DESIGN AND ANALYSIS OF NOVEL STRUCTURAL GEOMETRIES FOR PIEZORESISTIVE PRESSURE SENSORS ALLOWING IMPROVED MEASUREMENT SENSITIVITY

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Chuang Li

Madrid, Mar. 2018
RESUMEN

Los sensores de presión piezorresistivos (MEMS por sus siglas en inglés) son ampliamente utilizados en las industrias automotriz, aeroespacial y petroquímica debido a su pequeño tamaño, circuito de lectura simple, capacidad de fabricación por lotes y bajo precio. Sin embargo, existen dos inconvenientes para los sensores estructurales tradicionales de sustrato de silicio. Una es el compromiso entre sensibilidad y linealidad que es difícil de mejorar simultáneamente para la membrana plana tradicional. Otro es la corriente de fuga a más de 125 °C de temperatura a través de la unión p-n, lo que degrada significativamente el rendimiento. Por lo tanto, las últimas investigaciones se han centrado en el desarrollo de nuevas estructuras y nuevos materiales para desarrollar sensores de presión de alta precisión y alta temperatura.

En la presente tesis doctoral, se propone un nuevo sensor de presión piezorresistivo estructural con membrana de ranura anular combinada con viga de rodadura para mediciones de baja presión basadas en el sustrato SOI. Tecnicas para el diseño mecánico y el análisis han sido utilizadas para poner de manifiesto de forma teórica el mecanismo de trabajo de las ranuras anulares y el travesaño. Solo mediante la disposicion de un perfil de tensión altamente concentrado (HCSP por sus siglas en inglés) y una membrana parcialmente rigidizada (PSM por sus siglas en inglés), se alivió la contradicción entre sensibilidad y linealidad. Debido al doble perfeccionamiento tanto para la sensibilidad como la linealidad creado por esta nueva estructura, la sensibilidad y la linealidad lograron una mejora simultánea.

Uno de los desarrollos logrados en esta tesis fue que las dimensiones de la membrana sensible fueron optimizadas mediante la combinación de la simulación FEM y el método de ajuste polynomial de curvas. Sobre la base del ajuste multivariable, se dedujeron las relaciones entre las variables de dimensión estructural y el rendimiento mecánico, lo que hizo que los procesos de fabricación fueran más eficientes. De acuerdo con la teoría estadística, el coeficiente de
determinación $R^2$ y la suma residual de cuadrados $RSS$ se introdujeron para indicar si las ecuaciones de ajuste y las curvas coincidían bien con la simulación. En base a esos resultados, se determinó una serie de las dimensiones óptimas.

Además de estudiar el efecto de las dimensiones de la membrana en el rendimiento mecánico, también se analizó el efecto de la estructura de la membrana en las características de salida. La forma, el tamaño y la ubicación de los piezoresistores jugaron un papel importante en las características de salida. En comparación con otras membranas estructurales, el sensor propuesto logra varias ventajas, como el tamaño micro, alta sensibilidad, baja no linealidad y comportamiento dinámico estable.

De acuerdo con el análisis mecánico anterior y la optimización de la estructura, se determinó la estructura del sensor propuesto. El chip sensor se realizó con éxito mediante los procesos de micro fabricación de MEMS. Después del ensamblaje, el sensor propuesto se probó a diferentes temperaturas y se evaluó su rendimiento. Los resultados experimentales mostraron que el sensor obtuvo una sensibilidad de 30.9 mV/V/psi, una presión no lineal de 0.25% FSS y una precisión de 0.34% en el rango operativo de 0~1 psi a temperatura ambiente. En términos de tamaño micro, sensibilidad y linealidad, el sensor propuesto obtuvo un buen rendimiento, por lo que fue adecuado para ser aplicado en la medición de una presión absoluta menor a 1 psi.
ABSTRACT

MEMS piezoresistive pressure sensors are widely used in automobile, aerospace and petrochemical industries due to their small size, simple reading circuit, batch fabrication capability and low price. However, there are two drawbacks for the traditional structural silicon substrate sensors. One is from the trade-off between sensitivity and linearity which are difficult to be improved simultaneously for the traditional flat membrane. Another is from the leakage current above 125 °C temperature across the p-n junction, which will significant degrade the performance. Thus, researchers have focused on the development of novel structures and new materials for developing high accuracy and high temperature pressure sensors.

In this thesis work, a novel structural piezoresistive pressure sensor with four-grooved membrane combined with rood beam has been proposed for low pressure measurements based on silicon on insulator (SOI) substrate. The mechanical design and analysis were used to theoretically illustrate the working mechanism of the grooves and the rood beam. Just because of the forming of a high concentrated stress profile (HCSP) and partially stiffened membrane (PSM), the contradiction between sensitivity and linearity was alleviated. Owing to the double improvements for the sensitivity and linearity, the accuracy of the sensor achieved a large improvement.

One of the developments accomplished in this thesis was that the dimensions of the sensitive membrane were optimized by the combination of FEM simulation and curve fitting method. On the basis of multivariate fitting, the relationship between structural dimension variables and mechanical performance was deduced, which made the fabrication processes more efficient. According to statistics theory, the coefficient of determination $R^2$ and residual sum of squares RSS were introduced to indicate whether the fitting equations and curves matched well with the simulation. After that, a series of the optimal dimensions were determined.
In addition to studying the effect of the membrane dimensions to the mechanical performance, the effect of membrane structure on output characteristics was also analyzed. The shape, size and location of piezoresistors played an important role in the output characteristics. Compared with other structural membranes, the proposed sensor achieves several advantages including micro size, high sensitivity, low nonlinearity, and stable dynamic behavior.

According to the previous mechanical analysis and structure optimization, the structure of the proposed sensor was determined. The sensor chip was successfully realized by MEMS micromanufacture processes. After the assembling, the proposed sensor was tested at different temperatures and the results were evaluated. Experimental results showed that the sensor obtained a sensitivity of 30.9 mV/V/psi, a pressure nonlinearity of 0.25% FSS and an accuracy of 0.34% in the operating range of 0~1 psi at room temperature. In terms of micro size, sensitivity and linearity, the proposed sensor obtained a good performance, so it was suitable to be applied in measuring absolute low pressure.
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<th>Full name</th>
<th>Units</th>
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<tr>
<td>BMDI</td>
<td>Beam-membrane-dual-island</td>
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</tr>
<tr>
<td>BMMI</td>
<td>Beam-membrane-mono-island</td>
<td></td>
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<tr>
<td>BMQI</td>
<td>Beam-membrane-quad-island</td>
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<tr>
<td>CAGR</td>
<td>Compound Annual Growth Rate</td>
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<tr>
<td>CBM</td>
<td>Cross beam membrane</td>
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<tr>
<td>CNT</td>
<td>Carbon Nanotubes</td>
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<tr>
<td>CTE</td>
<td>Coefficients of thermal expansion</td>
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<tr>
<td>DRIE</td>
<td>Direct reactive ions etching</td>
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<tr>
<td>FEM</td>
<td>Finite element method</td>
<td></td>
</tr>
<tr>
<td>FSS</td>
<td>Full-scale span</td>
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<tr>
<td>HCSP</td>
<td>High concentrated stress profile</td>
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<tr>
<td>IC</td>
<td>Integrated circuits</td>
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<tr>
<td>ICP</td>
<td>Inductively coupled plasma</td>
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<tr>
<td>LPCVD</td>
<td>Low pressure chemical vapor deposition</td>
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<tr>
<td>IoT</td>
<td>Internet of Things</td>
<td></td>
</tr>
<tr>
<td>MEMS</td>
<td>Microelectromechanical Systems</td>
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</tr>
<tr>
<td>MTBF</td>
<td>Mean time between failures</td>
<td></td>
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<tr>
<td>MTTF</td>
<td>Mean time to failure</td>
<td></td>
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<tr>
<td>PECVD</td>
<td>Plasma enhanced chemical vapor deposition</td>
<td></td>
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<tr>
<td>PSM</td>
<td>Partially stiffened membrane</td>
<td></td>
</tr>
<tr>
<td>PNL</td>
<td>Pressure nonlinearity</td>
<td></td>
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<tr>
<td>$R^2$</td>
<td>Coefficient of determination</td>
<td></td>
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<tr>
<td>RSS</td>
<td>Residual sum of squares</td>
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<tr>
<td>RIE</td>
<td>Reactive ion etching</td>
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<tr>
<td>SOI</td>
<td>Silicon on isolator</td>
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<tr>
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<td>Silicon on Sapphire</td>
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<tr>
<td>SCRs</td>
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<tr>
<td>SiC</td>
<td>Silicon Carbide</td>
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<td>Silicon oxide</td>
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<tr>
<td>Si₃N₄</td>
<td>Silicon nitride</td>
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<tr>
<td>TCR</td>
<td>Temperature coefficient of resistivity</td>
<td>%</td>
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<tr>
<td>TCS</td>
<td>Temperature coefficient of sensitivity</td>
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<tr>
<td>S</td>
<td>Sensitivity</td>
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<td>mV</td>
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<tr>
<td>Uᵢn</td>
<td>Input voltage</td>
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<tr>
<td>Vᵢoff</td>
<td>Offset voltage (or zero output)</td>
<td>mV</td>
</tr>
<tr>
<td>πᵢl</td>
<td>Longitudinal piezoresistive coefficient</td>
<td>Pa⁻¹</td>
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<tr>
<td>πᵢt</td>
<td>Transversal piezoresistive coefficient</td>
<td>Pa⁻¹</td>
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<tr>
<td>σᵢl</td>
<td>Longitudinal stress</td>
<td>MPa</td>
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<td>σᵢt</td>
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Chapter 1: Introduction

1. INTRODUCTION

1.1 Motivation of this thesis

In some movies or TV series, we always watch such a storyline like the following: “A heart disease patient lay quietly in an ambulance, his wife sat next to him in the rear of the ambulance and held his hand in silence, but her eyes could not hide her concern and fear. Then, the paramedic clipped onto the patient’s left arm a small device from which a flexible cable wire led to a digital display that was showing the irregular cardiac waveform. Suddenly, a warning sign in the upper right-hand corner of the display was flashing next to the low blood-pressure reading. At this moment, the experienced paramedic expertly removed an adhesive path from a plastic bag and attached it to the patient’s right arm. With minutes, the display was showing a recover of the screen. The paramedic looked with a smile at the wife, who thanked with a deep sigh of relief.”

This short fictional story illustrates how technology can touch our daily lives in so many different ways. In the story, there are several sensors working to help the patient safely reach the hospital as soon as possible. For example, the modern blood pressure sensor clipped onto the patient’s arm allowed the paramedic to monitor blood pressure and cardiac output. Meanwhile, the micro yaw-rate sensor in the vehicle stability system ensured that the ambulance did not skid on the icy highway. In the event of accident, the crash acceleration sensor guaranteed the airbags would deploy just in time to protect the passengers. Also, the silicon manifold absolute pressure sensor in the engine compartment helped the engine electronic control unit maintain at the location’s high altitude the proper proportions in the mixture of air and fuel. Finally, the microneedles in the adhesive patch ensured the immediate delivery of medication to the minute blood vessels under the skin. Just because the vehicle was safely traveling, together with equally advanced technology, the patient’s life was saved.
Through a simple story, we can find that sensors are so close to our lives, and meanwhile, improve our lives. A sensor is an element of a measuring system that is directly affected by the phenomenon, body, or substance carrying the quantity to be measured. For example, bourdon tube of a pressure gauge, rotor of a turbine flow meter, and sensing coil of a platinum resistance thermometer, etc. Sensor is the heart of a measurement system. It is the first element that comes in contact with environmental variables to generate an output. Referring to sensors, we have to mention Microelectromechanical Systems (MEMS).

“MEMS are one way of making things”. These “things” combine the functions of sensing and actuation with computation and communication to locally control physical parameters at the microscale, yet cause effects at much larger scales. Like many other emerging techniques with significant future potential, MEMS are subject to a rising level of excitement and publicity. Applications of MEMS in aerospace, automotive, biotechnology, defense, environment, healthcare and portable devices prompted many experts and engineers to spend more energy on it. As it evolves and ends markets develop. Present markets are primarily in pressure and inertial sensors. Especially the pressure sensors, they take up about 25% market shares among all the MEMS devices.

The first high-volume production of a pressure sensor began in 1974. Since then, Pressure sensing has grown to a large market with an estimated $ 9.8 billion market size in 2022. The vast majority use silicon piezoresistive sense elements to detect stress in a thin sensitive membrane in response to a pressure load. For the working situation lower than 100 °C, silicon piezoresistive pressure sensors achieve huge success owning to their small scale, easy integration and direct signal transduction mechanism. In the fields of automobile, aerospace, petrochemical, even biomedical, microphones, atomic force microscope probes, etc., they are widely used. However, there are two weaknesses for silicon piezoresistive pressure sensors. One is from leakage current above 125 °C temperature across the p-n junction, which will
significant degrade the performance. Another is from the trade-off between sensitivity and linearity for the traditional flat membrane of the sensor. For the first problem, we planned to utilize SOI technology to resolve it. For the second one, we tried to design and optimize a novel structural sensitive membrane to eliminate the contradiction between sensitivity and linearity.

1.2 Objectives

The specific objectives pursued in this scientific work are summarized as:

- Development and fabrication of a novel structural piezoresistive pressure sensor based on SOI substrate. The sensor should possess the characteristics such as micro size, high sensitivity, low pressure nonlinearity error, high accuracy, etc.

- Designed sensor should be worked more than 125 °C and avoid the leakage current due to the p-n junctions at high temperatures.

- The working mechanism of the grooves and rood beam should be studied and discussed based on mechanical theory. Utilize both finite element method and mechanical calculation to verify the merits of the proposed structure.

- On the basis of multivariate fitting method, the relationships between structural dimension variables and mechanical performance should be deduced. Through statistics, the coefficient of determination $R^2$ and residual sum of squares RSS are introduced to indicate whether the fitting equations and curves match well with the simulation results. Finally, a series of the optimal membrane dimensions should be determined.

- After the mechanical analysis and structure optimization, the proposed sensor should be realized by MEMS micromanufacture processes. The assembled sensor should be tested at different temperatures. By analyzing measurement results, it should be stated whether the sensor has the ability to work in high temperature
and whether the sensitivity and low nonlinearity error of the sensor can be improved simultaneously. Besides, the accuracy and thermodynamic parameters should also be studied.

1.3 Why chose SOI piezoresistive mechanism as research object?

As we all know, any product that wants to be recognized and promoted by the market must meet some necessary conditions, such as excellent performance, market requirements, suitable cost, etc. SOI piezoresistive pressure sensors just meet such conditions to really become the dominant technology of the commercial detection systems:

- **High performance**: Besides the merits of simple reading circuit and low energy consumption for such device, the other advantages such as good linearity over a wide dynamic range and moderately high pressure sensitivity further expand the applied range.

- **Stable and reliable**: SOI substrate usually has a stable built-in insulation SiO$_2$, which makes the device dimensionally stable even at elevated temperatures. Besides, the transition mechanism is based on a Wheatstone bridge, which can further improve the stability and reliability of the sensor.

- **Batch production capacity**: Nowadays, MEMS benefit, to a degree that no other sensor technology has, from the developments in silicon processing and modeling for the integrated circuits (IC) industry. Technological advances in the fabrication of ICs can ensure the batch production capacity.

- **Cost effectiveness**: SOI chips have a production price in laboratory conditions of around dozens of euros, which could be potentially decreased if they are produced at mass scale.
1.4 Structure of this thesis

This thesis is divided in seven different chapters.

Chapter 1 Introduction: Is a general introduction to the topic of the thesis and a description of the structure of the thesis. Meanwhile, list a number of aims to be accomplished. Also there are explained the reasons of the choice of SOI piezoresistive pressure sensor.

Chapter 2 MEMS pressure sensors: state of the arts: Describes the state of the art for the MEMS pressure sensors, including the working principle of the sensors, the different classifications of the pressure sensors, the research status, and the main figures of merit of a pressure sensor.

Chapter 3 Design and analysis of a novel structural membrane: By theoretical mechanical, the working mechanisms of the four grooves and rood beam are studied. Some design methods, including model design, dimensions optimization, structure design, are presented. Then, a series of the optimal membrane dimensions are determined. Based on the optimized structure, the output performances affected by the piezoresistors and membrane dimensions are also discussed. Finally, the sensor is compared with other reported structures in static and dynamic performances.

Chapter 4 Microfabrication and key processing issues study: Based on the optimized dimensions of the sensitive membrane, the sensor chip is realized by MEMS processing. The principle for each process is introduced. Besides, the key processing issues are studied and a standardized pressure sensor production line is determined. Finally, the sensor chip is fabricated and assembled into the pressure sensor.

Chapter 5 Assembling and measurements: In this part, the assembling process and measurement are carried out. The performances of the proposed sensor at different temperatures are evaluated. The comparison between the test results and simulation results are compared and the accuracy and thermodynamic parameters are studied.
Chapter 1: Introduction

Chapter 6 General conclusions and future prospects: It summarizes the conclusions drawn from this scientific work and proposes some future plans on the basis of the current results.

Chapter 7 Bibliography: Reports the scientific publications of the author and the references used for the development of this thesis.
2. MEMS PRESSURE SENSORS: STATE OF THE ARTS

Nowadays is the information society, which is characterized by informatization for the social activities and production activities. The three significant aspects of modern information science include information collection (sensing technology), measuring transducer (communication technology), and measurement system (computer technology). Sensors are the element of a measurement system that is directly transferred physical signal to electric signal, and they are also the basis of human survival and development [Hsu, 2008], [Bao, 2005]. Since modern industry puts forward a higher request for sensors. Microelectromechanical Systems (MEMS), due to their micro size, low energy consumption, batch production capacity, etc. have been developed greatly and applied widely in recent decades [Meng, 2016].

2.1 MEMS

MEMS, the acronym of Micro Electro Mechanical Systems, are generally considered as micro systems consisting of micro mechanical sensors, actuators and electronic circuits. MEMS contains component of sizes ranging from one micrometer (μm) to one millimeter (mm). Since the advancement in silicon substrate manufacturing technology along with device processing have contributed to a large growth in the field of MEMS [Chang, 2016]. Initially, MEMS borrowed the processes from the integrated circuit (IC) industry, but there has been a major evolution in processes specifically used in MEMS, like surface micromachining [Bustillo, 1998] and bulk micromachining [Kovacs, 1998].

The core element in MEMS usually consists of two principal components: a sensing or an actuating element and a signal transduction unit. The relationship between these components in a MEMS sensor is illustrated in Figure 2.1. MEMS sensors are used to sense the existence and the intensity of certain physical, chemical, or biological quantities such as pressure, temperature, moisture, sound, light, force, magnetic, and chemical and biological compositions [Wang, 2015]. Recently, with the
market expansion of electronic devices including automobiles, aerospace and portable electronics, MEMS experiences a huge increase due to the advantages of being micro size, sensitive and accurate with minimal amount of required sample substance.

![Figure 2.1 Schematic diagram for a MEMS device.](image)

Currently, the Internet of Things (IoT) represents a magnificent future internet development tendency that can incorporate various IT artifacts, information sources, and recently, even people and services [Liu, 2015]. If the IoT is seen as the Milky Way, sensors are the countless stars, and the network constituted by sensors is just like the Solar System. With the concept of “Industry 4.0”, actualization of the IoT in the industrial field will accelerate the emergence of the next industrial revolution [Wang, 2016], [Perera, 2015]. Thus, MEMS technology and application will achieve great development and change the lifestyle of human. Based on the latest report from Transparency Market Research in the United States, the global sensor market will continue to grow at a compound annual growth rate of 45.2% by 2020. Yole Development also indicated that the demand of MEMS sensor products had experienced an annual growth rate of 12%~13% since 2014, the production value of industry was predicted to reach 24 billion US dollars [Research TM]. Among MEMS devices, it's worth noting that pressure sensors are the most widely used and take up the largest market share.
2.2 MEMS pressure sensors

MEMS pressure sensors are widely used in automobile, aerospace and petrochemical industry. Most of these sensors function on the principle of mechanical deformation and stress of thin membrane induced by the measured pressure. Mechanically induced membrane deformation and stress are then converted into electrical signal output through several means of transduction [Wei, 2012a], [Niu, 2014].

The demand for pressure sensors over a wide range of pressures varying from hundreds of Pa to dozens of MPa and the growing demand for pressure tests in high-temperature environments (>200 °C) have spurred the development of robust and reliable MEMS-based pressure sensor technologies involving silicon, Silicon on Insulator (SOI), Silicon on Sapphire (SOS), Silicon Carbide (SiC) and Carbon Nanotubes (CNT) [Bhat, 2013], [Suja, 2015]. As a result, the wide applications further promote the development of the micro-manufacturing technology. There exist many different pressure acceptors, such as thin film, PZT/PMT, Wheatstone bridge, etc. to “translate” the physical events into a quantifiable signal as shown in Figure 2.2. After process circuits with specific software, the signals can be read easily.

![Figure 2.2 Elements as part of MEMS pressure sensors for various requirements.](image-url)
In 1935, Cookson first utilized the term piezoresistance to the change in conductivity with stress, as distinct from the total fractional change of resistance [Cookson, 1935]. Three years later, Clark and Datwyler chose a bonded wire to monitor strain in a stressed member [Clark, 1938]. In the same year, Arthur Ruge independently reinvented the bonded metallic strain gauge to measure pressure. Until 1950, Bardeen and Shockley predicted relatively large conductivity changes with deformation in single crystal semiconductors [Bardeen, 1950]. Just from then, the pressure sensors started to design and fabricate with single crystal semiconductors.

In 1957, Mason and Thurston first reported silicon strain gauges for measuring displacement, force, and torque [Mason, 1957]. They found that semiconductor strain gauges, with sensitivity more than fifty times higher than conventional metal strain gauges, were considered a leap forward in sensing technology. As expected, developments in the manufacture of semiconductors, especially Hoerni’s invention of the “planar” transistor in 1959, resulted in an improved method of manufacturing piezoresistive sensors [Hoerni, 1960]. Piezoresistive pressure sensors were the first commercial devices requiring three-dimensional micromachining of silicon. This kind of technology was a singularly important precursor to the MEMS technology that emerged in the 1980’s. In 1982, Petersen’s paper “Silicon as a Mechanical Material” reviewed several micromachined silicon transducers, which helped drive the growth in innovation and design of micromachined silicon devices over the subsequent years [Petersen, 1982].

Nowadays, MEMS benefited, to a degree that no other sensor technology has, from developments in silicon processing for the integrated circuits (ICs) industry. Technological advances in the fabrication of ICs including etching, doping, bonding and thin film deposition methods, have allowed significant improvements in device sensitivity, resolution, bandwidth, and miniaturization. To understand the success of this technology, both at a research and at a commercial level it is significant to know the principles of operation of the pressure sensors and also their final applications.
2.3 Applications of MEMS pressure sensors

There exists a wide range of MEMS pressure sensors as shown in Figure 2.3. Some of these applications are aerospace and automobile fields, industry in different domains, like hydraulics, hydroelectric, traffic, and many other situations, also such sensors are applied in medical fields.

Today, pressure sensors are playing an important role in modern industries. MEMS pressure sensors are already widely utilized in many different applications for their high-performance, low cost and small size. According to Yole Développement’s new report “MEMS Pressure Sensor”, automotive applications are still dominating the MEMS pressure sensor market as shown in Figure 2.4. Besides, medical, industrial and high-end markets are growing 4% to 7% however the consumer market is growing 25% in value (38% in volume) because of new opportunities in smartphones and tablets. Due to the consumer electronic applications, its market is expected to show a 22% Compound Annual Growth Rate (CAGR). The top 5 players (Bosch, Denso, Sensata, GE Sensing and Freescale) take up about 50% of the total global market. Automotive, medical, industrial, and high-end markets have their mature leaders and smaller companies following. The consumer electronic market is still attracting some attentions from conventional MEMS sensor companies.

*Figure 2.3 Applications of MEMS pressure sensors.*
2.4 Types of MEMS pressure sensors

In the family of MEMS pressure sensors, there are many different kinds of pressure sensors according to different classification methods. At the present, there exist several of pressure sensors typologies, all of them operating under the same principle, but combining various pressure receptors, transducing materials and signals.

For almost all types of pressure sensors, the basic sensing element is the thin membrane, which deforms under the applied pressure from the measured fluid. The thickness of the membrane usually varied from a few micrometers to tens of micrometers. A constraint base made of ceramics or glass supports the silicon dies. The deformation of the membrane by the applied pressure is transduced into electrical signals by suitable transduction mechanism. In the following, this section will in turn introduce several kinds of pressure sensors based on reference pressure, transducing mechanism and substrate material.
2.4.1 Classification based on reference pressure

According to the reference pressure with respect to which the measurement is carried out, MEMS pressure sensors can be categorized as absolute, gauge and differential types. The detailed introductions are seen as follows:

- **Absolute pressure sensors**: Absolute pressure sensors measure the absolute value with vacuum as the reference pressure as shown in Figure 2.5 (a). Absolute pressure is referred to the vacuum of free space (zero pressure). In practice absolute pressure sensors measure the pressure relative to a high vacuum reference sealed behind its sensing diaphragm. The vacuum has to be negligible compared to the pressure to be measured. Such devices are always applied in the fields of aerospace, automobile and airflow control systems. The pressure sensors used for cabin pressure control, launch vehicles, and satellites also belong to this category [Keulemans, 2012], [Liu, 2013], [Je, 2016].

- **Gauge pressure sensors**: Gauge pressure sensors measure pressure relative to atmospheric pressure. The cavity of the sensor chip is filled with atmospheric pressure as shown in Figure 2.5 (b). Changes of the atmospheric pressure due to weather conditions or altitude directly influence the output of a gage pressure sensor. A gage pressure higher than ambient pressure is referred to as positive pressure. If the measured pressure is below atmospheric pressure it is called negative or vacuum gage pressure. Gage pressure sensors only offer one pressure port. The ambient air pressure is directed through a vent hole or a vent tube to the back side of the sensing element and thus compensated. They are usually used for blood pressure, intra-cranial pressure, gas cylinder pressure and most of ground-based pressure [Pelletier, 2007], [Schwerter, 2014].

- **Differential pressure sensors**: Differential pressure sensors measure pressure difference between inside and outside membrane, and hence need two pressure inlets as shown in Figure 2.5 (c). Therefore, differential pressure sensors must
offer two separate pressure ports with tube or threaded connections. They find applications in high pressure oxidation systems, small differential pressures superimposed on large static pressure and internal pressure in spacecraft [Piotto, 2016], [Venugopal, 2014]. Here, the outside of the membrane is maintained at a slightly higher gas pressure. The pressure difference is monitored by a differential pressure sensor which ensures the chip won’t experience a differential pressure greater than its rupture stress of 1 atmosphere ($10^5$ Pascal).

The differential pressure sensors are also used in some fields where it is desirable to detect small differential pressures superimposed on large static pressures.

![Figure 2.5 Classification of MEMS pressure sensors based on reference pressure.](image)

**Figure 2.5 Classification of MEMS pressure sensors based on reference pressure.**

### 2.4.2 Classification based on transducing mechanism

In almost all types of MEMS pressure sensors, the basic sensing element is the membrane, which deforms in response to the applied pressure. As the deformations in membrane-based sensors are small they can’t be directly measured. This mechanical deflection in the membrane is converted ultimately into electrical signals using suitable transduction mechanisms, namely, capacitive, piezoelectric, and piezo resistive techniques, which are usually employed as candidates for the pressure sensors as described in the following.

#### 2.4.2.1 Capacitive pressure sensor

Capacitive type pressure sensors usually utilize capacitance changes for pressure measurements. Two electrodes made of thin metal films are attached to the bottom
of the top cover and the top of the membrane as shown in Figure 2.6 (a). Any deformation of the membrane due to the applied pressure will narrow the gap between the two electrodes, which leads to a change of capacitance across the electrodes. The capacitance $C$ in a parallel-plate capacitor can be related to the gap $d$ between the plates by the expression [Akar, 2001]:

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d} \quad (2.1)$$

where $\varepsilon_r$ is the relative permittivity of the dielectric medium, $\varepsilon_0$ is the permittivity of free space (vacuum, $\varepsilon_0 = 8.85 \text{ pF/m}$), $A$ is the area of the electrode plate.

![Figure 2.6 Schematic diagram of a capacitive pressure sensor.](image)

Capacitors are common transducers as well as actuators in microsystems. The capacitance variation in a capacitor can be measured by simple circuits, such as that illustrated in Figure 2.6 (b). The variable capacitance can be measured by measuring the output voltage $U_0$. The relationship between the capacitance variation and output voltage can be presented by:

$$U_0 = \frac{\Delta C}{2(2C + \Delta C)} U_{in} \quad (2.2)$$

where $\Delta C$ is the capacitance change in the capacitor in the micro pressure sensor, $C$ is the capacitance of the other capacitors in the circuit and $U_{in}$ is the constant supply voltage for the circuit.

The merits of capacitive pressure sensors are their high sensitivity, which is practically invariant with temperature. However, a major shortcoming of capacitance
transducers is the nonlinear relationship between the capacitance and displacement and hence a force-balancing and linearizing electronic circuit is essential to capture a wide range of pressures.

### 2.4.2.2 Piezoelectric pressure sensor

Piezoelectric type pressure sensors utilize piezoelectric effect as the measurement mechanism. Certain crystals, such as lead zirconium titanate (PZT) or Zinc Oxide (ZnO), that exist in nature deform with the application of an electrical voltage. An electric voltage can be generated across the crystal when an applied force deforms the crystal, which is called the piezoelectric effect. The working principle of a piezoelectric pressure sensor is illustrated in Figure 2.7.

![Figure 2.7 Schematic diagram of a piezoelectric pressure sensor.](image)

One of the most commonly used sensing materials in MEMS and microsystems is piezoelectric crystal. Piezoelectric crystals are the solids of ceramic compounds that can produce electric charge when a mechanical force is applied between their faces. The quantity of electric charge $q$ of the crystal can be expressed by:

$$q = dF$$

(2.3)

where $d$ is the piezoelectric coefficient, $F$ is the applied force. The reverse situation, the application of voltage to the crystal, can also change its shape. This conversion of mechanical energy to electronic signals and vice versa is unique characteristic for the piezoelectric material [Tseng, 2013]. Some other synthesized crystals, such as
Rochelle salt (Na\textsubscript{4}KC\textsubscript{4}H\textsubscript{4}O\textsubscript{6}-4H\textsubscript{2}O), barium titanate (BaTiO\textsubscript{3}) and sodium potassium tartrate, also possess such characteristics.

For a crystal to exhibit the piezoelectric effect, its structure should have no center of symmetry. A mechanical load applied to such a crystal will alter the separation between the positive and negative charge sites in each elementary cell, leading to a net polarization at the crystal surface. Then, an electric field with voltage potential is thus created in the crystal because of such polarization. The output voltage of a piezoelectric type pressure sensor can be deduced by:

\[ U_0 = \frac{q}{C_a + C_c + C_i} = \frac{q(R_a + R_i)}{R_a R_i} \]

where \( R_a \) is the leaking resistance, \( R_i \) is the input resistance of the preamplifier, \( C_a \) is the self-capacitance, \( C_c \) is the connecting capacitance, \( C_i \) is the input capacitance of the preamplifier.

Piezoelectric pressure sensors generate detectable charge signal by itself in responding to applied load, showing several advantages, such as self-powering, fast response of highly dynamic load, and relatively simple readout circuit [Liang, 2015], [Akiyama, 2006]. Often a membrane configuration is adopted in the piezoelectric sensor design that results in high sensitivity for pressure sensing. Besides, it is ideal for measuring fluctuating input pressure signal [Kalange, 2007], [Je, 2011]. However, piezoelectric material are fragile, this brittleness will lead to a rupture of the thin film during the work. Thus, this kind of sensor is not suitable for working in wide-range pressure measurements [Qin, 2012], [Tseng, 2013].

