UNIVERSIDAD POLITÉCNICA DE MADRID

**ESCUELA TÉCNICA SUPERIOR
DE INGENIEROS DE TELECOMUNICACIÓN**

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**DESIGN OF MULTICHANNEL FREEFORM OPTICAL
SYSTEMS FOR IMAGING APPLICATIONS**

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RESUMEN

El problema de la formación de imagen perfecta de un objeto es muy antiguo y se ha desarrollado a lo largo de los años, desde la formulación paraxial de Newton hasta las técnicas recientes de optimización multiparamétrica. Esta evolución ha sido constantemente apoyada a través de desarrollos de las nuevas técnicas de fabricación e impulsada por las demandas impuestas por otros campos, en particular los campos derivados de las necesidades del consumidor. Las soluciones iniciales de estos problemas de formación de imagen fueron elegantes y en su momento encajaban con las necesidades de campos específicos Sin embargo, con la creciente necesidad de hacer que los sistemas resultantes fuesen cada vez más y más compactos, con diseños parecidos a los de los dispositivos comúnmente utilizados, ha llegado el momento en que las soluciones triviales no parecían ser una opción.

Los avances en las técnicas de fabricación llevaron primero a uso de las superficies no-esféricas o asféricas para resolver los problemas comunes de formación de imagen. Por ejemplo, Schwarzschild en 1906 demostró analíticamente que dos formas asféricas son necesarias para resolver el problema de aplanatismo. Esto abrió el camino para que más sistemas empleen estas formas asféricas para llegar a soluciones elegantes. Con una complejidad creciente en los requisitos de sistemas tanto como otras limitaciones de la formación de imagen, el diseñador llegó a necesitar grados de libertad adicionales. Esto, a su vez, podría resolverse utilizando un número elevado de superficies ópticas, lo cual conduciría a soluciones mucho más voluminosas. Afortunadamente, gracias a los avances en las técnicas de moldeo por inyección, la solución parece ser el uso de las superficies anamórficas (*freeform*). Las superficies anamórficas se hicieron prácticas por primera vez con su introducción para la resolución de problemas de óptica de *nonimaging* donde las restricciones de diseño no son tan estrictas en comparación con sus contrapartes del campo de la formación de imagen, sino que era más un problema de transporte masivo.

Los sistemas anamórficos resultantes eran compactos y demostraron así la importancia en hacer los sistemas ópticos significativamente más pequeños. Esto también dio lugar a formulación de muchas técnicas de diseño de superficies anamórficas directas. El siguiente paso lógico en la comunidad óptica fue adaptar esta introducción de las superficies anamórficas a problemas de formación de imagen. El cuello de botella en esta introducción fue la indisponibilidad de sistemas que se podrían utilizar como puntos de partida. Así vino la gran dependencia de las técnicas de optimización asistidas por ordenador. Esto llevó a muchas investigaciones sobre la representación matemática de las superficies anamórficas que podrían tener consecuencias directas en las limitaciones de fabricación que son la clave para hacer que los sistemas resultantes sean más prácticos. Uno de esos métodos de diseño directo fue el método de Superficies Múltiples Simultaneas (SMS), que se introdujo inicialmente como una técnica de diseño directo en el campo de *Non-imaging*, para diseño de las superficies anamórficas. Posteriormente fue extendido a las aplicaciones de formación de imagen, donde ayudó inmensamente a los diseñadores a llegar a un buen punto de partida para la optimización posterior.

Esta tesis doctoral sigue la evolución de las superficies anamórficas y se presenta en el Capítulo 1 con una introducción básica de diseño óptico como tal, junto con las métricas utilizadas para la cuantificación del rendimiento de sistema. Este capítulo también proporciona una breve introducción de varios términos utilizados en la óptica de Realidad Virtual, que constituye el enfoque principal de capítulos posteriores.

Capítulo 2 continúa con la exploración del método SMS y su extensión para diseñar 3 frentes de onda de entrada que constituyen un plano para acoplarse con tres frentes de onda de salida que constituyen un plano. Esta extensión del método SMS puede ser utilizada como una técnica directa de diseño de sistemas ópticos anamórficos. También demostramos una forma simple y directa de derivar la condición senoidal bajo prescripción anamórfica, cuya violación resulta en sistemas con aberraciones ópticas con una dependencia de campo lineal. Después demostramos matemáticamente la conexión

entre el método SMS y un sistema clásico libre de aberración esférica y coma, conocido como “sistema aplanático”. Este trabajo es uno de los primeros que formalmente presenta el vínculo entre el método SMS y los sistemas aplanáticos anamórficos.

Capítulo 3 detalla ampliamente la formulación de sistemas aplanáticos de tres superficies utilizando la formulación de ecuaciones diferenciales y cómo, en general, tres superficies ópticas anamórficas son necesarias para obtener un aplanatismo anamórfico. Esto se consigue utilizando la condición de integralidad para demostrar que no hay una solución cuando se utilizan solo dos superficies anamórficas. Hay una notable excepción de este criterio aproximado que son los sistemas aplanáticos afocales, donde se necesitan solo dos superficies anamórficas.

Capítulo 4 introduce el concepto de ThinEyes® en el entorno de los sistemas ópticos de realidad virtual. Esta estrategia de diseño se explica detalladamente con el uso de un sistema óptico tradicional llamado “pancake”, y como el uso de la tecnología ThinEyes® puede significativamente incrementar la resolución de la imagen aparente sin a la vez comprometer otros parámetros de sistema. Además introducimos otros modelos de diseños ópticos anamórficos compactos de multicanal: (a) Diseño híbrido de dos canales refractivos-difractivos (b) Diseño anamórfico de nueve canales.

Capítulo 5 está enfocado en el diseño, desarrollo y caracterización de un sistema anamórfico de dos canales pensado para el uso en un dispositivo de realidad virtual. El capítulo detalla ampliamente las reglas de diseño y la posterior caracterización de la óptica fabricada utilizando la determinación experimental de función de distorsión (*mapping*) que se debe de corregir con el software.

ABSTRACT

The problem of coupling an object to be imaged perfectly through an optic is an ancient one and has evolved through the ages, from Newton's paraxial formulation to the very recent multi-parametric optimization techniques. This evolution has been constantly aided by developments in fabrication techniques and driven by demands posed by other fields particularly from those arising out of consumer needs and not just strictly research oriented. Initial solutions to these imaging problems were elegant and matched to the needs of specific fields at that point in time. But, with increasing demands to make the resulting systems more and more compact with form factors resembling to those of commonly used consumer devices, solving these problems with trivial solutions no longer seemed to be an option.

Advancements in manufacturing techniques first led to the use of non-aspherical or aspherical surface shapes in solving common imaging problems. For example, Schwarzschild analytically proved in 1906 that two aspherical shapes are required to solve the problem of aplanatism. This paved way for more systems employing these aspherical shapes to come up with elegant solutions. With increasing complexity in the system demands and other imaging constraints, additional degree of freedoms were needed by the designer. This could in turn be solved by using more number of surfaces leading to bulkier solutions. Thankfully, with advancements in injection moulding techniques, the use of freeform surfaces seem to be the solution. Freeform surfaces were first made practical with their introduction in solving non-imaging problems where the design constraints are not so strict when compared to their imaging counterpart but was more of a mass transport problem. The resulting freeform systems were compact and thus demonstrated the significance of freeform surfaces in miniaturising optical systems. This also led to many direct design techniques to be formulated for the design of freeform

optical surfaces. The next logical step in the optics community was to adapt this introduction of freeform surfaces into imaging problems. The bottleneck in this introduction was the unavailability of systems to be used as a starting point. Thus came the heavy reliance on computer-aided optimization techniques. This led to many investigations into the representations of freeform surfaces shapes which could have direct consequence in fabrication constraints to making the resultant systems more practically realisable. One such direct design method was the Simultaneous Multiple Surface method (SMS), which was initially introduced as a Non-Imaging direct design technique for the design of freeform optical surfaces. This was later extended to imaging applications where it immensely helped the designers in arriving at a good starting point for further optimisation.

This thesis tracks this evolution of freeform surfaces and is introduced in Chapter 1 with a basic introduction of optical design as such, along with the metrics used for the quantification of system performance. This chapter also provides a brief introduction of various terminology used with respect to virtual reality optics which forms the main focus of later chapters.

Chapter 2 continues with the exploration of the SMS method and its extension in designing three input wavefronts constituting a plane to be coupled onto three output wavefronts constituting a plane. This particular extension of the SMS method can be used as a good direct design technique in designing freeform optical systems. We also demonstrate a simple and a straight forward way of deriving the sine condition under freeform prescription, whose violation will result in systems suffering from aberrations having linear field dependency. Thereafter, we mathematically prove the connection between the SMS method and a classical system free from spherical aberration and coma known as “Aplanatic systems”. This work is one of the first to report formally, the link between SMS method and freeform aplanatic systems.

Chapter 3 details extensively with the formulation of three surface aplanatic systems through the use of differential equation formulation and how, in general, three freeform optical surfaces are needed in obtaining freeform aplanatism. This is established through the use of integrability condition to demonstrate how there is no solution when two freeform optical surfaces are used. There is a noticeable exception to this rule of thumb, namely, afocal freeform aplanatic systems where only two freeform surfaces is enough.

Chapter 4 introduces the concept of ThinEyes® in the context of virtual reality optical systems. This design strategy is explained in detail with use of a traditional pancake optic and how the use of ThinEyes® technology can significantly increase the apparent image resolution, at the same time without compromising on other system parameters. We also introduce other compact novel multichannel freeform optical design models namely: (a) Hybrid refractive-diffractive two channel freeform optical design (b) Nine fold refractive freeform optical design.

Chapter 5 focuses on the design, development and characterization of a two channel freeform optical design to be used for a virtual reality headset. The chapter details extensively, the design rules and also the subsequent characterization of the fabricated optic through experimental determination of the distortion function to be software corrected.

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Chapter 1

INTRODUCTION TO OPTICAL DESIGN AND VIRTUAL REALITY OPTICS

1.1. Introduction to Optical Design

Optical design is a wide area in optics which deals with the problem of realizing an optical surface(s) to perform a specified imaging function. In this context, imaging can be defined to be a perfect coupling of a light constituting an input wavefront to a specific output wavefront. This is of course an idealistic definition as, in practical realizations of any optical system, there is seldom perfect coupling between the input and the output wavefronts. These practical realizations lead to imperfect imaging functions caused due to the occurrence of optical aberrations. These can be seen as any deviation from ideality and the challenge now becomes more towards limiting or keeping these “deviations from idealities” to a minimum.

To understand better the challenges in optical design, it becomes necessary to understand the nature of light itself, since the ultimate end goal is to image this light from one point to another point. In general, light is usually defined as an electromagnetic wave which obeys the laws described in the Maxwell's equations with regards to its propagation. However, this electromagnetic view of light can be replaced by a much simpler geometric theory when the wavelength of light is sufficiently small enough so

that the spatial variations of the electromagnetic field are much higher than the wavelength. This theory is termed as “Geometric Optics”, and these curvilinear trajectories of light can now be referred to as “rays”. Since the trajectories of light is now referred to as rays, this theory is sometimes also referred to as “Ray optics”. Since in most practical realizations of optical systems, the above conditions are met, Geometric optic theory has taken a much significant place in the field of optical design. The condition of small wavelength is usually satisfied in most real-life optical applications where the size of optical elements is much bigger than the wavelength. That is why traditional optical system design uses geometrical optics approach as it is simpler and at the same time effective in defining the system. In the following sections, we will examine some basic definitions and concepts pertaining to geometric optics.

1.2. Introduction to Geometric optics

Since geometric optics involves the study of ray paths, it becomes important to examine what happens when a ray intercepts a boundary of different refractive indices, such as a ray impinging a lens surface from air. The basic law which lays the foundation of geometric optics is the law of refraction. Pierre de Fermat was a mathematician who formulated that light takes the path, from one point to another, which requires the least amount of time. This path length was given by the following integral [1-2]:

$$L_{opt} = \int_a^b n(x, y, z) dl \quad (1.1)$$

Where $n(x, y, z)$ is the refractive index of the media at a given point and dl is in the differential length element along the path from a to b . It is to be noted that, in this thesis, we will be assuming the use of homogeneous media, and because of this all ray paths will be straight lines. The above formulation was later experimentally validated by Wilibord Snel which can be expressed mathematically as:

$$n' \sin i' = n \sin i \quad (1.2)$$

Where n, n' are refractive indices of media before and after refraction. The incident and refracted angles are given by i and i' respectively.

Figure 1.1. Illustration of Snell's law for a simple lens configuration.

To apply the law of refraction or Snell's law as it is commonly referred to, we simply have to calculate the tangent at the point the ray meets the lens surface, which subsequently can give us the normal. The angle of refraction then can be calculated using Eq.1.2.

Another law which pertains with the interaction of light rays with a smooth reflector/mirror is the law of reflection, which states that, incident ray with an incident angle i_1 and the reflected ray with a reflected angle of i_2 are related as in Eq. 1.2 and are coplanar.

$$i_1 = i_2 \quad (1.3)$$

Another important formulation that often appears while addressing rays is the term "Wavefront". Wavefronts can be defined as surfaces which have constant optical path length. They have an important relationship with rays, which is, the rays are trajectories that are normal to this constant optical path length surface. To reiterate, if we have a point source, the wavefronts emanating from this point source will all be spheres with origin at the location of the point source. Another important result associated with two wavefronts was first formulated by Malus and later on by Dupin. This theorem was termed as "Theorem of Malus and Dupin" [3], which states that the optical path length between two wavefronts is constant. This important result is the basis of many optical

design techniques to calculate surfaces to produce perfect imaging (example, Cartesian Ovals), will be detailed later in the thesis.

1.3. Quantifying optical systems through system definitions

Optical design involved the realization of an optical system consisting of reflecting, refracting and diffracting surfaces to perform a specific imaging function. To quantitatively describe the resulting system and also to guide the designers in the pursuit of such a system, it becomes important to define and assign certain terminology to the optical system. Such a terminology assignment also helps the end user to get a quick see through of the optical system without having the need to go in depth in its analysis.

Any system consists mainly of three components, namely: (1) source (2) target and (3) the system itself. The source is, as the name suggests, consists of input parameters which needs to transformed to a desired state which are described by the target. This transformation from the source to target is done by the system. In the same manner, for an imaging system, the source now takes the role of the “object”. Object is nothing but the source of light energy, which can be described in terms of rays, waves or energy itself, depending on the theory used for its treatment, which then needs to be transformed to a suitable output. This suitability, in an imaging system context is the formation of a perfect image of the object itself. Since, in this thesis we majorly use geometric optics for the subject treatment, the light energy is described in terms of rays. This suitable transformation of the object results in the image, which is the output. The system which performs this transformation, in our context *imaging*, consists of lenses, mirrors and diffractive surfaces. The role of this thesis is to provide an in depth description of these three components of an imaging system over a wide range of applications specifically to those of Virtual Reality Headsets or Head Mounted Displays (HMDs).

Source as mentioned earlier is the input light energy, in our context described in terms of rays which pass through the optical system to be transformed into the image. Source description itself can be described in a number of ways.

Figure 1.2. Illustration of a simple lens system along with its various defining parameters.

Above figure shows a simple lens system from where various system definitions can be inferred. It can be observed that the amount of light entering into the system from the object of height Y is limited by the *Aperture stop*. This is a physical stop which can be a mechanical piece to limit the light entering the system. As it can be observed, chief ray is that ray which passes through the centre of this aperture stop to reach the final image whose height in this case is $(-Y')$. Another significant ray which defines the source is the marginal ray, which as denoted in the illustration is the edge ray that passes through the edge of the aperture stop. For system having rotational symmetry, these two rays can be used to fully represent the source. But in practise, we require five reference rays, namely, one chief ray and four marginal rays, each defining the extent of the source in two unique dimensions and passing through the respective edges of the aperture stop, are required to fully quantify the source extents. Other important quantities to specify the aperture of the system are the f-number ($f/\#$), entrance pupil diameter (EPD) and numerical aperture (NA). The entrance pupil diameter defines the beam print in the object space and is used to characterise source in ray tracing software such as CodeV®. The ratio of this EPD to the system focal length (f) is known as the f-number. This sort of specification

can be found in camera lenses. Numerical aperture or NA is a quantity used to specify the amount of light entering the system and is given by the product of image refractive index times the sine of the half-angle cone of light θ . This sort of specification can be found in microscopic objectives.

$$f / \# = \frac{f}{EPD} \quad (1.4)$$

$$NA = n' \sin \theta' \quad (1.5)$$

Another important quantifying measure for the source is Field of View (FOV), which can be either angular or spatial at a certain object distance. The commonly used angular field of view, which is the maximum object/image height or angle addressed by the system, in case of systems performing rectilinear mapping between the object and image is given by:

$$Y' = f' \tan \theta' \quad (1.6)$$

Where Y' is the image height and f' is the focal length of the system. Additionally, objects can be characterized by their spectral distribution around a central wavelength λ_0 with a bandwidth of $\Delta\lambda$. With this, most system parameters will also have a dependency on the wavelength and this nature is termed as polychromatic.

1.4. Optical aberrations in imaging

As discussed earlier, the aim of an optical imaging system is to image an object with perfect image quality. In this context, it becomes important to define what is perfect? According to Maxwell [4],

- All the rays emanating from object point O, must get imaged at image point O' after passing through the lens.

- Every portion of the object plane perpendicular to the optical axis must be imaged to a corresponding portion of the image plane perpendicular to the optical axis.
- The image height h' must deviate from the object height h by a constant multiple alone.

Figure 1.3. Illustration of the appearance of optical aberrations in a real lens.

The above three conditions gives us a brief insight into how a perfect imaging lens should perform. Deviation from any of the above listed points results in optical aberrations leading to imperfect image formation. Specifically, violation of the first condition results in image aberrations followed by a deviation in the second condition presents itself in the form of image curvature and lastly, violation of the third condition presents itself with an imperfect mapping between the object and the image plane resulting in unwanted distortion. It is to be noted that when all the above conditions are met and the optical aberrations are sufficiently small, which is to say that, all the wavefronts at the exit pupil of the system seem to be spherical, centred at the image point, then we will have perfect imaging. Even under this condition, we will still see diffraction at the limiting edge of the pupil creating what is called as the *Airy disk*.

As it can be observed from the previous section, optical aberrations arise when there is a deviation in the wavefront shape at the imaging plane from being spherical,

centred at the image point. This deviation is called as the optical path difference and is often quantified by what is called as the wavefront aberration polynomial [5], which is expressed as the difference between the real wavefront and a spherical reference wavefront.

Figure 1.4. Figure illustrating the ray coordinate assignment.

We will keep this treatment to rotationally symmetric systems, although can be extended to any system lacking inherent symmetry, a.k.a , freeform systems [6]. Under this assumption, any ray in the system can be represented using (ρ, r, θ) , and hence the wave aberration function can be also represented using the same parameters. This can be expressed explicitly in the following manner [7-9]:

$$\begin{aligned}
 W = & {}_0W_{20}\rho^2 && \text{Defocus} \\
 & + {}_1W_{11}r\rho\cos\theta && \text{Lateral image shift} \\
 & + {}_2W_{00}r^2 && \\
 & + {}_0W_{40}\rho^4 && \text{Spherical aberration} \\
 & + {}_1W_{31}r\rho^3\cos\theta && \text{Coma} \\
 & + {}_2W_{22}r^2\rho^2\cos^2\theta && \text{Astigmatism} \\
 & + {}_2W_{20}r^2\rho^2 && \text{Field curvature} \\
 & + {}_3W_{11}r^3\rho\cos\theta && \text{Distortion} \\
 & + {}_4W_{00}r^4 && \\
 & + {}_0W_{60}\rho^6 && \text{Spherical aberration} \\
 & + \dots &&
 \end{aligned}
 \quad
 \left.
 \begin{array}{l}
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 \right\}
 \begin{array}{l}
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 \\
 \text{Third order aberration} \\
 \\
 \\
 \\
 \\
 \text{Higher order aberration}
 \end{array}$$

The consequence of the occurrence of these aberrations is that it introduces displacements in the ray intersection of a real ray (aberrated) from the location of the ideal image point. This lateral distance is referred to as the transverse ray error and can be obtained through the partial derivative of the wave aberration function with respect to its pupil coordinate ρ [7].

1.4.1. Third order Seidel Aberrations

The third order aberrations presented previously, also goes by Seidel aberrations, who in 1857, assigned five primary aberration types as departure from first order theory [8]. They are illustrated in terms of ray traces below. These aberrations are also referred to as primary aberrations.

Figure 1.5. Seidel's third order aberrations – Figure source [8]. Note that the subscripts P, M, S and T refer to paraxial, marginal, sagittal and tangential respectively.

Spherical aberration is the result of non-constant dependence in the bending imparted to the rays crossing different parts of the lens at different heights from the optical axis. Thus, the magnitude of spherical aberration depends on the height of the ray in the entrance pupil and is proportional to the cube of the ray height incident on the lens.

Coma is an off – axis aberration resulting because of annular zones in the aperture having different magnifications. As a result, systems suffering from this type of aberration have spot shapes that resemble of a comet. This is observed typically in off-axis tilted systems. The presence of spherical and coma together in a system is commonly observed in microscopic objectives as well and a solution to get rid of them was proposed by Abbe [7]. This will be dealt in detail in the subsequent chapters.

Astigmatism arises in an optical system when tangential ray fans and sagittal ray fans have different foci. The magnitude of astigmatism is linearly proportional to the lens aperture and to the square of the field angle. Systems suffering from astigmatism have spot shapes resembling to that of ellipse which in direct consequence of difference in focus positions for the tangential and sagittal ray fans.

Figure 1.6. Ray trace through a positive lens to illustrate the presence of field curvature.

As seen above, an optical system made up of spherical components naturally focuses on a curved image surface as opposed to a desirable flat image surface. This departure is described through the aberration type field curvature. One simple way of removing this is to make use of positive and negative elements in the optics.

Distortion appears in systems where the magnification is a function of image height. As a consequence of this, straight lines appear curved at the very edge of the field of view. Commonly observed distortion types are pin cushion and barrel distortion observed in wide angle cameras.

