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An Approach to Incremental and Modular Context-sensitive Analysis

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Abstract

The flexibility of dynamic languages often comes at the cost of having to perform at run time a number of additional tasks when compared to static languages. This includes for example detecting property violations in calls to built-ins and libraries (as well as in user code) or performing dynamic specialization and other optimizations. Analyzing programs statically can help reduce the amount and cost of these dynamic tasks. However, performing global analysis can in turn be costly at compile time for large code bases, and this can be a special burden if it is performed at each development iteration. On the other hand, changes between development iterations are often isolated within a few components, and analysis cost can be reduced by reusing the results of previous analyses. This has been achieved to date through modular analysis, which reduces memory consumption and often localizes the computation during reanalysis mainly to the modules affected by changes, and through global incremental fixpoint analysis techniques, that achieve cost reductions at finer levels of granularity, such as changes in program lines, but which are not directly applicable to modular programs. This paper describes, implements, and evaluates a context-sensitive fixpoint analysis algorithm aimed at achieving both inter-modular (coarse-grain) and intra-modular (fine-grain) incrementality, solving the problems related to propagation of the fine-grain change information and effects across module boundaries, for additions and deletions in multiple modules. The implementation and evaluation of our algorithm shows encouraging results: the expected advantages of fine-grain incremental analysis carry over to the modular analysis context. Furthermore, the fine-grained propagation of analysis information of our algorithm improves performance with respect to traditional modular analysis even when analyzing from scratch.

CCS Concepts • Theory of computation → Invariants; Pre- and post-conditions; Program analysis; Program semantics; Abstraction;

Keywords Program Analysis, Dynamic Languages, Incremental Analysis, Modular Analysis, Constraint Horn Clauses, Abstract Interpretation, Fixpoint Algorithms, Logic and Constraint Programming

ACM Reference Format:

1 Introduction and motivation

Dynamic programming languages are a popular programming tool for many applications, due to their flexibility. They are often the first choice for web programming, prototyping, and scripting. However, this large degree of flexibility often comes at the cost of having to perform at run time a number of additional tasks when compared to static languages. This includes performing dynamic specialization and other optimizations, e.g., for dynamic method selection, or performing run-time checks for detecting property violations in calls to built-ins and libraries, as well as in user code (which are covered, at least in part, by full static typing or other forms of full static verification in static languages).

Static analysis can infer information that can help reduce the amount and cost of these dynamic tasks by specializing, eliminating dynamic decisions, or discharging assertion checks as much as possible at compile time, but at a cost: large, real-life programs typically have a complex structure combining a number of modules with other modules coming from system libraries, and context-sensitive global analysis of such large code bases can be expensive. This cost can be specially problematic in interactive uses of analyzers during agile program development and prototyping, one of the contexts where dynamic languages are particularly popular.
A practical example is detecting and reporting bugs, such as, e.g., assertion violations, back to the programmer as the program is being edited, by running the analysis in the background. This can be done at small intervals, each time a set of changes is made, when a file is saved, when a commit is made on the version control system, etc. In large programs, triggering a complete reanalysis for each such change set is often too costly. Another typical scenario is when updating the analysis information after some source-to-source transformations and/or optimizations. This usually involves an analyze, perform transformation, then reanalyze cycle for which it is clearly inefficient to start the analysis from scratch.

A key observation is that in practice, each development or transformation iteration normally implies relatively small modifications which are isolated inside a small number of modules. This property can be taken advantage of to reduce the cost of re-analysis by reusing as much information as possible from previous analyses. Such cost reductions have been achieved to date at two levels, using relatively different techniques:

- Modular analyses have been proposed which obtain a global fixpoint by computing local fixpoints on one module at a time. Such modular techniques are aimed at reducing the memory consumption (working set size) but can also localize analysis recomputation to the modules affected by changes, even in the context-sensitive setting, thus achieving a certain level of coarse-grained incrementality [9, 14, 17, 18, 23, 50].
- In parallel, context-sensitive (non-modular) incremental fixpoint analyses identify, invalidate, and recompute only those parts of the analysis results that are affected by these fine-grain program changes. They achieve incrementality at finer levels of granularity and for more localized changes (such as at the program line level) [3, 4, 32, 36, 51, 58].

The main problem that we address is that the context-sensitive, fine-grained incremental analysis techniques presented to date are not directly applicable to modular programs. Since these algorithms are not aware of the module boundaries, and at the same time the flow of analysis information through the module interfaces is complex and requires iterations, since the analysis of a module depends on the interface and analysis of other modules in complex ways, through several paths to different versions of the exported procedures. In order to bridge this gap, we propose a generic framework that performs context-sensitive fixpoint analysis while achieving both inter-modal (coarse-grain) and intra-modal (fine-grain) incrementality. Our analysis algorithm is based on the generic (i.e., abstract domain-independent), context-sensitive PLAI algorithm [44, 46] that has been the basis of many analyses for both declarative and imperative programs, and also the subject of many extensions –some recent examples are [3, 15, 24]. In particular, we build on the previously mentioned extensions to make it incremental [32, 51] or modular [9, 14, 50]. Addressing the issues mentioned before, we solve the problems related to delaying the propagation of the fine-grain change information across module boundaries, devising the additional bookkeeping required. We also describe the actions needed to recompute the analysis fixpoint incrementally after multiple additions and deletions across different modules. The new setting modifies the cost-performance tradeoffs, and this requires experimentation to determine if and when the new algorithm offers clear advantages. To this end we have implemented the proposed approach within the Ciao/CiaoPP system [30, 31] and provide experimental results.

For generality, we formulate our algorithm to work on a block-level intermediate representation of the program, encoded using (constrained) Horn clauses [26, 43], i.e., we assume that programs are converted to this representation, on a modular basis. While the conversion itself is beyond the scope of the paper (and dependent on the source language), the process is of course trivial in the case of (C)LP programs or (eager) functional programs: mostly eliminating all syntactic sugar – conditionals, loops, macros, negation, grammars/DCGs, etc.– as done normally by the compiler. For imperative programs we refer the reader to [26, 29, 43] and the references below, and recall some characteristics (and advantages) of this representation: all iterations are represented uniformly as (tail or last call) recursions, non-deterministic or unknown choices are coded through multiple definitions (multiple clauses for the same procedure), all conditional forms are represented uniformly through clause guards, multiple input and output arguments are represented directly as clause head arguments, and clause literals represent either calls to other clauses/blocks or constraints. Such constraints encode primitive operations such as assignment (for source representations), or bytecodes or machine instructions (in lower-level representations), and correspond to transfer functions in the abstract domain. Horn Clauses have been used successfully as intermediate representations for many different programming languages and compilation levels (e.g., bytecode, llvm-IR, or ISA), in a good number of analysis and verification tools [2, 5–7, 19, 20, 27, 28, 33–35, 39–41, 48] (see Section 5 for related work).

2 Preliminaries and notation

Abstract Interpretation. The formalism that our analyses are based on is Abstract Interpretation [16], a technique for static program analysis in which execution of the program is simulated on a description (or abstract) domain \((D_a)\) which is simpler than the actual (or concrete) domain \((D)\). Values in the description domain and sets of values in the actual domain are related via a pair of monotonic mappings \((\alpha, \gamma)\): abstraction \(\alpha: D \rightarrow D_a\), and concretization \(\gamma: D_a \rightarrow D\).
which form a Galois connection. A description (or abstract value) \( d \in D_a \) approximates an actual (or concrete) value \( c \in D \) if \( a(c) \sqsubseteq d \) where \( \sqsubseteq \) is the partial ordering on \( D_a \). The correctness of abstract interpretation guarantees that the descriptions computed (by calculating a fixpoint through a Kleene sequence) approximate all of the actual values or traces which occur during any possible execution of the program, and that this fixpoint calculation process will terminate given some conditions on the description domains (such as being finite, or of finite height, or without infinite ascending chains) or by the use of a widening operator [16].

Intermediate Representation. A Constrained Horn Clause program (CHC) is a set of rules of the form \( A : \leftarrow L_1, \ldots, L_n \), where \( L_1, \ldots, L_n \) are literals and \( A \) is an atom said to be the head of the rule. A literal is an atom or a primitive constraint. We assume that each atom is normalized, i.e., it is of the form \( p(x_1, \ldots, x_m) \) where \( p \) is an \( m \)-ary predicate symbol and \( x_1, \ldots, x_m \) are distinct variables. A set of rules with the same head is called a predicate (procedure). A primitive constraint is defined by the underlying abstract domain(s) and is of the form \( c(e_1, \ldots, e_m) \) where \( c \) is an \( m \)-ary predicate symbol and the \( e_1, \ldots, e_m \) are expressions. Also for simplicity, and without loss of generality, we assume that each rule defining a predicate \( p \) has identical sequence of variables \( x_{p1}, \ldots, x_{pn} \) in the head atom, i.e., \( p(x_{p1}, \ldots, x_{pn}) \). We call this the base form of \( p \). Rules in the program are written with a unique subscript attached to the head atom (the rule number), and a dual subscript (rule number, body position) attached to each body literal, e.g., \( H_k \leftarrow B_{k1}, \ldots, B_{kn} \) where \( B_{ki} \) is a subscripted atom or constraint. The rule may also be referred to as rule \( k \), the subscript of the head atom.