2.4.2.3 Piezoresistive pressure sensor

The phenomenon by which the electrical resistance of a material changes in response to mechanical stress is known as piezoresistivity. Piezoresistivity in semiconductors is widely applied in different sensors including pressure sensors,
accelerometers, cantilever force sensors, and inertial sensors [Barlian, 2009]. Piezoresistive pressure sensors utilize the piezoresistive effect as the detection mechanism. Then a Wheatstone bridge is built up through electric connections with four piezoresistors to transduce the resistance change into output voltage when pressure is applied on the membrane surface, as shown in Figure 2.8 (a).

![Figure 2.8 Schematic diagram of a piezoresistive pressure sensor.](image)

The relative change in resistance of a rectangular conductor is given by:

$$\frac{\Delta R}{R} = (1 + 2\nu)\varepsilon + \frac{\Delta \rho}{\rho}$$

(2.5)

where $R$ is the resistance, $\rho$ is the resistivity, $\nu$ is the Poisson’s ratio and $\varepsilon$ is the strain in the resistor. The relative change in resistance is constituted by two terms. The first part is due to the geometrical changes in resistance because of stress and the second is due to the phenomenon of piezoresistivity [Kanda, 1991]. For metals, the first term is dominant, with little or no effect of the second term. In semiconductors, the first term is negligible compared to the second term and therefore the effect of the first term is neglected [Bao, 1991]. Consider the Wheatstone bridge circuit with voltage and current as shown in Figure 2.8 (b). The output voltage ($U_0$) of the Wheatstone bridge with input voltage ($U_{in}$) can be written as:

$$U_0 = U^+ - U^- = \left(\frac{R_3}{R_1 + R_3}\right)U_{in} - \left(\frac{R_4}{R_2 + R_4}\right)U_{in} = \left[\frac{R_2R_3 - R_1R_4}{(R_1 + R_3)(R_2 + R_4)}\right]U_{in}$$

(2.6)
Sometimes the input voltage source is replaced with a current source \((I_{in})\), then:

\[
I = i_{1-3} + i_{2-4} = \frac{U_{in}}{R_1 + R_3} + \frac{U_{in}}{R_2 + R_4} = \left[ \frac{R_1 + R_2 + R_3 + R_4}{(R_1 + R_2)(R_2 + R_4)} \right] U_{in}
\] (2.7)

Substituting Equation (2.7) in Equation (2.6), we can get:

\[
U_0 = \frac{R_2R_3 - R_1R_4}{(R_1 + R_2 + R_3 + R_4)} I_{in}
\] (2.8)

Piezoresistive pressure sensors have a wide range of applications leading to the highest amount of sales volume in the pressure sensor market [Zhang, 2016]. Also, the other advantages for the sensors, such as good linearity over a wide dynamic range and moderately high pressure sensitivity, further expand the applied range. Among all kinds of material with piezoresistive effect, single-crystal silicon is the most widely used materials for MEMS and microsystems. The popularity of silicon for such application is primarily due to the followings:

- Its mechanical performance is stable and can be integrated into electronics on the same substrate. For example, \(p\)- or \(n\)-type piezoresistive can be readily integrated with the silicon substrate [Kumar, 2016a].
- It has the same Young’s modulus as steel \((2 \times 10^5 \text{ MPa})\) but is as light as aluminum, with a mass density of \(2.3\text{g/cm}^3\). Materials with a higher Young’s modulus are better to maintain a linear relationship between applied pressure and induced deformation [Lin, 2012].
- It possesses a high melting point at 1400 °C, which makes silicon dimensionally stable even at elevated temperatures. Also, the coefficient of the thermal expansion is about 4 times smaller than that of steel and about 9 times smaller than that of aluminum [Ho, 2002].

According to the excellent performance and mature MEMS process technology as discussed above, this thesis is focused on a novel structural piezoresistive pressure sensor design and fabrication.
2.4.3 Classification based on substrate materials

Strictly speaking, it’s unscientific to classify MEMS pressure sensors according to substrate materials. However, substrate materials indeed determine the sensors where they can be worked, so some commonly used materials will be introduced in this section.

2.4.3.1 Silicon

Silicon is the earliest study of semiconductor materials to be used in fabricating of piezoresistive pressure sensor. It is both an extraordinary mechanical and electronic material for sensor applications. Among the pressure sensors, the silicon piezoresistive types using MEMS technology have received great attention because they find applications in everyday life involving sensing, monitoring and controlling pressure, and they therefore constitute 60% to 70% of the market shares. However, pure silicon sensors containing p-n junctions for electrical isolation exhibit a serious drawbacks, which results that they can’t be used at temperatures more than 125 °C, as the leakage current across the junctions drastically increases at high temperatures. Besides, once the temperature is above 500 °C, even good mechanical properties of silicon begin to deteriorate as the material becomes plastically deformable when the stress is loaded. These realities substantially weaken the field of applications and the accessible markets [Kroetz, 1999].

Generally, a single crystal silicon piezoresistive pressure sensor chooses a square or rectangular membrane of n-type silicon, which acts as the substrate. Figure 2.9 illustrates the cut away isometric view, and the top view of the pressure sensor chip. The model represents the general structure and working principle of silicon piezoresistive devices. However, it should be noted that p-n junctions created by individual p-type piezoresistors with the n-type membrane provide the isolation required between the resistors. Though it would be reasonable to utilize circular membranes to avoid unwanted stress concentration, silicon membranes turn out to
be square or rectangular when they are fabricated using anisotropic wet chemical etching on silicon wafers of (100) orientation. It is also easier to place the resistors parallel and perpendicular to the edges of the membrane which are in the <110> direction, thus ensuring that the piezoresistive coefficients $\pi_l$ and $\pi_t$ are maximum along this direction.

![Silicon piezoresistive pressure sensor](image)

**Figure 2.9 Silicon piezoresistive pressure sensor [Bhat, 2013].**

Nevertheless, there has been an increasing demand for electronic control concepts appearing from the automotive and aerospace industries, where sensor components of such control systems have to work under harsh situation [Krötz, 1998]. Sensors of such control units have to bear high temperatures between 200 °C and 800 °C or hostile media like coolants, oil, exhaust gases and humidity, as well as mechanical loads like high pressures and strong vibrations. Therefore, some “stronger” devices have to be designed and applied in such environments.

2.4.3.2 Silicon on insulator (SOI)

Silicon on insulator (SOI) is an advanced variation of the conventional silicon wafer. In SOI wafer, a thin film of active single-crystalline silicon lies on a silicon dioxide dielectric layer at the top of a silicon wafer. SOI is based on the wafer bonding on a thin layer of single-crystalline silicon, cut from its bulk material by a proton implantation, and high-temperature annealing. This process is drastically less
expensive than oxygen ion implantation, as well as being adapted to yielding large volumes in industrial production. Therefore, an impressive decrease in the SOI wafer price is expected soon. This structure solves many drawbacks of conventional integrated circuits, such as sensibility to radiation, current leakage, instability at high temperature, etc. During the recent ten years, it has been established that SOI is a very convenient substrate to provide single-crystalline silicon microstructures in surface micromachining processes. The use of SOI wafers for piezoresistive devices drastically enhances the sensor performances. As the strain gauges are electrically insulated from the bulk of the substrate, the temperature range is increased, the upper limit being pushed up from 125 °C to more than 250 °C [Diem, 1990].

In this approach the SOI layer itself serves as the sensitive membrane, with the handle wafer etched using anisotropic etching from the backside. P-type silicon resistors are arrayed on the oxide grown on the SOI layer as shown in Figure 2.10. Essentially speaking, the mechanical performance and working principle of SOI type are almost same to silicon device. This is because that SiO$_2$ are formed by ion implantation or deposition technologies in the silicon substrate. The resistors are created by doping followed by annealing. With this approach, piezoresistors are isolated from each other by the oxide layer. Therefore, the isolation between the resistors is maintained even at temperatures in excess of 250 °C.

![Figure 2.10 SOI piezoresistive pressure sensor [Bhat, 2013].](image-url)
The ultimate success of silicon micromachining in smart systems depends on the capacity to integrate mechanical components with electronics on the same chip. Vinoth Kumar et al. reported that the possibility of the monolithic integration of piezoresistive pressure sensors with electronics had been succeeded. In this approach, the starting wafer was an SOI wafer and resistors were grown by the deposition as shown in Figure 2.11 [Kumar, 2006].

![Figure 2.11 Processing schematic of a SOI piezoresistive pressure sensor chip.](image)

![Figure 2.12 Assembling schematic of a SOI pressure sensor and test results.](image)
Figure 2.12 illustrates the photographs of the integrated chip, the chip die mounted on the header and wire bonded, and the final assembling of the SOI piezoresistive pressure sensor, respectively. The test results show that the overall sensitivity of 270 mV/bar has been achieved with this SOI integrated piezoresistive pressure sensor. And, more remarkable, the sensitivity of the sensor remains nearly constant when working environment is changing from 30 °C to 70 °C, which is just due to the use of SOI material [Kumar, 2006].

2.4.3.3 Silicon carbide

Three silicon compounds are often used in piezoresistive pressure sensors: silicon dioxide, SiO$_2$; silicon carbide, SiC; and silicon nitride, Si$_3$N$_4$. SiO$_2$ and Si$_3$N$_4$ are usually worked as thermal and electric insulator in the sensor chip. Only SiC can be utilized as the substrate and sensitive membrane to measure the pressure. Although SOI along with oxide isolated piezoresistive elements has enabled their use up to about 250 °C. However, some higher-temperature (>600 °C) and harsh situation (aerospace applications, gas turbine control, oil industry, nuclear power, etc.) have spurred the development of more robust and reliable pressure sensors [Casady, 1996]. Thus, a mass of efforts were directed towards taking advantage of the superior thermos-mechanical properties of SiC to develop pressure sensors that were able to extend the sensing ability to 600 °C and beyond.

In spite of its several merits, the progress of SiC pressure sensors is still slow owing to the difficulties involved in its microfabrication. At the very beginning stage, SiC piezoresistors were fabricated by oxide grown on a silicon membrane, but silicon became the limiting factor. Until 2003, this had, indeed, been made possible, as reported by Lide et al. [Lide, 2003]. By micromachining N-type SiC in the DRIE system with a gas plasma mixture of SF$_6$ and oxygen, SiC diaphragm was then realized. From then, Vandelli et al. reported their SiC MEMS devices achieved a repeatable hysteresis less than 2.5% at 300 °C [Vandelli, 2008]; Fraga et al. achieved much progress in SiC film deposition by improving the processing technology [Fraga, 2010];
Du et al. [Du, 2003] designed and fabricated a SiC MEMS pressure sensor with a sensitivity of 7.7 fF/Torr at 400 °C. The typical chip based on SiC piezoresistive elements is shown in Figure 2.13. The piezoresistors are usually realized on the oxide grown on the SiC by depositing, doping and patterning the polysilicon.

![Figure 2.13 Photos of a SiC pressure sensor and test results [Fragaa, 2010].](image)

Above results have indeed demonstrated the benefits of using SiC membranes for high temperature operations and a capacitive sensing approach for low-pressure operations and this has opened up new avenues for future work for sensors using SiC. However, the cost of material and processing for SiC piezoresistive pressure sensors is high, so only some high precision and advanced fields will choose SiC as the substrate material [Shor, 1993].

2.4.3.4 Diamond

Along with SiC, diamond is a leading material for MEMS application in harsh environment. It is commonly known as nature’s hardest material, an ideal property
for high-wear environments. Diamond has a very large electronic bandgap (5.5 eV) that is well suited for stable high-temperature operation. Beside, diamond is a very high-quality insulator with a dielectric constant of 5.5. Due to enormous progress in the epitaxial growth of polycrystalline CVD diamond layers on silicon substrates, piezoresistive sensing elements can be produced easily by standard silicon processing technologies once the diamond layers are deposited.

If we only consider the effect of mechanical stress on the electronic band structure of p-doped diamond films, a gauge factor larger than 500 is expected for moderately doped diamond films [Werner, 1998]. However, it is hard to obtain reproducible measurements of gauge factors of expected size. This is mainly because the strongly varying quality of the diamond layers and the effects of bad ohm contacts on the measured piezoresistive data [Taher, 1994]. Structuring of the electrically active diamond layers is usually finished by selective deposition. It can be got by deposition on a substrate which is structured by an insulating mask layer silicon compounds (SiO₂, Si₃N₄). Microsensors based on diamond still suffer from irregular material properties and processing technic, although they exhibit excellent physical properties and a far developed processing technology.

2.4.3.5 Brief summary

Different types of substrate materials have been discussed and compared with respect to their sensor properties, their compatibility to standard silicon processing, and their state of technological development. Silicon is the most widely and earliest used in microsystems, but its current leakage at high temperature restricts the development. SOI seems to be a very promising material to overcome the restrictions of standard silicon micro sensors if only high temperatures up to 350 °C are concerned. Although SiC and diamond pressure sensors can work above 600 °C and beyond, consider the cost and process technic, they will be used only in a certain field of harsh environment.
MEMS piezoresistive pressure sensors must meet some requirements to really become the dominant technology of the commercial detection systems:

- High performance
- Stability
- Low cost
- Miniaturization
- Compatibility
- Batch production capacity
- Reliability

By comprehensive consideration, SOI piezoresistive pressure sensors will be the most worthy of research and promotion in industry fields in the future. In this thesis, a 4 inch n-type (100) oriented SOI wafer with 30 μm top silicon, 2 μm buried SiO₂ layer and 300 μm bottom silicon was chosen as the substrate of the sensor chip, hoping the proposed sensor could be worked steadily at high temperature environments.

2.5 SOI piezoresistive pressure sensors

Although SOI is chosen as the substrate of the proposed sensor chip, the physical properties of the material still derive from silicon. As the nature of SOI is still belong to silicon. In the following, the characteristics of SOI piezoresistive pressure sensor will be studied. SOI was chosen as the material for making the sensitive membrane due to its desirable characteristics like excellent mechanical properties and reproducible elastic deformations. Meanwhile SOI is also free from hysteresis and creep [Timoshenko, 1959]. SOI based piezoresistive pressure sensors have been highly popular in applications spanning industrial, aerospace, automobile, biomedical equipment and portable electronic devices, etc. [Wei, 2012b], [Park, 2016], [Shaby, 2015]. In this section, a detailed introduction of the piezoresistive pressure sensors will be proposed.
2.5.1 Development history

- **Invention stage (1945-1960):** C.S. Smith discovered that silicon and germanium owned the characteristics of piezoresistive effect in 1954, namely, the resistance will be significantly changed when there is an external force on the semiconductor material. According to this principle, the structure of early sensor can be divided two types: one is semiconductor strain gauge; another is doping \( p \)-type silicon on \( n \)-type silicon. For above two types, the sensor chips have to be affixed to the elastic component to feel the change of stress. Therefore, some disadvantages, such as low sensitivity, small natural frequency and severe creep, limit the application fields. Besides, the minimum size of the sensor at this stage is about 1 cm, which makes it difficult to miniaturize and integrate.

- **Technological development stage (1960-1970):** With the development of the silicon diffusion technology, the strain resistance is diffused into the certain crystal plane and direction. After that, wet etching process is used to deeply etch the bottom of the silicon wafer to form a cavity. Then, a thin film fixed by surrounding silicon cup is produced. For this structure, the thin membrane not only works as the substrate of the resistors, but also plays an important role in feeling stress change. Because of the integration of the elastic element and transition element, the resistors and various compensation circuits can be integrated in one piece of silicon. Thus, this type of structure achieves a high sensitivity, small size, light weight and high reliability, which provides the possibility of large-scale commercial development.

- **Commercial integration fabrication stage (1970-1980):** On the basis of silicon diffusion technology, the anisotropic etching technology is developed. Silicon anisotropic chemical corrosion technology gradually becomes the main aspect of the diffused silicon sensor processing, such as “V” groove method, automatic suspension by strong boron, automatic suspension by anodic oxidation and automatic suspension by computer control. Because thousands of pressure films
can be produced in one silicon wafer at the same time, the possibility of mass 
production will become true. Meanwhile, the strong market demand has made 
these sensors become highly popular in applications spanning automobile, 
aerospace, biomedical equipment, petroleum, and portable electronic devices. 
The technology development and market demand both promote the study and 
fabrication of this kind of sensor.

- **Microfabrication stage (1980-Now):** With the rapid development of micro/nano 
  processing technology, recently, there have been studies trying to reduce the 
  sizes of conventional piezoresistive pressure sensors and enhance their low 
  sensitivity by using a wide variety of nanomaterials and nanostructures as 
  sensing elements. Since then, the pressure sensors enter the stage of microns. 
The advent of the micromachining of silicon to form mechanical microstructures 
in silicon and the already existing expertise in manufacturing microelectronic 
devices and integrated circuits in silicon together open the doors of the highly 
interdisciplinary area of MEMS and microsystems.

2.5.2 Piezoresistive effect

Piezoresistance is defined as the change in electrical resistance of solids when 
subjected to stress fields. For silicon semiconductor, it can be divided into p-type 
silicon and n-type silicon, namely, doping boron to the silicon lattice produces p-type 
silicon crystal while doping arsenic or phosphorus results in n-type silicon crystal. 
Both p- and n-type silicon exhibit excellent piezoresistive effect. Generally speaking, 
p-type silicon is adopted as piezoresistors and n-type silicon is chosen as substrate for 
pressure sensors. The relationship between the change of resistance and the existent 
stress is shown below:

\[
\{\Delta R\} = [\sigma ] [\sigma ]
\]

where \(\{\Delta R\} = \{\Delta R_{xx}, \Delta R_{yy}, \Delta R_{zz}, \Delta R_{xy}, \Delta R_{xz}, \Delta R_{yz}\}^T\) represents the change of resistance in an infinitesimally small cubic piezoresistive crystal element with relevant stress.
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components \{\Delta \sigma\} = \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}\}^T$, as shown in Figure 2.14. For silicon, the piezoresistive coefficient matrix \(\pi\) can be simplified by symmetric operations to have only three non-zero independent components of piezoresistive coefficient tensor:

\[
[\pi] = \begin{bmatrix}
\pi_{11} & \pi_{12} & 0 & 0 & 0 \\
\pi_{12} & \pi_{11} & 0 & 0 & 0 \\
\pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 \\
0 & 0 & \pi_{44} & 0 & 0 \\
0 & 0 & 0 & \pi_{44} & 0 \\
0 & 0 & 0 & 0 & \pi_{44}
\end{bmatrix}
\] (2.10)

Substituting Equation (2.10) into Equation (2.9), we can obtain:

\[
\begin{bmatrix}
\Delta R_{xx} \\
\Delta R_{yy} \\
\Delta R_{zz} \\
\Delta R_{xy} \\
\Delta R_{xz} \\
\Delta R_{yz}
\end{bmatrix} = \begin{bmatrix}
\pi_{11} & \pi_{12} & 0 & 0 & 0 \\
\pi_{12} & \pi_{11} & 0 & 0 & 0 \\
\pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 \\
0 & 0 & \pi_{44} & 0 & 0 \\
0 & 0 & 0 & \pi_{44} & 0 \\
0 & 0 & 0 & 0 & \pi_{44}
\end{bmatrix} \begin{bmatrix}
\Delta \sigma_{xx} \\
\Delta \sigma_{yy} \\
\Delta \sigma_{zz} \\
\Delta \sigma_{xy} \\
\Delta \sigma_{xz} \\
\Delta \sigma_{yz}
\end{bmatrix}
\] (2.11)

![Figure 2.14 Silicon piezoresistance subjected to a stress field.](image)

Utilizing matrix technique to calculate, we can get the relation as the following:

\[
\begin{align*}
\Delta R_{xx} &= \pi_{11}\Delta \sigma_{xx} + \pi_{12}(\Delta \sigma_{xy} + \Delta \sigma_{xz}) \\
\Delta R_{yy} &= \pi_{11}\Delta \sigma_{yy} + \pi_{12}(\Delta \sigma_{xx} + \Delta \sigma_{zz}) \\
\Delta R_{zz} &= \pi_{11}\Delta \sigma_{zz} + \pi_{12}(\Delta \sigma_{xx} + \Delta \sigma_{yy}) \\
\Delta R_{xy} &= \pi_{44}\Delta \sigma_{xy} \\
\Delta R_{xz} &= \pi_{44}\Delta \sigma_{xz} \\
\Delta R_{yz} &= \pi_{44}\Delta \sigma_{yz}
\end{align*}
\] (2.12)
Based on Equation (2.12), it is apparent that the coefficients $\pi_{11}$ and $\pi_{12}$ are associated with the normal stress components, whereas the coefficient $\pi_{44}$ is related to the shearing stress components. The actual values of these three coefficients rely on the angle of the piezoresistor with respect to the silicon crystal lattice. The empirically determined piezoresistive coefficients under low doping concentration for high resistivity $p$- and $n$-type silicon at room temperature are shown in Table 2.1.

### Table 2.1 Piezoresistive coefficients under low doping concentration.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ (Ω·cm)</th>
<th>$\pi_{11}10^{11}$ (Pa$^{-1}$)</th>
<th>$\pi_{12}10^{11}$ (Pa$^{-1}$)</th>
<th>$\pi_{44}10^{11}$ (Pa$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-type silicon</td>
<td>7.8</td>
<td>+6.6</td>
<td>-1.1</td>
<td>+138.1</td>
</tr>
<tr>
<td>N-type silicon</td>
<td>11.7</td>
<td>-102.2</td>
<td>+53.4</td>
<td>-13.6</td>
</tr>
</tbody>
</table>

We can realize that the maximum piezoresistive coefficient for $p$-type silicon $\pi_{44} = +138.1 \times 10^{-11}$ Pa$^{-1}$, and the maximum piezoresistive coefficient for $n$-type silicon is $\pi_{11} = -102.2 \times 10^{-11}$ Pa$^{-1}$. Therefore, many silicon piezoresistors are made of $p$-type materials with boron as the dopant for achieving high performance. In practical application, the piezoresistive coefficients for prediction are $\pi_l$ and $\pi_t$. $\pi_l$ donates the piezoresistive coefficient along longitudinal direction, whereas $\pi_t$ represents the piezoresistive coefficient in the transversal direction. For monocrystalline silicon, the most common orientation in pressure sensor involves using a resistor along the [110] direction on a (100) crystal plane [Kanda, 1982]. As the thickness of piezoresistors is very small compared to its length and width, the stresses normal to the plane can be neglected. As it is known, the longitudinal direction is parallel to the direction of current and the transverse direction is perpendicular to it. For piezoresistors along the [110] direction on a (100) crystal plane, the shear stress component can be omitted and the relative change in resistance can be given by [Chen, 2009]:

$$\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t$$

(2.13)

where $\sigma_l$ is the longitudinal stress and $\sigma_t$ is the transverse stress. The expressions of $\pi_l$ and $\pi_t$ for any arbitrary crystallographic orientation for piezoresistors are given by Equation (2.14) and Equation (2.15):
\[ \pi_l = \pi_{11} - 2(\pi_{11} - \pi_{12} - \pi_{44})(l_1^2 m_1^2 + l_1^2 n_1^2 + m_1^2 n_1^2) \] (2.14)

\[ \pi_t = \pi_{12} + (\pi_{11} - \pi_{12} - \pi_{44})(l_2^2 l_2^2 + m_2^2 m_2^2 + n_2^2 n_2^2) \] (2.15)

where \((l_1, m_1, n_1)\) are the sets of direction cosines between the longitudinal resistor direction and the crystal axis and \((l_2, m_2, n_2)\) are the sets of direction cosines between the transverse resistor direction and the crystal axes. Figure 2.15 illustrates the piezoresistive coefficients in the (100) plane of \(p\)-type silicon and \(n\)-type silicon under low doping concentration at room temperature. It shows that the resistors must be aligned along the \([110]\) direction and \([100]\) direction for \(p\)-type and \(n\)-type piezoresistors respectively for maximum sensitivity [Kanda, 1982]. Then, for resistors aligned along \(\langle 110 \rangle\) direction on (100) wafers, expressions for \(\pi_l\) and \(\pi_t\) can further simplify as:

\[ \pi_{l[110]} = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44}) \] (2.16)

\[ \pi_{t[110]} = \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44}) \] (2.17)

Thus, the above expressions can be used for calculating the relative change in resistance under a given stress field.

**Figure 2.15** Piezoresistive coefficients in the (100) plane of (a) \(p\)-type silicon (b) \(n\)-type silicon under low doping concentration [Kanda, 1982].
2.5.3 Transducing mechanism

After providing a description of the relative change in resistance for a silicon piezoresistor under mechanical stress field and piezoresistive coefficients under pressure, four piezoresistors connecting to a Wheatstone bridge on the membrane is presented in Figure 2.16 (a). Initially, when the membrane is in unstressed condition and all the resistors have same resistance, the bridge is balanced and the output voltage is equal to zero.

![Figure 2.16 Wheatstone bridge on a silicon membrane under stress.](image)

Here, we take a square membrane as an example for describing the concept but the working principles are also equally valid for a circular or rectangular membrane. We assume that the sensor chip is fabricated on (100) silicon wafer and the longer axis of resistor is along [110] direction. As shown in Figure 2.16 (b), the maximum stress is emerged on the center of the four edges, so the resistors should be placed in the maximum stress region to experience more stress change from the membrane. Four resistors experience both longitudinal stress and transverse stress. Let $\pi_l$ and $\pi_t$ represent longitudinal and transverse stress experienced by $R_2$ and $R_4$. Then, the stress experienced by $R_1$ and $R_3$ can be approximated as longitudinal stress, $\sigma_l$, and transverse stress, $\sigma_t$, (which is rotated by an angle of 90°, compared to stresses in $R_2$ and $R_4$).
In the beginning, \( R_1 = R_2 = R_3 = R_4 = R_0 \). When \( R_2 \) is under stress due to \( \sigma_l \) and \( \sigma_t \), the relative resistance change is followed by:

\[
\gamma_1 = \frac{\Delta R_2}{R_2} = \pi_l \sigma_l + \pi_t \sigma_t \tag{2.18}
\]

Similarly, the relative resistance change for \( R_1 \) is given by:

\[
-\gamma_2 = \frac{\Delta R_1}{R_1} = \pi_l \sigma_l + \pi_t \sigma_t \tag{2.19}
\]

where \( \gamma_1, \gamma_2 > 0 \) (as \( \Delta R_2 > 0 \) and \( \Delta R_1 < 0 \)). By symmetry, \( \Delta R_3/R_3 = \gamma_1 \) and \( \Delta R_4/R_4 = -\gamma_2 \). Then, the output of the sensor can be presented by:

\[
U_{\text{out}} = \left[ \frac{R_2 R_4 - R_1 R_3}{(R_1 + R_3)(R_2 + R_4)} \right] U_{\text{in}} = \left[ \frac{(1 + \gamma_1)^2 - (1 - \gamma_2)^2}{(2 + \gamma_1 - \gamma_2)^2} \right] U_{\text{in}} = \left[ \frac{\gamma_1 + \gamma_2}{2(1 + \gamma_1 - \gamma_2)} \right] U_{\text{in}} \tag{2.20}
\]

When \((\gamma_1 - \gamma_2)\) is neglected:

\[
\frac{U_{\text{out}}}{U_{\text{in}}} = \frac{\gamma_1 + \gamma_2}{2} \tag{2.21}
\]

Based on the piezoresistive coefficients and stresses on the resistors, the output of the sensor can be estimated. P-type piezoresistors are preferred over n-type resistors owing to their higher piezoresistive coefficients and consequently higher sensitivity.

The relative change in resistance can also be expressed by [Li, 2012]:

\[
\frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = \pi_{[110]} \sigma_{[110]} + \pi_{[110]} \sigma_{[\bar{1}0]0} = \frac{1}{2} \pi_{44}(\sigma_{[110]} - \sigma_{[\bar{1}0]0}) = \frac{1}{2} \pi_{44}(\sigma_x - \sigma_y) \approx \frac{1}{2} \pi_{44} \sigma_{[110]} \tag{2.22}
\]

\[
\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = \pi_{[110]} \sigma_{[110]} + \pi_{[\bar{1}0]} \sigma_{[\bar{1}0]} = \frac{1}{2} \pi_{44}(\sigma_{[110]} - \sigma_{[\bar{1}0]}0) = \frac{1}{2} \pi_{44}(\sigma_y - \sigma_x) \approx \frac{1}{2} \pi_{44} \sigma_{[110]} \tag{2.23}
\]

Finally,

\[
U_{\text{out}} = \frac{1}{2} \pi_{44}(\sigma_x - \sigma_y) U_{\text{in}} \tag{2.24}
\]

Thus, it can be concluded that the differential stress on resistors is an important indicator of the output voltage of the sensor.
2.5.4 Structure of the sensitive membrane

As it is known, sensor chip is the most important part in one device. This is because it directly determines the performance of the sensor. In the structure of a sensor chip, the sensitive membrane plays a key role in displaying specific parameters, such as output, sensitivity, accuracy, etc. With the increasing demand of sensitivity and accuracy for the sensor, more and more engineers contribute their efforts to the design and optimization of the membrane structure. Thus, many novel structural membranes were developed in past decades.

2.5.4.1 C-type membrane

C-type membrane, which formed by a cavity on the silicon wafer like the alphabet “C” from the side face as shown in Figure 2.17, was the first proposed structure and was widely used for measuring the pressure of gas or water [Samaun, 1973]. C-type structural membrane is the simplest mechanical structure suitable for use of pressure sensing elements. They are usually used as a sensor element in both traditional and MEMS technology pressure sensors [Rajavelu, 2014]. Due to the planar nature of many established fabrication processes, this membrane is the main form of sensor element.

Pressure applied to one (or both) side(s) of the membrane will cause it to deflect until the elastic force balances the pressure. The pressure range of a given membrane will depend upon its dimensions (surface area and thickness), geometry, edge conditions, and the material from which it is made. In early stage, this kind of structural membrane achieved huge development because of its simple processing technic and low cost. However, a thin C-type membrane utilized in the low pressure measurement usually results in a dramatic drop in the mechanical nonlinearity error. Severe pressure nonlinearity may cause a high-sensitivity device of little practical value once it is beyond a certain range [Aravamudhan, 2008]. This is because sensitivity is proportional to the (membrane length)/(membrane thickness) ratio.
\[(L/H),\] so it can be increased by a larger ratio of that quantity. Unfortunately, the pressure nonlinearity increases with this ratio at a much faster rate, as the pressure nonlinearity of the pressure-to-stress conversion is proportional to \((L/H)^4\) [Marco, 1996]. Thus, the contradiction between sensitivity and linearity is always irreconcilable for C-type membrane structure. To solve this problem, some novel structures, such as beam combined with bossed membrane, are developed.

![Design and fabrication of C-type membrane structural pressure sensor](image)

**Figure 2.17 Design and fabrication of C-type membrane structural pressure sensor**

[Herrera-May, 2009], [Kumar, 2016b].

2.5.4.2 Beam combined with bossed membrane

To achieve advanced performance on sensitivity and linearity, some novel structural membranes were proposed. Take beam combined with bossed membrane as an example, it is worth mentioning that beam structure works like a cantilever when the mechanical load is applied on the surface of the membrane as illustrated in Figure 2.18. For the traditional C-type membrane, the stress is concentrated in a relatively larger area, and consequently disperses the total strain energy. In contrast, the beams-structured-membrane gives a non-symmetric geometry along the y-direction, and thereby generates a local enhancement of the mechanical stress to form the
stress concentration regions (SCRs). Therefore, the strain energy in the SCRs is increased remarkably within a narrower area for the proposed structure. Based on the theory of transducing mechanism in 2.4.2.3, the output of the sensor will be improved.

