GENERATOR REGIME OF SELF BALANCED ELECTRODYNAMIC BARE TETHERS

J. Peláez* and M. Sanjurjo*

Abstract

The generator regime of electrodynamic bare tethers working at inclined orbits is analyzed. Without damping or some kind of control the attitude dynamics of these tethers is unstable. However, the instability disappears, or reduces drastically, when the tether is self balanced, that is, when the Lorentz torque about the system center of mass vanishes. The aim of the paper is to describe in detail the main parameters involved in the dynamics of this kind of tethers and also their influence on the balance condition. In addition a strategy will be proposed to keep the tether balanced taking into account the whole trajectory followed by the system during the deorbiting process.

INTRODUCTION

DE-ORBITING of satellites from LEO orbits at the end of their operational life has been suggested as a feasible and attractive application of electrodynamic tethers, specially in the field of space debris mitigation. There, the significant advantages of the electrodynamic tethers will play a decisive role in the near-term.

There are two basic configurations for an electrodynamic tether: *i)* **the insulated tether**, with current collection and emission only at end-masses, and *ii)* **the bare tether**, with current collection along its anodic portion. Moreover, an electrodynamic tether has two basic operation regimes: **the thruster mode**, and **the generator (deboost) mode**.

Apart from the polar band (orbital inclination $i \simeq 85 - 90^\circ$) where the vertical tether is not appropriate, the largest satellite occupation takes place in the midinclination band ($i \simeq 40 - 60^\circ$). However, when an electrodynamic tether operates in an *inclined* orbit, its attitude is affected by a dynamic instability due to the electrodynamic forces acting on the system. This *skip rope type* instability appears for any tether configuration and for any operation regime. It has been studied in several works (see Refs. [8, 7, 9, 13, 12, 11, 15, 14, 16]), and it was detected in several dynamic simulations [2, 10]. Since the averaged tether current is considered constant along the orbit and, in particular, independent of the actual tether position, the analysis of papers [8, 7, 9, 13, 12, 11, 15, 14] can be applied to both tether configurations and both operation regimes.

The instability appears in the motion relative to the system center of mass G and depends on the Lorentz torque about G which is proportional to a nondimensional parameter ε (see eq. (4)). This parameter, gauges the Lorentz torque versus the torque produced by the gravity gradient and inertia forces, and it depends on the tether current and the mass distribution of the system. In tether dynamics simulations, it should be obtained solving, in general, a numerical problem. Obviously, if the tether current is zero, i.e., for the inert tether, the parameter ε vanishes and the instability disappears, but this rather trivial situation is completely lacking in interest.

It is clear that the tether current closely depends on the device used to collect electrons. Since the instability increases with the tether current, it seems appropriate to focus the analysis in **the**

*Grupo de Dinámica de Tethers (GDT). ETSI Aeronáuticos, Universidad Politécnica de Madrid, Madrid, Spain.
E-mail address: jpelaez@faia.upm.es, msanjurjo@cmusanjuan.com

bare tether concept because produce the largest tether currents. Indeed, the analysis of [3] for the generator mode, found only a discrete gain by adding an electron-collecting device at the end of a bare tether. When a bare tether is used as a deorbiting device the electrodynamic parameters of the tether must be tuned to obtain a high drag efficiency and a good adaptiveness to variable ambient conditions. In the analysis of Ref. [6] the current-voltage response of the bare tether shows two ideal regimes: the short and the long tether regimes.

Recently a new concept has been introduced in this matter: the *Self Balanced Electrodynamic Tether (SBET)* (see [17, 19]). This concept takes advantage of the properties of the tether current profile, specially in the case of bare tethers. In a *SBET* the Lorentz torque about the system center of mass becomes zero, or takes negligible values, with tether current different from zero, *even with high values of the tether current*. Thus, for a *SBET*, the parameter ε vanishes with current flowing in the wire. The balance condition is obtained by adjusting appropriately the system mass distribution and it results, in a first approximation, independent of the tether current. This is the key point of the concept that successfully combines high currents in the tether with very small values of the Lorentz torque.

This concept, which has been used in a recent work for ESA (see [20]), is especially interesting in the analysis of the dynamics of satellite orbit descent/raise using electrodynamic tethers as thrusters, particularly, when a high tether current is expected. For example, the concept would be advantageous to keep big masses, as the ISS, in LEO orbit, compensating the aerodynamic drag with a thrusting tether. Also in missions to the giant planets, where high orbital velocities and high magnetic fields suggest high tether currents.

The balance condition requires a detailed design of the end masses and it only fulfills for nominal conditions which must be specified for each particular mission. Fortunately, for the bare tethers, certain *stiffness* of the current profile makes easy to operate close to the nominal conditions, i.e., with very low values of the Lorentz torque and the instability takes a long time to develop. It would not be necessary to use additional devices to control the system attitude dynamic. However, if control is required for safety reasons, the *SBET* will be readily controllable.

The *SBET* concept offers an additional advantage. If the tether is *flexible*, the balance condition will facilitate the analysis of the dynamic response of the tether to this kind of instabilities, and the influence of the electrodynamic forces on the tether vibrations will appear sharply. Note that a zero Lorentz torque makes difficult the transfer of energy between the rigid body motion of the tether and its vibrational modes.

However the concept has some limitations. It is not clear, at present, that the *SBET* concept would remove the instability from the vibrational motion of the tether, specially when *small* end masses are involved. Some simulations performed in our group show a smooth behaviour in this sense, but this is an open matter which must be studied in detail. Moreover, in some cases the mass distribution is given beforehand or totally determined by other mission requirements, the balance condition would not fulfill, in general, and the *SBET* concept would be unusable.

