Rotational Instability of a Long Liquid Column

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Abstract

A liquid column is, apart from its intrinsic interest from the basic science point of view, a good mechanical model of a crystal growth process known as the floating zone technique. In this technique, rotation of the supporting rods is used to uniformize the usually non-axisymmetric temperature field otherwise produced by the directional heating.

The theory of the influence of solid rotation on the stability limit is already available and early experimental results showed the existence of two different kinds of unstable shapes (in absence of other perturbations): the amphora mode and the C-mode.

The existence of the amphora mode was realized in SL-D1 experiments but C-mode breakages could not be obtained, probably due to unexpected existence of body forces that always excited the amphora mode breakage.

As none of these modes can be realistically simulated on Earth, an experiment in microgravity conditions (TEXUS-23) was performed in order to obtain C-mode deformations that achieved up to now in a reproducible way.

However, experiments on Earth using the Plateau Tank Technique have been performed in order to obtain more insight in the problem and to prepare experiments aboard sounding rockets.

Results of the experiment aboard TEXUS-23 show a reasonably good agreement with the theoretical predictions.

1. Introduction

In the last years a large effort has been devoted to the study, both theoretical and experimental, of the behaviour of liquid bridges, due to the relevance of this configuration to a crystal growth technique known as the floating zone technique. A review of the early work in this field can be found in [1].

Rotation, either of one of the supports or of both, and in the latter case of both disks in counterrotation or isorotation is normally implemented in this technique to achieve a uniform temperature field. In the case of isorotation, the main dimensionless parameters appearing in the problem are the Weber number \( W = \rho \Omega^2 R^3 / \sigma \) (where \( \rho \) is the liquid density, \( \Omega \) is the rotation rate, \( R \) is the radius of the disks and \( \sigma \) is the surface tension) and the liquid bridge slenderness \( \Lambda = L / (2R) \) (where \( L \) is the separation of the disks). The effect of isorotation around the common disk axis in the stability limit of a liquid bridge of cylindrical volume \( V_c = \pi R^2 L \) has been studied in [2] and the main results are summarized in Fig. 1. It can be shown that for not very slender ridges \( \Lambda < \Lambda_b = \pi \sqrt{3} / 2 \) and relatively high spin rates, an unstable asymmetric mode (C-mode) appears, which leads to the breakage of the bridge as was experimentally found for the first time in Skylab IV [3]. For larger values of the slenderness, the axisymmetric mode (amphora mode) would occur. The observation of a controlled C-mode breakage was one of the goals of the Spacelab missions SL-1 and SL-D1 [4,5].

In the SL-1 mission the C-mode could only be excited on non-cylindrical liquid bridges of odd shape, as a consequence of some problems while establishing the liquid bridge. In the SL-D1 mission four test points close to point B (Fig. 1) were planned in order to observe the transition between the amphora mode and the C-mode.
Figure 1. Stability diagram for zero eccentricity. Curve AB represents loss of stability with non-axisymmetric breakage and curve BC with axisymmetric breakage. $W$ is the Weber number and $A$ is the liquid bridge slenderness.
breakages. In the two cases in which A > A, the amphora mode was observed but due to an unexpectedly strong residual acceleration, the amphora mode was excited also for the case A < A and a breakage appeared preventing the observation of the C-mode deformation. A further trial in which A should have been sufficiently smaller than A had also been planned but could not be carried out due to lack of time.

The existence of such an important residual acceleration is under discussion. The deformation of the liquid bridge indicates the presence of a force field and the most obvious explanation seemed to be the aerodynamic drag. However, such large values of residual acceleration are not consistent with the shuttle orbit and other effects should be investigated. In the meantime this point remains obscure.

Be it as it may, the theoretical study performed [2] showed that an external force field can modify the bifurcation leading to breakage. The diagram in Fig. 1 corresponds to the case of perfect bifurcation in which a stable equilibrium shape (the cylinder) suddenly changes its character becoming unstable at the points indicated by lines AB and BC. This sudden change would give rise to problems both in experimental observation (quick unexpected motion) and also in the significant difference from expected results that might be produced by other external uncontrolled perturbations (as in SL-D1). Thus, if a controlled perturbation is introduced in the system to produce a deformation of the same character (either symmetric or asymmetric with respect to the mid plane) as the equilibrium shape, otherwise always unstable, then this mode is excited in a stable form (imperfect bifurcation). The change of stability can thus be controlled more easily in experiments. Indeed, a stable deformation can be monitored and increased up to the breaking point.

