Correlation of spacecraft thermal mathematical models to reference data

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ABSTRACT

Model-to-test correlation is a frequent problem in spacecraft-thermal control design. The idea is to determine the values of the parameters of the thermal mathematical model (TMM) that allows reaching a good fit between the TMM results and test data, in order to reduce the uncertainty of the mathematical model. Quite often, this task is performed manually, mainly because a good engineering knowledge and experience is needed to reach a successful compromise, but the use of a mathematical tool could facilitate this work. The correlation process can be considered as the minimization of the error of the model results with regard to the reference data.

In this paper, a simple method is presented suitable to solve the TMM-to-test correlation problem, using Jacobian matrix formulation and Moore-Penrose pseudo-inverse, generalized to include several load cases. Aside, in simple cases, this method also allows for analytical solutions to be obtained, which helps to analyze some problems that appear when the Jacobian matrix is singular. To show the implementation of the method, two problems have been considered, one more academic, and the other one the TMM of an electronic box of PHI instrument of ESA Solar Orbiter mission, to be flown in 2019. The use of singular value decomposition of the Jacobian matrix to analyze and reduce these models is also shown. The error in parameter space is used to assess the quality of the correlation results in both models.

1. Introduction

Model-to-test correlation is a frequent problem in spacecraft-thermal control design. The idea is to determine the parameters of the thermal mathematical model (TMM) that makes a good fit between the TMM results and test data, thus reducing the uncertainty of the mathematical model. In a way, it seeks to determine some thermal properties from test data, which is known as a parameter identification problem (Curry and Williams, 1972) [1].

A very similar problem is the determination of the parameters of a reduced thermal mathematical model (RTMM, typically with a size less than 20 nodes), obtained from a detailed model (DTMM, typically with a size of some thousand nodes), in such a way that the difference between the values predicted by the RTMM and the DTMM are inside some given bounds. We will use here the term “reference results” for DTMM results or test data, indistinctly. Model reduction in the spacecraft thermal modelization field is a relevant discipline from the engineering point of view (Deimi et al., 2015) [2], (Fernández-Rico et al., 2016) [3], as the RTMM are used for the spacecraft thermal integration responsible to build up the complete spacecraft TMM. Subsystem RTMMs should have been previously adequately validated.

Quite often, this correlation process is performed manually, mainly because a good engineering knowledge and experience is needed to reach a successful compromise, but this work could be helped by the use of a mathematical tool [4] (Frey et al., 2015) [5], (Akito et al., 2012).

For a given device, the correlation process of its reduced TMM to the reference results (DTMM and/or test results) is facilitated by the previous existence of the respective mathematical model (which has been developed during the design phase or test preparation phase). In fact, the values of the parameters of these models are available beforehand and can be used as starting values to initiate the correlation process. From a practical point of view, the improvement of the process by using a mathematical tool is to avoid a lengthy and costly manual correlation process.

This correlation process can be considered as the minimization of the error of the model results with regard to the reference results (either a DTMM results or experimental data). Therefore, it can be classified as an optimization problem, a very frequent problem in the broad field of engineering, which can be solved in a number of more or less complex ways.

Fortunately, in this regard, some simplifications can be considered. As above mentioned, the correlation process can be started at a point which is not far from the optimum solution, and besides, the solutions of the thermal mathematical models show a smooth and monotonous dependence on the parameters of the model. These characteristics suggest that the use of methods based on the gradient or Jacobian matrix (Milman and Petrick, 2000) [6] could be appropriate, thus avoiding the need for more generic and therefore complex optimization methods.

In spacecraft thermal control discipline, to verify the TMM, it is usual to perform several thermal tests with different load cases (hot

Nomenclature

A_i	radiative surface of the node
c_0	initial condition factor
E_T	error in the results space
E_k	error in the search space
k_{ij}	thermal conductance between node i and j
k_{lr}	thermal conductance obtained from linearization of radiation term
k_{iw}	thermal conductance to wall
k_j	free parameter fluctuation around the initial value
k^{**}	solution for zero error
$[K]_{ij}'$	derivative of matrix $[K]$ with regard to free parameter x_{ij}
$[K], K_{ij}$	conductance matrix
$[K_a]$	adjoint matrix of $[K]$
$[M], M_{ij}$	Jacobian matrix
$[M_c]$	square matrix
N_{k^*}	norm of free parameter correction vector
N_N	number of data points or model nodes
N_P	number of free parameters
N_{TRM}	norm of temperature error vector (difference between reference and model temperatures)
N_{TRO}	norm of vector difference between reference and initial temperatures
N_{x0}	norm of free parameter vector at initial conditions
N_{xR}	norm of free parameter vector at reference conditions
N_{xRM}	norm of free parameter error vector
q_i	heat load at node i , including contribution of radiation linearization

q_{id}	dissipated heat load at node i
q_{lr}	contribution of the linearized radiative heat load at node i
$\{Q\}$	heat load vector
r_x	relative error in free parameters
t_i	temperature fluctuation of the mathematical model node i around the initial solution
T_j	temperature of node j of mathematical model
t_{Ri}	temperature fluctuation of the reference model i with regard to the initial value
T_{Rj}	temperature of node j of reference model (data points)
T_i^0	temperature of the mathematical model for the initial value of the free parameters
x	free parameter of the model
x_{Ri}	free parameter value in the reference model
x_j^0	initial values of the free parameters
$ x $	norm of vector x
ε	small dimensionless parameter $\varepsilon \ll 1$, emissivity
σ	Stefan-Boltzmann constant

Subscripts

C1, C2	load cases
R	reference model
0	initial condition
Si	result of the iteration step i
M	results from the model

Superscripts

0	initial condition
*	solution of equation

operational, cold operational, etc ...) corresponding to different heat input profiles. The method proposed here is not limited to the correlation of only one load case, which is a usual limitation of other proposed methods. The reason is that each load case has an associated Jacobian matrix, and it has not been found yet how to solve the least-square equation system in this case, solution which is given here.

The correlation problem, is in fact one of optimization. As reviewed by Galski et al., 2007 [7], there are many methods for global optimization to solve very general problems of science and engineering, which are non-linear, multimodal, and subject to several restrictions on the variables. A short review can be found in this reference. But the implementation of these methods can be difficult. Fortunately, in the case of thermal mathematical model correlation, the mathematical model is known, the behavior is monotonous, and a rough assumption on the value of the parameters is also known beforehand. Therefore, a local optimization of the mathematical model should be suitable, and a global optimization is not needed, whose associated complexity can be avoided.

However, a large effort in the past has been devoted to global optimization methods, and for reference a few ones are considered here concerning thermal model correlation problems. In Klement et al., 2016 [8] two methods are compared: a genetic algorithm and a method based on non-linear equations solving algorithms of the Broyden class. Also a review of the efforts on automatic thermal model test correlation is presented, including gradient based minimization, and stochastic algorithms (genetic, particle swarm, and others). A method based on quasi-Newton equation solving algorithms of the Broyden class is proposed. As an advantage of this method, among others, it uses a vector of information at each iteration, instead of a single variable.

This method linearly approximates the model at the initial step using the gradient (Jacobian matrix) of the temperature results as a function of the model free parameters. The optimum of this linear approximation can be analytically obtained in one step. If there is a difference between the reference values and the obtained values (in nonlinear problems) the

linear optimum value can be used as starting point for the next iteration; typically from 3 to 20 iterations would be needed to reach a RMS of the temperature deviations below 10^{-4} K. For the genetic algorithm used, the details can be found in Garmendia & Anglada 2016 [9]. Four models were used to implement the comparison (including TriboLab results, a materials experiment facility on board the International Space Station). They conclude that the algorithms of the Broyden class analyzed are faster than genetic algorithms, although the limitation is that it will require monotone functions, which fortunately is the normal situation in thermal models, although the influence of the configuration on the convergence of quasi-Newton methods is an open point to be investigated. The main advantage of genetic algorithms is that they are general, in the sense that they are independent of the properties of the model and of the function to be optimized, and therefore can be applied to almost any problem, however the authors pay attention to the error in the temperature fitting but did not check the error in the solution obtained for the model parameters (the error in the search space).

The work reported by Zijl et al., 2013 [10] is focused on correlation of thermal balance test results with a TMM using evolutionary algorithms (Monte Carlo Simulation, Genetic Algorithm, Adaptive Particle Swarm Optimization APSO). The experimental set-up included 129 nodes and 24 relevant temperatures. Four tests were performed, two at Dutch Space and two at ESTEC, on a Solar panel with HDRM (Hold Down and Release Mechanism). 16 parameters were considered in the correlation process. APSO gave the best results, an average of 1.1 °C residual error was found.

Genetic algorithms are also used by Garmendia and Anglada 2016 [9] and Anglada and Garmendia [11] to correlate thermal tests with TMM results in the case of TriboLab. The TMM in Ref. [9] includes 47 nodes (21 mass nodes, 7 MLIs nodes and 19 sink nodes). Hot and cold cases were considered. 6 temperature points were measured in the tests. An in-house developed genetic algorithm GAC-TM was employed together with 117 free parameters (21 MCs, 42 GLs, 54 GRs). The correlation target established a difference of less than 0.5 °C in each node.

Another TMM correlation to thermal balance test (STM of Bepi Colombo laser altimeter (BELA) is reported by Beck et al., 2012 [12], using a genetic algorithm, iterative optimization method (adaptive particle swarm optimization, APSO). The thermal model gives results similar to thermal test to an average accuracy of 4.2 °C with a standard deviation of 3.2 °C, after using 600 iteration steps. Their TMM simulates 5 different environments, 10 variables were left for optimization. Concerning test reference results 79 temperature data points were considered (15 points for three cases and 17 for the other two cases). As explained by these authors, manual thermal model correlation is very time consuming, and requires great effort from experienced engineers with a good knowledge of the thermal system being studied. The idea of research in automatic correlation methods is not only to reduce this work, but also to achieve results with a better accuracy.