The balance condition has been obtained in two cases (see Refs. [17, 19]):

- for an isolated tether terminated with a classical spherical collector
- for a floating bare tether

For the *floating* bare tether, the balance condition does not exhibit sensitivity with the ionospheric plasma density. In other words, the balance condition is very robust for this type of tethers. As consequence, for a balanced tether, the orbit of the center of mass could undergo significant height variations without any substantial change of the Lorentz torque, that is, keeping the tether balanced.

However, the balance condition exhibits a small sensitivity with E_m the electric field induced by the Earth magnetic field. Since E_m changes along the orbit, with the tether position into the orbital frame and with the orbit itself, the balance condition only fulfills exactly for a particular value of E_m (nominal conditions). Taking as nominal condition the average value of E_m along the orbit with the tether *frozen* on the local vertical, during the tether motion the Lorentz torque would be oscillating around zero and the maximum excursion will be bounded by, approximately, 1° in the mass angle

defined in (6-7). Such a residual torque which has no significant effects on the tether dynamics, could be readily controlled if needed. In fact, in simulations carried out in our group with balanced tethers they never have been destabilized.

The balance condition can be extended without significant difficulties to other kinds of bare tethers (non floating). Such extension requires a detailed knowledge of the mission to be carried out by the tether system. In what follows, we extend the analysis to obtain the balance condition for electrodynamic tethers working in the generator regime. The goal of this paper is to describe in detail the influence on the balance condition of the main parameters involved in the analysis. We select the deorbiting regime of the electrodynamic tether by two reasons:

- in this regime the mathematics are a little bit simpler than in the thruster regime
- there are significant applications behind it, specially in the space debris mitigation field

In addition, a strategy will be proposed to keep the tether balanced taking into account the whole trajectory followed by the system during the deorbiting process. We finish this section underlining that to balance the tether is a solution to overcome the dynamic instabilities of the electrodynamic tethers, but it is not the only one. There are other ways to gain control on this instability. However, we believe that this solution has interest by itself and it is worthy of to be analyzed.

DUMBBELL MODEL AND GOVERNING EQUATIONS

We will use the dumbbell model to introduce the main features of the SBET concept. A detailed deduction of the tether governing equations using the dumbbell model can be found in Refs. [18, 1], for the general elliptic case.

For the circular case, the relevant points involved in the system dynamics can be found in Refs. [17, 19]. We will assume a circular inclined orbit and for the sake of brevity we will use the same nomenclature as in [17, 19]. In the orbital frame $Gxyz$ (see Fig. 1), the position of the tether and its unit vector $\vec{u}$ are defined by the *in-plane* angle θ and the *out-of-plane* angle φ , that will be taken as generalized coordinates.

The tether governing equations take the form

$$\ddot{\theta} - 2(1 + \dot{\theta})\dot{\varphi}\tan\varphi + \frac{3}{2}\sin 2\theta = f_\theta \quad (1)$$

$$\ddot{\varphi} + \sin\varphi\cos\varphi\{(1 + \dot{\theta})^2 + 3\cos^2\theta\} = f_\varphi \quad (2)$$

where the electrodynamic forcing terms (f_θ, f_φ) are given by

$$\begin{aligned} f_\theta &= -\varepsilon (\sin i \tan \varphi [2 \sin \nu \cos \theta - \cos \nu \sin \theta] + \cos i) \\ f_\varphi &= \varepsilon \sin i (2 \sin \nu \sin \theta + \cos \nu \cos \theta) \end{aligned}$$

Here, the true anomaly ν is the independent variable. They must be integrated from the initial conditions

$$\nu = \nu_0 : \quad \theta = \theta_0, \quad \varphi = \varphi_0, \quad \dot{\theta} = \dot{\theta}_0, \quad \dot{\varphi} = \dot{\varphi}_0 \quad (3)$$

when the tether current is *switched on*. Equations (1-2) involve two free parameters, the inclination i and ε . This parameter gauges the Lorentz torque versus the torque produced by the gravity and inertia forces

$$\varepsilon = \frac{J_1 \mu_m}{I_s \mu_E} \quad (4)$$

The other parameters which appears in (4) are:

Figure 1: Orbital frame and tether position

- μ_E the gravitational constant of the Earth
- μ_m the strength of the magnetic dipole used to describe the geomagnetic field
- I_s the moment of inertia relative to a line perpendicular to the tether through G
- J_1 an integral expression appearing in the Lorentz torque

Note that the Lorentz torque about G produced by the tether current profile $I_e(h)$ is

$$\vec{M}_E = \vec{u} \times (\vec{u} \times \vec{B}) J_1 \quad \text{where} \quad J_1 = \int_0^L (h_G - h) I_e(h) dh \quad (5)$$

Here L is the tether length, $\vec{B}$ is the Earth magnetic field, which will be considered constant along the tether and equal to its value in G . The details can be found in Refs. [8, 7].

Instead of (m_1, m_2, m_t) , we use the parameters (m, ϕ, Λ_t) because are more advantageous. Here, $m = m_1 + m_2 + m_t$ is the total mass of the system, $\Lambda_t = m_t/m$ is a nondimensional tether mass and ϕ the mass angle. It is defined by

$$\cos^2 \phi = \frac{1}{m} \{m_1 + \frac{1}{2}m_t\} \quad (6)$$

$$\sin^2 \phi = \frac{1}{m} \{m_2 + \frac{1}{2}m_t\} \quad (7)$$

and it is a measure of the mass distribution between both end masses. In (5) h is the distance from the upper mass of the system and $h_G = L \cos^2 \phi$ the position of the system center of mass G . The mass angle ϕ belongs to the interval $\mathcal{I} = [\phi_{min}, \phi_{max}]$ where

$$\phi_{min} = \arcsin(\sqrt{\frac{\Lambda_t}{2}}), \quad (m_2 = 0),$$

$$\phi_{max} = \arccos(\sqrt{\frac{\Lambda_t}{2}}), \quad (m_1 = 0)$$

Figure 2: Mass distribution of the system

If $\Lambda_t = 0$, $\mathcal{I} = [0, \pi/2]$. For fixed values of Λ_t and m , when ϕ covers the interval $\mathcal{I}$ from ϕ_{min} to ϕ_{max} , mass is re-distributed from the lower particle to the upper one. The total mass m does not change in this process. If both end masses are equals, $\phi = \pi/4$.

