FORCED OSCILLATIONS OF ISOTHERMAL LIQUID BRIDGES

José Meseguer[1], José M. Perales[2] and Ana Laverón-Simavilla[2]
E.T.S.I. Aeronáuticos, Universidad Politécnica de Madrid
28040 Madrid, Spain

ABSTRACT

The forced oscillations of a liquid column held by surface tension forces between two solid supports in a zero gravity environment are analysed by using a linear three-dimensional model in which viscosity effects are not considered. Forced oscillations are imposed to the liquid column by vibrating the supporting disks (with particular attention to the in phase or in counter-phase cases), and the response of the liquid bridge (interface deformation and pressure and velocity fields) is obtained. The results are compared with those obtained with one-dimensional models.

INTRODUCTION

A liquid bridge is an idealisation of the fluid configuration appearing in the crystal growth process known as floating zone. When only mechanical aspects of the floating zone problem are considered, the simplest modelisation of the molten zone consists of an isothermal mass of liquid, of constant and uniform properties, filling the gap between two parallel solid disk placed a distance L apart, as sketched in Fig. 1.

Because of its interest not only as a mechanical model of the molten zone appearing in the floating zone process but its own scientific interest, liquid bridges have paid the attention of many scientists during the last two decades. Leaving apart the aspects of the liquid bridge problem related to the statics (a review of the literature concerned with this topic can be found in Meseguer, Slobozhanin and Perales[1]), a number of papers devoted to the dynamics of liquid columns have been published, most of them dealing with the oscillation of liquid bridges[2]. Because of the complexity of the problem formulation, most of the available results related to the liquid bridge dynamics are based on simplified formulations which somehow reduce the range of validity of them. In some papers the analysis is performed by using one-dimensional models, which are of application only when it is assumed that the liquid bridge slenderness is large enough or, when three-dimensional models are used, other restricting hypothesis (dealing with the fluid properties - inviscid liquid- with the geometry of the fluid configuration -cylindrical liquid bridges- or both) are introduced in order to get treatable formulations.

1: Professor, 2: Associate Professor
Tel.: 34 - 1 - 336 63 53
Fax: 34 - 1 - 336 63 63
E-mail: meseguer @ sat.cide.upm.es

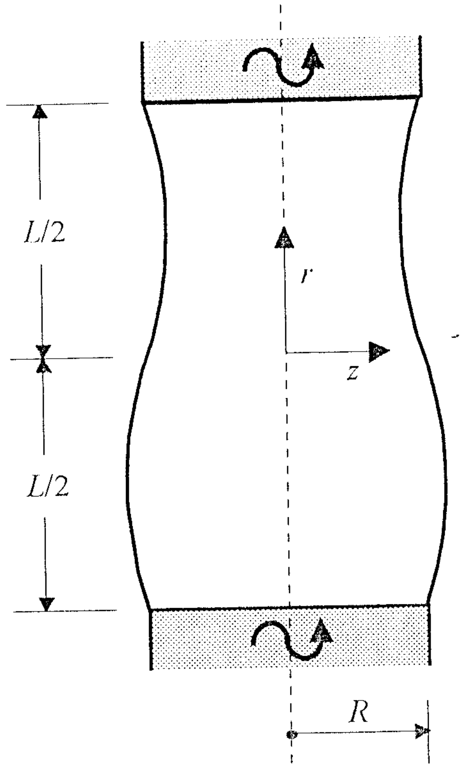

Fig. 1. GEOMETRY AND COORDINATE SYSTEM FOR THE LIQUID BRIDGE PROBLEM.

The resonant frequencies have been calculated in the past by using three-dimensional models (which allow a precise consideration of inertial terms) in the inviscid case and considering a small viscosity for cylindrical liquid bridges[3]. If the volume is not cylindrical or the disks are not of equal diameter and concentrical, the analytical solutions based on three dimensional models are no longer possible and numerical schemes are to be used[4,5].

Concerning the forced response of a liquid bridge when one (or both) of the supporting disks are oscillated or when the gravity field changes with time, only one-dimensional models have been used up to now. These models, although easily allow the consideration of the viscous effects, introduce uncertainties in the consideration of inertial effects. In effect, they are based in averaging the velocity field and in the assumption of a given radial variation of it. This hypothesis can be justified as far as the bridge is slender enough but precludes the application of the model for short liquid bridges of large excitation frequencies.