A similar problem arises in the field of structural dynamics, denoted as “model updating” (Mottershead and Friswell, 1993 [13]). In this case, the objective is the correlation of the eigenvalues and eigenvectors of the modes, leading to techniques that are not (at least until now) easily transferred to thermal correlation methods.

A special attention has been devoted by the authors to Klement 2014 [14] study on “Model-to-test correlation” (MTTC). It is based on the use of quasi-Newton algorithms of the Broyden class. As outlined in this paper, a significant number of methods developed to solve MTTC problems are based on stochastic algorithms that often require several hundred iterations to converge, while methods based on Broyden class algorithms need considerably less iteration steps. The algorithm proposed is based on searching for the roots of the equation that establishes the identity between the model results and the reference values of temperatures. He pointed out some of the problems and limitations of the method (common to the methods based on Jacobian matrix), which in fact are requirements to be considered when choosing these methods, and also for both the selection of parameters of the TMM, and of the reference model results to be used for comparison. These requirements are:

- Monotony and differentiability. If the function $T(x)$ does not fulfill this requirement, the convergence will not be achieved.
- Observability. Each parameter should affect at least one result. Otherwise the respective column of the Jacobian matrix would be zero (or close to zero, within the accuracy of the measurement data). In this case, the best solution is to eliminate this parameter from the analysis.
- Controllability. Each result should be affected by at least one parameter. Otherwise the respective row of the Jacobian matrix would be zero (or close to zero). The best solution is to eliminate this temperature from the analysis, or to change the TMM architecture.
- Accuracy of measurements and model inaccuracies.
- Constraints in parameter range.

In Klement's paper, two examples are analyzed: a simple 4-node model, and a complex model of a real piece of hardware. In the simple 4-node model example, three conditions of the problem were considered:

- Undetermined system (6 parameters and 4 temperatures).
- Determined system (4 parameters and 4 temperatures).
- Overdetermined system (3 parameters and 4 temperatures).

From this model, one relevant conclusion is that the convergence of Klement's algorithm for undetermined systems is faster than determined and overdetermined systems. In any case, a few iterations (between 5 and 20) are needed to achieve a small temperature error.

In this regard, it should be pointed out that the correlation problem is not a well determined problem (the number of unknowns is different from the number of data). Therefore, in the overdetermined case, the minimization of the error (in the least square sense) can be considered as a suitable method to obtain an approximated solution. In the current

paper, a way to translate this problem to the inversion of a non-square matrix is presented, the solution is obtained using the Moore-Penrose (MP) pseudo inverse, and it is generalized to take into account several load cases, which requires the generalization of the MP pseudo inverse.

The proposed method, based on the Jacobian matrix (or Taylor series expansion, Marquardt 1963) [15], is presented following in part the paper of Klement et al., 2016 [4], including the analysis of several reduced TMM (which helps to analyze the mathematical issues of the formulation). Furthermore, to show the implementation problems, is applied to a real problem, the Electronic Unit (PHI-ELE) part of instrument PHI to be flown in the European Space Agency's (ESA) Solar Orbiter mission in 2019.

The proposed method also allows for a physical interpretation of the process to be obtained, identifying the optimized error as the minimum distance (in the result space) between the reference data and the sub-space image of the free parameter space (search space). In the over-determined case, it is possible to obtain a solution that is relatively close to the starting condition (as the optimum is not far from the initial guess). However, in the undetermined case, multiple solutions can be found that minimize the temperature error, but do not necessarily reduce the error in the parameter space; however, and there is no criteria to decide what is the best solution. To overcome this problem, in this study it has been very useful to take into account the error in the parameter space (search space) in addition to the usual practice of only considering the error in the result space (temperature data), thus facilitating the evaluation of the results in a more straightforward and conclusive way, in overdetermined problems. To classify the problems, the rank of the generalized pseudo-inverse matrix obtained by singular value decomposition algorithm has been used, which also gives information to reduce the order of the mathematical models.

The above mentioned mathematical models are obtained by applying the lumped parameter method. There are also other methods: finite element based methods, which takes advantage of the detailed meshes coming from structural models (Jacques et al., 2017) [16], or zone averaged equations (Liu et al., 2010) [17]. However, the method developed here is most suitable for the analysis of models with a reduced number of temperature data.

The method proposed here can also be used to solve design problems where fixed temperature requirements are imposed, as for instance the surface temperature value and uniformity at a blackbody for on-board calibration of IR radiometers (Oh and Shin 2012) [18].

Following these lines, in the present paper a method based on the use of a generalized Jacobian matrix and Moore-Penrose pseudo inverse to solve the model-to-reference correlation problem by least square minimization of the error, is presented. In section 2, the definition of the problem is established. In section 3, the sensitivity analysis of a simple 2-node model is considered and analytical solutions are obtained in order to develop the concepts introduced in section 2, which help to analyze the problem. In section 4 the minimization problem is solved, and generalized for several load cases. In section 5 this method is used to analyze the Klement's 4-node model with only one load case, including the influence of the distance between the initial and reference conditions. In section 6, the method is used to analyze a real space thermal design problem, the Electronic unit (ELE) of “Polarimetric and Helioseismic Imager” PHI instrument (to be flown in Solar Orbiter mission) with two load cases. Finally, in section 7 conclusions are drawn.

Following Klement et al., 2014 [14] paper, to measure the difference between two vectors $\{x_1\}$, $\{x_2\}$ we adopted the norm defined as

$$N_{x12} = |\{x_1\} - \{x_2\}| = \left[\sum_{i=1}^{N_c} (x_{1i} - x_{2i})^2 \right]^{1/2}$$

or the root sum square. N_c is the dimension of the vectors.

2. Problem definition. Jacobian matrix

The correlation of a RTMM (reduced thermal mathematical model) to the associated Reference Model results (either thermal data or DTMM results) can be transformed to an optimization problem, trying to minimize the sum squared deviation E_T between Reference Model results T_{Rj} and Reduced TMM results $T_j(x)$, which depends on the free parameters of the model x (search space), which is denoted here as error for short,

$$E_T(x) = \sum_{j=1}^{N_y} (T_{Rj} - T_j(x))^2. \quad (2.1)$$

The error gradient $\nabla_x E$ is given by

$$\frac{\partial E_T}{\partial x_i} = \sum_{j=1}^{N_y} \frac{\partial E_T}{\partial T_j} \frac{\partial T_j}{\partial x_i} = -2 \sum_{j=1}^{N_y} (T_{Rj} - T_j(x)) \frac{\partial T_j(x)}{\partial x_i}, \quad i = 1 \dots N_p \quad (2.2)$$

where N_p is the number of free parameters and N_N is the number of data points. The sensitivity to parameter x_i (Jacobian matrix columns) is

$$M_{ji} = \frac{\partial T_j(x)}{\partial x_i}. \quad (2.3)$$

The error minimization is given by the condition

$$\nabla_x E_T(x) = 0, \quad (2.4)$$

which is a system of N_p equations and N_p unknown parameters to be determined. Even if the equation system could be explicitly written, it is nonlinear, and therefore the determination of the solution can be a complex problem. Often, the way of solving the problem is to use an iterative process (e.g. steepest descent, Broyden class algorithms) until the condition

$$|\nabla_x E_T(x^*)| < \varepsilon,$$

is fulfilled, where $\varepsilon \ll 1$ is the tolerance for error, and $\{x^*\}$ is the set of free parameters that gives the minimum error E_T . $|x|$ is the norm of vector x .

In a linear thermal problem, the sensitivities (2.3) can be determined using the TMM model equation for steady problems

$$[K]\{T\} = \{Q\}, \quad (2.5)$$

where $[K]$ is the conductance matrix, $\{T\}$ the node temperature, and $\{Q\}$ the node heat load vector. The elements of $[K]$, $K_{ij} = f(k_{ij})$, are functions of the conductances between nodes $k_{ij}(x_i)$, and x_i are the free parameters of the model. By differentiation of (2.5) with regard to the free parameter x_i , the following expression for the sensitivities is obtained

$$\begin{aligned} \frac{\partial}{\partial x_i} \{T\} &= [K]^{-1} \left(-\frac{\partial}{\partial x_i} [K] \{T\} + \frac{\partial}{\partial x_i} \{Q\} \right) \\ &= [K]^{-1} \left(-\left(\frac{\partial}{\partial x_i} [K] \right) [K]^{-1} + \frac{\partial}{\partial x_i} \right) \{Q\} = [S_i] \{Q\} \end{aligned} \quad (2.6)$$

where (2.5) has been used to substitute for $\{T\}$. This vector is the column " i " of Jacobian matrix $[M]$. $[S_i]$ is a matrix operator based on the conductance matrix $[K]$.

It is worth noting the linear dependence of the sensitivity vectors (Jacobian columns) on the heat load $\{Q\}$. This result can be used to speed up the calculation process in problems where the number of load cases is large, allowing for the sensitivity to unitary loads to be determined, and then combine them with the different loads of each case. The rows are linearly independent if the operator $[S_i]$ has an appropriate structure.