SUMMARY OF PREVIOUS STABILITY ANALYSES

If $\varepsilon = 0$, eqs. (1-2) exhibit stable equilibrium positions, $(\theta = 0, \pi, \varphi = 0)$, with the tether aligned along the local vertical. But, if $\varepsilon \neq 0$, instead of equilibrium positions they exhibit *periodic solutions* with the orbital period $(2\pi$ in nondimensional variables). They have been analyzed in Refs. [8, 7] using asymptotic techniques in the limit $\varepsilon \rightarrow 0$ and also in Refs. [11, 15] for ε of order unity. For each pair of values (ε, i) , the analysis yields: *i*) a special 2π -periodic solution which reduces to the equilibrium position $(\theta = \varphi = 0)$ when $\varepsilon = 0$, *ii*) their monodromy matrix, and *iii*) the moduli of its eigenvalues, which give the stability properties of the orbit. Figure 3 shows the periodic solution when $i = 25^\circ$ for different values of ε . The monodromy matrix eigenvalues are two couples of complex conjugate numbers, whose moduli are given, to order ε^3 , by:

$$|\lambda_{1,2}| = 1 + \frac{\pi}{9} \cos i \sin^2 i \varepsilon^3 + O(\varepsilon^4), \quad \text{if } \varepsilon > 0 \Rightarrow |\lambda_{1,2}| > 1$$

$$|\lambda_{3,4}| = 1 - \frac{\pi}{9} \cos i \sin^2 i \varepsilon^3 + O(\varepsilon^4), \quad \text{if } \varepsilon < 0 \Rightarrow |\lambda_{3,4}| > 1$$

Figure 3: Periodic solutions given by the asymptotic (a) and the numeric (b) analysis, for $i = 25^\circ$ and different values of ε : 0.47324, 0.70986, 1.18310, 1.65634, 1.89296 and 1.95212

As a consequence, **without damping or control the periodic orbit is always unstable**. The Lorentz torque about G , $\propto \varepsilon$, gives place to a periodic excitation of the equations (1-2). A non-linear resonance 1:2 is produced then, and the system becomes unstable due to a mechanism which ends by pumping a net flux of energy into the system. However, for small values of ε the instability is weak because the difference

$$f_g = ||\lambda| - 1| = \frac{\pi}{9} |\cos i| \sin^2 i \varepsilon^3 + O(\varepsilon^4) \quad (8)$$

is small. Note that any initial deviation Δ from the basic 2π -periodic solution is roughly multiply, in each orbit, by the growth factor f_g , that is, after n orbits, the initial deviation becomes, approximately, $(1 + f_g)^n \Delta \approx (1 + n f_g) \Delta$. The growth factor (8) exhibits a cubic dependence on ε which must be underlined. If, for example, ε decreases by a factor two, the instability declines by a factor eight. Thus, a design goal of a given electrodynamic tether would be to reach values of ε as small as possible, within the limits imposed by the system where the tether will operate.

Assuming ε constant and independent of i , $f_g \propto \cos i \sin^2 i$, the instability mechanism grows with i and reaches a maximum at $i = \arctan \sqrt{2} \simeq 54.74$ deg. For actual tethers, however, ε depends on the inclination i and this result is not realistic.

TETHER CURRENT PROFILE

To describe the electrodynamic performances of the bare tethers, we will follow closely the analysis of [6]. If the diameter of the tether is smaller than the Debye length the electron collection takes place in the Orbital Motion Limited regime (OML) (see [5]). There are two tether segments; in the first one (length L_B) the tether collects electrons and in the second one (length $L - L_B$) collects ions. In the deorbiting regime of the tether, the governing equations for the

Figure 4: Bare tether

electron collection are:

$$\underline{h < L_B} : \quad \frac{dI_e}{dh} = en_\infty \frac{p}{\pi} \sqrt{\frac{2e}{m_e}} \Phi \quad (9)$$

$$\frac{d\Phi}{dh} = \frac{I_e}{\sigma A_t} - E_m \quad (10)$$

$$\underline{h > L_B} : \quad \frac{dI_e}{dh} = -en_\infty \frac{p}{\pi} \sqrt{\frac{2e}{m_e}} |\Phi| \mu (1 + \gamma_1 |\Phi|) \quad (11)$$

$$\frac{d\Phi}{dh} = \frac{I_e}{\sigma A_t} - E_m \quad (12)$$

Here, the tether current profile, $I_e(h)$, and the potential drop between tether and plasma, $\Phi = V_t - V_p$, are the dependent variables. The other parameters are: σ electrical conductivity, e electron charge, p perimeter of the tether cross-section, A_t tether cross-section of the conductive part, γ_1 secondary emission yield, m_e mass of the electron, m_i mass of the ions, $\mu = \sqrt{m_e/m_i}$, E_m the induced electric field and n_∞ the ionospheric plasma density. Note that, the external fields provide the free parameters E_m and n_∞ which are the most important drivers of the electron collection process.

Equations (9 - 12) should be integrated together with the boundary and initial conditions (13 - 15):

$$\text{at } h = 0 : I_e = 0 \quad (13)$$

$$\text{at } h = L : I_e = I_C \quad (14)$$

$$\text{at } h = L_B : \Phi = 0, I_e = I_B \quad (15)$$

In these equations, L_B , I_B and I_C are unknowns and they should be obtained as a part of the solution. Assuming that I_C is known, the boundary value problem (9-15) turns out to be mathematically closed and it provides the profiles $I_e(h)$, $\Phi(h)$ and the parameters L_B and I_B . However, I_C is unknown and therefore we need one additional relation in order to determine this value: *the tether circuit equation* which is considered in what follows.

