle x_i(t)^2 \rangle}} \quad (4.23)$$

Where τ is the time shift between the state vectors of the interacting systems. The lower the minimum of the similarity function $\delta = \min_{\tau} S(\tau)$, the better the synchronization.

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Figure 4.12: Numerical (a) peak value of y_2 , (b) IPI, (c) NSD of peak y_2 , and NSD of IPI as a function of natural frequency of slave oscillator ω_2 for $\omega_2 = 0.2$ and $\omega_1 = 1$.

Figure 4.14 shows how the minimum similarity between two coupled oscillators, δ_{12} , depends on both the natural frequency of the slave oscillator $\omega_2 = 1$ [Figure 4.14(a)] and the coupling strength σ [Figure 4.14(b)]. The slump of the dependence in Figure 4.14(a), close to $\omega_2 = 1$, results from phase synchronization when the natural frequency of the slave oscillator ω_2 approaches the natural frequency of the master oscillator $\omega_1 = 1$. When the oscillators' natural frequencies exactly match ($\omega_2 = \omega_1 = 1$), complete synchronization is observed ($\delta_{12} = 0$) because in this case we deal with identical systems. Another, local minimum at $\omega_2 = 1.14$ coincides with the deterministic coherence resonance in Figure 4.12. This means that enhanced coherence is accompanied by improving phase synchronization. Note that changes in the similarity function associated with coherence resonance are only present in the frequency dependence in Figure 4.14(a) but not in the dependence on the coupling strength. Indeed, as one can see from Figure 4.13, the maximal coherence occurs for $\sigma = 0.2$ without qualitative changes for this value in

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Figure 4.13: Normalized standard deviations of (a) peak y_2 and (b) interpeak intervals in the (ω_2, σ) -parameter space. The violet (dark) regions are associated with increasing coherence.

Figure 4.14: Minimum similarity as a function of (a) natural frequency of slave oscillator ω_2 for $\sigma = 2$ and (b) coupling σ for $\omega_2 = 1.1$.

Figure 4.14(b). Evidently, synchronization measures cannot be used to characterize coherence because the latter occurs in one (slave) oscillator only,

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while synchronization refers to a relative behavior between two oscillatory systems.

Next, we analyze the Lyapunov exponent spectrum in order to demonstrate how the control parameters affect the system stability. Since the dynamics of the master oscillator does not depend on the control parameters, the stability of the whole system is determined by the dynamics of the slave oscillator only. All six Lyapunov exponents $\lambda_{(1-6)}$ are plotted in Figure 4.15 in the (ω_2, σ) -parameter space.

Figure 4.15: Lyapunov exponents [(a)-(f)] $\lambda_{(1-6)}$ of two coupled oscillators in (ω_2, σ) -parameter space. Deterministic coherence resonance is associated with negative λ_3 and λ_4 in the region of the central blue (dark) spots in (c) and (d).

Since the master oscillator is chaotic, the first largest Lyapunov exponent λ_1 [Figure 4.15(a)] is always positive for all control parameters. Depending on the control parameters, the second exponent λ_2 [Figure 4.15(b)] is either positive or zero. The most important information about the system stability can be extracted from the divergence or convergence of the phase-space trajectory towards the directions of the slave oscillator variables, i.e., the third and fourth Lyapunov exponents, λ_3 and λ_4 . One can see from Figure 4.15(c)

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and (d) that these exponents take negative values and reach minima in the dark (blue) region in the parameter range $1.1 < \omega_2 < 1.15$ and $\sigma > 0.13$, corresponding to the best coherence of the slave oscillator. Lower Lyapunov exponents, i.e., higher stability of the system, are associated with coherence enhancement. However, in the region of complete synchronization when the oscillators are identical ($\omega_2 = \omega_1 = 1$), these exponents are close to zero. By comparing the NSD diagrams in Figure 34 with the Lyapunov exponents in Figure 4.15 we can see their strong correlation in the region of the coherence resonance where the minimum NSD matches the minima of the third and fourth Lyapunov exponents. The fifth and sixth Lyapunov exponents, λ_5 and λ_6 , do not yield additional information because they are always negative in the explored parameter range [Figure 4.15(e) and (f)].

4.2.6 Simulations of three coupled oscillators

For numerical simulations of the three Rössler oscillators unidirectionally coupled in a ring we choose the parameters $a = 0.165$, $b = 0.2$ and $c = 10$ because for these parameters the uncoupled oscillator exhibits a chaotic regime in a wide range of the natural frequency, as shown in the bifurcation diagram in Figure 4.16. This allows us to vary significantly the distance between the oscillators (frequency mismatch) as long as they remain chaotic.

Figure 4.16: Numerical bifurcation diagram of peak x with respect natural frequency of the uncoupled Rössler oscillator.

For simplicity, we consider a symmetric case where the distances between the oscillators 1 and 2 and the oscillators 2 and 3 are the same, i.e., $\Delta_{21} = \Delta_{32} = \Delta_{13}/2 = \Delta$. To study the influence of the detuning Δ on the coherence, we fix the natural frequency of the oscillator 2 to $\omega_2 = 0.9$ and use Δ as a control parameter. Figure 4.17 shows the time series, phase portraits, and power spectra of the x variable of the uncoupled [Figure 4.17 (a) and (b)]

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and coupled oscillators with the coupling strength $\sigma = 0.2$ [Figure 4.17 (c) and (d)].