Concerning experiments, data exist for slender configurations as well as for short configurations. These data have been obtained in orbital laboratories[6] (Spacelab D-2), in sounding rockets[7] and on ground by simulating microgravity conditions (either with liquid bridges of millimetric dimensions or by using the neutral buoyancy technique). While for slender bridges the agreement between experimental results and one-dimensional ones is good, for shorter bridges some discrepancies appear. One of the aims of this paper is to explain these differences.

In this paper the forced oscillations of a cylindrical liquid column, held by surface tension forces between two equal-in-diameter disks under zero gravity conditions, is analysed by using a linear three-dimensional model in which viscosity effects are not considered. Such axisymmetric fluid configuration is uniquely identified by two dimensionless parameters which are related to the geometry of the liquid column (Fig. 1), namely: the slenderness $\Lambda = L/(2R)$, and the dimensionless volume of liquid, $V = \mathbf{V}/(\pi L R^2)$, where $\mathbf{V}$ stands for the physical volume of liquid (although in the following only the case $V = 1$ is considered).

PROBLEM FORMULATION

Let us assume an axisymmetric, inviscid liquid bridge having cylindrical volume ($V = 1$) and slenderness Λ, whose supporting disks can be axially vibrated in an arbitrary form, although two particular cases will received more attention, namely when the disks are vibrated either in counter-phase (symmetric excitation) or in phase (antisymmetric excitation). Under the simplifying assumptions already introduced, the set of differential equations of the three-dimensional axisymmetric, non-rotating, inviscid flow are the following:

Continuity equation:

$$u_r + u/r + w_z = 0 \ . \tag{1}$$

Momentum equations:

$$u_t + \tfrac{1}{2}(u^2 + w^2)_r + w(u_z - w_r) = -p_r \ , \tag{2}$$

$$w_t + \tfrac{1}{2}(u^2 + w^2)_z + u(w_r - u_z) = -p_z \ , \tag{3}$$

and the boundary conditions which express the regularity of the speed at the liquid bridge axis:

$$u(0,z,t) = w_r(0;z,t) = 0 \ , \tag{4}$$

the anchorage of the liquid bridge interface to the edges of the disks:

$$f(\Lambda_{\pm}(t),t) = 1 \ , \tag{5}$$

zero normal velocity at the disks:

$$w(r,\Lambda_{\pm}(t),t) = \frac{d}{dt}\Lambda_{\pm}(t) \ , \tag{6}$$

and the kinematic and dynamic boundary conditions at the interface:

$$f_t(z,t) = u(f(z,t),z,t) - f_z(z,t)w(f(z,t),z,t) \ , \tag{7}$$

$$p(f(z,t),z,t) + \frac{f_{zz}(z,t)}{\left(1+f_z^2(z,t)\right)^{3/2}} - \frac{1}{f(z,t)\left(1+f_z^2(z,t)\right)^{1/2}} = 0 \ , \tag{8}$$

plus the condition of volume preservation:

$$\int_{\Lambda_-(t)}^{\Lambda_+(t)} f^2(z,t)dz = 2\Lambda \ . \tag{9}$$

In the formulation (1)-(9), u and w are the radial and axial components of the dimensionless velocity, respectively, f stands for the shape of the liquid bridge interface and p is the pressure. To write down these dimensionless expressions R and $(\rho R^3/\sigma)^{1/2}$, where ρ is the liquid density and σ the surface tension, have been used as characteristic length and time, respectively. The subscripts t, r and z mean partial derivatives with respect to time, r-coordinate and z-coordinate of the respective functions, and the subscript $\pm$ refers to conditions either in the upper disk (+) or in the lower one (−).