Example 2.1. Factoring program in imperative C-style implementation (left) and its translation to CHC (right):

```
1 int fact(int N){
2     int R = 1;
3     while(N > 0){
4         R *= N;
5         N--;
6     }
7     return R;
8 }
```

```
1 fact(N, F) :-
2     while(N,1,F).
3 while(N, R, R) :- N <= 0.
4 while(N, A, R) :- N > 0.
5 A is A + N.
6 N1 is N - 1.
7 while(N1, A1, R).
```

Modular partitions of programs. A partition of a program is said to be modular when its source code is distributed in several source units (modules), each defining its interface with other modules of the program. The interface of a module contains the names of the exported predicates and the names of the imported modules. Modular partitions of programs may be synthesized, or specified by the programmer, for example, via a strict module system, i.e., a system in which modules can only communicate via their interface. We will use \( M \) and \( M' \) to denote modules. Given a module \( M \), we will use:

- \( \text{exports}(M) \) to express the set of predicate names exported by module \( M \).
- \( \text{imports}(M) \) to denote the set of modules which \( M \) imports.
- \( \text{depends}(M) \) to refer to the set generated by the transitive closure of \( \text{imports} \).

We also define \( \text{mod}(A) \) to denote the module in which the predicate corresponding to atom \( A \) is defined.

3 The Algorithm for Modular and Incremental Context-sensitive Analysis

We assume a setting in which we analyze successive “snapshots” of modular programs, i.e., at each analysis iteration, before the analyzer is called, a snapshot of the sources is taken and used to perform the next analysis. We also assume that we can have information on the changes in this snapshot with respect to the previous one (e.g., by comparing the two sources), in the form of a set of added or deleted clauses.

The algorithm is aimed at analyzing separately the modules of a modular program, using context-sensitive fixpoint analysis, while achieving both inter-modular (coarse-grain) and intra-modular (fine-grain) incrementality. Each time an analysis is started, the modules will be analyzed independently (possibly several times) until a global fixpoint is reached. Although in practice we use basically the module partition defined by the programmer our algorithm works with any partition of the sources.

Program analysis graph. We use analysis graphs, similarly to the PLAi algorithm [44], to represent the analysis results. They represent the (possibly infinite) set of (possibly infinite) and-or trees explored by a top-down (SLDT) exploration of the CHC program on the concrete domain for all possible input values—a representation of all the possible executions and states of the original program. Given an analysis graph it is straightforward to reconstruct any program point annotation.

An analysis graph is represented in the algorithm via a pair of data structures \((AT, DT)\). The answer table \((AT)\) contains entries of the form \( A : \lambda^c \mapsto \lambda^t \), with \( A \) is always a base form, representing a node in the analysis graph. It represents that the answer pattern for calls with calling pattern \( \lambda^c \) to \( A \) is \( \lambda^t \). Note that for a given base form \( A \), the \( AT \) can contain a number of different entries for different call patterns. As usual, \( \bot \) denotes the abstract substitution such that \( \gamma(\bot) = \emptyset \). A tuple \( A : \lambda^c \mapsto \bot \) indicates that all calls to predicate \( A \) with substitution \( \theta \in \gamma(\lambda^c) \) either fail or loop, i.e., they do not produce any success substitutions. The dependency table \((DT)\) contains the arcs of the program analysis graph that go from atoms in a rule body to the corresponding analysis node \((AT\) entry). An arc is of the form \( H_k : \lambda^c \Rightarrow [\lambda^p] B_{ki} : \lambda^{c'} \). This represents that calling rule \( H_k \) with calling pattern \( \lambda^c \) causes literal \( B_{ki} \) to be called with calling pattern \( \lambda^{c'} \). The remaining part \( [\lambda^p] \) is the abstract state at the program point just before \( B_{ki} \) and contains information about all variables in rule \( k \). The \( \lambda^p \) field is not really necessary, but is included
for efficiency. In fact, the DT itself is not strictly necessary; it is used to speed up convergence.

Example 3.1. Analyzing the following program that calculates the parity of a message with an abstract domain that captures whether variables take values of 0 or 1:

```
:- module(parity, [par/2]).
par((X, 0), 0).
par((X, X0), X) :- xor(X, X0).
```

Will produce an analysis graph of this shape:

```
\[ \text{par}(M, X) : \quad \top \mapsto M/\top, X/b \]
\[ \text{xor}(M, X0, X) : \quad \top \mapsto (X/b, X0/b, X/b) \]
\[ \text{par}(M, X) : \quad X/b \mapsto M/\top, X/b \]
\[ \text{xor}(M, X0, X) : \quad X/b \mapsto (M/b, X0/b, X/b) \]
```

The arrows in this graph represent the DT and the nodes the AT.

We use several instances of the analysis graph. The global analysis graph \(G\), represents intermodular (global) information. \(G = (G_{\text{AT}}, G_{\text{DT}})\), where the global answer table \(G_{\text{AT}}\) is used to store the results of the boundaries of the modules (exported predicates), and the global dependency table \(G_{\text{DT}}\) is used to store the relations between modules with predicate and call pattern precision. \(G_{\text{DT}}\) entries are of the form \(A : \lambda^c \Rightarrow H : \lambda^{c'}\) meaning that in module \text{mod}(A) a call to exported predicate \(A\) with description \(\lambda^c\) may produce a call to imported predicate \(H\) with \(\lambda^{c'}\). Local analysis graphs, \(\mathcal{L} = (\mathcal{L}_{\text{AT}}, \mathcal{L}_{\text{DT}})\), represent intra-modular information. The local answer table \(\mathcal{L}_{\text{AT}}\) keeps the results of a module during its analysis, i.e., of the predicates defined in that module and the (possibly temporary) results of its imported predicates, and the local dependency table \(\mathcal{L}_{\text{DT}}\) contains the arcs between rules defined in the module being analyzed. We use \(\mathcal{L}^M\) to denote the local analysis graph of partition \(M\).

\[ \text{getdef}(\mathcal{L}_{\text{AT}}, A : \lambda^c) \triangleq \sigma^{-1}(\lambda^c) \text{ if there exists a renaming } \sigma \text{ s.t. } \sigma(A : \lambda^c) \mapsto \lambda^c \in \mathcal{L}_{\text{AT}}. \] This partial function will obtain, if it exists, a renamed answer pattern for \(A : \lambda^c\) from some \(\mathcal{L}_{\text{AT}}\) (any \(\mathcal{L}_{\text{AT}} \text{ or } G_{\text{AT}}\)).

Domain operations. The algorithm is parametric on the abstract domain \(D_a\), which is defined by providing four abstract operations which are required to be monotonic and to approximate the corresponding concrete operations:

- \(\text{Aproject}(\lambda^c, L)\) which performs the abstract restriction of a calling pattern \(\lambda^c\) to the variables in the literal \(L\);
- \(\text{Aadd}(C, \lambda^c)\) which performs the abstract operation of conjoining the primitive constraint \(C\) with the description \(\lambda^c\);
- \(\text{Acombine}(\lambda^c_1, \lambda^c_2)\) which performs the abstract conjunction of two descriptions;
- \(\text{Alub}(\lambda^c_1, \lambda^c_2)\) which performs the abstract disjunction of two descriptions.

Events. The algorithm is centered around processing tasks triggered by events. They are separated in two priority queues, one for “coarse-grain” global tasks and the other for “fine-grain” local tasks. Global events will trigger actions that involve the \(G\), and similarly, local events will be used to update the \(\mathcal{L}\). There is one kind of global event, \(\text{updmod}(M)\), which indicates that module \(M\) has to be reanalyzed. There are three kinds of local events:

- \(\text{newcall}(A : \lambda^c)\) indicates that a new call \(\lambda^c\) for atom \(A\) has been encountered.
- \(\text{arc}(D)\) means that recomputation needs to be performed starting at program point (literal) indicated by dependency \(D\).
- \(\text{updated}(A : \lambda^c)\) indicates that the answer pattern to \(A : \lambda^c\) has changed.

To add events to any of the queues we use \(\text{add\_event}(E)\), which inserts the event in the corresponding queue (global or local) depending on its type.