Figure 4.17: Numerical [(a) and (b)] time series, phase portraits, and [(c) and (d)] power spectra of three [(a) and (c)] uncoupled and [(b) and (d)] coupled Rössler oscillators for $\sigma = 0.33$ and $\Delta = 0.2$. The coupled oscillators behave periodically.

Being uncoupled, the oscillators are chaotic with different dominant frequencies in their power spectra [Figure 4.17(c)]. As the coupling strength σ is increased, the oscillators' phases synchronize, resulting in the same dominant frequency in their power spectra [Figure 4.17(d)]. When σ is further increased, they oscillate in a period-2 regime [Figure 4.17(b)]. These results are in good agreement with the experiment.

Figure 4.18 shows NSD of the peak value of the variable x for every oscillator as a function of both frequency mismatch (distance) Δ and coupling strength σ . While for small distances, all oscillators are incoherent (chaotic) [left-hand (green) tongues], they become highly coherent for intermediate distances and coupling [right-hand (dark blue tongues)]. Therefore, there exists a preferential coupling for which the coherence is maximized. High NSD for $\Delta > 0.2$

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Figure 4.18: NSD of peak (a) x_1 , (b) x_2 , and (c) x_3 in (ω_2, σ) -parameter space.

and $0.2 < \sigma < 0.4$ does not mean low coherence; this occurs because for these parameters the oscillators are involved in a period-doubling regime where the difference between amplitudes of subsequent peaks is large.

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Figure 4.19: NSD of peak (a) x_1 , (b) x_2 , and (c) x_3 in (ω_2, σ) -parameter space.

The resonance behavior with respect to both the coupling strength and the distance is clearly seen in Figure 4.19, where we present one-dimensional plots of NSDs for fixed $\Delta = 0.03$ [Figure 4.19(a)] and fixed $\sigma = 0.2$ [Figure 4.19(b)]. In Figure 4.19(a), NSDs of the peak x of all oscillators take minimal values at $\sigma \approx 0.08$ displaying the coherence resonance behavior. In Figure 4.19(b), the coherence enhancement is observed in a wide range of distances ($0.1 < \Delta < 0.23$). In the previous section we have shown that in the case of two coupled oscillators the coherence resonance was accompanied by improving phase synchronization. Now we are interested in how the enhancing coherence affects synchronization in the case of three ring-coupled oscillators.

In the previous section we have shown that in the case of two coupled oscillators the coherence resonance was accompanied by improving phase synchronization. Now we are interested in how the enhancing coherence affects synchronization in the case of three ring-coupled oscillators.

In Figure 4.19(c) we plot the minimum similarity δ_{ij} for each pair of the coupled oscillators as a function of the coupling strength for the same detuning $\Delta = 0.03$ as in Figure 4.19(a). When the coupling is small ($\sigma < 0.05$), δ_{ij} is very large, meaning that the oscillators are asynchronous. A sudden slump in δ_{ij} near $\sigma = 0.05$ indicates the onset of phase synchronization. By comparing Figure 4.19(a) with Figure 4.19(c) one can see that the coher-

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ence resonance occurs near the onset of the phase synchronization. As the coupling increases, δ_{ij} slowly creases, meaning there is improved phase synchronization. The evident situation, that synchronization of the oscillators 1 and 3 with the largest frequency mismatch is worse than the synchronization of other pairs of oscillators with smaller mismatch, is illustrated in Figure 4.19(c).

The effect of the frequency mismatch on synchronization is demonstrated in Figure 4.19(d). When $\Delta = 0$, the oscillators are identical and therefore completely synchronized. As Δ is increased, Δ_{ij} also increases, leading to desynchronization. For small Δ the minimum similarity of the oscillators 1 and 3 (δ_{31}) with the largest frequency mismatch is higher than δ_{ij} of other, closer located oscillators. The maxima of δ_{ij} at $\Delta \approx 0.2$ mean partial desynchronization due to coherence enhancement, i.e., the oscillators are less similar in spite of higher coherence.

Figure 4.20: Largest Lyapunov exponent of three ring-coupled oscillators in (Δ, σ) -parameter space. The blue dark tongue indicates the region of coherence resonance and periodicity for large Δ and intermediate σ .

It is important to check whether the coupled system does become stable in a certain region of the coupling strength and frequency mismatch. For this purpose we analyze the system stability through Lyapunov exponents. In Figure 4.20, we plot the largest Lyapunov exponent in the parameter space of distance and coupling σ . When the oscillators' frequencies are very close to each other, they are chaotic for any coupling strength, whereas for larger distances their dynamics become more regular. The comparison of Figure 4.20 and Figure 4.17, shows that the coherence enhancement is associated

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with increasing system stability in the dark blue tongue. Indeed, for relatively large distances ($\Delta > 0.1$) and intermediate coupling ($0.1 < \sigma < 0.4$) the largest Lyapunov exponent takes negative values; in this parameter region a period-2 orbit is stabilized.