Another important aberration that appears due to the wavelength dependence of refractive index of the materials used in the optics is called as Chromatic aberration. This is in direct consequence of different wavelengths of light from a single object point getting focussed at different positions on the image plane. Common ways to handle this type of aberrations is to employ colour correcting elements such as doublet type lens elements and diffractive elements.

1.5. Quantifying system performance in the presence of aberrations

It must be obvious now that the existence of a perfect imaging optical system is of seldom occurrence in practical realizations of the same. Hence, optical designers aim at realizing a system which has aberrations down to a minimum level and it becomes significant in this context to assign some performance metrics to quantify the occurrence of the aberration types described previously. This thesis will majorly use the following three performance metrics:

1.5.1. Root Mean Square Spot size

The root mean square spot radius / diameter is a standard metric that is used by modern optical designers. This is a common option available in most ray tracing softwares, where for each object point, a specific number of rays is traced to image it at the imaging plane. The deviation of these real ray hits at the imaging point from the paraxial image point is computed as [10]:

$$\sigma^2 = \frac{\sum_{\lambda} \left\{ W_{\lambda} \iint \left[(x(\lambda; x_p, y_p) - \bar{x})^2 + (y(\lambda; x_p, y_p) - \bar{y})^2 \right] dx_p dy_p \right\}}{\sum_{\lambda} W_{\lambda}} \quad (1.7)$$

Where $x(\lambda; x_p, y_p)$ is the image plane x coordinate of the ray, traced at wavelength λ , that passes through the pupil point (x_p, y_p) , and $y(\lambda; x_p, y_p)$ is the image plane y coordinate of the ray, traced at wavelength λ , that passes through the pupil point (x_p, y_p) . Also, $\bar{x}, \bar{y}$ are the x and y centroids of the rays at the image plane at wavelength λ .

1.5.2. Ray aberration curves

The ray aberration curves or plot gives an insightful feel for how the tangential (y) and sagittal (x) ray fan behave relative to the chief ray in the image plane. A 2D grid of rays are traced for different ray heights along the pupil and their deviation from the chief ray position is plotted. This kind of a representation helps the designer in knowing the type of aberration present in the system by looking at the shapes of these plots. Commonly occurring Seidel aberrations present themselves as follows:

Figure 1.7. Ray aberration plots for third order aberrations. Image source [11]

1.5.3. Modulation Transfer function (MTF)

The sole purpose of virtually all imaging systems is to resolve a given minimum sized object over a desired field of view. This information can be conveyed through a contrast metric which is often termed as a Modulation Transfer Function (MTF). It is probably the most valuable piece of information available to the designer as it gives clues as to how the optical resolution evolves along the field of view. It is often expressed as the magnitude of the Optical Transfer Function which is given as ratio of the image to object contrast. The MTF itself is expressed as [12-13]:

$$MTF_{diff}(v) = \frac{2}{\pi} \left[\arccos\left(\frac{v}{v_0}\right) - \left(\frac{v}{v_0}\right) \sqrt{1 - \left(\frac{v}{v_0}\right)^2} \right] \quad (1.8)$$

$$v_0 = \frac{1}{\lambda(f \#)}$$

Where v is the spatial frequency of interest and v_0 is the cut off-frequency. The plot of the diffraction MTF with respect to normalised cut-off frequency is given below.

Figure 1.8. MTF plotted as a function of normalized cut-off frequency.

The usual practice is to observe the MTF as the image sine wave modulation to the object sine wave modulation as a function of spatial frequency, which is typically specified in lines/mm or cycles/mm. A cycle is defined by a period of peak and trough of the input sine wave as shown below. A complete MTF prescription is denoted by the plot of MTF values for individual fields that make up the field of view.

Figure 1.9. Illustration showing the input sine wave modulation and its image [14].

1.6. Introduction to freeform surfaces

We now have defined source specifications along with how to analyse a system under the presence of optical aberrations. Next, in this section, we will shift our focus to the optical surfaces which perform the imaging function of transforming the object to the image.

Traditional optical designs aiming to achieve perfect imaging of an object evolved from the basis, that for perfect imaging, optical path length from the objects point(s) to the image points(s) must be the same along all the rays that make up the individual fields. The first attempt in solving this was proposed by René Descartes in the 17th century [15]. He had discovered that one single optical surface could be used to perfectly couple one object to one image point using what he termed as Cartesian Oval and was later generalised by Levi-Civita [16].

This was the starting point for other designers to start formulating direct design methods to come up with surfaces that could perfectly couple an entire object to an image. One important design strategy, which this thesis will majorly focus on, is the Simultaneous Multiple Surfaces (SMS) design method, first developed in 1990s for a Non-imaging design application by Miñano et al, and later developed by Pablo et al [17-21]. The evolution of this design scheme will be detailed in the later chapters.

As it can be observed, even though the first point of origin in the development of optical surfaces proposed a surface free of any symmetry, traditional designs still were restricted by the use of optical surfaces having inherent symmetry in them. In fact, it was not until 1880s that the art of optical design transformed into actual science with the teaming up of Abbe, Schott and Zeiss. Schott, especially is credited to be the one to have created a viable industry out of engineering refractive index and dispersion of optical glasses. Thereafter, the optics industry consisting of different phases in the development cycle, namely, design, fabrication, testing and assembly, was fully emergent by 1990 and was still revolving around the use of rotationally symmetric surface shapes. These surfaces were usually, spheres, conic sections and later aspheres (even though aspheres may have an infinite number of descriptive parameters, they still possess rotational symmetry). This was warranted due to the limitations in fabrication techniques and of course, the computation power required to calculating these surfaces were limited to manual, on-paper ray trace calculations. Advances in computer aided designs and improvements in fabrication techniques such as injection molding etc., paved way for “real aspheres” which had significant departures from their spherical counterparts. The usual development cycle consisted of use of Seidel’s aberration theory and or other direct design techniques to come up with an initial estimate, followed by a few manual ray traces to confirm their validity and finally their fabrication and experimental verification/characterisation. With the introduction of optimisation engines in commercial ray tracing software packages, the need for orthogonal representation for the aspheres became important, since other non-orthogonal surface descriptors when introduced into an optimization engine resulted in wild aspheres and never had any direct relationship to the final fabrication and tolerancing metrics. This all changed when G. Forbes introduced the Q-polynomial type of aspheric representation [22-25], a new form of aspheric surface description where the coefficients that describe the aspheres are in terms of sag units.

With vast improvements in field of computer aided designs, it has drastically equipped designers with fast ray trace calculations and resulted in designs solving complex optical problems. These complex optical problems such as those in head-lamps and others in [26-28], warranted a new class of optical surfaces namely freeform optical surfaces, i.e. surfaces having no inherent symmetry. The introduction of freeform surfaces equipped the designers with additional degree of freedoms to ensure more control over the rays resulting in designs successful in solving optical problems having rotationally asymmetric prescription.

Freeform optics is not a relatively new term, in fact it is almost 135 years old [29-30]. One of the first instances of a commercial product using freeform surfaces was the Polaroid's SX-70 camera [31]. With advances in the field of single point diamond turning, many freeform shapes that were unimaginable a couple of decades back, can be manufactured with sufficient accuracy that the application demands and along with injection molding techniques, mass production is now instantly available. This prompted the increasing use of freeform surfaces, first in the field of non-imaging optics since its requirements of manufacturing accuracy is less strict than its imaging counterpart. Its use in the field of automotive lighting, illumination, solar concentrators is unprecedented and is extensively documented in [32-33].

The use of freeform surfaces in imaging applications are ever expanding with reports of increased compactness in off-axis imaging spectrometers [34], Head Mounted Displays [35-37], Wide angle cameras [38], all resulting in compact devices and increased imaging resolution. This thesis will particularly explore, how the use of freeform surfaces in the field of Head Mounted Displays result in compact, high resolution imaging devices much different to those in offering currently.

1.6.1. Designing with freeform surfaces – commercial ray tracing software

We have seen how the use of freeform surfaces can aid in reducing the overall volume of the resulting optical device without any compromise in its imaging performance. However, this lack of any inherent symmetry in the surface geometry can pose some very intriguing challenges. For starters, the direct design of freeform surfaces is still a very nascent field except for a few methods such as SMS 3D and [39-40]. The lack of a fully developed aberration theory (Nodal Aberration theory [41] is still not widely accepted in the imaging community for freeform systems) specifically catering to freeform surface design is another bottle neck along with no pre-existing freeform designs in the literature to be used as a starting point, as in the case of its rotationally symmetric counterpart. All of this makes an optical designer to come up with a starting design, arising out of the previously mentioned direct design techniques such as SMS 3D and thereafter, heavily depend on computer aided optimization schemes to get a system closely matching the specifications. Computer aided optimization, which is based on ray tracing is much more effective in case of freeform systems, as it overcomes the limitations of aberration theory in that when designing with aberration theory, very rarely a specific aberration affects the system performance alone. In a freeform system with tilted and decentred components, non-conventional surface shapes, wavelength dependent performance drivers can be accurately modelled using ray tracing [10].

Common steps in an optimization scheme involves the designer initially coming up with a starting system either from pre-existing patents or literature and identifying system variables such as Lens thicknesses (inter and intra), material parameters, and the most important of them all : *surface descriptor parameters*. This is then input to an optimization engine, whose sole objective is to minimise an error function or a merit function which is a single value denoting the various performance operands such as RMS spot sizes, MTF values along with system constraints such as focal length, distortion etc.

This merit function can also include user-defined constraints to customise the overall design process. A common definition for the merit function is given below [10]:

$$\xi = \sum_Z \sum_F \sum_{\lambda} \sum_R [(WTX(Z, F) * WTW(Z, \lambda) * WTA(Z, R) * \Delta X)^2 + (WTY(Z, F) * WTW(Z, \lambda) * WTA(Z, R) * \Delta Y)^2 + (WTC, \Delta c)^2] \quad (1.9)$$

Where WTX, WTY refers to the weight associated with x and y components of the input fields, WTW refers to the weights associated with the input spectrum, WTA refers to the aperture entry, and the letters Z, F, λ, R refer to the zoom, field, wavelength, ray respectively. Once this is set up, the optimization engine searches for a minimum of this error function based on commonly available search algorithms such as Damped Least Squares method (DLS) or a Global search (global synthesis in CodeV® [10]). Damped Least Squares method is the most commonly used optimization algorithm which also allows the use of Lagrange multipliers [42] to optimize a starting system to find a local minima for the error function. This local minima is heavily dependent on the starting system and hence it is very important to find a good starting point [43]. The characteristic feature of this method is that it allows only for downhill movements in the merit function parameter space while the Global optimization algorithm allows for both uphill and downhill movements and thus provides a more comprehensive search for a good minima in the parameter space. The downside of this method is that it is more computationally exhaustive and hence DLS is more preferred by the designers.

It was mentioned previously that the most important variable in a freeform optimization process is the surface descriptor. The most commonly used surface descriptor is the polynomial notation of freeform surfaces such as the standard XY power series and other anamorphic representations. Since the introduction of Q-type aspheric polynomials, its extension to cover freeform surfaces were also introduced [44]. This representation along with other orthogonal representations have widely impacted freeform optimization routines and their usefulness has been extensively reported in [45-

46]. Other local representative notations such as the freeform NURBS have also been proposed and have been found effective in certain cases where local control of rays is needed [47] but they have not yet found wide acceptance in the imaging community and their representation in commercial ray tracing packages is still far-fetched.

The designs presented in this thesis are classified into two sections. The first section involves the extension of a well-known classical theory of Aplanatism to a freeform prescription case and formally proving the link between the SMS design method and freeform aplanatic systems. The second section is devoted to exploring freeform optical design models to be used with Head Mounted Devices for virtual reality applications. As a precursor to Virtual reality optics, the next section introduces some basic terminology which will be used throughout the course of this thesis.

1.7. Virtual Reality Optics - Glossary of terms

A basic layout of virtual reality optical system is shown below. It consists of a digital display which is then addressed by an optic forming a virtual image at a specified distant imaging plane on to the eye.

Eye relief: Eye relief is the distance between the eye pupil and the first optical surface facing the eye. This quantity depends on if the design takes into account the end user's prescription glasses. Without it, a standard eye relief of 16-20mm is acceptable.

Total track length: It is the distance between the digital display and the eye pupil. For tilted and decentred systems, the maximum horizontal distance is taken as the reference. Typical numbers for this quantity in current headsets are somewhere between 50-60 mm.

Figure 1.10. Basic layout a VR optical system.

Field of View: It is the angular extent of the virtual image of the digital display as seen by the end user. It is usually described either by the diagonal extent or the horizontal and vertical extent of the virtual image, either in degrees or in size at a certain distance. This quantity needs to be as high as possible to ensure highly immersive feeling imparted to the end user. This is due to the fact that the monocular FOV of a human eye is around 190 deg and the end user feels fully immersive only if FOV of the VR headset matches to that of human eye. Typical FOV numbers for some of the headsets are somewhere between 100 to 120 deg only.

Resolution: This is a quantity which is used to describe the image quality in a VR environment. It is the number of display pixel allocated per degree of Field of view. This relationship between the number of display pixels (also known as o-pixel, the physical pixel of the digital display) to the FOV will give a measure of how pixelated a certain object appears inside the VR scene. This quantity is usually expressed in terms of pixels per degree or PPD and the human eye is usually able to resolve upto 50-60 PPD. Current VR headsets reach only up to a maximum of 24 PPD. Another shortcoming of current headsets is that they try accomplish to achieve uniform resolution throughout the field of view without exploiting the characteristics of the human eye. As it will be seen in

Chapter 4, the resolving power of the human eye is not constant throughout its field of view and in fact drops considerably from its peak value at the centre as it moves towards the edge of its periphery. This fact is something the designs presented in this thesis takes advantage of and will be detailed extensively later in chapter 4. Often, when mentioning resolution, it is important make the distinction between the display resolution (often represented in terms of its pixel count) and the VR resolution as seen by the end user

(often expressed in terms of pixel per deg, i.e. $\text{Res}_{\text{VR-PPD}} = \frac{1}{\tan^{-1}(o - \text{pixel} / f)}$).

Eye box or Exit pupil: it is the distance the end user's pupil can be allowed to move in both directions without losing the display, at the eye relief. Ideal characteristic would be is to have a large eye box with very thin optics. An eyebox of 10mm in horizontal and 8mm in vertical is usually acceptable.

Eye direct gazing and Eye looking frontwards: Eye looking frontwards is the case when the pupil is looking forward toward the optic as shown above, and the entirety of the scene is seen peripherally. In the case of direct gazing, the eye pupil is rotated and is directly gazed on. These two scenarios are very important as they define the image quality of the optics, which will be dealt with in detail in chapter 4.

Figure 1.11. Illustration of Eye box.

1.7.1. Mapping function

Mapping in the context of virtual reality optics, is the transformation done by the optics on the object pixels (o-pixels) to obtain the virtual reality pixels (i-pixels). The treatment presented in this section follows the terminology as in []. Given that (ρ, ϕ) are the polar coordinates of a point r on the digital display and (θ, ϕ) are the polar and azimuthal angles respectively, of the spherical coordinates on the virtual screen. The mapping function is given by:

$$r(\theta, \phi) = (\rho(\theta, \phi) \cos(\phi(\theta, \phi)), \rho(\theta, \phi) \sin(\phi(\theta, \phi))) \quad (1.10)$$

The focal length along the radial direction at the virtual screen direction (θ, ϕ) is

$$f_{rad} = \left| \frac{\delta r}{\delta \theta} \right| \text{ and the focal length along the sagittal direction is } f_{sag} = \left| \frac{\delta r}{\delta \phi} \right| * \frac{1}{\sin \theta} .$$

The focal length directly points towards the compression or expansion imparted by the optics to the object pixels to obtain the virtual pixels. Current VR headsets usually employ rotationally symmetric surfaces to perform this mapping and if we assume that the axis of this symmetry is along $\theta = 0$ and that this direction is imaged on to the digital display at the point $\rho = 0$, then the resulting mapping function is such that $\rho = \rho(\theta)$ alone. The radial and sagittal focal lengths are $f_{rad} = \left| \frac{\delta \rho}{\delta \theta} \right|$ and $f_{sag} = \rho * \frac{1}{\sin \theta}$. The standard rectilinear mapping which can be found in most optical imaging systems takes the form:

$$\rho(\theta) = f \tan \theta \quad (1.11)$$

, where f is the focal length and is constant leading to $f_{rad} = f_{sag} = \frac{f}{\cos \theta}$.

On the other hand, standard VR headsets employ a slight variation of the above relationship called the linear mapping:

$$\rho(\theta) = f \theta \quad (1.12)$$

Both of the above mapping schemes makes the ipixels progressively smaller towards the edge of the peripheral field of view. Because of this, the pixels appear smaller than those at $\theta = 0$ at the edge of the gazing field of view (around $\theta = 20^\circ$), and this is not required since the eye will never directly gaze those pixels and this hinders the increase in resolution that could have been otherwise imparted to the centre of the field of view where it is more needed. To accomplish this, the optical models that are introduced in this thesis use a new form of mapping scheme, called as the *foveated mapping*, in which the radial focal length is a decreasing function with an approximately constant slope with θ outside of the directly gazed region, so the ipixels appear significantly larger in this direction, showing a better adaptation to the human eye resolution. An in-detail analysis of the above three mapping schemes is presented later in Chapter 4.

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Chapter 2

FREEFORM APLANATIC SYSTEMS AS A LIMITING CASE OF SMS

2.1. Introduction

Simultaneous Multiple Surface (SMS) design methods have been studied extensively in the past [1-3]. There has been no significant investigation into the link between the SMS method and aplanatic systems. In this chapter, we present an overview of the SMS design method as a solution to the coupling of three input wavefronts to three output wavefronts, giving a detailed mathematical insight into its limiting case when the input wavefronts approach each other and the output wavefronts do approach each other. We prove that an optical system obtained as a result of this scenario becomes aplanatic in the limiting case, meaning that such a system forms a perfect image of the infinitesimal area around the point of interest.

Aplanatic systems with rotational symmetry are a result of optical systems corrected for spherical aberration, satisfying the Abbe sine condition. Two mirror rotationally symmetric aplanatic systems have already been extensively investigated in [4-7] and were primarily designed for telescopes which required superior aberration-free imaging characteristics. The surface profiles were analytically formulated by Schwarzschild, 1905 in [8] for rotationally symmetric systems and has been quite well known since. Recently Lynden-Bell et al. gave an analytic formulation of any two-mirror

aplanatic design with rotational symmetry, which henceforth will be called 2D aplanatic since only the 2D cross section intervenes in those designs.

2.1.1. Freeform surfaces for Non-imaging and Imaging systems

For the majority part of early 21st century, optical systems were primarily designed using rotationally symmetrical, spherical shapes. This was mainly due to limitations in computing and also manufacturing techniques present during that time. Recent advances in manufacturing techniques such as multiple axis diamond turning and increased use of plastics has opened up new avenues that were seldom available to optical designers before. Automated computing has taken the design process into the next evolutionary stage with powerful optimization engines that are included as a part of some of the optical design software available today. As a consequence of which, with rapidly evolving freeform manufacturing techniques, any shape with high precision is practically feasible.

Freeform surfaces allow for considerable reduction in the number of surfaces required for performing a certain desired optical function, while compensating for some higher order aberrations as well, making it more compact when compared to its non-freeform counterparts. They also provide additional degrees of freedom to the designers which enables the introduction of additional functionalities to the system overall. Recently, as a consequence of increasing demand for customized irradiance tailoring, freeform design methods and principles have become important. Specific cases being that of street lighting, laser beam shaping etc.

Off axis optical systems in general make the overall optical system more compact (thinner, lighter) than their on axis counterparts. Tilted and decentred reflective systems have been studied thoroughly in the past and have been observed to offer increased field of view and speed as compared to on axis systems [9-10]. So, one can imagine the additional capabilities a freeform off-axis non-imaging system might have.

Similar trends have been observed in the imaging community as well, with the introduction of new freeform surface descriptors to be used in imaging applications and its adoption in commercial ray tracing softwares [11]. This has led to the introduction of freeform surfaces in various imaging applications such as head mounted displays, imaging spectrometers etc [12-13].

2.2. SMS method – An overview

SMS method was developed as a non-imaging optic design tool during the 1990s. It was initially developed for 2D cases by Miñano and later was extended for three dimensions by Benítez and Miñano. SMS design method computes N optical surfaces used for focusing N input wavefronts onto N output wavefronts, as, for instance, the 2 plane input wavefronts normal to $\mathbf{v}_1$ and $\mathbf{v}_2$ shown in Fig.2.1 which are focused onto 2 spherical wavefronts centred on points $\mathbf{A}_1$ and $\mathbf{A}_2$. This correspondence between the number of optical surfaces and that of wavefronts to couple is no longer consistent when the footprints of the design bundles do not occupy the full SMS surfaces, as demonstrated in [14].

Figure 2.1. Illustration of the SMS design principle in 2D and 3D scenarios.

2.2.1. SMS 2D method

To give more insight into the above described principle, let us assume that we want to couple two input wavefronts namely: $\mathbf{WF}_{i1}$, $\mathbf{WF}_{i2}$ to two put wavefronts namely: $\mathbf{WF}_{o1}$, $\mathbf{WF}_{o2}$. The optical path lengths between the two respective pairs of wavefronts can be

chosen by the designer (L_1, L_2) along with one starting point P_0 along with its normal vector N_0 . The illustration below describes the same:

Figure 2.2. Illustration of the SMS 2D process – SMS chain generation.

Ray₀ that originates from $\mathbf{WF}_{i1}$ is deflected at point P_0 with its normal vector N_0 . This ray is propagated further and the point P_1 with its normal vector N_1 is calculated using the constant optical path length L_1 which was chosen earlier. Subsequently, Ray₂ that originates from $\mathbf{WF}_{i2}$ is deflected at point P_1 with its normal vector N_1 and propagated (in a reverse manner) to calculate point P_2 with its normal vector N_2 using the constant optical path length L_2 which was chosen earlier. This process is iterated until we obtain the full SMS chain. This SMS chain generation process results in two sets of points and normal pertaining to the two optical surfaces to be generated. These two sets are then interpolated to get the 2D curves. If the rays that were chosen to generate the above chain all came belonging to a single plane, then the resulting curve from the interpolation can be revolved along with optical axis, also the symmetry axis, to obtain the full surface. Note that this procedure only couples rays that belong to a single plane and hence the entire wavefront (in 3D) is not coupled. To obtain full coupling of the wavefronts, SMS 3D procedure is used and is extensively detailed in [1]. An extension of this SMS 3D procedure to couple three, on-axis design points has already been reported. An extension

of this method to couple three design points belonging to a plane will be reported in the next section.