LINEAR ANALYSIS

The position of the disks is assumed to be of the form

$$\Lambda_{\pm}(t) = \pm\Lambda + \varepsilon\lambda_{\pm}e^{i\Omega t} \ , \tag{10}$$

where Λ is the zero amplitude position, $\lambda_{\pm}$ stands for the amplitude of the oscillation of the disks, ε is a small parameter that gives the order of magnitude of the oscillations and Ω is the frequency of the forced oscillations. If only small amplitudes for the disks oscillation are considered, $\varepsilon \ll 1$, the variables involved in the problem formulation can be expressed as

$$f(z,t) = 1 + \varepsilon F(z)e^{i\Omega t}, \quad p(r,z,t) = 1 + \varepsilon P(r,z)e^{i\Omega t},$$

$$u(r,z,t) = \varepsilon U(r,z)e^{i\Omega t}, \quad w(r,z,t) = \varepsilon W(r,z)e^{i\Omega t} \ ,$$

and the introduction of these expressions in (1)-(9), neglecting higher order terms, gives the following irrotational linearised problem:

$$U_r + U/r + W_z = 0 \ , \tag{11}$$

$$i\Omega U = -P_r \ , \tag{12}$$

$$i\Omega W = -P_z \ , \tag{13}$$

$$U(0,z) = W_r(0,z) = 0 \ , \tag{14}$$

$$F(\pm\Lambda) = 0 \ , \tag{15}$$

$$W(r,\pm\Lambda) = i\lambda_{\pm}\Omega \ , \tag{16}$$

$$i\Omega F(z) = U(1,z) \ , \tag{17}$$

$$P(1,z)+F_{zz}(z)+F(z)=0 \ , \qquad (18)$$

$$\int_{-\Lambda}^{\Lambda} F(z)dz = -\frac{1}{2}(\lambda_+ - \lambda_-) \ . \qquad (19)$$

Momentum equations (12) and (13) give the velocities U and W in terms of the pressure P. The introduction of these expressions for U and W in the continuity equation (11) and in the boundary conditions at the liquid bridge axis, expression (14), and at the disks, (16), gives the following formulation for the pressure problem

$$P_{rr} + P_r / r + P_{zz} = 0 \ , \qquad (20)$$

$$P_r(0,z) = P_{rz}(0,z) = 0 \ , \qquad (21)$$

$$P_z(r,\pm\Lambda) = \lambda_\pm \Omega^2 \ . \qquad (22)$$

Expressions (20) and (21) are equal to the corresponding ones appearing in the non-forced oscillation problem solved by Sanz[8], but the third one is different (the quoted problem has an homogeneous formulation and thus only the eigenproblem was solved there). If we now define a new function $Q(r,z)$ by

$$P(r,z) = \lambda_+ \frac{\Omega^2}{2\Lambda}\left[\frac{z^2}{2} + \Lambda z - \frac{r^2}{4}\right] + \\ + \lambda_- \frac{\Omega^2}{2\Lambda}\left[-\frac{z^2}{2} + \Lambda z + \frac{r^2}{4}\right] + Q(r,z) \ , \qquad (23)$$

the following formulation for $Q(r,z)$ results

$$Q_{rr} + Q_r / r + Q_{zz} = 0 \ , \qquad (24)$$

$$Q_r(0,z) = 0 \ , \qquad (25)$$

$$Q_z(r,\pm\Lambda) = 0 \ . \qquad (26)$$

Therefore, the problem for $Q(r,z)$ is now the same problem solved by Sanz[8] and the same solution technique used by him will be used. A general solution of the differential equation (24) fulfilling the boundary condition (25) and boundary condition (26) can be found by looking a solution in terms of a function only of r times a function only of z, the solution being:

$$Q(r,z) = g + \sum_{n=1}^{\infty} a_n \Omega^2 \, I_0(\mu_n r)\cos(\mu_n(z+\Lambda)) \ , \qquad (27)$$

where $\mu_n = n\pi/(2\Lambda)$ and g and a_n are unknown constants; I_0 is the modified Bessel function of first kind of zero order.