4.3 Chaos in unidirectionally coupled neural oscillators

Besides synchronization, in this section we study the influence of coupling on the complexity of neural dynamics [85] in the sense of its sensitivity to initial conditions. This research was also motivated by recently discovered deterministic coherence resonance in unidirectionally coupled Rössler oscillators [58]. It was found that chaotic slave oscillator exhibits more regular (almost periodic) behavior under the influence of another chaotic oscillator in the presence of a small mismatch between their parameters. To check whether or not a similar effect occurs in a neural model, we explore two coupled HR oscillators in a master-slave configuration. Choosing the parameters so that the uncoupled neurons are in a periodic spiking regime, we study how the dynamics of the slave neuron depends on the coupling strength by analyzing bifurcation diagrams of the peak membrane potential and interspike intervals (ISI), calculating the normalized standard deviation (NSD) of ISI and the largest Lyapunov exponent.

The majority of papers on coupled neural oscillators are devoted to a study of their synchronization through numerical simulations [7, 12, 15, 82, 85]. Significantly less attention was paid to the electronic implementation of the HR model [126], which can provide experimental evidence of numerical results in a natural system, where parameter tolerances and noise are always present. Therefore, another aim of this research is to check whether or not the predicted theoretical findings take place in real experimental conditions in noisy environment.

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4.3.1 Numerical simulations

The HR neuron model is given by a system of three-dimensional nonlinear differential equations (2.23), (2.24) and (2.25), with the same parameter meanings and values for each one (see 2.2.3).

Figure 4.21: Bifurcation diagram of ISI as a function of external current. The vertical dashed line shows the value of the external current $I_{ext} = 1.4$ explored in numerical simulations. The inset shows the time series of the periodic spiking regime realized for this current.

The dynamics of the membrane potential x is determined by the external current I_{ext} , as seen from the time series and bifurcation diagram of interspike intervals (ISI) shown in Figure 4.21.

For intermediate and large currents, ($1.4 < I_{ext} < 2.9$ and ($I_{ext} > 3.4$), the dynamical behavior of the membrane potential is periodic, whereas for small currents, ($I_{ext} < 1.3$), and intermediate currents, ($I_{ext} \in (2.9, 3.4)$), the dynamics is chaotic. In this work, we are interested in the regime where the uncoupled neurons generate regular spikes. Therefore, we fix the external current to $I_{ext} = 1.4$ where the period-1 spiking regime takes place.

When the neurons are unidirectionally coupled, the dynamics of the slave

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Figure 4.22: Bifurcation diagrams of (a) peak x_s and (b) ISI of x_s versus coupling strength σ at $I_{ext} = 1.4$. The diagrams are constructed by varying randomly initial conditions. The upper horizontal line in both diagrams corresponds to a period-1 regime, which is similar to the attractor of the master oscillator, while other branches represent a new attractor induced by the coupling.

neuron can be described by the equations (4.24), (4.25) and (4.26).

$$\dot{x}_s = y - ax_s^3 + bx_s^2 - z_s + I_{ext} - \sigma \cdot (x_s - x_m) \quad (4.24)$$

$$\dot{y}_s = c - dx_s^2 - y \quad (4.25)$$

$$\dot{z}_s = r[s(x_s - x_0) - z_s] \quad (4.26)$$

Where the sub-indexes m and s are referred, respectively, to the master and slave oscillators and σ is the coupling strength. In this section we consider the case when the master and slave neurons are identical and, being uncoupled, they both are in a spiking periodic regime.

To study how the dynamics of the slave oscillator depends on the coupling strength, we plot the bifurcation diagrams of the peak value of x_s (local maxima in the time series) and ISI of x_s in Figure 4.22(a) and (b), respectively,

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as a function of the coupling strength σ . The bifurcation diagrams display branches corresponding to two coexisting attractors. The upper horizontal branch (marked by the arrow) belongs to the period-1 regime illustrated in in Figure 4.23 (a) and (c) with the time series and phase portraits, respectively. This regime is similar to the period-1 attractor of the master oscillator (see the inset of Figure 4.21). The lower multiple branches in Figure 4.22 correspond to a new attractor which for small and intermediate coupling ($\sigma < 0.7$) coexists with the period 1. Thus, one can see that the coupling induces bistability in the slave oscillator. The periodicity of the induced attractor is higher than the periodicity of the original attractor. For certain coupling strengths ($0.18 < \sigma < 0.20$) the induced attractor is chaotic. The coexisting periodic and chaotic attractors are shown in Figure 4.23 (b and d) with the time series and phase portraits.

Figure 4.23: (a,b) Time series and (c,d) phase portraits of coexisting (a,c) periodic and (b,d) chaotic regimes of the slave oscillator for coupling $\sigma = 0.1928$. The periodic orbit in (a) is similar to the attractor of the master oscillator.