![Design and fabrication of beam with bossed membrane sensor](image)

**Figure 2.18 Design and fabrication of beam with bossed membrane sensor [Meng, 2016], [Herrera-May, 2009].**

There are two factors that can affect the linearity, one comes from the high upload pressure which may damage directly the linearity relationship of the piezoresistive theory, and another is from the oversize deflection of the membrane that converts the linearity between deflection and pressure into nonlinearity [Tian, 2010]. Once the value of the deflection exceeds 1/5 thickness of the membrane, the large deflection theory will work and the relationship between the deflection and pressure has to turn nonlinear [San, 2013]. The advantage of the bossed membrane is to avoid an oversize deflection. This is because the membrane is stiffened by the bossed structure. Then, the deformation of the membrane is limited within a narrow range. Finally, experimental results show that the sensor in Figure 2.18 has a sensitivity of as high as 17.339 µV/V/Pa in the range of 500 Pa at room temperature. Due to its excellent property, this sensor can be used in measuring pressure lower than 500 Pa.
2.5.4.3 Peninsula membrane

Besides the beamed and bossed membranes, some other structures also can alleviate the trade-off between sensitivity and linearity, such as the peninsula structure, as shown in Figure 2.19. Compared with the flat membrane, the peninsula structure sensor can achieve a sensitivity increase by 11.4%, nonlinearity reduction of 60% and resonance frequency increase of 41.8% [Huang, 2014]. In addition, the modified peninsula-structured membranes combined with a center boss have been optimized to achieve ultra-low nonlinearities of 0.018%FSS and 0.07%FSS for the 5 kPa and 3 kPa pressure ranges respectively. Further, its sensitivity is higher than reported CBM (cross beam membrane) and hollow stiffening structures. Finally, the fabricated sensor with the peninsula-structured membrane achieved a high sensitivity of 18.4 mV/V full-scale output and a nonlinearity error of 0.36%FSS in the pressure range 0~5 kPa. Thus, it indicates that this sensor structure is potentially a better choice for designing low pressure.

Figure 2.19 Design and fabrication of the peninsula structural membrane sensor [Huang, 2014].
2.5.4.4 Performance comparisons with different membrane

To improve the accuracy of the sensor, namely, obtain high sensitivity and low pressure nonlinearity synchronously, previous efforts were mainly focused on the following two aspects. On the one hand, create stress concentration regions (SCRs) and localize strain energy within a relatively narrow space. Piezoresistors can be placed on the SCRs and the piezoresistive sensitivity can be enhanced. The most typical example of the SCRs is the beam structure. To date, several new membranes, such as cross-beam membrane (CBM) [Tian, 2012], beam-membrane-mono-island (BMMI) [Yu, 2013], beam-membrane-dual-island (BMDI) [Meng, 2016], and beam-membrane-quad-island (BMQI) [Yu, 2015], all have achieved high sensitivity compared with the conventional structure sensors. On the other hand, an alternative option is to stiffen partially membrane of the sensor and then reduce its deflection. According to the definition of the pressure nonlinearity error, a small membrane deflection will lead to a decrease in the gradient of the output curve, thereby reducing the difference between the real output and the ideal linear output. In this situation, the PNL tends to be decreased. The typical such structural pattern is the bossed membrane and grooved membrane [Zhang, 2014], [Nambisan, 2015]. Usually, the membrane structure with partially stiffened membrane (PSM) can make the PNL reduce to less than 1%. Also, one solution for alleviating the contradiction between sensitivity and nonlinearity may be found by incorporating partially stiffened membrane in the membrane design process. PSM is to locally stiffen the membrane thereby restricting partially the deformation, but not affecting the stress concentration in the sensitive areas. Thus, it is possible to combine the advantages between SCRs and PSM to achieve a good performance device with high sensitivity and low nonlinearity error. Comparisons with typical sensor structures are conducted and summarized in terms of sensitivity, pressure nonlinearity error and first resonant frequency as shown in Table 2.2 [Herrera-May, 2009], [Tian, 2012], [Yu, 2013], [Meng, 2016], [Yu, 2015], [Zhang, 2014], [Nambisan, 2015], [Kinnell, 2010].
Table 2.2 Performance comparisons of different membrane structures.

<table>
<thead>
<tr>
<th>Group</th>
<th>Membrane types</th>
<th>Membrane structures</th>
<th>Performance comparisons</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Sensitivity (mV/kPa)</td>
</tr>
<tr>
<td>#1</td>
<td>C-type</td>
<td><img src="image1" alt="C-type membrane" /></td>
<td>9.76</td>
</tr>
<tr>
<td>#2</td>
<td>E-type</td>
<td><img src="image2" alt="E-type membrane" /></td>
<td>4.8</td>
</tr>
<tr>
<td>#3</td>
<td>CBM</td>
<td><img src="image3" alt="CBM membrane" /></td>
<td>7.081</td>
</tr>
<tr>
<td>#4</td>
<td>BMMI</td>
<td><img src="image4" alt="BMMI membrane" /></td>
<td>11.098</td>
</tr>
<tr>
<td>#5</td>
<td>BMDI</td>
<td><img src="image5" alt="BMDI membrane" /></td>
<td>17.339</td>
</tr>
<tr>
<td>#6</td>
<td>BMQI</td>
<td><img src="image6" alt="BMQI membrane" /></td>
<td>17.795</td>
</tr>
<tr>
<td>#7</td>
<td>Hollow</td>
<td><img src="image7" alt="Hollow membrane" /></td>
<td>30.1</td>
</tr>
<tr>
<td>#8</td>
<td>Grooved</td>
<td><img src="image8" alt="Grooved membrane" /></td>
<td>12.31</td>
</tr>
<tr>
<td>#9</td>
<td>Peninsula</td>
<td><img src="image9" alt="Peninsula membrane" /></td>
<td>18.4</td>
</tr>
</tbody>
</table>

By comparing the different structures of the sensor membrane, it is found the beams structure is good for achieving high sensitivity, meanwhile creating partially stiffened membrane is beneficial to improve the linearity of the sensor. Thus, when combining with the characteristics of both structures, it is possible to obtain a device with high sensitivity and low nonlinearity at the same time.
2.5.5 Figures of merit of a pressure sensor

There exist different magnitudes commonly studied in order to assess and compare the performance of different piezoresistive pressure sensors.

- **Sensitivity (S):** The sensitivity of a pressure sensor is defined as the relative change in the output voltage per unit of applied pressure \( [Hsieh, 2010]: \)

\[
S = \frac{\Delta U}{\Delta P} \frac{1}{U_{in}} = \frac{\Delta R}{\Delta P} \frac{1}{R} \tag{2.25}
\]

where \( \Delta U \) is the change of the output voltage in pressure test range \( \Delta P \). It can be seen that sensitivity depends on \( \Delta R/R \) which depends on the gauge factor. The sensitivity reflects the ability of transferring pressure variation to voltage. Thus, it is one of the most important parameter of a pressure sensor.

- **Offset voltage (or zero output) (\( V_{off} \)):** The output voltage of a pressure sensor without any load is called an offset voltage, as presented in Figure 2.20. Normally, the output should be equal to zero when no pressure is applied on the membrane. However, some residual stress on the membrane or variability in the four resistors both will result in the offset voltage. Of course, the offset voltage can be compensated by connecting external resistors. It can be compensated using compensating resistors along with electronics.

*Figure 2.20 Illustrations for calculation of \( V_{off} \), PNL, R and H.*
• **Pressure nonlinearity (PNL), Repeatability (R) and Hysteresis (H):** Pressure nonlinearity is defined as the deviation of the specified calibration curve from the experimentally determined output of the sensor. The definition of the PNL can be followed as [Albert Chiou, 2008]:

\[
PNL = \frac{\Delta U_1}{FSS} \times 100\% \tag{2.26}
\]

\[
R = \frac{\Delta U_2}{FSS} \times 100\% \tag{2.27}
\]

\[
H = \frac{\Delta U_3}{FSS} \times 100\% \tag{2.28}
\]

where \( \Delta U \) is the voltage output difference of various range as shown in Figure 2.20, the full-scale span (FSS) is the voltage range of the full pressure range.

• **TCS, TCO and TCR:** Silicon piezoresistors are sensitive to temperature variation, which influences the mobility and number of carriers, leading to a change in piezoresistive coefficients (sensitivity), zero output, and conductivity (or resistivity), namely temperature coefficient of sensitivity (TCS), temperature coefficient of voltage offset (TCO) and temperature coefficient of resistivity (TCR) [Wieczorek, 2007].

\( TCS \) is calculated by the span signals \( U_T \) at measurement temperature \( T \) and reference temperature \( T_R (25^\circ C) \) by:

\[
TCS = \frac{1}{U_{TR}} \frac{U_T - U_{TR}}{T - T_R} \times 100\% \tag{2.29}
\]

\( TCO \) is followed by the zero output \( U_O \) at measurement temperature \( T \) and reference temperature \( T_R (25^\circ C) \) by:

\[
TCO = \frac{1}{U_{OR}} \frac{U_{UO} - U_{OR}}{T - T_R} \times 100\% \tag{2.30}
\]
\( TCR \) is also deduced by the span signals \( R_T \) at measurement temperature \( T \) and reference temperature \( T_R \) (25°C) by:

\[
TCR = \frac{100}{R_{TR}} \left( \frac{R_T - R_{TR}}{T - T_R} \right) \times 100\%
\]  

MEMS devices generally present small size, high precision and complex preparation and other characteristics. It is difficult to test them during the design and fabrication. Because of the development of the simulation methods, many difficult measurement parameters, such as deformation, stress, voltage and current, can be deduced directly, which provides great help for the researchers. Many problems can be resolved at early stage of the design. By simulation, the relative performance of the device can be predicted preliminarily, meanwhile, the time can be reduced because the number of test is decreased. As early as 1990s, researchers began to use the finite element software for device design and simulation, study the stress and deflection distribution on the sensor chip [Werner, 1995]. Lv et al. analyzed the sandwich structure of the sensor chip by simulation software, obtained the deflection and stress distribution, and finally designed and fabricated the capacitive pressure sensor [Lv, 2012]. Hsieh Chichang et al. used the simulation software to design and establish a piezoresistive pressure sensor model, with the help of simulation, the optimal arrangement of resistors was determined [Hsieh, 2013].

For mechanical engineers, a major effort in fabricating a product includes suitable selection and application of manufacturing techniques such as machining, welding, casting, milling, molding, stamping and drilling. It is quickly realized that none of the above traditional techniques can be used in MEMS manufacturing because of the extremely micro size of these devices. Some of these traditional fabrication methods, however, are used in the packaging of MEMS products. The fabrication techniques used in MEMS consist of the conventional processing developed for integrated circuit techniques and a variety of techniques developed specifically for MEMS. Generally, the essential elements in conventional silicon processing include (a) bulk
microfabrication, (b) surface microfabrication, (c) LIGA process, and (d) anodic bonding.

Bulk micromanufacturing is widely used in the production of microsensors and accelerometers. It was first used in microelectronics in the 1960s. Bulk microfabrication involves the removal of materials from the bulk substrate, usually silicon wafers, to form the desired three-dimensional geometry of the microstructures. In contrast to bulk micromachining in which substrate material is removed by physical or chemical means, the surface micromachining technique builds the microstructure by adding materials layer by layer on top of the substrate. LIGA is an acronym for the German terms meaning lithography (Lithographie), electroforming (Galvanof ormung), and molding (Abformung). The single most important feature of this process is that it can produce “thick” microstructures that have extremely flat and parallel surfaces, such as micro gear trains, micro motors, and micro turbines made of metals and plastics. These unique advantages are the primary reasons for its increasing popularity in the MEMS industry. Anodic bonding is reliable and effective for attaching silicon wafers to several other materials. It provides a hermetic seal and is an inexpensive way for die bonding. Anodic bonding is popular in microsystems packaging because of the relatively simple setup with inexpensive equipment.

In this thesis, the main fabrication techniques we chose involved deposition, etching, lithography and bonding, together with a broad range of micromachining techniques. The detailed processes for fabricating a sensor chip used above techniques will be introduced in Chapter 4.
3. DESIGN AND ANALYSIS OF THE NOVEL STRUCTURAL MEMBRANE

The structure of piezoresistive pressure sensors always plays an important role in the performance. Design a novel structure with stress concentration regions (SCRs) and partially stiffened membrane (PSM) is an efficient way to improve sensitivity and linearity, simultaneously. Mechanics, by a traditional definition, is a branch of engineering science that studies the relationship between the applied forces to a substance and the resulting motions. In the beginning, we started with the mechanical analysis to illustrate the working principle. For most case, these membranes, either in circular, square, or rectangular shapes, can be treated approximately as thin plates subjected to lateral bending by uniformly applied pressure.

3.1 Mechanical design and analysis

3.1.1 Deflection theory of thin plate

For piezoresistive pressure sensors, elastic sensitive diaphragm is the most important part. Almost all the properties of the sensor rely on the membrane, so the theory of thin plate is usually used for analysis of the membrane. According to the ratio between membrane deformation and thickness, the deflection theory can be divided into small-deflection theory and large-deflection theory. The small-deflection theory is always chosen to describe the principle of the sensor. The large-deflection theory is usually adopted to illustrate the phenomenon of nonlinearity.

There are two reasons for non-linearization of the pressure sensor, one is related to the excessive load on the membrane which directly destroys the linearity principle of the piezoresistive effect, and another is the oversized membrane deflection that changes linearity to nonlinearity in the relation between pressure and deformation. The case of excessive load on the membrane is not discussed in this work. After all, it belongs to an accident. Therefore, the case for oversized deflection will be the only
discussed here. When the deflection exceeds the definite value compared to the thickness of membrane, the large-deflection theory works, and then, the linearity relationship between deflection and pressure will change to nonlinearity. To resolve the problem of large deflection, a combination method developed by Foppl is chosen [Lin, 1999]. This method combines small deflection theory on clamped boundary conditions and the membrane theories for large deflection problems on simply supported boundary conditions. Then, the total loading $P$ is divided into two parts, namely, the bending stress and shearing stress represented by $P_1$ and membrane stress represented by $P_2$ ($P = P_1 + P_2$).

- The loading $P_1$ is derived from the small deflection theory as:

$$P_1 = 71.3 \frac{\omega EH^3}{L^4}$$  \hspace{1cm} (3.1)

- The loading $P_2$ is derived from the large deflection theory as:

$$P_2 = 31.1 \frac{\omega^3 EH}{L^4}$$  \hspace{1cm} (3.2)

These two equations are deduced separately and combined together in the end. For the large deflection, the equation can be shown as below:

$$\frac{PL^4}{EH^4} = 71.3 \left( \frac{\omega}{H} \right) + 31.1 \left( \frac{\omega}{H} \right)^3$$ \hspace{1cm} (3.3)

where $PL^4/EH^4$ is the relative load, $\omega/H$ is the deflection relative to membrane thickness. By comparing the small and large deflection theories, the dimensionless deflection with respect to dimensionless load is presented in Figure 3.1.

From the large deflection curve, it can be seen the relationship between the load and deflection is no longer linear when the relative deflection $\omega/H$ is exceeded 0.2. It indicates that the small deflection theory works only when the deflection is smaller than 1/5 thickness of membrane. Thus, the maximum deflection should be less than 1/5 thickness of the membrane, which is beneficial for obtaining low nonlinearity.
For silicon piezoresistive pressure sensors, there are several kinds of shape can be chosen as the membrane, such as circular, square and rectangular shapes. In early stage, circular shape was always adopted due to the convenient fabrication and stable performance. With the development of the microfabrication technique, other two types are usually used to achieve higher properties.

### 3.1.2 Stress analysis of square membrane

As it is known, stress analysis of the square membrane is based on the deflection theory that includes small deflection theory and large deflection theory according to the deformation degree. Normally, small deflection theory works only when the deflections are smaller than 1/5 thickness of membrane. Otherwise, the large deflection theory works. Besides, the residual stresses play an important role and alter the total stress and load limitation, especially for diaphragms with large length to thickness ratio [Zhao, 2016].

#### 3.1.2.1 Stress analysis based on small deflection theory

Based on the small-deflection theory, the membrane displacement in the center is small enough (namely, \( w_0/H < 1/10 \)), compared to the thickness of the membrane under transverse load pressure. The total stress usually includes bending stress and tensile stress, whereas the tensile stress derived from
the mid-plane of diaphragm strain is small enough to be neglected. Considering a uniformly loaded, clamped, square shaped membrane with residual stress, the load deflection relation with small deflection is followed as \cite{elwenspoek2012}:

\[
P = 3.41 \sigma_0 \frac{H^2}{L^2} \left( \frac{\omega_b}{H} \right) + 4.13 \frac{EH^3}{L^4(1-\nu^2)} \frac{\omega_b}{H} \tag{3.4}
\]

where \(P\) is the uniform load pressure, \(\omega_b\) is the maximum deflection and \(\sigma_0\) is the residual stress. Here, we care more about the relationship between internal stress and load pressure. It is presented by the following:

\[
\sigma_{total \, max} = \sigma_{bend \, max} = 1.76 \frac{EH^2}{L^2} \frac{\omega_b}{H} \tag{3.5}
\]

where Young’s modulus \(E = 166\) GPa, membrane length \(L = 3600\) \(\mu\)m, thickness \(H = 30\) \(\mu\)m. The maximum total stress is proportional to \(\omega_0/H\), when \(\omega_0/H\) is equal to 1/10, \(\sigma_{total \, max}\) is \(2 \times 10^6\) Pa. Obviously the maximum total stress is far less than the fracture stress of silicon (7\(\times\)10\(^9\) Pa) \cite{petersen1982} with small deflection under uniform pressure. Then, the relationship between the maximum stress \(\sigma_{total \, max}\) and uniform pressure \(P\) can be deduced by combining with Equation (3.4) and Equation (3.5):

\[
\sigma_{total \, max} = \frac{1.76P}{3.41 \sigma_0 \frac{H^2}{E} + 4.54 \frac{H^2}{L^2}} \tag{3.6}
\]

when the residual stress is neglected in the calculation of the maximum total stress, That is to say, the first term on the right of Equation (3.4) is neglected. The maximum total stress of the membrane free from residual stress under small deflection can then be easily simplified from Equation (3.6) \cite{hin-leung1987,palik1998}:

\[
\sigma_{total \, max} = \frac{1.76P}{4.54 \frac{H^2}{L^2}} = 0.31 \frac{L^2}{H^2} P \tag{3.7}
\]

If the residual stress is taken as the middle value between 30 MPa and 100 MPa, namely 65 MPa, and the ratio of membrane length to thickness \(L/H\) as 3600:30, it will result to \cite{bourouina1995,maseeh1990}:
Chapter 3: Design and analysis of the novel structural membrane

$$\sigma_{\text{totalmax}} = 0.52 \frac{E}{\sigma_0} P$$  \hspace{1cm} (3.8)

However actually, to facilitate the calculation, the residual stress is usually neglected during the process of simulation. Though there will be some errors between the calculation and actual value, it is more convenient to predict the performance of the pressure sensor.

3.1.2.2 Stress analysis based on large deflection theory

Since the ratio of the membrane length to thickness is huge and the thickness of membrane is very thin, the flexural rigidity of the plate is deemed to be zero approximately. The solution of load deflection relation with large deflection can be expressed by [Tabata, 1989]:

$$P = 3.04 \frac{\sigma_0 H^2}{L^2} \left( \frac{\alpha_0}{H} \right) + 1.88 \frac{EH^4}{L^4(1-\mu^2)} \left( \frac{\alpha_0}{H} \right)^3$$ \hspace{1cm} (3.9)

The total stresses $\sigma_{\text{total max}}$ for membrane includes the bending stress $\sigma_{\text{bend}}$ and the mid-plane tensile stress $\sigma_{\text{tensile}}$. The tensile stress distribution on the membrane is shown in Figure 3.2. It can be found that the maximum tensile stress occurs at the center of an edge. $\sigma_{\text{bend}}$ and $\sigma_{\text{tensile max}}$ with large deflection are presented as follows:

$$\sigma_{\text{bend}} = 6.5 \frac{EH^2}{L^2} \frac{\alpha_0}{H}$$ \hspace{1cm} (3.10)

$$\sigma_{\text{tensile max}} = 0.864 \frac{EH^2}{L^2} \left( \frac{\alpha_0}{H} \right)^2$$ \hspace{1cm} (3.11)

Then, the total maximum stress $\sigma_{\text{total max}}$ is obtained as following:

$$\sigma_{\text{total max}} = \sigma_{\text{tensile max}} + \sigma_{\text{bend}} + \sigma_0$$

$$= 0.864 \frac{EH^2}{L^2} \left( \frac{\alpha_0}{H} \right)^2 + 6.5 \frac{EH^2}{L^2} \frac{\alpha_0}{H} + \sigma_0$$ \hspace{1cm} (3.12)
Comparing Equation (3.10) with Equation (3.11), it can be concluded that the tensile stress is in proportion to \((\omega_0/h)^2\), whereas the bending stress is proportional to \(\omega_0/h\). Obviously, the tensile stress makes more contributions to the total stress under large deflection. We assume that the typical parameters \(L\), \(H\), \(E\), \(\sigma_0\) are 3600 \(\mu\)m, 30 \(\mu\)m, 166 GPa, 65 MPa. If the maximum deflection \(\omega_0=300\ \mu\)m, the maximum tensile stress \(\sigma_{\text{tensile max}}\) and bending stress \(\sigma_{\text{bend}}\) are 996 MPa and 750 MPa, respectively. It can be seen that the two items are much bigger than the residual stress \(\sigma_0\). Then, the residual stress can be neglected. However, the actual maximum deflection of the proposed membrane \(\omega_b\) is only 77.6 \(\mu\)m [Li, 2017b], the membrane will be fractured when deflection is much more than 77.6 \(\mu\)m. Where \(\omega_b/H=2.6\), the three stresses, residual stress (65 MPa), tensile stress (67.3 MPa) and bend stress (194.8 MPa) are comparable. Here, the residual stress cannot be neglected. Thus, the residual stress should not be neglected in calculating the total stress for boron-doped silicon diaphragms with large deflection. Just because the residual stress cannot be neglected, so the nonlinearity for large deflection is so large that the test values are far from the true values.

### 3.1.3 Structure design of proposed membrane

A novel structure featuring a four-grooved membrane with rood beam structure was designed for the sensor chip to measure micro pressure less than 1 psi (1psi =
6.895 \times 10^3 \text{ Pa}). N-type SOI wafer was chosen as the substrate of the sensor chip owing to its desirable characteristics like excellent mechanical properties and reproducible elastic deformations.

On the front side of the membrane, there are four grooves around the surface as shown in Figure 3.3 (a). Moreover, four ribs are located between every two grooves which are just on the top of the gap between each beam and membrane edge. Then, a Wheatstone bridge is built up through electric connections with the four piezoresistors on the surface of the rib regions.

![Figure 3.3 Sketch of the proposed membrane: (a) Front view; (b) Rearview; (c) Partial section view; (d) Detailed structure around the rib place.](image)

On the rear side, a rood beam structure is placed as shown in Figure 3.3 (b). The end of each rood beam is not connected with membrane edge but remain at a distance between them, which can be seen clearly in Figure 3.3 (c). Meanwhile the rib width is equal to the groove width, the rib length is equal to the rood beam width, which not only simplifies the fabrication processes, but facilitates the dimensions optimization. By incorporating four grooves and rood beam into the membrane, high concentrated
stress profile (HCSP) is expected to be formed to maximize the sensitivity. Meanwhile, the partially stiffened membrane (PSM) is good for reducing the deflection of the membrane. As a result, the nonlinearity is reduced.

### 3.1.4 Mechanics analysis for front side of the membrane

The membrane deformation model is based on the solution for a rectangular and continuous plate on elastic foundation with uniform load. This model assumes that the intensity of the reaction $p$ at any point of the bottom plate is proportional to the deflection $\omega$ at that point, so that $p = k\omega$, $k$ is the modulus of the foundation [Ren, 2013]. Based on this assumption, a series of Navier trigonometric equations about the membrane deformation are deduced as follows [Timoshenko, 1959]:

\[
\frac{\partial^4 \omega}{\partial x^4} + 2\frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{q}{D} - \frac{k\omega}{D} \tag{3.13}
\]

\[
q = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{L} \tag{3.14}
\]

\[
\omega = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{L} \tag{3.15}
\]

where $q$ is the intensity of the lateral load; $\omega$ is the membrane deformation; $D$ is the flexural stiffness of the membrane, $D =EH^3/[12(1-\mu^2)]$, $E$ is the Young modulus, $\mu$ is the Poisson ratio, $H$ is the membrane thickness, $L$ is the side length of the square membrane, $A_{mn}$ and $a_{mn}$ are the constants related to the membrane structure. Also, $A_{mn}$ and $a_{mn}$ can be expressed by follows:

\[
A_{mn} = \frac{a_{mn}}{\pi^4 D \left(\frac{m^2}{L^2} + \frac{n^2}{L^2}\right)^2 + k} \tag{3.16}
\]

\[
a_{mn} = \frac{4P}{L^2} \frac{\sin \frac{m\pi x}{L}}{L} \frac{\sin \frac{n\pi y}{L}}{L} \tag{3.17}
\]
where \((\zeta, \eta)\) represents a certain point of the membrane, \(m\) and \(n\) are the odd number and even number of the series term, respectively. By substituting Equation (3.16) and Equation (3.17) into Equation (3.15), the final \(\omega\) is obtained:

\[
\omega = \frac{4P}{L^2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin \frac{m\pi \zeta}{L} \sin \frac{n\pi \eta}{L}}{m^4 D \left(\frac{m^2}{L^2} + \frac{n^2}{L^2}\right)^2 + k} \sin \frac{m\pi \alpha}{L} \sin \frac{n\pi \alpha}{L} \tag{3.18}
\]

Having the deformation of the membrane produced by a concentrated force, the deformation produced by any kind of lateral loading is obtained by the method of superposition. Substitute \(q\), \(d\zeta\) and \(d\eta\) for \(P\) into Equation (3.18) and integrating between the limits 0 and \(L\), \(\omega\) is followed as:

\[
\omega = \frac{16q}{\pi^2} \sum_{m=1,3,5,\ldots}^{\infty} \sum_{n=1,3,5,\ldots}^{\infty} \frac{\sin \frac{m\pi \alpha}{L} \sin \frac{n\pi \alpha}{L}}{\pi^4 D \left(\frac{m^2}{L^2} + \frac{n^2}{L^2}\right)^2 + k} \tag{3.19}
\]

According to the Navier theory, the loading condition of the proposed structure can be seen as the superimposition of three basic loading conditions. They are uniform load on a simply supported membrane, centrosymmetric load on a simply supported membrane and bending moment along four edges on a simple supported membrane, respectively [Zhao, 2016].

3.1.4.1 Uniform load on a simply supported membrane

For the simply supported membrane loaded with a uniform pressure, the deflection of the membrane can be presented as [Timoshenko, 1959]:

\[
\omega_{y} = \frac{4qL^4}{\pi^5 D} \sum_{m=1,3,5,\ldots}^{N} \frac{1}{m^3} \left(1 - \frac{\alpha_m\tan \alpha_m}{2\cosh \alpha_m} + 2\frac{\alpha_m y}{2\cosh \alpha_m} \frac{2y}{L} \sinh \frac{2\alpha_m y}{L} \right) \sin \frac{m\pi \alpha}{L} \tag{3.20}
\]

where \(\alpha_m\) is equal to \(m\pi/2\), \(N\) is the largest value of \(m\). By using Equation (3.20), the deflection at any point on the membrane can be calculated. In the case of a uniform load, the maximum deflection of the membrane emerges at its center. By substituting \(x=L/2, y=L/2\) in the Equation (3.20), \(\omega_{\text{max}}\) is shown as the following:
\[
\alpha'_{\text{max}} = \frac{4qL^4}{\pi^5D} \sum_{m=1,3,5,\ldots}^{N} \left( \left(1 - \frac{\alpha_m \tanh \alpha_m + 2}{2 \cosh \alpha_m} \right) \right) \quad (3.21)
\]

The series in this expression converges very rapidly, and sufficient accuracy can be obtained by taking only the first term. Due to \(\alpha_1 = \pi/2, \alpha_3 = \pi/3\), so \(\omega_{\text{max}}\) can be expressed finally by:

\[
\omega_{\text{max}} = \frac{5}{384} \frac{qL^4}{D} - \frac{4qL^4}{\pi^5D} (0.68526 - 0.00025 + \ldots) = 0.0040qL^4/D \quad (3.22)
\]

3.1.4.2 Centrosymmetric load on a simply supported membrane

For the simply supported membrane loaded with a centrosymmetric concentration force, the loading condition can be derived from the superimposition of the loading condition of a simply supported membrane loaded with cross-shaped partial load. Using Navier’s method an expression in double-series form has been introduced. Take the point \(x = \xi, y = \eta\) as an example, the deflection at that place can be expressed by [Timoshenko, 1959]:

\[
\omega_c = \frac{qL^2}{\pi^3D} \sum_{m=1,3,5,\ldots}^{N} \left( 1 + \beta_m \coth \beta_m \right) \left( \coth \frac{\beta_m y_1}{L} - \coth \frac{\beta_m \eta}{L} \right) \left( \sinh \frac{\beta_m \eta}{L} \sinh \frac{m \pi \xi}{L} \sin \frac{m \pi \eta}{L} \right) \left( \sinh \frac{\beta_m y_1}{L} \sin \frac{m \pi \eta}{L} \right) \left( \sin \frac{m \pi \eta}{L} \right) \left( \sin \frac{m \pi \eta}{L} \right) \quad (3.23)
\]

where \(\beta_m = m \pi, y_1 = L - y\), and \(y \geq \eta\). In the case of \(y < \eta, y_1\) will be replaced by \(y\) and \(\eta\) by \(\eta_1 = L - \eta\). Consider more closely the particular case of a load \(q\) concentrated at a point \(A\) on the axis of symmetry of the membrane, which may be used as the \(x\) axis. With \(\eta = L/2\) and \(\alpha_m = \beta_m/2\), the deflection of the membrane becomes:

\[
\omega_c = \frac{qL^2}{2\pi^3D} \sum_{m=1,3,5,\ldots}^{N} \left( 1 + \alpha_m \tanh \alpha_m \right) \left( \frac{\alpha_m}{L} (L - 2y) - \frac{\alpha_m}{L} (L - 2y) \coth \frac{\alpha_m}{L} (L - 2y) \right) \left( \sin \frac{m \pi \xi}{L} \sin \frac{m \pi \eta}{L} \right) \left( \sin \frac{m \pi \eta}{L} \right) \left( \sin \frac{m \pi \eta}{L} \right) \quad (3.24)
\]

In the case of a load \(q\) applied at the center of the membrane, the maximum deflection, which is at the center, is obtained by substituting \(x = \xi = L/2\) in Equation (3.24). In this way, maximum deflection can be followed as:

\[
\omega_{c,\text{max}} = \frac{qL^2}{2\pi^3D} \sum_{m=1,3,5,\ldots}^{N} \frac{1}{m^2} \left( \tanh \alpha_m - \frac{\alpha_m}{\cosh^2 \alpha_m} \right) = 0.0116qL^2/D \quad (3.25)
\]
3.1.4.3 Bending moment along four edges on a simple supported membrane

Take account of the fixed boundary condition of the membrane, the membrane rotation along the four edges should be equal to zero [Barlian, 2009]. To compensate the rotation along the membrane edges produced by the pressure, bending moment is introduced to the model. When the pressure is loaded on the surface of the plate, the bending moments loaded on each edge can be shown as follows [Timoshenko, 1959]:

\[
(M_x)_{x=\pm L/2} = \sum_{m=1,3,5...}^{N} (-1)^{(m-1)/2} E_m \cos \frac{m\pi x}{L} \quad x \in (-\frac{L}{2}, \frac{L}{2}) \tag{3.26}
\]

\[
(M_y)_{y=\pm L/2} = \sum_{m=1,3,5...}^{N} (-1)^{(m-1)/2} F_m \cos \frac{m\pi y}{L} \quad y \in (-\frac{L}{2}, \frac{L}{2}) \tag{3.27}
\]

where \(E_m, F_m\) are the coefficient decided by the load and interaction force. Due to the constraint of the two edges, the magnitude of the deflection in the membrane is increased quickly when the uniform pressure is loaded on the membrane surface. According to the Navier theory, the deflection of the membrane can be followed as [Timoshenko, 1959]:

\[
\omega_B = \frac{4qL^4}{\pi^2 D} \sum_{m=1,3,5...}^{N} (-1)^{(m-1)/2} \frac{m\pi}{m^5} \cos \frac{m\pi x}{L} (1 - \frac{\alpha_m \tanh \alpha_m + 2}{2 \cosh \alpha_m} \cosh \frac{m\pi y}{L} + \frac{1}{2 \cosh \alpha_m} \frac{m\pi y}{L} \sinh \frac{m\pi y}{L}) \tag{3.28}
\]

Taking the case of a centrally loaded membrane and assuming that the edges \(x=\pm L/2, y=\pm L/2\), the maximum deflection of the membrane can be expressed by the following Equation (3.29):

\[
\omega_{B_{\text{max}}} = \frac{qL^2}{2\pi^2 D} \left[ \sum_{m=1,3,5...}^{N} \frac{1}{m} (\tanh \alpha_m - \frac{\alpha_m}{\cosh^2 \alpha_m}) - \frac{\pi^2}{4} \sum_{m=1}^{\infty} \frac{1}{m \sinh \alpha_m \cosh \alpha_m + \alpha_m} \right] = 0.00214 \frac{qL^2}{D} \tag{3.29}
\]
3.1.4.4 Superimposing for the loading condition

The superimposing for the loading condition of a four-edge fixed membrane with a centrosymmetric concentration force is illustrated in Figure 3.4. By synthesizing the sections presented in the section of 3.1.4.1−3.1.4.3, the condition of the four-edge membrane loaded with a centrosymmetric concentration force can be obtained. Then, the deflection of the four edge fixed membrane under pressure can be presented as:

$$\omega = \omega_U - \omega_C - \omega_B \quad x, y \in \left( -\frac{L}{2}, \frac{L}{2} \right)$$

(3.30)

where $\omega$ is the membrane deflection for the proposed structure. It means that the final deformation can be calculated by superimposing the above three loading conditions.