3.1 Operation of the algorithm

The pseudocode of the algorithm is detailed in Fig. 1. The algorithm takes as an input a (partitioned) program, a set of program edits in the form of additions and deletions, some entries \(Es\), i.e., the set of initial \(A : \lambda^c\), and, implicitly, the result of the analysis of the program before the changes. We assume that the first time a program is analyzed all data structures are empty. Each module with changes is scheduled to be reanalyzed. If there are recursive dependencies between modules, the modules in each clique will be grouped and analyzed as a whole module (after doing the corresponding renamings). Finally, the analysis loop for the global queue begins.

Processing global events. This task is performed by \(\text{process}(\text{updmod}(M))\) and consists in analyzing the module \(M\) for some previously annotated entries \(\text{Enf}(M)\) and/or updating a previous analysis with new information on the

\[ \text{A worked example illustrating the algorithm(s) is provided in App. B.} \]

\[ \text{Typically computed w.r.t. the previous snapshot for each module.} \]

\[ \text{Event ordering in the global queue can affect fixpoint convergence speed. Some scheduling policies were studied in [14]; in our experiments we use the “top-down” strategy defined there.} \]
imported rules. \texttt{load\_local\_graph}(M) ensures that the set of clauses of \(M\) and \(\mathcal{L}^M\) (initially empty) are loaded and available. Then, \(\mathcal{L}^M\) is updated (\texttt{update\_local\_graph}) with possibly new results of the imported predicates. Since the algorithm works with one \(\mathcal{L}^M\) at a time, we simply write \(\mathcal{L}\).

If there were pending source changes they are processed. Then, for all the entries of \(\text{Enf}[M]\) that were not in the \(\mathcal{L}\), a \texttt{newcall} event is added and the local analysis loop is performed, i.e., the module is analyzed.

Next, \(\mathcal{G}\) is updated. In the \(\mathcal{G}_{\text{DT}}\), for all the newly analyzed entries we add (or update) tuples of the form \(P : \lambda^c \Rightarrow H : \lambda^{c'}\) for each imported \(H\) reachable with \(\lambda^{c'}\) from the entry \(P : \lambda^c\). Note that this can be determined using the \(\mathcal{L}_{\text{DT}}\). To update the \(\mathcal{G}_{\text{AT}}\), we add or replace tuples whose answer pattern (\(\lambda^c\)) has changed, and add events to (re)analyze the affected modules. Finally, the set of pending entries (\(\text{Enf}[M]\)) is emptied and the current \(\mathcal{L}\) (\(\mathcal{L}^M\)) is ensured to be committed by \texttt{store\_local\_graph}.

\textbf{Processing local events.} The process\((\texttt{newcall}(A : \lambda^c)\)) procedure initiates the processing the rules in the definition of atom \(A\). If \(A\) is defined in the module an \texttt{arc} event is added for the first literal of each of the rules. The \texttt{initial\_guess} function returns a guess of \(\lambda^c\) to \(A : \lambda^c\). If possible, it reuses the results in the ATs, otherwise returns \(\bot\). For imported rules, if the exact \(A : \lambda^c\) was not present in the \(\mathcal{G}_{\text{AT}}\), an \texttt{updmod} event is created (adding the corresponding entry).

The process\((\texttt{updated}(A : \lambda^c))\) procedure propagates the information of new computed answers across the analysis graph by creating \texttt{arc} events with the program points from which the analysis has been restarted.

The process\((\texttt{arc}(H_k : \lambda^c \Rightarrow [\lambda^p]\ B_{k, i} : \lambda^{c'}))\) procedure performs the core of the module analysis. It performs a single step of the left-to-right traversal of a rule body. If the literal \(B_{k, i}\) is a primitive, it is added to the abstract description; otherwise, if it is an atom, an arc is added to the \(\mathcal{L}_{\text{DT}}\) and \(\lambda^x\) is looked up (a process that includes creating a \texttt{newcall} event for \(A : \lambda^x\) if the answer is not in the \(\mathcal{L}_{\text{AT}}\)). The obtained answer is combined with the description \(\lambda^p\) from the program point immediately before \(B_{k,i}\) to obtain the description for the program point after \(B_{k,i}\). This is either used to generate an \texttt{arc} event to process the next literal (if there is one), or otherwise to update the answer of the rule in \texttt{insert\_answer\_info}. This function combines the new answer with the previous, and creates \texttt{updated} events to propagate the new answer if needed.

\textbf{Updating the local analysis graph.} The assumptions made for imported rules (in \texttt{initial\_guess}) may change during analysis. Before reusing a previous \(\mathcal{L}\), it is necessary to detect which assumptions changed and how these changes affect analysis graph, and either propagate the changes or delete the information which may no longer be precise (\texttt{remove\_invalid\_info}).

\textbf{Adding clauses to a module.} If clauses are added to a module, the answer patterns for those rules have to be computed and this information used to update the \(\mathcal{L}\). Then these changes need to be propagated. The computation and propagation of the added rules is done simply by adding \texttt{arc} events before starting the processing of the local queue. The propagation will come by adding and later processing \texttt{updated} events.

\textbf{Removing clauses from a module.} Unlike with incremental addition, we do not strictly need to change the analysis results at all. Since the inferred information is an over-approximation it is trivially guaranteed to be correct if a clause is deleted. However, this approach would obviously be very inaccurate. We have adapted one of the strategies proposed in [32] for getting the most precise analysis in the modular setting. The \texttt{delete\_clauses} function selects which information can be kept in order to obtain the most precise semantics of the module, by removing all information in the \(\mathcal{L}\) which is potentially inaccurate, i.e., the information related to the calls that depend on the deleted rules (\texttt{remove\_invalid\_info}). Function \texttt{depends\_call} gathers transitively all the callers to the set of obsolete \texttt{Calls}, and the \(A : \lambda^c\) generated from literals that follow in a clause body any \texttt{Calls}, because they were affected by the answers \(\lambda^c\).

When a module is edited and the program is reanalyzed, the source update procedures will be performed only the first time that the module is analyzed in the intermodular fixpoint, because the sources will not change between iterations.

\textbf{SCC guided deletion strategy.} The proposed deletion strategy is quite pessimistic. Deleting a single rule most of the times means reusing only a few dependency arcs and answers. However, it may occur that the analysis does not change after removing a clause or some answers/arcs may still be correct and precise. We would like to partially reanalyze the program without removing these potentially useful results, for example, using information of the strongly connected components. Our proposed algorithm allows performing such partial reanalysis, by running it (within the algorithm) partitioning the desired module into smaller partitions. Concretely, this can be achieved by replacing the call to \texttt{remove\_invalid\_info} (line 40) in \texttt{delete\_clauses} by running the algorithm, taking as an input the partition of the current module into “submodules” of its SCCs, setting the initial \(Es\) of this modular analysis as \(Cs\), and initializing the \(\mathcal{G}\) with the \(\mathcal{L}\) of the current module.

\textbf{3.2 Fundamental results for the algorithm}

We now state the fundamental results for our algorithm. The proofs of the following theorems and propositions are provided in App. A. We assume that the domain \(\langle D_\alpha, \preceq, \cup, \cap, \top, \bot \rangle\) is finite, that \texttt{initial\_guess} returns a value below the \textit{least fixed point (lfp)}, that modules which have recursive dependencies in exported and imported predicates
Algorithm Modular Analyze

input: P = {M₁, ..., Mₙ}, Diff, InitEs
global: (G_AT, G_DT), (Δ_G_AT, Δ_G_DT), CurrM, Ent