2.2.2. A general mathematical description of the SMS method

Let us define an SMS design as that optical design which images stigmatically two or more points of the object space into corresponding points of the image space. Stigmatic imaging of one object point is achieved using Cartesian ovals. The general expression of the image coordinate $x' = A(x, y, p, q)$ for an SMS design stigmatically imaging 3 object points: $\alpha = (x_\alpha, y_\alpha)$, $\beta = (x_\beta, y_\beta)$ and $\gamma = (x_\gamma, y_\gamma)$ onto 3 image points: $\alpha' = (x'_\alpha, y'_\alpha)$, $\beta' = (x'_\beta, y'_\beta)$ and $\gamma' = (x'_\gamma, y'_\gamma)$ can be written as :

$$x' = A_0 + A_1 x + A_2 y + F(x, y, p, q, x_\alpha, y_\alpha, x_\beta, y_\beta, x_\gamma, y_\gamma) \quad (2.1)$$

The constants A_0, A_1, A_2 can be calculated as:

$$\begin{pmatrix} A_0 \\ A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} 1 & x_\alpha & y_\alpha \\ 1 & x_\beta & y_\beta \\ 1 & x_\gamma & y_\gamma \end{pmatrix}^{-1} \begin{pmatrix} x'_\alpha \\ x'_\beta \\ x'_\gamma \end{pmatrix} \quad (2.2)$$

F is an arbitrary continuous function nulling at the points α, β and γ . Its general expression is:

$$F(x, y, p, q, x_\alpha, y_\alpha, x_\beta, y_\beta, x_\gamma, y_\gamma) = \sum_{i,j,k=\{0,1\}} A_{ijk} (x - x_\alpha)^i (y - y_\alpha)^{1-i} (x - x_\beta)^j (y - y_\beta)^{1-j} (x - x_\gamma)^k (y - y_\gamma)^{1-k} \quad (2.3)$$

where A_{ijk} are arbitrary continuous functions of x, y, p, q taking finite values at the points α, β and γ (i, j, k are Boolean variables taking values 0 or 1, then the sum in the above equation has 8 addends). Note that:

$$\begin{aligned}
\left. \frac{\partial F}{\partial x} \right|_{\substack{x=x_\alpha \\ y=y_\alpha}} &= \sum_{j,k=\{0,1\}} A_{1jk} (x_\alpha - x_\beta)^j (y_\alpha - y_\beta)^{1-j} (x_\alpha - x_\gamma)^k (y_\alpha - y_\gamma)^{1-k} \\
\left. \frac{\partial F}{\partial y} \right|_{\substack{x=x_\alpha \\ y=y_\alpha}} &= \sum_{j,k=\{0,1\}} A_{0jk} (x_\alpha - x_\beta)^j (y_\alpha - y_\beta)^{1-j} (x_\alpha - x_\gamma)^k (y_\alpha - y_\gamma)^{1-k}
\end{aligned} \tag{2.4}$$

The function $y' = B(x, y, p, q)$ can be written in a similar fashion as x' . Let us now examine the case when the three points $\alpha = (x_\alpha, y_\alpha)$, $\beta = (x_\beta, y_\beta)$ and $\gamma = (x_\gamma, y_\gamma)$ are coincident with the origin and $x' = 0$ for this coincident point. In this case $A_0 = 0$, as seen from Eq. 2.2.

It is clear from Eq. 2.4 that the quantities $\partial F / \partial x$ and $\partial F / \partial y$ at the point $\alpha = (x_\alpha, y_\alpha)$ are 0 in this limit case. Then, according to Eq. 2.1:

$$\begin{aligned}
\left. \frac{\partial x'}{\partial x} \right|_{\substack{x=x_\alpha \\ y=y_\alpha}} &= A_1 & \left. \frac{\partial x'}{\partial y} \right|_{\substack{x=x_\alpha \\ y=y_\alpha}} &= A_2
\end{aligned} \tag{2.5}$$

Consider now the case when $(x_\alpha, y_\alpha) = (0, 0)$, $(x_\beta, y_\beta) = (\Delta_x, 0)$, $(x_\gamma, y_\gamma) = (0, \Delta_y)$ and $(x_{\alpha'}, y_{\alpha'}) = (0, 0)$, $(x_{\beta'}, y_{\beta'}) = (\Delta_{x'}, 0)$, $(x_{\gamma'}, y_{\gamma'}) = (0, \Delta_{y'})$ ($\Delta_x, \Delta_y, \Delta_{x'}$ and $\Delta_{y'}$ are constants). According to Eq. 2.2 we have:

$$\begin{aligned}
\begin{pmatrix} A_0 \\ A_1 \\ A_2 \end{pmatrix} &= \begin{pmatrix} 1 & 0 & 0 \\ 1 & \Delta_x & 0 \\ 1 & 0 & \Delta_y \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \Delta_{x'} \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ -\Delta_x^{-1} & \Delta_x^{-1} & 0 \\ -\Delta_y^{-1} & 0 & \Delta_y^{-1} \end{pmatrix} \begin{pmatrix} 0 \\ \Delta_{x'} \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ \Delta_{x'} / \Delta_x \\ 0 \end{pmatrix}
\end{aligned} \tag{2.6}$$

The coefficients corresponding to y' can be found in a similar way to be:

$$\begin{pmatrix} B_0 \\ B_1 \\ B_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \Delta'_y / \Delta_y \end{pmatrix} \quad (2.7)$$

From Eqs. 2.1 and 2.6, and their equivalents in the y' coordinate, we get that in this limit case, the expansion (in terms of x, y) of the functions $x' = A(x, y, p, q)$ and $y' = B(x, y, p, q)$ at the point $x = y = 0$ is (only terms until the first order are explicitly written):

$$\begin{aligned} x' &= \frac{\Delta'_x}{\Delta_x} x + \dots \\ y' &= \frac{\Delta'_y}{\Delta_y} y + \dots \end{aligned} \quad (2.8)$$

The above equation is very significant in proving its link to an aplanatic system.

2.2.3. Extension of SMS 3D method to couple 3 points on a plane

With increasing occurrence of usage of freeform systems to solve a variety of imaging problems, there occurs a need in coming up with direct design techniques which facilitates the designer with a better starting system for further optimization. One powerful direct design method was introduced by Miñano et al. and subsequently extended by Benitez et al. [1], which under its current form SMS 3D, can be used for directly calculating more than one freeform surface(s) in an imaging context. Latest developments include, the calculation of three freeform surfaces for the coupling of three input source points positioned along a line. The current development carried out in this thesis will extend this to couple three source points defining a plane and how this can lead to the design of freeform aplanatic systems.

Figure 2.3. A schematic illustration of SMS 3D optical system coupling three pairs of freeform wavefronts

A schematic illustration of such a problem is shown below. Even though these surfaces and wavefronts could in general be freely chosen, here we will limit the problem to the geometry in which the surfaces are plane-symmetric with respect to two orthogonal planes ($x = 0$ and $y = 0$), and the input wavefronts are spherical and positioned along a plane as shown in the next illustration.

Figure 2.4. Illustration of the object points positioned along a plane and its corresponding image points. Note that points A and C are chosen to be symmetric.

The SMS design procedure for 3 freeform surfaces is based on SMS 3D algorithm for two freeform surfaces, which have already been detailed in [15]. The known parameters of the system include, the refractive indices of the four media that separate the surfaces, the object points A, B and C and its corresponding image points A', B' and C' (in turn their respective optical path lengths). The design procedure presented here is divided in two phases: (1) calculation of the seed patches and (2) the calculation of the rest of the SMS surfaces.

The first step in phase 1 consists of choosing a bottom surface patch. Next, choose a point Q_0 (along with its normal N_{Q_0}) corresponding to the middle surface (All points on the middle surface will be denoted by Q). We can propagate a ray from C to Q_0 and (since the optical path length from C to C' is known) calculate the point P_0 and its normal N_{P_0} satisfying this path, as shown in Fig.2.5 (a). The next step (Fig. 2.5(b) is then to propagate a ray from the source point B through Q_0 and since we know the path length from B to B', we can analogously calculate the point P_1^* and its normal $N_{P_1^*}$. Through symmetry with respect to plane $x=0$, the symmetric point P_1 and its normal N_{P_1} can be calculated (Fig. 2.5(c)). The next step (Fig. 2.5(d)) is to propagate a ray from C to C', through P_1 and its normal N_{P_1} , to calculate the point Q_1 and its normal N_{Q_1} using the optical path length. Note that all the above ray propagations go through the bottom surface patch as dictated by the respective optical path length conditions. By iterating the above steps, a chain pair of points $((P_0, P_0^*), (P_1, P_1^*), \dots)$ & $((Q_0, Q_0^*), (Q_1, Q_1^*), \dots)$ are obtained, as shown in Fig.2.5 (e). These are known as the SMS chains. By interpolating a line (with points and normal vectors) between Q_0 and Q_1 , and using those as initial points the procedure described above, intermediate points between the chain points can be calculated and lines laying on the surfaces are thus found, as shown in Fig. 2.5 (f). Notice that the rays from C to C' passing through the P and Q lines also define a line on the bottom surface, which we can call R (and its symmetric with respect to $x=0$, R^*). Finally, by interpolating a surface patch between lines P and P^* which is consistent with the normal vectors along those lines, and

analogously between lines Q and Q^* (its symmetric one), the two portions of top and middle surfaces are found to complete this Phase 1. This surface interpolation, can be performed using NURBS as the basis as explained in [15].

Figure 2.5. Illustration of Phase I of the SMS 3D design procedure for three on-axis freeform surfaces.

Using the seed patches obtained in Phase 1, the surfaces can now be advanced in the following manner: Propagate a ray fan originating from source point A to A' through the bottom surface patch, such that it passes through the previously obtained patch for the top and middle surface. By growing this ray fan in the x direction, the entire top surface can be obtained. In a similar manner, with ray fans originating from B and C to B' and C' respectively, the middle and bottom surfaces can be calculated as well. This has also been well documented in phase 2 of the SMS 3D construction method in [14].

To demonstrate the above method, we designed an all refractive system with the following parameters:

Table 1 System parameters

System parameters	
Effective focal length	4.89 mm
F number	2.44
Design points (x,y) mm	(0,1.058),(0,-1.058),(1.058,0) mm
Material	PMMA
Freeform representation	XY polynomial

The design points are all spherical wavefronts centred at points mentioned above. The first surface is prescribed as a plane.

Figure 2.6. A 2d plot representing rays along y axis along with the perspective view of the entire system.

Figure 2.7. RMS spot diameter map for the entire field of view of the system. It can be noted how there is a sharp drop off or zeroes corresponding to the design points and rapid degradation when moved away from this.

2.3. Aplanatism and its link to SMS design method

Ernst Karl Abbe demonstrated in 1873 [16] that rotational symmetric optical systems adhering to the sine condition were free of circular coma. Any offence against this resulted in aberrations which had linear dependence over the field angle. Many other recent studies have quantified this effect and have also derived the sine condition under more generalized circumstances [17-20]. The treatments presented in these papers involved the formulation of ray aberrations as a function of the offense against the sine condition. We provide an alternate treatment to the derivation of the sine condition under rotationally asymmetric prescription between the source and target using the

étendue-conservation theorem [21]. This treatment does not assume any symmetry in the optical system and does not involve the computation of ray and or wave aberrations of the generalized system as shown below.

Figure 2.8. Nomenclature used in establishing the link between SMS and aplanatism.

Consider a ray characterization by the Cartesian coordinates (x, y) when intersecting the plane $z = 0$ (object plane) and by the optical direction cosines (p, q) at this point of intersection, where (p, q, r) are optical direction cosines with respect to x, y and z axes respectively. The refractive indices of the medium in the object and image plane are denoted by n and n' respectively. The optical direction cosines fulfill $p^2 + q^2 + r^2 = n^2(x, y, z)$. A similar characterization is used in the image plane x', y' as shown in Fig. 2.8. Let's group the coordinates into 2 vectors, $\mathbf{p}$ and $\mathbf{t}$, defined as follows:

$$\begin{aligned} \mathbf{p} &= (x, y, 0) & \mathbf{t} &= (p, q, r) \\ \mathbf{p}' &= (x', y', 0) & \mathbf{t}' &= (p', q', r') \end{aligned} \quad (2.9)$$

A two parametric bundle of rays can be defined by a couple of vector functions $\mathbf{p}(u, v)$ and $\mathbf{t}(u, v)$ where u and v are the 2 parameters of the bundle. This same set of rays can also be characterized at the image plane by another couple of functions $\mathbf{p}'(u, v)$ and $\mathbf{t}'(u, v)$. Let us recall now Herzberger's Fundamental Optical Invariant [22-24]. This invariant establishes that $\mathbf{p}'_u \cdot \mathbf{t}'_v - \mathbf{p}'_v \cdot \mathbf{t}'_u = \mathbf{p}_u \cdot \mathbf{t}_v - \mathbf{p}_v \cdot \mathbf{t}_u$, for any two-parametric set of rays

where u and v are the parameters (sub-indices denote partial derivative). This invariant is the same as the étendue conservation for 2-parametric bundles [21].

Now consider the particular case of a two-parametric set of rays where the parameters u and v coincide with x and p . Then $\mathbf{p}_x \cdot \mathbf{t}_p - \mathbf{p}_p \cdot \mathbf{t}_x = 1$. Choosing other couples of input coordinates as parameters we can get the following equations:

$$\begin{aligned}
 \mathbf{p}'_x \cdot \mathbf{t}'_p - \mathbf{p}'_p \cdot \mathbf{t}'_x &= \mathbf{p}_x \cdot \mathbf{t}_p - \mathbf{p}_p \cdot \mathbf{t}_x = 1 \\
 \mathbf{p}'_x \cdot \mathbf{t}'_q - \mathbf{p}'_q \cdot \mathbf{t}'_x &= \mathbf{p}_x \cdot \mathbf{t}_q - \mathbf{p}_q \cdot \mathbf{t}_x = 0 \\
 \mathbf{p}'_y \cdot \mathbf{t}'_q - \mathbf{p}'_q \cdot \mathbf{t}'_y &= \mathbf{p}_y \cdot \mathbf{t}_q - \mathbf{p}_q \cdot \mathbf{t}_y = 1 \\
 \mathbf{p}'_y \cdot \mathbf{t}'_p - \mathbf{p}'_p \cdot \mathbf{t}'_y &= \mathbf{p}_y \cdot \mathbf{t}_p - \mathbf{p}_p \cdot \mathbf{t}_y = 0
 \end{aligned} \tag{2.10}$$

Let us now consider the image space coordinates x', y', p', q' as functions of the object space variables. Expanding the image coordinates in terms of x and y , we get (only terms until the first order in x or y have been explicitly written):

$$\begin{aligned}
 x' &= A(x, y, p, q) = a_{00}(p, q) + a_{10}(p, q)x + a_{01}(p, q)y + \dots \\
 y' &= B(x, y, p, q) = b_{00}(p, q) + b_{10}(p, q)x + b_{01}(p, q)y + \dots \\
 p' &= C(x, y, p, q) = c_{00}(p, q) + c_{10}(p, q)x + c_{01}(p, q)y + \dots \\
 q' &= D(x, y, p, q) = d_{00}(p, q) + d_{10}(p, q)x + d_{01}(p, q)y + \dots
 \end{aligned} \tag{2.11}$$

One of the conditions required to achieve aplanatism is that all rays satisfy the stigmatic condition between one object and one image point. Coordinate systems are chosen so these points are the origins, so in mathematical terms this condition is just $a_{00}=b_{00}=0$.

$$\begin{aligned}
 x' &= A(0, 0, p, q) = a_{00}(p, q) = 0 \\
 y' &= B(0, 0, p, q) = b_{00}(p, q) = 0
 \end{aligned} \tag{2.12}$$

Using Eqs. 2.10, 2.11 and Eq. 2.12 for the point $x = y = 0$ we get,

$$\begin{pmatrix} a_{10} & b_{10} \\ a_{01} & b_{01} \end{pmatrix} \begin{pmatrix} c_{00p} & c_{00q} \\ d_{00p} & d_{00q} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.13}$$

The other condition of aplanatism, by definition, is first order sharp imaging around the origin which means that the 4 terms of the first matrix in Eq. 2.13 are independent of p and q , i.e., $a_{10\,p} = a_{10\,q} = a_{01\,p} = a_{01\,q} = b_{10\,p} = b_{10\,q} = b_{01\,p} = b_{01\,q} = 0$, ($a_{10\,p}$ represents the derivative of a_{10} with respect to p) i.e., a_{10} , a_{01} , b_{10} , b_{01} must be constants. Additionally we can assume without loss of generality that the x, y plane is rotated around the z axis to get $a_{01} = b_{10} = 0$. The other constants are called lateral magnifications M along x and y axes respectively: $a_{10} = M_X$, $b_{01} = M_Y$. Thus, using Eq. 2.13 for the aplanatic designs we get:

$$\begin{pmatrix} c_{00p} & c_{00q} \\ d_{00p} & d_{00q} \end{pmatrix} = \begin{pmatrix} M_X^{-1} & 0 \\ 0 & M_Y^{-1} \end{pmatrix} \quad (2.14)$$

From which: $c_{00} = p'_0 + p/M_X$ and $d_{00} = q'_0 + q/M_Y$, which leads to the generalized Abbe sine condition which states that, in an aplanatic system, the rays linking the origin ($x=y=0$) with the other origin ($x'=y'=0$) must also fulfill

$$\begin{aligned} p' &= p'_0 + \frac{p}{M_X} \\ q' &= q'_0 + \frac{q}{M_Y} \end{aligned} \quad (2.15)$$

Where p'_0 and q'_0 are constants that represent the variables p' and q' corresponding to the ray originating from $x=y=0$ with direction $p=q=0$. We can now write again Eq. (2.9) with the expressions of the coefficients a_{00} , a_{10} , a_{01} , b_{00} , b_{10} , b_{01} , c_{00} , c_{10} , c_{01} , d_{00} , d_{10} , d_{01} calculated in Eqs (2.10) – (2.13), to get

$$\begin{aligned} x' &= M_X x + \dots & y' &= M_Y y + \dots \\ p' &= p'_0 + \frac{p}{M_X} + \dots & q' &= q'_0 + \frac{q}{M_Y} + \dots \end{aligned} \quad (2.16)$$

Where again only the terms until the first order have been explicitly written. Terms in x and y of order 2 and higher are represented by the suspension points. The coefficients of those higher order terms are arbitrary functions of p, q .

We can also prove this relationship through the use of the conservation of étendue applied to the above system. This treatment is as follows:

Let us now consider the image space coordinates x', y', p', q' as functions of the object space variables. Expanding the image coordinates in terms of x and y , we get (only terms until the first order in x or y have been explicitly written):

$$\begin{aligned} x' &= A(x, y, p, q) = a_{00}(p, q) + a_{10}(p, q)x + a_{01}(p, q)y + \dots \\ y' &= B(x, y, p, q) = b_{00}(p, q) + b_{10}(p, q)x + b_{01}(p, q)y + \dots \\ p' &= C(x, y, p, q) = c_{00}(p, q) + c_{10}(p, q)x + c_{01}(p, q)y + \dots \\ q' &= D(x, y, p, q) = d_{00}(p, q) + d_{10}(p, q)x + d_{01}(p, q)y + \dots \end{aligned} \quad (2.17)$$

In order to achieve aplanatism, all rays have to satisfy the stigmatic condition between one object and one image point. Coordinate systems are chosen so these points are the origins, therefore:

$$\begin{aligned} x' &= A(0, 0, p, q) = a_{00}(p, q) = 0 \\ y' &= B(0, 0, p, q) = b_{00}(p, q) = 0 \end{aligned} \quad (2.18)$$

Aplanatism also refers to strict first order imaging between the object and image points: $a_{01}(p, q) = 0; b_{10}(p, q) = 0$. Using étendue conservation theorem for bi-parametric bundle ($y=0, q=\text{constant}$) for the point $x = y = 0$, we get

$$\begin{aligned} \varepsilon_{image} &= \begin{vmatrix} A_x & A_p \\ C_x & C_p \end{vmatrix} dx dp + \begin{vmatrix} B_x & B_p \\ D_x & D_p \end{vmatrix} dx dp \\ \varepsilon_{object} &= dx dp \\ \left| \begin{vmatrix} A_x & A_p \\ C_x & A_p \end{vmatrix} + \begin{vmatrix} B_x & B_p \\ D_x & D_p \end{vmatrix} \right| &= 1 \end{aligned} \quad (2.19)$$

Computing the various partial derivatives mentioned above and expanding the terms in it, we get:

$$\begin{aligned}
 c_{10}(p, q) \cdot c_{00, p}(p, q) &= 1, \\
 \frac{\partial c_{00}(p, q)}{\partial p} &= \frac{1}{a_{10}(p, q)} \\
 \Rightarrow c_{00}(p, q) &= \frac{p}{a_{10}(p, q)} + \theta(q) \\
 \Rightarrow c_{00}(p, q) &= \frac{p}{M_X} + \theta(q)
 \end{aligned} \tag{2.20}$$

Similarly, doing the same above routine for the other ray bundle: $x = 0, p = \text{constant}$, we get

$$d_{00}(p, q) = \frac{q}{M_Y} + \Omega(p) \tag{2.21}$$

Additionally, applying the étendue conservation for the ray bundle: $y = 0, p = \text{constant}$ leaving the parameters to be x and q and following a similar routine as above, we get:

$$\begin{vmatrix} A_x & A_q \\ C_x & A_q \end{vmatrix} + \begin{vmatrix} B_x & B_q \\ D_x & D_q \end{vmatrix} = 0 \tag{2.22}$$

$$\begin{aligned}
 m_x \cdot c_{00, q}(p, q) &= 0, \\
 \frac{\partial c_{00}(p, q)}{\partial q} &= 0 \\
 \theta(q) &= \text{constant}
 \end{aligned} \tag{2.23}$$

Following a similar route for the other ray bundle: $x = 0, q = \text{constant}$, we get

$$\begin{aligned}
 m_x \cdot d_{00, p}(p, q) &= 0, \\
 \frac{\partial d_{00}(p, q)}{\partial p} &= 0 \\
 \Rightarrow \Omega(p) &= \text{constant}
 \end{aligned} \tag{2.24}$$

Substituting the various terms obtained in Eq.2.17 we get:

$$\begin{aligned}
x' &= M_X x + \dots & y' &= M_Y y + \dots \\
p' &= p'_0 + \frac{p}{M_X} + \dots & q' &= q'_0 + \frac{q}{M_Y} + \dots
\end{aligned} \tag{2.25}$$

Comparing Eq.2.8 and Eq.2.25, it is clear that a three point SMS design at the limit (when $\Delta_x, \Delta_y, \Delta_x'$ and $\Delta_y' \rightarrow 0$) is an aplanatic one with $M_X = (\Delta_x' / \Delta_x)$ and $M_Y = (\Delta_y' / \Delta_y)$. From the preceding result we also conclude that, in general, an aplanatic freeform design must contain a minimum of three optical surfaces since the SMS design for three points needs three surfaces. Only in particular cases, such as those resulting in rotational symmetry, the aplanatic design requires just two.