It should be noted that the bistability arises already for a very weak coupling ($\sigma \approx 0.01$) and exists for all coupling strengths $\sigma \leq 0.68$. The branch corresponding to the new attractor for very small σ does not exist in the

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diagram in Figure 4.22(b) because the ISI does not change, only the amplitude. The new attractor is only distinguished by the peak's amplitude of the neuron spikes.

The complexity of the induced attractor is analyzed with the largest Lyapunov exponent. In Figure 4.24, we plot the largest Lyapunov exponent of the coupled system as a function of the coupling strength. Since the uncoupled oscillators are periodic, the largest Lyapunov exponent is zero, and the positive exponents mean chaos.

Figure 4.24: Largest Lyapunov exponent of the coupled neurons as a function of the coupling strength. The coupling induces chaos for $\sigma \in (0.18, 0.20)$.

The coexisting attractors of the slave oscillator have different synchronization properties, as can be seen from the synchronization diagrams in Figure 4.25. While the original orbit of the slave oscillator is completely synchronized with the master oscillator (straight line), the induced state is in generalized synchronization with the master (thin curve).

The coherent properties of the induced attractor depend on the coupling strength. In physics, a coherent state is known as a periodic state which dynamics is close to the dynamics of a classical harmonic oscillator [83]. The closer is the dynamics to harmonic oscillations, the higher the coherence. Instead, noisy and chaotic oscillators exhibit highly incoherent dynamics.

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Figure 4.25: Synchronization diagram for slave and master oscillators when the slave oscillator is the period-1 regime (straight line) or in the higher periodic regime (curve) for coupling $\sigma = 0$. The straight line means complete synchronization.

Among the many measures of coherence (or ordering), the most popular ones are the normalize autocorrelation function and the normalized standard deviation (NSD) [58, 116, 118], in this section we use the latter.

In Figure 4.26 we plot the NSD of ISI as a function of the coupling strengths. The zero NSD means that the dynamics of the slave oscillator is strongly coherent because it is in a period-1 regime, whereas nonzero NSD can be a consequence of higher periodicity or chaos. The coupling always worsens the coherence in the slave oscillator. Although we do not provide the case when the master HR oscillator is in a chaotic regime, even in this case we did not observe the coherence enhancement in the slave oscillator. This coherent property of the coupled neurons differs from the coherent enhancement observed in the coupled chaotic Rössler oscillators [58, 118], where deterministic coherent resonance occurs for intermediate coupling. Thus, one can see that the intermediate coupling induces the additional incoherent state which can be either periodic (with higher periodicity than the coexisting original orbit) or chaotic.

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Figure 4.26: Normalized standard deviation of ISI of the slave oscillator as a function of the coupling strength. The horizontal zero line corresponds to the period-1 attractor, while the upper branches correspond to the coexisting chaotic and higher periodic regimes.

4.3.2 Electronic implementation

We already saw how to implement the HR neuron model as a real physical system in 3.3. Taking into account that, we can represent the equations as shown in (4.27), (4.28) and (4.29).

$$\dot{V}_x = T[-AV_x^3 + BV_x^2 + CV_y - DV_z + EI_{ext}] \quad (4.27)$$

$$\dot{V}_y = T[F - GV_x^2 - HV_y] \quad (4.28)$$

$$\dot{V}_z = TR[S(V_x - V_0) - V_z] \quad (4.29)$$

Where V_x , V_y , and V_z are the voltages in the electronic circuit associated with three variables, $T = 10^4 s$ is the time scale factor, and $I_{ext} = V_{ext}/R$ is the external current V_{ext} , (V_{ext} and R being the external voltage and load resistance, respectively). To satisfy the theoretical model saw in section 3.3, we use the following scaling coefficients: $A = 10^6 V^{-2}$, $B = 3 \times 10^6 V^{-2}$, $C = D = H = 10^4$, $E = 10^4 V/A$, $F = 10^4 V^{-1}$, $R = 50$, $S = 4$, and $V_0 = 1.6V$.

In Figure 4.27, we plot the experimental bifurcation diagrams of ISI of V_x of the master and slave oscillators versus external voltage V_{ext} and the time series in the period-1 regime. The difference between these diagrams appears

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because the master and slave oscillators are not completely identical due to a small tolerance of their electronic components. Therefore, the period-1 orbits in the master and slave oscillators occur for different values of the external voltages.

Figure 4.27: Bifurcation diagrams of ISI of V_x of (a) master and (b) slave oscillator with external voltage V_{ext} used as a control parameter. The insets show the time series of the regimes explored in the experiment. The experimental time series are recorded using the data acquisition card (National Instruments USB-6363) controlled by LabView.

To be able to compare the experimental results with the numerical ones, we have to explore the same regime (period 1) in both oscillators. For this purpose, we fix $V_x = 1.9V$ for the master and $V_x = 2.3V$ for the slave oscillator, where they are in the period-1 regimes shown in the insets. One can see very close similarity between these period-1 dynamics of the uncoupled oscillators.