1: for all M ∈ P do
2:   if Diff[M] ≠ ∅ then
3:     Es := {A : λ ∈ A :: T → ∅ ∈ G_AT, mod(H) = M}
4:   if En[M] = InitEs(M) ∪ Es
5:      add_event(updmod(M))
6:   analysis_loop(global)
7:   procedure analysis_loop(Queue)
8:     while E := next_event(Queue) do process(E)
9:   procedure process(updmod(M))
10:  Es := En[M], CurrM = M
11:  load_local_graph(M), update_local_graph(M)
12:  if Diff[M] = (Dels, Adds) ≠ (0, 0) then
13:     delete_clauses(Dels)
14:     add_clauses(Adds)
15:   if Diff[M] = (0, 0) then
16:     for all A :: λ ∈ Es do
17:       if A :: λ ∈ Δ_G_AT then
18:         add_event(newcall(A :: λ'))
19:     analysis_loop(local)
20:     remove(G_DT, E := E ∪ {A : λ})
21:     for all A :: λ ∈ Es do
22:       if all reachable H :: λ', mod(H) ≠ M do
23:         set(G_DT, A :: λ', H := λ')
24:       λ' := getans(G_AT, A :: λ')
25:       λ := set(G_AT, A :: λ', λ')
26:       if λ' ≠ λ then
27:         for all A' :: λ' ∈ Δ_G_DT do
28:           E := En[mod(A')]
29:         if E' = 0 then add_event(updmod(mod(A')))
30:     store_local_graph(M), En[M] := 0
31:   procedure add_clauses(Rs)
32:     for all A_k :: B_k, ..., B_k, n_k ∈ Rs do
33:       for all A :: λ' ∈ Δ_G_AT do
34:         A :: λ' := A::λ', B_k, n_k
35:       add_event(newcall(A_k :: λ := [λ, B_k, n_k]))
36:     procedure delete_clauses(Rs)
37:     Cs := {A :: λ' : A :: λ' ∈ Δ_G_AT, (A :: B) ∈ Rs}
38:     remove_invalid_info(Cs)
39:   procedure remove_invalid_info(Calls)
40:     T := depends_call(Calls)
41:     for all A :: λ ∈ T do
42:       remove(Δ_G_AT, A :: λ' := _)
43:     remove(Δ_G_DT, A :: λ := [λ, _])
44: procedure process(newcall(A :: λ'))
45:   if mod(A) = CurrM then
46:     for all rule A_k :: B_k, ..., B_k, n_k do
47:       A :: λ' := Aproj(λ', vars(B_k, n_k))
48:       add_event(arc(A_k :: λ := [λ', B_k, n_k]))
49:     λ := initial_guess(A :: λ')
50:     if λ' ≠ λ then
51:       add_event(updated(A :: λ'))
52:       set(Δ_G_AT, A :: λ', λ')
53:     procedure process(updated(A :: λ'))
54:     for all Arc = H_k :: λ' := [λ'] B_k, i :: λ'i ∈ Δ_G_DT
55:       where there exists σ s.t. A :: λ' = σ(B_k, i :: λ'i) do
56:         add_event(arc(A :: λ'))
57:     procedure process(arc(H_k :: λ' := [λ'] B_k, i :: λ'i))
58:       if B_k, i is a primitive then
59:         λ' := Aadd(B_k, i :: λ'i)
60:       else
61:         set(Δ_G_DT, H_k :: λ' := [λ'] B_k, i :: λ'i)
62:       procedure insert_info(H :: λ' := λ')
63:     procedure lookup_answer(A :: λ')
64:       if λ' ∈ Δ_G_AT then
65:         return λ'
66:       else
67:         λ' := getans(Δ_G_AT, A :: λ')
68:       return λ'
69:     procedure update_local_graph(M)
70:     for all A :: λ' ∈ Δ_G_AT, mod(A) ≠ M do
71:       A :: λ' := getans(Δ_G_AT, A :: λ')
72:       A :: λ := Aflip(λ', λ')
73:       if λ' ≠ λ then
74:         set(Δ_G_AT, A :: λ', λ')
75:       procedure update_local_graph(M)
76:     for all A :: λ' ∈ Δ_G_AT, mod(A) ≠ M do
77:       A :: λ' := getans(Δ_G_AT, A :: λ')
78:       return λ'
79:     else
80:       add_event(newcall(σ(A :: λ')))
81:     where σ is a renaming s.t. σ(A) is in base form
82:     return T
83:   procedure update_local_graph(M)
84:     for all A :: λ' ∈ Δ_G_AT, mod(A) ≠ M do
85:       A :: λ' := getans(Δ_G_AT, A :: λ')
86:     if λ' ⊆ λ' then
87:       set(Δ_G_AT, A :: λ', λ')
88:     add_event(updated(A :: λ'))
89:     else
90:       remove_invalid_info(A :: λ')
91: Figure 1. The generic context-sensitive, modular, incremental fixpoint algorithm.

are analyzed jointly, and that we analyze snapshots of programs, i.e., sources cannot be modified during the analysis.

We represent executing the modular incremental analysis algorithm with the function:

\[ G' = \text{ModIncAnalyze}(P, Es, G, \Delta P') \]

where \( P \) is the (partitioned) program, \( G \) is the analysis result of \( P \) for \( Es \), and \( \Delta P' \) is a pair of (additions, deletions) to \( P \) for which we want to incrementally update \( G \) to get \( G' \), the analysis graph of \( P' \). Note that while \( G \) is not an explicit input parameter in the pseudocode, we use it and \( G' \) to model the update.

Similarly, we represent the analysis of a module within the algorithm (lines 10 to 19 in the pseudocode), with the function:

\[ \text{LocIncAnalyze}(M, Es, G, L_M, \Delta M') \]
where $M$ is a module, $ℒ^M$ is the analysis result of $M$ for $Es$, $ΔM'$ is a pair of (additions, deletions) with which we want to incrementally update $ℒ$ to get $ℒ'$, the analysis graph of $M'$, and $G$ contains the (possibly temporary) information for the predicates imported by $M'$.

**Proposition 3.2** (Analyzing a module from scratch).

*If module $M$ is analyzed for entries $Es$ within the incremental modular analysis algorithm from scratch (i.e., with no previous information available):*

$$ℒ^M = \text{LocIncAnalyze}(M, Es, G, (∅, ∅), (∅, ∅))$$

$ℒ^M$ will represent the least module analysis graph of $M$ and $Es$, assuming $G$.

**Proposition 3.3** (Adding clauses to a module).

*Given $M$ and $M'$ s.t. $M' = M \cup C_i$,*

$$ℒ^M = \text{LocIncAnalyze}(M, Es, G, (∅, ∅), (∅, ∅))$$

*then $LocIncAnalyze(M', Es, G, (∅, ∅), (∅, ∅)) = \text{LocIncAnalyze}(M, Es, G, ℒ^M, (C_i, ∅))$*

I.e., if module $M$ is analyzed for entries $Es$, obtaining $ℒ^M$ with the local incremental analysis algorithm, and $ℒ^M$ is incrementally updated adding clauses $C_i$, the result will be the same as when analyzing $M'$ from scratch. Note that the analysis of module $M$ for $Es$ from scratch ($\text{LocIncAnalyze}(M', Es, G, (∅, ∅), (∅, ∅))$) can be seen also as adding incrementally all the clauses of module $M$ to the analysis of an empty program ($\text{LocIncAnalyze}(∅, Es, G, (∅, ∅), (M, ∅))$).

**Proposition 3.4** (Removing clauses from a module).

*Given $M$ and $M'$ s.t. $M' = M \setminus C_i$,*

$$ℒ^M = \text{LocIncAnalyze}(M, Es, G, (∅, ∅), (∅, ∅))$$

*then $LocIncAnalyze(M', Es, G, (∅, ∅), (∅, ∅)) = \text{LocIncAnalyze}(M, Es, G, ℒ^M, (C_i, ∅))$*

I.e., if module $M$ is analyzed for entries $Es$, obtaining $ℒ^M$ with the local incremental analysis algorithm, and $ℒ^M$ is incrementally updated removing clauses $C_i$, the analysis result of $M'$ will be the same as when analyzing it from scratch.

**Adding and deleting clauses at the same time.** The results above hold also when combining additions and deletions of clauses, since the actions performed when adding and deleting clauses are compatible: when adding clauses the local analysis graph is re-used as is. Deleting clauses erases potentially inaccurate information, which will only imply, in the worst case, some unnecessary recomputations.

**Proposition 3.5** (Updating the $ℒ$).

*Given $ℒ^M = \text{LocIncAnalyze}(M, Es, G, (∅, ∅), (∅, ∅))$ if $G$ changes to $G'$:

$$ℒ^M = \text{LocIncAnalyze}(M, Es, G', (∅, ∅), (∅, ∅)) = \text{LocIncAnalyze}(M, Es, G', ℒ^M, (∅, ∅))$$*

I.e., if module $M$ is analyzed for $Es$ assuming some $G$ obtaining $ℒ^M$, then if the assumptions change to $G'$, incrementally updating these assumptions in $ℒ^M$ will produce the same result as when analyzing $M$ with assumptions $G'$ from scratch.

**Computing the intermodular lfp.** So far we have seen that $\text{LocIncAnalyze}$ calculates the lfp of modules. This guarantees:

**Proposition 3.6** (Analyzing modular programs from scratch).

*If program $P$ is analyzed for entries $Es$ by the incremental modular analysis algorithm from scratch (with no previous information available):*

$$G = \text{ModIncAnalyze}(P, Es, (∅, ∅), (∅, ∅))$$

$G$ will represent the least modular program analysis graph of exports($M$), s.t. $M \in P$.