The tether circuit equation

The cathodic contactor at the lower end of the tether adjust the tether potential to the plasma potential. Let V_{cc} be the potential drop in the cathodic contactor which includes two contributions: *i)* the potential drop due the contact impedance for the plasma contactor and ionosphere and *ii)* the wave radiation losses. In addition, let Z_T be any interposed load at the cathodic end of the tether. Figure 5 is an sketch of the tether profiles in the ideal case in which $V_{cc} = 0$.

Figure 5: Tether current and potential profiles

The tether circuit equation, see figure 5, turns out to be:

$$V_{cc} + I_C Z_T + \Delta V_{BC} = E_m(L - L_B) \quad (16)$$

where ΔV_{BC} , the potential drop due to ohmics effects along the BC segment of the tether, is given by

$$\Delta V_{BC} = \frac{1}{\sigma A_t} \int_{L_B}^L I_e(h) dh \quad (17)$$

The equation (16) is the additional relation that we need to close mathematically the boundary value problem (9-15).

Non-dimensional equations

To integrate the equations the following **non-dimensional variable** will be introduced:

$$\begin{aligned} \xi = \frac{h}{L_*} \in [0, \ell_t], \quad \ell_t = \frac{L}{L_*}, \quad L_* = \frac{(m_e E_m)^{1/3}}{e^{2/3}} (3\pi \frac{\sigma h_t}{n_\infty})^{2/3} \\ I_e = I_{sc} i_e(\xi), \quad \varphi_e = \Phi / (E_m L_*), \quad I_{sc} = \sigma E_m A_t \end{aligned}$$

where L_* is a characteristic length typical of the bare tethers, I_{sc} is the short circuit current and $h_t = 2A_t/p$ is a characteristic transversal length (for a cylindrical tether coincides with the radius of the wire).

In nondimensional variables equations (9-12) take the form

$$(\xi \leq \xi_B) \quad \frac{di_e}{d\xi} = \frac{3}{4} \sqrt{\varphi_e} \quad (18)$$

$$(\xi \geq \xi_B) \quad \frac{di_e}{d\xi} = -\frac{3}{4} \mu \sqrt{|\varphi_e|} \left(1 + \frac{\delta}{\ell_t} |\varphi_e| \right) \quad (19)$$

$$\frac{d\varphi_e}{d\xi} = i_e - 1 \quad (20)$$

where $\xi_B = L_B/L_*$ and $\delta = \gamma_1 E_m L$ is associated with the secondary emission. In non-dimensional variables, the boundary conditions take the form

$$\xi = 0 : \quad i_e = 0 \quad (21)$$

$$\xi = \xi_B : \quad \varphi_e = 0, \quad i_e = i_B \quad (22)$$

$$\xi = \ell_t : \quad i_e = i_C \quad (23)$$

The potential drop ΔV_{BC} can be written as

$$\frac{\Delta V_{BC}}{E_m L_*} = \int_{\xi_B}^{\ell_t} i_e(\xi) d\xi = \int_{\xi_B}^{\ell_t} \left(1 + \frac{d\varphi_e}{d\xi} \right) d\xi = \ell_t - \xi_B + \varphi_C$$

and the tether circuit equation (16) takes the following non-dimensional form

$$(\Omega i_C + \tilde{V}_{cc}) \ell_t + \varphi_C = 0 \quad (24)$$

where $\tilde{V}_{cc} = V_{cc}/(E_m L)$ is the non-dimensional form of V_{cc} and $\Omega = Z_T/R_T$ is the non-dimensional form of the interposed load Z_T (here, $R_T = L/(\sigma A_t)$ is the electrical resistance of the tether).

Thus, the determination of the tether current profile requires to solve the boundary value problem given by (18-24). The problem must be tackle numerically to obtain:

- the profiles $i_e = i_e(\xi; \ell_t, \Omega, \mu, \tilde{V}_{cc}, \delta)$ and $\varphi_e = \varphi_e(\xi; \ell_t, \Omega, \mu, \tilde{V}_{cc}, \delta)$
- the maximum current $i_B = i_B(\ell_t, \Omega, \mu, \tilde{V}_{cc}, \delta)$

- the current at the cathodic end $i_C = i_C(\ell_t, \Omega, \mu, \tilde{V}_{cc}, \delta)$
- the distance of the tether anodic segment $\xi_B = \xi_B(\ell_t, \Omega, \mu, \tilde{V}_{cc}, \delta)$

The details on the integration of the equations (18-24), which do not pose significant problems, can be found in Ref. [4].

Notice that, in a bare tether appropriately tuned to be an effective deorbiter, the influence of the secondary emission is very small. Therefore the term $\frac{\delta}{\ell_t} |\varphi_e|$ which appears in the right hand side of (19) can be neglected. We will go back on this point later on. The value of the parameter $\mu = \sqrt{m_e/m_i}$ is small and it depends on the presence of the ions of the different species. In the Earth, the most abundant ion in the ionosphere is the atomic oxygen O^+ for which $\mu \approx 1/172$. At present, the more promising cathodic contactor are hollow-cathode devices. For them the potential drop V_{cc} takes values in the range 15-30 V, that is, very small compared with the voltage $E_m L$ which can be expected in a tether several kilometers long. As consequence, we will take $\delta = 0$, $\mu = 1/172$ and $\tilde{V}_{cc} = 0$ in the analysis what follows.