Depending on the coupling strength, the dynamics of the slave oscillator can be either periodic or chaotic. The time series of V_x and corresponding power

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Figure 4.28: (a,b) Experimental time series and (c, d) corresponding power spectra of slave oscillator for coupling $\sigma = 0.66$ (a, c) and $\sigma = 0.99$ (b, d).

spectra W_s of the chaotic and periodic regimes are shown in Figure 48 . While the intermediate coupling induces chaos in the slave oscillator (Figure 4.28(a,c)), the strong coupling synchronizes the slave oscillator to the period 1 (Figure 4.28(b,d)).

Figure 4.29: Synchronization diagrams for (a) uncoupled $\sigma = 0$ and (b,c) coupled oscillator with coupling $\sigma = 0.66$ and (c) $\sigma = 0.99$.

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It is interesting to know how the oscillators synchronize when they are in a similar regime or in different regimes. For this aim, in Figure 4.29 we plot the synchronization diagrams for uncoupled (Figure 4.29(a)) oscillators and for coupled oscillators with intermediate (Figure 4.29(b)) and strong (Figure 4.29(c)) coupling. Evidently, the uncoupled oscillators are not synchronized (Figure 4.29(a)). For intermediate coupling, the oscillators are in generalized synchronization (Figure 4.29(b)), whereas for strong coupling they are completely synchronized (Figure 4.29(c)).

Figure 4.30: Experimental NSD of ISI for slave oscillator as a function of the coupling strength. The intermediate coupling reduces coherence.

Our experiments confirm that the intermediate coupling induces a chaotic attractor in the slave oscillator. This can be seen in the bifurcation diagram of ISI of V_x in Figure 4.30. In a similar way as in the numerical simulations, the coherence of the slave oscillator is studied using NSD of ISI. The dependence of NSD of ISI on the coupling strengths is shown in Figure 4.31. The larger NSD means lower coherence. One can see that at certain coupling strength ($\sigma \approx 0.8$) NSD reaches its maximum value, i.e. the coupling induces disorder in the slave oscillator.

Due to inevitable internal noise, we do not observe bistability in the experiments. Only the chaotic attractor releases because its basin of attraction is much larger than the basin of the periodic orbit.

4.4 Neural Dynamics Simulator

In this section, we present an application, based on solving ordinary differential equations, to help students, engineers and scientists in dynamic systems.

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Figure 4.31: Experimental NSD of ISI for slave oscillator as a function of the coupling strength. The intermediate coupling reduces coherence.

Particularly, in the biomedical technology field, where we have developed the application focused on the Hindmarsh-Rose neuron model.

4.4.1 Problem formulation

Nowadays, computers help us to solve ordinary differential equations which model many dynamical systems. Moreover, the fast development of computers makes them powerful enough to visualize the solution of very complex dynamical systems. Then, could we take advantage of them to show easily some neuronal behavior? I believe it by developing a software application based on the ordinary differential equations in a neuronal model. Consequently, we can also ask ourselves how we can bring it to the society closer. We have tried to answer this question designing a software application as intuitive as possible. Suddenly, this idea came out after performing one of the sessions of the Madrid Science Week, "... Its main aim is to disseminate results of research, opening spaces where it is performed and foster scientific careers and entrepreneurship among young people" ¹.

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¹Fundación para el Conocimiento *madri+d*; Madrid Science Week. [on line] [http : //www.madrimasd.org/scientific - culture/science - week?lan = en](http://www.madrimasd.org/scientific-culture/science-week?lan=en). [query: 24-1-2017]

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4.4.2 Design of Neural Dynamics Simulator

To keep a correlation with the last study during this thesis, we took the Hindmarsh-Rose neuron model to be used as the dynamical system. Due to our knowledge got during this thesis period, we used a visual programming language as development environment because of it offers many ways for showing a user-friendly interface. The interface is shown in Figure 4.32.

Figure 4.32: Neuron dynamics simulator interface.

This software application simulates the dynamic behavior of two neurons connected through the electrical and chemical synapse. Let us show you the interface step by step. On top of the interface, users can find on tags “Neuron 1” and “Neuron 2” the ordinary differential equations for both neurons (Figure 4.33). In order to solve these equations, we use numerical methods based on Runge-Kutta of fourth-fifth order.

Figure 4.33: The information space on the user interface. User’s information to read more details (left); “Information tag”. The responsible mathematical equations from the dynamical system (right); tags “Neuron 1” and “Neuron 2”.

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On the left-hand side, there is a space for the parameters. Users have the opportunity of changing the value of the model parameters under study (Figure 4.34).

Figure 4.34: Parameters space on the user interface. V_c is synaptic reverse potential and the value of X_0 and Y_0 is related with the fact that each neuron must receive an input every time the other produces a spike [72, 157].

Below the parameters space is located the controls for managing values of the external current/stimulus for each neuron (Figure 4.35).

Figure 4.35: External current space on the user interface. Input 1 and Input 2 are associated with the "Neuron 1" (black) and the "Neuron 2" (brown) respectively.

Moving a little bit to the right-hand side there are controller spaces for both chemical and electrical synapse. In the former case, users can change from excitatory to inhibitory synapse and also the coupling strength. In the latter

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case, users can tune the unidirectional and bidirectional coupling strength (Figure 4.36).

