Detrended fluctuation analysis for spatial characterisation of landscapes

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The interactions among abiotic, biotic, and anthropic factors and their influence at different scales create a complex dynamic in landscape evolution. Scaling and multifractal analysis have the potential to characterise landscapes in terms of the statistical signature of the selected measure, in this case, altitude. This work evaluates the multifractality of altitude data points along transects that are obtained in several directions using Detrended Fluctuation Analysis (DFA) in a protected area adjacent to Madrid. The study data set consist of a matrix 2048×2048 pixels obtained at a 5 m resolution and extracted from a digital terrain model (DTM) using a Geographic Information System (GIS). We found that the distribution of altitude fluctuations at small scales revealed a non-Gaussian character in the statistical moments, indicating that Fractional Brownian modelling is not appropriate. Generalised Hurst dimensions ($H(q)$) were calculated on several transects crossing the area under study, all of which exhibited multifractality within a certain scale range. The results show a persistent behaviour in all directions because all of the $H(q)$ values exceeded 0.5 and because there were differences in the intensities of the multifractality.

The analysis of the directionality by means of a generalised Hurst rose plot showed differences in the scaling characteristics both along and across rivers and reservoirs. This indicates a clear anisotropy that is mainly due to the directions of the two river basins located in the area and the basement movement as a consequence of gradual tectonic displacement, which must be considered in two-dimensional DFAs.

1. Introduction

Landscape is created and modified by human and natural processes. The type of rock and soil, shape of the land, amount

of rainfall, type of vegetation, river shape, size and flow, slope influence and drainage pattern are factors that may act individually or together to produce gradual changes in the dynamics of landscape morphology. Therefore, landscape

topography is the result of the overall complex interactions among them (Veneziano & Niemann, 2000). The study of these dynamics is very complex. Various scientific disciplines have contributed to its understanding (Gupta, Castro, & Over, 1996).

Digital elevation models (DEMs) provide the information basis in many geographic applications, for example, topographic and geomorphologic studies and landscape analyses employing geographic information systems (GIS). The information obtained from those models has been combined with powerful mathematical methods such as fractal geometry to study landscape dynamics (Aguado, del Monte, Moratí, & Tarquis, 2014; Cheng, Russell, Sharpe, Kenny, & Qin, 2001; Lovejoy & Schertzer, 2007; Lovejoy, Lavalée, Schertzer, & Ladoy, 1995). Topography has often been cited as an example of scaling processes in nature; when the topographic surface over a small region is properly magnified, it becomes indistinguishable from the topographic surface over a larger region (Mandelbrot, 1983; Mark & Aronson, 1984; Voss, 1985). Fractional Brownian motion (fBm), which has stationary first-order increments, has been used to model realistic topographic profiles (Mandelbrot & Van Ness, 1968). The interest in fBm is due to its ability to represent a wide class of non-stationary and statistically self-similar signals based on a few parameters, many of which are applied in image processing when modelling natural landscapes and textures (Jennane & Harba, 1994; Pentland, 1984; Zachevsky & Zeevi, 2014).

Based on Mandelbrot's work (Evertsz & Mandelbrot, 1992), the development of multifractal (MF) theory, which was introduced in the context of turbulence, has been applied in many areas, including earthquake distribution analysis (Hirata & Imoto, 1991), soil pore characterisation (Kravchenko, Boast, & Bullock, 1999; Tarquis, Gimenez, Saa, Diaz, & Gasco, 2003), local-level environmental applications (Roering, Kichner, & Dietrich, 1999), image analysis (Sanchez, Serna, Catalina, & Afonso, 1992) and remote sensing (Cheng & Agterberg, 1996; Turiel, Isern-Fontanet, Garcia-Ladona, & Font, 2005). MFs are scaling fields; fields at different scales are related only by a transformation that involves the scale ratio, and locally different scaling laws have been found (Pachepsky & Ritchie, 1998). Based on several parameters extracted from this MF analysis, multifractal transects or multifractal surfaces (two-dimensional) can be generated (Mandelbrot, 1974; Meneveau & Sreenivasan, 1987, 1991; Novikov, 1990).

There are several MF methods that can be used to characterise scaling properties, and several relations among them can be found in the literature (Moratí, Castellanos, Bird, & Tarquis, 2016). In the context of soils, the most popular method applied to soil transect data, including altitude, is the moment method developed by Halsey, Jensen, Kadanoff, Procaccia, and Shraiman (1986). This type of MF analysis can be directly applied on original data if the variable under study does not present any significant trend with distance. However, many authors do not check that condition, which can lead to inaccuracies (Tarquis et al., 2017). Detrended Fluctuation Analysis (DFA) is a MF method that includes the elimination of trends to properly analyse the scaling properties of local fluctuations.

DFA is commonly used to study long-term correlations in time/space series. This method is simply based on fluctuation

analysis (FA), which consists of the calculation of fluctuation functions $F(s)$ for different scales s . For long-term-correlated data, $F(s)$ behaves like a power law. In a typical fluctuation analysis, the differences between the ends of the profiles of the segments are calculated. The squares of those differences represent the squares of the fluctuations in the segments. The FA does not eliminate trends, which is also true of conventional spectral analysis (Govindan et al., 2002). DFA has been applied in several fields, including studies on DNA sequences (Yu, Anh, & Lau, 2004), meteorological data (Lin & Fu, 2008; Tarquis, Moratí, Castellanos, & Perdigones, 2008) and topographic data (Cao et al., 2017). In the last study, the authors applied MF-DFA on topographic data series extracted from shoulder lines in three areas on the Loess Plateau of China. Recently, the DFA algorithm has been extended to two dimensions (2D), assuming isotropy, for studying multifractality on 2D synthetic surfaces (Wang, Zou, 2014; Wang, Fan, & Stanley, 2016) and for classifying leaf textures (Wang, Liao, Li, & Liao, 2015).