Balance condition

Introducing the non-dimensional variables, the integral J_1 defined in (5) becomes

$$\frac{J_1}{\sigma E_m A_t L_*^2} = \int_0^{\ell_t} (\ell_t \cos^2 \phi - \xi) i_e(\xi) d\xi \quad (25)$$

and the parameter ε can be written as

$$\varepsilon = \varepsilon_0 \cdot \hat{f} \quad (26)$$

where

$$\varepsilon_0 = \frac{E_m}{L} \frac{12\Lambda_t}{(3\sin^2 2\phi - 2\Lambda_t)} \cdot \frac{\mu_m}{\mu_E} \frac{\sigma}{\rho_v}, \quad (27)$$

$$\hat{f} = \int_0^{\ell_t} (\cos^2 \phi - \frac{\xi}{\ell_t}) i_e(\xi) \frac{d\xi}{\ell_t}. \quad (28)$$

This way ε has been split in two factors: ε_0 and $\hat{f}$. Apart from the ratio (μ_m/μ_E) , the factor ε_0 depends on the tether material (σ/ρ_v), (ρ_v is the material density), the mass distribution (through ϕ and Λ_t) and the ratio E_m/L . It takes values of order unity, except when ϕ is close to the ends of the interval $\mathcal{I} = [\phi_{min}, \phi_{max}]$. As example, let us consider a tether 20 Km long made of a tape with the following dimensions

$$\begin{aligned} t &= 0.18 \text{ mm (thickness)} \\ p &\simeq 24 \text{ mm (perimeter)} \end{aligned} \quad (29)$$

Figure 6 shows ε_0 versus ϕ for several values of Λ_t . In this plot the following values

$$\begin{aligned} \sigma &= 3.5 \cdot 10^7 \text{ } \Omega^{-1}\text{m}^{-1} \\ \rho_v &= 2700 \text{ Kg/m}^3 \\ E_m &= 165 \text{ V/Km} \end{aligned}$$

have been used. They correspond to a tether made of aluminum orbiting in LEO inclined 25° . However, different configurations do not introduce substantial changes in the values of ε_0 , which remains of order unity.

Figure 6: Parameter ε_0

The Lorentz torque and the parameter ε depend on the ionospheric plasma density n_∞ . This dependence takes place essentially through the parameter $\hat{f} = \hat{f}(\phi, i_e(\xi))$ which should be calculated numerically together with the current profile $i_e(\xi)$. Taking into account the non-dimensional equations (18-20) the parameter $\hat{f}$ can be rewritten as

$$\hat{f} = \cos^2 \phi \frac{U_1(\ell_t, \Omega)}{\ell_t} - \frac{U_2(\ell_t, \Omega)}{\ell_t^2}$$

where $U_1(\ell_t, \Omega)$ and $U_2(\ell_t, \Omega)$ are defined by the following integrals

$$U_1(\ell_t, \Omega) = \int_0^{\ell_t} i_e(\xi) d\xi, \quad U_2(\ell_t, \Omega) = \int_0^{\ell_t} \xi i_e(\xi) d\xi \quad (30)$$

Therefore, $\hat{f}$ is a function $\hat{f} = \hat{f}(\phi, \ell_t, \Omega)$ which must be determined for the different regimes appearing in the problem.

The balance condition takes the simple form $\hat{f} = 0$ and leads to the following mass angle

$$\cos \phi^* = \sqrt{\frac{U_2(\ell_t, \Omega)}{\ell_t U_1(\ell_t, \Omega)}} \quad (31)$$

assuring the Lorentz torque vanishes.

Power

For a bare tether working in the generator regime there are two significant parameters related to the power developed by the electrodynamic forces acting on the system:

- the power dissipated in the whole system, $\dot{W}_d$
- the power generated in the interposed load Z_T , $\dot{W}_g$

The first one gives the loss of mechanical energy associated with the de-orbiting process. The second one is the energy that can be used on board, i.e. on the orbiter, to perform some task.

The electrodynamic force acting on a tether element and the power dissipated by this force are

$$\vec{f}_e^{dh} = I_e(h) \vec{u} \times \vec{B} dh, \quad \dot{W}_d^{dh} = \vec{v}_G \cdot (\vec{u} \times \vec{B}) I_e(h) dh$$

where, in approximation, the velocity of the element is assumed to be equal to the velocity $\vec{v}_G$ of the system center of mass. The overall dissipated power will be

$$\dot{W}_d = \vec{v}_G \cdot (\vec{u} \times \vec{B}) \int_0^L I_e(h) dh = -[\vec{u}, \vec{v}_G, \vec{B}] \int_0^L I_e(h) dh = E_m \int_0^L I_e(h) dh$$

because the component of the electric field induced by the Earth magnetic field along the tether direction is given by

$$E_m = \vec{u} \cdot (\vec{v}_G \times \vec{B}) = [\vec{u}, \vec{v}_G, \vec{B}]$$

In terms of the non-dimensional variables it takes the form:

$$\dot{W}_d = -E_m L I_{sc} \frac{1}{\ell_t} \int_0^{\ell_t} i_e(\xi) d\xi = -E_m L I_{sc} \frac{U_1(\ell_t, \Omega)}{\ell_t}$$

where $U_1(\ell_t, \Omega)$ is defined in (30) and can be calculated as follows

$$U_1(\ell_t, \Omega) = \int_0^{\ell_t} i_e(\xi) d\xi = \int_0^{\ell_t} \left(1 + \frac{d\varphi_e}{d\xi}\right) d\xi = \ell_t - \varphi_A + \varphi_C$$

In non-dimensional terms we have

$$\frac{\dot{W}_d}{E_m L I_{sc}} = -\frac{U_1(\ell_t, \Omega)}{\ell_t} = -\left\{1 - \frac{\varphi_A - \varphi_C}{\ell_t}\right\}, \quad \eta_t = \left\{1 - \frac{\varphi_A - \varphi_C}{\ell_t}\right\} \quad (32)$$

and the last parameter η_t can be taken as a good measure of the tether efficiency as deorbiter.

