Loewner Approach Model Order Reduction in Hybrid BI-FEM Solution for the Design of Frequency Selective Surfaces

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Abstract—A Loewner matrix data-driven model order reduction technique for fast frequency sweep in full-wave solution in large finite frequency selective surfaces is detailed. A hybrid Boundary Integral-Finite Element Domain Decomposition Method is addressed to accurately predict the electromagnetic behavior of these structures. A reduced-order model is built up by means of simulation scattering parameters. A Loewner matrix approach is taken into account to provide a tangential interpolation to output data. As a result, a dynamical system representation fitting the simulation data is obtained, and further electromagnetic insight can be obtained from this system representation.

Index Terms—Boundary element methods, discontinuous Galerkin methods, finite element methods, Loewner framework, reduced order systems.

I. INTRODUCTION

Frequency selective surfaces (FSS) [1] take part in many electromagnetic devices. Full-wave solution of electrically large finite Frequency Selective Surfaces (FSS) is a challenging numerical problem that can be addressed by a hybrid Boundary Integral-Finite Element Method (BI-FEM) Domain Decomposition approach [2], [3], [4]. It should be pointed out that an appropriate preconditioner must be applied for accurate solution of the large electromagnetic problem at its corresponding operating frequency.

The computational effort taken into account for a single frequency analysis is already large enough, however, a specific electrical behavior in a given frequency band is required for engineering purposes. In order to achieve this goal, multiple analyses have to be carried out until the final design is obtained. In order to compute the frequency behavior of these electrically large structures, a reduced-order model is desirable instead of computationally expensive frequency point analyses. Model order reduction stands upon replacing a rather complex mathematical model by a much simpler approximated one that still maintains certain aspects of the original model [5].

In this solution setting, the frequency dependency in the preconditioned numerical matrix operators to be solved is complex enough to apply standard model order reduction techniques, such as Asymptotic Waveform Expansion (AWE),

Padé via Lanczos (PvL), Proper Orthogonal Decomposition (POD) or Reduced Basis Method (RBM) [6], [7], [8], [9], where a reduced-order projection space is appropriately built up taking advantage of the specific parameter dependence. As a result, we propose to build up a reduced-order model by the only means of the output scattering data provided by the hybrid BI-FEM solver. This solver will be therefore used as a black box, unlike other model order reduction techniques, which gives rise to a data-driven model order reduction.

This data-driven reduced-order model is based on the Loewner matrix approach. This technique builds a linear dynamical system fitting the data and, not only the frequency band behavior of the FSS is determined, but also, we place special emphasis in getting a linear dynamical system description of the physics in the FSS such that we can use this information for design purposes.

II. PROBLEM STATEMENT

The time-harmonic Maxwell's equations can be written in variational form over an appropriate admissible discrete function space $\mathcal{X}_h$, viz.

$$\begin{aligned} &\text{Find } E \in \mathcal{X}_h \text{ such that} \\ &a(E, v) = f(v) \quad \forall v \in \mathcal{X}_h. \end{aligned} \quad (1)$$

where $a(\cdot, \cdot)$ is an appropriate bilinear form taken the hybrid BI-FEM Domain Decomposition discretization of Maxwell's equations and $f(\cdot)$ is a linear functional. For a detailed definition of these forms, field variables and approximation space we refer to [2], [3], [4].

Taking frequency as a parameter, we are interested in solving the following discrete variational problem:

$$\begin{aligned} &\text{Find } E(\omega) \in \mathcal{X}_h \text{ such that} \\ &a(E(\omega), v; \omega) = f(v; \omega) \quad \forall v \in \mathcal{X}_h. \end{aligned} \quad (2)$$

for every frequency ω in a given frequency band of interest $[\omega_{\min}, \omega_{\max}] \subset \mathbb{R}$. Nevertheless, we go a step further and we just need the input-output relation as a function of frequency, namely, the transfer function behavior of the electromagnetic structure.

A. Loewner Matrix Approach

Let us denote the frequency domain S parameter data provided by BI-FEM Domain Decomposition solver as $S(s)$, where the complex frequency variable s will be sampled in a specific frequency band. $S(s_i)$ stands for the matrix-valued transfer function, i.e., S parameters, sampled at complex frequency s_i . The goal is to fit a descriptor system state space model of the form

$$\Sigma : \begin{cases} E \frac{d}{dt} x(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \quad (3)$$

Σ is a linear dynamical system with m -inputs, m -outputs and N dimensional internal variable, where $u(t)$ is the input, $y(t)$ is the corresponding output and $x(t)$ is the internal variable. $E, A \in \mathbb{C}^{N \times N}$, $B \in \mathbb{C}^{N \times m}$ and $C \in \mathbb{C}^{m \times N}$. E matrix may be singular. The transfer function for the linear dynamical system (3) is

$$S(s) = C(sE - A)^{-1}B. \quad (4)$$

The Loewner matrix approach builds a state space model $[E, A, B, C]$ up fitting the S parameter data $S(s_i)$ [10], [11].