Based on the studies described above, the present study uses Multifractal DFA (MF-DFA) to evaluate the multifractality of altitude data points along transects and for comparison with other works. The transects present several directions for studying the isotropic characteristics of the scaling properties to determine whether the DFA algorithm can be extended to 2D.

First, a statistical analysis of selected altitude transects and their increments was undertaken using several lags to study their stationarity. The MF-DFA technique was used to assess the scaling characteristics of the altitudinal transects for different directions using generalised Hurst dimensions and a Hurst Rose.

2. Materials and methods

2.1. Site description

The study area is known as “Monte de El Pardo”, which encloses 10537 ha. It is located a short distance from Madrid city at an altitude ranging from 908.2 to 595.3 m and with UTM zone 30N coordinates (Northern Hemisphere), X: 424303.456 to 434563.431 and Y: 4494559.721 to 4484299.529 (Fig. 1).

The location has its genesis as a continental detrital formation derived from the erosion of the granites of the Central System. This is clearly seen in the northwest of the area, where there is contact between the area of detritus and granite. According to the drainage network and slope map, two units that are clearly defined by the Manzanares river and correspond to each of its margins are distinguished in this zone (Monte del, 1982, p. 464). The area studied in this work corresponds to the right or western margins of the river. In this zone, two geomorphological units corresponding to two “watersheds” are distinguished. The small basin, which belongs to Trofa creek (a tributary of the Manzanares river) is located on the left side of the area (Fig. 1), and the larger basin is the Manzanares river. The watershed between the basins can be observed. The area of the Manzanares river basin has a long drainage network. The basin has a uniform SE–NW slope and a topography in which soft forms predominate without

large differences in height and with average slopes between 0 and 10%, although the peak heights in the study area are reached at its NW end. The Trofa basin stream has greater slopes and a drainage network lower than that of the Manzanares river.

In this area, different types of soils are found that reveal different degrees of evolution due to erosion. The main types are Entisols, Inceptisols and Alfisols (Table 1). However, a mix of those types is found in the area, as shown in Fig. 2 (Comunidad de Madrid, 2017).

The vegetation of this zone consists of evergreen oaks in different states (cleared forests-wooded pastures), and *Quercus rotundifolia* Lam. is the majority and dominant species. In 1973, a water reservoir with a capacity of 43 hm³ was constructed in the eastern part of the study zone (see Figs. 1 and 2), and some local streams were modified.

2.2. Digital Terrain Model (DTM)

ArcGis 10.2 (ESRI) was used to make the different rasters and shapefiles used in this work. A topographic map was constructed that joined four vectorial maps (0533-2, 0533-4, 0534-1 and 0534-3) at 1:25000 scales that were downloaded from the Spanish National Geographic Institute's (GNI, 2016) topographic collection (GNI-a, 2016).

The joined area was clipped using a polygon shapefile whose size and coordinates were the same as those of the abovementioned study area in order to make a digital terrain

model (DTM) of the study zone. Those DTMs were obtained by the interpolation of LIDAR data with resolutions of 5 m pixel⁻¹, 1 m height resolution and RMS error <2 m. The geodesic reference system used was ETRS89 in the UTM projection in zone 30. Both sheets were joined using the raster tools in ArcGis 10.2 (ESRI) to make a DTM file of 11602 × 7762 pixels (112.568 ha).

A DTM of 2048 × 2048 pixels that included only the study area was made using Arc Toolbox's "Extract by polygon". That DTM was converted to a point shapefile using the Arc Toolbox "Raster to point" tool. The attribute table of the shapefile contained the height values of all points in the file. The values were exported to a dbf file format. A Visual-Basic 6.0 (Microsoft) program was developed to create an ASC file.

The ASC file was exported to Excel 2010 (Microsoft). Twenty horizontal series each comprised of 2048 values were selected from the matrix, and the values of the first series contained those of the matrix from (1, 1) to (1, 2048). The remaining horizontal series were obtained by consecutively increasing the number of the first row by 100 pixels (500 m). Twenty vertical series were similarly selected. The first vertical series contained the matrix values from (1, 1) to (2048, 1). The remaining vertical series were obtained by consecutively increasing the number of the column by 100 pixels (500 m). The main diagonals of the matrix were also selected (Fig. 1). The series had different slopes. The diagonal series fulfilled the condition of going through the centre of the matrix.

Figure 1 – Digital Terrain Model of the 10537 Ha “Monte de El Pardo” study area at a spatial resolution of 5 m × 5 m. Altitude transects selected to analyse the scaling characteristics and detrended fluctuation analysis. Transects used in Figures 4–6 are labelled N–S, W–E, SE–NW and SW–NE.

Table 1 – Main soil types in the study area, their main characteristics and grade of evolution (Comunidad de Madrid, 2017).