Two different errors can be considered. One, given by (2.1), is the error in the temperature space (results space, dependent variables), and the other, the error E_k in the model free parameters space (search space) is defined as

$$E_k = \sum_i^{N_p} (x_{Ri} - x_i)^2, \quad (2.7)$$

where x_{Ri} are the values of the free parameters in the reference model. In the case of thermal mathematical model (TMM) correlation, with monotonous dependence of $\{T\}$ on $\{x\}$, a local linear transformation can be defined between both sets of coordinates (Taylor series expansion, Marquardt 1963) [15],

$$\begin{aligned} t_i &= T_i - T_i^0 & k_j &= x_j - x_j^0 \\ \{t\} &= [M]\{k\}, \end{aligned} \quad (2.8)$$

$$t_i = \sum_{h=1}^{N_p} \frac{\partial t_i}{\partial k_h} k_h; \quad i = 1 \dots N_N, \quad (2.9)$$

where $\{t\}$ and $\{k\}$ are the fluctuations of both temperature and free parameter values around the solution of (2.5) obtained for the initial values of the free parameters $\{T^0\} = \{T(x^0)\}$ and $[M]$ is the Jacobian matrix (2.3). In matrix form

$$[M] = \begin{bmatrix} \frac{\partial t_1}{\partial k_1} & \frac{\partial t_1}{\partial k_2} & \dots & \frac{\partial t_1}{\partial k_{N_p}} \\ \dots & \dots & \dots & \dots \\ \frac{\partial t_{N_N}}{\partial k_1} & \frac{\partial t_{N_N}}{\partial k_2} & \dots & \frac{\partial t_{N_N}}{\partial k_{N_p}} \end{bmatrix} \quad (2.10)$$

where

$$\{t\}^T = \{t_1 \dots t_{N_N}\}^T; \quad \{k\}^T = \{k_1 \dots k_{N_p}\}^T \quad (2.11)$$

$[M]_{N_N \times N_p}$ is a constant matrix for a given load condition, according to (2.6). If $[M]$ is a square and non-singular matrix, (2.8) represents a one-to-one transformation and, therefore, the problems in the results space (2.1) and the search space (2.7) are equivalent. As shown in the example of Fig. 1, in an overdetermined problem, as the dimension of $\{k\}$ is smaller than that of $\{t\}$, the image of space $\{k\}$ is only a subspace of $\{t\}$, and varying $\{k\}$ only a subset of the whole $\{t\}$ space is accessible. This subset may or not include the reference data. However, if in the TMM the relevant free parameters $\{k\}$ have been taken into account, the image of $\{k\}$ should include the value of the target solution $\{t_R\}$ of the problem, and thus a null error can be obtained. On the contrary, the existence of an optimum solution with non-zero error can be due to two situations:

- 1 The mathematical model does not have the right architecture, that is, it does not include the appropriate k_{ij} links in matrix $[K]$.
- 2 The right free parameter set, $\{x\}$ or $\{k\}$ (search space) has not been chosen for the correlation process, even if the right architecture has been considered.

It should be remembered that we are interested in thermal reduced mathematical models (RTMM), that is, with a number of parameters as low as appropriate for an engineering analysis. Generalization to a larger scenario is underway.

3. Sensitivity analysis of a simple 2-node model

The use of simple models is helpful to analyze the influence of the parameters in a given problem (Tsai 2004) [19]. In this case we are interested in discussing the influence of the free parameters in the sensitivities or Jacobian matrix. To do this, let us consider a simple TMM with only two nodes, with linearized effect of radiation (Fig. 2),

$$k_{12} (t_2 - t_1) - k_{1r} t_1 = q_1 \quad (3.1a)$$

$$k_{12} (t_1 - t_2) - k_{2r} t_2 = q_2 \quad (3.1b)$$

Fig. 1. Sketch showing the relation between free parameter space $\{k\}$ (search space, left, dimension 2) and temperature space $\{t\}$ (results space, right, dimension 3). In the results space, the image of search space is the plane labeled k_1 - k_2 . The points t_R (t_{R1}, t_{R2}, t_{R3}) and k_R (k_{R1}, k_{R2}) are the reference values.

Fig. 2. Sketch of the 2-node model. k_{12} , thermal conductance; k_{1r} , k_{2r} linearized radiation terms; q_1 , q_2 heat loads.

where

$$q_r = q_{id} + q_{ir} \quad (3.2a)$$

$$q_{ir} = \varepsilon_i \sigma A_i T_0^4 \quad ; \quad k_{ir} = 4\sigma \varepsilon_i A_i T_0^3 = 4 \frac{q_{ir}}{T_0} \quad (3.2b)$$

and k_{12} is the thermal conductance between nodes 1 and 2, k_{1r} and k_{2r} are the linearized radiation terms (Tsai 2004) [19]; t_1, t_2 are the temperature fluctuations around T_0 (the temperature used to linearize the radiations terms in (3.1) and (3.2)); and q_1 and q_2 are the heat load terms corresponding to nodes 1 and 2, respectively, which include both the dissipated heat load q_{id} and the contribution of linearization of radiation terms q_{ir} . In the systems that we are studying, the fluctuations of temperature are close to the initial guess value. In this condition the radiation terms can be linearized, thus obtaining a linear system, with some equivalent radiative conductance linear terms. The relative error in these terms is of order

$$\frac{3}{2} \frac{\Delta T}{T_R} \simeq 1.5 \frac{6 \text{ K}}{300 \text{ K}} \simeq 0.03$$

where ΔT is the temperature variation and T_0 the temperature used as a reference to linearize the radiative terms. A 3% conductance error does not have a significant influence in the conclusions of the analysis performed in this paper.

Eqs (3.1) and (3.2) can be written in matrix form

$$\begin{bmatrix} -k_{12} - k_{1r} & k_{12} \\ k_{12} & -k_{12} - k_{2r} \end{bmatrix} \begin{Bmatrix} t_1 \\ t_2 \end{Bmatrix} = \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \quad [K] \{t\} = \{Q\} \quad (3.3)$$

To obtain an overdetermined system, the variable k_{12} is chosen as the only free parameter, in order to minimize the error (2.1), which is rewritten now as

$$E_T = (t_{R1} - t_1)^2 + (t_{R2} - t_2)^2 \quad (3.4)$$

that is

$$\frac{\partial E_T}{\partial k_{12}} = (t_{R1} - t_1) \frac{\partial t_1}{\partial k_{12}} + (t_{R2} - t_2) \frac{\partial t_2}{\partial k_{12}} = 0 \quad (3.5)$$

which is one equation to determine the value of k_{12} . To determine the factors $\partial t_i / \partial k_{12}$ the algorithm (2.6) should be used, and taking into account that $\{Q\}$ does not depend on k_{12} , the sensitivity can be written as

$$\frac{\partial}{\partial k_{12}} \{t\} = -[K]^{-1} \left(\frac{\partial}{\partial k_{12}} [K] \right) [K]^{-1} \{Q\} \quad (3.6)$$

that is

$$\begin{aligned} \frac{\partial}{\partial k_{12}} \{t\} &= -[K]^{-1} [K]_{12}' \{t\} \\ &= -[K]^{-1} [K]_{12}' [K]^{-1} \{Q\} \\ &= -\frac{1}{|K|} [K_a] [K]_{12}' \frac{1}{|K|} [K_a] \{Q\} \end{aligned} \quad (3.7)$$

where $[K_a]$ is the adjoin of $[K]$, and

$$[K]_{12}' = \frac{\partial}{\partial k_{12}} [K] = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \quad (3.8)$$

and

$$[K]^{-1} = \frac{1}{|K|} \begin{bmatrix} -k_{12} - k_{2r} & -k_{12} \\ -k_{12} & -k_{12} - k_{1r} \end{bmatrix} = \frac{1}{|K|} [K_a] \quad (3.9)$$

where $|K|$ is the determinant of $[K]$

$$|K| = (-k_{12} - k_{2r})(-k_{12} - k_{1r}) - k_{12}^2 = k_{1r}k_{2r} + k_{12}(k_{1r} + k_{2r}) \quad (3.10)$$

Finally one obtains from (3.7),

$$\begin{aligned} \frac{\partial}{\partial k_{12}} \{t\} &= \frac{1}{|K|^2} [H_{12}] \{Q\} \\ &= \frac{1}{|K|^2} \begin{Bmatrix} -k_{2r}^2 q_1 + q_2 k_{2r} k_{1r} \\ k_{1r} k_{2r} q_1 - k_{1r}^2 q_2 \end{Bmatrix} \\ &= \frac{(-k_{2r} q_1 + k_{1r} q_2)}{|K|^2} \begin{Bmatrix} k_{2r} \\ -k_{1r} \end{Bmatrix} \end{aligned} \quad (3.11)$$

Condition (2.4) to determine the solution $\{k\}$ of the problem can be written as

$$\{t_R - t\}^T \left\{ \frac{\partial t}{\partial k_{12}} \right\} = 0 \quad (3.12)$$

The temperature predicted by the model from (2.5) is given by

$$\begin{aligned} \{t\} &= [K]^{-1} \{Q\} = \frac{1}{|K|} [K_a] \{Q\} \\ &= \frac{1}{|K|} \begin{bmatrix} -k_{12} - k_{2r} & -k_{12} \\ -k_{12} & -k_{12} - k_{1r} \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} \\ &= \frac{1}{|K|} \begin{Bmatrix} (-k_{12} - k_{2r})q_1 - k_{12}q_2 \\ (-k_{12}q_1 + (-k_{12} - k_{1r})q_2 \end{Bmatrix} \end{aligned} \quad (3.13)$$