Figure 4.36: (Top) Chemical synapse space:excitatory (left) and inhibitory (right). You can tune coupling strength for the chemical synapse with the controls below. (bottom) Electrical synapse space for tuning the coupling strength.

Afterward, users can run the application and visualize the neurons behavior on the display (Figure 4.36) and change the control values while the application is running.

Finally, users have a choice to save the data in a text file. The "File number" box offers you to put a number at the end of the text file name. Besides, there is a red button for stopping the execution at any time (Figure 4.37).

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Figure 4.37: Neuronal behavior display. You can observe the amplitude of "Neuron 1" (blue line) and "Neuron 2" (yellow line) over time.

Figure 4.38: The number that a user writes on "File number" box (upper-left) replaces the "#" symbol in the text file name. "Store Directory" box (lower-left) gives the option to change the file directory. "Save file" button (center) looks bright green when you push the button to save a text file. And the last, the "stop" button (right). To push twice: once to stop the simulation and once again for the next simulation.

Chapter 5

CONCLUSIONS AND PERSPECTIVES

5.1 Conclusions

In order to reach the results, (see chapter 4), we have used our background in electronics. An experimental setup has been designed and implemented to check theoretical predictions. This implementation allowed the investigation of the robustness from the predicted phenomena by our numerical simulations. On one side, we have shown the general behavior of a small network, based on the physical implementation of the Chua electronic oscillator, driven by oscillatory input signals. Particularly, we have constructed several binary logic gates with electronic circuits operating in a chaotic regime. We also have experimentally shown that, a network built by coupling two of these gates through a hub, is able to process information as expected.

On another side, we have presented the experimental evidence of the appearance of an order in a chaotic system, under the influence of a chaotic signal generated by another similar system. Such coherence enhancement has been demonstrated in two and three unidirectionally coupled Rössler oscillators with the small mismatch between their natural frequencies. The enhanced

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coherence has a resonant character in respect of the coupling strength and frequency mismatch.

Keeping in mind how to characterize this deterministic coherence resonance, we have used the normalized standard deviations of the peak amplitude and inter-peak interval. For instance, in the case of two coupled oscillators, the coherence resonance is accompanied by improved phase synchronization, while in the ring of three oscillators it is associated with partial desynchronization. In the latter case, the stabilization of a periodic orbit has been observed. Moreover, the results of the numerical simulations are in good agreement with the experimental results.

The analysis of the Lyapunov exponent spectra has shown that in two coupled oscillators, the coherence enhancement is associated with negative third and fourth Lyapunov exponents, while in the ring of three oscillators, all Lyapunov exponents take negative values in a certain range of frequency mismatch and coupling strength. Accordingly, the last phenomenon is the manifestation of self-stabilization of coupled chaotic systems due to their interaction.

Besides, we have studied the dynamics of two unidirectionally electrical coupled Hindmarsh-Rose oscillators. Choosing the parameters, so that the uncoupled neurons are in a periodic spiking regime, we have found that intermediate coupling induces bistability in the slave neuron. The additional attractor is more complex than the original one; it is either of higher periodicity or chaotic, depending on the coupling strength. The complexity of the neural dynamics was analyzed with the bifurcation diagrams of the peak membrane potential and ISI, NSD of ISI, and the largest Lyapunov exponent. When the slave neuron is in an attractor, similar to that, of the master neuron, the neurons are completely synchronized. Whereas, when the slave neuron is in a new attractor, they are in generalized synchronization. The experimental study with the electronic HR circuits confirmed the results of the numerical simulations.

It is worth pointing out that the induced attractor has a larger basin of at-

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traction than the original attractor. Although we did not measure the basin sizes, the random variation of the initial conditions revealed that the new attractor appeared much more frequently than the original one. Moreover, the original attractor of the slave oscillator was never found in the experiments. It allows us to conclude that the basin of attraction of the induced attractor is much larger than the basin of attraction of the original attractor, and that inherent experimental noise does not allow the system to be in the original attractor. These results imply that asymmetric electrical coupling in a real (noisy) neural network may cause a disorder in neural spikes.

Finally, we have presented a new software application based on the Hindmarsh-Rose neuron model. The application simulates two neuron coupling through a chemical and electrical synapse. Moreover, users can change different parameters of each neuron, as well as the stimulus for both. Our interface controls how the chemical and electrical synapse are. Offering the possibility to change both, the kind of synapse (inhibitory or excitatory in the chemical synapse, and unidirectional or bidirectional in the electrical synapse) and the coupling strength between them.

5.2 Future Work

On one hand, we can study synchronization and resonance effects in other complex dynamical systems. In order to analyze the route to synchronization and how extended the resonance effects are.

On the other hand, we can also extend the results of this thesis into the field of networks. More specifically: investigating synchronization in networks, how clusters or hubs arise and how resonance effects propagate inside networks.

An interesting option would be a study of all these aspects from both a numerical and experimental point of view. Particularly, experiments contributing to pass from an ideal environment to a real system; a physical system that you can touch.