Soil Type	Characteristics	Evolution
Entisols	Absence of marks in the soil of any major set of soil-forming processes. Dominance of mineral soil materials and absence of distinct paedogenic horizons.	0
Inceptisols	One of more paedogenic alteration or concentration horizons. Little accumulation of translocated materials other than carbonates or amorphous silica. Texture finer than loamy sand	+
Alfisols	More evolved soil. Mark of processes that translocate silicate clays (Argilic horizon)	++

2.3. Statistics on fluctuations

Transects with west to east (W–E), north to south (N–S), southwest to northeast (SW–NE) and southeast to northwest (SE–NW) directions, as marked in Fig. 1, were selected to calculate the first four statistical moments – average, variance, kurtosis and asymmetry (or skewness) – and study whether those statistical moments were close to the ones presented by a Gaussian distribution.

The same calculations were performed on each of the transects after differentiating the series at several non-overlapping lags from 4 until 128, equivalent to 20–640 m for the W–E and N–S transects and 28.8–904.96 m for the SW–NE and SE–NW transects. In this way, we could study the statistical moments of the frequency distribution of the values obtained in each lag.

When a differentiation with a lag of 4 was applied, the statistics were calculated on 512 values for each transect. For a differentiation with a lag of 128, 16 values were obtained. All of the selected transects presented 2048 altitude data points.

For all calculations, the XLStat-Pro software program (Addinsoft, 2008) was used.

2.4. Multifractal Detrended Fluctuation Analysis (MF-DFA)

The main feature of multifractals is that they are characterised by high variabilities over wide ranges of temporal or spatial scales that are associated with intermittent fluctuations and long-range power-law correlations. To undertake a multifractal analysis, Kantelhart et al. (2002) developed

Figure 2 – Map of soils located in the area study and the locations of urban soil and the water reservoir (Comunidad de Madrid, 2017).

'Multifractal Detrended Fluctuation Analysis' (MF-DFA). A brief description of the algorithm is provided in this section.

The DFA operates on $x(i)$, where $i = 1, 2, \dots, N$ and N is the length of the series. We represent the mean value with $\bar{x}$:

$$\bar{x} = \frac{1}{N} \sum_{k=1}^N x(k) \quad (1)$$

We assume that $x(i)$ are increments of a random walk process around the average $\bar{x}$, and the 'trajectory' or 'profile' is therefore given by the integration of the signal

$$y(i) = \sum_{k=1}^i [x(k) - \bar{x}] \quad (2)$$

Furthermore, the integration will reduce the level of measurement noise present in observational and finite records. Next, the integrated series is divided into $N_s = \text{int}(N/s)$ non-overlapping segments of equal lengths s . Because the length N of the series is often not a multiple of the considered timescale s , a short part at the end of the profile $y(i)$ may remain. To avoid disregarding that part of the series, the same procedure is repeated starting from the opposite end. Thereby, $2 N_s$ segments are obtained altogether. We then calculate the local trend for each of the $2 N_s$ segments by a least-squares fit of the series. We then determine the variance

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^s \{y[(v-1)s + i] - y_v(i)\}^2 \quad (3)$$

for each segment v , where $v = 1, \dots, N_s$ and

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^s \{y[N - (v - N_s)s + i] - y_v(i)\}^2 \quad (4)$$

for $v = N_s + 1, \dots, 2N_s$. Here, (i) is the fitting line in segment v . After detrending the series, we average over all segments to obtain the q th-order fluctuation function

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum [F^2(s, v)]^{\frac{q}{2}} \right\}^{\frac{2}{q}}, \quad (5)$$

where in general, the index variable q can take any real value except zero. In our case, the series lengths were multiples of s , and Eq. (4) was not applied.

Repeating the procedure described above for several timescales s , $F_q(s)$ will increase with an increasing s . By analysing the log-log plots of $F_q(s)$ versus s for each value of q , we can determine the scaling behaviour of the fluctuation functions. If the series x_i is long-range power-law correlated, $F_q(s)$ increases for large values of s as a power law:

$$F_q(s) \propto s^{H(q)} \quad (6)$$

$H(q)$ is the generalised Hurst exponent (or self-similarity scaling exponent) (Davis, Marshak, Wiscombe, & Cahalan, 1994). As mentioned above, monofractal series with compact support are characterised by $H(q)$ independent of q . The different scalings of small and large fluctuations will yield a significant dependence of $H(q)$ on q . The difference in scaling increases with increasing dependency.

Estimating the Hurst exponent ($H(2)$) from the given data is an alternate and effective way to determine the nature of the

correlations in it (Hurst, 1951). Hurst exponents have been successfully used to quantify long-range correlations in plasma turbulence (Yu, Peebles, & Rhodes, 2003; Gilmore, Yu, Rhodes, & Peebles, 2002), finance (Moody & Wu., 1995, pp. 26–30; Weron & Przybylowicz, 2000), network traffic (Erramilli, Roughan, Veitch, & Willinger, 2002), and physiology (Ivanov et al., 1999).

Calculation of $H(q)$ allows the straightforward identification of persistence, or long-time correlations, as well as the stationary/nonstationary and monofractal/multifractal nature of the data (Lovejoy, Schertzer, & Stanway, 2001). Stationary processes have scale-independent increments, and $H(q) \equiv 0$ due to invariance under translation. Processes with constant $H(q)$ are non-stationary and monofractal; otherwise, they are non-stationary and multifractal.

3. Results and discussion

3.1. Transect characteristics

From the visual observation of the four transects alone (see Fig. 3), different trends can be observed. The SE–NW transect shows a clear trend of increasing altitude in that direction, and a decreasing trend is seen in the W–E transect. However, the other two transects do not show clear trends.