Substituting (3.11) and (3.13) into (3.12) leads to

$$(t_{R1} - t_1) \frac{\partial t_1}{\partial k_{12}} + (t_{R2} - t_2) \frac{\partial t_2}{\partial k_{12}} = 0 \quad (3.14)$$

that is

$$\left[t_{R1} - \frac{1}{|K|} [(-k_{12} - k_{2r})q_1 - k_{12}q_2] \right] \cdot [-k_{2r}q_1 + k_{1r}q_2]k_{2r} + \left[t_{R2} - \frac{1}{|K|} [-k_{12}q_1 + (-k_{12} - k_{1r})q_2] \right] \cdot [k_{2r}q_1 - k_{1r}q_2]k_{1r} = 0 \quad (3.15)$$

Equation (3.15) is a linear equation that needs to be solved to determine the free parameter k_{12} as a function of the other parameters of the model. Before proceeding to do that, let us analyze the Jacobian matrix (3.11), which leads us to some interesting conclusions (see Table 1). If both sensitivities are $\frac{\partial t_1}{\partial k_{12}} = \frac{\partial t_2}{\partial k_{12}} = 0$, obviously it is not possible to use k_{12} as a free parameter. If one of them is zero, then the corresponding temperature cannot be fitted using k_{12} . For instance, let us consider the case $k_{2r} = 0$ and $q_1 = 0$, sketched in Fig. 3. Then $\partial t_1 / \partial k_{12} = 0$ and (3.15) leads to

$$t_{R2} - \frac{1}{|K|} (-k_{12} - k_{1r})q_2 = 0 \quad (3.16)$$

Therefore, from (3.16)

$$\frac{t_{R2}}{q_2} = \frac{k_{12} + k_{1r}}{-k_{12}k_{1r}} = \frac{1}{k_{1r}} - \frac{1}{k_{12}} ; \quad \frac{1}{k_{12}} = \frac{1}{k_{1r}} - \frac{t_{R2}}{q_2} ; \quad (3.17)$$

and one obtains the solution

$$k_{12} = \frac{1}{\frac{1}{k_{1r}} - \frac{t_{R2}}{q_2}} \quad (3.18)$$

Now the temperatures of the model can be determined from (3.13), using (3.18)

$$\{t\} = \frac{1}{k_{12}k_{1r}} \begin{Bmatrix} k_{12}q_2 \\ (k_{12} + k_{1r})q_2 \end{Bmatrix} \quad (3.19)$$

$$t_1 = \frac{q_2}{k_{1r}} ; \quad t_2 = \frac{k_{12} + k_{1r}}{k_{12}k_{1r}} q_2 = t_{R2} \quad (3.20)$$

It can be remarked that t_2 is equal to t_{R2} (that is, is determined without error), but t_1 cannot be fitted at all. This result is a consequence of the physical characteristics of the system shown in Fig. 3: t_1 is independent of k_{12} , as the thermal flux through k_{1r} is q_2 (as $q_1 = 0$). But t_2 does depend on both t_1 and k_{12} .

It is clear that the use of the Jacobian matrix can help to analyze the election of the free parameters for optimization. Additional information can also be obtained concerning the modelization of the system. For instance, a linear dependency between rows (or between columns) in $[M]$ is an indication of an excess of parameters (node temperatures or free parameters), as is shown in Sections 5 and 6.

Table 1
Values of the parameters leading to singularities of the Jacobian matrix.

	q_1	q_2	k_{1r}	k_{2r}	$-k_{2r}q_1 + k_{1r}q_2$
$\frac{\partial t_1}{\partial k_{12}} = 0$	0	0	-	-	-
	0	-	0	-	-
	-	-	-	0	-
	-	-	-	-	0
$\frac{\partial t_2}{\partial k_{12}} = 0$	0	0	-	-	-
	-	-	0	-	-
	-	0	-	0	-
	-	-	-	-	0

Fig. 3. Sketch of the 2-node model. Singular conditions, case $k_{2r} = 0$ and $q_1 = 0$. k_{12} , thermal conductance; k_{1r} , linearized radiation; q_2 heat load.

4. Model parameter updating

4.1. Algorithm

In TMM correlation, the mathematical model is known, and the aim of the task is to determine the relatively small adjustments in the free parameters $\{k\}$ needed to correct the error $\{t_R\}$ between the TMM and the reference model (either experimental data or DTMM results). The mathematical model has been linearized in the proximity of the initial solution as in (2.8). The minimum error condition from (2.2) and (2.4) is given by

$$\sum_{j=1}^{N_N} (T_{Rj} - T_j) \frac{\partial T_j}{\partial k_i} = 0 ; \quad \sum_{j=1}^{N_N} \left(T_{Rj} - T_j^0 - \sum_{h=1}^{N_P} \frac{\partial T_j}{\partial k_h} k_h \right) \frac{\partial T_j}{\partial k_i} = 0. \quad (4.1)$$

Therefore

$$\sum_{j=1}^{N_N} \left(t_{Rj} - \sum_{h=1}^{N_P} \frac{\partial T_j}{\partial k_h} k_h \right) \frac{\partial T_j}{\partial k_i} = 0; \quad i = 1 \dots N_P. \quad (4.2)$$

After some algebra, this equation system can be rewritten in matrix form (Curry and Williams 1972) [1],

$$[M]^T \{t_R\} = [M]^T [M] \{k\}. \quad (4.3)$$

This N_P equation system can be solved for $\{k\}$ by left multiplying it by the inverse of $[M_c] = [M]^T [M]$, if it is non-singular. $[M_c]$ is a square symmetric matrix, $N_P \times N_P$ size. Therefore, the solution $\{k^*\}$ is (Marquardt 1963) [15],

$$\{k^*\} = [M_c]^+ \{t_R\} ; \quad [M_c]^+ = ([M]^T [M])^{-1} [M]^T. \quad (4.4)$$

If $N_P > N_N$, $[M_c]$ is rank N_N at most, and Eq. (4.3) is an undetermined system, with multiple solutions that give zero error, which is of no much interest for the correlation problem. If $N_P < N_N$, $[M_c]$ is rank N_P at most, and Eq. (4.3) is an overdetermined system, with no solution in a general case. However, this is the interesting situation, because a best fit result can be obtained using $[M_c]^+$, the classical Moore-Penrose pseudo-inverse of $[M]$ (MacAusland 2014 [20]). This result can be generalized to any number of load cases, where a different Jacobian matrix $[M]$ is associated to each load case according to (2.6). In a space project, the heat load cases to be considered in the design can be large, as a result of the operation modes, as well as the thermal test conditions. For instance, four kinds of thermal environments are considered for the thermal design of a solar array in (Li et al., 2013) [21]. Let us consider here just two load cases, C1, C2 (e.g. hot case, cold case). Condition (4.1) can be rewritten as

$$\left(\sum_j T_{Rj} - T_j \right) \frac{\partial T_j}{\partial k_i} \Big|_{C_1} + \left(\sum_j T_{Rj} - T_j \right) \frac{\partial T_j}{\partial k_i} \Big|_{C_2} = 0, \quad (4.5)$$

The same procedure to obtain (4.3) can be followed to get to

$$[M]^T \{t_R\} \Big|_{C_1} + [M]^T \{t_R\} \Big|_{C_2} = ([M]^T [M] \{k\}) \Big|_{C_1} + [M]^T [M] \{k\} \Big|_{C_2}, \quad (4.6)$$

or

$$[M]^T \{t_R\}_{C_1} + [M]^T \{t_R\}_{C_2} = ([M]^T [M])_{C_1} + [M]^T [M]_{C_2} \{k\}, \quad (4.7)$$

where $\{k\}$, a common factor which appears in the two terms of RHS of (4.6), is the same for both cases, C1 and C2. Eq. (4.7) is analogue to (4.3). Finally, we obtain the solution for the multiple load case

$$\{k^*\} = [M_c]_{C_1 C_2}^{-1} ([M]^T \{t_R\}_{C_1} + [M]^T \{t_R\}_{C_2}), \quad (4.8)$$

where

$$[M_c]_{C_1 C_2} = [M]^T [M]_{C_1} + [M]^T [M]_{C_2}, \quad (4.9)$$

and $[M_c]_{C_1 C_2}$ is a square matrix which combines the Jacobian matrices of both load cases, $[M]_{C_1}$ and $[M]_{C_2}$. Eq. (4.8) can be regarded as a kind of average of the influence of both test cases C1 and C2. This equation can be easily generalized to any number of load cases. If the phenomena under consideration depend on the parameters x_i in a non-linear way, it is possible to use the result (4.8) or (4.4) to obtain the correction of the values of conductance matrix $[K(x)]$, and then applying iteratively the algorithm to the configuration obtained from the previous step. The analysis of iteration process in some numerical test performed is presented in sections 5 and 6.