In the same way as axial microgravity can stably excite amphora mode deformation, it is possible to stably excite an otherwise unstable C-mode by shifting the axis of the disks and the rotation axis. This suggests a way to achieve a well controlled excitation of this mode.

In the following, a short description of the theoretical model is shown, a summary of experiments on Earth follows and then experimental results in microgravity are presented and compared with the theoretical model.

2. Theoretical Model

The configuration considered is the one sketched in Fig. 2a: a liquid column is held by surface tension forces between two circular disks of radius R placed a distance of L apart. Both disks are parallel and coaxial. The volume of the bridge is that corresponding to a cylindrical shape \((V = \pi L^2 R^2)\). The liquid and the disks are solidly rotating at an angular speed \(\Omega\) around an axis which is parallel to the axis of the disks, and is placed a small distance \(E\) apart from this line.

The theory governing the behaviour of the rotating liquid column has been developed to find that the radial deformation \(H\), of an initially cylindrical zone near its stability limit can be deduced [6] in first approximation from:

\[
W = \left( \frac{\pi}{2A} \right)^2 + C \left( H - \frac{E}{R} \right)^2 - \frac{\pi E}{A} H
\]

where \(E\) is the distance between the rotation axis and the centre of the disks, \(H\) is the lateral displacement from the undisturbed shape (at the middle of the bridge) and \(C\) is a function of \(A\). If linear deformations are assumed, the following expression applies:

\[
\frac{H}{E} = \frac{1 - \cos \sqrt{W} A}{\cos \sqrt{W} A}
\]
Figure 2. Geometry of the liquid bridge off-axis rotation problem. (a) Solid body rotation. Side view. (b) Top view of a liquid bridge section and disk position in a general case. O, rotation centre. O', disk centre. O", liquid bridge section centre. E, position of the disk centre. H, lateral displacement of the liquid bridge section from the disk centre. d, lateral displacement of the liquid bridge section centre from the rotation centre. FV: Front view direction; SV: Side view direction.
which, if small rotation rates are assumed can be approximated by

\[
\frac{H}{E} = \frac{W A^2}{2}
\]

For the geometry of the experiment reported here \((A=2.5)\) the value of \(C\) can be easily computed \([2,6]\) the result being

\[
C = -0.17.
\]

The predicted response of the liquid bridge for the case \(A=2.5, E=0.001\ m\) and \(R=0.015\ m\) has been plotted in Fig. 3.

3. Ground Tests

In order to prepare for the analysis of the microgravity experiments, ground tests were performed producing a liquid bridge with an outer shape similar to the expected one, although not in solid-body rotation. The Plateau Tank Facility (PTF) \([7]\) was used for this purpose. The two supporting disks, 30 mm in diameter, rotate around an axis shifted 2 mm from their centres. Due to both centrifuge acceleration and hydrodynamic effects, a C-mode shape appeared. In order to achieve different amplitudes, slightly different rotation rates were imposed to both disks. Fig. 4 shows the outer shape in two different positions.

The image is recorded with a CCD-camera and digitized. The digitized image can be enhanced and the outer shape and mean line automatically found (Fig. 5a). It is possible also to follow the temporal evolution of the position of the disks and of several sections of the interface in real time.

Fig. 6 shows the temporal evolution of the disks' position and of the bridge section shown in Fig. 5b. The position of this section vs. the position of the disks has been plotted in Fig. 7 to show the phase shift.

Microgravity conditions cannot be suitably simulated by using the Plateau technique. If the bath is not rotating, viscous force effects are very large, and if solid rotation is imposed on both the bath and the bridge, some density difference should exist in order to obtain centrifugal accelerations: this density mismatch will mix gravity effects with rotation effects.