One such instance of a freeform aplanatic system is shown below, more can be found in [25] and will also be detailed in the next chapter. It is a system containing 3 reflective surfaces with $M_X = 2$, $M_Y = 1$, $p'_0 = 0$ and $q'_0 = 0.86$. The following chapter will provide a more extensive treatment to the design of 3 surface freeform aplanatic systems.

Figure 2.9. A three mirror freeform aplanatic system showing rays linking the origin of the object space with the origin of the image space.

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Chapter 3

DESIGN OF FREEFORM APLANATIC SYSTEMS

3.1. Introduction

Traditional optical designs consisted, in general, of rotationally symmetric shapes owing to limitations of fabrication technologies. Advances in manufacturing of optical systems has paved way for the fabrication of optical systems having no rotational symmetry which have come to be known as freeform surfaces [1]. Introduction of freeform surfaces have aided the designers with additional degrees of freedom to ensure more control over the rays resulting in designs successful in solving optical problems having rotationally asymmetric prescription such as those in head-lamps and other situations described in [2-4]. On the other hand, optical systems that are free from spherical aberration and circular coma are known as aplanatic systems since Ernst Abbe demonstrated that adherence to the Abbe sine condition can eliminate circular coma for a microscopic objective [5]. Aplanatic systems can also be defined as optical systems which image stigmatically one point in the object space to a point in the image space and satisfy the Abbe sine condition. Aplanatism here refers to full aplanatism, i.e. the aberrations vanish for all orders. Many studies have already reported extensively about aplanatism in the last century. An excellent summary of such studies can be found in [6].

Two-mirror rotationally symmetric aplanatic systems have already been extensively studied before [7-8] and were primarily designed for telescopes which required superior aberration-free imaging characteristics. With increasing use of freeform surfaces in optical designs, it becomes significant to explore aplanatism under freeform prescription. In a previous manuscript [9], we have addressed freeform aplanatism and its link to the SMS design method. In this chapter, which can be treated as a follow up to the previous, we give a mathematical formulation defining a three-surface freeform aplanat. Also, we formally prove that a two-surface configuration cannot, in general, satisfy the conditions required for a freeform aplanatic system.

3.2. Aplanatism in rotationally symmetric systems

The sine condition is a mapping prescription for the rays imaging stigmatically 2 points which in general, are the origins of coordinates in object and image spaces. Systems failing to adhere to sine condition suffer from aberrations which have a linear dependency over field [10].

Figure 3.1. Illustration of a system having rotational symmetry.

This sine condition establishes that $p = M p'$ for the rays emanating from the origin of the object plane, which must be stigmatically imaged on the origin of the image plane. p and p' are the optical direction cosines of the ray with respect to the x axis at the object

and at the image space respectively. M is a constant called magnification. Because of the rotational symmetry the same equation holds for q and q' (direction cosines with respect to y axis), i.e., $q = M q'$. The variables p, q , and p', q' are the optical direction cosines with respect to the x, y axes (object plane) and the x', y' axes (image plane).

We have explained in chapter 1 about how for perfect imaging, the constant optical path length condition among all rays in the pupil should be fulfilled. Any deviations in it produces aberrations, which can be described through the use of the wave aberration polynomial. Let us examine the below optical system through the use of the wave aberration polynomial. The notations followed are from [11].

Figure 3.2. Illustration of notations in an optical with rotational symmetry for the wave aberration polynomial.

The wave aberration polynomial, for the above system can be expressed as:

$$\begin{aligned}
 W(r, Q, \theta) = & {}_0C_{40}Q^4 + {}_1C_{31}rQ^4 \cos \theta \\
 & + {}_2C_{22}r^2Q^2 \cos^2 \theta + {}_2C_{20}r^2Q^2 \\
 & + {}_3C_{11}r^3Q \cos \theta + {}_0C_{60}Q^6 \\
 & + {}_1C_{51}rQ^5 \cos \theta + \dots
 \end{aligned} \tag{3.1}$$

Where C_{xx} represent the various coefficients of the wave aberration polynomial. The above equation after rearranging the terms in ascending powers of r , for small fields and wide pencils, we get [12]:

$$W(r, \rho, \theta) = f_{00}(\rho) + (f_{11}(\rho) \cos \theta) r + O(r^2) \quad (3.2)$$

For a system to be stigmatic, the first term $f_{00}(\rho) = 0$. This results in the system being free from spherical aberration. In addition to this, if we make $f_{00}(\rho) = f_{11}(\rho) = 0$, then system is free from spherical aberration and coma for all orders. These systems will be termed aplanatic of all orders or simply aplanatic. One important characteristic of this is now the wave aberration polynomial (which also contains the metric for geometric spot blur for different object points in the pupil) is now dependent solely on higher orders of r^2 . This gives us a clue to characterise a given aplanatic system in terms of its RMS spot blur distribution across object points, which now can be expected to have a smooth parabolic nature to it.

The above characteristic of an aplanatic system can also be obtained through observing the image coordinate of a 2D system imaging point x to x' . If we expand the image coordinate through a Taylor's series expansion around point $x=0$, we will get:

$$x' = A|_{x=0} + \left. \frac{\partial A}{\partial x} \right|_{x=0} x + \left. \frac{\partial^2 A}{\partial x^2} \right|_{x=0} x^2 + \dots \quad (3.3)$$

For sharp imaging we should have:

$$\frac{\partial x'}{\partial p} = 0 \Leftrightarrow \frac{\partial}{\partial p} (A|_{x=0}) = \frac{\partial}{\partial p} \left(\left. \frac{\partial A}{\partial x} \right|_{x=0} \right) = \frac{\partial}{\partial p} \left(\left. \frac{\partial^2 A}{\partial x^2} \right|_{x=0} \right) = \dots = 0 \quad (3.4)$$

From where, for the system to be stigmatic at $x=0$ we have: $\frac{\partial}{\partial p} (A|_{x=0}) = 0$, and for it to

be aplanatic at $x=0$, we have: $\frac{\partial}{\partial p} \left(\left. \frac{\partial A}{\partial x} \right|_{x=0} \right) = 0$.

Figure 3.3. An example for a single point imaging device (a) along with its characteristic RMS spot map for different object positions (b).

We have seen how a single optical surface can sharply image one object point to another image point. A classic example for the same would be a Cartesian oval. One such instance is shown above for a system which was designed for a pupil size of 30mm and a focal length of 85mm. As expected, the imaging is sharp only for the design point and degrades rapidly when moved away from it. The above problem of imaging one single object point, when extended for two object points to be imaged for two image points, can be solved by SMS method as was previously explained in Chapter 2. The system below illustrates the same, which was designed for object directions of +5 deg and -5 deg for an entrance pupil of 30mm and a focal length of 85mm.

Figure 3.4. A two surface SMS design (a) along with its characteristic RMS spot map for different object positions (b).

As expected, the imaging is sharp only for the design points and degrades rapidly when moved away from it and has a characteristic inverted “u” shape for its RMS spot map.

Figure 3.5. Illustration of aplanatic systems derived from SMS systems at limit along with their characteristic RMS spot map.

It was demonstrated in Chapter 2, how in a SMS design method when the two design points or the object points approach each other at the limit, the resultant design becomes aplanatic in nature. The system shown above is an illustration of the same where a 2 surface SMS system in 2D at the limit exhibits aplanatic characteristic. We also show systems for different magnification factors (refer chapter 2) along with the RMS map for the system with $M_x = 5$ units. The figure below makes the distinction between a system which is stigmatic, to a system which is aplanatic. We can see how in the case of a stigmatic one, sharp imaging happens only at the design object point but in the case of the aplanatic one, sharp imaging is performed for not only the imaging point but also close to its neighbourhood.

Figure 3.6. Plot of RMS spot diameter for different object position for a system which is stigmatic and aplanatic.

3.3. Freeform Aplanatism – How many surfaces does it take?

We have already established the link between SMS (Simultaneous Multiple Surfaces) design method and freeform aplanatic systems [9] and using that perspective we proved that in general two optical surfaces do not provide enough degrees of freedom to satisfy the aplanatism condition in freeform problems. In this section, we show how two surfaces are insufficient from a perspective that does not involve the relationship between the SMS method and aplanatic systems.

The Abbe sine condition generalized to the freeform case establishes that the rays stigmatically imaging the two origins must also fulfill $p' = p_0' + p/M_x$, $q' = q_0' + q/M_y$, where p_0 , q_0 , M_x , and M_y are constants (the last two are called magnifications, see [Eq. 7] in [9]). This mapping prescription appears as a necessary condition when requiring imaging properties for a point at the object and its neighborhood. These types of constraints were analyzed by Stone and Forbes for a general asymmetric system when the system is required to possess a prescribed set of first-order imaging properties [13].

The case we are considering here is first order only in the spatial coordinates but full order in p , q , and p' , q' . Unlike the rotationally symmetric case, now the coordinate systems at the object and image space have an arbitrary location and orientation one with respect the other, i.e., they do not have to share the z, z' axis and nor they need to have parallel x, x' and y, y' axes.

Consider an example of optical system formed by 2 mirrors as shown below, and let's try to design an aplanatic system with the same magnification factor M ($M = M_Y = M_X$) for both axes (a similar analysis can be done by substituting one or both mirrors by refractive surfaces) and with $p_0' = q_0' = 0$. We will investigate this configuration to see if it can constitute a freeform aplanatic system. Let's call L as the optical path length of the rays connecting $\mathbf{O}$ and $\mathbf{O}'$, which are the origins of the object and image planes (the orientation of the object coordinate systems with respect to the one at the image space is arbitrary).

Figure 3.7. Two mirror aplanat under freeform prescription and (Right) plot of the integrability condition of the same illustrating it is not zero.

The nomenclature in this treatment follows the notation shown above, bold letters refer to vectors (e.g. $\mathbf{N}$), small bold letters as $\mathbf{u}$ refers to unit vectors, while italic refers to a

scalar as $u=(x^2+y^2+z^2)^{1/2}$. Consider an arbitrary point (x,y,z) on the first mirror. Using the path length condition, the distance of the image origin u' from the second optic can be deduced.

$$u + |\mathbf{OO}' + u'\mathbf{u}' - u\mathbf{u}| + u' = L \rightarrow u' = f(\mathbf{u}, \mathbf{u}', u, L) \quad (3.5)$$

Now, let's calculate the normal $\mathbf{N}$ at a point on the first mirror using the reflection law:

$$\mathbf{N} = \mathbf{u} - \frac{\mathbf{OO}' + u'\mathbf{u}' - u\mathbf{u}}{|\mathbf{OO}' + u'\mathbf{u}' - u\mathbf{u}|} \quad (3.6)$$

Note that $\mathbf{u} = (x,y,z)/u$, and because of the Abbe sine condition $M p' = p$ and $M q' = q$, i.e., $M \mathbf{u}' \cdot \mathbf{x}' = \mathbf{u} \cdot \mathbf{x}$ and $M \mathbf{u}' \cdot \mathbf{y}' = \mathbf{u} \cdot \mathbf{y}$. Then $\mathbf{u}'$ can be written as $\mathbf{u}' = (x \mathbf{x}' + y \mathbf{y}') / (Mu) + g \mathbf{z}'$, where the $g(x,y,z)$ is making $|\mathbf{u}'| = 1$, i.e., $g^2 = 1 - (x^2 + y^2) / (Mu)^2$. Since $u, u', \mathbf{u}, \mathbf{u}'$ can be written as functions of (x,y,z) and $\mathbf{OO}', \mathbf{x}', \mathbf{y}', \mathbf{z}'$ are constants, then $\mathbf{N}$ in [Eq. (3.6)] can be written as a function $\mathbf{N}(x,y,z)$, ($\mathbf{N}$ is then a vector field). If a solution of this 2-surface design problem exists, then this vector field must be integrable. The integrability condition is $\mathbf{N} \cdot \nabla \times \mathbf{N} = 0$ ($\nabla \times \mathbf{N} \equiv \text{curl } \mathbf{N}$). We do not have degrees of freedom to impose the integrability condition on $\mathbf{N}$.

Figure 3.8. Plot of the integrability condition of the system shown in Fig.3.5

In Fig.3.8 we have plotted the function $|\mathbf{N} \cdot \nabla \times \mathbf{N}|$ for the above example. This function is equal to 0 only at a plane in this example. Thus, there is no first mirror solving the problem, i.e., in general, the problem has no solution with just two surfaces. Then, we conclude that *freeform aplanatism* cannot be achieved, in general, using two surfaces. Of course there are notable exceptions to this rule. In particular, in rotationally symmetric problems (around the z axis) the vectors $\mathbf{u}$, $\mathbf{N}$ and $\mathbf{z}$ are contained in meridian planes and $\nabla \times \mathbf{N}$ is perpendicular to $\mathbf{N}$ and then the integrability condition $\mathbf{N} \cdot \nabla \times \mathbf{N} = 0$ is fulfilled everywhere, which just confirms that the rotationally symmetric aplanats need only 2 surfaces, but can only solve problems with rotational symmetric constraints. This result is well known from previous works [8]. The two aplanatic points of any refractive sphere gives us a notable example of a single surface aplanatic system [14]. There are also notable exceptions among pure freeform devices: In [15] examples of 2 freeform surface afocal aplanatic systems are given.

It may seem surprising that freeform aplanats need in general 3 surfaces while rotational symmetric ones need just 2. This is because we are not comparing same things: A freeform aplanat comprises a set of conditions broader than that achievable by a rotational symmetric aplanat. For instance there is no solution for a rotational symmetric aplanat with different magnifications in x and y while the problem is in general solvable for the freeform case. We can also view the above scenario from the point of view of an SMS design. We have already established that, freeform aplanatic systems can be seen as a limiting case of SMS design method for three coincident points in [9]. The three (non collinear) points in the object space define the object plane and their images define the image plane. It is well known that for a k -point SMS design (where the footprint of the design ray bundles occupies the entire optical surface), we require k optical surfaces. Following the same logic, we also conclude that for a freeform aplanatic system, seen as a limit case of a three point SMS design, we require a minimum of three freeform optical

surfaces, which at the end is caused by the need of at least three points to define the object (or image) plane.

From the design point of view, the rotational symmetric aplanat is a 2D geometry design (all the rays involved in the design are contained in a meridian plane). In 2D geometry only two points are needed to define the object or image straight lines, and consequently the SMS designs converging into aplanat designs need only two optical surfaces.

3.4. Semi Aplanatic systems

It is possible to design a freeform system, with just 2 optical surfaces, which fulfills a single scalar equation relating the coordinates p, q, p', q' : For instance we can choose one of the equations defining the Abbe sine condition for freeform prescription, i.e., $p' = p_0' + p/M_x$ or $q' = q_0' + q/M_y$, but not both. Observe that in this case one of the components of u' (either p' or q') is not determined as a function of (x,y,z) and we can use this degree of freedom to force $\mathbf{N} \cdot \nabla \times \mathbf{N} = 0$. We call such designs as semi-aplanatic systems. From the point of view of an SMS design, a semi-aplanatic design corresponds to the limiting case for two coincident points. These two points define one straight line in the object plane and a corresponding one in the image plane. Except for the points along that straight line, aberrations have linear dependency over the distance to the origin at the object plane, i.e., except for the points along that straight line the design is not aplanatic. An instance of this is described later in this chapter.

The rotational symmetric case is not a semi-aplanatic case but a full aplanat one even though it needs just two surfaces. The 2 equations of the Abbe sine condition for freeform prescription become coincident for the rotational symmetric problems and hence the need of just 2 surfaces.

3.5. Three surface freeform aplanatic system formulation

This section will be devoted to the differential formulation of a three surface “freeform” aplanat. The concept of using a system of differential equations to determine an aplanatic rotational symmetric system was first used by Schwarzschild [8] and later by Wassermann and Wolf [16] and many others. We introduce an intermediate optic, as shown in Fig.3.9, (surface containing point **M**) to the configuration previously used, while preserving the same nomenclature.

Figure 3.9. Illustration of a three surface freeform aplanatic system.

Using the same rules as before we will call $\mathbf{v}\mathbf{v}$ as the vector $\mathbf{AM}$ and $\mathbf{v}'\mathbf{v}'$ as the vector $\mathbf{A'M}$. We consider $u, v, u', v', \mathbf{u}, \mathbf{v}, \mathbf{u}', \mathbf{v}'$ functions of the initial direction cosines (p, q) . For example, $\mathbf{u} = (p, q, r)$ where $r^2 = 1 - p^2 - q^2$ is the third direction cosine. All the vectors are addressed in the x, y, z coordinate system. All these functions but $\mathbf{u}$ are unknown functions of p, q . Since the aplanatic system must image $\mathbf{O}$ in $\mathbf{O}'$, then the following two conditions are fulfilled (optical path length from $\mathbf{O}$ to $\mathbf{O}'$ and relative position between $\mathbf{O}$ and $\mathbf{O}'$):

$$\begin{aligned} u + v + v' + u' &= L \\ u\mathbf{u} + \mathbf{v}\mathbf{v} - \mathbf{v}'\mathbf{v}' - u'\mathbf{u}' &= \mathbf{OO}' \end{aligned} \quad (3.7)$$

Using Eq. (3.7) (4 scalar equations) we can reduce the list of unknown functions to $u, u', \mathbf{v}, \mathbf{u}'$ (for instance). The conditions for aplanatism are [9]:

$$\mathbf{u}' \cdot \mathbf{x}' = p'_0 + \frac{p}{M_X} \quad \mathbf{u}' \cdot \mathbf{y}' = q'_0 + \frac{q}{M_Y} \quad (3.8)$$

Since $|\mathbf{u}'| = 1$ and the vectors $\mathbf{x}'$ and $\mathbf{y}'$ are assumed to be known, these 2 equations determine $\mathbf{u}'$ as function of p, q , so the list of unknown functions may reduce to $u, u', \mathbf{v}$ (remember that M_X, M_Y, p'_0 and q'_0 are constants). Subsequently, as explained in [15] we can set the reflection law using Herzberger's formulation as (first line is reflection at point **A**, and second at **A'**, sub-indices p and q denote partial derivatives.):

$$\begin{aligned} u \mathbf{u}_p \cdot \mathbf{v} &= u_p (1 - \mathbf{u} \cdot \mathbf{v}) & u \mathbf{u}_q \cdot \mathbf{v} &= u_q (1 - \mathbf{u} \cdot \mathbf{v}) \\ u' \mathbf{u}'_p \cdot \mathbf{v}' &= u'_p (1 - \mathbf{u}' \cdot \mathbf{v}') & u' \mathbf{u}'_q \cdot \mathbf{v}' &= u'_q (1 - \mathbf{u}' \cdot \mathbf{v}') \end{aligned} \quad (3.9)$$

Reflection at **M** is implicit in the constant optical path length equation once the other 2 reflections are imposed. These set of 4 (scalar) partial differential equations along with the corresponding contour conditions, determine the last unknown functions (remember that $\mathbf{v}$ is a unit vector) with which the 3 surface aplanatic system can be calculated. The present formulation can be easily extended to refracting surfaces by just taking into account the refractive indices in the optical path length calculation [Eq. (3.7)], in the optical direction cosines p, q, p', q' and in both deflections of [Eq. (3.9)].

3.6. Freeform aplanatism – Examples

The first example shown in Fig. 3.10 is a semi-aplanatic system composed of two freeform mirrors. It was designed such that the rays emanating from **O** perform stigmatic imaging from **O** to **O'** and are forced to satisfy the equation $p' = p'_0 + p/M_X$ ($M_X = 0.5, p'_0 = 0$) but not the equation $q' = q'_0 + q/M_Y$.

Figure 3.10. (Left) Illustration of a semi aplanatic system formed by two freeform mirrors and (Right) Plot describing the dependence of RMS spot size with object position in x and y directions.

As we can see from Fig.3.10, the distribution of RMS spot size at the image is parabolic for object positions along x direction exhibiting aplanatic characteristics, whereas it has a V shape cross-section along y direction as indicated by the facets in the plot (See also Figure 3.11 for more clarity). Additionally, we have evaluated the adherence of the equation of the Abbe sine condition used in the design and for the above system the quantity: $|p' - p_0' - p / M_x| < 1.435 \cdot 10^{-8}$ for the rays emanating from O . We call these kind of systems as “semi aplanatic” alluding to the fact that it is aplanatic only in one direction.

Figure 3.11. 2D cross sections of the RMS map shown in Fig.3.10 to illustrate the distinction between stigmatic and aplanatic behavior in a semi aplanatic system.

The following system shown in Fig.3.12 was designed for $M_x = 0.2$, $p_0' = 0$, $M_y = 2.85$ and $q_0' = -0.40674$. The entrance pupil diameter was set at 350mm. The sizes of the three mirrors are as follows (1) Primary mirror: 275 mm (circular aperture) (2) Secondary mirror: 465.0798 mm (circular aperture) (3) Tertiary mirror: 503.0588 mm in y direction and 100 mm in x direction (for better representation). As indicated by the magnification factors, we can see the ray bundle expanded along the y direction and compressed in the x direction. It is a 3 freeform mirror aplanatic design. We can also see from the distribution of RMS spots along two directions of the object position that the system has a parabolic behavior along any direction. Additionally, we have evaluated the adherence of Abbe sine condition and for this system the quantity: $|p' - p_0' - p/M_x| < 4 \cdot 10^{-10}$; $|q' - q_0' - q/M_y| < 3 \cdot 10^{-9}$. We call these kinds of systems as aplanatic or “full aplanatic” alluding to the fact that it is aplanatic in any direction.