B. Tangential Interpolation

Let us take into account the following frequency points in the complex plane $P = \{s_1, \dots, s_p\}$, and its corresponding matrix-valued transfer function evaluations $\{H(s_1), \dots, H(s_p)\}$. P can be divided into two groups

$$P = \{\lambda_1, \dots, \lambda_k\} \cup \{\mu_1, \dots, \mu_h\}. \quad (5)$$

The rational interpolation method gives rise to a controllable and observable state space model from the sampled data $S(s_i)$ provided by the FEM analysis of microwave circuits. The tangential interpolation method uses the following right interpolation data

$$\begin{aligned} A &= \text{diag}\{\lambda_1, \dots, \lambda_k\} \in \mathbb{C}^{k \times k} \\ R &= [r_1, \dots, r_k] \in \mathbb{C}^{m \times k} \\ W &= [w_1, \dots, w_k] \in \mathbb{C}^{m \times k} \end{aligned} \quad (6)$$

and the left interpolation data

$$L = \begin{bmatrix} l_1 \\ \vdots \\ l_h \end{bmatrix} \in \mathbb{C}^{h \times m}, V = \begin{bmatrix} v_1 \\ \vdots \\ v_h \end{bmatrix} \in \mathbb{C}^{h \times m} \quad (7)$$

r_i and l_j stand for right and left tangential directions respectively. The tangential interpolation method sets a minimal dynamical system realization $[E, A, B, C]$ up such that the corresponding transfer function $S(s)$ satisfies the following right and left constraints

$$\begin{aligned} S(\lambda_i)r_i &= S(\lambda_i)r_i = w_i \\ l_j S(\mu_j) &= l_j S(\mu_j) = v_j \end{aligned} \quad (8)$$

Loewner matrix and shifted Loewner matrix are specially well-suited to face this tangential interpolation problem since

these matrices, by construction, give rise to a system realization fitting the simulation data [10]. The Loewner matrix of $S(s)$ can be built as

$$\mathcal{L} = \begin{pmatrix} \frac{v_1 r_1 - l_1 w_1}{\mu_1 - \lambda_1} & \cdots & \frac{v_1 r_k - l_1 w_k}{\mu_1 - \lambda_k} \\ \vdots & \ddots & \vdots \\ \frac{v_h r_1 - l_h w_1}{\mu_h - \lambda_1} & \cdots & \frac{v_h r_k - l_h w_k}{\mu_h - \lambda_k} \end{pmatrix}. \quad (9)$$

Similarly, the shifted Loewner matrix can be obtained by setting the Loewner matrix of $sS(s)$

$$\sigma\mathcal{L} = \begin{pmatrix} \frac{\mu_1 v_1 r_1 - \lambda_1 l_1 w_1}{\mu_1 - \lambda_1} & \cdots & \frac{\mu_1 v_1 r_k - \lambda_k l_1 w_k}{\mu_1 - \lambda_k} \\ \vdots & \ddots & \vdots \\ \frac{\mu_h v_h r_1 - \lambda_1 l_h w_1}{\mu_h - \lambda_1} & \cdots & \frac{\mu_h v_h r_k - \lambda_k l_h w_k}{\mu_h - \lambda_k} \end{pmatrix}. \quad (10)$$

Finally, recalling fundamental results from Loewner framework, cf. [11], the function

$$S(s) = W(\sigma\mathcal{L} - s\mathcal{L})^{-1}V \quad (11)$$

interpolates the S parameter simulation data.

III. NUMERICAL RESULTS

In this section, we apply the Loewner matrix approach described earlier to a Frequency Selective Surface made out of a finite size PEC where 8x8 circular rings are taking into account to build up a stop-band FFS. The dimension of the finite FSS is detailed in Fig. 1. The geometry details of the circular rings for each of the 64 unit cells are shown in Fig. 2. The hybrid BI-FEM Domain Decomposition solver is used to provide S parameter data within the 5-20 GHz band. Fig. 3 compares the full-wave response and the Loewner response. Good agreement in the S parameter behavior is found in the stop-band. Fig 4 detail another example unit cell, namely the unit cell in a 8x8 Swiss cross dipoles FSS where the cross is 10 mm long and 1 mm wide. Fig. 5 compares the results obtained by the full-wave solver and Loewner approach. Good agreement is once again found.

IV. CONCLUSION

A Loewner matrix data-driven model order reduction technique for fast frequency sweep in full-wave solution in large finite frequency selective surfaces has been detailed. A hybrid Boundary Integral-Finite Element Domain Decomposition Method has been used as a black box to provide S parameter data. The Loewner matrix approach has been used for tangential interpolation of the simulation data. Finally, a minimal dynamical system realization has been shown to provide good accuracy results in the frequency band of interest.

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Fig. 1. 8x8 circular slot FSS.

Fig. 2. Unit cell circular ring geometry in the FSS.

Fig. 3. Wide-band S parameter response comparison between data-driven reduced-order model and full-wave hybrid BI-FEM Domain Decomposition analysis in the circular ring FSS.

Fig. 4. 14x14 mm unit cell Swiss cross dipole in the FSS.

Fig. 5. Wide-band S parameter response comparison between data-driven reduced-order model and full-wave hybrid BI-FEM Domain Decomposition analysis in the Swiss cross dipole FSS.

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