5.3 Achievements

5.3.1 List of Publications

During this thesis, the following scientific articles were published:

- M. A. García-Vellisca, R. Jaimes-Reátegui, A. N. Pisarchik. *Chaos in Neural Oscillators Induced by Unidirectional Electrical Coupling*. Math. Model. Nat. Phenom., (accepted/in press).
- M. A. García-Vellisca, A. N. Pisarchik, and R. Jaimes-Reátegui. *Experimental evidence of deterministic coherence resonance in coupled chaotic systems with frequency mismatch*. PRE, 94(1):012218,07/15 2016. ID: 10.1103/PhysRevE.94.012218.
- D. Malagarriga, M. A. García-Vellisca, A. E. Villa, J. M. Buldú, J. García-Ojalvo, and A. J. Pons. *Synchornization-based computation through networks of coupled oscillators*. *Frontiers in computational neuroscience*, 9:97, Aug 4 2015b.

During the course of the doctorate, other works were published(not included in the body of this thesis):

- Luis Armando Carvajal Ahumada, Oscar Leonardo Herrera Sandoval, Nuria Peña Perez, Felipe Andrés Silva Gómez, Mariano Alberto García Vellisca, José Javier Serrano Olmedo. *Quartz Crystal Resonator for Real-Time Characterization of Nano-Scale Phenomena relevant for Biomedical applications*. Chapter in the 7th volume of Springer book series.(accepted/in press).
- Alexander N. Pisarchik and Mariano Alberto García-Vellisca. Chapter in *Nonlinear Dynamics of Complex System: Theory and Applications*,Springer.(accepted/in press).
- De Santos-Sierra D, Sanchez-Jimenez A, Garcia-Vellisca MA, Navas A, Villacorta-Atienza JA. *Effects of Spike Anticipation on the Spiking Dy-*

5.3. ACHIEVEMENTS

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namics of Neural Networks. Frontiers in Computational Neuroscience. 2015;9:144. doi:10.3389/fncom.2015.00144.

The following talk and posters were also presented:

- Talk: *Coupling-Induced bistability in Hindmarsh-Rose neurons*. September, 19th - October, 16th 2016. Max Planck Institute for the Physics of Complex Systems (mpipks), Germany.
- Poster: *Route to synchronization in a bistable chaotic oscillator with chaotic driving: Experimental study*. Multistability and Tipping: From Mathematics and Physics to Climate and Brain. Max Planck Institute for the Physics of Complex Systems (mpipks). International Seminar & Workshop. September, 19th - October, 14th, 2016. Dresden, Germany.
- Poster: *Deterministic coherence in a ring of coupled chaotic oscillator*. Recent Advances in Bioinformatics and Neuroscience. UPM - BBVA Workshop. June, 9th-11th 2015. Madrid, Spain.

5.3.2 Registration of the intellectual property and Awards

- * *Diseño de software para apoyo académico y científico*. actúaupm: Economic award and diploma to the best business idea. April, 6th 2017. Technical University of Madrid (UPM), Spain.
- * *Simulador de la Dinámica Neuronal*. Registration of the intellectual property. December, 2016. Spain.(submitted).

5.3. ACHIEVEMENTS

Appendix A

Passive components

A.1 Fixed resistor

Roughly speaking, we could say that an ideal resistor is a component that obstructs the flow of current at all frequencies and which releases heat following Joule's law, see equation [4.27](#):

$$P = v * i \tag{A.1}$$

Where, P is rate of dissipation of heat in watts, v is voltage across the resistor in volts and i is current through the resistor in amperes [\[138\]](#). Moreover, the relationship between voltage and current through a resistor is very well known as Ohm's law. Taking into account the passive sign convention used in [Figure A.1](#) the equations describing the linear resistor element are as shown in equation [A.2](#).

$$v = R * i \tag{A.2}$$

Where, R is called resistance and has the unit in ohms, Ω . Consequently, the device described in [Figure A.1](#) is the linear model for a fixed resistor.

A.2. VARIABLE RESISTOR

Accordingly to $i - v$ characteristic of this device a straight line is a good approximation if we operate the device within its linear range. However, to model this curve across the full operating range would require at least a cubic equation [149].

Figure A.1: The resistor circuit symbol.

A.2 Variable resitor

Inside the electronic components field, there is another sort of resistors called variable resistor. Those are provided with a third contact which is able to change the amount of resistance between this third contact and either of the fixed contacts [138].

Figure A.2: The variable resistor circuit symbol.

Particularly, these variable resistors should not be used in frequency sensitive radio-frequency circuits, because they will introduce inductive and capaci-

A.3. CAPACITOR

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tance effects ¹.

A.3 Capacitor

A capacitor is a dynamic element involving of an electric field produced by a voltage. We can express the element in terms of voltage and current as follow:

$$i_C(t) = C \frac{dv_C(t)}{dt} \quad (\text{A.3})$$

Figure A.3: The capacitor circuit symbol showing current and voltage.

Assuming that, the reference marks for the current and voltage follow the passive sign convention ² shown in Figure A.3.