The SE–NW transect began at the Manzanares river, crossed its drainage network in an almost parallel direction and was close to the drainage divide of both rivers. The predominant type of soil in the transect is Entisols (Fig. 2), which is the less evolved soil (Table 1). The W–E transect, which had a lower altitude average than did the SE–NW transect (Table 2), crossed the beginning of the Trofa river basin and then crossed the Manzanares river basin at its middle point, where a reservoir is located (Fig. 1). A mix of Alfisols/Entisols form the predominant soil type (Fig. 2). A brief observation of Fig. 2 will reveal that from this transect to south, the most developed soils predominate, unlike in the remainder of the study area.

The SW–NE transect crossed the Trofa river basin almost perpendicularly and then crossed the Manzanares river basin in a diagonal direction (Fig. 1). The transect presented a high variety of soil types (Fig. 2), the most predominant of which were Entisols and a mix of Alfisols/Entisols. The N–S transect, however, began at the end of the Trofa river basin at the location of a reservoir, crossed the drainage device and partially crossed the Manzanares' basin (Fig. 1). The type of soils in the transect were exclusively Entisols and a mix of Alfisols/Entisols (Fig. 2).

The first four statistical moments were calculated for the altitude ($x(i)$) of the four selected transects (Table 2). From the original values, the SE–NW transect had the highest variance, followed by the W–E transect, as expected after observing their values in Fig. 3, and were much lower in the SW–NE and N–S cases. Regarding the higher-order moments, the asymmetry and kurtosis values were closer to the values corresponding to a normal distribution, except for the W–E kurtosis (−1.110) and the SE–NW asymmetry (1.358). In the first three transects (W–E, N–S and SW–NE), the kurtosis was negative, whereas that for the SE–NW transect was positive,

Figure 3 – Altitude original data for several transects: SE–NW, transect in the southeast to northwest direction, SW–NE, transect in the southwest to northeast direction, N–S, transect in the north to south direction, and W–E, transect in the west to east direction. All the transects had 2048 data points, the N–S and W–E transects were equidistant by 5 m, and the other two transects were equidistant by 7.07 m.

which indicated a ‘peaked’ distribution. Studying the measured asymmetry, only the asymmetry for the SW–NE transect was negative and therefore slightly skewed left; the remaining three asymmetries had higher positive values, indicating that the distribution was skewed right. SE–NW asymmetry was the largest. The observation of these results indicates that the altitude distributions did not present null kurtosis values, as the Gaussian distribution did, and the N–S transect values were closest (–0.66). The asymmetries were quite close to zero, except for those for the SE–NW transect, which had a higher frequency at the higher altitudes.

3.2. Statistics of the altitude fluctuations

Focussing our attention on the differentiated values at different lags (Tables 3 and 4), we observed that lags larger than 128 lacked enough data points to provide a good estimation of the statistical moments of the frequency distributions; therefore, we concentrated on the lags from 4 to 128. In all transects, the average values were close to zero at small lags and varied at larger lags. From Table 3, we observe that from lag 4 to 16, the averages were almost zero and began to increase from lags 32 to 128. The W–E and N–S transects showed linear increases in the averages of the differences in altitudes when they were obtained for distances ranging from 80 m to 640 m. This finding indicates that at that range of scales, the trends were revealed and were positive for both

transects. However, the increase in average was higher in the W–E transect, as we could more readily visually perceive that trend in Fig. 3, than in the N–S transect.

As listed in Table 4, the value for the SW–NE transect decreased from lags 32 to 128, and the sign was negative. This implies that a decreasing trend appeared from 226 m to 905 m. The SE–NW transect also presented averages with negative values that constantly varied from lags 8 to 128. That variation was higher than that for the SW–NE transect, and both showed linear relations, as observed in the other two transects. In other words, as the lag increased from 4 to 128, the average values for the SW–NE and SE–NW transects tended to decrease with different intensities, whereas those for the W–E and N–S transects increased, indicating a positive trend.

The variances for the four transects tended to increase over all of the lag ranges used in this study. In this case, the relations between the variance and distance or lag were nonlinear.

The N–S and SW–NE transects presented higher kurtosis values than those of the Gaussian distribution and decreased as the lag increased, becoming negative at lags 64 and 128. For the W–E transect, the behaviour was similar, but the kurtosis was positive at all lags. Finally, the SE–NW transect presented kurtosis values lower than those of the other three transects, there were no clear tendencies with lag, and the values were very close to those of a Gaussian distribution, except for lags 32 and 128.

All of the transects and lags from 4 to 128 showed asymmetry values that were negative and close to zero, with the exception of the SE–NW transect. In the last case, the values from lags 4 to 16 were positive, and from lag 128, they were negative and had magnitudes greater than 1.

Observing the combinations of kurtosis and asymmetry at each lag for the four transects, there was always a lag (or distance) at which the obtained values were very close to those obtained from a Gaussian distribution. The W–E transect’s altitude differences at lag 128, or a distance of 640 m (Table 3), showed third and fourth statistical moment values

Table 2 – Descriptive statistics using the first four moments (average, variance, kurtosis and asymmetry) of the data series corresponding to the four transects with directions W–E, N–S, SW–NE and SE–NW.