4.2. Example. Overdetermined system

The idea of this section is to analyze the 2-node TMM model given in (3.3), using (4.4). However, written in this form would give rise to a too complex formulation. Thus, eq. (2.8) for this system can be better written as

$$\begin{Bmatrix} t_1 \\ t_2 \end{Bmatrix} = \begin{bmatrix} M_{11} \\ M_{21} \end{bmatrix} k_1; \quad [M] = \begin{bmatrix} M_{11} \\ M_{21} \end{bmatrix}. \quad (4.10)$$

Using (4.4) one obtains

$$\begin{aligned} \{k_1^*\} &= \left([M_{11} \ M_{21}] \begin{Bmatrix} M_{11} \\ M_{21} \end{Bmatrix} \right)^{-1} [M_{11} \ M_{21}] \begin{Bmatrix} t_{R1} \\ t_{R2} \end{Bmatrix} \\ &= (M_{11}^2 + M_{21}^2)^{-1} (M_{11} t_{R1} + M_{21} t_{R2}) = \frac{M_{11} t_{R1} + M_{21} t_{R2}}{M_{11}^2 + M_{21}^2} \end{aligned} \quad (4.11)$$

and the correlation result for the model temperatures is obtained from (4.10) by using (4.11),

$$\begin{aligned} t_{M1}^* &= M_{11} k_1^* = M_{11} \frac{t_{R1} M_{11} + t_{R2} M_{21}}{M_{11}^2 + M_{21}^2}, \\ t_{M2}^* &= M_{21} k_1^* = M_{21} \frac{t_{R1} M_{11} + t_{R2} M_{21}}{M_{11}^2 + M_{21}^2}. \end{aligned} \quad (4.12)$$

This situation is sketched in Fig. 4, where the points representing the correlation result $P_M^*(t_{M1}^*, t_{M2}^*)$ and reference condition $P_R(t_{R1}, t_{R2})$ are shown. The error in the results space is an ellipsoid (Marquardt 1963) [15], which is given by

$$E_T = (t_{R1} - M_{11} k_1^*)^2 + (t_{R2} - M_{21} k_1^*)^2, \quad (4.13)$$

that can be rewritten as

$$\sqrt{E_T} = \frac{p - q}{(1 + p^2)^{1/2}} \quad (4.14)$$

where $p = M_{21}/M_{11}$ and $q = t_{R2}/t_{R1}$. Expression (4.14) is the distance from point $P_R(t_{R1}, t_{R2})$ to the straight line OM (results from the model), as shown in Fig. 4. The OM line normal components (a, b) are given by $a = 1/M_{11}$, $b = -1/M_{21}$, and the distance is

$$d = \frac{(x_p - x_0)a + (y_p - y_0)b}{(a^2 + b^2)^{1/2}} \quad (4.15)$$

in this case with $x_0 = y_0 = 0$. This expression can be used to improve the fitting of the model to the reference data. The error can be made zero if the condition $p = q$ is fulfilled

$$\frac{M_{21}}{M_{11}} = \frac{t_{R2}}{t_{R1}} \quad (4.16)$$

Then, the solution (4.11) with zero error (using (4.16)) is

$$k_1^{**} = \frac{t_{R1}}{M_{11}} = \frac{t_{R2}}{M_{21}} \quad (4.28)$$

But the fulfillment of condition (4.16) may imply a change in the Jacobian itself, not only in the values of the parameters that are involved in the conductance matrix $[K]$. One problem is how to find this change, not in an arbitrary form but in a way that keeps the physical meaning of the configuration.

From a mathematical point of view, this optimization problem can be modeled as the orthogonal projection of point P_R onto the line OM, which represents the model results (Fig. 4). It can be easily generalized first to the case of a hyperplane, where $N_N - N_P = 1$, and then to any dimension by using a recursive process.

In the case of a nonlinear mathematical model, the correlation results can be improved by iteration, starting with the result of the first iteration step S1, as shown in Fig. 4b.

5. Klement's 4-node model (K4 model)

In this section, the concepts developed in previous sections are

Fig. 4. a) Sketch of the error variation E_T in the result space (t_1, t_2) . $E_{Tmin} = 0$ minimum value of E_T , is reached at the reference point $P_R(t_{R1}, t_{R2})$; $P_M^*(t_{M1}^*, t_{M2}^*)$, correlation result. OM, subspace of model results. b) As a), for two iterations, S1 and S2.

applied to solve the Klement's 4-node model [14], by determining the correction $\{k^*\}$ needed to minimize the error, using (4.4). Klement's 4-node model is sketched in Fig. 5. It consists of four nodes placed inside an environment at 0°C . Each node radiates to the environment with an emissivity $\varepsilon_i = 1$ and has a surface area $A = 0.1\text{ m}^2$ leading to a linearized radiative thermal conductance k_{ir} as defined in (3.2). A heat load $q_{id} = 10\text{ W}$ is dissipated at each node. Each node is conductively connected to each other through a thermal conductance k_{ij} ($i = 1..4, j = 1..4, i \neq j$). Node 1 is connected to the environment wall with a thermal conductance $k_{w1} = 1\text{ W/K}$.

The free parameters are the thermal conductances between nodes, k_{ij} , or a subset, depending of the problem considered, as follows $x_1 = k_{12}$, $x_2 = k_{13}$, $x_3 = k_{14}$, $x_4 = k_{23}$, $x_5 = k_{24}$, $x_6 = k_{34}$. In Klement's paper, the nomenclature is $GL_i = x_i$. The reference temperatures are defined as those obtained from the model for a given value of $\{x\}$ denoted as $\{x\}_R$, where $x_{Ri} = (0.1 + i \cdot 10^{-2})\text{ W/K}$.

As explained in the introduction, Klement considered three types of problems (taking into account the four node temperatures to be correlated):

- Undetermined system (six free parameters: x_1 to x_6).
- Determined system (four free parameters; x_1, x_2, x_4 , and x_5 ; x_3 and x_6 are frozen at the reference values $x_3 = 0.13\text{ W/K}$, $x_6 = 0.16\text{ W/K}$).
- Overdetermined system (three free parameters: x_1, x_2, x_4 , and $x_3 = 0.13\text{ W/K}$, $x_5 = 0.6\text{ W/K}$, $x_6 = 0.16\text{ W/K}$). As x_5 is not the reference value, zero error cannot be achieved.

The slowest convergence rate was shown by the overdetermined system (which does not reach zero error). Because of both characteristics (and the lack of a unique solution of undetermined system), we have chosen this kind of system to analyze the problem with the help of algorithm (4.4). Let us first make some comments. In Klement's paper, the starting conditions to initiate the iteration is $\{x\}_0$, with $x_{0i} = 0.5\text{ W/K}$. In the case of the undetermined system, these starting conditions lead to a singular Jacobian matrix, as the last three columns are zero (Table 2).

The reason for the Jacobian matrix singularity is the symmetry of the configuration, the initial values x_{0i} , and heat load conditions. In the Jacobian matrix, the last 3 columns are zero, showing that there is no dependence on parameters x_4, x_5 and x_6 (observability problem).

In an attempt to avoid this symmetry, we have made a slight change in the initial conditions with regard to x_{0i} , ($x_1 = x_3 = x_5 = 0.4\text{ W/K}$, $x_2 = x_4 = x_6 = 0.6\text{ W/K}$), but the Jacobian matrix thus obtained was still

Fig. 5. Sketch of Klement's 4-node model. k_{ij} , thermal conductances; k_{w1} , node 1-wall thermal conductance; k_{ir} , linearized radiation terms; q_i , heat loads.

ill-conditioned (Table 2), as the last three columns still have values close to zero. This result is a consequence of the symmetry of the problem as x_4, x_5 and x_6 have no much influence on the temperatures T_1 to T_4 in this load case (which is also symmetric).

In fact, the singular value decomposition of $[M]$ gives only three non-zero singular values, instead of four, which in principle should have been the rank of the system. However, from the geometry of the system it is deduced that the thermal links x_4, x_5 and x_6 play a little role in the heat load case considered, as the heat flow through them should be very small.

Therefore, we considered appropriate to discuss the system obtained when only the three relevant free parameters (x_1, x_2, x_3) are used for correlation, which will make the system better conditioned, and more sound. In fact, this choice also changes the character of the system to an overdetermined one.

Additionally, it was observed that the Klement's initial conditions were too far from the reference temperatures to obtain physically suitable values of the correction for free parameters $\{k^*\}$. In order to clarify this point, the influence of the distance from the starting point to the reference conditions was considered. To this end, the initial conditions $\{x\}_0$ were defined as proportional to the reference conditions $\{x\}_R$

$$\{x\}_0 = c_0 \{x\}_R \quad (5.1)$$

where c_0 is the parameter that gives the distance between initial and reference conditions, $0 < c_0 < 2.5$. Obviously, the exact solution of the problem is obtained when $c_0 = 1$. To facilitate the comparison, five auxiliary vectors are considered (Table 3), which measure the quality of the correlation results.

In order to allow a scalar comparison, we consider the norm of these vectors N_{xR} , N_{xRM} , N_k^* , N_{TRO} , and N_{TRM} . The results obtained are shown in Fig. 6. Results for $c_0 = 0$ are summarized in Table 4 (step S1).

For $c_0 > 2.5$, the correlated result for the parameters $\{x\}_M$ obtained from the algorithm gets outside the physically significant range, so that the range $c_0 > 2.5$ is not explored in this case.

In all cases considered, the correlation temperature error is $N_{TRM} < 10^{-6}$ at the first step, therefore, in principle, no further iteration would be considered necessary, concerning temperature correlation. This is the reason why this parameter has not been plotted in Fig. 6. However, this point is discussed further below.

As shown in Fig. 6, the parameter correction norm N_k^* , the initial temperature difference norm N_{TRO} , and the thermal conductance error N_{xRM} decreases as c_0 approaches 1, from both sides, $c_0 < 1$ and $c_0 > 1$.

Both N_k^* and N_{TRO} follow an almost linear variation, while the thermal conductance error N_{xRM} follows a parabolic path.

It is worth noting that even if the temperature error obtained N_{TRM} is already very small in the initial iteration, the error in thermal parameters N_{xRM} is a function of the distance of the initial condition to the reference values, c_0 ; e.g. for $c_0 = 0$ is $N_{xRM} \approx 0.03$, and $N_{xRM} \approx 0.20$ for $c_0 = 2.5$. Unfortunately, the error N_{xRM} in the calculated parameter value $\{x\}_M$ can be significantly larger than temperature error, even if this error N_{TRM} is quite small. Therefore, an apparently good convergence in temperature terms is not as good in parameter $\{x\}$ terms. In fact, to obtain a good final correlation (a small error N_{xRM}) the correction vector $\{k^*\}$ should also be small enough.