---

**Figure 3.4 Composition of the loading condition of a four-edge fixed membrane under a centrosymmetric concentration force.**
Chapter 3: Design and analysis of the novel structural membrane

The four edge fixed membrane separately loaded with an axisymmetric concentrated force or a pressure is the basic model to study the proposed novel structural membrane. The bending moment and stress of the membrane are both from the deformations, the deflections of the membrane under different loads should be discussed. The detailed material properties and the geometry parameters are shown in Table 3.1 [Maluf, 2000], [Yu, 2001], [Hopcroft, 2010].

**Table 3.1 Material property and geometry parameters in the basic loading condition.**

<table>
<thead>
<tr>
<th>Items</th>
<th>Uniform load on a simply supported membrane</th>
<th>Centrosymmetric load on a simply supported membrane</th>
<th>Bending moment along four edges on a simple supported membrane</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(L(\mu m))</td>
<td>(3.6 \times 10^3)</td>
<td>(3.6 \times 10^3)</td>
<td>(3.6 \times 10^3)</td>
</tr>
<tr>
<td>(H(\mu m))</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td><strong>Material properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(E) (GPa)</td>
<td>166</td>
<td>166</td>
<td>166</td>
</tr>
<tr>
<td>(\mu)</td>
<td>0.28</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>(\rho) (g cm(^{-3}))</td>
<td>2.33</td>
<td>2.33</td>
<td>2.33</td>
</tr>
<tr>
<td><strong>Loading conditions</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(F) (N)</td>
<td>-</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>(q) (Pa)</td>
<td>6895</td>
<td>(F/dx\ dy)</td>
<td>(F/dx\ dy)</td>
</tr>
</tbody>
</table>

The deflection of the membrane in different cross sections can be obtained from Equation (3.29) as well as FEM simulation as shown in Figure 3.5. The theoretical results are calculated by superimposing the deflection of four edge fixed membrane under various axisymmetric concentrated forces. Figure 3.6 illustrates the relative errors of the deflections between the theoretical calculation and FEM simulation across the section line \(y = 0\) in Figure 3.5. It can be seen that the relative errors are small and the value is controlled within 1.5%, which illustrates that the mechanics analysis is sufficiently accurate and reliable.
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Figure 3.5 Deflections results by theory calculation and FEM simulation in different section line.

Figure 3.6 Relative errors between theory and FEM simulation results.

To prove the feasibility of the analytical model in section 3.5, the results of the membrane deflection value are provided by both the theoretical calculation and the finite element method (FEM), as shown in Figure 3.7. In order to observe the deformation of the membrane, a three-dimensional working sketch is displayed. The relative error between the analytic and FEM is lower than 10%, which validates the theoretical calculation is consistent with the simulation results. Under the load of 1 psi, the maximum deflection of 2.65 μm is far less than membrane thickness. Based on small deformation theory, a low nonlinearity error will be achieved when the maximum deflection is under 1/5 thickness of membrane.
3.1.5 Mechanics analysis for rear side of the membrane

3.1.5.1 Mechanics model for the rood beam

For the proposed sensor structure, the rood beam plays the main role for the strain energy concentration at the HCSP. To interpret the working mechanism of the rood beam structure, the Euler-Bernoulli beam theory is introduced to describe the deformation mechanism of the membrane combined with a rood beam structure [Barber, 2004]. A fundamental assumption of this theory is that the cross section of the beam is infinitely rigid in its own plane, i.e., no deformations occur in the plane of the cross-section. This means the Euler-Bernoulli beam theory works only when the cross-section of a beam remains plane after deformation.

The rood beam’s width of 200 μm is quite small compared to the membrane length of 3600 μm as shown in Figure 3.8 (a). When the pressure is applied to the front surface of the membrane, the deflections in the same cross section along the width direction of the corresponding rood beam are assumed to retain the same value. Meanwhile, a distributed reactive force along the rood beam length will be produced by each beam structure to maintain the membrane balance. The four membrane...
edges can be regarded as a rigid body compared with the membrane as shown in Figure 3.8 (b). When pressure is applied to the membrane, the distributed reactive force can be simplified as a concentrated load $P$ and a bending moment $M$ at the end of the rood beam [Shimazoe, 1981].

Under the action of the bending moment, this segment deforms into a circular segment with ends defined by the cross-sections $S'$ shown in in Figure 3.8 (c). After the deformation, the beam is symmetric with respect to any plane perpendicular to its deformed axis. Because the deformed cross-section must satisfy this symmetry requirement, it must remain planar and perpendicular to the deformed axis of the beam.

![Figure 3.8 Schematic diagram of the rood beam under bending moment.](image-url)
3.1.5.2 Working principle for the rood beam

The rood beam structure is thicker than other places and can be regarded as a rigid body compared with the membrane. When the pressure is applied to the membrane, it will experience a deformation, and a distributed reactive force along the rood beam length will be produced by each beam structure to maintain the membrane balance [Xu, 2016]. The distributed reactive force will induce stress in the interface between the membrane and the rood beam. As shown in Figure 3.9, when 1 psi pressure is loaded to the membrane, the stress is calculated for different rood beam thickness and concentrated with dozens of MPa at the end of the rood beam structure. The figure indicates that the thickness of the rood beam affects the value of the stress but does not change the variation trend of the stress distribution.

![Maximum von Mises stress variation between the membrane and rood beam structure.](image)

**Figure 3.9 Maximum von Mises stress variation between the membrane and rood beam structure.**

The distributed reactive force can be simplified as a concentrated load $F$ and a bending moment $M$ at the end of the rood beam structure as shown in Figure 3.10. The bending moment $M$ ensures a rotation of zero at the end of the beam structure along the beam length. The loading condition of the membrane under the pressure in the first step can be equivalent to a single pressure, four concentrated forces, and four moment couples, as shown in Figure 3.10 (a).
Figure 3.10 Maximum von Mises stress variation between the membrane and rood beam structure [Zhao, 2016].

Figure 3.10 (b) shows the simplified process for the loading condition of the membrane in the half cross section view of the mechanics model. Both $F$ and $M$ can be further decomposed to a couple of concentrated force $F_1$ and $F_2$ with the relationship:

$$
\begin{align*}
F &= F_1 - F_2 \\
M &= F_2 \cdot dl
\end{align*}
$$

Besides the boundary condition of the membrane with four fixed edges, the other boundary condition induced by the rood beam structure is presented as:

$$
\begin{align*}
(w_{1})_{x=-l} &= 0 \\
(\frac{d w_{1}}{dx})_{x=-l} &= 0
\end{align*}
$$
where $w_1$ is the deflection of the membrane with rood beam structure only and $l$ is the distance from the membrane center to the end of the rood beam structure along the X axis [Zhao, 2016].

Figure 3.11 presents a schematic diagram of the membrane with rood beam structure in various colored regions from the front view. The compensation moment $M$ is very small compared with that produced by the pressure, so the effect of $M$ for the interaction force at the interface between regions B and C can be ignored. Then, the membrane deflection caused by $M$ to region C can be further simplified and the value of $M$ is calculated by the beam theory [Timoshenko, 1959]:

$$
M_1 = \frac{(\omega_2)_{x=-l_1} - (\omega_2)_{x=-l_2} + (l_1 - l_2)(\frac{d\omega_2}{dx})_{x=-l_1}}{I_{12} + (l_1 - l_2)(l - L)} E.
$$

where $I_{34}$ and $I_{12}$ are the inertial moments for the $L_3L_4$ and $L_1L_2$ segments respectively. $E$ is the Young’s modulus of the material.

Figure 3.11 Schematic diagram of membrane with rood beam structure in the front view.

It is difficult to obtain the deformation of the proposed membrane with rood beam structure directly at a single pressure. However, it is feasible to calculate the individual membrane deflection in the simplified mechanics model with the pressure and the bending moment separately. Then, the final membrane deflection in the $L_1L_2$
segment is obtained by superimposing the deflection generated by $M$ on the original deflection as deduced by Equation (3.34):

$$\omega_{L_1L_2} = \omega_2 + \omega_{ML}$$  \hspace{1cm} (3.34)

where $\omega_{ML}$ is the compensation deflection by $M_L$ and is derived from the model shown in Figure 3.10.

3.1.5.3 Double improvements principle for the accuracy

On the rear side of the membrane there is a rood beam structure located around the membrane center. The free end of the rood beam is perpendicular to the membrane edge, which is not connected with membrane wall but remains at a distance between them. Figure 3.12 introduces the performance comparison between with and without rood beam structure, which can further prove the significance of the rood beam. When the membrane is combined with a rood beam, the stress increases about 67.5% and the deflection reduces about 45.2%. It means the sensitivity and linearity may both experience a rise with the combination of the rood beam. The high accuracy of the sensor will be finally achieved.

**Figure 3.12 Performance comparison between with and without rood beam structure: (a, c) With rood beam structure; (b, d) Without rood beam structure.**
Based on the performance comparison between with and without rood beam structure, it is noted that the groove creates a stiffness mutation at the rib region along the transversal direction of the rood beam structure. It further enhances the stress concentration at the top surface of the rib structure to form a high concentrated stress profile (HCSP). Besides, this structure also lowers the pressure nonlinearity of the sensor by increasing the constraint of the partial membrane due to the relatively thick thickness of the rib region.

For the rood beam, it plays a role in partially stiffening the membrane, which is beneficial to avoid oversize deformation. Then, the pressure nonlinearity will be improved. For another, a stiffness mutation is created above the gap position along the longitudinal direction of rood beam structure to localize the strain energy precisely in the rib region above the gap. From the point of energy transmission, the rood beam works like a transmission pipeline which transfers continuously the strain energy to the end of the resistance sensitive area. Then, the piezoresistors will experience more stress-strain and the performance of the sensor will be improved. Owing to the double improvements for the sensitivity and linearity created both by grooves and rood beam as shown in Figure 3.13, the sensitivity and pressure nonlinearity of the sensor both experience twice enhancements when pressure is loaded on the membrane. Thus, the proposed sensor is expected to achieve a breakthrough in the performance.

![Figure 3.13 Theoretical analysis diagram about improving sensitivity and linearity.](image-url)
3.2 Structure design

A piezoresistive pressure sensor is usually consisted of sensor chip, shell and connecting plug. The sensor chip is the most important part because it directly determines the performance of a device. The sensor chip is formed by sensitive membrane, piezoresistors and cavity. It's worth noting that the dimension and structure of the membrane play an important role in the range, sensitivity, linearity and reliability for the device. Thus, this chapter will emphatically discuss membrane optimization, sensor structure design and reliability design. In this chapter, structural models of sensor are built by Solidworks and the performances of the membrane are simulated using COMSOL Multiphysics 5.2 and OriginLab 9.0.

3.2.1 Sensitive membrane dimensions

3.2.1.1 Design principles

In the process of the dimensional design, several principles should be considered: (a) Principle of the sensitivity. The sensitivity should be improved as far as possible within the pressure; (b) Principle of the linearization. The deflection of the membrane should obey small-deflection theory; (c) Principle of reliability. The sensor chip should work safely in the range of measurement. As the design parameters including working range \(P = 0\sim 1\, \text{psi}\), sensitivity \((\geq 150\, \text{mV}/(5\, \text{V}\ 1\, \text{psi}))\), nonlinearity error \((\leq 0.5\%\text{FSS})\), overload pressure \((P_{\text{max}}=25\, \text{kPa})\), resistance \((R=6.7\, \text{k}\Omega)\) and input voltage \((U_{\text{in}}=5\, \text{V})\) have been considered. Considering the former investigations and reviews on the MEMS piezoresistive pressure sensor [Chou, 2009], [Kanda, 1997], determining that the thickness and side length of a square membrane should be a high-priority job. Based on the above design principles and requirements, the thickness \(H\) and side length \(L\) of a square diaphragm must meet the following requirements [Tian, 2009]:

- Sensitivity of 150 mV/(5V 1psi) needs the relative change of the resistance to be higher than 3.0%;
• Linearity of 0.5% FSS requires the maximum deflection in the center of the membrane to be less than 1/5 thickness of membrane;

• The overload pressure $P_{\text{max}}$ means that the difference between the longitudinal stress $\sigma_l$ and the transverse stress $\sigma_t$ should not be larger than the material yield stress $\sigma_m$ of 30% ($\sigma_m = 2 \times 10^9 \text{ N/m}^2$ for silicon). According to above theories, three design requirements can be expressed from Equation (3.35) to (3.37):

\[
\frac{\Delta R}{R} = \frac{1}{2} \pi_{44} (\sigma_l - \sigma_t)_{\text{max}} = \frac{0.308}{2} \pi_{44} P \frac{L^2}{H^2} (1 - \mu^2) \geq 3.0\% \quad \text{(3.35)}
\]

\[
\omega_{\text{max}} = 0.0152 \frac{PL^4(1-\mu^2)}{E H^3} \leq \frac{1}{5} H \quad \text{(3.36)}
\]

\[
P_{\text{max}} = 0.308 \frac{L^2}{H^2} (1 - \mu) \leq 30\% \sigma_m \quad \text{(3.37)}
\]

where $\pi_{44}$ is the shear piezoresistive coefficient ($\pi_{44} = 138.1 \times 10^{-11} \text{ m}^2/\text{N}$), $E$ is the Young’s elastic modulus ($E = 166 \text{ GPa}$) and $\mu$ is the Poisson ratio ($\mu = 0.28$). From above three equations, it can be computed that the range of $L/H$ should meet $102.8 \leq L/H \leq 142.2$.

For the pizoresistive pressure sensor, the trade-off between sensitivity and linearity is always irreconcilable during processes of design and fabrication, especially for low pressure measurement [Thanh-Vinh, 2014]. Since sensitivity is proportional to $(\text{membrane length})/(\text{membrane thickness})$, ratio $(L/H)$, it can be increased by a larger value of that quantity. Unfortunately, the nonlinearity error increases with this ratio at a much faster rate, since the nonlinearity error of the pressure-to-stress conversion is proportional to $(L/H)^4$. Previous studies called this effect “balloon effect”. A thinner membrane is effective to obtain high sensitivity, but it may induce unsatisfactory performances such as low linearity and inferior reliability due to the large deflection and instability. Severe nonlinearity might cause a high-sensitivity device of little practical value once $L/H$ is beyond a certain range. Thus, the $L/H = 120$ is determined finally to achieve a high sensitivity and linearity.
3.2.1.2 Membrane structure

Based on the mechanical design and analysis, a novel structure featuring a four-grooved membrane with rood beam structure is designed for the sensor chip to measure micro pressure less than 1 psi (6.895 kPa). N-type SOI wafer was chosen as the substrate of the sensor chip owing to its desirable characteristics like excellent mechanical properties and reproducible elastic deformations at the high temperature. In SOI wafer, a thin film of active single-crystalline silicon lies on a silicon dioxide dielectric layer at the top of a silicon wafer. This structure solves many of the drawbacks of conventional integrated circuits including leakage currents, high temperature failure and sensibility to radiation, etc. Thus, it is a suitable material for piezoresistive pressure sensors working in harsh environment.

There are four grooves on the front side of the membrane as shown in Figure 3.14 (a). Meanwhile, four ribs are placed between every two grooves which are just on the top of the gap between each beam and membrane edge. According to the working principle, the Wheatstone bridge is built up through electric connections with the four piezoresistors on the surface of the rib regions.

A rood beam structure is located on the rear side of the membrane as shown in Figure 3.14 (b). The end of each rood beam is not connected with the membrane edge, but remains at a distance between them, which can be seen clearly in Figure 3.14 (c). It's worth noting that the rib width is equal to the groove width $b$, the rib length is equal to the rood beam width $a$, which not only simplifies the fabrication processes, but facilitates the dimensions optimization. High concentrated stress profile (HCSP) is expected to be formed to maximize the sensitivity by incorporating four grooves and rood beam into the membrane. The partially stiffened membrane (PSM) is beneficial for reducing the deflection of the membrane, and then minimizes the nonlinearity. By combining the above merits from HCSP and PSM, the high accuracy with good sensitivity and linearity is hopefully obtained.
3.2.1.3 Model analysis

According to the working principle and structural features of the proposed sensor, it works like the EI-type sensor. EI-type structure is featured by grooved and bossed membrane for sensor chip. Since the proposed structure with four grooves and bossed membrane presents a closed geometry with EI-type sensor, the central deflection $\omega$ can be expressed by Equation (3.38) [Yasukawa, 1989]:

$$
\omega = \frac{\pi^2 b^3}{8bg^3E(1+\beta)} \left[1 + \beta \left(\frac{Hg}{h}(1 + \beta) \right) \right] \omega^3
$$

(3.38)

$$
\beta = \frac{2gbE + g(\pi L - 2bE)}{\pi LbD}
$$

(3.39)

$$
D = \frac{EH^3}{12(1-\mu^2)}
$$

(3.40)
where \( \omega \) is the membrane deflection, \( \beta \) is the coefficient as defined by Equation (3.39), \( D \) is the support stiffness, such stiffness can be characterized by flexural rigidity as expressed in Equation (3.40), \( L \) and \( H \) are membrane length and thickness, \( b \) and \( g \) are groove width and depth, respectively. \( P \) is the applied pressure, \( E \) is the Young’s elastic modulus, and \( \mu \) is the Poisson ratio.

From the definition of the EI-type membrane deflection, \( \omega \) can be treated as a subtraction of the rib bending strain (the 1st part of the Equation (3.38)) minus the membrane deforming strain (the 2nd portion of Equation (3.38)) [Yasukawa, 1989]. It is noted that \( \omega \) will be decreased by reducing the groove depth \( g \) and groove width \( b \). In other words, the linearity can be improved by reducing the volume of the groove.

According to previous studies, there are several theories should be considered: the membrane deformation should be under 1/5 thickness of membrane based on small deformation theory; a plate is called “thin” when its ratio of thickness to the smaller span length is less than 1/20 [Ugural, 1981], [Cao, 2000]; the thickness is usually above 10\( \mu \)m for a good linearity according to the literature [Albert Chiou, 2008], especially for those with the central bossed structure. If all factors are taken into consideration, the groove depth should be as close as possible to half of central membrane.

Additionally, one of initial conditions for the equations of bossed structure assumes a negligible bending moment at the central bossed-region under a certain pressure. The stiffness at the membrane center has to be much higher than that of the groove region because of the different thickness of the plate. Then, this stiffness mutation promotes the appearance of the HCSP at the rib region, which makes it possible to achieve high sensitivity. Since the thickness of the membrane edge is thinner than the center, there will be some losses in terms of stress. However, the introduction of the rood beam at the rear side of the membrane not only can compensate the losses of stress, but also play a role in partially stiffening the membrane. It is an efficient way to resolve the contradiction between sensitivity and linearity.
3.2.2 Geometry optimization

3.2.2.1 FEM analysis

Based on the previous design experiences [Tian, 2009], [Zhao, 2016], [Zhao, 2017], the scope for each structural dimension variable is followed by:

\[
\begin{align*}
3000 & \leq L \leq 4000 \\
20 & \leq H \leq 40 \\
20 & \leq h \leq 50 \\
160 & \leq a \leq 240 \\
40 & \leq b \leq 120 \\
0 & \leq g \leq 20
\end{align*}
\]

In the scope for each structural dimension variable, take some fixed size as an example. The performances of the proposed sensor chip are calculated under 1 psi by non-linear static analysis and modal analysis using the commercially-available finite element method software COMSOL Multiphysics. Only a quarter of the finite element model is established for the sensor chip due to the symmetry as shown in Figure 3.15. In accordance with the previous discussion, the stress is mainly concentrated at the hinge area which is located at the rib surface, as indicated by the dotted square, where is called as the HCSP.

![Figure 3.15 Von-Mises stress distribution for a 1/4 model of the membrane.](image)

For the P-type [110] oriented piezoresistors, sensitivity is actually determined by the magnitude of stress difference \((\sigma_x - \sigma_y)\) between transversal stress \(\sigma_x\) and longitudinal stress \(\sigma_y\). Generally speaking, the transversal stress is several times larger than longitudinal stress for single crystal silicon when resistors are placed in right direction.
Hence, the $\sigma_x$ will be taken as an example to discuss the stress distribution on the membrane. **Figure 3.16** reflects the transversal stress distribution along x-path from one edge to another under 1 psi uniform pressure and illustrates the relationship between stress and x-path. It can be observed that the maximum stress along x-path appears at HCSP, but the stress in other regions of membrane is close to zero. It means that the strain energy is strictly limited in a narrow area on the surface of the rib region and the energy is not spread easily outside the HCSP.

![Figure 3.16 Transversal stress distribution of the proposed membrane along x-path.](image)

As discussed before, a stiffness change was formed at the mutation region on the surface of the membrane to concentrate more strain energy in this region [Bashir, 2000]. Based on the small deformation theory introduced above, the rood beam can make a contribution to the stiffness enhancement and reduce the membrane deflection under pressure. Thus, the proposed structure will be a proper choice for maximizing the sensitivity and minimizing the nonlinearity.

3.2.2.2 Optimization process

The performance of a pressure sensor depends on the initial resistance of the piezoresistors, which in turn is dependent on the geometry dimension variables of the membrane. In order to determine and optimize the structural dimensions, it is necessary to derive the relationship between structure and stress for the proposed
sensor designing. The simulation analysis for the transversal stress and deflection with different structural dimension variables will be analyzed below. According to the formulas of traditional C-type membrane structure, the maximum stress and deflection of the membrane are the power functions of each single dimension variable [Young, 2002]. Theoretically, the mathematical function relationship between the proposed structure and C-type should be similar. Based on Yu’s study [Yu, 2013], Young’s elastic modulus \( E \) had almost no influence on the stress \( \sigma \), but revealed the inverse proportional relationship to the deflection \( \omega \). Consequently, the formulas of the proposed membrane are expressed as Equation (3.41) and (3.42):

\[
\sigma = Q_1 \cdot P \cdot L^{i_1} \cdot H^{k_1} \cdot h^{m_1} \cdot a^{n_1} \cdot b^{r_1} \cdot g^{s_1} \quad (3.41)
\]

\[
\omega = Q_2 \cdot P \cdot E^{-1} \cdot L^{i_2} \cdot H^{k_2} \cdot h^{m_2} \cdot a^{n_2} \cdot b^{r_2} \cdot g^{s_2} \quad (3.42)
\]

where \( L, H, h, a, b, g \) are the structural dimensions as shown in Figure 3.14; \( \sigma \) and \( \omega \) are the maximum transversal stress and maximum deflection under pressure; \( Q_1, Q_2, i_1, i_2, k_1, k_2, m_1, m_2, n_1, n_2, r_1, r_2, s_1, s_2 \) are the coefficients.

To derive the coefficients in Equation (3.41) and (3.42), each single dimension variable should be discussed separately. It means that other variables have to be seen as constants when one variable is studied. For instance, when the influence of membrane length \( L \) is focused on, other variables have to be assumed as constants. Of course, the values of these variables are given initially and arbitrarily in the ranges of actual processing. As shown in Figure 3.17 (a), with the membrane length adding, the stress and deflection increase quickly. The relationships between the sensor chip performance and membrane length satisfy Equation (3.43) and (3.44):

\[
\sigma = Q_{1l} \cdot L^{i_1} \quad (3.43)
\]

\[
w = Q_{2l} \cdot L^{i_2} \quad (3.44)
\]
where $Q_1$, $Q_2$, $Q_3$, $Q_4$, $i_1$, $i_2$ are the coefficients for the variable $L$, and other parameters are as mentioned. Since the variation of the membrane length $L$, a series of $\sigma$ and $\omega$ will be got by COMSOL numerical calculation. Then, fitting curves and the coefficients $Q_1$, $Q_2$, $i_1$, $i_2$ are obtained by utilizing the software Origin in accordance with simulation results. Therefore, Equation (3.43) and (3.44) are followed below:

\begin{align*}
\sigma &= 7.83965 \times 10^{-8} L^{2.51041} \\
\omega &= 4.54742 \times 10^{-15} L^{4.15924}
\end{align*}

\(3.45\) \hspace{2cm} \(3.46\)

Figure 3.17 (a) Relationship between stress and membrane length; (b) Relationship between deflection and membrane length.

In order to verify the goodness-of-fit between these equations and the simulation results, the residual curves of stress and deflection are also illustrated in Figure 3.17 (b). The residual curve is drawn by residual error points that reflect the difference between the actual value of each point on the scatterplot and the regression equations’ predicted value. To achieve the best goodness-of-fit, the coefficient of determination ($R^2$) and residual sum of squares (RSS) are introduced.

In statistics, the coefficient of determination, denoted $R^2$, is a number that indicates the proportion of the variance in the dependent variable that is predictable from the independent variable. $R^2$ is a statistic that will give the information about the goodness-of-fit of a model. In regression, the coefficient of determination $R^2$ is a
statistical measure of how well the regression line approximates the real data points. An $R^2$ closer to 1 indicates that the regression line perfectly fits the data. So the coefficient of determination of stress and deflection can be derived respectively as Equation (3.47) and (3.48):

$$R^2_{\sigma} = 1 - \frac{\sum (\sigma(x_i) - \bar{\sigma})}{\sum (\sigma_i - \bar{\sigma})}$$  \hspace{1cm} (3.47)

$$R^2_{\omega} = 1 - \frac{\sum (\omega(x_i) - \bar{\omega})}{\sum (\omega_i - \bar{\omega})}$$  \hspace{1cm} (3.48)

where $\sigma_i$ and $\omega_i$ are the actual values of $i_{th}$ point, $\sigma(x_i)$ and $\omega(x_i)$ are the regression equations’ predicted values, and $\bar{\sigma}$ and $\bar{\omega}$ are the arithmetic means.

$RSS$ means the deviations predicted from actual empirical values of data. It is a measure of the discrepancy between the data and an estimation model. A small $RSS$ indicates a tight fit of the model to the data. Then, the residual sum of squares of stress and deflection can be expressed respectively as Equation (3.49) and (3.50):

$$RSS_{\sigma} = \sum_{i=1}^{n} (\sigma_i - \sigma(x_i))^2$$  \hspace{1cm} (3.49)

$$RSS_{\omega} = \sum_{i=1}^{n} (\omega_i - \omega(x_i))^2$$  \hspace{1cm} (3.50)

where $\sigma_i$ and $\omega_i$ are the actual values of $i_{th}$ point, $\sigma(x_i)$ and $\omega(x_i)$ are the regression equations’ predicted values. Based on Equation (3.49) to (3.50), the stress $R^2_{\sigma}$ of Equation (3.47) and deflection $R^2_{\omega}$ of Equation (3.48) are equal to 0.99788 and 0.99986, the $RSS_{\sigma}$ and $RSS_{\omega}$ are equal to 4.17353 and 0.0009, respectively. The results indicate that the fitting equations and curves match well with the simulation results.

Using the same approach, the fitting equations and curves related to other membrane dimensions can also be deduced. Figure 3.18 shows the stress and deflection change trends with the membrane thickness, the fitting and residual curves are also presented in this figure. The corresponding fitting equations and analysis for goodness-of-fit are derived as Equation (3.51) and (3.52):
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\[ \sigma = 7.26461 \times 10^4 H^{-2.0675} \]  
(3.51)

\[ \omega = 8.04527 \times 10^3 H^{-2.3447} \]  
(3.52)

The $R^2_{\sigma}$ of stress and $R^2_{\omega}$ of deflection are equal to 0.99811 and 0.99884, the stress RSS$_{\sigma}$ and deflection RSS$_{\omega}$ are equal to 2.07594 and 0.03633, respectively. The results indicate that the curves fit well.

\begin{align*}
\sigma &= 32.20326 g^{0.32476} \quad (3.53) \\
\omega &= 2.12076 \times 10^3 g^{0.12861} \quad (3.54)
\end{align*}

From the above analysis, the stress $R^2_{\sigma}$ is 0.91507, the deflection $R^2_{\omega}$ is 0.93637, the stress RSS$_{\sigma}$ is 1.39643 and the deflection RSS$_{\omega}$ is 0.02867, which shows a high goodness-of-fit.

**Figure 3.18 (a) Relationship between stress and membrane thickness; (b) Relationship between deflection and membrane thickness.**

The relationship between groove depth and the corresponding performances are presented in Figure 3.19. Moreover, the figure also illustrates the fitting and residual curves. The corresponding fitting equations and analysis for goodness-of-fit are followed as Equation (3.53) and (3.54):
Figure 3.19 (a) Relationship between stress and groove depth; (b) Relationship between deflection and groove depth.

In the same manner, the function relationship between groove width and the corresponding performances are shown in Figure 3.20. Equations about groove width are listed as follows:

\[
\sigma = 131.97b^{-0.16719} 
\]

(3.55)

\[
\omega = 1.21898b^{0.20466} 
\]

(3.56)

Here, the stress \( R^\sigma \) is 0.97308 and the deflection \( R^\omega \) is 0.98777, the stress \( RSS_\sigma \) is 3.10571 and the deflection \( RSS_\omega \) is 0.00426. The results reflect that the curves are well fitted.

Figure 3.20 (a) Relationship between stress and groove width; (b) Relationship between deflection and groove width.
The equations relating to the rood beam width are presented as Equation (3.57) and (3.58) based on the relationship between the beam width and corresponding performances as shown in Figure 3.21.

\[
\sigma = 218.33701a^{-0.22859} \quad (3.57)
\]

\[
\omega = 20.64224a^{-0.37743} \quad (3.58)
\]

The \(R^2\) of stress and \(R^2\) of deflection are equal to 0.98041 and 0.99475, the stress \(RSS\) and deflection \(RSS\) are equal to 0.59563 and 0.0008, respectively. The results indicate that the fitting equations and curves match well with the simulation results.