4. Flight Experiment

The module T.E.M. 06-9, already flown in TEXUS-10, TEXUS-12 and TEXUS-18, was used with a new mechanism added which allowed to spin both end disks synchronously. Silicone oil AK20 was selected as a working fluid with \(\rho = 930\ \text{kg.m}^{-3}, \nu = 20 \times 10^{-6}\ \text{m}^2/\text{s}\) and \(\sigma = 0.02\ \text{N/m}\). The geometric characteristics are \(L = 0.075\ m, R = 0.015\ m\) and \(E = 0.001\ m\). The diagnosis of the experiment is based on image recording as in the ground tests. A high-resolution CCD-camera was used to achieve the required spatial resolution. In the image, two mutually perpendicular side views of the column and the top view were simultaneously recorded with the help of mirrors. The image also includes, for synchronization purposes, a clock showing the time from lift-off, and the readings of selected and actual rotation speed. The experiment procedure was as follows: First, a cylindrical liquid column 75 mm long was established after the beginning of the microgravity period. The disk separation rate was initially 2 mm/s until a length of 65 mm was reached, then it was decreased to 1 mm/s, and when a length of 70 mm was reached the rate was dropped to 0.5 mm/s. This procedure lasted 47.5 seconds in total. The column was left quiet for 30 s
Figure 3. Bifurcation diagram. $H$ is the lateral displacement of the mid section, $E$ is the eccentricity of the disks, $R$ is the radius of the disks and $W$ is the Weber number. The symmetric bifurcation ($E=0$, dotted line), and the stable (solid line) and unstable (dashed line) equilibrium shape deformations for the imperfect bifurcation ($E=1$ mm, $R=15$ mm) are shown. The solid line passing through the origin is the linear approximation.

Figure 4. Ground tests. Two different instants of the cycle.
Figure 5. Ground tests. (a) Treated image showing computed outer shape and mean line. (b) Section whose lateral displacement is measured.
Figure 6. Ground tests. Lateral displacement of the section shown in Figure 5b (upper curve) and of both disks (lower curves), as a function of time.

Figure 7. Ground tests. Lateral displacement of the section considered as a function of the lateral displacement of the disks.
to allow for residual motions to stop. The rotation rate was then increased stepwise
to 3, 6, 8, 10, 11, 12, 13, 14, 15 rpm consecutively at an interval of 30 seconds
between each step. The rotation speed profile is shown in Fig. 8.

The experiment was successfully carried out on board TEXUS 23 launched on 25
November 1989. The experimental sequence described above was nominally
followed.

Some of the images taken are shown in Fig. 9. If the camera is assumed to be at
$\theta_i=0$ direction (see Fig. 2b), the left and right images are the side and front views
respectively. Frames in Fig. 9 have been selected to correspond to $\theta_i=0$. Thus, side
views clearly show the C-mode deformation. Front views show no interface
deformation in Fig. 9a and b (which agree with linear predictions), a small
deformation already appears in Fig. 9c, and a large (and predictable [6]) deformation
is shown in Fig. 9d. Rotating breakage evolution shapes are shown in Fig. 9e and f.
Figure 9. Images taken from the video-tape recorded during the performance of the experiment. The last two images (e) and (f) correspond to the breaking in the C mode.
(a) $r = 145.03$ s, $\Omega = 0$ rpm.
(b) $r = 327.27$ s, $\Omega = 12$ rpm.
(c) $r = 359.52$ s, $\Omega = 13$ rpm.
(d) $r = 377.47$ s, $\Omega = 14$ rpm.
(e) $r = 386.96$ s, $\Omega = 14$ rpm.
(f) $r = 387.00$ s, $\Omega = 14$ rpm.
5. Results

Theoretical results can be summarized by the evolution of the interface deformation (lateral displacement, \(d\), as a function of \(\Omega\)). As an intermediate step, \(d\) has been measured as a function of time. This deformation is given using a rotating frame fixed to the disks as a reference, according to the theory developed. In a non-rotating frame (that of the camera) the position of the bridge is modulated by the position of the disks.

To obtain the three-dimensional position of both the centre of a section of the liquid bridge, \(\hat{d}\), and the centre of a disk, \(\hat{E}\), the use of two orthogonal lateral views is needed, and to obtain the position of the liquid section centre with respect to the disk centre, \(H\), the phase between them, \(\phi\), has to be measured in the same image.