Incidentally, a question arises concerning the election of the initial value. In a real case the reference values $\{x\}_R$ are unknown, therefore the distance c_0 cannot be defined using (5.1). However, according to the previous results ($N_{xRM} \approx 0.03$ for $c_0 = 0$, $N_{xRM} \approx 0.20$ for $c_0 = 2.5$) the initial value $\{x\}_0 = 0$ seems to be a reasonable choice, better than larger values of $\{x\}_0$, and easier to define, because is independent of $\{x\}_R$.

To improve the results, an iteration process is considered. To analyze this iteration process, let us consider the initial condition $\{x\}_0 = 0$ denoted S1, thus $c_{0-S1} = 0$. As shown in Table 4 one obtains the correction $\{k^*\}_{M-S1} = [0.098, 0.102, 0.106]$ and temperature error $N_{TRM-S1} = 8.7 \cdot 10^{-15}$, but the parameter error is $N_{xRM-S1} = 0.031$, even if $\{k^*\}_M$ is already close to the reference value $\{x\}_R$.

Table 2Jacobian matrix $[M]_{4 \times 6}$ for starting values that lead to a singular condition.

Klement's initial conditions $x_{0i} = 0.5 \text{ W/K}$						Modified initial conditions $x_{0i} = (0.5 + 0.1 \cdot (-1)^i) \text{ W/K}$					
0.97	0.97	0.97	0	0	0	1.12	0.98	1.12	0	0	0
-2.23	-0.43	-0.43	0	0	0	-2.58	-0.55	-0.47	-0.13	0	0
-0.43	-2.23	-0.43	0	0	0	-0.51	-2.01	-0.51	0.11	0	0.11
-0.43	-0.43	-2.23	0	0	0	-0.47	-0.55	-2.58	0	0	-0.13

Table 3

Quality parameters considered in the assessment of the correlation process.

Definition	Vector	Norm	Units
Reference parameter vector	$\{x\}_R$	N_{xR}	"
Free parameter error, where $\{x\}_M = \{x\}_0 + \{k^*\}_M$	$\{x\}_R - \{x\}_M$	N_{xRM}	"
Free parameter calculated correction vector	$\{k^*\}_M$	N_{k^*}	"
Temperature difference between reference and initial conditions, $\{t\}_R = \{T\}_R - \{T\}_0$	$\{t\}_R$	N_{TR0}	°C
Correlation temperature error	$\{T\}_R - \{T\}_M$	N_{TRM}	°C

^a No unit is defined as the parameters used may be different. **Fig. 6.** Variation as a function of the initial condition factor c_0 , of the quality parameters: free parameter error N_{xRM} (+), calculated correction vector norm N_{k^*} (*), and initial temperature difference norm N_{TR0} (triangle). Klement's 4-node model. Free parameters x_1, x_2, x_3 . Units are described in Table 3.

To improve the determination of $\{x\}$, a new iteration can be started (S2), considering as new initial condition the result of iteration step S1, $\{x\}_{0,S2} = \{x\}_{M,S1} = \{k^*\}_{S1}$. In this case, the initial condition factor for the second step can be estimated as

$$c_{0-S2} \approx \frac{N_{x0-S2}}{N_{xR}} = \frac{N_{k^*-S1}}{N_{xR}} = \frac{0.177}{0.208} = 0.85$$

as the norm of reference parameter vector $\{x\}_R$ is $N_{xR} = 0.208$. As shown in Fig. 6 this value $c_{0-S2} = 0.85$ is much closer to the exact solution than the starting conditions of step S1. The exact solution is obtained when the initial conditions satisfy (5.1) with $c_0 = 1$. In this second step, the temperature error is $N_{TRM-S2} = 1.8 \cdot 10^{-15}$ and the parameter error is $N_{xRM-S2} = 0.007$, much larger, but the relative error is small $N_{xRM-S2}/N_{xR} = 0.007/0.208 = 3\%$. The initial temperature difference is $N_{TR0-S2} = 0.52^\circ\text{C}$, which compared to N_{TR0-S1} gives $N_{TR0-S2}/N_{TR0-S1} \approx 0.52/5.1 \approx 0.10$, that is, the initial temperature difference is reduced to 10%, because of the initial parameters in step S2 are much

Table 4First two iterations, S1 and S2, of the correlation process for Klement's 4-node model. Free parameters considered x_1, x_2, x_3 .

Reference values					
$[K]_R =$			$\{T\}_R =$		$\{x\}_R =$
-1.82	0.11	0.12	0.13	9.2660	0.1100
0.11	-0.86	0.14	0.15	19.2073	0.1200
0.12	0.14	-0.88	0.16	19.1081	0.1300
0.13	0.15	0.16	-0.90	19.0163	
					$N_{xR} = 0.2083$
Iteration S1 ($c_{0-S1} = 0$)					
$[K]_0 =$			$\{T\}_0 =$		
-1.46	0	0	0	6.8423	
0	-1.46	0	0	21.6691	
0	0	-1.46	0	21.6691	
0	0	0	-1.46	21.6691	
					$N_{TR0} = 5.05$
$[M] =$			$[M_c] =$		
7.89	7.89	7.89	687	62	62
-25.00	0	0	62	687	62
0	-25.00	0	62	62	687
0	0	-25.00			
$\{k^*\}_M$	$\{T\}_M$	$\{T_R - T_M\}$	$\{x\}_0$	$\{x\}_M$	$\{x_R - x_M\}$
		$10^{-14} \times$			
0.098	9.266	0.710	0	0.098	0.011
0.102	19.207	0.355	0	0.102	0.017
0.106	19.108	0.355	0	0.106	0.023
	19.016	0			
N_{k^*}		N_{TRM}	N_{x0}		N_{xRM}
		$10^{-15} \times$			
0.177		8.70	0.031		0.031
Iteration S2 ($c_{0-S2} = 0.85$)					
$[K]_0 =$			$\{T\}_0 =$		
-1.76	0.09	0.10	0.10	9.0173	
0.09	-0.56	0	0	19.4437	
0.10	0	-0.56	0	19.3716	
0.10	0	0	-0.56	19.3041	
					$N_{TR0} = 0.5195$
$[M] =$			$[M_c] =$		
4.11	4.06	4.01	230	-3	-4
-14.55	0.71	0.70	-3	223	-4
0.74	-14.35	0.73	-4	-4	217
0.76	0.76	-14.16			
$\{k^*\}_M$	$\{T\}_M$	$\{T_R - T_M\}$	$\{x\}_0$	$\{x\}_M$	$\{x_R - x_M\}$
		$10^{-14} \times$			
0.018	9.266	0.177	0.098	0.116	-0.006
0.020	19.207	0	0.102	0.122	-0.002
0.022	19.108	0	0.106	0.128	0.001
	19.016	0			
N_{k^*}		N_{TRM}	N_{x0}		N_{xRM}
		$10^{-15} \times$			
0.035		1.77	0.177		0.007

closer to the reference values.

Concerning the Jacobian matrix $[M]$ in case S1, the first row indicates that $\partial T_i / \partial x_i$ ($i = 1, 2, 3$) is the same for x_1, x_2 and x_3 (see Table 3), as can be easily deduced from the configuration symmetry. The same is true in case S2, but with some small deviations. The rest of Jacobian matrix (rows 2 to 4) forms a diagonal submatrix, that is, null except the diagonal

$\partial T_i / \partial x_i \neq 0$, which can be easily explained, based on the absence of heat flux between nodes 2, 3 and 4. Besides, the values are the same for all nodes $i = 1, 2, 3$, which can also be easily explained, based on the symmetry of the configuration (thermal links and heat loading).

In case S2 the situation is quite similar, as the elements outside the diagonal are quite small compared to the diagonal elements.

In Klement's paper, temperature errors are reported, but there is no information on parameter errors, therefore the convergence of the parameters cannot be evaluated, as it has been presented in Table 4. In fact, undetermined systems can converge in temperature, but the solution obtained in parameter $\{x\}_M$ can be significantly different from the reference value $\{x\}_R$.

6. PHI -ELE model

Solar Orbiter is a European Space Agency (ESA) mission whose main scientific objective is to understand how the Sun creates and modifies the heliosphere. With the perihelion of the orbit at 0.28 AU, it will receive a solar load equivalent to 13 solar constants, what makes the thermal design a very challenging task.

PHI (Polarimetric Helioseismic Imager) is one of the remote sensing instruments of Solar Orbiter. The PHI Electronics Unit (PHI-ELE) is a separated unit that carries all the electronics that manage the data and the subsystems, and provide it with the necessary power. Is a modular assembly consisting of six horizontally stacked modules, each 200 mm $\times$ 200 mm $\times$ 20 mm. Each module consists of a dissipating (or two) PCBs, a frame to support it, and the connectors needed to communicate with the others. The box is sitting on one of the spacecraft panels with a strong thermal conductive coupling. The PCB frames, baseplate and top lid are made from Aluminum alloy 7075 T7351 with alodine protection, and are painted black with black conductive coating to maximize thermal radiation to the environment.

6.1. Model

The analysis is based on a DTMM with 4657 nodes, from which a RTMM is manually derived. The geometrical model nodal diagram (GMND) of the RTMM is depicted in Fig. 7, as taken directly from ESATAN output.

The thermo-optical materials employed are Aeroglaze Z306 and conformal coating, both with IR emissivity 0.90. The load cases are defined in Table 5. The couplings between the nodes are sketched in Fig. 8.