Data statistics	W–E	N–S	SW–NE	SE–NW
Average	683.673	696.135	689	702
Variance	1454.420	1060.697	1177.736	5327.278
Kurtosis	–1.110	–0.662	–0.962	0.937
Asymmetry	0.020	0.212	–0.112	1.358

Table 3 – Descriptive statistics using the first four moments (average, variance, kurtosis and asymmetry) of the differences in the values of the data series corresponding to the two transects with directions W–E and N–S at different lags, with their distance equivalents.

Statistics		W–E					
Lag	4	8	16	32	64	128	
Average	0.191	0.381	0.762	1.524	3.049	6.098	
Variance	1.507	4.724	15.105	48.327	124.879	434.447	
Kurtosis	5.711	6.302	7.866	3.420	0.576	0.182	
Asymmetry	−0.092	−0.196	0.128	−0.108	−0.141	−0.145	
Data points	512	256	128	64	32	16	
Statistics		N–S					
Lag	4	8	16	32	64	128	
Average	0.122	0.243	0.486	0.973	1.946	3.891	
Variance	3.185	9.376	26.892	67.672	199.021	628.816	
Kurtosis	4.985	2.151	0.744	0.002	−0.336	−0.089	
Asymmetry	−0.826	−0.547	−0.485	−0.315	−0.471	−0.487	
Data points	512	256	128	64	32	16	

close to those of a Gaussian distribution. This occurred at lag 32 (160 m) for the N–S transect, lag 32 (226 m) for the SW–NE transect and lag 8 (56.6 m) for the SE–NW transect. Therefore, if the altitude measurements were estimated at that resolution in each transect, the altitude increments would be represented by a Gaussian distribution; otherwise, simple fBm modelling would be chosen.

By obtaining the measurements using this technology at higher resolutions, the probability distributions of transects' altitude increments were revealed to be quite symmetrical and to have heavy tails that described a non-Gaussian probability distribution, and a more complex scaling model is needed. Other authors have pointed this out in several contexts (Guadagnini, Neuman, Schaap, & Riva, 2014; Neuman, Guadagnini, Riva, & Siena, 2013).

3.3. Generalised Hurst exponents

The MF-DFA was applied to all twelve transects indicated in Fig. 1, but in this section, the four that are marked W–E, N–S, SW–NE and SE–NW are discussed in detail as representative of all calculations. We would like to remark that in the DFA method, the trend was removed at each scale in the study,

which yielded different information from that provided by a straight fluctuation analysis.

The first step in the multifractal analysis is to determine whether there is a linear relationship between the double-log plots of $F(q,s)$ versus s (see Fig. 4). This was found to be the case from lags 4 to 128, which corresponded to distances of 20–640 m for transects W–E and N–S (Table 3) and to distances of 28.8–904.96 m for transects SW–NE and SE–NW (Table 4). The coefficients of determination for a linear fit in all cases were between 0.95 and 1.00. Such relationships indicate the presence of scaling laws (Hu, Ivanov, Chen, Carpena, & Stanley, 2001).

The result of the MF-DFA procedure is the family of the generalised Hurst exponents $H(q)$ (Fig. 5). For an actual multifractal signal, $H(q)$ is a decreasing function of q , whereas for a monofractal signal, $H(q)$ is a constant value. It can be seen from Fig. 5 that the $H(q)$ vs q curves, when performing the calculations from $q = 0.5$ to 5, indicate a dependence of $H(q)$ on q , which suggests that the altitude profiles are characterised by multifractality. Furthermore, the four transects are characterised by long-term persistence because the values of $H(2)$ are equal to or greater than 0.8 (Feder, 1988). Similar values were obtained by Cao et al. (2017). The above results indicate

Table 4 – Descriptive statistics using the first four moments (average, variance, kurtosis and asymmetry) of the differences in the values of the data series corresponding to the two transects with directions SW–NE and SE–NW at different lags, with their equivalent distance.

Statistics		SW–NE					
Lag	4	8	16	32	64	128	
Average	−0.080	−0.160	−0.320	−0.641	−1.281	−2.563	
Variance	6.508	18.794	54.865	155.313	332.983	949.996	
Kurtosis	4.608	2.431	3.394	0.493	−0.972	−1.267	
Asymmetry	−0.918	−0.812	−1.150	−0.639	−0.154	−0.422	
Data points	512	256	128	64	32	16	
Statistics		SE–NW					
Lag	4	8	16	32	64	128	
Average	−0.533	−1.066	−2.133	−4.266	−8.531	−17.063	
Variance	7.744	22.894	69.407	197.722	238.322	556.863	
Kurtosis	1.758	0.982	1.228	2.406	0.482	3.763	
Asymmetry	0.363	0.320	0.066	−0.431	−0.397	−1.575	
Data points	512	256	128	64	32	16

Figure 4 – Detrended Fluctuation (F_q) for $q = 1, 2, 3, 4$ and 5 for different transects: a) W–E, b) N–S, c) SW–NE and d) SE–NW. The scales ranged from lags of 4–128.

Figure 5 – $H(q)$ curves obtained by Multifractal Detrended Fluctuation Analysis (MFDFA) for the a) W–E, b) N–S, c) SW–NE and d) SE–NW transects.

Figure 6 – Rose plot of the directional generalised Hurst dimension values ($H(q)$) for $q = 0.5, 1, 2, 3, 4$ and 5 . The values of the radius axis range from 0.5 to 1.1 .

that the altitude series of the four transects in the study area are non-stationary multifractal altitude profiles.