The solution for P is:

$$P(r,z) = \lambda_+ \frac{\Omega^2}{2\Lambda}\left[\frac{z^2}{2} + \Lambda z - \frac{r^2}{4}\right] + \lambda_- \frac{\Omega^2}{2\Lambda}\left[-\frac{z^2}{2} + \Lambda z + \frac{r^2}{4}\right] + \\ + g + \sum_{n=1}^{\infty} a_n \Omega^2 \, I_0(\mu_n r)\cos(\mu_n(z+\Lambda)) \ , \qquad (28)$$

and taking into account the equation (12) and the boundary condition (17) it is deduced that $F(z) = P_r(1,z)/\Omega^2$, so that, according with the expression (28) for the pressure, the expression for the liquid bridge interface shape becomes:

$$F(z) = -\frac{1}{4\Lambda}(\lambda_+ - \lambda_-) + \sum_{n=1}^{\infty} a_n \mu_n \, I_1(\mu_n)\cos(\mu_n(z+\Lambda)) \ . \qquad (29)$$

On the other hand, $F(z)$ must fulfil boundary condition (18), which now can be written as

$$F + F_{zz} = -P(1,z) = -\lambda_+ \frac{\Omega^2}{2\Lambda}\left[\frac{z^2}{2} + \Lambda z - \frac{1}{4}\right] - \\ -\lambda_- \frac{\Omega^2}{2\Lambda}\left[-\frac{z^2}{2} + \Lambda z + \frac{1}{4}\right] - \\ -g - \sum_{n=1}^{\infty} a_n \Omega^2 \, I_0(\mu_n)\cos(\mu_n(z+\Lambda)) \ , \qquad (30)$$

as well as (15) and (19), that is

$$F(\pm\Lambda) = 0 \ , \qquad (31)$$

$$\int_{-\Lambda}^{\Lambda} F(z)dz = -\frac{1}{2}(\lambda_+ - \lambda_-) \ . \qquad (32)$$

Expressions (30)-(32) are the formulation to be solved to calculate $F(z)$ and particularly to determine the constants g and a_n. Once the constants are known, the pressure results from equation (28) and the function $Q(r,z)$ from (27). The velocity field can then be obtained by using equations (12) and (13).

To calculate the constants g and a_n appearing in (30) the procedure is as follows: the general solution of the differential equation (30) can be written as

$$F(z) = a\sin(z) + b\cos(z) + (\lambda_+ - \lambda_-)\frac{\Omega^2}{4\Lambda}\left[\frac{5}{2} - z^2\right] - \\ -(\lambda_+ + \lambda_-)\frac{\Omega^2}{2} z - \\ -g - \sum_{n=1}^{\infty} \frac{a_n \Omega^2 \, I_0(\mu_n)}{1-\mu_n^2}\cos(\mu_n(z+\Lambda)) \ , \qquad (33)$$

and from boundary conditions (31) and (32) one obtains:

$$-a\sin(\Lambda) - b\cos(\Lambda) + g + \sum_{n=1}^{\infty} \frac{a_n \Omega^2 \, I_0(\mu_n)}{1-\mu_n^2}\cos(n\pi) = \\ = (\lambda_+ - \lambda_-)\frac{\Omega^2}{4\Lambda}\left[\frac{5}{2} - \Lambda^2\right] - (\lambda_+ + \lambda_-)\frac{\Omega^2}{2}\Lambda \ , \qquad (34)$$

$$a\sin(\Lambda)-b\cos(\Lambda)+g+\sum_{n=1}^{\infty}\frac{a_n\Omega^2\,\mathrm{I}_0(\mu_n)}{1-\mu_n^{\,2}}=$$

$$=(\lambda_+-\lambda_-)\frac{\Omega^2}{4\Lambda}\left[\frac{5}{2}-\Lambda^2\right]+(\lambda_++\lambda_-)\frac{\Omega^2}{2}\Lambda\ , \qquad (35)$$

$$g\Lambda-b\sin(\Lambda)=(\lambda_+-\lambda_-)\left[\frac{1}{4}+\Omega^2\left(\frac{5}{8}-\frac{\Lambda^2}{12}\right)\right]. \qquad (36)$$

