Comparative Saturn-Versus-Jupiter Tether Operation

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Abstract Saturn, Uranus, and Neptune, among the four Giant Outer planets, have magnetic field $\mathbf{B}$ about 20 times weaker than Jupiter. This could suggest, in principle, that planetary capture and operation using tethers, which involve $\mathbf{B}$ effects twice, might be much less effective at Saturn, in particular, than at Jupiter. It was recently found, however, that the very high Jovian $\mathbf{B}$ itself strongly limits conditions for tether use, maximum captured spacecraft-to-tether mass ratio only reaching to about 3.5. Further, it is here shown that planetary parameters and low magnetic field might make tether operation at Saturn more effective than at Jupiter. Operation analysis involves electron plasma density in a limited radial range, about 1–1.5 times Saturn radius, and is weakly requiring as regards density modeling.

1. Introduction

All Giant Outer planets have magnetic field $\mathbf{B}$ and corotating plasma, allowing *nonconventional* exploration. Electrodynamic tethers, which are thermodynamic (dissipative) in character, can

1. provide propellantless drag both for deorbiting spacecraft in Low Earth Orbit at end of mission and for planetary spacecraft capture and operation down the gravitational well, and
2. generate accompanying, useful electrical power, or store it to later invert tether current (Sanmartin et al., 1993; Sanmartin & Estes, 1999).

At Jupiter, tethers could be effective because its field $\mathbf{B}$ is high (Sanmartin et al., 2008). Tethers would allow a variety of science applications (Sanchez-Torres & Sanmartin, 2011). The Saturn field is 20 times smaller, however, *and* tether operation involves field $\mathbf{B}$ *twice*, which makes that thermodynamic character manifest:

1. The S/C velocity $\mathbf{v}'$ relative to the corotating magnetized planetary plasma induces in it a *motional* electric field $\mathbf{E}_m = \mathbf{v}' \wedge \mathbf{B}$, in the S/C reference frame, a *Faraday* effect in XIX-century language, previous to Relativity.
2. The field $\mathbf{B}$ exerts *Lorentz* drag per unit length $\mathbf{l} \wedge \mathbf{B}$ on tether current $\mathbf{l}$ driven by $\mathbf{E}_m$.

For given tether length and mass, a thin tape has larger perimeter and can thus collect more current than a wire. Planetary capture is more effective (S/C-to-tether mass ratio M_{SC}/m_t is higher), the thinner and longer a tape, and the lower the incoming S/C periapsis, $\mathbf{B}$ dipole value decaying very rapidly with distance to the planet. But *Jovian* tethers cannot operate at extreme optimal conditions because of the very high $\mathbf{B}$ value itself, which might lead to strong tether heating and to electrons crossing the tape with *penetration range* greater than thickness h , escaping collection; maximum mass ratio for effective capture by Jupiter might only reach to about 3.5 (Sanmartin et al., 2017).

In a two-body problem, all orbits around a planet, whether circular, elliptical, parabolic, or hyperbolic, keep a simple relation between eccentricity e , periapsis radius r_p , and (uniform and constant) specific energy ε ,

$$\varepsilon = \frac{\mu}{2r_p} (e - 1), \quad (1)$$

where μ is the planet gravitational constant. For the hyperbolic orbit of a S/C, with incoming velocity v_∞ in the *Saturn* frame, we have $\varepsilon_h = \frac{1}{2} v_\infty^2$, with $v_\infty = 5.42$ km/s, for a *Hohmann* transfer (Battin, 1999) from heliocentric circular orbit at *Earth* to circular orbit at *Saturn*, leading to

$$e_h - 1 = v_\infty^2 r_p / \mu \approx 0.046 \quad (2)$$

using $r_p \approx$ *Saturn* radius R , a numerical value we will consider in what follows.

Planetary capture requires the Lorentz drag work $W_d < 0$ to take down eccentricity to a value $e_c < 1$, and a negative energy, $W_d/M_{SC} = \Delta\epsilon < 0$, yielding

$$\frac{|W_d|}{m_t v_\infty^2/2} = \frac{M_{SC}}{m_t} \times \frac{e_h - e_c}{e_h - 1}, \quad (3)$$

where we used equations (1) and (2). The normalized drag-work in (3) conveniently exhibits tether mass because Lorentz drag, involving the product of tether length L and length-averaged current I_{av} , which is bounded by the *short-circuit* value, $\sigma_t E_m \times hw$ with σ_t and w tether conductivity and width, thus involves tether volume. For further convenience, we write the LHS of (3) as product of two dimensionless factors

$$|W_d|/\frac{1}{2}m_t v_\infty^2 \equiv B_s^{*2} \times W_d^* \quad (4)$$

where

$$B_s^{*2} = \sigma_t B_s^2 a_s v_s / 2^{5/6} \rho_t v_\infty^2 \quad (5)$$

with ρ_t tether density and B_s and v_s reference values at radius $a_s = (\mu/\Omega^2)^{1/3}$ ($\approx 1.84R$ for Saturn) of a *stationary* equatorial circular orbit, where velocities of SC and plasma co-rotating with a planetary spin Ω are equal.