Comparing the decrease in $H(q)$ with q , there are some differences among the four transects. For the N–S transect (Fig. 5b), $H(q)$ varied from 0.95 (for $q = 0.5$) to 0.59 (for $q = 5$), yielding a difference in the curve extremes of 0.36 , whereas for the SW–NE transect (Fig. 5c), $H(q)$ presented a difference of 0.26 (from 0.97 for $q = 0.5$ to 0.71 for $q = 5$). The other two transects (W–E and SE–NW) presented values between those. Therefore, among the four transects, the strength of the multifractal character varied.

3.4. Directional generalised Hurst exponents

In addition to the issues discussed above, a fractal/multifractal surface may present different types of behaviour. For example, for only the four transects discussed above, the calculated $H(q)$ exhibited values that varied with direction, thus clearly indicating anisotropy. The study of oriented topography through generalised Hurst exponents has revealed that relief features change significantly with direction for a variety of reasons. In many cases, the most common cause of anisotropy was some directionality in the processes that produced or modified the landscape. In this area, anisotropy is clearly related to the directions of both river basins, which can be appreciated from Fig. 1; further explanations are provided in this section.

Figure 6 shows a rose plot of generalised Hurst exponents, $H(q)$, which were calculated for each of the transects drawn in Fig. 1. Once MF-DFA was applied, the localised trends were removed, but the $H(q)$ values obtained still show oriented roughness. The directional $H(q)$ analysis revealed that transect W–E had the highest values. The smoothness of the roughness in that transect, once that the trend was removed, is explained by the gradual movement of the basement as a

consequence of tectonic movements over centuries (Cadavid & Hernández, 1967), which produced a gradual change in the direction of the Trofa river until the river reached its current position. That movement favoured an erosive process for the Trofa river, which created a different drainage morphology and network than that developed by the Manzanares river and positioned in another direction.

The features presented by $H(q)$ in the SW–NE and SE–NW transect point out the erosion processes both river basins were undergoing, and both transects presented similar values, although they differed from those of the W–E transect. However, between them (the 135° – 45° transects, including the N–S transect), lower and similar $H(q)$ values can be observed in the Hurst rose (Fig. 6). It is in this section where we can contemplate a closer isotropic behaviour. However, all of the studied transects presented a strong persistence character or positive long memory because all of the $H(q)$ values exceeded 0.5 (Morató et al., 2016).

The multifractal strengths in all the studied directions, measured as the difference in the extreme values of the $H(q)$ function, were higher in the N–S transect. They then decreased gradually as the direction turned to the SW–NE, presented a minimum, and increased again at the W–E transect (perpendicular to the river basins). Continuing clockwise, the multifractality strength diminished until the SE–NW transect was reached. Hence, the strength of the multifractality also showed anisotropy.

4. Conclusions

The purpose of this manuscript is to provide an evaluation of the multifractality of topography data along transects obtained along several directions in the region known as “Monte El Pardo”, which is adjacent to Madrid City (Spain).

First, the study of the statistical moments of the four selected transect altitude increments (N–S, W–E, SW–NE and SE–NW) were close to those of a Gaussian distribution for most lags, except for higher resolutions (small lags), where they revealed a different probability distribution with high symmetry and heavy tails, making fBm modelling a poor choice. This indicated the importance of obtaining high-resolution topography data, which would allow us to obtain an accurate description of the statistical distributions of the altitude fluctuations.

The DFA results revealed a multiscaling property (multifractal type) over several scales for all the altitude series. The degree of multifractality changed with direction. The highest scaling heterogeneity (multifractality) was observed for transect N–S. That scaling heterogeneity gradually decreased, turned to the SW–NE, and increased again at the W–E transect, perpendicular to the two river basins.

The analysis of the directionality using a generalised Hurst rose plot showed differences in scaling characteristics with direction that revealed non-isotropy and that still remained after removing the local trends in the analysis. This was consistent with the directions of the two river basins and the gradual change in the direction of the Trofa river towards the Manzanares river over the course of centuries, until reaching its current position.

To perform two-dimensional space detrending fluctuation analyses, the anisotropies in $H(q)$ values, that expressed different multifractal strengths, should be considered when designing the algorithm rather than simply extending it in two dimensions.