The measurement of \(\hat{E}\) and \(\hat{d}\) has been performed by using a video image processing system (as in the analysis of the ground tests). The lateral displacement, \(d\), of the centre of a section is obtained in each view from the position of the edges of the liquid bridge by analysing the corresponding line in the video image by an edge detection algorithm. The detection algorithm is helped by the illumination background that was chosen after some trials in the ground tests performed. This analysis has to be made in real time instead of counting images in order to avoid time lag errors generated by losses of images. An example of measurements performed is reported in Fig. 10. Some data points have been eliminated by a rejection algorithm in order to eliminate as much as possible spurious data generated by noisy images. Thus, in this and in the following figures, experimental data are represented as individual points and rejected measurement appears as white spaces in between.

From the previous results, the lateral displacement of each section can be represented in a Lissajous plot (\(d \sin \theta\) vs. \(d \cos \theta\)). This plot has been calculated for the section placed \(L/4\) from the bottom disk and is shown in Fig. 11. This particular section has been selected instead of that placed at \(L/2\) because measurements can be made even at large interface deformations.

The circular shape of the traces at \(\Omega = \text{cte}\.), except for \(\Omega = 14\) rpm, shows the absence of other significant perturbations, or of strong coupling at the change of \(\Omega\), although oscillation around the circular behaviour can be observed in Fig. 11. The spiral behaviour for \(\Omega = 14\) rpm clearly shows the unstable character of this configuration.

Another interesting result is the relation between the phase of the lateral displacement and the phase of the disk rotation which is shown in Fig. 12. Despite the high noise level it can be observed that the phase delay is small when the configuration is stable (\(\Omega < 14\) rpm) but that this delay begins to increase when \(\Omega\) changes from 13 rpm to 14 rpm until it reaches 180° at breaking. The noise is attributable to the error in determining the position of the disk due to the low quality of the video images. By subtracting the eccentric displacement produced by the rotation of the disks, \(\hat{E}\), to the lateral displacement, \(d\), the radial deformation \(H\) can be calculated. Results for the \(L/4\) section are shown in Fig. 13 together with the theoretical predictions for the same section.
Figure 10. Measurements obtained from the video tape of the experiment. Position (in pixels) of the two edges of the LI4 section video line No. 348 of the liquid bridge in both views (four upper curves), and positions of the edge of the lower disk (two lower curves), as a function of time, t. Pixel 0 and 12 correspond to the left and right borders of the image, respectively.

Figure 11. Lissajous plot of the projections of the lateral displacement, \(d \sin \theta\) vs. \(d \cos \theta\), measured in the two views of the liquid bridge, at the section placed LI4 (video line No. 348) from the bottom disk, for different values of the rotation rate, \(\Omega\).
(a) \(\Omega=11\) rpm, (b) \(\Omega=12\) rpm, (c) \(\Omega=13\) rpm, (d) \(\Omega=14\) rpm.
6. Conclusions

Several observations can be made:

First, the expected breakage occurs for $\Omega$ somewhere between 13 rpm (where a stable shape was obtained) and 14 rpm (where the liquid bridge broke), as predicted by the theory.

Second, in the linear regime both theoretical and experimental results seem to match. However, larger resolution or specially dedicated experiments seem to be needed to fully support this statement.

Third, a matching region of linear regime to non-linear bifurcation regime appears at $\Omega$ between 10 to 12 rpm. However, the validity of the results of the bifurcation analysis is only approximative as the related error increases with the difference between $\Omega_{|\Delta=0}$ and $\Omega_{|\Delta=\infty}$ due to the asymptotic character of the analysis.

Acknowledgements

This work has been supported by the Spanish Comision Interministerial de Ciencia y Tecnologia (CICYT) and is part of a more general endeavour for the study of fluid physics and materials processing under microgravity (Project No. ESP88-0359).

The authors wish to thank Prof. J. Meseguer and Prof. I. Martinez for their helpful comments.

References

Figure 12. Phase delay, $\phi$, between the lateral displacement of the liquid bridge, $H$, and that of the disk, $E$, as a function of time.

Figure 13. Theoretical results: bifurcation diagram. $H'$ is the radial deformation of the $LI4$ section. Experimental results: rectangles, measurements from video copy paper; solid bars, measurements from results in Figure 11.