The power generated in the interposed load is given by

$$\dot{W}_g = I_C^2 Z_T = E_m L I_{sc} i_C^2 \Omega$$

and taking into account the tether circuit equation (24) can be written as

$$\frac{\dot{W}_g}{E_m L I_{sc}} = i_C \left(\frac{|\varphi_C|}{\ell_t} - \tilde{V}_{cc} \right) \approx \frac{i_C |\varphi_C|}{\ell_t} \quad (33)$$

DEORBITING IN THE IDEAL CASE

For deorbiting applications the interposed load Z_T must be as small as possible in order to minimize the power generated (33) which is not interesting. The limit in which $Z_T = 0$ is denominated the *ideal case* and it has been analyzed in [6]. In this section we will summarize the main characteristics of this regime following closely the analysis of [6] and assuming $\tilde{V}_{cc} = \delta = 0$.

Since μ is small, equations (19-23) for $\xi > \xi_B$ have the solution $\varphi_e(\xi) \approx -(1 - i_B)(\xi - \xi_B)$, $i_e(\xi) \approx i_B \approx i_C$. Thus, the final value φ_C turns out to be $\varphi_C = -(1 - i_B)(\ell_t - \xi_B)$. However, when $\Omega = 0$ the tether circuit equation (24) provides $\varphi_C = 0$ and this condition holds in only two different ways: either $i_B = 1$ or $\ell_t = \xi_B$. They give place to two different regimes

- if $\ell_t < 4$ the condition $\varphi_C = 0$ fulfils with $\ell_t = \xi_B$ and this is the *short tether regime*
- if $\ell_t \geq 4$ the condition $\varphi_C = 0$ fulfils with $i_B = 1$ and this is the *long tether regime*

Short tether regime

In the short tether regime the segment BC disappears, i.e., the whole tether is inside the anodic segment AB . Therefore, in this regime the relation $\ell_t = \xi_B$ is always satisfied.

Figure 7 shows, on the left side, the current and the potential profiles of the tether for different values of the maximum tether current $i_B = 0.25, 0.5, 0.85, 1.0$.

The integrals U_1 and U_2 defined in (30) must be extended to the interval $[0, \xi_B]$ and take the following values

$$U_1(\ell_t, 0) = \ell_t - \varphi_A = \xi_B - [i_B(2 - i_B)]^{\frac{2}{3}}$$

$$U_2(\ell_t, 0) = \frac{1}{2}\xi_B^2 - \frac{4}{3}(1 - i_B)^{\frac{5}{3}} \int_0^{v_0} \sinh^{\frac{5}{3}} u \, du$$

where v_0 is defined by the relation $\cosh v_0 = (1 - i_B)^{-1}$. Since $\xi_B = \xi_B(i_B)$ both parameters only depends on i_B . Notice that the secondary emission does not take place because there is no cathodic segment in the tether. The power dissipated by the system in this regime is

$$\frac{\dot{W}_d}{E_m L I_{sc}} = -\left\{1 - \frac{[i_B(2 - i_B)]^{\frac{2}{3}}}{\xi_B}\right\}$$

and is also a function of i_B .

The upper extreme of this segment appears for $\ell_t = \xi_B = 4$, $i_B = 1$ where the transition to the long tether regime takes place.

Figure 7: Current and potential profiles of the tether. *Short tether regime* in the left, *long tether regime* in the right

Long tether regime

In the long tether regime $i_B = 1$ ($\xi_B = 4$) and the solution for the cathodic segment, $\xi > \xi_B$, is trivial: $\varphi_e(\xi) \equiv 0$ and $i_e(\xi) \equiv 1$.

Figure 7 shows, on the right side, the current and the potential profiles of the tether for a given value of the tether length: $\ell_t = 10$. For different values of $\ell_t > 4$ the picture are the same with only one difference: the extension of the cathodic segment.

The integrals U_1 and U_2 defined in (30) must be extended now to the whole tether $[0, \ell_t]$ and take the following values

$$U_1(\ell_t, 0) = \ell_t - 1$$

$$U_2(\ell_t, 0) = \frac{36}{5} + \frac{1}{2}(\ell_t^2 - 16) = \frac{1}{2}(\ell_t^2 - \frac{8}{5})$$

The power dissipated by the system in this regime is

$$\frac{\dot{W}_d}{E_m L I_{sc}} = -\{1 - \frac{1}{\ell_t}\}$$

and is only a function of ℓ_t .

Note that the secondary emission does not take place because in the cathodic segment the potential drop between tether and plasma vanishes ($\varphi_e \equiv 0$). Therefore the assumption $\delta = 0$ is completely justified when the tether is mainly used as deorbiter.

Deorbiting tether efficiency

The parameter η_t defined in (32) is a good measure of the tether efficiency as deorbiter. The expressions obtained for the short and long regimes permit to plot this parameter as a function of the tether length ℓ_t . Figure 8 shows this plot. Note that:

- the tether efficiency in the *long tether regime* extends down until $\ell_t \approx 2$, approximately
- the efficiency grows clearly with the tether length ℓ_t

Figure 8: Dissipated power vs. ℓ_t : (1) *long tether regime*, (2) *short tether regime*

Balance condition

The critical value of the mass angle ϕ as given for (31) is only function, in this ideal case, of the tether length ℓ_t . Figure 9 shows the plot of this function taking into account the expressions obtained above for the functions U_1 and U_2 .