![Figure 3.21](image)

**Figure 3.21 (a) Relationship between stress and rood beam width; (b) Relationship between deflection and rood beam width.**

To further study the relationship between the rood beam thickness dimensions and the performance of the sensor, their relationship is shown in Figure 3.22 and the equations about rood beam thickness are described as Equation (3.59) and (3.60):

\[
\sigma = 116.51832h^{-0.17377} \quad (3.59)
\]

\[
\omega = 17.79247h^{-0.52592} \quad (3.60)
\]

The \(R^2\) of stress and \(R^2\) of deflection are equal to 0.8612 and 0.97765, the stress \(RSS\) and deflection \(RSS\) are equal to 1.84235 and 0.05112. A small stress \(R^2\) means discrepancy between the data and fitting curve is relatively large. This is because the correlation between the rood beam thickness and stress is not high, results in a large undulation for the stress fitting curve.
3.2.2.3 Optimization results

By combining each single variable from Equation (3.45) to (3.60), the particular equations of Equation (3.41) and (3.42) can be determined. The relative main equations can be derived as follows:

\[
\sigma = \frac{Q_1}{H} \frac{PL^{2.51041} \frac{E}{h}^{0.32476} \frac{g}{a}^{0.22859} \frac{b}{a}^{0.16719}}{L^{2.0675} \frac{h}{a}^{0.17377} g^{0.02466} b^{0.02466}} \quad (3.61)
\]

\[
w = \frac{Q_2}{EH} \frac{PL^{4.15924} \frac{a}{h}^{0.20466} \frac{g}{a}^{0.12861}}{h^{2.3447} \frac{a}{b}^{0.52592} g^{0.37743}} \quad (3.62)
\]

From Equation (3.61) and (3.62), it can be concluded that with the increase of the membrane length \(L\), the maximum stress and deflection, both, experience a rise, namely, improve sensitivity, but worsen linearity. Besides, the impacts for the membrane thickness \(H\), rood beam thickness \(h\), rood beam width \(a\) and groove depth \(g\) on the sensitivity and linearity are same with the exception of the groove width \(b\). The groove width is the only variable which is in inversely proportional to the stress and directly proportional to the deflection. It means that the sensitivity and linearity can be improved synchronously when the groove width is chosen appropriately. Thus, it is hopeful to eliminate the clash between sensitivity and linearity when proper...
dimensions are determined. Besides, the indexes of the $L$ and $H$ are larger than other variables, which mean that these two dimensions are more sensitive in affecting the performance of the sensor, so the accuracy for fabricating these two variables should be higher. To verify the rationality and accuracy of the hypothesis about functional forms, the values of the coefficient of determination $R^2$ and residual sum of squares $RSS$ are listed in Table 3.2. A series of $R^2$ and $RSS$ demonstrate that good curves fitting have been achieved for the corresponding equations except Equation “Stress related to $h^*$”, where the $R^2$ for rood beam thickness is a little farther to 1. This is because the correlation between this dimension variable and stress is not high, which leads to an undulation for the fitting curve. Overall, however, the results certify the goodness of fit.

Table 3.2 Curve fitting values of the proposed membrane dimensions.

<table>
<thead>
<tr>
<th>Equation</th>
<th>$R^2$</th>
<th>$RSS$</th>
<th>Equation</th>
<th>$R^2$</th>
<th>$RSS$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress related to $L$</td>
<td>0.99788</td>
<td>4.17353</td>
<td>Stress related to $a$</td>
<td>0.98041</td>
<td>0.59563</td>
</tr>
<tr>
<td>Deflection related to $L$</td>
<td>0.99986</td>
<td>0.0009</td>
<td>Deflection related to $a$</td>
<td>0.99475</td>
<td>0.0008</td>
</tr>
<tr>
<td>Stress related to $H$</td>
<td>0.99861</td>
<td>2.07594</td>
<td>Stress related to $b$</td>
<td>0.97308</td>
<td>3.10571</td>
</tr>
<tr>
<td>Deflection related to $H$</td>
<td>0.9984</td>
<td>0.03633</td>
<td>Deflection related to $b$</td>
<td>0.98777</td>
<td>0.00426</td>
</tr>
<tr>
<td>Stress related to $h$</td>
<td>0.8612</td>
<td>1.84235</td>
<td>Stress related to $g$</td>
<td>0.91507</td>
<td>1.39643</td>
</tr>
<tr>
<td>Deflection related to $h$</td>
<td>0.97765</td>
<td>0.05112</td>
<td>Deflection related to $g$</td>
<td>0.93637</td>
<td>0.02867</td>
</tr>
</tbody>
</table>

The impacts of structural dimension variables on the stress are presented in Figure 3.23. It can be regarded as the estimation for the performance of the sensor, as each dimension variable has an effect on the stress. An intersection line is also introduced in this figure. For clear observation of the relationships between different variables, the partially enlarged pictures of each structural dimension variable are drawn in Figure 3.24. Through a comprehensive consideration of various variables, a series of structural dimension variables are determined based on crossover points between variable curves and intersection line as listed in Table 3.3. The membrane structure based on the above optimized dimensions is presented by Figure 3.25.
Figure 3.23 Stress variations versus the structural dimension variables.

Figure 3.24 Partially enlarged diagrams of structural dimension curves.

Table 3.3 Optimized dimensions for the proposed membrane.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>L</th>
<th>H</th>
<th>h</th>
<th>a</th>
<th>b</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension (μm)</td>
<td>3600</td>
<td>30</td>
<td>35</td>
<td>200</td>
<td>80</td>
<td>10</td>
</tr>
</tbody>
</table>

On the basis of parameters for the proposed membrane presented in Table 3.3, $E$ is 166 GPa, and $P$ is 1 psi. Then, the simulation results are as follows: $\sigma$ is equal to 65.1 MPa, and $\omega$ is 2.64 μm. Then, $Q_1$ and $Q_2$ are equal to $7.36 \times 10^{-2}$ and $5.64 \times 10^{-3}$, respectively. Finally, the main stress and deflection equations specific to the proposed membrane are expressed by Equation (3.63) and (3.64):
\[
\sigma = 7.36 \times 10^{-2} \frac{P L^{2.51041} g^{0.32476}}{H^{2.0675} R^{0.17377} a^{0.22859} b^{0.16719}} 
\]

\[
w = 5.64 \times 10^{-3} \frac{P L^{4.15924} b^{0.20466} g^{0.12861}}{E H^{2.3447} h^{0.52592} a^{0.37743}} 
\]

Figure 3.25 Schematic diagram of the sensor chip for the front view and partial cross-section view.

By above analysis, we not only determine the optimal membrane dimension variables, but also deduce the relationships between the structural dimensions and mechanical performances of the sensor. The design and fabrication processes can be executed more efficiently when the dimension variables are determined in advance. Yet the ranges of all the variables discussed are constrained by actual demand and processing, and it means the equations are effective in certain scopes.

3.2.3 Sensor structure design

By integrated circuit technology, the monocrystalline silicon membrane and resistors are integrated together to form a silicon piezoresistive chip. Then, the chip is installed within stainless steel to fabricate the final sensor. For typical piezoresistive pressure sensors, there are two pressure cavities on both sides of the membrane. One is high pressure cavity that is connected with the measured pressure. Another is the vacuum cavity which is formed in the sensor chip as shown in Figure 3.26.
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Figure 3.26 Assembling and interconnecting structure of the sensor.

MEMS piezoresistive pressure sensors usually have a small volume and work at harsh environment. It is necessary to process the structural strength check of the shell. From the point of mechanical engineering, a simple shape of the shell is usually chosen. In the proposed structure, the weakest point in strength happens at the reentrant groove of the shell, where has the thinnest thickness compared with other parts as presented in Figure 3.26. Besides, this place has a bending angle which is easily form stress concentration. The shear stress at the reentrant groove is calculated by:

\[
[\tau] = \frac{F}{A_3} = \frac{P A}{\pi D l} = \frac{P \left(\frac{D}{2}\right)^2 \pi}{\pi D l}
\]  

(3.65)

where \(A_3\) is the shear area of the reentrant groove, \(D\) is the diameter at the place of the reentrant groove, \(l\) is the depth of the reentrant groove. Put all the parameters into Equation (3.65), the shear stress at the reentrant groove is equal to 50.2 MPa. The relationship between the allowable shear stress and allowable tensile stress can be expressed by the following:

\[
[\tau] = 0.5[\sigma]
\]  

(3.66)

The safety factor for the alternating stress parts is in the range of 1.3~1.8. Then, the allowable tensile stress can be obtained by 100.4 MPa. By checking the strength of the weakest point, it meets the design requirements.
3.2.4 Sensor reliability design

MEMS piezoresistive pressure sensors are commonly worked in hazardous environments. Once they are damaged, the output signals will not reflect the true situation. Therefore, reliability is one of the most important indexes in the design process. The definition of reliability is the capacity of a device to complete a specified task after a period of working. There are several indexes to evaluate the reliability of the device, namely, reliability rate, failure rate, MTBF and MTTF.

(a) Reliability rate: Reliability rate means that the probability of some product completing the required task within the stipulated time. Reliability probability is inversely proportional to the time; (b) Failure rate: Failure rate means the probability of some product does not work when it is operating at a certain time. The failure rate of a system usually depends on time, with the rate varying over the life cycle of the system; (c) MTBF and MTTF: Mean time between failures (MTBF) is the predicted elapsed time between inherent failures of a mechanical system, during normal system operation. MTBF can be calculated as the arithmetic mean (average) time between failures of a system. Mean time to failure (MTTF) is extremely similar to MTBF. The difference between these terms is that while MTBF is used for products than that can be repaired and returned to use, MTTF is used for non-repairable products. When MTTF is used as a measure, repair is not an option.

For some star products developed by famous global companies, MTBF is usually chosen to judge the average life of piezoresistive pressure sensors. The MTBF of some excellent devices can reach 3 to 5 years whose reliability is high. In this paper, we chose stainless steel as the shell structure. The connecting between different shell parts is used by laser welding. The wire connecting the sensor and aviation plug is the polytetrafluoroethylene aviation cable, which can resist high temperature more than 200 °C. To protect the welding spot, the fluororubber heat shrink tube is covered at the connection between every two wires. When satisfying the basic functions, comprehensively consider various factors that will impact the reliability.
The purpose is to improve the level of reliability. In the manufacturing, the environmental conditions, such as temperature, humidity, light, cleanliness, etc. must meet the requirements. In the design, fully consider the reliability indexes, the maximum reliability should be set up. To understand the failure model and improve the mean time between failures, the fault tree analysis should be done in the process of design as shown in Figure 3.27 and Table 3.4.

![Fault tree analysis of the proposed sensor.](image)

**Table 3.4 Failure modes of the proposed sensor.**

<table>
<thead>
<tr>
<th>Parts name</th>
<th>Function</th>
<th>Failure mode</th>
<th>Local failure result</th>
<th>Final failure result</th>
<th>Precaution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor chip</td>
<td>Form voltage signal</td>
<td>Without signal or drifting</td>
<td>Without signal or errors</td>
<td>Failure</td>
<td>Check strictly the fabrication process</td>
</tr>
<tr>
<td>Plug</td>
<td>Connecting</td>
<td>Disconnection</td>
<td>Without signal or errors</td>
<td>Failure</td>
<td>Select long Life products</td>
</tr>
<tr>
<td>Shell</td>
<td>Protection and installation</td>
<td>Fracture or metal failure</td>
<td>Without signal or errors</td>
<td>Failure</td>
<td>Filter strictly</td>
</tr>
<tr>
<td>Connect Wire</td>
<td>Transfer voltage signal</td>
<td>Disconnection</td>
<td>Without signal or errors</td>
<td>Reduction or failure</td>
<td>Check strictly the welding points and line</td>
</tr>
</tbody>
</table>
3.3 Performance optimization

There are many factors to impact the output of the pressure sensor, such as membrane length, membrane thickness, piezoresistive coefficient, piezoresistors dimensions and distributions, etc. To improve the sensitivity, accuracy and reliability of the sensor, it is necessary to study the influences from membrane dimensions, piezoresistors size and shape. Meanwhile, the proposed membrane should compare with other reported models to illustrate its advantage in the performance of the sensor. In last chapter, the mechanical performance related to the structural dimensions has been discussed and the mechanical structure has been determined. This chapter will study the static and dynamic characteristics of the sensor with the help of simulation. The purpose is further to improve the performance of the sensor by optimizing the electrical structure.

3.3.1 Theory

In this thesis, a Wheatstone bridge was chosen for transferring pressure to electric signal as shown in Figure 3.28 (a). Initially, when the membrane is in unstressed condition and all the piezoresistors have equal resistance, the bridge is balanced and the output voltage is zero.

For simplicity, we assume that the membrane is fabricated on (100) silicon and all the piezoresistors have their longer axis along (110) direction. The output of the pressure sensor is calculated by approximating the stresses on the piezoresistors. The maximum stress regions of the membrane and the placement of the piezoresistors on the membrane are shown in Figure 3.28 (b). Each piezoresistor experiences both longitudinal and transverse stresses simultaneously. Let $\sigma_l$ and $\sigma_t$ denote the longitudinal and transverse stress experienced by $R_1$ and $R_3$. Then, the stress experienced by $R_2$ and $R_4$ can be approximated as longitudinal stress, $\sigma_l$ and transverse stress, $\sigma_t$ (which is rotated by an angle of 90°, compared to stresses in $R_1$ and $R_3$).
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**Figure 3.28 (a) Piezoresistors on top of diaphragm. (b) Placement of piezoresistors and stress distribution on the diaphragm surface.**

Initially, \( R_1 = R_2 = R_3 = R_4 = R_0 \). When \( R_1 \) experiences stress due to \( \sigma_l \) and \( \sigma_t \), the relative resistance change is given by:

\[
\alpha_1 = \frac{\Delta R_1}{R_1} = \pi_l \sigma_l + \pi_t \sigma_t \quad (3.67)
\]

where \( \pi_l \) and \( \pi_t \) are the longitudinal and transverse piezoresistive coefficients of silicon, respectively. Similarly, the relative resistance change for \( R_2 \) is given by:

\[
\alpha_2 = \frac{\Delta R_2}{R_2} = \pi_t \sigma_l + \pi_l \sigma_t \quad (3.68)
\]

Also, by symmetry, \( \Delta R_3/R_3 = \alpha_1 \) and \( \Delta R_4/R_4 = -\alpha_2 \). The output voltage of the sensor can then be given by:

\[
V_{out} = \left[ \frac{R_2 R_3 - R_1 R_4}{(R_1 + R_2)(R_3 + R_4)} \right] V_{in} \quad (3.69)
\]

By the piezoresistive coefficients and stresses on the piezoresistors, the output of the sensor can be estimated. P-type resistors are preferred over n-type resistors because of their higher piezoresistive coefficients and consequently higher performance. Sometimes in p-type piezoresistors the change in resistance is approximated as:

\[
\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = \pi_{[110]}[\sigma_{[110]} + \pi_{[110]}] + \frac{1}{2} \pi_{44} \left( \sigma_{[110]} - \sigma_{[1\overline{1}0]} \right) \\
= \frac{1}{2} \pi_{44} \left( \sigma_x - \sigma_y \right) \approx \frac{1}{2} \pi_{44} \sigma_{[110]} \quad (3.70)
\]
Then,

\[ V_{\text{out}} = \frac{V_{\text{in}}}{2} \pi_{44}(\sigma_x - \sigma_y) \]  

(3.72)

Thus, the differential stress (\(\sigma_x - \sigma_y\)) on resistors is an important indicator of the output voltage of the sensor. The sensitivity of the sensor for a certain mechanical pressure \(P\) (1 psi) and input voltage \(U_{\text{in}}\) (5 V) is then calculated by Equation (3.73):

\[ S = \frac{\Delta V}{U_{\text{in}}} \frac{1}{P} \]  

(3.73)

On the basis of the definition of pressure nonlinearity (PNL), the relationship between PNL and sensor output voltage is revealed by Equation (3.74):

\[ PNL = \frac{\Delta U_{\text{max}}}{FSS} \times 100\% \]  

(3.74)

### 3.3.2 Output characteristics

The silicon crystal possesses a diamond structure. The basic unit of the structure is a face-centered cubic cell. In the crystallographic coordinate system of the crystal, there are only three non-zero independent components for the piezoresistive coefficient tensor. Namely, \(\pi_{11}=\pi_{22}=\pi_{33}, \ \pi_{12}=\pi_{21}=\pi_{13}=\pi_{31}=\pi_{23}=\pi_{32}\) and \(\pi_{44}=\pi_{55}=\pi_{66}\). Therefore, the piezoresistive coefficient tensor of silicon has a simple form in a crystallographic coordinate system. The three non-zero independent components were found experimentally by C. S. Smith for high resistivity silicon material. The data is given in Table 3.5. According to the data in the table, the approximation often chosen is: \(\pi_{11}=\pi_{22}=0\) for p–Si, and \(\pi_{44}=0\) and \(\pi_{11}=-2\pi_{22}\) for n–Si.
Table 3.5 Piezoresistive coefficients under low doping concentration [Hsu, 2008].

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$(Ω·cm)</th>
<th>$\pi_{11}10^{-11}$(Pa$^{-1}$)</th>
<th>$\pi_{12}10^{-11}$(Pa$^{-1}$)</th>
<th>$\pi_{44}10^{-11}$(Pa$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P-type silicon</td>
<td>7.8</td>
<td>+6.6</td>
<td>-1.1</td>
<td>+138.1</td>
</tr>
<tr>
<td>N-type silicon</td>
<td>11.7</td>
<td>-102.2</td>
<td>+53.4</td>
<td>-13.6</td>
</tr>
</tbody>
</table>

By the analysis of the piezoresistive effect, the potential and current distribution caused by pressure can be determined. Meanwhile, the deflection and stress on the membrane can also be studied. With the help of simulation, the maximum deformation of the membrane is 2.8 μm that is smaller than 1/5 thickness of the membrane as shown in Figure 3.29 (b). From the stress distribution, the stress concentration happens at the center of the membrane edge, the maximum value reaches 71.5 MPa as presented in Figure 3.29 (c). The potential distribution is illustrated in Figure 3.29 (d). Different colors represent different potential. For example, the blue means 0 V and the red means 5 V. After simulation, the output of the sensor can be calculated.

![Figure 3.29 Output simulation process: (a) Mesh generation; (b) Deflection of the membrane; (c) Stress distribution on the membrane; (d) Voltage distribution on the metal wires.](image)
The material properties of the silicon, Au and doped silicon (piezoresistors) are listed in Table 3.6. Actually, the experiment of sputtering chose Cr-Au as the wires for the Wheatstone bridge. Here, to simplify the algorithm, only Au was used in the simulation of the output characteristic. Figure 3.30 shows the electric potential distribution and current density of the piezoresistors and wires with a 5 V power supply on the Wheatstone bridge. The output voltage of the proposed sensor is equal to $V_1 - V_2$ as shown in Figure 3.30 (a). Then, the output and sensitivity can be calculated, respectively. In addition, a terminal current can also be obtained as presented in Figure 3.30 (b). The current flow “spreading out” into the sense electrodes (resistor place), this phenomena is defined as the “short circuit” effect. The asymmetry in the potential, which is induced by the silicon piezoresistive effect, is also apparent in this figure.

**Table 3.6 Physical properties for the materials.**

<table>
<thead>
<tr>
<th>Material</th>
<th>Young’s modulus (GPa)</th>
<th>Resistivity (Ω·cm)</th>
<th>Density (g/cm³)</th>
<th>Piezoresistive coefficient ($10^{11}$/Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>166</td>
<td>2.3</td>
<td>2.3</td>
<td>/</td>
</tr>
<tr>
<td>Au</td>
<td>78</td>
<td>2.05×10⁻⁸</td>
<td>19.32</td>
<td>/</td>
</tr>
<tr>
<td>P-doped silicon</td>
<td>168</td>
<td>2.3</td>
<td>2.3</td>
<td>$\pi_{11} = 6.6; \pi_{12} = -1.1; \pi_{44} = 138.1$</td>
</tr>
</tbody>
</table>

**Figure 3.30 Electromechanical behaviors for the proposed sensor under 5V power supply on the Wheatstone bridge:** (a) Electric potential; (b) current density.
3.3.3 Effect of size on the output performance

Sensitive membrane is the most important part of piezoresistive pressure sensors. The size determines the sensitivity, range and linearity, etc. Therefore, the membrane length and thickness should be the primary consideration. Reasonable sensitive membrane size can effectively improve the performance and life of the sensor. In this section, five sets of data are compared in the output characteristics of the proposed sensor. The dimensions of the sensitive membrane are shown in Table 3.7. Meanwhile, the length, width and thickness of the piezoresistors are set as 160 μm, 5 μm and 1 μm respectively.

<table>
<thead>
<tr>
<th>Number</th>
<th>Membrane length</th>
<th>Membrane thickness</th>
<th>Membrane length/thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>3400×3400</td>
<td>20</td>
<td>170</td>
</tr>
<tr>
<td>#2</td>
<td>3500×3500</td>
<td>25</td>
<td>140</td>
</tr>
<tr>
<td>#3</td>
<td>3600×3600</td>
<td>30</td>
<td>120</td>
</tr>
<tr>
<td>#4</td>
<td>3700×3700</td>
<td>35</td>
<td>105</td>
</tr>
<tr>
<td>#5</td>
<td>3800×3800</td>
<td>40</td>
<td>95</td>
</tr>
</tbody>
</table>

The output characteristic of different membrane dimensions is compared under 1 psi (6.895 kPa) load as shown in Figure 3.31. In this comparison, the materials, applied pressure and main dimensions of the three models piezoresistors are totally same. As can be seen from the figure, all the output curves keep a linear relation and the signal has a larger value. The bigger the ratio of membrane length/membrane thickness is, the larger the output achieves. However, pressure nonlinearity just shows an opposite trend, namely, when the ratio of membrane length/membrane thickness increase, the linearity of the output reduces as shown in Figure 3.32. This phenomenon is consistent with the theory and reported papers [Zhang, 2016], [Kumar, 2016a], [Lin, 2012]. To balance the contradiction between sensitivity and linearity, the final membrane length 3600 μm and thickness 30 μm are determined.
3.3.4 Effect of Piezoresistors on the output performance

In this thesis, the total resistance of each piezoresistors was taken as 6.7 kΩ. The piezoresistors was planned to fabricate by ion implantation of silicon. P-type piezoresistors was used as they gave better sensitivity than n-type piezoresistors. The size, shape and placement will affect the sensitivity and nonlinearity of the sensor. In order to balance the sensitivity and nonlinearity and obtain a higher sensitivity and a lower nonlinearity, the doping, size and location of the piezoresistors deserve to be studied and optimized.
3.3.4.1 Doping concentration

Temperature influences the properties of electronic systems in many ways. Since the change in electrical conductivity can affect the sensitivity of the system, so the change in electrical conductivity of piezoresistors with respect to temperature is considered. Usually, doping of p-type piezoresistors is also considered which immensely contributes to the resistance and thus needs to be considered in design optimization.

The piezoresistive coefficient $\pi_{44}$ is often considered as the most dominant factor while determining the sensitivity of pressure sensor in p-type piezoresistors. Thus, often only the variation of this piezoresistive coefficient with temperature and doping concentration is calculated [Bao, 2005]:

$$\pi(N_A, \theta) = P(N_A, \theta)\pi_{ref}$$  \hspace{1cm} (3.75)

For $\pi_{44}$ the piezoresistance factor is followed by:

$$P(N_A, \theta) = \theta^{-\nu} \left[ 1 + \left( \frac{N_A}{N_B} \right)^\alpha \theta^{-\beta} + \left( \frac{N_A}{N_C} \right)^\gamma \theta^{-\delta} \right]^{-1}$$  \hspace{1cm} (3.76)

where $\theta = T/T_0$, $T_0 = 300$ K, $N_A$ is the doping concentration.

The relative parameters in Equation (3.76) are shown in Table 3.8. The plot drawn by using Equation (3.76) is presented in Figure 3.33. It can be found from the plot that at lower doping concentration, higher values of piezoresistive coefficient are obtained leading to better output sensitivity, so a lower doping concentration is always adopted in the process of ion implantation. When doping concentration is higher, there is a drop in the value of piezoresistive coefficient but the plots of different temperature tend to converge at higher doping, leading to less temperature drift. The temperature sensitivity of the sensor must be properly understood and compensated. This can be done by using signal-conditioning circuitry. It is also advantageous to sometime choose highly doped resistors because the loss in sensitivity can be compensated by using amplification circuitry. However, the
nonlinearity at lower doping concentration is more difficult to compensate. Thus, a suitable doping concentration should be determined. According to previous process experience, a doping concentration in the range of $1 \times 10^{14} \sim 1 \times 10^{18}$ cm$^{-3}$ is usually chosen.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_b$</td>
<td>$6 \times 10^{19}$ cm$^{-3}$</td>
</tr>
<tr>
<td>$N_c$</td>
<td>$7 \times 10^{20}$ cm$^{-3}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.9</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.43</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.6</td>
</tr>
<tr>
<td>$\delta$</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 3.8 Fitting parameters used in Equation (3.76).

Figure 3.33 Piezoresistance factor with doping concentration and temperature [Suja, 2015].

3.3.4.2 Size of piezoresistors

Piezoresistors were formed by the ion implantation process in the thesis, the doping concentration was usually in the range of $1 \times 10^{14} \sim 1 \times 10^{18}$ cm$^{-3}$. The sheet resistance
was set as \(210\pm 10\ \Omega/\square\). The relationship between piezoresistor resistance and sheet resistance is followed by:

\[
R = R_s \frac{L}{b}
\]  
(3.77)

where \(R\) is the piezoresistor resistance, \(R_s\) is the sheet resistance, \(L\) and \(b\) are length and width of the piezoresistor respectively. Besides, the maximum power consumption per unit area \(P_{\text{max}}\) is \(5\times 10^{-3}\ \text{mW}/\mu\text{m}^2\), the definition of the \(P_{\text{max}}\) is shown as:

\[
P_{\text{max}} = \frac{i^2 R}{b L} = \frac{i^2 R_s L}{b L} = \frac{i^2 R_s}{b^2}
\]  
(3.78)

By Equation (3.78), we can obtain:

\[
\frac{i}{b} = \sqrt{\frac{P_{\text{max}}}{R_s}}
\]  
(3.79)

Assume \(i\) is equal to 1 mA, the width of the piezoresistor is about 5.2 \(\mu\text{m}\). We take the integer \(b=5\ \mu\text{m}\). Then, length of the piezoresistor is 160 \(\mu\text{m}\).

3.3.4.3 Shape of piezoresistors

In order to place the complete piezoresistors within the high stress regions, different resistor shapes are designed and compared to achieve the best performance. The high stress regions can even slightly extend outside the membrane boundary but in the case of thin membrane it can be assumed that the high stress regions are present at the center of the four edges of the membrane. Consequently, the four piezoresistors are placed at the center of the edges of the diaphragm.

The total resistance of each piezoresistors was taken as 6.7 k\(\Omega\). The piezoresistors would be fabricated by ion implantation of silicon. P-type piezoresistors would be used as they give better sensitivity than n-type piezoresistors. The effect of implanting boron atoms with a dosage of \(4.86\times 10^{14}\ \text{atoms/cm}^2\) and energy of 70 keV on n-type substrate with resistivity 5 \(\Omega\)-cm is simulated using Silvaco Athena®. This
gives a sheet resistivity of $210 \pm 10 \Omega / \square$ for the piezoresistors. The total length of the piezoresistor arms must be equal to 70 μm i.e. this length has to be distributed between the different arms of the piezoresistors. The connecting arms will be formed using metal lines so that they do not contribute to the piezoresistive effect. In our simulations, we consider four resistor configurations consisting of zero, one, two and three turns as shown in Figure 3.34. The parameter definitions for different shaped piezoresistors are presented in Table 3.9.

![Figure 3.34 Different piezoresistors configurations](image)

**Figure 3.34 Different piezoresistors configurations (a) no turn (b) one turn (c) two turns (d) three turns.**

**Table 3.9 Dimensions for different shaped piezoresistors.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square membrane</td>
<td>Length $L=3600 \mu m$</td>
</tr>
<tr>
<td></td>
<td>Thickness $H=30 \mu m$</td>
</tr>
<tr>
<td>Meander shaped</td>
<td>S0 (No turn)</td>
</tr>
<tr>
<td></td>
<td>S1 (One turn)</td>
</tr>
<tr>
<td>Piezoresistor</td>
<td>S2 (Two turns)</td>
</tr>
<tr>
<td></td>
<td>S3 (Three turns)</td>
</tr>
</tbody>
</table>
Chapter 3: Design and analysis of the novel structural membrane

The structure obtained after optimization of various dimensions is simulated for different values of pressures from 0 to 1 psi in steps of 0.1 psi and the differential output across the Wheatstone bridge is tested. An input voltage of 5 V was used for the simulations. Table 3.10 gives the details of the simulation results for the four different configurations of piezoresistors. The results show that the output voltage experiences an improvement when the resistor turns are increased.

Table 3.10 Output characteristics for four different piezoresistor configurations.

<table>
<thead>
<tr>
<th>Pressure (psi)</th>
<th>Output voltage $\Delta V = V_1 - V_2$ (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no turn</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td>14.56</td>
</tr>
<tr>
<td>0.2</td>
<td>29.12</td>
</tr>
<tr>
<td>0.3</td>
<td>43.68</td>
</tr>
<tr>
<td>0.4</td>
<td>58.24</td>
</tr>
<tr>
<td>0.5</td>
<td>72.80</td>
</tr>
<tr>
<td>0.6</td>
<td>87.36</td>
</tr>
<tr>
<td>0.7</td>
<td>101.92</td>
</tr>
<tr>
<td>0.8</td>
<td>116.48</td>
</tr>
<tr>
<td>0.9</td>
<td>131.04</td>
</tr>
<tr>
<td>1.0</td>
<td>145.60</td>
</tr>
</tbody>
</table>

The combined plot of differential performance for all the four piezoresistor configurations was studied. In order to obtain the non-linearity, the curve is shifted so as to pass through the origin. With the number of turns increasing, the sensitivity of the sensor improves about 18.6% as presented in Figure 3.35 (a). This is because the piezoresistors of more turns can experience more stress change when the membrane deforms. Consider technical processing capacity, too much turns of the piezoresistive will bring a challenge to engineers. Besides, the piezoresistor with two turns has the best pressure nonlinearity, 0.37%, as shown in Figure 3.35 (b).
Chapter 3: Design and analysis of the novel structural membrane

pressure nonlinearity of the piezoresistor with no turn and one turn are 0.74% and 0.65%, respectively. Generally, compensating an electronic nonlinearity is usually much more difficult than amplifying an electrical signal. Thereby, the pressure nonlinearity of 0.54% for three turns is a medium choice. Finally, three turns are determined for the proposed membrane.

\[\text{Figure 3.35 Performance for different piezoresistor configurations: (a) Sensitivity; (b) Pressure nonlinearity.}\]

After the design of the membrane and piezoresistors, the dimensions of the membrane and piezoresistors are determined for measuring pressure less than 1 psi. The length and thickness of the membrane are 3600 μm and 30 μm and the length and width of the piezoresistors are 160 μm and 5μm, respectively. The length of one single resistor for three turns is 40 μm. In order to more clearly understand the structure of the sensor chip, the planar construction and the cross-sectional view along A-A of the proposed structure marked with structural dimension variables are presented in Figure 3.36 (a) and 3.36 (b). Four piezoresistors are located in the HCSP on the surface of the rib regions and connected with each other to form a Wheatstone bridge. The patterns of piezoresistors in place 1 and place 2 are described in detail in Figure 3.36 (c). Since both piezoresistors in two places are symmetric positioned to the rib center, the performance degradation induced by alignment errors between the piezoresistor layer and rib layer can be decreased. Then, the non-uniform and rapidly changing stress in sensitive regions will have little impact on the fabrication tolerance.
3.3.5 Comparison with other structures

In this section, it has been analyzed and compared the static and dynamic performance of different membrane structures for piezoresistive pressure sensors through FEM, including the C-, E-type, CBM, BMQI and proposed membranes. By comparing with some reported structure, we hope to prove that the proposed membrane structure indeed has some advantages on size and performance.