**Theorem 3.7** (Modular incremental analysis).

*Given modular programs $P$, $P'$ s.t. $ΔP = (C_i, C_j), P' = (P \cup C_j) \setminus C_i$, entries $Es$, and $G = \text{ModIncAnalyze}(P, Es, (∅, ∅), (∅, ∅))$:

$$\text{ModIncAnalyze}(P', Es, G, (∅, ∅)) = \text{ModIncAnalyze}(P, Es, G, ΔP')$$*

I.e., if $P$ is changed to $P'$ by editions $ΔP'$ and it is reanalyzed incrementally, the algorithm will return a $G$ that encodes the same global analysis graph as if $P'$ is analyzed from scratch.

Finally, note that these results also hold for the SCC-guided deletion strategy (this follows from Theorem 3.7).

**4 Experiments**

We have implemented the proposed approach within the Ciao/CiaoPP system [30, 31]. We have selected some well-known benchmarks that have been used in previous studies of incremental analysis and show different characteristics.5 E.g., Ann (a parallelizer) and boyer (a theorem prover kernel), are programs with a relatively large number of clauses located in a small number of modules. In contrast, e.g., bid is a more modularized program.

The tests performed consist in analyzing the benchmarks with the different approaches. For all our experiments we have used a top-down scheduling policy as defined in [14]. The exported predicates of the main module of each benchmark were used as starting point, with $⊤$ or the initial call patterns, if specified. We use the well-known sharing and freeness abstract domain [45] (pointer sharing and uninitialized pointers).6

As baseline for our comparisons we use the non-modular incremental and modular algorithms of [32] and [50]. We would like to be able to compare directly with the non-modular framework but it cannot be applied directly to

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5 Some relevant data on the benchmarks is included in App. C.

6 App. D provides also results for the def domain (dependency tracking via propositional clauses [21]).
The modular benchmarks. Instead, we use the monolithic approach of [50], which is equivalent: given a modular program $P$ it builds a single module $m_{flat}$ which is equivalent to $P$, by renaming apart identifiers in the different modules in $P$ to avoid name clashes. In summary, we perform the experiments for four approaches: the monolithic approach, the incremental approach described in [32] (after transforming the benchmarks to $m_{flat}$ form), the modular approach described in [50], and the proposed modular incremental approach.

To perform the tests we have developed a front end that computes the differences between two states of source files at the level of transformed clauses using Myers algorithm [47]. Note however that this step is independent of the algorithm, and could also be obtained, for instance, from the IDE. We are also factoring out the time introduced by the current implementation of $load\_local\_graph$ and $store\_local\_graph$, since this depends really on factors external to the algorithm such as representation, disk speeds, etc. In any case, the measured cost of loading and storing $\mathcal{L}$ in our setup is negligible (a few milliseconds) and the overall module load cost is low (40-110 ms per module). The experiments were run on a MacBook Pro with an Intel Core i5 2.7 GHz processor, 8GB of RAM, and an SSD disk.

Discussion.
We first study the analysis from scratch of all the benchmarks for all approaches, in order to observe the overhead introduced by the bookkeeping in the algorithm. The results are shown in Fig. 2. For each benchmark four columns are shown, corresponding to the four analysis algorithms mentioned earlier: monolithic (mon), monolithic incremental (mon_inc), modular (mod), and modular incremental (mod_inc). The bars are split to show how much time each operation takes: analyze is the time spent processing local events, incAct is the time spent updating the local analysis results (procedure $update\_local\_graph$ in the algorithm), preProc is the time spent processing clause relations (e.g., calculating the SCCs), and updG is the time spent updating the $G$. In Fig. 2 warplan, witt, and cleandirs use the scale on the right hand side of the graph. In the monolithic setting, the overhead introduced is negligible. Interestingly, incremental modular performs better overall than simply modular even in analysis from scratch. This is due to the reuse of local information specially in complex benchmarks such as ann, peephole, warplan, or witt. In the best cases (e.g., witt) performance competes with monolithic thanks to the incremental updates, dropping from 23s in the modular non incremental to 3s in modular incremental.

Clause addition experiment. For each benchmark and approach, we measured the cost of analyzing the program adding one rule at a time. That is, the analysis was first run for the first rule only. Then the next rule was added and the resulting program (re)analyzed. This process was repeated until all the rules in all the modules were added.

Clause deletion experiment. We timed the case where the program rules are deleted one by one. Starting from an already analyzed program, the last rule was deleted and the resulting program (re)analyzed. This process was repeated until no rules were left. The experiment was performed for all the approaches using the initial top-down deletion strategy (td) and the SCC-guided deletion strategy of Section 3.1 (scc).

Fig. 3 shows these two experiments for warplan. Each point represents the time taken to reanalyze the program.
These results are encouraging both in terms of response times and scalability. The incremental settings (mon_inc, mod_inc) are always faster than the corresponding non-incremental settings (mon, mod) (except in aiakl where mod_inc is essentially the same as mod). Furthermore, while the traditional modular analysis is sometimes slower than the monolithic one (for the small benchmarks: hanoi, qsort, and witt), our modular incremental algorithm always outperforms both, obtaining 10× speed-up over monolithic in the best cases (boyer and cleandirs). Furthermore, in the larger benchmarks modular incremental outperforms even the monolithic incremental algorithm.

Fig. 5 shows the results of the deletion experiment. The analysis performance of the incremental approaches is in general better than the non-incremental approaches, except for small programs. Again, our proposed algorithm shows very good performance, in the best cases (ann, peephole, and read) we obtain a speed-up of 5×, competing with monolithic incremental scc and outperforming in general monolithic incremental td. The SCC-guided deletion strategy seems to be more efficient than the top-down deletion strategy. This confirms that the top-down deletion strategy tends to be quite pessimistic when deleting information, and modular partitions limit the scope of deletion.

5 Related work

Modular analysis [17] is based on splitting large programs into smaller parts (e.g., based on the source code structure). Exploiting modularity has proved essential in industrial-scale analyzers [18, 23]. Despite the fact that separate analysis provides only coarse-grained incrementality, there have been surprisingly few results studying its combination with fine-grained incremental analysis.

Classical data-flow analysis: Since the first algorithm for incremental analysis was proposed by [54], there has been considerable research and proposals in this topic (see the bibliography of [52]). Depending on how data flow equations are solved, these algorithms can be separated into those based on variable elimination, which include [10], [11], and [55]; and those based on iteration methods which include [13] and [49]. A hybrid approach is described in [42]. Our algorithms are most closely related to those using iteration. Early incremental approaches such as [13] were based on restarting iteration. That is, the fixpoint of the new program’s data flow equations is found by starting iteration from the fixpoint of the old program’s data flow equations. This is always safe, but may lead to unnecessary imprecision if the old fixpoint is not below the Ifp of the new equations [56]. Reinitialization approaches such as [49] improve the accuracy of this technique by reinitializing nodes in the data flow graph to bottom if they are potentially affected by the program change. Thus, they are as precise as if the new equations had been analyzed from scratch. These algorithms are generally not based on abstract interpretation. Reviser [4] extends the.
more generic IFDS [53] framework to support incremental program changes. However IFDS is limited to distributive flow functions (related to condensing domains) while our approach does not impose any restriction on the domains. **Constraint Logic Programs**: Apart from the work that we extend [32, 51], incremental analysis was proposed (just for incremental addition) in the Vienna abstract machine model [37, 38]. It was studied also in compositional analysis of modules in (constraint) logic programs [8, 12], but it did not consider incremental analysis at the level of rules. **Horn clause-based representations**: The frameworks defined in this line are based on abstract interpretation of Constraint Logic Programs (CLP), which by definition are sets of Horn clauses built around some constraint theories. The CLP representation has been successfully used in many analysis frameworks, both as a kernel target language for compiling logic-based languages [30] or used as an intermediate representation for imperative programs. As mentioned in the introduction, our approach within this framework is based on mapping the program to be analyzes (either in source or binary form) to predicates, preserving as much as
possible the original program structure. This internal representation may be a straightforward desugared version or include more complex transformations. It may include effectful computations, which may require special treatment from the domains or additional program transformations, such as static single assignment (SSA). Other frameworks implementing a similar top-down abstract interpretation approach support incremental updates [3], based on the same principles [32], but not modular analysis. Other Horn clause-based approaches restrict to a pure subset, more recently denoted as Constrained Horn Clauses (CHC). Verifiers using CHCs [6, 19, 28, 33, 34] are based on the generation of specific encodings for some properties of interest. These encodings may not be easy to map to the original program. To the best of our knowledge, CHC-based solvers are focused on verification problems and none of them deal with modularity and incremental updates.

**Datalog and tabled logic programming:** In a related line to the previous one, other approaches are based on Datalog and tabled logic programming. FLIX [41] uses a bottom-up semi-naive strategy to solve Datalog programs extended with lattices and monotone transfer functions. This approach is similar to CLP analysis via bottom-up abstract interpretation. However, it has not been extended to support incremental updates. Incremental tabling [57] offers a straightforward method to design incremental analyses [22], when they can be expressed as tabled logic programs. While these methods are much closer to our incremental algorithm, they may suffer similar problems than generic incremental computation, as it may be difficult to control.