Figure 3.12. (Left) Perspective of a three mirror freeform aplanatic system and (Right) distribution of RMS spot diameter along two directions.

Additionally, to prove the effectiveness of a freeform aplanatic system as compared to a conventional system as reported in [17], we present the following system which consists of three reflectors and designed for an effective pupil diameter of 30mm and effective focal length of 42mm. The design parameters are: $M_x = 4.54$, $p_0' = 0$, $M_y = 4.54$ and $q_0' = 0.08112$.

Figure 3.13. (a) A perspective view of the 3 surface all reflective freeform aplanatic system followed by (b) y-z view (c) RMS map in 3D (d) 2D cross section of the RMS map.

As observed in the previously presented cases, this system also exhibits smooth imaging behaviour in 3D in its design point and in its neighbourhood. To quantify its performance further, the following figure is a comparison in terms of its RMS wavefront error maps. It is very clear how close each system is in terms of imaging performance despite the fact the freeform aplanatic system was not a system obtained as a result of local optimization technique as is the case of the system in [17]. This shows the effectiveness of the

freeform aplanatic system, which is obtained as a result of a direct design method and can possibly result in even more effective system after further optimization.

Figure 3.14. RMS wavefront error maps for (a) Our freeform aplanatic system presented in Fig.3.13 and (b) system presented in [17].

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Chapter 4

FREEFORM MULTICHANNEL OPTICS FOR VIRTUAL REALITY APPLICATIONS

4.1. Introduction

We have seen significant developments in digital technologies that have made its way to our everyday life making it simpler and much more enjoyable. One such development is the use of virtual reality (VR) in day to day life. As the name suggests, virtual reality points to an instance which does not exist in reality but the user experiencing it is made to feel as if it is in fact real. A host of visual manipulation techniques employed by the device conveying the VR information tricks the brain into believing that it is indeed experiencing the scene in reality. The means to deliver this experience is done through what is called as Virtual Reality Headsets. Virtual reality headsets are classified under Head Mounted Displays (HMDs) along with augmented reality headsets.

A classical definition of an HMD was termed by Melzer [1-2] in 1997 as a device consisting of “image source and collimating optics in a head mount”. Of course this definition was framed with the military applications it was intended to target but still holds good in the current scenario to a certain extent. Rash [3] in 2000 extended this definition to include a coupling system that employs head or motion tracking information

relayed on to aircraft systems. Basically, these definitions amounted to a user wearing a huge helmet which protected the user's head and also served as a means to mount the display electronics and the optics to convey VR information. VR information here refers to a more primitive version of what is available today thanks to ever growing developments in digital display technologies and electronics. This chapter though however focuses on the optics scheme of things and how proper design of optics can ensure in a much pleasant experience of virtual reality. The definition of "pleasant" will become clearer to the reader as we progress further down the chapter.

4.1.1. Head Mounted displays in its earlier versions

The history of HMDs can be traced back to this first ever instance in 1916 when Albert Bacon Pratt of Vermont was issued a series of patents during the first world war for a gun adapted to be mounted on and fired from the head of the user [4]. It does not take a lot of effort to notice the striking similarities in its conception with the current HMDs. Also the fact that this helmet gun was intended to be used as a weapon to find its use in military applications makes it a point of origin for HMD design development since one major application area for currently existing HMDs are in the area of military applications as well, specifically in virtual aircraft cockpits to help interface aviation specialists with a variety of specific information related with aircraft operations. This definition is still primitive and a more comprehensive definition was coined by Manning [5] to consist of 4 building blocks namely (1) A mounting platform (2) Image source (3) Relay optics (4) Head tracker. This definition is very much valid when considering current versions of HMDs and a tradeoff is required while considering the above blocks to achieve a design that will be acceptable overall to an end user. However, this chapter will focus more on the optics side of things specifically to the lens design and conception and analysis thereafter concerning its optical performance. Previously mentioned stereoscopes [6] can be also added to the above description since they demonstrated the potential to display images with depth perception added to it. Both of the above

mentioned developments combined with some latest developments in motion tracking and electronics can be said to constitute the current family head mounted displays.

A host of other developments in HMD, specific to military applications are summarized in [7].

Figure 4.1. Illustration of the helmet gun in 1916 which has a striking resemblance to current HMDs along with the Brewster type stereoscope.

4.1.2. Classification of HMDs based on constituent optics

This classification of HMD is based on the type of image formed by the optics employed in the headset. There are two types of images that can be formed by the underlying optics namely (a) Real image (b) Virtual image. A real image is formed when it is a direct recreation of an object on a surface- A classic example is the image seen in a cinema theatre. To put it in more specific terms, a real image is where light actually converges. On the contrary, a virtual image is a location from where light appears to have converged. Real image HMD designs are hard to come by. Most of the currently existing HMD designs consist of optics which produces a virtual image and it is well known that it poses lesser fatigue to the human eye which is a very important design goal of an HMD since the end user is us, human beings.

Figure 4.2. Classification of HMDs based on image delivery

HMDs are also classified on the mode of image delivery. *One-eye* occluded type of HMDs, as in [8] provide imagery to one eye while keeping the real world blocked whereas the see through *one-eye* type [9] does not hide the real world. In the same way, *Two-eye* see through type [10] imagery is presented to both eyes without blocking out real world information. This chapter focuses on *Two-eye* occluded type HMDs such as Oculus rift, HTC vive, OSVR etc.

Figure 4.3. Examples showcasing the classification of HMDs based on mode of image delivery.

Another classification that finds importance is the one based on the type of imagery delivery. Monocular refers to a HMD where the display is seen by a single eye alone. Binocular refers to a HMD where image from the display is seen by both the eyes which are exactly the same image. On the other hand, binocular refers to a HMD where the display(s) present(s) slightly different version of the same image to both the eyes for added depth perception. Most of the HMDs that will be discussed in this chapter fall under the binocular class. In these HMDs there is an overlap of the image that is being

presented from the two optical channels to both the eyes to create large field of views. Two eyes-Occluded-Binocular type HMDs are the ones which deliver Virtual reality to consumers and these types of HMDs along with new potential optical designs for them will be discussed in detail in the following sections. Since this chapter heavily focuses on two eyes-Occluded-Binocular type HMDs, the term “VR headset” and “HMD” will be interchangeably used.

4.2. Multi channeling optics in Head Mounted Displays

An ideal head mounted display is a device which has a high resolution with a large field of view, weighing as little as possible and supporting structures which measure low in terms of physical dimensions. For a VR headset, this usually is accompanied by a head tracker which tracks the head motion of the user to keep the immersive sensation alive throughout the experience. Many families of HMDs have successfully achieved individually the above mentioned qualities desired from an ideal HMD. This results in an incomplete experience for the end user with problems ranging from low immersive experience, physically uncomfortable headsets and reduced sense of believability of the VR environment resulting from a smaller field of view inside the VR environment due to the inferior quality of optical design consequently producing low image quality. One way to circumvent the above problem is to increase the field of view, which is from the point of view of VR is the observable environment at any given instance in the scene. The wider/larger this quantity is, the more the user feels transported into the scene thus giving the user a complete immersive experience. One basic methodology will be to make sure the eye relief, which is the distance between the pupil of our eye and the lens vertex is as short as possible.

Figure 4.4. Various HMD designs currently available in the market.

Current families of HMD headsets like Oculus Rift, HTC vive etc employ a single aperture imaging system to keep the optics side of things as simple as possible. We will, here, review briefly the optics make up of such systems, in particular the oculus rift, with the methodology in itself described in greater detail in the Chapter 5 before moving onto multi aperture systems.

Figure 4.5. Oculus rift Developer Kit 2 (DK2).

The Oculus rift is a virtual reality headset designed by Oculus VR [11] which was later taken over by Facebook. It had two developer versions of the headset namely, Dk1 and Dk2 in 2013 and 2014 respectively. The family of headsets from Oculus is seen as a major kick starter for most of the VR headsets in the last 7 years and is seen as the standard flag bearer for standards in the VR domain. We will briefly take a look into the Oculus Dk2 headset which is an example for single aperture optics employed in a VR headset. Oculus rift (Fig 4.5) consists of a low persistence 5.7" Super AMOLED display

with a resolution of 960 x 1080 per eye and was that of a Samsung galaxy Note 3 display. It consists of a 9 axis Inertial Measurement unit (IMU) with an update rate of 1000 Hz. In addition to this, the headset also comes with a positional tracking unit which is done with an external camera which equips this headset with a very stable motion tracking feature to track the eye and body movements inside the VR environment. This feature is very crucial to the entire VR experience since any tracking errors inside the VR frame will end up making the user lose the immersive feeling because of the time lag (latency between photons and the actual movement). The headset optics consists of an aspheric lens which was prescribed for two sets of users with varying myopic condition. In general, the lens was an aspheric with a field of view (claimed by Oculus) to be of 100 deg.

Figure 4.6. Teardown of Oculus rift Dk2 headset (Left, courtesy: ifixit)

We demounted the oculus dk2 headset ourselves to validate the parameters of the lens, as that is of interest to us within the scope of this chapter. To do this, we performed an experiment to determine the distortion function of the lens which gives us more details into the field of view and focal length of the lens. The experiment, in brief, consisted of displaying a grid of points on the display of the headset and capturing the image of it with the use of a go-pro camera (FOV 120 deg) to analyze the mapping function(also referred to as the distortion function) of the lens. Mapping here refers to the transformation performed by the lens on the display-pixels (o-pixels) to the VR pixels

(i-pixels). More details of this experiment will be presented in the final chapter but we present the results of that experiment for oculus DK2 lens here.

Figure 4.7. Plot illustrating the mapping function of various HMDs available in the market.

The results of the above mentioned experiment are shown in Fig 4.7. The plot describes how each pixel in the display is transformed or “mapped” on to the VR image. We have provided the details of the mapping functions for other headsets as well namely, HTC vive, Oculus CV1, Samsung Gear VR, all of which are radial, i.e., the systems have rotationally symmetry surfaces in their optics and hence the transformation that is done by the optic on the display to produce the virtual reality pixels have rotational symmetry inherently in them. We have provided the reference value of Note 4 display dimension. Note that dk2 headset came with a Note 3 display which is slightly less than Note 4 size. It is clear from the above plot that the field of view of Dk2 is not 100 but actually around 90 deg. The mapping function itself is pretty linear and it shows no sign of any adaptation (varying magnification factor for different angles which is a sign of *foveated* imaging) whatsoever.

Figure 4.8. Illustration of an image shot through DK2 lens.

The above image shows an instance of how the VR environment looks like of the Dk2 headset. It is highly pixelated and comes with very little color correction resulting in high color dispersion as seen in the image. The above results give us a clue as to how to improve the existing optical designs used in VR headsets. We have tried to improve on the above mentioned parameters and the rest of the chapter will focus on the design principles involving the same.

Achieving a wide field of view is important for a VR optic, since it is the means to deliver the VR information which should trick the user into believing that the scene that the user is visualizing is real. For this, the scene needs to be rendered at a high field of view matching that of the human eye (which is around 160 deg, detailed in the next section, and current headsets achieve around 100 deg). This is not the only parameter that needs to be high. The perceived resolution, which is the number of pixels allocated for every degree in the field of view needs to be high and possibly matching to that of the human eye's, which is around 60 ppd (pixels per deg). If a particular VR scene is rendered with a lower resolution than that of the human eye, i.e. less than 60 ppd, the end user will notice the objects appearing pixelated, leading to a break in the immersion. Another important factor, leading to break in the VR immersion, is the form factor of the headset the user is wearing. If this is not as compact as possible, the user will, at all times be aware that, the entire experience is artificial and never fully be immersed. Hence to sum it up, a VR headset needs to be highly compact, have a high field of view optic along with VR environment rendered with a high perceivable resolution.

There have been multiple approaches that were used to increase the field of view of VR optics, like in [12-13] which multiple displays per eye are arranged in a mosaic pattern which are not in the same plane. There have been other approaches [14] which used lenslet arrays to create a higher field of view and compact HMD designs but most of them suffered from the problem of requiring complex rendering systems to prevent cross talk between the lenslets. Traditional approaches also included the addition of newer optical components to compensate for the additional aberrations introduced in the system because of the scaling up of field of view. This resulted in complex systems without regard for its eventual increase in the overall volume of the system.

From a design point of view, it is important to note that these HMDs are intended to be used directly by users without any supervision. So characteristics resembling those in human beings are highly desirable. One such method that will be employed in our designs to be presented later in the chapter is what is known as “*foveated imaging*” [15]. This method takes advantage of the fact that the human eye, which in itself is a perfect example of wide field of view optic, makes use of parts of the field of view with varying resolution. This chapter discusses multi-channel freeform optical design which exploits this differential aspect of the human vision to provide for a HMD with wide field of view, compact optical designs to use with. This approach to optical design is seen as a solution in providing a VR headset which satisfies simultaneously in alleviating all the problems earlier posed by VR headsets to produce a HMD which is lightweight, has a high field of view, is compact and makes the user feel fully immersive in the VR environment.

4.2.1. Multi Channeling – What is it?

This chapter deals with the application of multi channeling techniques in reducing the volume/size of an optical imaging system. We take a brief look at what multi channeling means followed by some fundamental terminologies involved in describing multi channeling with some examples in nature where multi channeling is employed. It

needs to be mentioned that these terminologies have been introduced keeping in mind its eventual employment in describing optics systems suited for Virtual Reality applications.

Multi channeling goal for our applications is to decrease the volume or size of an optical system without compromising its overall performance. This is necessitated by the costs of fabrication of the system and the assembly of the system itself. Reduction of volume directly points towards the reduction of required volume of material needed for production in large scales in turn reducing the overall cost of production. This also results in increase in portability of the system which is advantageous in certain applications.

Figure 4.9. Multi channeling technique occurring in common insects (b) Apposition compound eye (c) Superposition compound eye [16].

One common way of implementing multi channeling is to divide up the system main channel into multiple sub channels with smaller apertures. This kind of an approach gives us more control over individual channels which can be designed carefully to achieve the design specifications using the same focal length to that of its single aperture counterpart. The paraxial nature of the ray sub bundles in these sub channels makes it easier to design for. There are multiple examples of multi channeling occurring in nature. One such example is the apposition compound eye of a common insect [16-18, Figure 4.9b]. In this kind of architecture, the entire system volume is split into multiple smaller eye volumes to achieve higher field of view but with lower spatial resolution. Also, the large number of micro lenses are arranged in a curved manner with single receptor at the focal plane of these individual micro lenses. In superposition compound eyes (Figure

4.9c), the light is focused by multiple neighbouring lenses combining on the retina to form a single real image. Superposition compound eyes are more light sensitive because the light bundles from one object point traveling through several adjacent channels are deflected towards the same image point increasing the effective light collecting aperture (several times larger than the diameter of a single micro lens). One of the first outline of an artificial compound eye was theoretically proposed by Dennis Gabor in 1940 and experimentally demonstrated more than 50 years after (Figure 4.10a). It follows the idea of superposition compound eyes, and it is known as the “Gabor Super lens”. Figure 4.10b shows an example of artificial apposition compound eye in the form of ultra-thin imaging sensor. Both of these techniques break the trade off between the focal length and the field of view. But owing to the requirement of high accuracy in their manufacturing, only few practical realizations of the above have been found [18-19]. Off- lately, with developments in high quality injection molding techniques, these multichannel approaches have become attractive again especially for use with head mounted displays.

Figure 4.10. (a) Schematic outline of a Gabor SuperLens (b) Ultra-thin imaging sensor using apposition compound eye principle and (c) illustration of an artificial apposition compound eye [19].

One design which finds use in VR application is the Near-To-Eye (NTE) light field display prototype developed by NVIDIA [20]. The NVIDIA NTE light field display is formed by a micro lens array and a digital display device where the image to be imaged on the retina is decomposed in small cluster images (one per micro lens). The micro lenses of the NVIDIA NTE light field display are identical and any micro lens can be obtained by copying

an original one with a simple translation movement. But this design still suffers from low spatial resolution images.

Figure 4.11. Head mounted near-eye light field display prototype (left). Comparison of the perceived image by the human eye when using a bare micro display and a near-eye light field display, showing clear advantage of using the second one (right).

Traditional imaging systems have always worked with single aperture systems forming high resolution images. But in many cases high resolution can be compromised to obtain a system much compact and a cheaper system. HMD designs existing today still use single channel imaging system and one of the consequence is that the final end product is bulky and huge.

Figure 4.12. Illustration of how bulky some of the currently existing headsets are.

Ultimate end goal of VR optics design is to design an imaging system capable of forming high resolution images on the human eye taking advantage of some characteristics of the human eye while simultaneously keeping in mind the overall requirements of a good VR headset. So, it becomes important to be aware of the human eye characteristics or the “imaging sensor” of our design problem.

4.2.2. Human eye – some fundamentals

The human eye, as such, can be considered a *complete* single aperture imaging system. As can be seen below, it is not exactly a sphere and it consists of cornea, iris and the lens. The interesting aspect to us is the lens itself. A brief literature survey would tell us that the parameters of this lens are as following [21]:

- Front to back length of the eye – around 25 mm
- Pupil diameter – 2mm contracted, 7-8mm when dilated
- Commonly used focal length – 17mm (Front focal length), 23mm (Back focal length).
- Field of view – 200 deg by 130 deg with a 120 deg overlap.

Figure 4.13. Illustration of the various constituting components of the Human eye.

The Field of view (binocular) of our eye is close to 200 deg by 130 deg with a 120 deg overlap. This does not mean that the FOV is uniform for different colours or the performance of the lens across the entire FOV is uniform. In fact there are areas which are referred to as the peripheral vision where this is different. Peripheral vision is that part of the vision which lies outside the center of sight. The main use of peripheral vision is to recognize well known shapes or boundaries without having the need to focus on it through the center of vision. There are also different parts of this peripheral vision which

varies significantly when compared to the central parts of the vision. Peripheral vision is generally considered weak in human beings and this aspect is taken advantage while designing VR lenses.

Another astonishing fact is the count of pixels on the retina which is about 130 million. To begin with, the retina itself is curved unlike the flat sensors that we are conventionally used to which results in the edges of the retina having the same distance to the center of the lens resulting in us having better sharpness at the edge of the image. The central part of retina (macula) contains the densest distribution of photoreceptors. The macula has about 150,000 pixels in each 1mm square resulting in the fact that the central part of our vision has the highest resolving power than any lens. This uneven distribution of pixels, denser in the centre and less dense towards the edges, is taken advantage of in lens design as well. The foveated imaging principle takes advantage of these spatially varying characteristics of the human eye in producing compact imaging optics without compromising on the imaging performance of the system.

Figure 4.14. Illustration showing different parts of the field of view of the human eye (left) [21] and human eye resolution for various peripheral angle (right).

4.3. ThinEyes® technology from LIMBAK

Virtual Reality headsets are viewed as a medium of delivering virtual information through a headset worn by the user, which transports them into a scene mimicking reality. So, a basic requirement for such an environment which is experienced by the user should be as close to being real. To achieve this, the headset worn should be as light as

possible, the optics present in them should perform image delivery with high field of view and high resolution in the virtual space. The overall weight of the headset is dictated by a number of factors, but we will limit ourselves to the constraint on the weight imposed by the optics alone. So for the headset to weigh less (470g for a typical headset [11]), the overall volume as imposed by the optics should be as little as possible pointing to the total track length being the minimum.

Another important parameter that dictates the quality of the image being perceived inside the VR space is the VR resolution. This quantity needs to be as high as possible since this directly denotes how “real” the objects are being perceived by the end user. Any undesired effect such as pixelation of these objects in VR space will lead to less sensation of immersion by the user and hence breaking the sense of reality. The VR resolution is described by the angular size of the VR pixel as seen by the user and can be quantified by its density ppd (pixels per deg), i.e. how many pixels of display are perceived per degree of field of view. The smaller the angular size is, higher the density is, which directly translates to lesser pixelation in the representation of the objects inside the VR space. If the optics is not designed properly, it may end up being the limiting factor for the VR resolution (typically 10-15 pixel/deg in current commercial headsets) since the eye itself can resolve a lot higher (typically around 50-60 pixels/deg). Thus there is an inherent need to increase the optical resolution in VR headsets.

4.3.1. ThinEyes® design principles

Traditional methods to increase the VR resolution mainly involves increasing the display pixel density itself while maintaining the other system parameters or to increase the focal length of the system and the display size while maintaining the pixel density of the display. Current limitations in display fabrication techniques prevent the first option in becoming effective presently. Also, increasing the focal length of the system makes it bulkier, thus making it more uncomfortable for the user wearing the headset. To

circumvent the above problem, we make use of optical foveation and ThinEyes® technology to achieve the resolution increase.

Conventional optical systems try to achieve uniform pixel distribution across the entire scene, compromising on imaging performance such as resolution. Foveation is a technique where significance is given to a central region where higher imaging characteristics is guaranteed accompanied by a wider peripheral portion with lower imaging performance. This sort of an approach especially comes in handy when systems employ gaze tracking [22] and the foveation is not fixed per se. This situation changes when there is a requirement of having fixed foveation as termed in the design challenge (no gaze tracking). This is where the application of ThinEyes® technology proves to be a game changer. This will be explained below.

The first principle of this strategy to increase the apparent resolution as seen in the VR space, we take into account the variable dependence [23] of the human eye acuity for various peripheral angles at the design stage itself accompanied by the eye rotations to make sure that the sharp image of the scene is formed on the fovea when directly gazed at. This variable dependence of the human eye acuity is shown in Fig 4.15. This makes sure that for a given pixel in the VR space when directly gazed at, we ensure maximum resolution (high ppd) and relax this condition when the same is peripherally viewed, i.e. imaged outside fovea.

Figure 4.15. Plot illustrating the variable nature of the human eye acuity with peripheral angle.