Expressions (34) and (35) can be replaced by the following two equivalent conditions:

$$a\sin(\Lambda)+\sum_{m=0}^{\infty}\frac{a_{2m+1}\Omega^2\,\mathrm{I}_0(\mu_{2m+1})}{1-\mu_{2m+1}^2}=(\lambda_++\lambda_-)\frac{\Omega^2}{2}\Lambda\ , \qquad (37)$$

$$-b\cos(\Lambda)+g+\sum_{m=1}^{\infty}\frac{a_{2m}\Omega^2\,\mathrm{I}_0(\mu_{2m})}{1-\mu_{2m}^2}=$$

$$=(\lambda_+-\lambda_-)\frac{\Omega^2}{4\Lambda}\left[\frac{5}{2}-\Lambda^2\right], \qquad (38)$$

and taking into account (36) the constant g can be replaced in (38), the resulting expression being

$$b\cos(\Lambda)\left[1-\frac{\tan(\Lambda)}{\Lambda}\right]-\sum_{m=1}^{\infty}\frac{a_{2m}\Omega^2\,\mathrm{I}_0(\mu_{2m})}{1-\mu_{2m}^2}=$$

$$=(\lambda_+-\lambda_-)\frac{1}{4\Lambda}\left[\frac{2}{3}\Omega^2\Lambda^2+1\right]. \qquad (39)$$

From expressions (37) and (39) the constants a and b are calculated in terms of the coefficients a_n. To calculate such coefficients, first of all the following expansions for $\cos(z)$, $\sin(z)$, z and z^2 are needed:

$$\sin(z)=2\frac{\cos(\Lambda)}{\Lambda}\sum_{m=0}^{\infty}\frac{1}{1-\mu_{2m+1}^2}\cos(\mu_{2m+1}(z+\Lambda))\ ,$$

$$\cos(z)=\frac{\sin(\Lambda)}{\Lambda}\left[1+2\sum_{m=1}^{\infty}\frac{1}{1-\mu_{2m}^2}\cos(\mu_{2m}(z+\Lambda))\right],$$

$$z=-\frac{2}{\Lambda}\sum_{m=0}^{\infty}\frac{1}{\mu_{2m+1}^2}\cos(\mu_{2m+1}(z+\Lambda))\ ,$$

$$z^2=\frac{\Lambda^2}{3}+4\sum_{m=1}^{\infty}\frac{1}{\mu_{2m}^2}\cos(\mu_{2m}(z+\Lambda))\ ,$$

which are introduced in the expression of $F(z)$ given by (33). Then, since we have now two different expressions for $F(z)$, equations (29) and (33), by equating them the following relationship results

$$2a\frac{\cos(\Lambda)}{\Lambda}\sum_{m=0}^{\infty}\frac{1}{1-\mu_{2m+1}^2}\cos(\mu_{2m+1}(z+\Lambda))+$$

$$+b\frac{\sin(\Lambda)}{\Lambda}\left[1+2\sum_{m=1}^{\infty}\frac{1}{1-\mu_{2m}^2}\cos(\mu_{2m}(z+\Lambda))\right]+$$

$$+(\lambda_+-\lambda_-)\frac{5\Omega^2}{8\Lambda}-$$

$$-(\lambda_+-\lambda_-)\frac{\Omega^2}{4\Lambda}\left[\frac{\Lambda^2}{3}+4\sum_{m=1}^{\infty}\frac{1}{\mu_{2m}^2}\cos(\mu_{2m}(z+\Lambda))\right]+$$

$$+(\lambda_++\lambda_-)\frac{\Omega^2}{\Lambda}\sum_{m=0}^{\infty}\frac{1}{\mu_{2m+1}^2}\cos(\mu_{2m+1}(z+\Lambda))-$$

$$-b\frac{\sin(\Lambda)}{\Lambda}-(\lambda_+-\lambda_-)\frac{1}{4\Lambda}\left[1+\Omega^2\left(\frac{5}{2}-\frac{\Lambda^2}{3}\right)\right]-$$

$$-\sum_{n=1}^{\infty}\frac{a_n\Omega^2\,\mathrm{I}_0(\mu_n)}{1-\mu_n^{\,2}}\cos(\mu_n(z+\Lambda))=$$

$$=-\frac{1}{4\Lambda}(\lambda_+-\lambda_-)+\sum_{n=1}^{\infty}a_n\mu_n\,\mathrm{I}_1(\mu_n)\cos(\mu_n(z+\Lambda))\ , \qquad (40)$$

and equating the coefficients of $\cos(\mu_n(z+\Lambda))$ on both sides of this last expression one obtains