3.3.5.1 Membrane size

To verify the proposed structure can achieve a smaller size, the mechanical performance comparisons among proposed membrane, C-type and E-type membranes are studied under 1 psi uniform pressure, including the maximum transversal stress and deflection by COMSOL. The three structures have the same membrane length and thickness as shown in Figure 3.37 (a). Compared with the other two structures, the proposed structure has the highest stress and an intermediate deflection. When the three membranes' length and thickness are all set as the same dimensions, the C-type structure gets the lowest stress, which means a small output (or sensitivity) for C-type. To obtain high stress, an efficient method is to increase the ration of membrane length/membrane thickness. For
example, the membrane length of C-type has to be 5776 \( \mu \)m, the stress can reach 65.1 MPa that is close to the proposed structure (see Figure 3.37 (b)), but the deflection will sharply increase to 15.7 \( \mu \)m which has exceeded 1/5 thickness of the membrane and not fit the small deflection principle anymore.

A kind of E-type structure pressure sensor is designed to decrease the large deflection of the C-type caused. As shown in Figure 3.37 (c), the deflection is reduced to 1.84 \( \mu \)m by the introduction of the central mass. It can be concluded that the deflection of the E-type is smaller than C-type but the stress is also declined because of the hard core. It means the E-type has to sacrifice sensitivity when a low nonlinearity is achieved. If the maximum stress of E-type reaches the level of the proposed membrane, the membrane length should be at least 6752 \( \mu \)m. Then, the deflection for the E-type structure is 3.6 \( \mu \)m, which will lead to a worse linearity than the proposed membrane.

![Figure 3.37 Comparison in size with C- and E-type structural membranes.](image)

According to the above analysis, the proposed membrane possesses the advantage in size. The size of the sensor is decided by the membrane length and depth of the
cavity. From Table 3.11, all the membranes are under 1 psi pressure. Cavity represents the etching depth, thickness is the membrane thickness and stress is the maximum transversal stress simulated by COMSOL. The table indicates, for example, when the membrane thickness is 30 μm, to achieve 65.1 MPa maximum stress, the membrane length of C- or E-type structures need almost 1.8 times the proposed structure, but the proposed structure just needs 3600 μm. It means that the proposed structure only requires approximate half size of the C- or E-type membranes for gaining the same stress.

Table 3.11 Comparison of the size among various structures.

<table>
<thead>
<tr>
<th>Cavity (μm)</th>
<th>Thickness (μm)</th>
<th>Stress (MPa)</th>
<th>C-type length (μm)</th>
<th>E-type length (μm)</th>
<th>Proposed type length (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>275</td>
<td>25</td>
<td>101.2</td>
<td>6028</td>
<td>7068</td>
<td>3600</td>
</tr>
<tr>
<td>270</td>
<td>30</td>
<td>65.1</td>
<td>5776</td>
<td>6752</td>
<td>3600</td>
</tr>
<tr>
<td>265</td>
<td>35</td>
<td>47.9</td>
<td>5525</td>
<td>6541</td>
<td>3600</td>
</tr>
<tr>
<td>260</td>
<td>40</td>
<td>41.8</td>
<td>5411</td>
<td>6322</td>
<td>3600</td>
</tr>
</tbody>
</table>

3.3.5.2 Static performance

The static performance of the four-grooved membrane combined with rood beam is compared with that of CBM structure [Tian, 2010] and BMQI structure [Yu, 2015] under 1 psi (6.895 kPa) load as shown in Figure 3.38. The piezoresistive pressure sensors with CBM and BMQI structures also achieved a high accuracy to detect the low pressure. In this comparison, the materials, applied pressure and main dimensions of the three models are same. Compared with other two types, the sensitivity of the proposed structure reaches the largest value of 34.5 mV/V/psi (see Figure 3.38 (d)), which almost improves 36.2% and 22.5% compared with CBM and BMQI structures. The main reason is illustrated by the stress distribution shown in Figure 3.38 (a). The stress of the proposed structure is concentrated in a smaller area than other two structures, even form a HCSP on the rib between every two grooves,
so the stress produced by the membrane deformation is also much bigger than other two types (see Figure 3.38 (c)). It indicates that the strain energy utilized by the resistors for the proposed structure is larger than others.

Based on the definition of the strain energy $U$, the $U$ stored in the membrane under load can be expressed by Equation (3.80) when the effective membrane area is substituted by the elementary area [Herrera-May, 2009]:

$$U = \frac{D}{2} \iint_{A} \left[ \frac{2(\mu - 1)(\sigma_x - \sigma_y)}{Eh} \right] dx dy - \iint_{A} \sigma(x, y) dx dy$$  \hspace{1cm} (3.80)

where $H$ is the membrane thickness, $A$ is the effective area of the membrane, and $\omega$ is the membrane deflection. Through the comparison among different structures around the stress concentration region, the stress for the proposed membrane is much larger than other structures as seen in Figure 3.38 (c). It also can be interpreted as the level of stress concentration for the proposed structure is higher than others as shown in Figure 3.38 (a). In addition, the deflection for EI-type sensor is always smaller than traditional structures. Thus, the difference value between the 1st part and 2nd part of the Equation (6.14) for the proposed sensor is relatively big.

In this improved membrane, the sensitivity is maximized by concentrating more strain energy at the position of the piezoresistors. This is successful achieved by two ways. On the one hand, the piezoresistors are placed on the narrow rib, which, due to its greater thickness than the groove, provides a major portion of the support for the membrane. The function of the groove is to transfer more strain energy to the rib region, so the sensitivity will be improved. On the other hand, there is a gap between the rood beam and the membrane edge, which results in a stiffness mutation on the reverse side of the membrane. When the membrane experiences a deformation, the strain energy will reach to the gap place along the rood beam. With the piezoresistors near the highest stress surface of this rib, sensitivity is secondary to be enhanced. Thus, the proposed sensor can obtain the highest sensitivity compared with the other two structures.
Figure 3.38 Comparison in performance with other two structural sensors.

From the Figure 3.38 (e), it is noted that CBM, BMQI and proposed structures all achieve a low nonlinearity value less than 0.3%. Especially for the BMQI structure, it gets the lowest nonlinearity of 0.12% FSS. This is because the deflection for BMQI reaches the smallest value of 1.2 μm as presented in Figure 3.8 (b). And yet, for the CBM, the nonlinearity is relative high due to the deflection almost reaching 3 μm. Non-linearization in the diaphragm-type piezoresistive pressure transducers happens when the diaphragm is deflected enough that it stretches. Then the geometry is deflected enough that the membrane starts to stretch in a non-linear manner and resists the applied pressure with membrane stress as in a balloon effect [Johnson, 1992].
In general, as the linearity improves, there is always some reduction in sensitivity. For the proposed membrane, four grooves are not only conducive to concentrate strain energy within a small area, but also create a greater deformation due to the stiffness change between the membrane center and edge. This is useful to obtain HCSP and then enhance the sensitivity, but, deteriorating linearity. However, the effect of rood beam on linearity assumes that it works like a strengthening rib. The rood beam reduces stretching of the rib at larger deflections, which in turn improves linearity. In contrast, the large open membrane area is relatively weak and free to stretch with minimum impact on linearity. Therefore, the conflict between high sensitivity and low nonlinearity is resolved by introducing grooves on the front side and rood beam on the rear side. Eventually, a high accuracy sensor with maximum sensitivity and minimum nonlinearity has been achieved.

3.3.5.3 Dynamic performance

In the field of high accuracy measurement, besides the sensitivity and linearity, the mechanical stability is also of significant factor for micro-high sensitive pressure sensors. The mechanical stability is depended upon the first natural frequency (1st) of the membrane structure [Kroetz, 1999]. To stabilize the membrane, normally a higher first natural frequency is often expected. This is because the structure of the sensor will be damaged due to the resonance effect when its natural frequency and measured signal are approximate [Jia, 2016].

As shown in Figure 3.39, the dynamic performance analysis results show that a high first resonance frequency of 42.1 kHz is achieved for the proposed membrane with an increment of 32.8% and 12.3% compared with the C-type and BMQI structures respectively; while for the CBM structure the first resonance frequencies almost equal with each other. The results mean that the proposed membrane is as good as the CBM structure and is better in comparison with the C-type and BMQI membranes in terms of stability.
Through modal analysis of the proposed membrane structure, the orders from mode 1\textsuperscript{st} to mode 6\textsuperscript{th} are determined and the frequencies of six orders are calculated as shown in Figure 3.40. Mode 1\textsuperscript{st} represents the membrane vibrating along the Z axis. Mode 2\textsuperscript{nd} and mode 3\textsuperscript{rd} show the vibration around the Y axis and X axis. Mode 4\textsuperscript{th} indicates the vibration along the Z axis in the X-Y plate. Mode 5\textsuperscript{th} reflects the motion that is vibrated around X axis in the X-Y plate. Mode 6\textsuperscript{th} displays the vibration along Y axis in the X-Y plate. Mode 1\textsuperscript{st}, mode 2\textsuperscript{nd} and mode 3\textsuperscript{rd} are often named the operational modes, and they can be used to estimate the dynamic performance of the membrane structures. The modal analysis for different membrane structures are shown in Table 3.12. The results indicate the dynamic performance of the proposed membrane is middle when compared with other types of the membrane.
Figure 3.40 Modal analysis of the proposed membrane.

Table 3.12 Mode analysis results of the different membrane structures.

<table>
<thead>
<tr>
<th>Natural frequency (Hz)</th>
<th>Mode 1&lt;sup&gt;st&lt;/sup&gt;</th>
<th>Mode 2&lt;sup&gt;nd&lt;/sup&gt;</th>
<th>Mode 3&lt;sup&gt;rd&lt;/sup&gt;</th>
<th>Mode 4&lt;sup&gt;th&lt;/sup&gt;</th>
<th>Mode 5&lt;sup&gt;th&lt;/sup&gt;</th>
<th>Mode 6&lt;sup&gt;th&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed structure</td>
<td>44176</td>
<td>86579</td>
<td>86677</td>
<td>112100</td>
<td>142550</td>
<td>158820</td>
</tr>
<tr>
<td>C-type structure</td>
<td>31684</td>
<td>64853</td>
<td>65070</td>
<td>93307</td>
<td>118000</td>
<td>118620</td>
</tr>
<tr>
<td>BMOI structure</td>
<td>37549</td>
<td>109950</td>
<td>110060</td>
<td>184970</td>
<td>202390</td>
<td>233360</td>
</tr>
<tr>
<td>CBM structure</td>
<td>40979</td>
<td>83620</td>
<td>83652</td>
<td>107160</td>
<td>139410</td>
<td>151990</td>
</tr>
</tbody>
</table>
3.4 Summary

In this chapter, the relationship between stress and deflection of the four edge fixed membrane has been deduced by Navier trigonometric equations. The relative error between the analytic and FEM was lower than 10%, which validated the theoretical calculation was consistent with the simulation results. Based on the Euler-Bernoulli beam theory, the deformation mechanism of the membrane combined with a rood beam structure was explained. Through the performance comparison between with and without rood beam structure, a double improvements principle for the sensitivity and linearity was also obtained.

Also, a design method, including model design, dimensions optimization and structure design of the novel structure sensor, was presented. FEM simulation and curve fitting method have been used to analyze the stress distribution of sensitive elements and the deflection of membrane. On the basis of multivariate fitting, the relationships between structural dimension variables and mechanical performance were deduced. According to statistics theory, the coefficient of determination $R^2$ and residual sum of squares $RSS$ were introduced to indicate that the fitting equations and curves matched well with the simulation results. After that, a series of the optimal membrane dimensions were determined.

According to the optimized structure, the output characteristics of the proposed sensor were studied by finite element method. By analyzing the effects from piezoresistors shape, size and location on the performance, the piezoresistors were achieved an optimization. To prove the superiority of the proposed sensor, it is compared with several reported sensors. In comparison with CBM and BMQI structures, the proposed structure not only stiffened partially the membrane to reduce the nonlinearity and keep high first natural frequency, but also transferred more strain energy to the HCSP to guarantee even higher sensing sensitivity. Besides, the proposed membrane would minimize the size of the sensor chip while having same maximum stress than C- and E-type.
4. MICROFABRICATION AND KEY PROCESSING ISSUES STUDY

The main goal of this doctoral thesis is the development of a novel structural piezoresistive pressure sensor. According to the previous mechanical analysis and structure optimization, the dimensions of the proposed membrane have been determined. In this section, the sensor chip will be realized by MEMS micro manufacture processes. To provide a reference for building up a standardized pressure sensor production line, this chapter also studied the key processes involved in the production of the proposed sensor chip.

A fabrication process for the proposed sensor is studied in detail as shown in Figure 4.1. A 4 inch n-type (100) oriented SOI wafer with 30 μm top silicon, 2 μm buried SiO$_2$ layer and 300 μm bottom silicon is chosen as the substrate of the sensor chip.

![Fabrication Processes Diagram]

Figure 4.1 Main fabrication processes used to manufacture the proposed sensor chip.
4.1 SOI wafer cleaning

SOI wafer is easily subject to a variety of pollutions such as dust particles, metal particles, organics and clathrates in the microfabrication process, which will inevitably deteriorate the subsequent process results. To obtain an ideal performance of the final sensor, wafer cleaning is indispensable before the processing. The general sequence of cleaning is presented as follows: organics, particles, metal and oxide layers pollutions \([\text{Kern}, 1970]\). The common methods used for cleaning is followed by:

- **H\textsubscript{2}SO\textsubscript{4} (98%): H\textsubscript{2}O\textsubscript{2} (30%) = 4:1, 90 °C, 10 min**: It is mainly used to remove heavy organics and dirty particles. For example, the photoresist is worked as the mask for etching Si\textsubscript{3}N\textsubscript{4} and SiO\textsubscript{2} in the RIE process. When etching time is longer than 5 minutes, the photoresist is possible to suffer from a denaturation so that it is difficult to be removed. Then, H\textsubscript{2}SO\textsubscript{4} (98%): H\textsubscript{2}O\textsubscript{2} (30%) mixture will be chosen to wipe off the pollutions.

- **HCl (36%): H\textsubscript{2}O\textsubscript{2} (30%): H\textsubscript{2}O = 1:1:6, 70 °C, 10 min**: This mixed solution is usually chosen to remove metal ions such as sodium, iron, aluminum, etc. After this cleaning process, metal contaminations can be reduced to about lower than 10\textsuperscript{10} atoms/cm\textsuperscript{2}.

- **NH\textsubscript{3} (28%): H\textsubscript{2}O\textsubscript{2} (30%): H\textsubscript{2}O = 1:1:5, 70 °C, 10 min**: In the above process, silicon surface is oxidized by H\textsubscript{2}O\textsubscript{2}, and then the oxide layer is removed by ammonium hydroxide. Particle removal is generally due to the electrostatic repulsion. When the silicon surface is positively charged, it is easy to absorb the particles and reduce the removal efficiency. On the contrary, when the silicon wafer is negatively charged, the particles are easily removed owing to the electrostatic repulsion. Besides, NH\textsubscript{3} (28%): H\textsubscript{2}O\textsubscript{2} (30%): H\textsubscript{2}O also can be applied to delete organics. Because of the strong oxidizing property of H\textsubscript{2}O\textsubscript{2}, the organics can react with it to produce CO\textsubscript{2} and H\textsubscript{2}O so as to achieve the purpose of cleaning.
- **HF: H$_2$O$_2$ = 1:50, 25 °C, 30 s**: This mixture liquid is usually used to remove oxide layers on the wafer surface. However, the silicon will be corroded during the cleaning process. To resolve this problem, the improved method, BHF (0.17% HF, 400 ppm surfactant, 17% NH$_4$F), may be better qualified for this cleaning.

To completely clean the silicon wafer and avoid weakening the product performance due to impurities, this thesis set up the cleaning steps as shown in Table 4.1. The schematic diagram of the cleaning process is illustrated in Figure 4.2.

**Table 4.1 Cleaning steps for the SOI wafer.**

<table>
<thead>
<tr>
<th>Step</th>
<th>Chemical solvent</th>
<th>Temperature</th>
<th>Time</th>
<th>Pollutant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4H$_2$SO$_4$: 1H$_2$O$_2$</td>
<td>120 °C</td>
<td>15 min</td>
<td>Organics</td>
</tr>
<tr>
<td>2</td>
<td>DI H$_2$O</td>
<td>25 °C</td>
<td>5 min</td>
<td>Cleanout</td>
</tr>
<tr>
<td>3</td>
<td>HCl: H$_2$O$_2$: 6H$_2$O</td>
<td>80 °C</td>
<td>15 min</td>
<td>Metal</td>
</tr>
<tr>
<td>4</td>
<td>DI H$_2$O</td>
<td>25 °C</td>
<td>5 min</td>
<td>Cleanout</td>
</tr>
<tr>
<td>5</td>
<td>NH$_3$: H$_2$O$_2$: 5H$_2$O</td>
<td>80 °C</td>
<td>15 min</td>
<td>Particles</td>
</tr>
<tr>
<td>6</td>
<td>DI H$_2$O</td>
<td>25 °C</td>
<td>5 min</td>
<td>Cleanout</td>
</tr>
<tr>
<td>7</td>
<td>HF: 50H$_2$O$_2$</td>
<td>25 °C</td>
<td>1 min</td>
<td>Oxide layer</td>
</tr>
<tr>
<td>8</td>
<td>DI H$_2$O</td>
<td>25 °C</td>
<td>5 min</td>
<td>Cleanout</td>
</tr>
</tbody>
</table>

**Figure 4.2 Schematic diagram of the cleaning process.**
4.2 Deposition process

4.2.1 SiO$_2$ deposition by thermal oxidation

SiO$_2$ is grown on silicon wafers in wet or dry oxygen ambient. This is processed in a furnace at high temperatures ranging from 750 °C to 1200 °C as shown in Figure 4.3. For oxides grown at atmospheric pressure, the thickness of the oxide will be as small as 1.5 nm to 2 μm. If some water vapor is added during the process (wet thermal oxidation), the SiO$_2$ deposition not only can become more compact, but also can save more processing time. The reaction mechanism of this technology is followed by:

$$\text{Si} + \text{O}_2 \rightarrow \text{SiO}_2$$

$$\text{Si} + 2\text{H}_2\text{O} \rightarrow \text{SiO}_2 + 2\text{H}_2$$

![Figure 4.3 Schematic diagram of the thermal oxidation](image)

Actually, thermal oxidation is a process of constantly consuming the original silicon atoms. SiO$_2$ is constituted by silica tetrahedron. In such structure, silicon atom locates in the center of the tetrahedron and oxygen atom lies on the four vertexes. Although the amorphous SiO$_2$ formed by thermal oxidation is constituted by silica tetrahedron, there is no regular pattern for the silicon and oxygen atoms. This is because most of oxygen atoms are connected with the neighbor silicon atoms to create covalent bonds. The number of covalent bonds determines the intensity of SiO$_2$ deposition.
For piezoresistive pressure sensors, SiO$_2$ is usually used for mask layer of wet etching or reactive ion etching and the insulation layer between metal wire and piezoresistors. The performance of SiO$_2$ is enough stable when its resistivity reaches $10^7$ V/cm. Then, it is suitable to work as insulation for sensitive structure and circuit of pressure sensors. Besides, SiO$_2$ is chemically stable and insoluble in water. It only reacts with HF at room temperature to form a water-soluble clathrate H$_2$SiF$_6$. SiO$_2$ also plays a role in protection and passivation of the device surface. Then, piezoresistors are isolated from outside environment to improve the stability and reliability of the sensor. In this experiment, a 300 ± 20 nm SiO$_2$ deposition layer was formed using wet oxidation at 1000 °C.

4.2.2 Si$_3$N$_4$ deposition by LPCVD

Si$_3$N$_4$ thin film can be realized by physical vapor deposition (PVD), ion beam enhanced deposition (IBED), chemical vapor deposition (CVD), etc. The working principle of CVD involves the flow of a gas with diffused reactants over a hot substrate surface, as shown in Figure 4.4. The gas that carries the reactants is called carrier gas. While the gas flows over the hot solid surface, the energy supplied by the surface temperature provokes chemical reactions of the reactants to form films during and after the reactants. After that, the by-products of the chemical reactants are then vented. Thin films of desired compositions can thus be created over the surface of the substrate.

![Figure 4.4 Typical CVD process over silicon substrates.](image)
Ammonia is a common carrier gas for depositing Si$_3$N$_4$ on silicon substrates. Three reactants can produce the thin Si$_3$N$_4$ films:

$$3SiH_4 + 4NH_3 \rightarrow Si_3N_4 + 12H_2$$ (4.3)
$$3SiCl_4 + 4NH_3 \rightarrow Si_3N_4 + 12HCl$$ (4.4)
$$3SiH_2Cl_2 + 4NH_3 \rightarrow Si_3N_4 + 6HCl + 6H_2$$ (4.5)

The CVD technology commonly includes atmospheric pressure chemical vapor deposition (APCVD), low pressure chemical vapor deposition (LPCVD), and plasma enhanced chemical vapor deposition (PECVD). In this thesis, LPCVD was chosen to create the Si$_3$N$_4$ layer. This is because LPCVD possesses some merits such as excellent purity and uniformity, larger wafer capacity, etc.

LPCVD unlike APCVD and PECVD, the pressure for LPCVD deposition gas is relatively low (0.25~4 torr). Since the ions have a large free path in such pressure, an ideal film is easier to be obtained. During the deposition process, the gas enters from one side of the system and discharges from the other side with a gradually decreased flow. Based on the thermodynamics, the molecule’s mean free path experiences an increase at low pressure area. Then, the deposition rate will also be improved. Besides, the possibility of generating contaminants is small at low pressure, which reduces the contamination of the film. The film fabricated by LPCVD possesses some advantages such as uniformity, high density, low pollution, strong resistance to acid and alkali, etc. The process parameters for LPCVD Si$_3$N$_4$ in this experiment were shown in Table 4.2. As a result, the final film thicknesses of 200 ± 20 nm Si$_3$N$_4$ film was achieved.

<table>
<thead>
<tr>
<th>Pressure (Torr)</th>
<th>Temperature (°C)</th>
<th>Deposition rates (nm/min)</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si$_3$N$_4$</td>
<td>4</td>
<td>600</td>
<td>10</td>
</tr>
</tbody>
</table>
4.3 Lithography

Lithography is one of the key technologies in the MEMS manufacturing process. This is because it is the only viable way to achieve the high-precision geometry at the microscale. By lithography, the pattern on the photomask can be transferred to the target substrate for subsequent processing. In the fabrication of the proposed sensor, six lithography processes need to be done. Namely, they are the pattern of resistor, the pattern of the groove on the front side, the pattern of rood beam and cavity on the rear side, the hole of connecting wire channel, the pattern of Cr-Au wire, respectively.

Normally, lithography process includes the processes such as surface clean, oven dry, spin coating with photoresist, prebaking, mask alignment, exposure, development, hard baking, cleaning, etc. The schematic diagram and main equipment in the lithography are shown in Figure 4.5.

Figure 4.5 Schematic diagram [Technology S] and real equipment of lithography.

Firstly, the photoresist is painted on the surface of the wafer by spin coating method. After that, the photoresist-containing substrate is exposed by a beam of light. Since
light can change the solubility of photoresist. Finally, the outside place of the photoresist-containing can be dissolved in some organic solution such as acetone. After corrosion in the organic solution, the pattern is retained on the photoresist mask layer. Photoresist can be divided into positive and negative types. After exposure process, the resist that is easy to dissolve is called positive photoresist. Otherwise, it is named as negative photoresist. In this thesis, the positive photoresist AZ 4620 is chosen for all the lithography processes. The required devices in the lithography process mainly include cleaning station, spin coating machine, drying cabinet, baking sheet, microscope, etc. In this work, the main process pictures for lithography of SOI wafer are shown in Figure 4.6.

**Figure 4.6 Main process pictures for lithography:** (a) Wafer cleaning; (b) Spin coating with photoresist; (c) Prebaking; (d) Exposure; (e) Photomask; (f) Alignment process; (g) Development; (h) Observation in the microscope.

- **Cleaning wafer.** In order to improve the cleaning of the SOI wafer and the adhesion of the photoresist, the wafer must be cleaned before lithography. The detailed processes are followed by section 4.1.
Chapter 4: Microfabrication and key processing issues study

- **Oven dry.** This process is carried out in an oxygen diffusion furnace. Meantime, the protective atmosphere of N\textsubscript{2} is added to prevent oxidation of silicon. Because of the photoresist shrinkage effect, the dried wafer should be placed at room temperature about 10 minutes before painting photoresist.

- **Spin coating with photoresist.** This technique is to paint a thin film with good adhesion, proper thickness and uniformity photoresist on the surface of the substrate. In the process of spin coating with photoresist, for one thing, the bubbles should be avoided; for another, the photoresist should drop to the middle of the substrate, covering 2/3 parts of the wafer surface. For the DRIE on the rear side of the substrate, a thicker photoresist is needed. This work utilized positive photoresist AZ 4620 in the lithography.

- **Prebaking.** The purpose of the prebaking is to evaporate the photoresist solvent to obtain a firm film, which is beneficial to avoid the “sticky” problem after exposure. The temperature and time of prebaking depend on the photoresist properties and lithography process. In this experiment, prebake temperature was set as 90 °C and the time was last about 1-2 min.

- **Exposure.** The most parameter exposure time depends on the intensity of light source, the thickness of photoresist layer, and the concentration of the development liquid. If exposure time is too short, the graphic boundary will be not clear. Otherwise, the graphics resolution will be reduced. In this work, SUSS MA6 lithography machine was chosen to complete the process (Mercury lamp power was set as 310 W, exposure time was set as 6 s).

- **Development.** Development is used to dissolve the exposure photoresist so as to obtain the desired pattern. Acetone solution is utilized for development, and the development time is 20 s. In the process of development, the wafer needs to be constantly shaken so that the dissolved photoresist can be separated from the substrate surface. After development, the wafer is cleaned in alcohol and water
respectively for 1 min. The results can be seen in the microscope. If the pattern lines are not clear enough, the developing time can be extended.

- **Hard baking.** The purpose of hard baking is to remove the residual solvents in the photoresist after development. Besides, this process can also improve the adhesive strength and resistant to corrosion. In this work, hard baking temperature was 90 °C and hard baking time was 2 min respectively.

- **Observation.** After the lithography process, the wafer with photoresist pattern is observed in the microscope to check whether the pattern is transferred from the mask to the wafer.

### 4.4 Piezoresistors pattern and fabrication

Piezoresistors in the thesis was formed by ion implantation process. Ion implantation is a low-temperature process by which ions of one element are accelerated into a solid target, thereby changing the physical, chemical, or electrical properties of the target. Ion implantation impurities have achieved good controllability and flexibility, which makes it widely used in MEMS device fabrication. But ion implantation also has its own shortcomings such as the damage to semiconductor lattices during the process. Thus, an annealing process is usually chosen to recover lattice damage and reduce square resistance.

#### 4.4.1 Ion implantation principle

Ion implantation is a non-equilibrium process. After the high-energy ions entering the target, they continually collide with atomic nuclei and extra nuclear electrons. With the energy consuming, some ions stop at lattice without any electric activity. The schematic diagram of the ion implantation and the equipment used in this work are shown in **Figure 4.7**. The movement of ions in the single crystal can be divided into two types. One is along crystal axis whose name is channel ions. Another is kept away from the crystal axis. Due to continual collision during the movement, the
energy loss is becoming larger, which leads to a short motion path. To delete the channel effect during the ion implantation, the ion beam always keeps a $7^\circ$ angle with the principal axis of crystal.

![Figure 4.7 Schematic diagram and the equipment of ion implantation.](image)

Figure 4.7 Schematic diagram [Implantation] and the equipment of ion implantation.

Since implantation energy is usually higher than $10^{12}$ ions/cm$^2$, the trajectories of the ions can be predicted statistically. Ion implantation distribution diagram is illustrated by Figure 4.8. $R$ means the total movement distance of the ions after implantation. $R_p$ means the projection of the depth of ion implantation. $\sigma_p$ and $\sigma_t$ means the deviation of total movement distance and criterion, respectively. The concentration of ion implantation can be expressed by:

$$n(x) = n_0 \exp \left\{ -\frac{(x-R_p)^2}{2\sigma_p^2} \right\} \quad (4.6)$$

where $n_0$ is the peak concentration. If the total dose of implantation is $Q$, the peak concentration can be calculated as follow:

$$n_p = \frac{Q}{\sqrt{2\pi}\sigma} \approx \frac{0.4Q}{\sigma_p} \quad (4.7)$$

where the maximum concentration location happens at $x=R_p$. Thus, the maximum concentration $n(R_p)$ is calculated as following:

$$n(R_p) = \frac{Q}{\sqrt{2\pi}\Delta R_p} \quad (4.8)$$
4.4.2 Piezoresistors pattern

The layouts for piezoresistors are shown in Figure 4.9 (a) and (c). The SiO$_2$ has been deposited on the surface by thermal oxidation process. Thus, RIE was utilized to etch the mask (SiO$_2$) of ion implantation. The detailed steps are presented as follows:

(a) The SOI wafer with SiO$_2$ was experienced a spin coating with photoresist with AZ 4620. The rotate speed was set as 1500 RPM and the time was 1 min;
(b) The wafer with photoresist was baked on the hot plate, and the prebake temperature was set as 90°C and the time was last about 1 min;
(c) In the acetone solution, the wafer was constantly shaken about 30 s. After that, the wafer was cleaned in alcohol and water respectively for 1 min;
(d) To evaporate the water vapor in the photoresist, the blow dried wafer was placed on the hot plate again to make the film harder;
(e) The wafer was placed in the RIE chamber. After setting a series of RIE parameters such as vacuum, pressure, time, flow and power, the process was processed automatically based on procedures. After etching, the pattern of piezoresistor was completed as shown in Figure 4.9 (b) and (d).
If the wafer is subjected to a long time reactive etching, the mask of photoresist will be changed. Then, it is difficult to remove the metamorphic photoresist only using acetone, which will bring a bad influence for the following Cr-Au sputtering. To resolve this problem, the photoresist must be deleted thoroughly after etching of connecting hole. After shaking in acetone for 5 min, the wafer is placed in the H$_2$SO$_4$/H$_2$O$_2$ solutions. Finally, the residual metamorphic photoresist after RIE process is removed completely.