**Generic incremental computation frameworks:** Obviously, the possibility exists of using a general incrementalized execution algorithm. Incremental algorithms compute an updated output from a previous output and a difference on the input data, which the hope that the process is (computationally) cheaper than computing from scratch a new output for the new input. The approach of [58] takes advantage of an underlying incremental evaluator, IncQuery, and implements modules via the monolithic approach. There exist other frameworks such as self-adjusting computation [1] which greatly simplify writing incremental algorithms, but in return it is difficult to control the costs of the additional data structures.

## 6 Conclusions

Dynamic languages offer great flexibility, but it can come at the price of run-time cost. Static analysis, coupled with some dynamic techniques, can contribute to reducing this cost, but it in turn can take excessive time, specially in interactive or program transformation scenarios. To address this we have described, implemented, and evaluated a context sensitive, fixpoint analysis algorithm aimed at achieving both inter-modular (coarse-grain) and intra-modular (fine-grain) incrementality. Our algorithm takes care of propagation of fine-grain change information across module boundaries and implements all the actions required to recompute the analysis fixpoint incrementally after additions and deletions in the program. We have shown that the algorithm is correct and computes the most precise analysis. We have also implemented and benchmarked the proposed approach within the Ciao/CiaoPP system. Our preliminary results from this implementation show promising speedups for programs of medium and larger size. The added finer granularity of the proposed modular incremental fixpoint algorithm reduces significantly the cost with respect to modular analysis alone (which only preserved analysis results at the module boundaries) and produces better results even when analyzing the whole program from scratch. The advantages of fine-grain incremental analysis—making the cost be ideally proportional to the size of the changes—thus seem to carry over with our algorithm to the modular analysis case.

### References


[38] Andreas Krall and Thomas Berger. 1995. The VAMₐₐ - an Abstract machine for Incremental Global Dataflow Analysis of Prolog. In ICLP’95 Post-Conference Workshop on Abstract Interpretation of Logic Languages, Maria García de la Banda, Gerda Janssens, and Peter Stuckey (Eds.). Science University of Tokyo, Tokyo, 80–91.


Appendix

A Proofs

We introduce some additional notation that will be instrumental in the proofs. Given a (finite) abstract domain \((D_\alpha, \sqsubseteq, \sqcup, \sqcap, \top, \bot)\), we express the abstract semantics of a program clause \(c\) with \(f_c : D_\alpha \rightarrow D_\alpha\). The abstract semantics of a computation step of program \(P\), given the set of clauses \(P\), is a function \(F\) collecting the meaning of all clauses: \(F(P) = \bigsqcup f_c \in P\). As a means to express assumptions on the semantics of the program (i.e., to express the semantics of builtins, to specify properties of the entries of the program, or of the imported predicates\(^7\)), we add a constant part \(X_0 \in D_\alpha\) to \(F\). \(X_0 \sqcup F(P)\). The most precise semantics of \(P\) and some assumptions \(X_0\) is the \(lfp\) \((X_0 \sqcup F(P))\). Note that the \(lfp\) exists because \(F\) is monotonic and \(X_0\) is a constant. Also note that the \(lfp\) operation of a fixed program \(P\), parametric on the assumptions, is monotonic. Also, we have that \(X_0 \sqsubseteq X'_0 \Rightarrow lfp(F(P) \sqcup X_0) \sqsubseteq lfp(F(P) \sqcup X'_0)\), since the \(lfp\) is a composition of monotonic functions.

Domain of analysis graphs. We build the domain of analysis graphs (parametric on \(D_\alpha\)) as sets of \((\text{pred}_\text{name}, D_\alpha, D_\alpha)\). This domain is finite, because it is the combination of finite domains. The set of predicate names may be infinite in general, but in each program it is finite. We do not represent the dependencies (DT) in this domain because they are redundant, only needed for efficiency. We define the partial order in this domain as:

\[
AG_1 \sqsubseteq_{AG} AG_2 \text{ if } \forall (P, CP, AP) \in AG_1 \exists (P, CP, AP') \in AG_2 \text{ s.t. } AP \sqsubseteq D_\alpha \rightarrow AP'
\]

For the sake of simplicity, \(D_\alpha\) in the following represents this analysis graph domain.

We recall the function definitions used in the theorems of section 3.2. We represent executing the modular incremental analysis algorithm with the function:

\[
G' = \text{ModIncAnalyze}(P, Es, G, \Delta P'),
\]

where \(P\) is the partitioned program, \(G\) is the analysis result of \(P\) for \(Es\), and \(\Delta P'\) is a pair of (additions, deletions) to \(P\) for which we want to incrementally update \(G\) to get \(G'\), the analysis graph of \(P'\). Note that while \(G\) is not an explicit input parameter in the pseudocode, we use it and \(G'\) to model the update.

Similarly, we represent the analysis of a module within the algorithm (lines 10 to 19 in the pseudocode), with the function:

\[
\mathcal{L}^M = \text{LocIncAnalyze}(M, Es, G, \mathcal{L}^M, \Delta M'),
\]

where \(M\) is a module, \(\mathcal{L}^M\) is the analysis result of \(M\) for \(Es\), \(\Delta M'\) is a pair of (additions, deletions) with which we want to incrementally update \(\mathcal{L}\) to get \(\mathcal{L'}\), the analysis graph of \(M'\), and \(G\) contains the (possibly temporary) information for the predicates imported by \(M'\).

In the following we show the proofs of the propositions and theorems of section 3.2. We assume that initial\_guess that a value below the \(lfp\). Let \(G[M] : D_\alpha \rightarrow D_\alpha\) be the function that represents the semantics of imports\((M)\), projected from \(G\).

Proposition 3.2 (Analyzing a module from scratch). If \(M\) is analyzed for \(Es\) within the incremental modular analysis algorithm from scratch (i.e., with no previous information available):

\[
\mathcal{L}^M = \text{LocIncAnalyze}(M, Es, G, (\emptyset, \emptyset), (\emptyset, \emptyset))
\]

\(\mathcal{L}^M\) will represent the least module analysis graph of \(M\) and \(Es\), assuming \(G\).

Assuming that all the modular structures are properly initialized, \(\text{LocIncAnalyze}\) of a module in our algorithm encodes the monolithic algorithm of \([32]\). We recall the basic result for that algorithm:

Theorem A.1. For a program \(P\) and initial \(x\)'s \(Es\), the PLAI algorithm returns an AT and a DT which represents the least program analysis graph of \(P\) and \(Es\).

This theorem is directly applicable to \(\text{LocIncAnalyze}\) using the techniques for expressing properties of built-ins in the monolithic algorithm to incorporate the semantics of external predicates.

Our algorithm, when analyzing a module \(M\) (starting from an empty local analysis graph) is obtaining the \(lfp\) by computing the supremum of the Kleene sequence of \(S(X) = G \sqcup Es \sqcup F(M)(X)\):

\[
\bot \sqsubseteq S(\bot) \sqsubseteq S(S(\bot)) \sqsubseteq \ldots \sqsubseteq S^n(\bot)
\]

Each composition of \(S\) involves applying each of the \(f_c \in M\). However, since the \(\sqcup\) operator is commutative, the order of computation of the \(f_c(X)\) to obtain each of the \(S^n(\bot)\) will not affect the final result. Furthermore, we can reorder the computation of the supremum in a way that we compose an \(f_c\) several times, and then apply an \(f_c\), allowing us not to compute exactly each of the \(S^n(\bot)\) intermediate steps, as long as all the \(f_c \in P\) are applied fairly. This is equivalent to obtaining the least fixed point by chaotic iteration \([16]\).

Adding clauses to a module. Let \(M\) and \(M'\) be two modules s.t., we add clauses \(C_i\) to module \(M\) to get \(M'\) and let \(G\) be some initial assumptions.