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REFERENCES

- Addinsoft. (2008). *XLSTAT-Pro Version 2008. Statistical software for MS Excel*. <http://www.xlstat.com/>.
- Aguado, P. L., del Monte, J. P., Moratitel, R., & Tarquis, A. M. (2014). Spatial characterization of landscapes through multifractal analysis of DEM. *Scientific World Journal*, 9. <https://doi.org/10.1155/2014/563038>, 563038.
- Cadavid, S., & Hernández, M. E. (1967). Estudio Megnetométrico del basamento de la hoja 583. *Arganda. Estudios Geológicos*, 23, 263–275.
- Cao, J., Na, J., Li, J., Tang, G., Fang, X., & Xiong, L. (2017). Topographic spatial variation analysis of loess shoulder lines in the loess plateau of China based on MF-DFA. *International Journal of Geo-information*, 6(5), 141. <https://doi.org/10.3390/ijgi6050141>.
- Cheng, Q., & Agterberg, F. P. (1996). Multifractal modelling and spatial statistics. *Mathematical Geology*, 28, 1–16.
- Cheng, Q., Russell, H., Sharpe, D., Kenny, F., & Qin, P. (2001). GIS based statistical and fractal/multifractal analysis of surface stream patterns in the Oak Ridges Moraine. *Computers and Geosciences*, 27, 513–526.
- Comunidad de Madrid. (2017). *WEB page of environmental cartography in the Madrid region*. http://www.madrid.org/cartografia_ambiental/html. (Accessed 14 March 2017).
- Davis, A., Marshak, A., Wiscombe, W., & Cahalan, R. (1994). Multifractal characterizations of nonstationary and intermittency in geophysical fields: Observed, retrieved, or simulated. *Journal of Geophysical Research*, 99, 8055–8072.
- Erramilli, A., Roughan, M., Veitch, D., & Willinger, W. (2002). Self-similar traffic and network dynamics. *Proceedings of the IEEE*, 90, 800–819.
- Evertsz, C. J. G., & Mandelbrot, B. B. (1992). Multifractal measures. Appendix B. In H. O. Peitgen, H. Jurgens, & D. Saupe (Eds.), *Chaos and Fractals: New frontiers of science*. New York, NY, USA: Springer.
- Feder, J. (1988). Random walks and fractals. In *Fractals*. New York: Plenum Press.
- Gilmore, M., Yu, C. X., Rhodes, T. L., & Peebles, W. A. (2002). Investigation of rescaled range analysis, the Hurst exponent, and long time correlations in plasma turbulence. *Physics of Plasmas*, 9, 1312.
- GNI. (2016). *Centro de Descargas GNI*. Available: <http://centrodedescargas.cnig.es/CentroDescargas/>.
- GNI-a. (2016). *WEB page of the national geographic Institute. Topographic map collection, MTN25 Vectorial format*. <http://www.centrodedescargas.cnig.es/CentroDescargas/catalogo>. (Accessed 6 June 2016).
- Govindan, R. B., Vyushin, D., Bunde, A., Brenner, S., Havlin, S., & Schellnhuber, H. J. (2002). Global climate models violate scaling of the observed atmospheric variability. *Physical Review Letters*, 89(2), 028501-1–028501–028504.
- Guadagnini, A., Neuman, S. P., Schaap, M. G., & Riva, M. (2014). Anisotropic statistical scaling of soil and sediment texture in a stratified deep vadose zone near Maricopa, Arizona. *Geoderma*, 214–215, 217–227.
- Gupta, V. K., Castro, S. L., & Over, T. M. (1996). On scaling exponents of spatial peak flows from rainfall and river network geometry. *Journal of Hydrology*, 187(1), 81–104.
- Halsey, T. C., Jensen, M. H., Kadanoff, L. P., Procaccia, I., & Shraiman, B. I. (1986). Fractal measures and their singularities: The characterization of strange sets. *Physical Review A*, 33, 1141–1151.
- Hirata, T., & Imoto, M. (1991). Multifractal analysis of spatial distribution of microearthquakes in the Kanto region. *Geophysical Journal International*, 107, 155–162.
- Hu, K., Ivanov, P. C., Chen, Z., Carpena, P., & Stanley, H. E. (2001). Effect of trends on detrended fluctuation analysis. *Physical Review E*, 64, 011114.
- Hurst, H. E. (1951). Long-term storage capacity of reservoirs. *Proceedings of American Society of Civil Engineering*, 116, 770–808.
- Ivanov, P. C., Amaral, L., Goldberger, A., Havlin, S., Rosenblum, M. G., Struzik, Z. R., et al. (1999). Multifractality in human heartbeat dynamics. *Nature*, 399, 461–465.
- Jennane, R., & Harba, R. (1994). Fractional brownian motion: A model for image texture. *EUSIPCO Signal Processing*, 3, 1389–1392 (Sept).
- Kantelhart, J. W., Zschiegner, S. A., Koscielny-Bunde, K., Havlin, S., Bunde, A., & Stanley, E. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. *Physica A*, 316, 87–114.
- Kravchenko, A. N., Boast, C. W., & Bullock, D. G. (1999). Multifractal analysis of soil spatial variability. *Agronomy Journal*, 91, 1033–1041.
- Lin, G. X., & Fu, Z. T. (2008). A universal model to characterize different multifractal behaviours of daily temperature records over China. *Physica A*, 387, 573–579.
- Lovejoy, S., Lavalée, D., Schertzer, D., & Ladoy, P. (1995). The $l^{1/2}$ law and multifractal topography: Theory and analysis.