### 4.4.3 Ion implantation process

#### 4.4.3.1 Mask layer thickness

When the percentage of the blocked implantation ions is determined, the minimum thickness of mask layer can be calculated based on the distance of ions movement. The injected ions distributed in the mask are shown in Figure 4.10. The injection dose in the area beyond depth $d$ can be expressed by Equation (4.8) [Nonogaki, 1998]:

$$S_d = \frac{s}{\sqrt{2\pi}a_p} \int_{d}^{\infty} \exp \left[-\left(\frac{x-R_p}{\sqrt{2}\sigma_p}\right)^2\right] dx$$  \hspace{1cm} (4.9)
According to the error function equation, the following can be deduced:

$$\int_{-\infty}^{\infty} e^{-y^2} dy = \frac{\sqrt{\pi}}{2} \text{erfc}(x) \quad (4.10)$$

Then, the ratio between injected dose and total dose can be calculated based on penetration coefficient:

$$T = \frac{S_d}{S} = \frac{1}{2} \text{erfc} \left( \frac{d-R_p}{\sqrt{2} \sigma_p} \right) \quad (4.11)$$

Once $T$ is determined, the thickness of mask $d$ can be deduced by the given $R_p$ and $\sigma_p$ shown in Table 4.3. When boron implantation energy was set as 70 keV, and suppose 99.99% injected ions needed to be blocked by the mask. Then, the required thickness of the mask should meet:

$$T \approx \frac{1}{2\sqrt{\pi}} \frac{e^{-\mu^2}}{\mu} \quad (4.12)$$

where $T=10^{-4}$, $\mu$ is equal to 2.8. When implantation energy was set as 70 keV, the thickness of the mask $d = R_p + 3.96 \Delta R_p = 0.3024 \mu m$. The results illustrated that SiO$_2$ coating layer formed by thermal oxidation met the requirements of the process.

**Table 4.3 Relationship between inject energy and projection range [May, 2006].**

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>120</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_p$ (Å)</td>
<td>1903</td>
<td>2188</td>
<td>2465</td>
<td>2733</td>
<td>2994</td>
<td>3496</td>
<td>3974</td>
</tr>
<tr>
<td>$\Delta R_p$ (Å)</td>
<td>556</td>
<td>601</td>
<td>641</td>
<td>677</td>
<td>710</td>
<td>766</td>
<td>813</td>
</tr>
<tr>
<td>$\Delta x$ (Å)</td>
<td>891</td>
<td>980</td>
<td>1061</td>
<td>1135</td>
<td>1203</td>
<td>1325</td>
<td>1431</td>
</tr>
</tbody>
</table>
4.4.3.2 Ion implantation depth

The energy range of the semiconductor doping ion implanter is usually in the range of 20–400 keV, and the implantation depth is commonly below 1 μm. To achieve specific implantation concentration and impurity distribution, we should choose proper ion dose and energy. In order to avoid channeling effect, the angle of sample placement should keep a 7° with the implantation beam. That is, ions are injected into the solid surface in random directions. Ion implantation in the sample surface is very close to the Gaussian distribution as shown in Figure 4.11, the distribution in the sample is followed by [Nonogaki, 1998]:

\[ N(x) = N_p e^{-\frac{1}{2} x^2} \]  

(4.13)

where the peak concentration \( N_p = N_s/(2\pi)^{1/2} \Delta R_p \), \( X = (x - \Delta R_p) \); \( N_s \) means the total number of ions in the unit area (total amount is the implantation dose); \( x \) means the ion implantation distance; \( R_p \) means the projected range of ion; \( \Delta R_p \) means the standard deviation of the projected range. When \( N/N_p = 1/e^{1/2} \), \( x = R_p + \Delta R_p \); When \( N/N_p = 10^{-2} \), \( x = R_p + 3\Delta R_p \). Then, the ion implantation depth can be calculated based above equations. In the thesis, the dose and energy of the ions implantation were 4.86×10^{14} atoms/cm² and 70 keV, respectively. Thus, the ion implantation depth is less than 1 μm.

![Figure 4.11 Concentration distribution of ion implantation in amorphous target](Nonogaki, 1998)
4.4.3.3 Implantation energy and dose

The energy of ion implantation mainly influences the implantation depth. A too large energy may cause the lattice deflect and thus reduces the performance of the device. The dose of ion implantation mainly affects the size of the sheet resistance and the temperature characteristics of the resistance. According to previous experiments, the sensitivity characteristics behave well when the impurity concentration is in the range of \(10^{14} - 10^{21}\) atoms/cm\(^2\). Electrical resistivity experiences a decrease trend with the increase of the impurity concentration as shown in Figure 4.12 [Barlian, 2009].

![Figure 4.12 Electrical resistivity variations with the doping concentration.](image)

In this work, a 4 inch n-type (100) oriented SOI wafer was chosen as the substrate. By boron ions implantation, p-type piezoresistors were created on the surface of the substrate. Through several times of experiment, the dose and energy of the ions implantation were determined by \(4.86 \times 10^{14}\) atoms/cm\(^2\) and 70 keV, respectively.

4.4.4 Annealing and measurement

Ion implantation is only the first step for the whole doping process. After a violent collision between ions, a large number of lattice defects will be appeared, leading to high resistivity and large square resistance. To recover lattice damage and reduce square resistance, high effective carrier density and carrier mobility are essential, namely, annealing process has to be done after ion implantation.
In the process of ions implantation, the implanted ions collide with the target atoms and transfer the energy to them. Eventually, the implanted ions consume their energy completely and remain in the target. When the injected energy is too large, it will cause movements for target atoms. Meanwhile, it will also cause other target atoms to shift so that a large amount of space and interstitial atoms are formed in the target. The existence of such defects reduces the mobility of carriers in the silicon wafer, and thereby affecting the performance of device.

Annealing process is very important in the fabrication of piezoresistive pressure sensors, which not only determines the depth of ions implantation, but also can recover the defects caused by high energy impact from ions. The most important parameters for annealing are temperature and time. A low temperature will lead to an inferior lattice recovery, but high temperature will bring additional impurities. It is difficult to get required implantation depth with a short time. On the contrary, a long annealing time will result in boron precipitation.

In this experiment, four-point probe method was utilized to measure the sheet resistance to estimate the final resistance of piezoresistors. Usually, to achieve targeted sheet resistance and recovery of the crystal lattice, we need to try many times to determine the annealing temperature and time. We chose CRESBOX four-point probe tester to measure sheet resistance as shown in Figure 4.13. Since the relationship between sheet resistance $R_b$ and piezoresistor resistance $R$ is followed by:

$$ R = R_b \frac{L}{b} $$ (4.14)

where $R$ in the thesis is set as 6.7 kΩ ± 3%. Then, $R_b$ should be equal to $210 \pm 10 \ \Omega /\text{Sq}$. After several experiments, a sheet resistivity of about $210 \pm 10 \ \Omega /\text{Sq}$. was obtained in the doped silicon by annealing at 1000 °C for 30 min in N$_2$ ambient.
Figure 4.13 CRESBOX four-point probe machine to measure sheet resistance.

After annealing process, the implantation area was measured to check the sheet resistance and implantation uniformity. With a probe interval of 1mm, the values of the sheet resistance before and after implantation were shown in Table 4.4. The results demonstrate that the sheet resistance after implantation is in the range of $210\pm10\ \Omega /\text{Sq}$, which meets the design requirement. In the section, the piezoresistors has been achieved by boron ion implantation.

Table 4.4 Sheet resistance values before and after implantation.

<table>
<thead>
<tr>
<th>Test position</th>
<th>Angle ($^\circ$)</th>
<th>Sheet resistance before implantation (Ω /Sq.)</th>
<th>Sheet resistance after implantation (Ω /Sq.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>61.22</td>
<td>213.5</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>62.04</td>
<td>209.9</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>64.11</td>
<td>206.4</td>
</tr>
<tr>
<td>4</td>
<td>90</td>
<td>64.22</td>
<td>207.0</td>
</tr>
<tr>
<td>5</td>
<td>90</td>
<td>64.46</td>
<td>210.0</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>62.41</td>
<td>212.7</td>
</tr>
<tr>
<td>7</td>
<td>180</td>
<td>61.99</td>
<td>212.2</td>
</tr>
<tr>
<td>8</td>
<td>270</td>
<td>65.30</td>
<td>213.5</td>
</tr>
<tr>
<td>9</td>
<td>270</td>
<td>6387</td>
<td>215.3</td>
</tr>
</tbody>
</table>
4.5 Cr-Au wires fabrication

Cr-Au wires provide two functions in this experiment. One is used to fabricate the Wheatstone bridge. The other is applied for the connecting with the piezoresistors. The quality of the wires will directly determine the final reliability of the sensor.

4.5.1 Material selection

There are many metal materials can be used for the Wheatstone bridge connecting, which include Al, Ti, Cr, Au, etc. Table 4.5 enumerates the coefficient of thermal expansion (CTE) of several materials at room temperature. As mentioned above, the difference in CTE values between the materials is mainly responsible for the residual stress induced in the multi-layered film structure. The larger the difference in CTE values between the materials, the higher the induced stress in the deposited film is. Thus, we had better choose the materials that have similar CTE values.

**Table 4.5 Coefficients of thermal expansion (CTE) for several materials [Lide, 2003].**

<table>
<thead>
<tr>
<th>Materials</th>
<th>CTE value (10^{-6}/^\circ C) at 25 (^\circ C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum (Al)</td>
<td>23.1</td>
</tr>
<tr>
<td>Gold (Au)</td>
<td>14.2</td>
</tr>
<tr>
<td>Titanium (Ti)</td>
<td>8.6</td>
</tr>
<tr>
<td>Chromium (Cr)</td>
<td>4.9</td>
</tr>
<tr>
<td>Silicon (Si)</td>
<td>2.6</td>
</tr>
<tr>
<td>LPCVD silicon nitride (Si(_3)N(_4))</td>
<td>1.6</td>
</tr>
</tbody>
</table>

For the aluminum (Al) metallization scheme on a LPCVD Si\(_3\)N\(_4\) membrane, a large CTE value \(23.1\times10^{-6}/^\circ C\) exists among the materials. This large difference in CTE value can be reduced to \(14.2\times10^{-6}/^\circ C\) by utilizing the gold (Au) metallization scheme. However, there is a potential adhesion reliability issue here as Au requires an adhesion layer between itself and the substrate surface. Chromium (Cr) and titanium (Ti) are two commonly used adhesion layer materials for Au film deposition.
Nevertheless, Cr is a better choice than Ti in the sensitive membrane applications as its CTE (4.9×10^{-6}/°C) is closer to that of LPCVD Si₃N₄ film (1.6×10^{-6}/°C) than that of Ti (8.6×10^{-6}/°C). Therefore, this dual-layered Cr/Au metallization scheme offers good adhesion property and potentially lower residual stress since a multilayered thin film structure tends to have a lower overall residual stress [Laconte, 2004]. As a result, a Cr-Au metallization scheme was selected for the connecting wires in this thesis.

### 4.5.2 Sputtering Cr-Au thin film

Here, we chose Cr-Au as the metal wires by thin film sputtering process. The schematic diagram and real equipment of thin film sputtering machine are presented in Figure 4.14. Thin film sputtering has many advantages including a wide choice of target materials, better step coverage, good uniformity, small shadow effect and strong adhesion. One significant feature of sputtering is its parameters that can be controlled to influence the film characteristics such as residual stress, film density and adhesion strength.

![Figure 4.14 Schematic diagram [Avada] and real equipment of sputtering machine.](image)

Typical control parameters involve the process pressure, substrate temperature, power and substrate bias voltage. By varying the process pressure, the film stress can
be altered. The transition region between the compressive and tensile stress is often very sudden and sharp, which can lead to unpredictable results. Thus, it is important to reduce the residual stress by controlling process parameters.

This experiment utilized JPG560BV type metal multilayer magnetron sputtering machine to form the Cr-Au wires for Wheatstone bridge and piezoresistors connecting. Inside the sputtering chamber, a SOI wafer was loaded on a revolving substrate holder plate while the target materials were stationary. It took about 15 s to complete a full revolution of the substrate holder plate. The process parameters for Cr-Au sputtering in this experiment are shown in Table 4.6. As a result, the final film thicknesses of the Cr and Au are 50 nm and 200 nm, respectively.

<table>
<thead>
<tr>
<th>Metal</th>
<th>Process pressure (Pa)</th>
<th>Argon flow (sccm)</th>
<th>Sputtering power (W)</th>
<th>Target base distance (mm)</th>
<th>Sputtering rate (nm/min)</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cr</td>
<td>0.2</td>
<td>50</td>
<td>300</td>
<td>30</td>
<td>30</td>
<td>1.5</td>
</tr>
<tr>
<td>Au</td>
<td>0.3</td>
<td>60</td>
<td>300</td>
<td>30</td>
<td>30</td>
<td>5</td>
</tr>
</tbody>
</table>

4.5.3 Etching Cr-Au wires

In the process of this thesis, wet etching was used to form the Cr-Au wires. The following chemical equations summarize the mechanism of Cr and Au etching with Ce(NH$_4$)$_2$(NO$_3$)$_6$ and I$_2$:

\[
3\text{Ce(NH}_4\text{)}_2\text{(NO}_3\text{)}_6 + \text{Cr} \rightarrow \text{Cr(NO}_3\text{)} + 3\text{Ce(NH}_4\text{)}_2\text{(NO}_3\text{)}_5
\] (4.15)

\[
2\text{Au + I}_2 \rightarrow 2\text{AuI}
\] (4.16)

To improve corrosion efficiency, HNO$_3$ and KI are usually joined in the above solvents. Finally, the etching solution for Cr was 15%Ce(NH$_4$)$_2$(NO$_3$)$_6$, 5%HNO$_3$ and 80%H$_2$O. For Au etching solution, the final etching liquid was 5%I$_2$, 10%KI and 85%H$_2$O. After thin film sputtering, lithography process was applied to create the pattern of the
Wheatstone bridge and piezoresistors connecting as shown in Figure 4.15 (a). The whole wet etching process was carried out at a temperature of 75 °C. The wafer was first placed in the etching solution of 5%I₂, 10%KI and 85%H₂O for 1 min. After that, the wafer was put in the etching solution of 15%Ce(NH₄)₂(NO₃)₆, 5%HNO₃ and 80%H₂O for 30s. In order to fully respond to the reaction, the etching solution needed constant shaking. When etching was completed, the reaction tended to be slow and the wires became clear. Removed the wafer to the cleaning tank for 1 min and observed it under a microscope as shown in Figure 4.15 (b).

![Figure 4.15 Process of etching Cr-Au wires: (a) Pattern of metal wires; (b) Metal wires after etching.](image)

In the process of Cr-Au etching, we need to pay attention to the following points. Firstly, the photoresist layer as a mask should be thick enough to prevent the etchant from damaging the metal wire; Moreover, the wafer must proceed the hard baking to remove the solvent from the photoresist after photolithographic development; At last, during the etching process, constant shaking is necessary to promote the full progress of the reaction.
4.6 Etching for sensitive membrane

Etching the sensitive membrane in this thesis was carried out in two steps. The first step was to etch the groove on the front side of the membrane by reactive ion etching (RIE) processing. The second step was to create the rood beam and cavity on the rear side of the membrane by deep reactive ion etching (DRIE) technique.

4.6.1 Etching principle

4.6.1.1 RIE

RIE is a plasma dry etching technique coexisting with physical and chemical effects. It combines the advantages of ion sputtering and plasma chemical etching. RIE not only possesses high resolution, but also has a fast etching rate. The working mechanism of RIE is shown in Figure 4.16. RIE uses chemically reactive plasma to remove material deposited on wafers. The plasma is generated under low pressure (vacuum) by an electromagnetic field. High-energy ions from the plasma attack the wafer surface and react with it. By the combination of physics and chemistry, the etching of the sample is completed.

*Figure 4.16 Schematic diagram of Reactive Ion Etching [Zelenograd]*.
4.6.1.2 DRIE

RIE, also known as plasma etching or dry etching, and its extension deep reactive ion etching (DRIE) are processes that combine physical and chemical effects to remove material from the wafer surface. The difference between RIE and DRIE processes can be done on the basis of etch rate, selectivity, aspect ratio or reactor type. DRIE reactors present some novel characteristics compared with RIE. They are typically equipped with at least two RF power generators for independent control of plasma generation and ion bombardment. High-density plasma is generated using an inductively coupled plasma (ICP) source. Using an ICP source with a plasma density one to two orders of magnitude higher can be achieved than with the capacitively coupled plasma (CCP) sources ($10^{10}$ vs. $10^{12}$ ions/cm$^3$). Therefore, the etch rate of DRIE is almost 1~2 orders of magnitude higher than RIE [Chutani, 2014], [He, 2011], [Wu, 2011]. The schematic diagram of DRIE process is introduced in Figure 4.17.

![Figure 4.17 Schematic diagram of DRIE process [Henry, 2010].](image)

Many DRIE applications call for structures tens or hundreds of micrometers deep, and etch rate is a very important consideration. DRIE, which is sometimes referred to as Bosch etching, relies on alternating cycles of ion-assisted chemical etching ($SF_6$) and polymer deposition ($C_4F_8$) to achieve parallel sidewalls. The main mechanism of the Bosch etching is achieved by alternating the process between etching and passivation cycles.
4.6.1.3 Bosch etching

The Bosch process is also known as a “switched process” or “time domain multiplexed process”. The relevant chemical reactions that occur during Bosch etching are given in simplified form in Equation (4.17) and (4.18). SF$_6$ and C$_4$F$_8$ are typically used as the etching and passivation gases, respectively. Equation (4.17) describes the dissociation of the etchant into ion (SxFy) and neutral (F) species. Equation (4.18) represents the chemical reaction which results in silicon removal. In the first step, a thin polymeric layer of CF$_2$ is deposited over a feature. After that, ions accelerated in the plasma sheath remove the polymer from the feature bottom, which is then chemically etched by the fluorine radicals in synergy with energetic ion interactions. Bosch steps typically range between 5 and 12 seconds in length.

$$SF_6 + e^- \rightarrow S_xF_y + S_xF_y^- + F^- + e^-$$  \hspace{1cm} (4.17)

$$Si + nF^- \rightarrow SiF_n$$  \hspace{1cm} (4.18)

Figure 4.18 illustrates the process mechanism of Bosch etching: Passivation and etching gases are introduced separately and alternately into the process chamber and exposed by high density plasma during the so-called passivation and etch cycles, respectively. Transport of the sidewall film, that is, removal and redeposition of Teflons-material, yields some local anisotropy of the etching steps which will otherwise be completely isotropic, in the vicinity of the sidewall, but does not adversely affect the main etching reaction away from the sidewall. Sidewall roughness is thus reduced despite the discontinuous process [Walker, 2001]. Since plasma polymerization is non-conformal, in narrow trenches film formation preferentially takes place at the trench opening, leaving deeper sidewall regions with less deposited passivating film, the flow of polymer material along the sidewalls toward the trench floor also provides a more uniform sidewall passivation over the trench depth. In this thesis, DRIE was chosen to etch the bear side of the membrane.
4.6.2 RIE for grooves

As introduced in chapter 3, there are four grooves around the surface on the front side of the membrane, which plays an important role in stress contraction. It is beneficial to improve the performance by this special structure. This section will illustrate how to realize it by RIE process. The detailed steps are presented as follows:

(a) The photomask of the groove was prepared to transfer the pattern to the SOI wafer by lithography process, as shown in Figure 4.19;

(b) The wafer with photoresist was baked on the hot plate and next put the wafer into acetone solution for development;

(c) To evaporate the water vapor in the photoresist, the blow dried wafer was placed on the hot plate again to make the film harder;

(d) The wafer was placed in the RIE chamber. After setting of a series of RIE parameters as shown in Table 4.7, the equipment was processed automatically based on procedures. After etching, the pattern of the groove was completed on the front side of the membrane as presented in Figure 4.20.

Table 4.7 Main process parameters for RIE process.

<table>
<thead>
<tr>
<th>Power (W)</th>
<th>Pressure (mTorr)</th>
<th>Etching/passivation (s)</th>
<th>Etching rate (μm/min)</th>
<th>Total time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>20–30</td>
<td>8/5</td>
<td>1.8</td>
<td>5</td>
</tr>
</tbody>
</table>
Figure 4.19 Photomask of the grooves pattern.

Figure 4.20 Photograph of the wafer after RIE process.

According to the micrograph of the groove, it is found that etching surface and uniformity both achieve a good result as shown in Figure 4.21. The depth of the groove was measured by Step Tester. To improve the accuracy of the measurement, three test points in the groove were measured and the results were presented in Table 4.8.

Table 4.8 Dimensions of the groove after RIE process.

<table>
<thead>
<tr>
<th>Item</th>
<th>Place 1</th>
<th>Place 2</th>
<th>Place 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groove depth (μm)</td>
<td>11.8</td>
<td>11.2</td>
<td>11.9</td>
</tr>
<tr>
<td>Groove width(μm)</td>
<td>91.2</td>
<td>87.5</td>
<td>90.4</td>
</tr>
</tbody>
</table>
4.6.3 DRIE for rood beam and cavity

Due to anisotropy of single crystal silicon, a 54.7° inclined wall is always left after deep etching for wet etching process. To obtain a vertical cavity wall, DRIE process was used to create the rood beam structure and cavity in this work. Owing to the special structure on the rear side of the membrane, the etching process had to be carried out in two steps. The first step was to etch a 35 μm depth rood beam on the back side of the membrane by the first mask as shown in Figure 4.22 (a). After that, we chose the second mask to etch the cavity as shown in Figure 4.22 (b). The total etching depth was 300 μm by the two steps. In the second step, there has been a 35 μm height difference between the surface of rood beam and the bottom of cavity. When the DRIE was completed, a 35 μm thickness rood beam and a 300 μm depth cavity were formed simultaneously. The detailed processed was introduced by the following:

(a) The photomask of rood beam was applied to transfer the pattern to the SOI wafer by lithography process as shown in Figure 4.22 (a);
(b) The front side of the membrane was adhered to a silicon wafer for protection. After that, the wafer was placed in the DRIE chamber and the process parameters were followed by the first step of Table 4.9;
(c) Removed the photoresist and transferred another pattern to the SOI wafer by lithography process as shown in Figure 4.22 (b);
(d) The wafer was put in the DRIE chamber for the second time and the process
parameters were followed by the second step of Table 4.9, the equipment was processed automatically based on procedures.

![Figure 4.22 Mask pattern for (a) the rood beam and (b) the cavity.](image)

Table 4.9 Main process parameters for DRIE process.

<table>
<thead>
<tr>
<th>Item</th>
<th>Power (W)</th>
<th>Pressure (mTorr)</th>
<th>Etching/passivation (s)</th>
<th>Etching rate (μm/min)</th>
<th>Total time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\textsuperscript{st} step</td>
<td>600</td>
<td>20–30</td>
<td>8/5</td>
<td>1.8</td>
<td>20</td>
</tr>
<tr>
<td>2\textsuperscript{nd} step</td>
<td>600</td>
<td>20–30</td>
<td>8/5</td>
<td>1.6</td>
<td>160</td>
</tr>
</tbody>
</table>

After DRIE process, the pattern of the rood beam and cavity were completed on the back side of the membrane as shown in Figure 4.23.

![Figure 4.23 Micrograph of the back side of the sensitive membrane.](image)

According to the micrograph of the rood beam, it is found that etching surface and uniformity both achieve a good result. The dimensions of the rear side of the membrane were also tested by Step Tester as shown in Figure 4.24. Three test points were measured and the results were presented in Table 4.10.
Figure 4.24 Dimension test for the rear side of the membrane by Step Tester.

Table 4.10 Dimensions for the rear side of the membrane after DRIE process.

<table>
<thead>
<tr>
<th>Item</th>
<th>Place 1</th>
<th>Place 2</th>
<th>Place 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Membrane depth (μm)</td>
<td>289</td>
<td>275</td>
<td>291</td>
</tr>
<tr>
<td>Membrane width (μm)</td>
<td>3560</td>
<td>3542</td>
<td>3583</td>
</tr>
<tr>
<td>Rood beam width (μm)</td>
<td>191</td>
<td>183</td>
<td>186</td>
</tr>
<tr>
<td>Rood beam thickness (μm)</td>
<td>28</td>
<td>25</td>
<td>30</td>
</tr>
</tbody>
</table>

4.7 Anodic bonding

4.7.1 Bonding principle

Bonding is a reliable and effective way to attach silicon wafer to several other materials. It provides a hermetic seal and is an inexpensive way for die bonding. Anodic bonding is popular in microsystems packaging due to the relatively simple setup with inexpensive equipment. It provides reliable hermetic sealing that is important for applications such as in micro valves and channels in a microfluidics network for micro pressure sensor dies. Another advantage of the anodic bonding technique is that it can be done at low temperatures in the range of 180~500 °C, which results in a low risk of residual stress and strain in the bonded materials.
The most common application of anodic bonding is to bond silicon wafers to glass wafers. Figure 4.25 illustrates the process for anodic bonding of glass (e.g., BF33 glass) to a silicon wafer. A modest weight is located on top of the BF33 glass wafer to ensure good contacting pressure. An electric field of 200~1000 V DC is applied to the system with the silicon wafer in contact with the positive electrode. The sandwiched dielectric BF33 glass and semiconductor silicon wafers between the two electrodes form an effective parallel-plate capacitor. Consequently, the voltage applied to the electrodes causes the two substrates to come into contact with the induced electrostatic force.

**Figure 4.25 Schematic diagram for anodic bonding of silicon to glass.**

### 4.7.2 Bonding process

At first, put the superimposed glass and silicon into the fixture and add the electrode. After that, the bonding chamber is filled with pressure at a suitable temperature. Since glass is similar to the electrolyte at high temperatures, sodium ions move toward glass surface where the negative electrode is applied. Then, non-bridging oxygen ions are left on the glass surface to form a thin depletion layer. Meanwhile, the electrostatic charge is generated on the surface of the silicon wafer. After the interaction between above two factors, the electrostatic field is formed between the depletion layer and glass surface. Finally, the internal and external potentials achieve the balance and silicon-glass is connected together. At high temperature, the reaction between oxygen ion in the depletion layer and silicon is occurred to form stable chemical bond O-Si-O. The reaction formulas are followed by:
In this experiment, a vacuum bonding process was utilized to bond the wafer and the glass together. The test equipment was an ELAN CB6L electrostatic bonding machine as shown in Figure 4.26. Since BF 33 glass had the similar thermal expansion coefficient with silicon, we chose it as the bonding material as this process.

Before bonding, the photoresist on the bonded surface was first removed. After that, the wafer was cleaned by alcohol and water for 10 min and then dried it with nitrogen. Finally, the wafer and BF33 glass were placed on the stationary fixture (wafer was on the top of glass) as shown in Figure 4.27. The process parameters for silicon-glass anodic bonding in this experiment are shown in Table 4.11.

Figure 4.26 Picture of real equipment of anodic bonding.

Figure 4.27 Alignment fixture for wafer and glass.
Table 4.11 Process parameters for silicon-glass anodic bonding.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Pressure (Bar)</th>
<th>Voltage (V)</th>
<th>Peak current (mA)</th>
<th>Time (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>360</td>
<td>5×10⁻⁶</td>
<td>600</td>
<td>1.5</td>
<td>20</td>
</tr>
</tbody>
</table>

4.7.3 Bonding current and voltage

In the bonding process, current and voltage are two important parameters for the technique. Figure 4.28 shows the current-time (I-t) curve of the silicon-glass electrostatic bonding. From the curve, it can be observed that the curve starts with ten seconds horizontal line. After that, the curve rapidly rises to a constant peak and then quickly decays to zero. According to the current changes, we can check the bonding condition. At first, the bonding starts with the contact point between silicon and glass, so the current is low. With the contacting area increasing, the current experiences a gradual rise and until reaches a peak. When the whole bonding is almost finished, the wafer and glass contact completely and the movable ions stop moving. Then, the current decreases to zero and the bonding process is completed.

![Figure 4.28 Current-time curve of the silicon-glass electrostatic bonding.](image)

The upper limit voltage of bonding is to prevent the glass from breakdown and the floor limit voltage is to create enough electrostatic attraction. A suitable voltage is beneficial to cause elastic and plastic deformation to achieve a good bonding result. For elastic deformation, the required voltage is followed by:
where \( \varepsilon_0 \) is the permittivity of vacuum, \( H \) is the distance between wafer and glass, \( L \) is the width of the distance, \( K \) is the elasticity modulus, and \( b \) is the thickness of the silicon wafer. For plastic deformation, the required voltage is followed by:

\[
V \geq \frac{H^3 K b^3}{80 \varepsilon_0 L^4} \frac{1}{2}
\]

(4.21)

where \( \sigma_r \) is the yield strength of the material, \( X_0 \) is the distance between the glass and anode. In the experiment, the thickness of the BF33 glass was 500 μm, the voltage was set as 600 V.

**4.7.4 Bonding pressure and temperature**

Since the bonding interface is in the side of the silicon cavity, and the bonding area is much smaller than the area of the entire wafer. The bonding pressure is set at \( 5 \times 10^{-6} \) Bar. Excessive pressure may cause the silicon rupture. Temperature plays an important role in the bonding process because electrical conductivity depends on the temperature. At high temperatures, the conductive Na\(^+\) in the glass move to the glass surface by the electric field leaving a negative landline on the surface of the glass next to the positive electrode. Near the room temperature, the basic circuit can’t measure the peak current due to a small number of conductive Na\(^+\). With the temperature rising, the number of the movable Na\(^+\) increase sharply, which brings a high conductivity. However, an excessive temperature will lead the glass to soften so that the bonding will be failed. Thus, the final temperature of the bonding process is set as 360°C.

Bonding occurs at a given temperature, but the cooling rate after bonding has a great influence on the bonding quality. Because of the difference in thermal expansion between silicon and glass, the degree of influence related to the temperature is different. If cooling rate is fast, the material may fracture due to the internal stress.
Therefore, the extent of cooling for the two materials must keep the same. When the temperature is lower than 400 °C, the thermal expansion for silicon and glass is closed. For the stress-free bonding, the bonding temperature is set as 360 °C. The cooling process for silicon-glass bonding is presented in Figure 4.29.

*Figure 4.29 Temperature curve for silicon-glass bonding.*

When the temperature is reduced to the room temperature, the wafer is taken out and observes the bonding effect. It is found that there is no obvious tiny cavity in the bonding interface, which illustrates the bonding is done as shown in Figure 4.30.

*Figure 4.30 Silicon-glass wafers after anodic bonding.*
4.8 Sensor chip fabrication

After the anodic bonding process, the microfabrication of the piezoresistive pressure sensor chip is almost completed. To separate the manufactured wafer, laser beam cut technique is utilized to execute this procedure. Finally, the individual sensor chip is finished as shown in Figure 4.31.

![Image of sensor chip]

*Figure 4.31 Microphotograph of the proposed sensor chip.*

4.9 Summary

In this chapter, a detailed fabrication process was proposed based on n-type (100) oriented SOI wafer to realize the sensor chip. The principle, processing points and process parameters were discussed.

Also, the involved processes were introduced deeply and the sensor chip completed the trial-produced. Meanwhile, the problems encountered in the processing have been solved. The optimized parameters for each process have been determined and a standard process schemes has been worked out. Finally, an ideal sensor chip was fabricated by MEMS microfabrication process.
5. ASSEMBLING AND MEASUREMENTS

In the last chapter, the fabrication of the proposed sensor chip has been completed. In this chapter, the assembling and measurements of the sensor will be carried out. The sensor will be tested at different temperatures and the results will be evaluated.

5.1 Assembling

The pressure sensor is assembled with the fabricated sensor chip as shown in Figure 5.1. The sensor is mainly constituted by chip, shell, soleplate, cable, aviation plug, etc. The metal material of the external structure is 304 stainless steel which is beneficial to improve the corrosion resistance. In order to increase the sealing and insulation characteristics of the sensor, the epoxy adhesive (60% ethoxyline resin, 30% quartz powder, 10% dioctyl phthalate) is filled in the shell and solidified for 24 h at room temperature. The connecting for the sensor and plug is polytetrafluoroethylene aviation cable, which can resist high temperature more than 200 °C. The process of the assembled sensor is presented in Figure 5.2. To protect the welding spot, the fluororubber heat shrink tube is covered at the connection between every two wires. Between the stainless steel shell and fixing ring, laser welding is chosen for connection, which can improve the long term stability and application of the sensor.

![Figure 5.1 Assembling and interconnecting structure of the sensor.](image-url)
5.2 Measurements

5.2.1 Resistance test of the piezoresistors

After the assembling of the sensor, connect the plug and socket to check the bridge resistance and ohm contacting situation, respectively, as shown in Figure 5.3. The resistances of three samples are listed in Table 5.1.