A.4 Inductor

The inductor is a dynamic circuit element involving the time variation of a magnetic field produced by a current. We can express the element in terms of voltage and current as follow:

$$v_L(t) = L \frac{di_L(t)}{dt} \quad (\text{A.4})$$

The time derivative means that the voltage across the inductor is zero unless the current is time varying. Under direct current excitation the current is constant and $v_L = 0$, so the inductor acts like a short circuit. The inductor is a dynamic element because only a changing current produces a nonzero voltage [149].

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¹ Henney, K., Walsh, C. (1957). *Electronic components handbook*. McGraw-Hill Book CO., INC., I

² *The analysis and design of linear circuits (7th ed.)*. Hoboken, N.J.: Wiley.

A.4. INDUCTOR

Figure A.4: The inductor circuit symbol showing current and voltage.

Appendix B

Active components

B.1 Operational Amplifier

The device itself is a complex array of passive and active components. Despite its complexity, the device can be modeled by rather simple $i - v$ characteristics.

Figure B.1: Operational Amplifier transfer characteristics.

The transfer characteristic is divided into three regions called +saturation, -saturation, and linear. In the linear region, the operational amplifier is a differential amplifier because the output is proportional to the difference between the two inputs. The slope of the line in the linear range is called

B.1. OPERATIONAL AMPLIFIER

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the voltage gain. In this linear region the input-output relation is as follow:

$$V_O(t) = A(V_P - V_N) \quad (\text{B.1})$$

Figure B.2: Operational Amplifier.

The voltage gain of the operational amplifier A is very large. As long as the net input ($v_P - v_N$) is very small, the output will be proportional to the input [149].

Appendix C

Building blocks with amplifiers

C.1 Voltage follower or buffer

As we see on Figure C.1 the feedback path is a direct connection from the output to the inverting input. It is used in interface circuits for isolating the source from the load.

Figure C.1: Voltage follower.

$$V_{out}(t) = V_{in} \quad (C.1)$$

C.2 Inverter configuration

The key feature of this configuration is that the input signal and the feedback are both applied to the inverting input. The non-inverting input is grounded.

C.3. ADDER CONFIGURATION

Figure C.2: Inverter configuration.

$$V_{out} = -\left(\frac{R_2}{R_1}\right) V_{in} \quad (C.2)$$

C.3 Adder configuration

The equations of a nonlinear system often include the sum of more than one term.

Figure C.3: Adder configuration.

$$V_{out} = \frac{R_f}{R_1} V_{in1} - \frac{R_f}{R_2} V_{in2} - \frac{R_f}{R_3} V_{in3} - \frac{R_f}{R_4} V_{in4} \quad (C.3)$$

The value of the resistor R_p is fixed in order to satisfy the following equation:

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_p} \quad (C.4)$$

C.4. DIFFERENTIAL AMPLIFIER

Then, the output of the circuit is given by the following equation:

$$V_{out} = \sum_i A_i V_i \quad (C.5)$$

with $A_i = R_f/R_i$. The output depends on each single input by means of only the associated input resistor and not of the other resistors.

C.4 Differential amplifier

This circuit has also two inputs, one applied at the inverting input and one at the non-inverting input of the operational amplifier. Using the superposition principle we can obtain the input-output relationship.

Figure C.4: Differential amplifier.

For the special case of $R_3/R_1 = R_4/R_2$ the output is proportional to the difference between the two inputs.

$$V_{out} = \frac{R_2}{R_1} (V_{in2} - V_{in1}) \quad (C.6)$$

C.5 RC integrator

Assuming that the node V_{out} is connected to high impedance, the current flowing in the resistor R_o can be considered equal to that in the capacitor.

C.5. RC INTEGRATOR

Figure C.5: RC integrator.

The current flowing through the resistor Ro is:

$$i = \frac{V - V_{out}}{Ro} \quad (C.7)$$

Taking into account the relationship between current and voltage across the capacitor:

$$i = C \frac{dV_{out}}{dt} \quad (C.8)$$

$$C \cdot Ro \cdot V_{out} = -V_{out} + AV_{in1} \quad (C.9)$$

If we properly choose the input V_{in} we implement a first-order generic differential equation of the type:

$$\dot{x} = -k(-x + f(x, t)) \quad (C.10)$$

It may be realized by taking $k = \frac{1}{CRo}$.

C.5.1 Miller integrator

This circuit allows an output corresponding to the integral of the input signal.

$$C \cdot R_1 \cdot \dot{V}_{out} = -V_{in1} \quad (C.11)$$

C.5. RC INTEGRATOR

Figure C.6: Miller integrator.

Appendix D

Software design

Figure D.1: Application to record data from the real experiment with the Chua electronic circuit. Labview Front Panel.

Figure D.2: Application to record data from the real experiment with the Chua electronic circuit. Labview Block Diagram.

Figure D.3: Application to record data from the real experiment with the Rössler electronic circuit. Labview Front Panel.

Figure D.4: Application to record data from the real experiment with the Rössler electronic circuit. Labview Block Diagram.

Figure D.5: Application to record data from the real experiment with the Hindmarsh-Rose electronic circuit. Labview Front Panel.

Figure D.6: Application to record data from the Hindmarsh-Rose electronic circuit. Labview Block Diagram.

Figure D.7: Flow chart for the application software seen in section 4.4.

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