Figure 9: Mass angle critical value ϕ^* versus ℓ_t : (1) *long tether regime*, (2) *short tether regime*

Figure 8 shows that the performances of the tether increase with the non dimensional tether length ℓ_t . But $\ell_t \propto n_\infty^{\frac{2}{3}}$ when the other parameters take constant values. Typically the ionospheric plasma density n_∞ undergoes changes along the orbit with variations of one order of magnitude. Thus, along the orbit, the ratio between the maximum and minimum values of ℓ_t is about $10^{2/3} \approx 5$. From the point of view of the effectiveness the tether must be designed to operate, approximately, in the range $\ell_t \in [3, 15]$; the values of η_t would range in the interval $[0.7, 0.95]$, in approximation. Figure 9 shows that the critical mass angle which keeps the tether balanced takes values in the range $[38.2^\circ, 43.2^\circ]$, that is, there are 5° of variation between the interval ends.

If we choose to balance the tether for the particular value $\ell_t \approx 6$, that is, $\phi \approx 40.7^\circ$ in the middle of the range, the tether will be unbalanced during almost the whole orbit. However, the magnitude of this unbalanced situation can be described with the help of the parameter $\hat{f}$ as defined in (28), taking $\cos \phi = \cos \phi^* \approx 0.758$. Figure 10 shows the variation of $\hat{f}$ with ℓ_t . Note that for the case analyzed here $\hat{f} \in [-0.03, 0.04]$, and considering $\varepsilon_0 \approx 1$, in approximation, (ε_0 is of order unity), the value of ε oscillates in the same interval $[-0.03, 0.04]$. The growth factor f_g given by (8) is, for $i = 45^\circ$, $f_g \approx 8 \cdot 10^{-6}$. The analysis associated with the growth factor f_g can not be directly applied because ε is not constant. However, assuming ε constant and in the worse case $\hat{f} = 0.04$, any initial deviation from the periodic solution takes a long time to double (≈ 10 years in LEO).

Figure 10: Values of $\hat{f}$ vs. ℓ_t : (1) *long tether regime*, (2) *short tether regime*

This way, during the tether mission the Lorentz torque will remain close to zero and the effects of the dynamic instability have been dropped. Simultaneously, the deorbiting effectiveness of the tether will be kept in high values. Obviously, it will be slightly lower than the effectiveness shown in figure 8 due to the librational motion of the tether which will be excited by the Lorentz torque. However, the tether could be balanced with more accuracy if we sacrifice some efficiency, as we show in the next section. Notice that the strategy to choose the critical mass angle ϕ^* would be optimized in several ways. For example, adjusting its value to the averaged expected values of the ℓ_t along the orbit.

DEORBITING IN THE GENERAL CASE

Some authors distinguish between the deorbiting mode, $\Omega = 0$ or very small, and the generator mode, $\Omega \approx \mathcal{O}(1)$. In the first case the tether is tuned to maximize the orbital decay and, in the second one, to maximize the power generated in the interposed load. In this section we assume $\Omega \neq 0$ but small enough to permit the tether works close to the deorbiting regime. As before, we will take

$\delta = 0$, $\mu = 1/172$ and $\tilde{V}_{cc} = 0$ in the analysis what follows.

The problem (18-24) must be solved numerically to obtain the profiles of current and potential along the tether

$$i_e = i_e(\xi; \ell_t, \Omega), \quad \varphi_e = \varphi_e(\xi; \ell_t, \Omega)$$

Figure 11: Profiles of current and potential for $\ell_t = 6$ and $\Omega = 0.0, 0.05$ and 0.10

Moreover, and as a part of the solution, the analysis provides the additional relationships

$$i_B = i_B(\ell_t, \Omega), \quad i_C = i_C(\ell_t, \Omega) \quad (34)$$

$$\xi_B = \xi_B(\ell_t, \Omega), \quad \eta_t = \eta_t(\ell_t, \Omega) \quad (35)$$

Figure 11 shows the tether current $i_e(\xi)$ and tether potential $V_t(\xi) = \varphi_e(\xi) + \xi$ profiles along the tether, for the value $\ell_t = 6$ which has been selected arbitrarily, and for three values of $\Omega = 0.0, 0.05$ and 0.1 . As it should be expected, $i_B - i_C \ll 1$, because the ion collection has very small effects in the neighborhood of the deorbiting ideal case. Moreover, the tether current profile shows that small variations of Ω provides a clear shortening of the anodic segment length (it passes from 4 to 2.5 when Ω changes from 0 to 0.05).

The draw of the potential profile helps us to understand why the length ξ_B of the anodic segment is so affected for small variation of the interposed load Ω . Notice that, for the ideal case $\Omega = 0$, the

tether potential profile is tangent to the plasma potential line at $\xi = \xi_B$ ($\xi_B = 4$ in the figure) and coincides with the plasma potential line for $\xi_B \leq \xi \leq \ell_t$. When $\Omega \neq 0$, the tether potential *crosses* the plasma potential line at $\xi = \xi_B$ and this point move down drastically. However, the deorbiting efficiency of the tether as defined in (32), is not too much affected by this behaviour, as we will show next.

Figure 12: Functions i_B and i_C (upper pictures), anodic segment ξ_B and deorbiting tether efficiency η_t (lower pictures) vs. ℓ_t for different values of $\Omega = 0, 0.01, 0.02, \dots, 0.1, 0.2$

Figure 12 shows the plots of the additional relationships (34-35) taking ℓ_t as independent variable and Ω as parameter. Apart from the ideal case, $\Omega = 0$, in the figure the following values of Ω have been considered: $\Omega = 0.01, 0.02, \dots, 0.1$ and $\Omega = 0.2$.

We can see in that figure, how the introduction of a non vanishing value for Ω separates the solution from the ideal case described in the previous section. This separation, however, is small for the small values of Ω considered here, as it should be expected. The most affected parameter is the length ξ_B of the anodic segment, as we said above, which changes drastically even for very small values of Ω . This drastic change is clearly noticeable in the corresponding plot. Notice that the deorbiting efficiency of the tether, η_t , decreases clearly with Ω , but it remains not too far away from the ideal case.