Proposition 3.3 (Adding clauses to a module). Given \(M\) and \(M'\) s.t., \(M' = M \cup C_i\),

\[
\mathcal{L}^M = \text{LocIncAnalyze}(M, Es, G, (\emptyset, \emptyset), (\emptyset, \emptyset))\]

then

\[
\text{LocIncAnalyze}(M', Es, G, (\emptyset, \emptyset), (\emptyset, \emptyset)) = \text{LocIncAnalyze}(M, Es, G, \mathcal{L}^M, (C_i, \emptyset))
\]

\(^7\)In CiaoPP this is done by means of assertions \([30, 31]\).
Proof of Proposition 3.3. The \( \text{Ifp}(\mathcal{M} \cup \mathcal{G}[M]) \) will be computed processing fairly all its clauses. Let us call \( f_A \) applying one random \( f_c \in (M \cup \{\mathcal{G}[M]\}) \) and \( f_B \) applying one random \( f_c \in (M' \cup \{\mathcal{G}[M]\}) \). There exists a valid sequence of computation of the Kleene sequence of \( F(M') \cup \mathcal{G}[M] \) that consists in applying first all the \( f_c \in M: \)

\[
\exists \ f_B(\bot) \subseteq f_B(f_B(\bot)) \subseteq \ldots \subseteq f_B^n(\bot) (\mathcal{L}_M) \\
\quad \text{equivalent computation steps} \quad \downarrow \text{reused} \\
\exists \ f_B(f_B(\bot)) \subseteq \ldots \subseteq f_B^n(\bot) (\mathcal{L}_M) \\
\quad \text{avoided steps} 
\]

Therefore it is safe to start the analysis of \( M' \) with initial assumptions \( \mathcal{G}[M] \) with \( \mathcal{L}_M = \text{Ifp}(\mathcal{G}[M] \cup F(M)) \), so that \( \text{Ifp}(\mathcal{L}_M \cup \mathcal{G}[M] \cup F(M')) = \text{Ifp}(\mathcal{G}[M] \cup F(M')) \). With this in mind, procedure \texttt{add\_clauses} in the algorithm does not remove any information, only adds events to process the new clauses. Note that thanks to this analysis reuse, the cost (in terms of abstract domain operations) of incrementally adding clauses to the analysis can be no worse than analyzing the module from scratch.

Removing clauses from a module. Let \( M \) and \( M' \) be two modules s.t. we remove clauses \( C_i \) from \( M \) to get \( M' \).

Proposition 3.4 (Removing clauses from a module). Given \( M \) and \( M' \) s.t. \( M' = M \setminus C_i \),

\[
\mathcal{L}^M = \text{LocIncAnalyze}(M, Es, \mathcal{G}, (\emptyset, \emptyset), (\emptyset, \emptyset)) \text{ and } \mathcal{L}^{M'} = \text{LocIncAnalyze}(M', Es, \mathcal{G}, (\emptyset, \emptyset), (\emptyset, \emptyset)) \\
\text{then} \quad \mathcal{L}^{M'} = \mathcal{L}^M \cdot \text{ModIncAnalyze}(M', Es, \mathcal{G}, (\emptyset, \emptyset), (\emptyset, \emptyset))
\]

Proof of Proposition 3.4 Let \( f_A \) and \( f_B \) be the same as in the proof of Proposition 3.3. As discussed, we know that there exists a sequence of applications of \( f_B \) s.t. by going back to the \( k - th \) step (backwards) of the Kleene sequence supremum computation we would have exactly the \( \text{Ifp} \) of \( f_A \),

\[
\exists \ f_B(\bot) \subseteq f_B(f_B(\bot)) \subseteq \ldots \subseteq f_B^n(\bot) (\mathcal{L}_M) \\
\quad \text{equivalent computation steps} \quad \downarrow \text{reused} \\
\exists \ f_B(f_B(\bot)) \subseteq \ldots \subseteq f_B^n(\bot) (\mathcal{L}_M) \\
\quad \text{avoided steps}
\]

However, this cannot be used in practice for two reasons. First, we do not store each of the intermediate steps of computation, and, second, and most important, this intermediate state \( k \) only exists if we specifically analyze in a sequence that leaves processing the \( f_c \in C_i \) to the end. In practice, we would like to remove any clause(s) from the program, not only the last processed. This is why we have encoded the abstract information of the program in each program point in a way that we can reconstruct \( any \) of the possible computing sequence the analysis graph. We reconstruct a state, by selecting the information from the tables in \texttt{depends\_ca11} which corresponds to an analysis sequence in which the processing of clauses that depend on the deleted clauses is left for the end.

\[
\downarrow \exists \ f_B(\bot) \subseteq \ldots \subseteq f_B^n(\bot) (\mathcal{L}_M) \quad \downarrow \text{reused} \\
\exists \ f_B(\bot) \subseteq \ldots \subseteq f_B^n(\bot) (\mathcal{L}_M) \\
\quad \text{avoided steps}
\]

Note that, as when adding clauses, reusing analysis causes the cost (in terms of abstract domain operations) of incrementally deleting clauses to the analysis to be no worse than analyzing the module from scratch.

Updating the local analysis graph.

Proposition 3.5 (Updating the \( \mathcal{L} \)). Given \( \mathcal{L}^M = \text{LocIncAnalyze}(M, Es, \mathcal{G}, (\emptyset, \emptyset), (\emptyset, \emptyset)) \) if \( \mathcal{G} \) changes to \( \mathcal{G}' \):

\[
\mathcal{L}^{M'} = \text{LocIncAnalyze}(M, Es, \mathcal{G}', (\emptyset, \emptyset), (\emptyset, \emptyset))
\]

Proof of Proposition 3.5. We can view updating the \( \mathcal{L} \) with a new \( \mathcal{G}' \) in two means adding or removing clauses, with the semantics abstracted as a constant \( \mathcal{G}[M] \). Let \( \mathcal{G}[M] \) and \( \mathcal{G}'[M] \) be the old and new assumptions, respectively, that are used to compute the semantics of module \( M \).

- If \( \mathcal{G}[M] \subsetneq \mathcal{G}'[M] \) we know that \( \mathcal{G}'[M] \cup \mathcal{G}[M] = \mathcal{G}'[M] \). The correctness and precision of this operation can be reduced to Proposition 3.3 when joining a constant \( f_c = \mathcal{G}'[M] \) to the program semantics.
- If \( \mathcal{G}[M] \not\subsetneq \mathcal{G}'[M] \), i.e., the assumptions are incompatible or more restrictive (concrete) with respect to the previous ones, we would have to remove \( \mathcal{G}[M] \) and add the new \( \mathcal{G}'[M] \), reconstructing analysis steps in a way that we can reuse some analysis information. Since we use the same procedure as deleting clauses, the correctness and precision of this operation follows from Proposition 3.4.

Proposition 3.6 (Analyzing modular programs from scratch). If program \( P \) is analyzed for entries \( Es \) by the incremental modular analysis algorithm from scratch (with no previous information available):

\[
\mathcal{G} = \text{ModIncAnalyze}(P, Es, (\emptyset, \emptyset), (\emptyset, \emptyset))
\]

\( \mathcal{G} \) will represent the least modular program analysis graph of exports(M), s.t. \( M \in P \).

Let us define \texttt{update} : \( D_a \times D_a \rightarrow D_a \), \texttt{update}(AG_1, AG_2) as:

\[
\text{for } a_{11} (P, CP, AP) \in AG_2 \quad \text{if } (P, CP, AP') \in AG_1 \quad \text{AG_1 = (AG_1 \setminus \{(P, CP, AP)\}) \cup \{(P, CP, AP')\})} \\
\text{else } AG_1 = AG_1 \cup \{(P, CP, AP')\}
\]

Note \( AG_1 \subseteq AG_2 \Rightarrow \text{update}(AG, AG_1) \subseteq \text{update}(AG, AG_2) \) and \( AG_1 \subseteq AG_2 \Rightarrow \text{update}(AG_1, AG) \subseteq \text{update}(AG_2, AG) \).
Proof of Proposition 3.6. The processing of an \textit{updmod} event (MA) is equivalent to analyzing a module plus updating newly computed information in initial \(G_0\):
\[
\text{MA}(M, Es, G_0, L^M, \Delta M') = \\
\text{update}(G_0, \text{LocIncAnalyze}(M, Es, G_0, L^M, \Delta M'))
\]
Given that \(lfp\) is monotonic w.r.t. the initial assumptions, and that \(update\) is also monotonic, MA is monotonic. Therefore, chaotic iteration of MA with the different modules of a program will reach a fixpoint which is the least fixed point, because the separated \(lfp\) of each of the modules was computed.

Theorem 3.7 (Modular incremental analysis).
Given modular programs \(P, P'\) s.t. \(\Delta P = (C_i, C_j)\), \(P' = (P \cup C_i) \setminus C_j\), entries \(Es\), and \(G = \text{ModIncAnalyze}(P, Es, (\emptyset, \emptyset), (\emptyset, \emptyset))\):
\[
\text{ModIncAnalyze}(P', Es, (\emptyset, \emptyset), (\emptyset, \emptyset)) = \\
\text{ModIncAnalyze}(P, Es, G, \Delta P')
\]
Proof of Theorem 3.7. We show by induction that Theorem 3.7 is true for any partition of program \(P\) in \(n\) modules with no recursive dependencies on predicates between modules. This condition ensures that if removing for some rule in \(L^M\) is needed all the dependent information for recomputing is indeed removed (nothing imprecise is reused from some other \(L^M\)).