All the conductors appearing in the RTMM represent a physical thermal path. The inner side of PHI_ELE_wall_frame, the PHI_ELE_boards

Table 5

Load cases definition. SC, spacecraft walls.
Cases: hot operational (C1); cold operational (C2).

Element	Node	C1	C2
Electronics dissipation	PCBs (Node 10800)	30.9	14.4
q_{7d} [W]			
Wall temperature	SC Radiative I/F (Node 90000)	50.0	-20.0
t_{SC} [°C]	SC Conductive I/F (Node 90001)	48.9	-21.1

Table 6

Linear couplings ($GL = k_{ij}$; thermal conductance). Label in TCS (see Figs. 7 and 8 and Table 8).

Elements	Linear couplings between nodes	Free parameter	Description
Structure internal	$k_{12} = k_{13} = k_{23} = k_{34}$	x_1	sidewall - sidewall
	$k_{15} = k_{25} = k_{35} = k_{45}$	x_2	sidewall - top shell
	$k_{16} = k_{26} = k_{36} = k_{46}$	x_3	sidewall - baseplate
Structure-equipment	$k_{17} = k_{27} = k_{37} = k_{47}$	x_4	board - sidewall
	k_{67}	x_5	board - baseplate
Interface couplings	$k_{6,SC}$	x_6	baseplate - spacecraft

and the PHI_ELE_IF_wall_frame shells are radiatively decoupled. This situation prevents the ESARAD tool to generate radiative conductors between them. The only radiative couplings used in the model are those from the external faces of the PHI_ELE_wall frame to the Spacecraft (SC).

The PHI-ELE Boards and the PHI-ELE interface (I/F) wall frames have been connected to each other by linear conductors. The conductive couplings GL (or k_{ij}) used are shown in Table 6, as a function of the parameters of the problem (x_1 to x_6). The values of these parameters are given in Table 7. They have been appropriately selected to match the results obtained from the detailed model.

Radiative couplings are calculated by ESATAN-ESARAD, and the results are $GR_{1,SC} = GR_{2,SC} = GR_{3,SC} = GR_{4,SC} = 0.03105 \text{ m}^2$ (sidewall-spacecraft), and $GR_{5,SC} = 0.03600 \text{ m}^2$ (top shell-spacecraft). Only the external faces of the Electronics Unit walls have radiative couplings. The inner face of the shells are radiatively decoupled between them. In the hot operational case (C1) electronics boards dissipate 30.9 W. The manual correlation of the model gives a maximum temperature difference between the RTMM and DTMM of 1.7 °C, which keeps the model inside the allowed margins. These margins are <3 °C for the equipment (PHI_ELE_Boards) and <5 °C for the external surfaces. Results are shown in Table 8. The heat fluxes obtained in the RTMM are also within the allowable margins, since the maximum error is 10%.

Fig. 7. PHI-ELE Geometrical Model Nodal Diagram. Figures indicate the node labeling in ESATAN model.

Fig. 8. PHI-ELE thermal coupling scheme (TCS). Figures: simplified node labeling (see Table 6). SC, spacecraft walls; q_{7d} , heat load.

Table 7
Values of the parameters of the model [W/K].

x_1	x_2	x_3	x_4	x_5	x_6
1.0	0.225	1.175	1.05	1.4375	12.2

In the cold operational case (Table 9), the correlated model results are very close to those obtained from the DTMM. The maximum difference in temperatures with the DTMM is 2.0 °C. This maximum error is far from the requirement of 3 °C or 5 °C depending on the part. The heat fluxes obtained in the RTMM are also within the allowable margins, since the maximum error is 10%.

In order to analyze this RTMM, looking for ways of improving it, a linearized version of the 7 nodes RTMM has been developed and used in the algorithm (4.8) to determine the correction of the free parameters $\{x\}$, as in the previous section.

As shown in Tables 6 and 7, although there are 18 GL (conductance couplings), only 6 parameters are independent. The radiative couplings GR are not included in the correlation process, and are kept constant. We analyze now the correlation process using the algorithm developed (4.8).

The first step is to determine: 1) the conductance matrix $[K]_{7 \times 7}$, 2) the load vectors $\{Q\}$ corresponding to hot and cold cases, $C1$ and $C2$, respectively, and 3) the respective Jacobian matrixes, $[M]_{C1}$ and $[M]_{C2}$, as required by (4.8). At this point some conclusions can be obtained from

the analysis of the two Jacobian matrixes (they are very similar to each other), one of them is (hot case loading, $[M]_{C1}$)

Table 9
Cold operational case. Temperatures and heat fluxes.

Temperatures Element	Detailed model [°C]	Reduced model [°C]	Error [°C]
PHI_ELE_wall_frame_plus_Xurf	-17.6	-18.3	-0.7
PHI_ELE_wall_frame_plus_Yurf	-17.7	-18.3	-0.6
PHI_ELE_wall_frame_minus_Xurf	-17.6	-18.3	-0.7
PHI_ELE_wall_frame_minus_Yurf	-17.9	-18.3	-0.4
PHI_ELE_wall_frame_plus_Zurf	-16.6	-18.6	-2.0
PHI_ELE_IF_wall_frame	-20.4	-20.0	0.4
PHI_ELE_boards	-15.8	-16.3	-0.5
Heat flux	Detailed model [W]	Reduced model [W]	Error [%]
PHI_ELE base to S/C by conduction	13.3	13.24	-0.4
PHI_ELE wall frame to S/C by radiation	0.9	0.96	5.9
Total heat flux (W)	14.2	14.2	0

Table 8
Hot operational case. Temperatures and heat fluxes. N_i , label in TCS (Fig. 8).

Temperatures Element	Detailed model [°C]	Reduced model [°C]	Error [°C]	N_i	ESATAN label
PHI_ELE_wall_frame_plus_Xurf	53.8	54.1	0.3	1	10700
PHI_ELE_wall_frame_plus_Yurf	53.8	54.1	0.3	2	10710
PHI_ELE_wall_frame_minus_Xurf	53.2	54.1	0.9	3	10720
PHI_ELE_wall_frame_minus_Yurf	53.6	54.1	0.5	4	10730
PHI_ELE_wall_frame_plus_Zurf	54.8	53.1	-1.7	5	10740
PHI_ELE_IF_wall_frame	49.7	51.0	1.3	6	10750
PHI_ELE_boards	57.1	58.8	1.7	7	10800
Heat flux	Detailed model [W]	Reduced model [W]	Error [%]		
PHI_ELE base to S/C by conduction	26.1	26.0	-0.1		
PHI_ELE wall frame to S/C by radiation	4.8	4.9	1.0		
Total heat flux (W)	30.9	30.9	0		

	0	-0.13	-1.58	0.60	-0.75	-0.13
	0	-0.13	-1.58	0.60	-0.75	-0.13
	0	-0.13	-1.58	0.60	-0.75	-0.13
$[M]_{C1} =$	0	-0.13	-1.58	0.60	-0.75	-0.13
	0	2.36	-1.21	0.46	-0.58	-0.10
	0	-0.04	0.15	-0.05	0.07	-0.16
	0	-0.11	-1.13	-2.68	-1.90	-0.14

Some preliminary conclusions worth attention. First, the parameter x_1 (the lateral conductances between sidewalls) plays no role in the results, as deduced from the first column, which is zero (this is a consequence of the symmetry of both the model and heat load arrangement). It does not fulfill the observability condition. Therefore, this parameter has not been included in the correlation process.

Second, in both load cases C1 and C2, the four first nodes 1 to 4 are not independent as their sensibilities are the same for all, $\partial T_i / \partial x_j$ for $i = 1, 2, 3$ and 4 (observe that the four first rows are identical). It is clear that in the load cases considered, these four nodes cannot be distinguished from each other. Therefore, only one of them is needed for the analysis.

From this result it can be concluded that, for the correlation process, the model can be further reduced by condensation of these four nodes into one. Also, this redundancy problem is the origin of the poor conditioning of matrix $[M]_{C1C2}$ (4.9).

6.2. PHI ELE 4-node model

In order to solve the problem just described, a 4-node model has been developed, by condensing nodes 1 to 4 just into one (node A in Fig. 9). The condensed model (CRTMM-4) allows the temperatures for $N_N = 4$ nodes to be determined for each load case, therefore the total number of reference results is $N_N = 8$. The number of free parameters is $N_P = 5$ (x_2 to x_6). Therefore the system is conceptually an overdetermined one, although the load cases are not fully independent.

In order to study the sensitivity of the correlation process results to the initial conditions, following the method in section 5, we have considered a reference problem and studied the variation of the quality parameters defined before N_{xR} , N_{xRM} , N_{k^*} , N_{TRO} , N_{TRM} (see Table 3), some of them extended now for the two load cases C1 and C2, when appropriate. We have considered as reference configuration R the RTMM

Fig. 9. PHI ELE CRTMM-4 thermal coupling scheme. x_2 to x_6 , free parameters; SC, spacecraft walls; q_{7d} , heat load; A, sidewalls; B, top shell; C, baseplate; D, board.

defined before (see Tables 6 and 7) and have started from an initial guess $\{x\}_0$ given by (5.1), looking for the influence of the initial condition factor c_0 . The norm N_{TRO} represents the difference between reference R and initial guess 0 configurations, e.g. in Fig. 10a at $c_0 = 0.5$, $N_{TRO-C1} = 9^\circ\text{C}$ (hot case) and $N_{TRO-C2} = 5^\circ\text{C}$ (cold case). These numerical values give us an idea of the sensitivity of the result, $\{t\}_M$, to the initial condition factor c_0 . The temperature error of the correlation is given by N_{TRM-Ci} (see Fig. 10b), e.g. at $c_0 = 0.5$ is $N_{TRM-C2} = 0.1$. These errors are small in the range $0.3 < c_0 < 1.5$.