- Nonlinear processes in geophysics. *European Geosciences Union (EGU)*, 2(1), 16–22.
- Lovejoy, S., & Schertzer, D. (2007). Scaling and multifractal fields in the solid earth and topography. *Nonlinear Processes in Geophysics*, 14, 465–502. <https://doi.org/10.5194/npg-14-465-2007>. <http://www.nonlin-processes-geophys.net/14/465/2007/>.
- Lovejoy, S., Schertzer, D., & Stanway, J. D. (2001). Fractal behavior of ozone, wind and temperature in the lower stratosphere. *Physical Review Letters*, 86, 5200–5203.
- Mandelbrot, B. B. (1974). Intermittent turbulence in self-similar cascades: Divergence of high moments and dimension of the carrier. *Journal of Fluid Mechanics*, 62, 331–358.
- Mandelbrot, B. B. (1983). *The fractal geometry of nature*. San Francisco, California: Freeman.
- Mandelbrot, B. B., & Van Ness, J. W. (1968). Fractional Brownian motion, fractional noises and applications. *SIAM Review*, 10(4), 422–438.
- Mark, D. M., & Aronson, P. B. (1984). Scale-dependent fractal dimensions of topographic surfaces: An empirical investigation, with applications in geomorphology and computer mapping. *Journal of the International Association for Mathematical Geology*, 16, 671–683.
- Meneveau, C., & Sreenivasan, K. (1987). Simple multifractal cascade model for fully developed turbulence. *Physical Review Letters*, 59, 1424.
- Meneveau, C., & Sreenivasan, K. (1991). The multifractal nature of turbulent energy dissipation. *Journal of Fluid Mechanics*, 224, 429.
- Monte del, J. P. (1982). *Estudio de los diferentes ecotopos y fitocenosis del bosque mediterráneo en el Monte de El Pardo*. Universidad Politécnica de Madrid. PhD thesis, in Spanish.
- Moody, J., & Wu, L. (1995). Price behavior and Hurst exponents of tick-by-tick interbank foreign exchange rates. *Proceedings of computational intelligence in financial engineering*. Piscataway, NJ: IEEE Press.
- Morató, M. C., Castellanos, M. T., Bird, N. R., & Tarquis, A. M. (2016). Multifractal analysis in soil properties: Spatial signal versus mass distribution. *Geoderma*. <https://doi.org/10.1016/j.geoderma.2016.08.004>.
- Neuman, S. P., Guadagnini, A., Riva, M., & Siena, M. (2013). Recent advances in statistical and scaling analysis of earth and environmental variables. In P. K. Mishra, & K. L. Kuhlman (Eds.), *Advances in hydrogeology* (pp. 1–15). New York: Springer.
- Novikov, E. A. (1990). The effects of intermittency on statistical characteristics of turbulence and scale similarity of breakdown coefficients. *Physics of Fluids A*, 2, 814. <https://doi.org/10.1063/1.857629>.
- Pachepsky, Y. A., & Ritchie, J. C. (1998). Seasonal changes in fractal landscape surface roughness estimated from airborne laser altimetry data. *International Journal of Remote Sensing*, 19(13), 2509–2516.
- Pentland, A. P. (1984). Fractal-based description of natural scene. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 6(6), 661–674.
- Roering, J. J., Kichner, J. W., & Dietrich, W. E. (1999). Evidence for nonlinear, diffusive sediment transport on hillslopes and implications for landscape morphology. *Water Resources Research*, 35, 853–870.
- Sanchez, A., Serna, R., Catalina, F., & Afonso, C. N. (1992). Multifractal patterns formed by laser irradiation in GeAl thin multilayer films. *Physical Review B*, 46, 487–490.
- Tarquis, A. M., Castellanos, M. T., Cartagena, M. C., Arce, A., Ribas, F., Cabello, M. J., et al. (2017). Scale and space dependencies of soil nitrogen variability. *Nonlinear Processes in Geophysics*, 24, 77–87. <https://doi.org/10.5194/npg-24-77-2017>.
- Tarquis, A. M., Gimenez, D., Saa, A., Diaz, M. C., & Gasco, J. M. (2003). Scaling and multiscaling of soil pore systems determined by image analysis. In Y. Pachepsky, D. E. Radcliffe, & H. Magdi Selim (Eds.), *Scaling methods in soil physics*. CRC Press.
- Tarquis, A. M., Morató, M. C., Castellanos, M. T., & Perdigones, A. (2008). Comparison of structure function and detrended fluctuation analysis in wind time series. *Nuovo Cimento C*, 31(5–6), 633–651. <https://doi.org/10.1393/ncc/i2009-10331-x>.
- Turiel, A., Isern-Fontanet, J., Garcia-Ladona, E., & Font, J. (2005). Multifractal method for the instantaneous evaluation of the stream function in geophysical flows. *Physical Review Letters*, 95, 104502.
- Veneziano, D., & Niemann, J. D. (2000). Self-similarity and multifractality of fluvial erosion topography. *Water Resources Research*, 36(7), 1937–1951.
- Voss, R. F. (1985). Random fractal forgeries. In R. A. Earnshaw (Ed.), *Fundamental algorithms for computer graphics* (pp. 805–836). New York: Springer-Verlag.
- Wang, F., Fan, Q., & Stanley, H. E. (2016). Multiscale multifractal detrended-fluctuation analysis of two-dimensional surfaces. *Physical Review E*, 93, 042213. <https://doi.org/10.1103/PhysRevE.93.042213>. Epub 2016 Apr 21.
- Wang, F., Liao, D., Li, J., & Liao, G. (2015). Two-dimensional multifractal detrended fluctuation analysis for plant identification. *Plant Methods*, 11, 12. <https://doi.org/10.1186/s13007-015-0049-7>.
- Wang, F., Wang, L., & Zou, R. B. (2014). Multifractal detrended moving average analysis for texture representation. *Chaos*, 24(3), 033127. <https://doi.org/10.1063/1.4894763>.
- Weron, R., & Przybyłowicz, B. (2000). Hurst analysis of electricity price dynamics. *Physica A*, 283, 462–468.
- Yu, Z. G., Anh, V., & Lau, K. S. (2004). Fractal analysis of measure representation of large proteins based on the detailed HP model. *Physica A*, 337, 171–184.
- Yu, C. X., Peebles, W. A., & Rhodes, T. L. (2003). Structure function analysis of long-range correlations. *Plasma Turbulence*, 10(2772).
- Zachevsky, I., & Zeevi, Y. Y. (2014). Single-image superresolution of natural stochastic textures based on fractional Brownian motion. *IEEE Transactions on Image Processing*, 23(5), 2096–2108. <https://doi.org/10.1109/TIP.2014.2312284>.