The balance condition (31) provides the critical value $\phi^* = \phi^*(\ell_t, \Omega)$. Figure 13 shows this function taking ℓ_t as independent variable and Ω as parameter. It is noticeable that for increasing values of Ω the mass angle that balance the tether increases, but for small values of Ω the differences are small.

An important point from figure 13: it is clear that the tether cannot be balanced keeping Ω constant. The change of the ionospheric plasma density n_∞ along the orbit induces the change of the non dimensional tether length ℓ_t and as consequence the critical value ϕ^* will change with ℓ_t . The

same problem appeared in the deorbiting ideal case described previously. However, we can improve the results that we obtained there by means of the following procedure:

- we select a critical value for the mass angle, ϕ_0^* , corresponding to a typical value of ℓ_t (for example, the average value of ℓ_t along the center of mass orbit, or the upper value expected in the orbit, would be appropriated)
- equation (31) provides then a relation

$$\cos \phi_0^* - \sqrt{\frac{U_2(\ell_t, \Omega)}{\ell_t U_1(\ell_t, \Omega)}} = 0$$

that gives, for each value of ℓ_t , the value of Ω which keeps the tether balanced:

$$\Omega^* = \Omega^*(\phi_0^*, \ell_t) \quad (36)$$

- the above relationship can be extended to any range $[a, b]$ of values of ℓ_t selecting appropriately the value of ϕ_0^*

Figure 13: Balance condition for different values of Ω

To fix ideas, we will develop an small example. Let us assume that $\phi_0^* = 42^\circ$. Moreover, we will choose to keep the tether balanced in the segment $\ell_t \in [a, b] = [6.18, 8.83]$. This way, the function $\Omega^* = \Omega^*(\phi_0^*, \ell_t)$ defined in (36) takes the form:

$$\Omega^*(\phi_0^*, \ell_t) = \begin{cases} \ell_t < 6.18 & \Rightarrow \Omega^* = 0.2 \\ 6.18 < \ell_t < 8.83 & \Rightarrow \Omega^* = 0.00122\ell_t^4 - 0.03845\ell_t^3 + 0.47\ell_t^2 - 2.69168\ell_t + 6.18134 \\ \ell_t > 8.83 & \Rightarrow \Omega^* = 0.0 \end{cases} \quad (37)$$

The expression appearing in the right hand side of (37) has been fitted from the numerical results by means of the least square method.

Thus, the tether will be exactly balanced in the segment $\ell_t \in [a, b] = [6.18, 8.83]$ but, outside of this segment, the tether is unbalanced. In order to have a measure of this unbalanced situation we can plot the parameter $\hat{f}$ versus ℓ_t using the law (37):

$$\hat{f} = \cos^2 \phi_0^* \frac{U_1(\ell_t, \Omega^*)}{\ell_t} - \frac{U_2(\ell_t, \Omega^*)}{\ell_t^2}$$

Figure 14 shows the plot of $\hat{f}$ versus ℓ_t for a tether balanced by means of the law (37). Comparing with the situation described in the ideal case, figure 10, two advantages are clearly noticeable

- when ℓ_t ranges in the interval $[3, 15]$, the extreme values of $\hat{f}$, $[-0.03, 0.04]$ in the ideal case, are substantially decreased: now they are, in approximation, $[-0.02, 0.02]$
- along the segment $[a = 6.18, b = 8.83]$ the tether is *exactly* balanced and the Lorentz torque is exactly zero

Figure 14: $\hat{f}$ vs. ℓ_t for the balanced tether

However, to stabilize the attitude motion of the system is not for free. We have to pay a price in the deorbiting tether efficiency. For the example that we are developing here the drag efficiency of the tether can be calculated as a function of the non dimensional tether length ℓ_t . Figure 15 shows the deorbiting tether efficiency η_t as a function of ℓ_t for the law (37). The upper line corresponds to the ideal case shown in figure 8; the lower line corresponds to the example. Both lines coincide for $\ell_t > b = 8.83$, but for $\ell_t < b$ some efficiency is lost.

Figure 15: η_t vs. ℓ_t for the balanced tether

Notice that the values ϕ_0^* and $[a, b]$ must be tuned for each particular mission, because the requirements on the level of dynamic stability of the tether are quite different in different missions.

CONCLUSIONS

This paper shows how to balance a bare electrodynamic tether operated in a circular inclined orbit which is working in the generator regime. For this kind of tethers, the balance condition depends on

- the geometric and material characteristics of the tether
- the external fields and orbital characteristics
- the system mass distribution
- the interpose load

When the tether is tuned to have good performances as deorbiter the interposed load is zero and the tether deorbiting efficiency can be maximized according to the missions requirements. In this deorbiting regime, to balance the tether implies to choose a mass distribution which can be selected to minimize the Lorentz torque. However, the large variation of the ionospheric plasma density along the orbit prevent to get zero Lorentz torque continually. If desired, the mass distribution can be tuned to get a Lorentz torque whose averaged value along the orbit be zero. With this design, the system improves its dynamic stability in such a way that for some missions requirements becomes an excellent solution to gain control on the attitude motion. However, there are more possibilities that have not been considered in this paper; they open wide range of solutions to be explored in the future.

For mission with stronger requirements on the dynamic stability of the attitude dynamic we offer a more sophisticated solution. Using a small interposed load and sacrificing some of the deorbiting tether efficiency it is possible to introduce a control system which permits to exactly cancel the Lorentz torque for a wide interval of ionospheric plasma densities. Some trade-off between the gained dynamic stability and the deorbiting efficiency lost should be reached for each mission in order to optimize the procedure.

Finally, we should underline that the real implementation of the control procedure that we propose in this pages is very simple. With the help of a potentiometer appropriately added to the tether circuit, the value of the interposed load can be readily controlled. The open control loop can be implemented measuring: 1) the tether current at the cathodic end (a simple ammeter), and 2) the ionospheric plasma density (with a cylindrical Langmuir probe).

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