$$a_{2m+1}\left[\Omega^2\,\mathrm{I}_0(\mu_{2m+1})+\mu_{2m+1}\left(1-\mu_{2m+1}^2\right)\mathrm{I}_1(\mu_{2m+1})\right]=$$

$$=2a\frac{\cos(\Lambda)}{\Lambda}+(\lambda_++\lambda_-)\frac{\Omega^2}{\Lambda}\frac{1-\mu_{2m+1}^2}{\mu_{2m+1}^2}\ , \qquad (41)$$

$$a_{2m}\left[\Omega^2\,\mathrm{I}_0(\mu_{2m})+\mu_{2m}\left(1-\mu_{2m}^2\right)\mathrm{I}_1(\mu_{2m})\right]=$$

$$=2b\frac{\sin(\Lambda)}{\Lambda}-(\lambda_+-\lambda_-)\frac{\Omega^2}{\Lambda}\frac{1-\mu_{2m}^2}{\mu_{2m}^2}\ . \qquad (42)$$

These last two equations express the coefficients a_n in terms of the constants a and b. The substitution of a_n as given by (41) and (42) in (37) and (39) finally yields

$$a\left\{\sin(\Lambda)+2\frac{\cos(\Lambda)}{\Lambda}\sum_{m=0}^{\infty}\frac{\Gamma_{2m+1}}{1-\mu_{2m+1}^2}\right\}=$$

$$=(\lambda_++\lambda_-)\left\{\frac{\Omega^2}{2}\Lambda-\frac{\Omega^2}{\Lambda}\sum_{m=0}^{\infty}\frac{\Gamma_{2m+1}}{\mu_{2m+1}^2}\right\}, \qquad (43)$$

$$b\left\{\cos(\Lambda)\left[1-\frac{\tan(\Lambda)}{\Lambda}\right]-2\frac{\sin(\Lambda)}{\Lambda}\sum_{m=1}^{\infty}\frac{\Gamma_{2m}}{1-\mu_{2m}^2}\right\}=$$

$$=(\lambda_+-\lambda_-)\left\{\frac{\Omega^2}{6}\Lambda+\frac{1}{4\Lambda}-\frac{\Omega^2}{\Lambda}\sum_{m=1}^{\infty}\frac{\Gamma_{2m}}{\mu_{2m}^2}\right\}, \qquad (44)$$

where

$$\Gamma_n = \frac{\Omega^2 \mathrm{I}_0(\mu_n)}{\left[\Omega^2 \mathrm{I}_0(\mu_n) + \mu_n\left(1-\mu_n^2\right)\mathrm{I}_1(\mu_n)\right]} .$$

Therefore, once a and b are determined by using (43) and (44), the coefficients a_{2m} and a_{2m+1} are obtained from (41) and (42), the constant g results from expression (36) and $F(z)$ can be obtained either from (29) or (33).

It is remarkable that responses to symmetric and antisymmetric (with respect to z = 0) excitations are fully decoupled. In effect, the coefficient a depends only on $\lambda_+ + \lambda_-$ and the coefficients b and g on $\lambda_+ - \lambda_-$. Thus an antisymmetric excitation produces $a_{2m+1} \neq 0$, $a_{2m} = 0$ (an antisymmetric response) and a symmetric excitation produces a symmetric response ($a_{2m+1} = 0$, $a_{2m} \neq 0$).

Before pursuing further it is convenient to compare the results here obtained with other results already published. The forced oscillation of long axisymmetric liquid bridges was analysed by Perales and Meseguer[4] by using a linearised one-dimensional model based on the Cosserat formulation for the liquid bridge problem. According to this study, when the frequency of the forcing perturbation vanishes ($\Omega \to 0$), in the case of in-phase oscillation the liquid bridge interface can be expressed as

$$F(z) = \frac{1}{4}(\lambda_+ - \lambda_-)\frac{\cos(z)-\cos(\Lambda)}{\Lambda\cos(\Lambda)-\sin(\Lambda)} . \quad (45)$$

According to the results here obtained, when $\Omega \to 0$ from expression (43) one gets a=0, and from (44) it is obtained

$$b\left[\Lambda\cos(\Lambda) - \sin\Lambda\right] = \frac{1}{4}(\lambda_+ - \lambda_-) ,$$

on the other hand, from (38) results $g = b\cos(\Lambda)$. Therefore, the substitution of these expressions in equation (33), which now is reduced to $F(z) = b\cos(z) - g$, yields to an expression for $F(z)$ identical to (45).