Exploiting this non-constant dependence of human eye acuity, ThinEyes® designs are comfortably above the human eye resolution curve, thus making sure that optics itself will not be the limiting factor for the resolution (pixel/deg or ppd). In most cases, for this case of direct gazing the limiting factor itself is the Nyquist frequency of the display used (Given by $F_{nyq} = 1/(2 * \text{pixel pitch})$). It is to be noted here that this design strategy ensures that the optics is well adapted to the eye resolution for every angle thus making the resolvable spatial frequency for larger peripheral angles low since it is not required.

Figure 4.16. Illustration showing the difference between a peripheral (left) and a directly gazed region (right) in a given scene. Note that the gazed region is limited by the Nyquist frequency of the display alone and the peripheral region is limited by the human eye resolution as explained earlier.

The second characteristic is that the variable VR pixel magnification is adapted to the eye resolution for different peripheral angles [24]. This results in designs where the VR pixels are denser when directly gazed at and coarser when peripheral. Since this variable magnification is well adapted to human eye resolution and the statistics of saccades (86% of eye rotation < 15 degs [25]), the user does not observe this varying size of the VR pixels in the periphery (see Fig 4.16). This is also aided by the fact that most eye rotations occur within 15 deg from normal direction and for a field of view of 100 deg, the eye rotations are limited to around 30 deg. A consequence of the above principle results in designs presenting strong distortion which must be compensated by software correction of the images projected through the display. The measurement of this distortion can be done in a conventional manner through a camera set up.

As an exercise to demonstrate the effectiveness of the above mentioned design principles, we apply the ThinEyes® design strategy to a standard pancake type optic as in [26].

LaRussa [27] introduced the use of polarized catadioptric optics, which are also known as pancake optics, to be used for avionic applications. The use of wire grid polarizers in a pancake configuration was described in [27]. The hybrid nature (reflective + refractive) of this configuration presented increased degrees of freedom for the designer. Although limited by the transmission efficiency of the polarizers and a semitransparent mirror (used in reflection and in refraction), the design still opened up a new class of optical system families to be explored. The addition of reflective optics to the existing refractive configuration in this design enabled us to reduce the total track length of the optics along with an increased resolution relative to a conventional pancake design.

The design consists of 4 aspheric surfaces arranged in a pancake configuration as shown below (Fig.4.17) occupying a total track length of 36mm only. This design assumes a standard 2.86 inch display having a resolution of 1440x1440, with a pixel pitch of 35.5 microns. The system has an eye relief of 15mm with a circular FoV of 100 deg accompanied by a 10 mm eyebox. The design assumes the use of a curved reflective polarizer (DBEF from 3M), which could be molded onto suitable aspheric geometries as in our case.

Table 2 System specification for ThinEyes® Foveated pancake optic

Basic system parameters	
Eye relief	15 mm
Distance Eye-Display	36 mm
Virtual Image	1.5 m
Pupil	4 mm
Wavelength band	Green Visible Central wavelength

	540nm FWHM 70nm
Eye rotation (gazing directions)	0° - 30°
Full field of view	Circular of 100°
Focal length and VR resolution (for 0° field)	49 mm, 24 PPD at the centre
Lens materials	Zeonex – “ZE48R” Polystyrene
Digital display: AMOLED	
Size	2.85” diagonal, square
Resolution	1440 x 1440
Display pixel pitch	35.5 μm

The presented 2D profiles (Fig. 4.17) are for a 4mm unvignetted pupil. The central focal length of the system was designed to be 49mm and gradually drops down to 13mm at the edge of FoV. The surfaces are represented through Qbfs representation as it was found to be suitable for the optimization process because of its orthogonal properties [29]. The optimization algorithm (reverse ray tracing, transverse ray aberration merit function) itself included user defined constraints to control the mapping function (explained next) for various field angles along with weights to control aberrations and mapping invariance across the eye box of 10mm to accommodate different pupil positions and, additionally, eye rotations (up to 30°).

Figure 4.17. Ray trace profile of the system when eye is looking frontwards for various peripheral angles (left) and the same when directly gazed at (right).

Fig.4.18 shows the mapping function (transformation of the object pixel to the VR pixel) for the system compared against other traditional mapping schemes namely constant focal length ($y - \theta$ mapping, typical in many commercial VR headsets) and rectilinear mapping ($y - \tan(\theta)$, which is the distortion-free one).

Figure 4.18. Plots illustrating the distortion observed over the entire FoV (left) and comparison of the mapping in the presented ThinEyes® design against other conventional optics (right).

It is to be noted that the derivative of these curves gives the information about the focal length of the system. As it can be observed from the plot, for the presented design, the derivative at the origin is larger in our design than the other conventional mapping types and thus points towards a larger focal length at the center and is smallest at the edges.

Consequently, our system has a focal length of 49mm, 30mm and 13mm at the center and periphery angles of 25 and 50 deg, respectively. This is in direct consequence of the system design being adapted to the human eye resolution and hence the pixels appear denser with a very high resolution at the center and becomes progressively coarser towards the edge of the periphery. In comparison, the rectilinear mapping has a focal length of only 22mm at the center resulting in a lower resolution of 10.92 pix/deg when compared to a constant focal length mapping which has a ppd of 14.4 (1440 pixel/100 deg) only, when compared to our design, which from the focal length of 49mm has a ppd as high as of 24. This is a gain of 2.2x relative to the rectilinear mapping. Figure 4 also shows how the distortion evolves across the full FoV. As mentioned earlier, this design presents a strong distortion (53% at 50 deg field angle) which can be compensated through software correction. But more importantly, it also gives us a measure of how the central region is emphasized more and is progressively deemphasized towards the periphery.

As explained before, for a given field, this design makes its imaging performance is the best when the eye is gazing it (so that VR pixel is imaged on the fovea), and gradually lower when the angle between the field and the eye lens optical axis (i.e. the so called peripheral angle) increases (so that VR pixel is imaged gradually further away from the fovea). Figure 4.19(a) shows on the MTF curves for the 15 deg field when the eye is directly gazing (with a 4 mm pupil), and on Figure 4.19 (b) the MTF curves for the same field when the eye is gazing frontwards. As can be appreciated, the MTFs in the gazed case are much better, particularly up to the Nyquist frequency of the display ($1/(2 \times 0.0355) = 14$ lp/mm (lines per mm)). For the case in which the eye is gazing frontwards, the eye is only capable to resolve about 15 arc min [30], or 2 cycles/deg, so the relevant spatial frequency (indicated in Figure 4.19 (b)) to compare with is 2.9 lp/mm, much lower than the display Nyquist frequency of 14 lp/mm. At 2.9 lp/mm, the MTFs on Figure 4.19 (b) are above 0.6.

In order to further explain this concept, Figure 4.19 (c) shows the system performance as RMS spot diameter versus the field angle for two situations. In Figure 4.19 (c), the 4 mm pupil is gazing the fields (as was shown in Figure 4.16 on the right), and the display pixel size is shown as a dashed line for reference. In Figure 4.19 (d), the pupil stays fixed looking frontwards (as was shown in Figure 4.16 on the left), and now the reference is a dashed curve we have computed as the pixel size that can be resolved by the eye in that peripheral vision. In this case, the solid design curve is below the dashed curve, indicating the adaptation of our design to how the eye works.

Figure 4.19. MTF curves of our pancake design for the field 15 deg for a 4 mm pupil when the eye is directly gazing the field (a) and when the eye is looking frontwards (b) along with plots illustrating RMS spot diameter for when directly gazed at (c) and when eye is looking frontwards (d).

The following illustrations shows the complete system performance analysis quantified through the ratio of the display pixel size to the RMS spot diameter map across the entire gazing FoV for the gazable fields. These are limited to $\pm 30^\circ$, since the aperture stop of the system is defined by the rim of the surface closest to the eye. As it can be observed

in Figure 4.20, the system when directly gazed at for the nominal position (zero pupil decenter) has superior imaging performance. The figure also illustrates the same through the polychromatic geometric MTF for the same gazing cone plotted up till the Nyquist frequency of the display. Fields exhibit excellent MTF values of above 0.4 for all frequencies up until 14 lp/mm, with a slight drop of the tangential MTF at 30 deg, which is very unlikely to be gazed in practice. Also, the field performance at the edge of the gazing cone is limited by chromatic aberration and this can also be inferred from the plot.

Figure 4.20. Normalized polychromatic RMS inverse spot diameter map across the gazing FoV (left) and polychromatic geometric MTF plot for a gazing cone of +/-30° (right).

To quantify the consistency in the performance of the system, the ratio of the pixel size to the polychromatic RMS spot diameter for different gazing directions for an angular cone of +/-30° was repeated for different pupil decentres across the eyebox. The ratio is truncated to a maximum value 1, so all spot diameters smaller than the display pixels will be represented with the value one. The pupil size itself was taken to be 4mm. The 10 mm targeted eye box implies eye sphere decentres of +/-3 mm. The results are shown in Figure 4.21. The plot shows a consistent performance across the eyebox and a little drop of 0.3 at the very edge of the eyebox (pupil decentre of 3mm, Fig.4.21 (d)), thus demonstrating superior imaging characteristics for the gazing directions. Notice that

besides this shift, our design allows the eye ball to rotate up to ± 30 degs. This means that, even without any eye ball centre shift, the eye is considered to be rotated so the envelope of all pupils within ± 30 degs defines an “enlarged” pupil of about 16 mm.

The above exercise demonstrates the effective of the ThinEyes® design strategy in improving the overall optical performance of any given system, adapting it to the human eye characteristics and thus making it a viable candidate to be used in a head mounted display. In particular, the above example presents an an advanced VR optical design based on ThinEyes® technology applied to a pancake optic resulting in an ultra-compact system weighing only 15g (optics only) with a short track length of 36mm for a circular FoV of 100° . Ray tracing simulations show consistent performance across the eye box of 10mm, adapted to human eye acuity resulting in a superior imaging performance.

Figure 4.21. RMS spot diameter map for a gazing cone of $\pm 30^\circ$ for different pupil positions across the eyebox of 10mm.

Near-Eye light field display is already an extremely compact multichannel system, although with a very limited spatial resolution. In the pursuit of a design that will manage to fulfil all the requirements of an ideal HMD design we present here a series of multiple channel optics which uses ThinEyes® technology developed by LIMBAK. This class of ThinEyes® devices consist of a display that generates an image and an optical system made of a lens array, arranged to generate an immersive virtual image from the display image. Each channel images one part of the full virtual image. An important feature of the following ThinEyes designs is that they use freeform surfaces instead of rotationally symmetric ones. Freeform geometry provides us with additional degrees of freedom that can be used to improve the resolution of our system or increase the field of view maintaining or even decreasing the volume. Each lens can be designed and addressed independently, having in mind cross-talk issues between channels. This is a very important feature, since different parts of field deal with different aberrations. For example, if we have a 5-channel system with a central lens, and four lenses surrounding it, the aberrations that appear in the central lens differ from the ones that appear in the other four. Size of each lens can also vary, as well as its local field.

In this section, we present various ThinEyes designs using the foveated approach namely (1) A diffractive-refractive hybrid two fold design (2) 2 nine-fold refractive designs named Omega1 and Omega 2, all exhibiting high field of view and spatial resolution as explained in previous sections pertaining to a VR headset.

4.3.2. Ultra-compact multichannel freeform optics for 4xWUXGA OLED micro displays

In this section, an advanced optical design for a high-resolution ultra-compact VR headset for high-end applications based on multichannel freeform optics and 4 OLED WUXGA micro displays developed under EU project LOMID is detailed [31]. Conventional optical systems in VR headsets require large distance between lenses and displays that directly leads to the rather bulky and heavy commercial headsets we have at present.

With the use of multichanneling technique, explained previously, the required display itself was reduced along with the total track length of the final optical system was only 36mm (to be compared to 60-75 mm in most conventional headsets). This ultra-compact optics allows reducing the headset weight and it occupies about a fourth of volume of a conventional headset with the same FOV. Additionally, this multichannel freeform optics provides an excellent image quality and a large field of view (FOV) leading to highly immersive experience. Unlike conventional micro lens arrays, which are also multichannel devices, this design uses freeform optical surfaces to produce sharp images even while operating at high angles of incidence to make sure that the optics is not the limiting factor when the eye is directly gazing at a particular portion of the scene.

The LOMID micro displays used in this headset are large-area high-resolution (WUXGA) micro displays with compact, high bandwidth circuitry, including special measures for high contrast by excellent blacks and low-power consumption. LOMID micro display diagonal is 0.98" with 16:10 aspect ratio. With two WUXGA micro displays per eye, this headset has a total of 4,800x1,920 pixels, i.e. close to 5k. As a result, this multichannel freeform optics provides a VR resolution 24 pixels/deg and FOV of 92x75 degs.

The architecture of the optics present in this prototype is based on a multichannel strategy, which can also be observed in nature and has been explained previously. To state briefly, the input bundle of rays is split up into multiple individual channels before the optical processing and "stitched" back for the final delivery. The advantage in doing so is to exploit the close to paraxial nature of the ray bundles in these sub-channels, which is easier to design for. Multichannel micro lens type solutions have been proposed before [20], but they suffer from low optical resolutions and consist of rotationally symmetric lenslets which have their own limitations while working at higher angle of incidences.

Parameter	Specification
OLED microdisplay	
Display diagonal	1"
Active display area	21.1 mm x 13.2 mm
Nominal display resolution	1920 x 1200 WUXGA (1920 x 2400 per eye)
Pixel pitch	11 μ m x 11 μ m
Optical system	
Type	Freeform 2-fold lens + 2 aspheric diffractive doublet lenses
Focal length, VR resolution (for field 0 deg)	15.5mm (adapted to human eye resolution),24.6 ppd
Number of freeform surfaces	2
Number of aspheric surfaces	3
Number of diffractive aspheric surfaces	1
Field of View	92° H x 75° V
Image quality	MTF > 20% at 45cycles/mm

The final freeform design consists of a multichannel freeform lens (traversing from the eye towards the display) followed by a hybrid diffractive-refractive aspheric doublet, i.e., a rotational symmetric aspheric doublet with a diffractive kinoform surface. The freeform lenslet itself was represented using orthogonal polynomials. Design strategy based on multichanneling makes optical tiling with two micro displays per eye (four in total for the entire headset). Hence, this optical system is composed of one 2-fold multichannel freeform lens and 2 aspheric diffractive doublet lenses per eye. The freeform lenslets have two refractive surfaces while aspheric doublet lenses have one diffractive kinoform surface and 2 refractive surfaces. Therefore, we have 5 optically active surfaces (i.e. rays undergo a total of five refractions while travelling from the display to the eye where one of them is inside of the doublet lens). The freeform lens was

manufactured using Zeonex (Z-E48R) and the doublets are made by cementing PMMA and PS simple lenses. The use of a diffractive surface in combination with a doublet lens provided the proper color correction.

Figure 4.22. Plot showing the evolution of the diffractive feature size along with radial coordinate of the diffractive surface (Left) and the ray trace profile of the entire system (Right).

It is well known [32] about the effect of the feature size on the diffractive efficiency when manufacturing constrains on the feature shape are considered. The diffractive features were strictly controlled in the design process and all local zone widths were maintained above a certain threshold to maximize the efficiency. This was accomplished through the introduction of a constraint in the optimization process which limited the gradient of the diffractive phase profile not to be above the inverse of the threshold size. The diffractive kinoform possesses rotational symmetry and is engraved on top of the positive lens of the aspheric doublet described using Qbf representation. The ray trace profile for the optical system along with the diffractive phase profile of the kinoform can be seen in Figure 4.22. Note that the profile shown constitutes the optics for only one eye and the freeform lens has plane symmetry respect to $y=0$ plane.

Figure 4.23. Normalized polychromatic RMS inverse spot diameter map across the gazing FoV (left) and polychromatic geometric MTF plot for a gazing cone of $\pm 30^\circ$ (right).

The above plot illustrates the performance of the system for different gazing directions within the gazing FOV. Since these directions are when the image is directly formed on the fovea, it is made sure that we have sharp image formation. From the plot, it is clear that consistent performance is achieved along the gazing field of view (elliptical in this case, $60^\circ \times 38^\circ$). Square wave polychromatic DMTF was evaluated across the entire visual field inside the directly gazed elliptical FOV of $60^\circ \text{ H} \times 38^\circ \text{ V}$ degrees. The polychromatic MTF plots were evaluated for the centered pupil of 4mm diameter at the spatial frequency of 45cycles/mm, which corresponds to the Nyquist frequency (i.e. the threshold spatial frequency of the LOMID microdisplay). The DMTF is 0.9 at 45cycles/mm for the central field (0,0), and for two FOV marginal fields (0,60) and (38,0) DMTF at Nyquist frequency is 0.2 and 0.3, respectively again reiterating excellent performance across the gazing field of view.

Figure 4.24. RMS spot diameter map for a gazing cone of ± 30 deg for different pupil positions across the eyebox of 10mm.

The above plot quantifies the system performance across the gazing field of view for different pupil positions in the eyebox. As it can be seen, the system has a consistent performance until 3mm pupil decentre where there is some drop for gazing angles greater than 10 deg. Notice that, this pupil position is a very rare occurrence and even so the performance drop observed here can be deemed acceptable. The same consistent performance can be seen for, when repeating the above exercise for the peripheral field of view (note that, in this exercise, the system performance bottleneck is dictated by the eye resolution unlike in the previous one where the bottleneck was the display pixel pitch itself).

Figure 4.25. RMS spot diameter map for the peripheral field of view for different pupil positions across the eyebox of 10mm.

4.3.3. Omega 1 – 9 fold freeform ThinEyes® design

Omega 1 freeform design was a proposal for a freeform optical imaging system for a virtual reality headset using a display with resolution of 1080x1080 pixels with a size of 2.18 inches in diagonal and a pixel pitch of 36 microns. Then the pixel density in pixels per inch (ppi) is $(2,54 \cdot 10^4 \text{ um/inch})/36\text{um} \approx 700 \text{ ppi}$, which can also be calculated as $1080 \cdot 2^{1/2}/2.18 \approx 700 \text{ ppi}$. This design was proposed for a display which was to be manufactured in the recent future and was conceptualized to be comprising of 9 lenslets/channels, with the central lens being made of rotationally symmetric surfaces and the surrounding lateral lenses of freeform surfaces. The material of the lens was fixed to be zeonex (Z-E48R) as it proved to be easier to work with, going by LIMBAK's earlier experience in dealing with zeonex. The 9 fold configuration was chosen so that we could get a much more compact design, in terms of eye relief (which is the distance between the eye pupil and the vertex of the first surface) and high field of view, and simultaneously having high performance metrics that is required for a VR headset. The central lens being

rotationally symmetric comprises of mapping the central portion of the display. The lateral lenses, thereafter, have to map the remaining portion of the display and owing to the geometry of the leftover portion of the display, the lateral lenses have to be freeform in nature. The lateral lens which is designed to be on top of the central lens is then duplicated by rotating it through multiples of 45 degrees to complete the mapping of the remaining area of the display. We will initially describe the design of the central lens and then proceed on to the lateral lens' design.

Figure 4.26. The 2d layout for the design of the central lens along with other system parameters.

The layout for the central lens is shown above. The eye relief, which is the distance between the eye pupil and the vertex of the first surface, comes out to be 10mm which is very compact when compared to its other counterparts. The focal length at the central part was designed for 11mm. A standard transverse wave aberration merit function (polychromatic) was used along with user defined constraints for the focal lengths of the system for different peripheral angles. The surface representation used was that of Qbfs asphere polynomials which are orthogonal is slope [29].

Table 3 System parameters for Omega 1

System parameters : Omega 1	
Eye relief	10 mm
Distance Eye-Display	38 mm
Virtual Image	1.5 m
Pupil	4 mm
Wavelength band	Green Visible Central wavelength 540nm FWHM 70nm
Eye rotation (gazing directions)	0° - 25°
Full field of view	Circular of 100°
Focal distance (for 0° field)	11 mm
Lens material	Zeonex
Digital display: AMOLED	
Size	55.4mm diagonal
Resolution	1080*1080
Opixel pitch	36 μ m
Nyquist frequency	13.9 cyc/mm($1/2*0.036$)

The central lens is designed up to an angle of 10 deg with the eye gazing directly. The lateral lens will then be designed starting from this point. One important design criteria for this step is the definition of the pupil range (region of the eye sphere illuminated by a single group of object pixels through the corresponding lenslet) which will limit the optical cross talk between the two lenses. For this purpose, we make use of the definition of the pupil range as described in [33] and we illustrate this in Figure 4.27.

Figure 4.27. Illustration of the definition of the pupil range condition.

The ray corresponding to eye gazing directly 15 degrees more than the cut point towards the intersection point and the ray corresponding to eye gazing directly 20 degrees below the cut point towards the intersection point needs to be parallel after the refraction by the lens. The corresponding tilts and decenters of the vertices for the continuation of the lateral lens is decided based on this definition. The incoming rays from the eye gazing directly above and below the cutting direction of 10 deg is represented by i_1 and i_2 in the Fig 4.27. The normal of the vertex of the lateral lens at the cut point is represented by n_1 . β is the angle of the incoming ray i_1 with respect to z axis, which is known. From the sine law we have:

$$\sin(\alpha + \beta) = n \sin(\gamma) \quad (4.1)$$

But from basic geometry: $\gamma + \delta = \alpha$. From these two equations, α can be deduced which gives the tilt that needs to be given to the vertex of the lateral lens to satisfy the pupil range condition. The lateral lens is then represented using standard x-y freeform polynomial and the same merit function used for designing the central lens is used here as well with appropriate user defined constraints for the focal length as we progress along the edge of the lens or until the mapping of the rays reaches the end of the display. After obtaining the lateral lens, it is then rotated 45 deg throughout to complete the entire system of lenslets.

Figure 4.28. Centroid position on the display (Left) and Focal length evolution (Right) for different peripheral angles.

The focal length distribution resembles that one of foveated imaging system. The central focal length is about 11mm and falls to 9.8mm for the central lens and has the extremities of 13.8mm, 11.6mm for the lateral lens. This adaptation results in the i-pixels looking bigger in the central part of the field of view and smaller as we progress to the edge of the display and this is not a desirable characteristic. This is addressed in the future iterations of this design.

Figure 4.29. (a) A front view illustration of the entire system along with some design rays and their hits on the display and 3D representation of polychromatic RMS for (b) different eye rotations (c) peripheral angles.