- If program \(P\) has one module, termination and precision of modular analysis is straightforward, it follows from Propositions 3.3 and 3.4.
- If program \(P = \{M_a, M_b\}\) is partitioned in 2 modules, since we do not allow recursive dependencies, let us assume that \(M_a\) imports \(M_b\). Let us assume that we reanalyze \(M_b\) first. We study the reanalysis cases of \(G' = \text{MA}(M_b, Es, G, L^M_b, \Delta M_a)\):
  1. If \(G' = G\) the procedure is equivalent if program \(P\) has one module.
  2. If \(G \sqsubseteq G'\), then analysis results need to be propagated to \(A\). Once the results of \(A\) are updated, the analysis iterations of \(A\) and \(B\) will be equivalent as when analyzing from scratch, because only new call patterns may appear (analysis results may not become incompatible nor smaller).
  3. If \(G' \sqsubset G\) these analysis results need to be propagated to the analysis of \(A\), which will be reanalyzed. Once \(A\) and \(B\) have updated their incompatible information the further (re)analyses can only become smaller, but since MA is monotonic and \(D_\alpha\) is finite, a fixpoint is reached, which is the \(lfp\) of \(P\), since the computation of each of the modules is the \(lfp\).
  4. Else, the information is incompatible. This can only happen if there were additions and deletions. This information needs to be propagated to \(M_a\) and the reanalysis of \(M_a\) will only lead to cases 1, 2, or 3.
- Program \(P\) is partitioned into \(N\) modules, we need to prove that if we finish analyzing \(N - 1\) modules, then we finish analyzing all \(N\) modules. Assuming that the analysis of the first \(N - 1\) modules finishes and it is the least fixed point, this \(N - 1\) result could be seen as one module, reducing this general case to the case of 2 modules.
B Worked example of the algorithm

In this section we provide a worked example to illustrate the operation of the algorithm. Consider the following modular program of Fig. 6 that computes the parity of a message, and a domain that recognizes the values that the parity can take: $D = \{ T, b, z, o, \perp \}$, $y(b) = \{ 0, 1 \}$, $y(z) = \{ 0 \}$, $y(o) = \{ 1 \}$, with the lattice structure shown in Fig. 7.

B.1 Analyzing form scratch

Assume that xor has not been fully implemented and only line 2 of module 11b is written.

We perform Modular Analyze($P, \emptyset, \text{main}(X,Y):T$), which proceeds as follows. Add entry for main($X,Y):T$, add updmod(main), start global fixpoint:

```
:- module(main, [main/2]).
:- use_module(lib, [xor/3]).
main(Message, Parity) :-
  Par = \emptyset,
  par(Message, Par, Parity).
par(Xs, Par#, Par) :-
  Xs = [],
  Par = Par#.
par(List, Par#, Par) :-
  List = [X]Xs,
  xor(X, Par#, Par1),
  par(Xs, Par1, Par).
```

```
:- module(lib, [xor/3]).
xor(0, 0, 0), % initial
xor(1, 0, 1), % clause added
xor(0, 1, 1).
```

![Figure 6. A simple two-module program.](image)

![Figure 7. Abstract domain to analyze program in Fig. 6.](image)
### Analysis graphs (analysis finished):

- **main**(X, Y): \( \top \mapsto (X/y, Y/z) \)
- **xor**(X, Y, Z): \( (Y/z) \mapsto X/z, Y/z, Z/z \)
- **par**(L, P, Y): \( (P/z) \mapsto (P/z, Y/z) \)

### Analysis result after finishing the algorithm:

- **main**(X, Y): \( \top \mapsto Y/z \)
- **xor**(X, Y, Z): \( (Y/z) \mapsto X/z, Y/z, Z/z \)
- **par**(X, P, Y): \( (P/z) \mapsto (P/z, Y/b) \)

### B.2 Adding xor clauses

We add incrementally the remaining clauses of xor:

**Modular Analyze**\( P, (\{ x_2, x_3, x_4, 0 \}, \text{main}(X, Y) : \top) \).

The algorithm proceeds as follows.

Module `lib` changed, so an event `updmod(lib)` is created with `Ent(lib) = xor(X, Y, Z) : Y/z`. Then:

- **process** | **actions**
  - updmod | `lib`
  - main | \( (X/y, Y/z) \mapsto \top \)
  - xor | \( (Y/z) \mapsto X/z, Y/z, Z/z \)
  - par | \( (P/z) \mapsto (P/z, Y/z) \)

### Analysis graphs after reanalyzing lib:

- **main**(X, Y): \( \top \mapsto P/z \)
- **xor**(X, P, P0): \( (P/z) \mapsto (X/z, Y/z, Z/b) \)
- **par**(X, P, Y): \( (P/z) \mapsto (P/z, Y/b) \)

### B.3 Deleting xor4 clause

Suppose that we want to delete the last clause of xor. We run

**Modular Analyze**\( P, (\emptyset, x_4), \text{main}(X, Y) : \top \).

The algorithm proceeds as follows. Module `lib` changed, so an event `updmod(lib)` is created with `Ent(lib) = xor(X, Y, Z) : Y/z`. Then:

- **process** | **actions**
  - updmod | `lib`
  - main | \( (X/y, Y/z) \mapsto \top \)
  - xor | \( (Y/z) \mapsto X/z, Y/z, Z/b \)
  - par | \( (P/z) \mapsto (P/z, Y/b) \)
Analysis result after deleting xor4:

C Additional information on the benchmarks

<table>
<thead>
<tr>
<th>Bench</th>
<th># Modules</th>
<th># Predicates; in each module</th>
<th># Clauses; in each module</th>
</tr>
</thead>
<tbody>
<tr>
<td>aiakl</td>
<td>4</td>
<td>(4 + 2 + 1 + 1)</td>
<td>(5 + 3 + 2 + 5)</td>
</tr>
<tr>
<td>ann</td>
<td>3</td>
<td>(24 + 32 + 13)</td>
<td>(43 + 69 + 117)</td>
</tr>
<tr>
<td>bid</td>
<td>7</td>
<td>(3 + 2 + 1 + 3 + 3 + 1 + 7)</td>
<td>(3 + 2 + 1 + 7 + 9 + 6 + 20)</td>
</tr>
<tr>
<td>boyer</td>
<td>4</td>
<td>(10 + 16 + 2 + 1)</td>
<td>(18 + 121 + 4 + 2)</td>
</tr>
<tr>
<td>cleandirs</td>
<td>3</td>
<td>(10 + 4 + 22)</td>
<td>(19 + 19 + 43)</td>
</tr>
<tr>
<td>hanoi</td>
<td>2</td>
<td>(3 + 1)</td>
<td>(4 + 2)</td>
</tr>
<tr>
<td>peephole</td>
<td>3</td>
<td>(11 + 18 + 4)</td>
<td>(26 + 59 + 84)</td>
</tr>
<tr>
<td>progeom</td>
<td>2</td>
<td>(4 + 6)</td>
<td>(9 + 9)</td>
</tr>
<tr>
<td>read</td>
<td>3</td>
<td>(6 + 14 + 5)</td>
<td>(8 + 74 + 12)</td>
</tr>
<tr>
<td>qsort</td>
<td>3</td>
<td>(2 + 2 + 4)</td>
<td>(4 + 5 + 8)</td>
</tr>
<tr>
<td>warplan</td>
<td>3</td>
<td>(4 + 4 + 29)</td>
<td>(33 + 4 + 77)</td>
</tr>
<tr>
<td>witt</td>
<td>4</td>
<td>(43 + 10 + 14 + 2)</td>
<td>(102 + 29 + 27 + 18)</td>
</tr>
</tbody>
</table>

Table 1. Benchmark characteristics
D  Results for the def domain
Figure 8. Addition experiments for def domain. The order inside each set of bars is: |mon|mon_inc|mod|mod_inc|.

Figure 9. Deletion experiments for def domain. The order inside each set of bars is: |mon|mon_inc|mod|mod_inc|mod_scc|. 
Detailed individual clause
addition/deletion results for other
benchmarks (shfr)
Figure 10. Analysis time for each edition test (aiakl).

Figure 11. Analysis time for each edition test (ann).
Figure 12. Analysis time for each edition test (bid).

Figure 13. Analysis time for each edition test (boyer).
Figure 14. Analysis time for each edition test (cleandirs).

Figure 15. Analysis time for each edition test (hanoi).
Figure 16. Analysis time for each edition test (peephole).

Figure 17. Analysis time for each edition test (progeom).
Figure 18. Analysis time for each edition test (read).

Figure 19. Analysis time for each edition test (qsort).
Figure 20. Analysis time for each edition test (rdtok).

Figure 21. Analysis time for each edition test (witt).