The transfer function of the liquid bridge interface, defined as the variation of the maximum interface deformation of the liquid bridge as a function of the frequency of the forcing perturbation, $\Delta F = (F_{max} - F_{min})/(\lambda_+ + \lambda_-)$ in the case of antisymmetric excitation and $\Delta F = (F_{max} - F_{min})/(\lambda_+ - \lambda_-)$ in the case of symmetric ones, has been represented in Figs. 2 and 3 for antisymmetric ($\lambda_+ = \lambda_- = 1/2$) and symmetric excitation ($\lambda_+ = -\lambda_- = 1/2$), respectively. In Fig. 2 as well as in Fig. 3 the responses of liquid bridges having different slendernesses (Λ = 1, 2 and 3) are represented. In these plots the transfer function resulting from the three-dimensional (3-D) model as well as the transfer function resulting from the Cosserat one-dimensional (1-D) model[4] are shown.

As it can be observed in both cases, symmetric and antisymmetric excitation, the differences between 3-D results and 1-D results increases as the slenderness of the liquid bridge decreases, as one could expect, since 1-D models show a decreasing accuracy as the slenderness decreases (in 1-D models it is assumed that the velocity does not depend on the radial coordinate r, and this hypothesis is only valid provided the slenderness is large enough[4], $\Lambda >> 1$).

Another aspect to be pointed out from Figs. 2 and 3 is that the differences between 1-D and 3-D models increases as the frequency of the forcing perturbation increases.

This fact can be easily explained as the characteristic length to be taken to compare with the radius is not really the liquid bridge length but the wavelength of the perturbation. Thus, for the one-dimensional models yielding accurate results it is needed not only to have $\Lambda >> 1$ but $k/R >> 1$, k being the wavelength. As the wavelength decreases as the excitation frequency increases, one-dimensional models yield worse results for higher frequencies.

CONCLUSIONS

A three-dimensional model to study forced oscillations of liquid bridges has been developed. This model (exact as long as neither viscosity effects nor non-linear effects are considered) compares well with previous results obtained using one-dimensional models (which, in turn, can easily include viscous effects and non-linear effects). Thus the range of validity of existing one-dimensional models has been determined developing a method to check its accuracy (limited to large slendernesses and small excitation frequencies).

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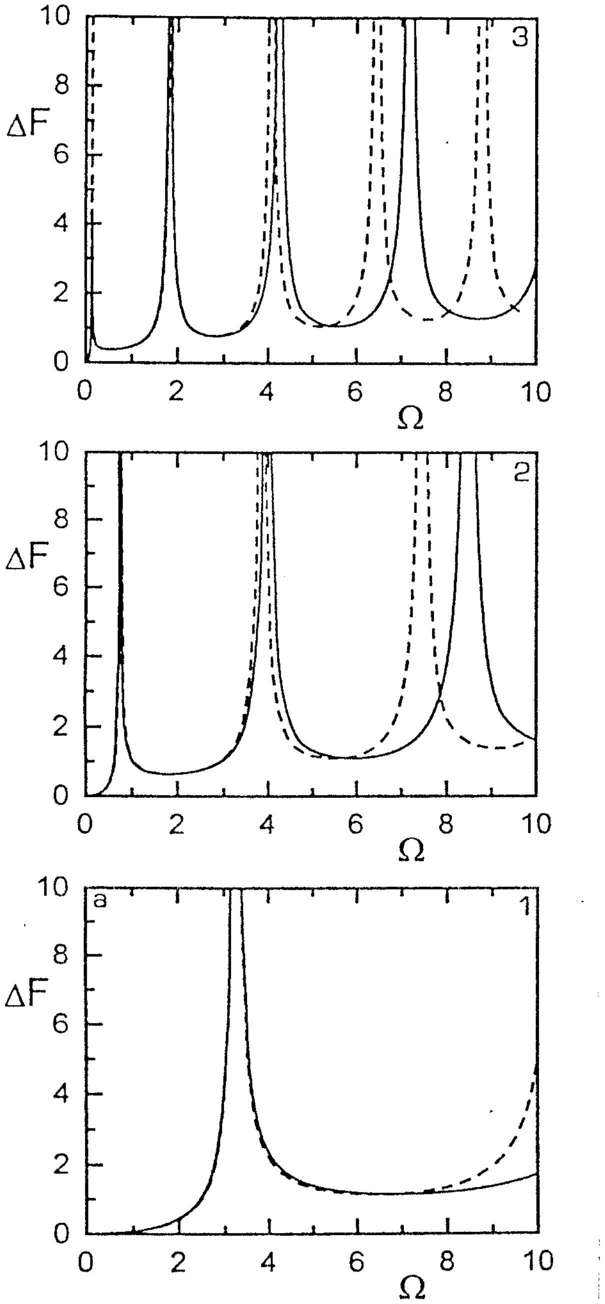