Normalized polychromatic (to o-pixel pitch of 36 μ m) 3D RMS spot size plots for gazing direction of $\pm 20^\circ$ of the resulting system for a pupil size of 4 mm, as well as the whole FOV of $\pm 50^\circ$ for the on-axis fixed pupil position are shown in Fig 4.29. These plots give an insight of how the image quality will degrade progressively when moving off-axis. It should be noted that the normalization is done taking into account the pixel pitch of the display and the resolution for peripheral angles of the human eye, as explained previously.

As it can be observed, the design rays seem to not have any cross talk when navigating between different folds of the lenses. Also, the overall RMS performance of the system seem to be above 0.4 for most of the field of view for different eye rotations and peripheral angles. This normalized value of 0.4 has been deemed satisfactory with LIMBAK's experience with its previous prototypes. As such the above design is a very good starting design for any future iterations of 9 fold freeform designs.

4.3.4. Omega 2 - 9 fold freeform design

The omega family of freeform designs were a proposal for 9 fold freeform designs for a VR optic having high resolution and high field of view with compact overall length. As such, Omega 1 was the first proof of concept for the same and was designed keeping in view of a futuristic display. Omega 2, on the other hand, is designed for a demonstrable display specifically for the display accompanying the Note 4 model of the famous Samsung's flagship mobile phones. The design rules of the design closely follow that of omega 1 and the characteristics of the display and the system itself can be found below.

Table 4 System parameters for Omega 2

System parameters : Omega 2	
Eye relief	13 mm
Distance Eye-Display	38 mm
Virtual Image	1.5 m

Pupil size	4 mm
Wavelength band	Green Visible Central wavelength 540nm FWHM 70nm
Eye rotation (gazing directions)	0° - 43°
Full field of view	Circular of 120°
Focal distance and VR resolution (for 0° field)	29 mm, 12 ppd
Lens material	Zeonex
Digital display: AMOLED	
Size	5.7 inches diagonal
Resolution	2560*1440
Opixel pitch	43 μm
Nyquist frequency	10.1 cyc/mm

The omega1 design still suffers from the concern of VR pixels appearing bigger when traversing from central to the lateral lens opposed to it become progressively becoming smaller when going from central to the lateral parts in the field of view of the VR space. This can be inferred from the focal length evolution plot, there is a small jump in the focal length when the transition happens from the central lens to the lateral lens. To overcome this challenge, we introduced a lens configuration similar to the one reported in [34], after the central lens making it a two lens system at the center of the FOV. The final design now consists of 2 rotationally symmetric lenses at the central region followed by a freeform lens to complete the rest of the field of view. This resulted in an overall increase in the focal length at the center and made the transition from central to the lateral parts of the field of view smoother. As a consequence of this smooth

decreasing transition of the focal length, VR pixel magnification also follows the same trend, i.e. the magnification becomes progressively less towards the edge of the periphery.

Figure 4.30. 2d layout of the system with super resolution added at the center (Left) along with its focal lengths (Right).

It is very clear that the addition of the super resolution configuration to the central part of the system resulted in the VR pixels appearing progressively smaller from central to the edges of the field of view of the system. This is demonstrated by the decreasing trend in the focal length (Fig 4.30) when compared to the focal length trend observed in Fig 4.28 for the design without the super resolution lens. This is believed to be because of the exchange in the degree of freedom between the two lenses in the central region resulting in the relaxing of the design constraints (point and normal) at the cutting point. This also resulted in a gain of 1.5x in terms of pixels per degree when compared to a conventional 9 fold design comprising of rotationally symmetric surfaces, having constant focal length without any foveation, for the same set of system parameters.

Figure 4.31. Plots illustrating the RMS performances for different eye rotations (Left) and various peripheral angles (Right).

In summary, these families of 9 fold freeform design models represent a new class of freeform optical models which can be used with VR headsets requiring compact track length along with high resolution and field of view. Further exploration in the above mentioned lines of optical designs can prove to result in superior – performing HMDs in optical terms.

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Chapter 5

DESIGN AND CHARACTERISATION OF A TWO CHANNEL FREEFORM OPTICAL DESIGN FOR A VIRTUAL REALITY HEADSET

5.1. Beta 3 – a two channel freeform optical design

We have seen the basics of the ThinEyes® technology [1] in the previous chapter. The current chapter deals with a finished prototype which was designed using the ThinEyes® design rules and was given the title Beta 3.

As we have seen in the earlier chapters, one of the biggest challenge in the design of optics for HMDs is its Total Track Length (TTL), which points to the overall length of the resulting system. Keeping in mind that these HMDs are to be worn by the end users who require it to be highly compact, thus also being lighter in terms of weight is a challenge that current optical designers face. Field of view (FOV) is the perceived angular size of the virtual screen and this value needs to be as high as possible to allow for the end user's full immersive experience. The immersive feeling is lost if the spatial resolution projected in the virtual screen is not high enough for the user to perceive individual pixels, thus, destroying the "real" feeling of the virtual reality. All the above mentioned quantities

make up for what will be described next as the “Trade-off triangle of virtual reality optics”.

5.1.1. The Trade-off triangle of FOV, Compactness and Resolution in virtual Reality optics

Traditional optical designs made use of rotationally symmetrical optical surfaces to compensate for aberrations plaguing the image quality. This was warranted by the limitations of the fabrication technologies that existed during earlier times. Presently existing optical models in commercial HMDs still make use of rotationally symmetric optical surfaces, thus falling into the so called trade-off triangle posed by Field of View (FOV), Compactness (often expressed as size^{-1}) and Optical Resolution of the VR pixels as seen by the user (see Fig 5.1).

Figure 5.1. Vicious triangle of VR optics : Tradeoff between field of view, compactness and resolution.

Many VR HMDs sacrifice one of the above quantities in the context of making a VR-HMD practically realizable. So far, there have been no reports of VR-HMDs simultaneously satisfying (an optimum value for all the three quantities resulting in a satisfying immersive VR experience for the end user) all of the above mentioned quantities in context of VR HMDs.

ThinEyes® technology aims at solving this tradeoff with the use of Multichannel – Freeform optical models to be used for VR-HMD optical models.

Multi-channeling is inspired from the nature as noted in [2-5], where a big FOV single channel is split into multiple smaller sub-channels with smaller FOVs. With this splitting, it becomes easier to correct for aberrations because of the paraxial nature of rays in the individual sub-channels. Stitching is then implemented finally to combine the individual sub-channels to obtain Big FOV with superior imaging characteristics. Splitting and/or stitching can be optical or electronical mainly depending on the application. In VR applications the electronic image is split electronically and so a split real image is shown in the digital display. This split image is the stitched optically restoring a virtual image without any split. The main goal of this technique is doing this restoring without any visible seam.

Figure 5.2. Illustration of multi channeling for systems with high FOV.

Even though Multi-channeling has existed for long, its use with freeform surfaces was not implemented in imaging systems due to the limitations in fabrication technologies. Thanks to the development of high quality injection molding manufacturing techniques, it was first tested out in phases in Non-imaging systems where the imaging constraints are not rather strict [6-8]. But with further developments in the area of freeform fabrication, the use of freeform optical surfaces has become a common sight in imaging systems as well. We have reported earlier about how this has prompted the exploration of some imaging theories now being extended for systems having no symmetry (or

freeform prescription) [9-11]. It has been very well documented how the use of freeform surfaces result in compact optical systems with reduced volume [12]. This was the main idea which prompted its use under the context of VR-HMDs along with multi-channeling which simultaneously satisfies the FOV and compactness criteria mentioned earlier.

5.1.2. Super Resolution and Foveation – Concepts and Principles

We have seen how “freeform – multichanneling” can achieve highly compact, large FOV optical designs which can be used with VR-HMDs. This leaves us to come up with a technique to also achieve high values for spatial resolution in the VR image space as perceived by the end user. The end user in the context of a VR headset is the human being who visualises the entirety of the VR scene through their eye. The typical properties of the human eye has already been explained in detail in the previous chapter, which in summary states that, there are regions in the eye with varying distribution/density of receptors. The central region where the density of receptors is higher is called the fovea.

Figure 5.3. Distribution of the photoreceptors (2D cross-section) along a line passing through the fovea. Image source [13].

There are two type of photoreceptors that are found in the human eye namely, rods and cones. Rods are the receptors that are responsible for vision at low light levels and they have a very low spatial acuity. On the other hand, cones which populate the fovea exclusively, have higher spatial acuity. Thus, it is clear there are different parts in the eye having different physical characteristics which directly make the human vision a

unique imaging system. Consider now a typical virtual reality optical imaging system consisting of a digital display whose virtual image is created by an optic, which is then *seen* by the eye. An efficient use of the o-pixels would be to assign the same ratio of pixels per eye sensor. The optimal scenario would be where there is one pixel per sensor unit. A higher ratio of pixels per sensor would be useless since each sensor cell detects only the radiant power but not where it comes from (it is a non-imaging detector), so the power from all the pixels assigned to a sensor is mixed and perceived as a single pixel. A lower ratio of pixels per sensor makes the image blurry. Since the density of pixels across a digital display (o-pixels) is constant, the optimal situation would assign most of the area of the digital display to the sensor located in the fovea. This could be achieved with an optical system imaging the digital display on the retina (through the eye) such that the magnification is proportional to the density of sensors on the retina. This means that the focal length corresponding to the pixels imaged on the fovea is much larger than that observed in the other scenarios.

Since in general the optical system is constant with time and, in VR applications, the optics is static relative to the human head, then the position of the fovea is not constant with respect to the optics (since the eye rotates inside the eye cavity, so the eye moves relative to the skull) and imaging a given portion of the digital display to the fovea is not possible. A possible solution to this problem is using an optics fixed to the eye (as contact lenses) combined with a pupil tracking system which give us the position of the fovea. Such a solution is nowadays expensive and considered invasive.

When there is no precise information about the position of the fovea there is still statistical information about this position. Then it is possible to distinguish portions of the digital display that may be imaged into the fovea from other portions which only reach the retina as peripheral vision. Then we can assign different magnifications to the regions of the digital display as a function of the region of the retina where it may be imaged. These different magnifications will give an optimum use of the pixel information

for the situation in which we only know a statistical information of the fovea position. Note that this different magnification will produce a distortion which should be electronically corrected, as explained later in the chapter. Gazeable regions of the virtual image will appear in the digital display greater than regular. The different magnification is characterized by the mapping function which gives the o-pixel to i-pixel correspondence established by the imaging system.

We term foveation as this weighting assignation in the optimization process which takes into account both where does the ray comes from in the digital display (and so the likelihood of that point of the digital display to be directly gazed) and the human eye acuity where the ray hits the retina.

The combination of foveation and the distortion given by the mapping function allows us to concentrate the scarce number of degrees of freedom in the design process and the scarce number of available o-pixels in improving the image quality seen by the user.

Figure 5.4. Plot illustrating foveation in ThinEyes® lenses exhibiting different magnification for different peripheral angles [14].

By taking into consideration the eye rotations while gazing at a portion of a scene, the image is sharply formed at fovea. But when the same portion of the scene is perceived “peripherally”, as explained in the previous chapter, the resolution that can be perceived

by the eye drastically drops depending on the human eye resolution for that peripheral angle. We take advantage of this phenomenon while designing our lenses, which is, we take into account the eye rotations and optimize for the sharpest image of the VR pixels to be obtained when they are directly gazed at. This also results in a lens which creates variable magnification to make VR pixels perceived by the user in the VR-space to be denser in the areas where they are directly gazed at and coarser when perceived peripherally. This can be seen from the plot shown above which describes the typical trend in the focal length (and hence the magnification of the VR pixels) of lenses using ThinEyes® technology. This was previously also observed in the freeform designs presented in the previous chapter.

5.1.3. Design Rules and parameter definitions

The aim of this section is to elaborate on the various rules that were adhered to during the design process of the beta 3 lenslets. A sample 2D cross section of the design concept is shown below. The entire digital display is equally divided into two sectors which is then catered to by each fold of the beta 3 lens. Note that the lens has plane symmetry along the YZ plane.

Figure 5.5. The digital display is divided into two equal sectors which is then addressed by each fold of the beta lens [14].

To better explain the rules of the design process, let us consider the 2-d cross section of the lenslets as shown below. For any given direction of the rays following the gaze vector, which is the ray traced from the center of the eye, the lens focuses the rays emitted by a given object pixel (o-pixel) on the display following a particular mapping between the object pixel and the image pixel (o-pixel and i-pixel respectively). In the designs presented in this thesis, the virtual screen where the i-pixels are visualized by the user is placed at 1500mm. The mapping function which takes the form $y=F(\theta)$, where y is the o-pixel coordinate on the display and θ is the angle i-pixel makes with respect to the optical axis (z). This mapping function is very crucial since this is the measure which directly contributes to the apparent increase in the resolution of the pixels as visualized by the end user when gazed directly. Thus the mapping function can be selected such that its slope $F'(\theta)$, which is proportional to the size of the i-pixel perceived in the VR space is larger for large values of θ and smaller for the smaller values of the same since that is where the human eye movements usually occur. This kind of a selection ensures that the pixel density expressed in pixel per degree (ppd) is high in the center of the FOV where human eye movements usually occur and progressively decreases towards the periphery of the FOV.

Figure 5.6. Rules for the design of the freeform lens in 2D (left) and addressing of the cross talk condition between the two folds (right).

The prevention of cross talk is very important when it comes to designing multi-channel systems. The definition of the rays pertaining to cross talk between the two folds is

illustrated above. The maximum pupil rotation of 25 deg has been assumed and this ray when looking towards the central part of the lens as indicated above, is made sure does not cross over to the other fold/channel. This value for the maximum pupil rotation has been deemed appropriate through the experience with our previous designs which can be found in [15-16].

The system parameters for the Beta 3 prototype can be found below:

Table 5.1 System parameters for Beta 3

System parameters : Beta 3	
Eye relief	13 mm
Distance Eye-Display	39 mm
Virtual Image	1.5 m
Pupil	4 mm
Wavelength band	Green Visible Central wavelength 540nm FWHM 70nm
Eye rotation (gazing directions)	0° - 25°
Full field of view	Rectangular of 90°X60°
Focal distance (for 0° field) and PPD	21 mm , 12 ppd
Lens material	Zeonex
Digital display: AMOLED	
Size	54mm diagonal
Opixel pitch	36 μ m
Nyquist frequency	13.9 cyc/mm

The actual design process, usually for a freeform system, initially involves the design of the 2d profile in one of the planes (in our case YZ). The 2D curve is obtained keeping in mind, the focal length evolution that the system demands to fulfil the FOV in the y

direction along with the constraints of the cross talk incorporated into it. After obtaining this curve, it is then fit to a freeform polynomial, as described in [17]. Specifically, in this case, Forbes Q-polynomial representation was used [18-20]. These surfaces are then optimized using a merit function which comprises of the system parameters, which in our specific case, is dominated by the control of focal lengths for various fields along with their RMS spot sizes for the individual fields. The mapping function was also constrained in the merit function which was evaluated for different fields starting from only fields in the YZ plane and progressing to the fields in the XZ plane as well. Once the surface has been grown to encompass the desired FOV along with satisfactory RMS performance throughout amounting to good imaging characteristics, the optimization procedure is said to have accomplished its objective.

Figure 5.7. Illustration of the obtained system with its cross sections in two orthogonal plane. Also shown here are some representative rays which were used in the optimization process.

5.2. Results and Analysis

The final system obtained is shown above. As you can see the single fold which was designed is mirrored to complete the entire system. The total track length of the system from the eye pupil to the digital display is around 39mm. The central focal length is 21mm and falls to 17mm. This shows strong adaptation which supports foveation as explained in the previous chapter. It can also be noted that, because of the geometry of the two folds, there is a “Dead zone/Hidden zone” of approximately 5mm in the centre warranted by the enforcement of the cross talk condition. But this is compensated for by the implementation of foveation for the rest of the display to reach a FOV of 90 deg in y direction. Also shown below are the polychromatic RMS spot map throughout the FOV. As it can be observed, for all the peripheral angles when the eye is looking frontwards, the performance is always above a value of 0.4 which has been deemed satisfactory with LIMBAK’s previous prototypes. Also, note that, for the eye looking frontwards, the normalization is done taking into account the pixel pitch and the human eye resolution for that field angle.

Figure 5.8. Plots describing the evolution of the focal length of the system in y direction (Left) and the trace of the position of the centroid position for various fields for different peripheral angles (Right).

For the case when the eye is rotated such that pupil plane is perpendicular to the field direction ,also known as the gazing direction, is more important, since this is the region where the image is formed directly on the fovea. For this, the normalization is done by

using the pixel pitch of 46 microns alone. This region represents the area where there is maximum VR resolution and as it can be seen the plot below, the RMS performance is always above 0.4 for ± 20 deg gazing directions.

Figure 5.9. Polychromatic normalized RMS map over the entire field of view for (Left) when the eye is looking frontwards and (Right) when eye is rotated so that the pupil plane is perpendicular to the field direction.

Figure 5.10. Polychromatic RMS spot map for peripheral field of view for different positions of the pupil in the eye box.

Above results point to a design satisfying comfortably the design objectives proposed earlier. A small array of 200 lenses was manufactured by conventional injection molding in Zeonex (z-e48r) to demonstrate its proof of concept. The mold insert was fabricated through multi-axis single point diamond turning and the injected parts showed surface figures better than 20 μ m peak-to-valley and roughness $R_a < 10$ nm. One such lens and the final assembled prototype is shown below. Adherence between the manufactured parts to the designed ones were already validated by an earlier student in his thesis [21] and thus will not be dealt with in this thesis.

Figure 5.11. The manufactured Beta 3 lens along with the final assembled prototype.

5.3. Measurement of Lens distortion

The freeform multichannel optical systems presented in this thesis closely resemble those of apposition compound eye. Apposition multichannel optics always require that the image on the display need to be processed (in real-time) to the VR image is projected on the retina without any discontinuity or distortion. This object-to-image mapping is implemented in the software exactly as the distortion correction of how any conventional lenses is done, just with a different piece-wise continuous function. This section will describe the experimental set up which was designed to measure the distortion function of the ThinEyes[®] lenslets. The distortion function can be calculated by two methods : 1)

Through simulation – Using ray tracing to estimate the o-pixel to i-pixel transformation and directly calculating the distortion function. 2) Through an Experimental set up. Method 1 relies on an ideal model of the freeform lens in the sense that it does not take into account the practical limitations that occur during the lens fabrication itself which might result in a slightly different mapping function. Also, the distortion function obtained through this method was found to be satisfactory only in the central part of the FOV. To get a correct estimate for the entire FOV, it became important to obtain it through the experimental method. For measuring the distortion, we need to capture the distorted images as seen through the lenslets by a camera which can mimic the wide field of view of the human eye (which is intended to be the end user in the real prototype). For this purpose, a GoPro camera was selected owing to the fact that it has a wide field of view of approximately 120 deg. The GoPro is made up of a fish eye lens which has an inherent barrel distortion associated with it. So, capturing the images through GoPro and to get the true distortion function of the ThinEyes lenslets alone, we need to measure the distortion introduced by the GoPro camera lens. This was done by capturing an image of a meter scale through the GoPro lens. Since we know the true image spacing, the transformation done by the GoPro camera was easily calculated. The distortion curve of the GoPro lens is shown below.

Figure 5.12. Distortion of GoPro camera. On the horizontal axis is the image position normalized to the size of the display, while the vertical axis represents the angles of the field of view.

Before explaining the actual experimental set up that was used in the measurement, the following discussion gives a brief insight into the mapping that a generic two fold freeform ThinEyes® lenslet introduces in the optical system. This is shown in Fig.5.13. The presented system creates a far-field image, so the lens is designed such that the pixels on the display are presented to the eye as plane wavefronts. The above image, represents the case when the eye is looking frontwards and the wavefront b_1 is the result of two different wavefronts originating from the display namely, a_{11} & a_{12} centered on the o-pixels op_{11} and op_{12} respectively. This situation creates an overlap of the resulting images, and the eye sees pixels op_{11} and op_{12} overlapped. The solution is to provide the same information to op_{11} and op_{12} , i.e., the regions of the digital display that the eye sees overlapped must have the same image.

Figure 5.13. A generic two fold lens along with illustrative rays that define the mapping between the o-pixel and i-pixel.

The figure 5.14 explains this “image overlapping” with a simple example. On the bottom right, the digital display shows two images, in which a portion of the image is repeated, with a shift. Each half of the display is addressed by the corresponding lenslet. On the left, the image shows the virtual image as seen by the eye in the VR space. The figure also shows the relevant rays to define the edges of the mapping between opixels and ipixels. In this drawing, r_{0a} , r_{1a} , r_{2a} and r_{3a} indicates the i-pixel positions on the image seen by the eye.

As there is a partial duplication of the information on the display, ipixel ip1 is formed by the projection of two o-pixels op11 and op12. Each one of these opixels are then projected into the eye through different optical channels, and form the same parallel wavefront and thus they share the same information such that the eye can create a complete image from the two segmented image on the screen. Ipixel ip2 is outside the overlapping region, thus it is only created by opixel op21. The correct mapping between opixels and ipixels is a function of the focal length of each channel of the lenses as explained in the design rules in the previous section.

Figure 5.14. Illustration showing the edge rays of the mapping function along with the image shown on the display and the corresponding image as seen by the end user in the VR space.

Figure 5.15. Experimental set up that was used to measure the distortion function of the ThinEyes lenslets.

As seen in the previous illustration, the GoPro is set up on a XY translation stage which is used to control the movement of the GoPro as we traverse across the lens thus capturing the entire FOV. To do this, a grid of points is shown in the object space which is then captured by the GoPro, as seen by the Human Eye. The same situation is illustrated below.

Figure 5.16. A rectangular points used for the mapping as shown in the object space (Left) and the distorted image of the same as seen through the lens at the "Eye looking frontwards" position. This image represents qualitatively the transformation done by the lens from opixel to ipixel (Right).