Figure 5.2 Schematic diagram for the process of the assembled sensor.

Figure 5.3 Test diagram of bridge resistance.
Test results show that the consistency of the resistance is not good enough. This is mainly because the connecting regions between piezoresistors and Cr-Au wires have some defects. Besides, the measured resistance is smaller than the designed value 6.7 kΩ. It is due to the errors involving in ion implantation and sputtering processes.

The performance of the pressure sensor is measured and the experimental schematic diagram for testing is shown in Figure 5.4. The pressure is from a pressure pump and the value can be controlled by the manometer with a 10 Pa measurement uncertainty. In order to ensure the airtightness of the measurement, the joint between the sensor port and connecting tube should be twined several laps using the rubber belt. A constant voltage of 5 V is provided to the Wheatstone bridge of the sensor using a High Current Switching DC Power Supplies (Model BK1694). The output of the sensor at different pressure loads is read by a digital multimeter (FLUKE 8845A).
5.2.2 Test results at the room temperature

The performance of the pressure sensor is usually expressed by static characteristics of technical indicators including full scale output, sensitivity, pressure nonlinearity error, repeatability, hysteresis, zero output, accuracy, etc. The definition of above parameters has been introduced in section 2.5.5. To obtain an average value for the proposed pressure sensor, three sensor devices were tested in sequence. They were numbered as device 1, device 2 and device 3. The measured voltage outputs and calculated pressure nonlinearity error are shown in Figure 5.5.

With the pressure increasing, the output of the sensor gradually increases and varies linearly. The average sensitivity of the three devices is 30.3 mV/V/psi with the average maximum nonlinearity error of 0.24% FSS. The zero output voltages for the three devices range between 8.6 mV and 21.1 mV, so the average value of 14.5 mV is
achieved finally. The detailed technical data of the device 2 (whose performance is closed to the average value) at room temperature are listed in Table 5.2. The results illustrate that the proposed sensor achieves a high sensitivity and a low linearity at the same time. Meanwhile, a high accuracy of 0.34% FSS is high compared with the publicly reported literatures.

Table 5.2 Technical data of the device 2 at room temperature.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full range pressure (psi)</td>
<td>1</td>
</tr>
<tr>
<td>Input voltage (V)</td>
<td>5</td>
</tr>
<tr>
<td>Resistance (kΩ)</td>
<td>6.7</td>
</tr>
<tr>
<td>Zero output (mV)</td>
<td>14.6</td>
</tr>
<tr>
<td>Full range output (mV)</td>
<td>169.1</td>
</tr>
<tr>
<td>Sensitivity (mV/V/psi)</td>
<td>30.9</td>
</tr>
<tr>
<td>Pressure nonlinearity (% FSS)</td>
<td>0.25</td>
</tr>
<tr>
<td>Repeatability (%FSS)</td>
<td>0.19</td>
</tr>
<tr>
<td>Hysteresis (%FSS)</td>
<td>0.14</td>
</tr>
<tr>
<td>Accuracy (%FSS)</td>
<td>0.34</td>
</tr>
<tr>
<td>TCO (%FSS/°C)</td>
<td>1.8</td>
</tr>
<tr>
<td>TCS (%FSS/°C)</td>
<td>-0.15</td>
</tr>
<tr>
<td>TCR (%FSS/°C)</td>
<td>0.19</td>
</tr>
</tbody>
</table>

5.2.3 Comparison between simulation and test

The measured results are drawn in Figure 5.6. The graph depicts the output voltage of the sensor as a function of the pressure varies from 0 to 1 psi at room temperature (25 °C). Meanwhile, the straight line is depicted for calculating pressure nonlinearity. In this figure, the lines for measured results and simulation results are very closed, which indicates a good agreement between the actual measurement and simulation.
According to the design results and analyses, the novel structural pressure sensors are fabricated and assembled, and the overall performances are also verified. It is confirmed from the experimental results that the actual sensitivity is similar to the estimated sensitivity, which proves the validity of the design process. However, there is still a deviation of 12.4% between the simulated result and experimental result as shown in Figure 5.6. The reasons mainly come from the following three aspects. One results from the poor consistency of the four piezoresistors in the Wheatstone bridge due to the inconsistent dopant concentration of the resistors involved in the process of ion implantation. Besides, the residual stresses on Si$_3$N$_4$ and SiO$_2$ passivation layers may also contribute to the relatively poor accuracy [Wei, 2012a]. Finally, the errors in the bulk- and surface microfabrication can’t be avoided. For example, the designed groove depth was 10 μm, but the actual value was 11.2 μm, so it is not possible to obtain a theoretical performance after many manufacture procedures.

The proposed sensor demonstrates that it does has the capacity of alleviating the contradiction between sensitivity and linearity to realize the micro measurement with high accuracy as shown in Table 5.3. Compared with the other three reported sensors, the sensitivity of the proposed sensor is the largest, which is more than two times bigger than that reported in [Peng, 2005]. At the same time, the pressure
nonlinearity for the sensor is intermediate among the four types of sensors, unlike [Kumar, 2016b] where a small nonlinearity error is obtained but its sensitivity is too low. Thus, it can be concluded that the proposed sensor achieves a high sensitivity and a low pressure nonlinearity when compared with the reported sensors.

**Table 5.3 Comparison with other pressure sensors.**

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Sensitivity (mV/V/psi)</th>
<th>Pressure nonlinearity (% FSS)</th>
<th>Accuracy (% FSS)</th>
<th>Full range (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed sensor</td>
<td>30.90</td>
<td>0.25</td>
<td>0.34</td>
<td>6.895</td>
</tr>
<tr>
<td>Sensor in [Tian, 2010]</td>
<td>10.14</td>
<td>0.19</td>
<td>0.24</td>
<td>10</td>
</tr>
<tr>
<td>Sensor in [Kumar, 2016b]</td>
<td>12.07</td>
<td>0.05</td>
<td>0.68</td>
<td>10</td>
</tr>
<tr>
<td>Sensor in [Shimazoe, 1981]</td>
<td>25.37</td>
<td>0.36</td>
<td>-</td>
<td>5</td>
</tr>
</tbody>
</table>

**5.2.4 Test results at the different temperatures**

The output voltage at different temperatures was measured in a high and low temperature test chamber (GD-4005) with the working range of −20~150 °C and the temperature fluctuation of ±1 °C. The equipment and measurement connection mechanism diagram are shown in Figure 5.7. To test the characteristics of the sensor at different temperatures, a total number of 9 points were tested in the range of −20~150 °C, namely −20°C, −10°C, 0°C, 25°C, 50°C, 75°C, 100°C, 125°C and 150°C, respectively. The purpose is to check the temperature adaptability of the device.

The designed pressure sensor range in the thesis was 0~1 psi. In the process of high temperature measurement, the test step change was set as 0.1 psi, namely, a total number of 11 test points was selected. Based on the pressure sensor calibration procedures, the pressure signals increase from low to high until 1psi, and then decrease from 1 psi to 0. The above process is called “one trip”. For each temperature point, three trips were carried out for a precise measurement. For each test point in each trip, the output data would be not read until the pressure was stable. Finally, the various indicators of the sensor were calculated.
The sensitivity and zero output analyses are the study of change in output voltage with respect to the applied pressure at different temperatures. Figure 5.8 (a) illustrates the relationship between the output of the sensor and the standard pressure under different temperatures. It can be observed that the lower the temperature is, the greater the output voltage is. It is also observed that the output increases linearly with the applied pressure. However, the output curves of the sensor at different temperatures are not coincident and there are also differences in the zero output voltage as well as the sensitivity, which demonstrates that the proposed sensor has a temperature drift and is consistent with the conclusion of the literatures [Song, 2015], [Zhang, 2016], [Zhao, 2017].

To be more intuitional, Figure 5.8 (b) and (c) displays the zero output changes and sensor’s sensitivity with temperature increasing. It is found that the zero output increases 15.5 times when the temperature is changing from 20 °C to 150 °C, which indicates the proposed sensor possesses an obvious temperature-sensitive character. By calculation, the TCO of the sensor is 1.8% FSS/°C. TCS is calculated using the full output at 150 °C and reference temperature 25 °C by Equation (2.29), where the TCS of the sensor is −0.15% FSS/°C, indicating that the sensitivity of the sensor has a negative temperature coefficient.
Chapter 5: Assembling and measurements

Figure 5.8 Performance under different temperatures: (a) Sensor input-output curves; (b) Zero output curves; (c) Sensitivity curves.

With the temperature increasing, the resistance experiences a rise until reaching the largest value at 150 °C as shown in Figure 5.9. Based on above resistance variation, TCR is calculated by 0.19% FSS/°C.

Figure 5.9 Curve of the resistance variation at different temperatures.
Meanwhile, the zero output voltage increases substantially when the temperature is increased. The zero output derives from two aspects. The first reason is due to some residual stress on the membrane. The second reason is because of the non-uniformity in the doping of the four piezoresistors. The high value of zero output at high temperature may be attributed to the difference in the shapes of piezoresistors experiencing longitudinal and transverse stress [Chou, 2009]. The linear fit function for the output of sensor at different temperatures is shown in Table 5.4. It may be stated here that the linear fit function can also be used as the calibration curve. In that case, the slope of the linear fit function is the sensitivity and the constant of the equation is the zero output. Besides, the sensitivity experiences a reduction as the temperature increases from \(-20 ^\circ C\) to \(150 ^\circ C\). This is mainly due to the fact that the piezoresistive coefficient decreases when temperature rises. However, the sensitivity is still as high as 21.2 mV/V/psi at \(150 ^\circ C\).

### Table 5.4 Zero output and linear fit function at different temperatures.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Zero output (mV)</th>
<th>Linear fit function (y=5\times\text{pressure (psi)}+2.2)</th>
<th>Root mean squared error (RMSE) for the linear fit function</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>2.2</td>
<td>(y=38.8x+2.2)</td>
<td>0.614</td>
</tr>
<tr>
<td>-10</td>
<td>5.1</td>
<td>(y=36.7x+5.1)</td>
<td>0.642</td>
</tr>
<tr>
<td>0</td>
<td>8.2</td>
<td>(y=34.7x+8.2)</td>
<td>0.716</td>
</tr>
<tr>
<td>25</td>
<td>11.2</td>
<td>(y=32.5x+11.2)</td>
<td>0.685</td>
</tr>
<tr>
<td>50</td>
<td>14.6</td>
<td>(y=30.9x+14.6)</td>
<td>0.791</td>
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<tr>
<td>75</td>
<td>19.5</td>
<td>(y=28.1x+19.5)</td>
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<tr>
<td>100</td>
<td>26.1</td>
<td>(y=25.8x+26.1)</td>
<td>0.814</td>
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<tr>
<td>125</td>
<td>32.8</td>
<td>(y=22.9x+32.8)</td>
<td>0.827</td>
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<tr>
<td>150</td>
<td>36.3</td>
<td>(y=21.2x+36.3)</td>
<td>0.805</td>
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Based on Table 5.4, it is found that the zero output increases 15.5 times and the sensitivity reduces almost by 50% when the temperature is changing from \(-20 ^\circ C\) to \(150 ^\circ C\), which indicates the proposed piezoresistive pressure sensor possesses an
obvious temperature-sensitive character. Besides, all the RMSE values for the fitting function are smaller than 1, which indicates a high accuracy between the test values and predicted values. Additionally, output has a close relationship with the pressure and temperature as indicated in Figure 5.10. To deduce one fitting formula which can directly displays the relationship between the load and temperature and pressure, a least squares fitting for the variables will be utilized as shown in the following:

\[ U(P, T) = C_0 + C_{10}P + C_{01}T + C_{20}P^2 + C_{11}PT + C_{02}T^2 \]  

(5.1)

where \( P \) is the corrected pressure, \( T \) is the working temperature, \( C_0, C_{10}, C_{01}, C_{20}, C_{11}, \) and \( C_{02} \) are the calibration coefficients. In order to search the fitting formula by least squares fitting method, Equation (5.1) is determined by fine adjustment of the MATLAB calculated results as followed by:

\[ U(P, T) = 6.327 + 178.6P + 0.127T + 0.042P^2 - 0.504PT + 0.000572T^2 \]  

(5.2)

where the confidence bound is 95%, the coefficient of determination is 0.9894, the root mean squared error is 0.839. Above the coefficients illustrate that the deduced formula possesses a goodness of fit.

Figure 5.10 Pressure-output 3D photograph under various temperatures.

Generally, two methods are usually used to improve temperature-sensitive character. On the one hand, since the piezoresistive coefficients are the function of the
temperature and doping concentration, namely, \( \pi(N,T) = \pi_0 \times P(N,T) \). According to the piezoresistance factor influenced by the temperature and impurity concentration for monocrystalline silicon, a lower doping concentration is commonly needed \([Bao, 2005]\). On the other hand, incorporate some compensation circuits and signal-conditioning circuits either by using them on the silicon die itself or by employing hybrid components/signal conditioning ICs/resistor to correct the output \([Gakkestad, 1995]\), \([Aryafar, 2015]\), \([Li, 2017c]\).

### 5.3 Summary

This chapter mainly focused on the measurement analysis of the proposed pressure sensor. The static performance index, testing system and testing method of sensor were introduced.

It can be found that the test values are basically consistent with the simulation results, which indicate the FEM simulation is reasonable. Though there is still a deviation of 12.4% between theoretical and practical, it still can provide enough guidance for design and fabrication.

Also, the performance of the sensor at different temperature was tested. When the applied pressure was 1 psi, the sensor achieved a sensitivity of 30.9 mV/V/psi, a pressure nonlinearity of 0.25% FSS and an accuracy of 0.34%, and thereby the contradiction between sensitivity and linearity was alleviated. In terms of micro size, accuracy and stability at high temperature, the proposed sensor obtained a good performance, so it was a proper choice for measuring micro-pressure less than 1 psi.
6. GENERAL CONCLUSIONS AND PROSPECTS

6.1 General conclusions

In this thesis work, a novel structural pressure sensor with four-grooved membrane combined with rood beam has been proposed for low pressure measurements based on SOI substrate. After mechanical analysis and geometry optimization, the proposed sensor was realized and proved it was suitable to test low pressure with high accuracy.

Several conclusions can be extracted from this work:

- According to mechanical formula derivation and simulation results, the working mechanisms of both grooves and rood beam were discussed. By Navier trigonometric equations, the relationship between stress and deflection of the four edge fixed membrane has been deduced. Based on the Euler-Bernoulli beam theory, the deformation mechanism of the membrane combined with a rood beam structure was explained.

- Based on the working mechanism of the proposed membrane, the groove created a stiffness mutation at the rib region along the transversal direction of the rood beam structure. It further enhanced the stress concentration at the top surface of the rib structure to form a high concentrated stress profile. Besides, this structure also lowered the pressure nonlinearity of the sensor by increasing the constraint of the partial membrane due to the relatively thick thickness of the rib region. The sensor got double improvements for the sensitivity and linearity created both by grooves and rood beam.

- Also, a design method, including model design, dimensions optimization and structure design of the novel structure sensor, was presented. FEM simulation and curve fitting method Origin have been used to analyze the stress distribution of sensitive elements and the deflection of membrane. On the basis of
multivariate fitting, the relationships between structural dimension variables and mechanical performance were deduced. The coefficient of determination $R^2$ and residual sum of squares $RSS$ were introduced to indicate whether the fitting equations and curves matched well with the simulation results. After that, a series of the optimal membrane dimensions were determined.

- By analyzing the effects from piezoresistors shape, size and location on the output performance, the piezoresistors were experienced an optimization. In comparison with CBM and BMQI structures, the proposed structure not only stiffened partially the membrane to reduce the nonlinearity and kept high first natural frequency, but also transferred more strain energy to the HCSP to guarantee even higher sensing sensitivity. Besides, the proposed membrane could minimize the size of the sensor chip while having same maximum stress than C- and E-type.

- A detailed fabrication process was proposed based on n-type (100) oriented SOI wafer to realize the sensor chip. The involved processes were introduced deeply and the sensor chip completed the trial-produced. Meanwhile, the problems encountered in the processing have been solved. The optimized parameters for each process have been determined and a standard process schemes has been worked out. Thus, an ideal sensor chip was fabricated by MEMS microfabrication process.

- Finally, it can be found that the test values are basically consistent with the simulation results, which indicate the FEM simulation is reasonable. The performance of the sensor at different temperature was tested. When the applied pressure was 1 psi, the sensor achieved a sensitivity of 30.9 mV/V/psi, a pressure nonlinearity of 0.25% FSS and an accuracy of 0.34%, and thereby the contradiction between sensitivity and linearity was alleviated. In terms of micro size, accuracy and stability at high temperature, the sensor obtained a good performance, so it was a proper choice for measuring micro-pressure less than 1 psi.
6.2 Future prospects

The results obtained in this thesis have left the door open to continue working in the development of this novel structural piezoresistive pressure sensor in different fields.

6.2.1 Signal processing and improvement

After finishing this doctoral thesis it is planned to continue working about signal processing and improvement. Although the proposed pressure sensor can alleviate the contradiction between sensitivity and linearity to achieve high accuracy, some inherent qualities such as zero output, obvious temperature-sensitive character, etc. are still existed. It limits its application in higher temperature environments. Thus, the compensation circuits or signal-conditioning circuits need to be introduced into the packaging of sensors to improve the signal-to-noise ratio and measurement accuracy in practical applications of long-term transmission.

The main function of the high-temperature signal-conditioning circuit is amplification of the output voltage signal from tens of millivolts to a few volts. Additionally, the circuit has a flexible adjustable range in offset voltage and magnification [Yao, 2016].

According to the high output impedance of the pressure-sensitive chip, the high-temperature signal-conditioning circuit should have high input impedance. Here, three operational amplifiers like Figure 6.1 are used to improve the circuit input impedance.

![Figure 6.1 Schematic of high-temperature signal-conditioning circuit [Yao, 2016.](image)
Chapter 6: General conclusions and prospects

The Comparison of the pressure-sensitive chip temperature compensation with the compensated circuit and uncompensated circuit is presented in Figure 6.2 (a). The sensor calibration curve compensated using the passive resistor temperature compensation (Figure 6.2 (b)) exhibits significantly better accuracy and smaller zero output over the entire temperature range. This is precisely the result we need.

![Figure 6.2 Comparison of the pressure-sensitive chip temperature compensation: (a) uncompensated sensor calibration curve; (b) compensated sensor calibration curve.](image)

6.2.2 New materials for this novel structural sensor

Many new raw materials for the sensor chip have been developed to improve the performance. For instance, nanomaterials are not only adopted for the transistor design, but also applied as new sensing element or detection platform. Moreover, some other novel materials, such as SiC, diamond and silicon nanowires are all chosen to fabricate sensor chips to get high precision.

In many industrial fields including oil drilling, avionics and spacecraft systems, engine, turbine, and industrial process control, etc., it is of primary importance to directly measure media pressures in excess of 2500 bar at temperatures up to 400°C. Then, a high temperature resistant substrate is urgently needed. SiC possesses excellent material properties that make it ideal for MEMS devices needed in variety of harsh environments. These properties include, but are not limited to, wider bandgap, higher breakdown voltage and Young’s modulus, and much higher heat conductivity.
and melting temperature than silicon. Thus, next step we will try to choose SiC as the substrate material to fabricate the proposed sensor chip. Up to now, the pressure sensor based on SiC material has been realized by microfabrication process as shown in Figure 6.3.

![Figure 6.3 Sensor chip and output based on the SiC substrate pressure sensor](Wieczorek, 2007).

The related reports about SiC substrate pressure sensor are becoming more with the development of microfabrication technique. However, there are still many challenges in the specific process due to the physical properties of SiC. Thus, it is significant to study the key processing issues about SiC microfabrication in the future.
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Appendix A: Static bending of thin plate

Piezoresistive pressure sensors work on the principle of converting the stresses in the deformed thin sensitive membranes, induced by the mechanical pressure, to the desired form of electronics output. For most cases, the membrane can be treated as a thin plate subjected to lateral bending by uniformly applied pressure as shown in Figure A.1. The governing differential equation for the deflection of a rectangular plate subject to lateral bending can be expressed as:

\[
\frac{D}{12(1-\mu^2)} \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) = \frac{P}{D}
\]

where \( \omega = \omega(x, y) \) is the deflection of a flat thin plate because of the uniformly distributed load \( P \). The parameter \( D \) is the flexural rigidity of the plate, which can be presented by:

\[
D = \frac{EH^3}{12(1-\mu^2)}
\]

where \( E \) and \( \mu \) are Young’s modulus and Poisson’s ratio of the thin plate material, respectively, and \( H \) is the thickness of the plate, as shown in Figure A.1.

Figure A.1 Schematic diagram of a rectangular-shaped membrane under the uniform normal pressure.

The components of bending moments about \( x \) and \( y \) axes (see Figure A.1) and the bending stresses can be calculated from the solution of the deflection obtained from Equation (A.1) as shown below.
The bending moments:

\[ M_x = -D \left( \frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2} \right) \] (A.3)

\[ M_y = -D \left( \frac{\partial^2 \omega}{\partial y^2} + \mu \frac{\partial^2 \omega}{\partial x^2} \right) \] (A.4)

\[ M_{xy} = D(1-\mu) \frac{\partial^2 \omega}{\partial x \partial y} \] (A.5)

The bending stresses:

\[ (\sigma_{xx})_{\text{max}} = \frac{6(M_x)_{\text{max}}}{H^2} \] (A.6)

\[ (\sigma_{yy})_{\text{max}} = \frac{6(M_y)_{\text{max}}}{H^2} \] (A.7)

\[ (\sigma_{xy})_{\text{max}} = \frac{6(M_{xy})_{\text{max}}}{H^2} \] (A.8)

By above equations, simplified formulas for maximum stresses or deflection owing to bending in beams can be deduced in several sources [Avellone, 1993]. The computed magnitude of maximum stresses and locations where these stresses occur in the deformed structure enable sensors to place measurement stations at these locations for maximum signal outputs.

For silicon piezoresistive pressure sensors, there are three kinds of shape can be chosen as the membrane, namely circular, square and rectangular shapes. In early stage, circular shape was always adopted due to the convenient fabrication and stable performance. With the development of the microfabrication technique, other two types are usually used to achieve higher properties.

A square membrane, in pressure sensors, is chosen on account of its higher stress when compared with rectangular and circular ones in the same conditions. On the basis of the theory of elasticity, the maximum stresses of the square, rectangular and circular membranes, respectively, are given by follows [Hsu, 2008]:

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\[
\sigma_{sm} = 0.308P \left( \frac{L}{H} \right)^2 (1 - \mu^2) \tag{A.9}
\]

\[
\sigma_{rm} = 0.383P \left( \frac{B}{H} \right)^2 (1 - \mu^2) \tag{A.10}
\]

\[
\sigma_{cm} = 0.75P \left( \frac{R}{H} \right)^2 (1 - \mu^2) \tag{A.11}
\]

where \( \sigma_{sm} \), \( \sigma_{rm} \) and \( \sigma_{cm} \) are the maximum stresses for the square, rectangular and circular membranes, respectively. \( P \) is the applied pressure; \( H \) is the thickness of the membrane; \( \mu \) is the Poisson ratio; \( L \) is the side length of the square membrane, \( B \) is the width of the rectangular membrane, and \( R \) is the radius of the circular membrane.

**Figure A.2 Maximum von Mises stresses for various membrane shapes.**

By assuming that they all have the same membrane thickness \( H \) and identical applied pressure \( P \), and additionally that, \( L \) is 1.2 times \( B \) and 2 times \( R \), the following relationships can be obtained by Equation (A.12):

\[
\sigma_{sm} = 1.16\sigma_{rm} = 1.64\sigma_{cm} \tag{A.12}
\]
which means an \( \sim 15\% \) and \( \sim 60\% \) higher stress can be achieved by utilizing square membrane rather than the other two types. The results can be demonstrated by the simulation as shown in Figure A.2. Thus, in this thesis, a square was chosen as the membrane of the novel structural sensor.
Appendix B: Finite element method

B.1 FEM theory

In semiconductors, piezoresistance results from the strain-induced alteration of the material’s band structure, and the associated changes in carrier mobility and number density. The general relationship between the electric field (\(\vec{E}\)) and the current density (\(\vec{j}\)) is given by Equation (B.1):

\[
\vec{E} = \rho \cdot \vec{j} + \Delta \rho \cdot \vec{j}
\]

(B.1)

where \(\rho\) is the resistivity tensor and \(\Delta \rho\) is the change of the resistivity tensor. Since silicon is cubic symmetric, the resistivity tensor is a symmetrical tensor (i.e., \(\rho_{ij} = \rho_{ji}\)), which can take the form of Equation (B.2):

\[
\rho = \begin{bmatrix}
\rho_{xx} & \rho_{xy} & \rho_{xz} \\
\rho_{xy} & \rho_{yy} & \rho_{yz} \\
\rho_{xz} & \rho_{yz} & \rho_{zz}
\end{bmatrix}
\]

(B.2)

For a single crystalline material with cubic symmetry like a silicon crystal, it can be proved that \(\rho_{xx} = \rho_{yy} = \rho_{zz} = \rho_0\) and \(\rho_{yz} = \rho_{xz} = \rho_{xy} = 0\) when the material is free from any deformation. When there is stress on silicon, the resistivity tensor \(\rho\) will change with six independent components (three normal stresses \(\sigma_i\) and three shear stresses \(\tau_i\)). Then Equation (B.3) is obtained by:

\[
\begin{bmatrix}
\rho_{xx} \\
\rho_{yy} \\
\rho_{zz} \\
\rho_{xy} \\
\rho_{xz} \\
\rho_{yz}
\end{bmatrix} =
\begin{bmatrix}
\rho_0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} + \frac{1}{\rho_0} \begin{bmatrix}
\Delta \rho_{xx} \\
\Delta \rho_{yy} \\
\Delta \rho_{zz} \\
\Delta \rho_{xy} \\
\Delta \rho_{xz} \\
\Delta \rho_{yz}
\end{bmatrix} = \begin{bmatrix}
\pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\
\pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\
\pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & \pi_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & \pi_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & \pi_{44}
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_1 \\
\tau_2 \\
\tau_3
\end{bmatrix}
\]

(B.3)

where \(\Delta \rho_i / \rho_0\) is the relative change of the resistivity component, and \([\pi_{ij}]\) are piezoresistive coefficients. The \(\Delta \rho\) vector computed from Equation (B.3) is assembled into matrix form in Equation (B.4):

\[
\begin{bmatrix}
\vec{E}_x \\
\vec{E}_y \\
\vec{E}_z
\end{bmatrix} =
\begin{bmatrix}
\rho_{xx} & \rho_{xy} & \rho_{xz} \\
\rho_{xy} & \rho_{yy} & \rho_{yz} \\
\rho_{xz} & \rho_{yz} & \rho_{zz}
\end{bmatrix} \begin{bmatrix}
\vec{j}_x \\
\vec{j}_y \\
\vec{j}_z
\end{bmatrix} + \begin{bmatrix}
\Delta \rho_{xx} & \Delta \rho_{xy} & \Delta \rho_{xz} \\
\Delta \rho_{xy} & \Delta \rho_{yy} & \Delta \rho_{yz} \\
\Delta \rho_{xz} & \Delta \rho_{yz} & \Delta \rho_{zz}
\end{bmatrix} \begin{bmatrix}
\vec{j}_x \\
\vec{j}_y \\
\vec{j}_z
\end{bmatrix}
\]

(B.4)
In the model, the Piezoresistivity and Boundary Currents interface are used to deduce the structural equations on the domain level and resolve the electrical equations on a thin layer coincident with a boundary in the model geometry. Then, the output voltage of the sensor can be obtained by the piezoresistance effect [Orthner, 2010]. For simplification, it is necessary to transform the piezoresistive equations to an arbitrary coordinate system. The largest piezoresistive coefficient can be obtained by using p-type silicon resistors oriented along <100> direction. In this case, the resistance change can be obtained as Equation (B.5) [Bao, 1991]:

\[
\frac{\Delta R}{R} = \frac{1}{2} (\pi_{11} + \pi_{12}) (\sigma_i + \sigma_t) + \frac{1}{2} \pi_{44} (\sigma_i - \sigma_t)
\]  

(B.5)

where \(\pi_{11}, \pi_{12},\) and \(\pi_{44}\) are piezoresistive coefficients. Since \(\pi_{44}\) is more significant than \(\pi_{11}\) and \(\pi_{12}\) in p-type. Therefore, Equation (B.5) can be simplified as Equation (B.6):

\[
\frac{\Delta R}{R} = \frac{1}{2} \pi_{44} (\sigma_i - \sigma_t)
\]  

(B.6)

In this case, the relationship between the output voltage \(U_{out}\) and input voltage \(U_{in}\) can be obtained as Equation (B.7):

\[
U_{out} = \frac{1}{2} \pi_{44} (\sigma_i - \sigma_t) U_{in} = \frac{1}{2} \pi_{44} \Delta \sigma R U_{in}
\]  

(B.7)

Once the stress \(\Delta \sigma R\) is calculated by the simulation, the output voltage, \(U_{out}\), of the sensor can be obtained. In the following, we will develop the finite element model to calculate the characteristics of the sensor.

**B.2 FEM process**

In the process of simulation, non-linear static analysis and modal analysis are chosen to calculate the structural response when the pressure is loaded constantly on the surface of the membrane. The deflection, stress and strain of the membrane will be obtained after analysis of the sensitive diaphragm. The detailed steps are described as below:
• **1st Modelling:** A novel structure featuring a four-grooved membrane with roof beam structure is designed by Solidworks 2012. By adding component, the three-dimensional graph is imported into COMSOL and the simulation model is generated automatically. Take chip size 6000μm×6000μm×330μm as an example, the back etching depth is 300μm, membrane thickness is about 30μm, as shown in Figure B.1 (a) and B.1 (b).

• **2nd Setting the material parameters:** The main properties of N-type silicon defined as chip substrate in the simulation model are shown in Table B.1. In the simulation process, the model considers a linear elastic material in a stationary case. The linear elastic material node adds the equations for a linear elastic solid and an interface for defining the elastic material properties.

• **3rd Meshing:** It is so significant in simulation, as rational and scientific division of grids will improve the reliability and authenticity of the calculation. The three-dimensional graph created by Solidworks is a solid model, and meshing can transform the solid model into finite element model. The model in the thesis is simple, so a “normal size” of the grids is chosen, as shown in Figure B.1 (c).

• **4th Adding constraint condition and boundary condition:** The thickness and material properties, such as the Young’s modulus, Poisson ratio, density, etc., are entered depending on the material for which the simulation is performed. Loads are applied on one boundary and the problem is solved to get the deflection of the membrane. A parametric solver is used to solve the problem with the applied load as the varying parameter. The electric field is determined as a function of the applied voltage and displacement of the membrane.

• **5th Results:** The performances of the proposed sensor chip are calculated by non-linear static analysis and modal analysis by the commercially-available finite element analysis software COMSOL Multiphysics. The maximum deflection is happened at the center of the membrane and the stress is mainly concentrated
at the SCRs (Stress Concentrated Regions) which locate at the center of each membrane edge as shown in Figure B.1 (d) and B.1 (e). The maximum deflection is about 4 μm and the maximum stress is about 29.1 MPa.

Figure B.1 Simulation process diagram: (a) the front side of the model; (b) the rear side of the model; (c) the meshing results; (d) the deflection of the membrane; (e) the stress distribution on the membrane; (f) the path definition; (g) the deflection along the path.
Table B.1 Properties of N-type silicon materials.

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<td>2330</td>
<td>Hardness (kg/mm³)</td>
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<td>Yield strength (GPa)</td>
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<td>Young elastic modulus (GPa)</td>
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