Fig. 2. TRANSFER FUNCTION, AS DEFINED IN THE TEXT, OF LIQUID BRIDGES SUBJECTED TO AN ANTISYMMETRIC EXCITATION. NUMBERS ON THE PLOTS INDICATE THE VALUE OF THE SLENDERNESS. SOLID LINES: 3-D, DASHED LINES: 1-D.

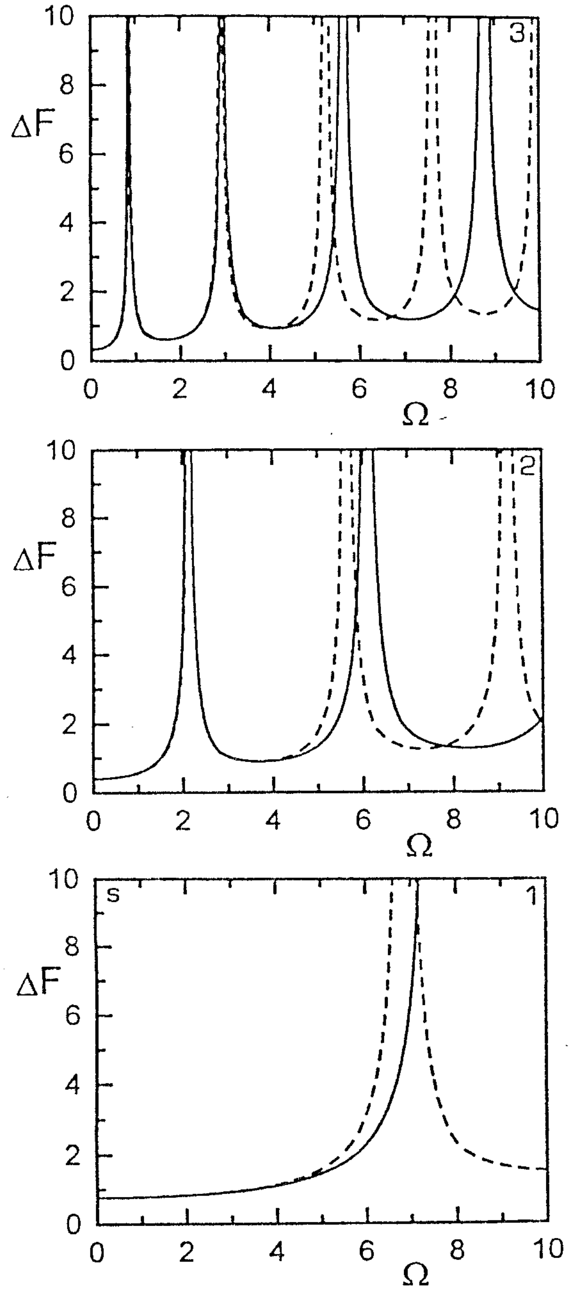

Fig. 3. TRANSFER FUNCTION, AS DEFINED IN THE TEXT, OF LIQUID BRIDGES SUBJECTED TO A SYMMETRIC EXCITATION. NUMBERS ON THE PLOTS INDICATE THE VALUE OF THE SLENDERNESS. SOLID LINES: 3-D, DASHED LINES: 1-D.