Before starting the discussion of the results, some previous comments are of interest. In order to investigate the poor conditioning of the matrixes involved in the calculations, we have paid attention to singular value decomposition (SVD) of the Jacobian matrixes. In the PHI study, two Jacobian matrixes $[M]$ are considered, one for each load case, C1 and C2. As the heat load configurations are very similar to each other (a load applied to node D), the respective Jacobian matrixes are not very different, and the influence of the case is not very significant. The minimum dimension of $[M]$ is $N_N = 4$ (the number of temperatures of each case) and therefore this is the maximum possible number of singular values (SV).

The influence of the number of free parameters included in the correlation process can be studied by paying attention to the number of non-zero SVs. If parameter x_2 (or x_6) is excluded of analysis, SV_3 (or SV_4) is lost from the set of singular values, the number of SVs is reduced, and the rank of the system is smaller. On the contrary, if any other parameter (x_3 , x_4 or x_5) is excluded, no SV is lost, and SVs values are only slightly modified. In this way, some free parameters can be associated to SVs: x_2 to SV_3 , and x_6 to SV_4 . It can be outlined that x_2 and x_6 represents the only thermal links without alternative heat path (Fig. 9).

Interestingly, two types of behavior appears in the quality parameters curves: B1 (Fig. 10) and B2 (Fig. 11), depending on the free parameters included in the analysis. The more frequent is B1, while B2 only appear when SV_4 is lost (x_6 parameter is not considered). On the contrary, if x_2 is excluded, and therefore SV_3 is lost, the behavior of the quality curves is not affected.

The most significant curve is $N_{k^*}(c_0)$, the norm of $\{k^*\}$, the parameter correction vector. It gives information about the amount of correction needed to attain the reference configuration. It is shown in Fig. 10a, together with N_{xRM} (error in $\{x\}_M$ with regard to the reference $\{x\}_R$), and N_{TRM-Ci} . All these quantities decrease as the condition $c_0 = 1$ is approached.

In the range $c_0 > c_{0m}$ (in this case $c_{0m} = 1.53$) at least one of the parameters x_i becomes negative, which is not physically possible. Although the results have no physical sense in that range, we have retained them, for the sake of completeness. In Figs. 10c and 11c, this condition is represented by the status signal "warning = 1".

The temperature errors in both load cases N_{TRM-Ci} ($i = 1, 2$) shown in Fig. 10b, are much smaller than the initial separation, e.g. for $c_0 = 0.5$ is $N_{TRO-C1} = 9^\circ\text{C}$, but the temperature error is $N_{TRM-C1} = 0.05^\circ\text{C}$.

In the behavior B1, the temperature error curves are parabolic around $c_0 = 1$, and are very similar for both hot and cold cases. A better parameter to analyze errors is the correlated parameter relative error, $r_x = N_{xRM}/N_{xR}$, shown in Figs. 10c and 11c.

To study the iteration process, it is useful to define the initial condition ratio for the second step, $c_{0-S2} = N_{xM-S1}/N_{xR}$, as the ratio of the parameter value obtained from the first iteration to the norm of the reference value $\{x\}_R$. As shown in Fig. 10c, in the range $c_0 < 1$, this curve is above the first diagonal (e.g. at $c_0 = 0.5$, $c_{0-S2} = 0.8$). In the next step (S2) the initial condition factor to be considered is $c_{0-S2}(0.8) = 0.95$, and so on. Thus, in each iteration step, the value of c_{0-Si} converges to $c_0 = 1$, that is, closer to the reference value. At the same time, the relative error $r_x = N_{xRM}/N_{xR}$ decreases to zero as $1 - c_{0-S2}$, in a parabolic way. In the range $1 < c_0 < 1.53$ the initial condition ratio is $c_{0-S2} < 1$ (e.g. at $c_0 = 1.5$ the value obtained is $c_{0-S2} = 0.75$), then in the next iteration step the evolution would be the one just described for initial condition $c_0 < 1$.

In the type B2 evolution (Fig. 11a), N_{k^*} and N_{xRM} are smaller than in

Fig. 10. Variation of the quality parameters a function of the initial condition factor c_0 . PHI-ELE, CRTMM 4-node model. Five free parameters (x_2, x_3, x_4, x_5, x_6). Two load cases. a) N_{k*} (*), N_{xR} (+), N_{TRM-C1} (square), N_{TRM-C2} (triangle); b) N_{TRM-C1} (square), N_{TRM-C2} (triangle), matrix conditioning (rhombus); c) N_{xRM}/N_{xR} (+), c_{0-S2} (o), warning (hexagon). Units are described in Table 3.

the B1 type (Fig. 10a), but the temperature error N_{TRM-C1} (Fig. 11b) is larger. But one of the most characteristic differences is the behavior of N_{TRM-C1} close to $c_0 = 1$, now the behavior is linear instead of parabolic.

Another striking difference is the change in the initial condition ratio $c_{0-S2} = N_{xM-S1}/N_{xR}$, which now is quasi-linear (Fig. 11c), close to the diagonal of first quadrant (slope close to 1) and slightly above it, and passing through point (1,1). This situation suggests that these points are close to be invariant, that is, if iterated a slow convergence rate to $c_0 = 1$ can be expected. Anyway, the value of the relative error N_{xRM}/N_{xR} is so small (even smaller than in B1 case) that no further iteration seems to be needed, at least in thermal TMM correlation applications.

Additional correlation examples have been studied using combinations of only three free parameters, and the behavior observed does not change significantly, including the existence of type B2 behavior if the

parameter x_6 is excluded of the search space.

7. Conclusions

The correlation of a reduced thermal mathematical model (RTMM) to the results of a reference model (experimental test results, or detailed TMM results) has been studied. Existing methods have been briefly summarized, and a new one, based on Jacobian matrix formulation (Taylor series expansion) and solved using the Moore-Penrose pseudo-inverse matrix, has been presented, and generalized to configurations with several load cases.

As the method is based on the Jacobian matrix formulation, a section is devoted to explore the structure of the Jacobian matrix in the case of thermal problems. An analytical study of a simple 2-node thermal model

Fig. 11. As Fig. 10, but for just four free parameters (x_2 , x_3 , x_4 , x_5). Behavior B2. Units are described in Table 3.

has been performed, focusing in the problems that can appear when the Jacobian matrix is singular. In such a case, Jacobian matrix can be analyzed looking for the singularity origin. The singularity can sometimes be avoided by reducing the size of model, e.g. disregarding free parameters when they have no influence (columns with small values), solving in this way the observability problem, or condensing nodes when their temperatures follow the same trends (rows linearly dependent) to solve the redundancy problem.

A geometrical explanation of the best fit problem formulation and minimum error size has been presented, explaining the effect of a correct choice of both the model architecture and the free parameter set.

The 4-node model example studied by Klement has been also considered, and the singularities of the model analyzed. Aside the classical correlation temperature error, a new parameter has been proposed to evaluate the quality of the correlation results, which is the error of the

solution obtained, compared to the reference model, in terms of the free parameter values. This error can be large even if the temperature error is small.

Finally, the method developed has been applied to the ELE equipment of PHI instrument of Solar Orbiter ESA mission. A 7-node reduced model has been analyzed, and further reduced in terms of both free parameters (one is found no relevant) and temperature nodes, condensing four contiguous nodes, thus reducing the model to a 4-node TMM.

The influence of the initial conditions and the number of free parameters in the errors obtained has also been analyzed, considering the rank of the Jacobian matrix obtained using the singular value decomposition method in both models, Klement's and PHI-ELE 4-node. In Klement's model the Jacobian matrix rank is 3, which transforms the problem in an overdetermined one. It is shown that the problem can be solved in one or two iterations depending on the error allowed. In ELE-

PHI model the rank is 4, but the problem can be solved even for 5 free parameters (formally an underdetermined problem), as two load cases are involved, and 8 temperature data are to be correlated, thus transforming the system in an overdetermined one. Different behaviors in the error curves and convergence rates have been observed, depending on the participation of one of the free parameters, x_6 , in the correlation process.

The work presented in this paper can help to advance in the process towards the automatization of the model-to-reference correlation methods, although it is shown that sound engineering experience is yet a premise to construct the mathematical models and to design the validation tests in a suitable way.

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List of acronyms

APSO	Adaptive Particle Swarm Optimization
AU	Astronomic Unit
BELA	Bepi Colombo Laser Altimeter
CRTMM	Condensed Reduced Thermal Mathematical Model
DTMM	Detailed Thermal Mathematical Model
ELE	Electronics Unit
ESA	European Space Agency
ESARAD	ESA Radiative tool
ESATAN	ESA Thermal Analysis Network
ESTEC	European Space Research and Technology Centre
GL	Linear Conductance
GMND	Geometrical Model Nodal Diagram
GR	Radiative conductance
HDRM	Hold Down and Release Mechanism
IF(I/F)	Interface
IR	Infrared
K4	Klement's 4-node model
MC	Nodal Thermal Capacity
MLI	Multi-Layer Insulation
MP	Moore-Penrose
MTTC	Model-to-test correlation
PCB	Printed Circuit Board
PHI	Polarimetric Heliosismic Imager
RHS	Right-Hand Side
RMS	Root Mean Square
RTMM	Reduced Thermal Mathematical Model
SC	Spacecraft
STM	Structural Mathematical Model
SV	Singular Value
SVD	Singular Value Decomposition

TCS	Thermal Coupling Scheme
TMM	Thermal Mathematical Model
URF	Unit Reference Frame

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