As it can be seen above, the grid is displayed only for one fold. But the entire mapping exercise is repeated for the other fold as well, since due to manufacturing limitations the two folds have the possibility of being distinct. This is the image as seen by the eye, for the ideal position of the eye. Since this position alone, does not represent all of the possible eye positions, the GoPro is then displaced along the Y direction (for the range of the eyebox) so that the position of the eye when the hidden zone (as described in the previous sections) is seen, is mapped as well. We have taken into account an eye box of $\pm 3\text{mm}$, for which the GoPro was displaced in both the positive and the negative directions along y axis, through the use of the translational stage, to map the hidden zone. It has to be noted that, while the positive displacement of the GoPro along the y axis is necessary to map the “Hidden zone”, the negative displacements are necessary so that more of the periphery part of the FOV is mapped as well. This is necessitated by the fact that the pupil of GoPro is smaller than the eye pupil of 4mm and hence it needs more displacements to map the entire FOV as seen by the Human eye. The following illustration shows the image as seen from the GoPro for different positive displacements in y.

Figure 5.17. Illustrations showing distorted image for different positive y displacements of the GoPro for (a) 0mm (b) 0.5mm (c) 1mm (d) 1.5 mm (e) 2mm (f) 2.5mm and (g) 3mm.

The above represented images are then fused together to get a resulting image which encompasses all of o-pixel transformed to i-pixels. This is done using Photoshop where all the images corresponding to different Y displacements of the GoPro are carefully interlaced together. The challenge in obtaining the above images, especially since the lenses are freeform in shape, was to get the perfect alignment of the GoPro with respect to the image shown as seen through the lens. For this purpose, an alignment marker in the form of a red point was projected on the o-pixel which was taken as a reference.

The above point cloud was then taken into account and processed for the distortion introduced by the GoPro itself (which has rotational symmetric prescription in its distortion function). After this, the obtained data is then fit with a 10th order polynomial,

as shown below, to analytically model the distortion introduced by the Beta 3 lens. The fitting is described as follows:

$$\begin{aligned}
X_i = & f_{00} + f_{01} \cdot X_o + f_{02} \cdot X_o^2 + f_{03} \cdot X_o^3 + f_{04} \cdot X_o^4 + f_{05} \cdot X_o^5 + f_{06} \cdot X_o^6 + f_{07} \cdot X_o^7 + f_{08} \cdot X_o^8 + f_{09} \cdot X_o^9 + f_{010} \cdot X_o^{10} \\
& + f_{10} \cdot Y_o + f_{20} \cdot Y_o^2 + f_{30} \cdot Y_o^3 + f_{40} \cdot Y_o^4 + f_{50} \cdot Y_o^5 + f_{60} \cdot Y_o^6 + f_{70} \cdot Y_o^7 + f_{80} \cdot Y_o^8 + f_{90} \cdot Y_o^9 + f_{100} \cdot Y_o^{10} \\
& + f_{11} \cdot X_o Y_o + f_{21} \cdot X_o Y_o^2 + f_{31} \cdot X_o Y_o^3 + f_{41} \cdot X_o Y_o^4 + f_{51} \cdot X_o Y_o^5 + f_{61} \cdot X_o Y_o^6 + f_{71} \cdot X_o Y_o^7 + f_{81} \cdot X_o Y_o^8 + f_{91} \cdot X_o Y_o^9 \\
& + f_{12} \cdot X_o^2 Y_o + f_{22} \cdot X_o^2 Y_o^2 + f_{32} \cdot X_o^2 Y_o^3 + f_{42} \cdot X_o^2 Y_o^4 + f_{52} \cdot X_o^2 Y_o^5 + f_{62} \cdot X_o^2 Y_o^6 + f_{72} \cdot X_o^2 Y_o^7 + f_{82} \cdot X_o^2 Y_o^8 \\
& + f_{13} \cdot X_o^3 Y_o + f_{23} \cdot X_o^3 Y_o^2 + f_{33} \cdot X_o^3 Y_o^3 + f_{43} \cdot X_o^3 Y_o^4 + f_{53} \cdot X_o^3 Y_o^5 + f_{63} \cdot X_o^3 Y_o^6 + f_{73} \cdot X_o^3 Y_o^7 \\
& + f_{14} \cdot X_o^4 Y_o + f_{24} \cdot X_o^4 Y_o^2 + f_{34} \cdot X_o^4 Y_o^3 + f_{44} \cdot X_o^4 Y_o^4 + f_{54} \cdot X_o^4 Y_o^5 + f_{64} \cdot X_o^4 Y_o^6 + f_{15} \cdot X_o^5 Y_o + f_{25} \cdot X_o^5 Y_o^2 + f_{35} \cdot X_o^5 Y_o^3 + f_{45} \cdot X_o^5 Y_o^4 + f_{55} \cdot X_o^5 Y_o^5 \\
& + f_{16} \cdot X_o^6 Y_o + f_{26} \cdot X_o^6 Y_o^2 + f_{36} \cdot X_o^6 Y_o^3 + f_{46} \cdot X_o^6 Y_o^4 + f_{17} \cdot X_o^7 Y_o + f_{27} \cdot X_o^7 Y_o^2 + f_{37} \cdot X_o^7 Y_o^3 + f_{18} \cdot X_o^8 Y_o + f_{28} \cdot X_o^8 Y_o^2 + f_{19} \cdot X_o^9 Y_o
\end{aligned} \tag{5.1}$$

$$\begin{aligned}
Y_i = & g_{00} + g_{01} \cdot X_o + g_{02} \cdot X_o^2 + g_{03} \cdot X_o^3 + g_{04} \cdot X_o^4 + g_{05} \cdot X_o^5 + g_{06} \cdot X_o^6 + g_{07} \cdot X_o^7 + g_{08} \cdot X_o^8 + g_{09} \cdot X_o^9 + g_{010} \cdot X_o^{10} \\
& + g_{10} \cdot Y_o + g_{20} \cdot Y_o^2 + g_{30} \cdot Y_o^3 + g_{40} \cdot Y_o^4 + g_{50} \cdot Y_o^5 + g_{60} \cdot Y_o^6 + g_{70} \cdot Y_o^7 + g_{80} \cdot Y_o^8 + g_{90} \cdot Y_o^9 + g_{100} \cdot Y_o^{10} \\
& + g_{11} \cdot X_o Y_o + g_{21} \cdot X_o Y_o^2 + g_{31} \cdot X_o Y_o^3 + g_{41} \cdot X_o Y_o^4 + g_{51} \cdot X_o Y_o^5 + g_{61} \cdot X_o Y_o^6 + g_{71} \cdot X_o Y_o^7 + g_{81} \cdot X_o Y_o^8 + g_{91} \cdot X_o Y_o^9 \\
& + g_{12} \cdot X_o^2 Y_o + g_{22} \cdot X_o^2 Y_o^2 + g_{32} \cdot X_o^2 Y_o^3 + g_{42} \cdot X_o^2 Y_o^4 + g_{52} \cdot X_o^2 Y_o^5 + g_{62} \cdot X_o^2 Y_o^6 + g_{72} \cdot X_o^2 Y_o^7 + g_{82} \cdot X_o^2 Y_o^8 \\
& + g_{13} \cdot X_o^3 Y_o + g_{23} \cdot X_o^3 Y_o^2 + g_{33} \cdot X_o^3 Y_o^3 + g_{43} \cdot X_o^3 Y_o^4 + g_{53} \cdot X_o^3 Y_o^5 + g_{63} \cdot X_o^3 Y_o^6 + g_{73} \cdot X_o^3 Y_o^7 \\
& + g_{14} \cdot X_o^4 Y_o + g_{24} \cdot X_o^4 Y_o^2 + g_{34} \cdot X_o^4 Y_o^3 + g_{44} \cdot X_o^4 Y_o^4 + g_{54} \cdot X_o^4 Y_o^5 + g_{64} \cdot X_o^4 Y_o^6 + g_{15} \cdot X_o^5 Y_o + g_{25} \cdot X_o^5 Y_o^2 + g_{35} \cdot X_o^5 Y_o^3 + g_{45} \cdot X_o^5 Y_o^4 + g_{55} \cdot X_o^5 Y_o^5 \\
& + g_{16} \cdot X_o^6 Y_o + g_{26} \cdot X_o^6 Y_o^2 + g_{36} \cdot X_o^6 Y_o^3 + g_{46} \cdot X_o^6 Y_o^4 + g_{17} \cdot X_o^7 Y_o + g_{27} \cdot X_o^7 Y_o^2 + g_{37} \cdot X_o^7 Y_o^3 + g_{18} \cdot X_o^8 Y_o + g_{28} \cdot X_o^8 Y_o^2 + g_{19} \cdot X_o^9 Y_o
\end{aligned} \tag{5.2}$$

Where X_i represents the vertical angle of the ipixel and Y_i represents the horizontal angle of the ipixel, X_o represents the vertical coordinate of the opixel and Y_o represents the horizontal coordinate of the opixel.

The transformation of the opixel to ipixel is defined by the pair of equations: $X_i = X_i(X_o, Y_o)$ and $Y_i = Y_i(X_o, Y_o)$. From these equations, we have accurate knowledge of which ipixel arises from a given opixel through the distortion function information. In other words, we can accurately assign ipixels (X_i, Y_i) on the virtual image which will be addressed by a given opixel (X_o, Y_o) on the object (the digital display). The values for the coefficients $f_{m,n}$ and $g_{m,n}$ depend largely on the geometry of the lens shape itself. Through different iterations of performing the above exercise for various freeform lens, it was

concluded that to get an accurate representation for the introduced distortion, a minimum order of ten is required. This is necessitated by the freeform nature of the lenses lacking any inherent symmetry. After knowing the functions $X_i = X_i(X_o, Y_o)$ and $Y_i = Y_i(X_o, Y_o)$, we can find the inverse functions $X_o = X_o(X_i, Y_i)$ and $Y_o = Y_o(X_i, Y_i)$. With these inverse functions we know how we should distort a real image on the display to obtain a virtual image without any distortion when viewed through the freeform lens. The table below tabulates the distortion coefficients obtained through the mapping exercise.

Table 5.2 Coefficients for the distortion function as described above for the two channels.

Left Fold				Right Fold			
f00	-0.08288	g00	0.301659	f00	-0.02852	g00	-0.27339
f01	0.052077	g01	0.000394	f01	0.049255	g01	0.002004
f02	-0.00029	g02	0.001126	f02	-0.00022	g02	-0.00145
f03	4.01E-06	g03	3.98E-05	f03	0.000101	g03	1.31E-05
f04	-3.17E-06	g04	-3.41E-07	f04	3.76E-06	g04	-3.95E-06
f05	1.75E-06	g05	-4.87E-07	f05	6.23E-07	g05	-6.72E-09
f06	1.14E-07	g06	-1.06E-07	f06	-8.31E-08	g06	8.69E-08
f07	-1.85E-09	g07	-3.32E-09	f07	-6.64E-10	g07	1.56E-09
f08	-9.88E-11	g08	1.68E-10	f08	-9.07E-11	g08	3.87E-10
f09	-6.59E-11	g09	2.42E-11	f09	-9.65E-11	g09	6.81E-12
f010	1.50E-12	g010	-4.11E-12	f010	1.02E-12	g010	2.73E-12
f10	-0.00145	g10	0.063921	f10	0.000865	g10	0.068673
f11	0.001359	g11	-0.0001	f11	-0.00213	g11	-0.00123
f12	-0.00011	g12	0.000127	f12	0.000208	g12	0.000361
f13	-2.1E-05	g13	1.3E-05	f13	-5.6E-05	g13	-2.3E-05
f14	-7.34E-07	g14	-4.26E-07	f14	2.63E-06	g14	1.91E-06
f15	1.42E-07	g15	-3.76E-07	f15	-2.31E-07	g15	-1.34E-07
f16	-1.34E-08	g16	-4.23E-08	f16	3.44E-08	g16	-5.87E-08
f17	-5.98E-09	g17	3.16E-10	f17	7.77E-09	g17	-9.23E-10
f18	7.05E-11	g18	-2.37E-10	f18	-3.05E-11	g18	-2.60E-10
f19	-3.56E-12	g19	1.77E-12	f19	6.10E-12	g19	-1.14E-12
f20	0.000618	g20	0.000782	f20	-0.00271	g20	-0.0082
f21	-0.00031	g21	-1.2E-05	f21	0.000985	g21	0.001134
f22	-5.4E-05	g22	1.97E-05	f22	-0.00015	g22	-0.00011
f23	-5.10E-07	g23	-9.62E-07	f23	2.22E-05	g23	1.24E-05
f24	-5.20E-07	g24	-7.47E-07	f24	-1.43E-06	g24	3.15E-07
f25	-9.34E-08	g25	-7.49E-08	f25	-1.19E-07	g25	5.28E-08
f26	-7.69E-10	g26	-1.31E-08	f26	-4.77E-09	g26	1.58E-08
f27	-6.83E-10	g27	6.45E-11	f27	-9.34E-10	g27	1.32E-10

f28	2.83E-12	g28	-1.13E-11	f28	1.04E-12	g28	1.24E-11
f30	0.000322	g30	-0.0011	f30	0.001336	g30	0.002376
f31	-0.00017	g31	4.27E-06	f31	-0.00035	g31	-0.00045
f32	-1.3E-05	g32	1.09E-06	f32	4.59E-05	g32	2.35E-05
f33	-2.75E-07	g33	-8.01E-07	f33	-2.90E-06	g33	-2.77E-06
f34	-9.29E-08	g34	-2.89E-07	f34	2.47E-07	g34	-2.47E-07
f35	-1.97E-08	g35	-7.11E-09	f35	2.60E-08	g35	-6.88E-09
f36	-3.01E-11	g36	-1.11E-09	f36	3.01E-10	g36	-1.29E-09
f37	-2.15E-11	g37	1.67E-12	f37	2.92E-11	g37	-4.23E-12
f40	8.4E-05	g40	-0.00038	f40	-0.00034	g40	-0.00047
f41	-3.5E-05	g41	2.81E-06	f41	6.58E-05	g41	9.07E-05
f42	-1.68E-06	g42	-1.38E-06	f42	-7.03E-06	g42	-1.61E-06
f43	-1.32E-07	g43	-1.23E-07	f43	9.71E-08	g43	3.05E-07
f44	-7.82E-09	g44	-3.57E-08	f44	-1.95E-08	g44	3.38E-08
f45	-1.28E-09	g45	-3.41E-10	f45	-1.67E-09	g45	3.43E-10
f46	-1.04E-12	g46	-2.87E-11	f46	-6.93E-12	g46	3.28E-11
f50	1.36E-05	g50	-6.7E-05	f50	4.93E-05	g50	5.62E-05
f51	-4.35E-06	g51	5.60E-07	f51	-6.73E-06	g51	-1E-05
f52	-1.23E-07	g52	-2.87E-07	f52	5.93E-07	g52	-6.12E-08
f53	-1.55E-08	g53	-8.71E-09	f53	6.88E-09	g53	-1.73E-08
f54	-3.20E-10	g54	-1.79E-09	f54	7.21E-10	g54	-1.76E-09
f55	-2.72E-11	g55	-6.47E-12	f55	3.44E-11	g55	-5.74E-12
f60	1.39E-06	g60	-7.04E-06	f60	-4.45E-06	g60	-4.00E-06
f61	-3.36E-07	g61	5.54E-08	f61	3.93E-07	g61	7.02E-07
f62	-5.09E-09	g62	-2.25E-08	f62	-2.79E-08	g62	1.30E-08
f63	-7.08E-10	g63	-2.99E-10	f63	-5.52E-10	g63	4.85E-10
f64	-5.07E-12	g64	-3.15E-11	f64	-1.00E-11	g64	3.15E-11
f70	8.97E-08	g70	-4.56E-07	f70	2.52E-07	g70	1.66E-07
f71	-1.55E-08	g71	2.96E-09	f71	-1.30E-08	g71	-2.79E-08
f72	-1.11E-10	g72	-7.96E-10	f72	6.88E-10	g72	-5.87E-10
f73	-1.13E-11	g73	-3.98E-12	f73	1.03E-11	g73	-5.30E-12
f80	3.51E-09	g80	-1.78E-08	f80	-8.67E-09	g80	-3.65E-09
f81	-3.85E-10	g81	8.20E-11	f81	2.22E-10	g81	6.05E-10
f82	-9.63E-13	g82	-1.06E-11	f82	-6.91E-12	g82	8.72E-12
f90	7.61E-11	g90	-3.81E-10	f90	1.67E-10	g90	3.14E-11
f91	-3.99E-12	g91	9.24E-13	f91	-1.49E-12	g91	-5.50E-12
f100	6.98E-13	g100	-3.47E-12	f100	-1.37E-12	g100	5.73E-14

The above coefficients are then fed into a shader file [22] which is input to a demo created in Unity. The details of the same are beyond the scope of this thesis. However, the below images show the software correction introduced into the image to compensate for the distortion produced by the lens.

Figure 5.18. Software corrected image using the distortion coefficients obtained through the mapping process (Left) and Undistorted image as seen through the lens (Right).

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CONCLUSIONS AND FUTURE WORK

The contents of the thesis are the reflection of the author's understanding of freeform optical surfaces developed in the context of imaging optics and specifically through the use of head mounted displays (Virtual reality headset) as an application. The practical aspects of the imaging design itself was gained through the use of the commercial ray tracing software package, CodeV®.

As in any thesis, the preliminary ground work for this thesis is carried out through the study of basics in the field of imaging optics. Chapter 1 summarises the various metrics that are used in evaluating a given optical imaging system. Additionally some elaboration is given towards the terminology that goes with the qualifying an optical source and image. The major focus was split into two sections (1) to the introduction of freeform optical surfaces, starting from the need for their introduction to the development of surface descriptors and their translation in a given ray tracing software package, namely in terms of polynomial type representations. (2) A glossary of terms associated with the design of virtual reality optics where various terminology associated with an optical imaging system along with its performance metrics, that is accepted as a standard in the VR community was given to make the understanding of the designs presented in later chapter easier.

In Chapters 2 and 3, a basic introduction to aplanatic systems followed by their characteristics under freeform prescription is detailed extensively. A straightforward way of deriving a generalised version of the Abbe sine condition which predominantly governs the aplanatic nature of a system is given through the use of Herzberger's invariant and the use of etendue conservation theorem. This treatment does not assume any symmetry

in the optical system and does not involve the computation of ray and or wave aberrations of the generalized system under consideration, thus making it a simple way of explaining how the adherence of the Abbe sine condition by a stigmatic imaging system makes it aplanatic. Chapter 2 also establishes a basic mathematical layout of the SMS method and how when examined under the limiting case of the design points converging to a single point, leads to aplanatic systems. This logic is also used to establish the fact that for a freeform aplanatic system, which can be seen as a limiting case of a 3 point SMS system, in general, we require three optical surfaces. This is warranted by the fact that a plane, which describes a 3D source requires a minimum of 3 points for its complete definition. This connection between the SMS method and aplanatic systems are shown in detail through the use of individual examples namely: (a) a single surface Cartesian oval (b) a two surface SMS method along with its characteristic RMS spot diameter curve (c) a two surface aplanatic system as a limiting case of the system in (b) along with its characteristic curve showing smooth RMS performance for the design point and its neighbourhood. We use this smooth behaviour as a general measure of how aplanatic systems behave for different object position displacements. To substantiate the claims for a three surface freeform aplanatic system, we show through the use of the integrability condition how a two surface system fails to achieve freeform aplanatism. Additionally we also provide the partial differential equation formulation for a three surface freeform aplanatic system along with illustrative examples. For future work, more exploration into examples which fail the rule stating, in general, a freeform aplanatic system requires three surfaces (apart from the previously stated exceptions in the chapter) needs to be comprehensively studied. Additionally, the systems shown in this chapter have been a result of designing them through the SMS method at the limit. Since freeform optical design itself relies heavily on computer optimization method, an indirect method, where a merit function comprising of the stigmatic and the Abbe sine condition, whose zero (since this is an analytical solution, as opposed to freeform optical models

where the merit function reaches a minimum value which is not necessarily zero) leads to a freeform aplanatic system needs to be formulated.

Chapter 4 is a continuation in the exploration of the use of freeform surfaces for a specific application, namely a virtual reality headset (also interchangeably referred to as head mounted display or HMD in this thesis). An extensive prior literature survey about the history of HMDs is given along with its various characteristics such as weight, size of currently existing HMDs in the market. An in depth analysis of the ThinEyes® technology which is developed at LIMBAK, where a part of this thesis was carried out, is presented through a case study where this design strategy is applied to a traditional polarized catadioptric optic and is demonstrated henceforth how its performance sees significant jump with use of ThinEyes® technology. Consequently, two more freeform optical design models designed using this principle are presented, namely (a) A hybrid diffractive – Refractive freeform model, which was used with an OLED microdisplay for a compact, high resolution (24 pixels per deg) VR headset is detailed. (b) A nine channel freeform optical design, which when compared to a conventional nine channel optical model comprising of rotationally symmetric surfaces shows 1.5x gain in terms of VR resolution along with superior imaging performance throughout its field of view.

Chapter 5 details the design, development and characterization of a two channel freeform optic for a virtual reality headset. This chapter tracks the design rules which were employed in the conception of the freeform optic along with illustrative details of the mapping employed (which is a characteristic of ThinEyes® optic, also extensively detailed in Chapter 4) is described. This chapter also addresses the problem of duplication of object pixels at the intersection of the two channels and how the cross talk between the two channels is avoided by strict definition of the pupil range condition. Additionally, extensive description about the experimental set up that was used in the determination of the distortion/mapping function of the optic are provided along with illustrative pictures of the experimental set up. The chapter ends with a comparison of the VR scene

which is software corrected through the data obtained in distortion function measurement and actual scene (undistorted) as seen through the optic.

In regards to future work pertaining to Chapter 4, the nine channel freeform optic needs to be optimized for a wider eye box as in the case of other optical designs presented in this thesis, which were optimized for an eye box of 10mm. Measurement of the distortion function and MTF measurements of the hybrid refractive-diffractive optic is currently being done and is challenging when compared to its other counterpart because of its complex electronic circuitry, which can pose alignment problems in the experiment. Another future challenge would be is to come up with a tolerance metric for the freeform surface descriptor (in terms of sag deviation expressed in terms of its polynomial coefficients) and its possible inclusion in the optimization process itself to allow for some tolerance monitoring in the final fabrication of the optic. A possible addition of these tolerance metrics in the design and optimization phase will indeed result in better optical systems.

PUBLICATIONS

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