With resulting eccentricity after capture $e_c < 1$ typically close to unity too, drag work may be approximately calculated along a parabolic orbit. The drag arc extends on both sides of the periapsis, from $r_p \approx R$ to a radius $r_M = a_s \times (2a_s/r_p)^{1/2}$, where Lorentz drag vanishes with the tangent relative velocity, leading to (Sanmartin et al., 2008)

$$W_d^* = r_M^{*8/3} \int_1^{r_M^*} \frac{(r_M^* - r^*)}{r^{*6} \sqrt{r^* - 1}} dr^* < 2i_{av}(r^*, \phi) \cos^2 \varphi >_\varphi \quad (r^* \equiv r/r_p) \quad (6)$$

where i_{av} is the ratio $I_{av}/\sigma_t E_m wh$, with W_d^* further involving ϕ and r averages. Averaging over angle ϕ between tether and $\mathbf{E}_m$ in W_d^* arises from tether spin introduced for Jupiter to limit tether bowing.

2. Tether Capture Efficiency at Saturn

Consider, for simplicity, capture leading to barely elliptical orbit, the eccentricity ratio in ((3)) then being about unity, and the mass ratio M_{SC}/m_t about $B_s^{*2} \times W_d^*$. The first factor for an Earth-to-Saturn *Hohmann* transfer is comparatively very small; for aluminum, $B_s^{*2} \approx 0.0083$, as against 2.11 for the corresponding capture analysis of *Jupiter* (Sanmartin et al., 2008). A Hohmann transfer may be not good enough option for Saturn, as leading to a heliocentric spacecraft velocity at the encounter with the planet well below the planet orbital velocity, resulting in that large incidence v_∞ value. A *gravity assist* (Battin, 1999) from Jupiter in the S/C route to Saturn, through a tail flyby, could reduce v_∞ from 5.44 to 2.5 km/s, thus increasing B_s^{*2} by a 4.8 factor, while allowing a decrease in launch specific impulse from 10.29 to 8.8 km/s, all which are worth the increase in total flight time from 6.05 to about 9 years.

As regards W_d^* , three effects concur to allow increasing the captured mass ratio:

1. No spin is required for the Saturn weak field, W_d^* then increasing in equation (6) above by about 2.
2. The normalized length-averaged tether current i_{av} , in work W_d^* , takes the form

$$I_{av}/\sigma_t E_m wh \equiv i_{av} (L/L_{ch}), \quad (7)$$

varying with a characteristic length $L_{ch}(r)$ that gauges ohmic and OML collection impedances (Sanmartin et al., 2008):

$$\frac{4}{3} e N_e \frac{2wL_{ch}}{\pi} \sqrt{\frac{2eE_m L_{ch}}{m_e}} \equiv \sigma_t E_m wh \rightarrow L_{ch}(r) \propto h^{2/3} \times (v'B/N_e^2)^{1/3} \quad (8)$$

At very small L/L_{ch} values, current i_{av} takes a limit form

$$i_{av} \approx 0.3 \times (L/L_{ch})^{3/2} \ll 1, \quad (9)$$

corresponding to negligible ohmic effects; at large L/L_{ch} , for dominant ohmic effects, i_{av} approaches unity. Accurate overall approximations for the average-current law are

$$i_{av} \approx 1 - L_{ch}/L, \quad L/L_{ch} \geq 2, \quad (10a)$$

$$L/4L_{ch}, \quad L/L_{ch} \leq 2. \quad (10b)$$

Saturn and Jupiter operations present an important difference involving i_{av} and the issues raised in (Sanmartin et al., 2017). Consider how maximum tether point temperature varies with L and B (Sanmartin et al., 2008),

$$T_{max} \propto (LB)^{3/8}.$$

Higher field B will require lower length L , and higher L_{ch} ($\propto B^{1/3}$). The result is that tethers operate in Jupiter at very small values of both L/L_{ch} and current i_{av} , whereas the weaker magnetic field of Saturn allows operating at values $i_{av} = O(1)$, making possible in principle, a much larger drag work W_d^* value.

3. The very short reach of Lorentz drag at Saturn (low both a_s/R , and $r_M/R \approx 3.53$ —as against 4.74 for Jupiter) would make possible a further W_d^* gain of about 2, the incoming S/C being made to approach facing the corotating plasma. This requires just changing $r_M - r$ to $r_M + r$ in the W_d^* integral, involving a weak change in i_{av} .

$$W_d^* = 2r_M^{*11/3} \times \int_1^{r^*} \frac{(1 + r^*/r_M^*)}{r^{*6} \sqrt{r^* - 1}} dr^* \times i_{av}(r^*) \quad (11)$$

With $i_{av} = O(1)$, the integral converges rapidly due to the r^{-6} power, whatever the upper limit; writing

$$W_d^* = \langle i_{av} \rangle_{r^*} \times 2r_M^{*11/3} \times \int_1^{r^*} \frac{(1 + r^*/r_M^*)}{r^{*6} \sqrt{r^* - 1}} dr^* \quad (12)$$

the above integral increases from 0.971 to just 1.008 in taking the upper limit $\tilde{r}_\infty$ from 1.5 to 2. Overall, the particular Saturn effects discussed above lead to an energy equation

$$\frac{e h^{-e} c}{e h^{-1}} \times \frac{M_{SC}}{m_t} = \frac{|W_d|}{m_t v_\infty^2 / 2} \approx \langle i_{av} \rangle_r \times 8.1, \quad (13)$$

which is greater than 4 for $\langle i_{av} \rangle_r$ about 0.5.

3. Plasma Density Modeling Issues

Operation analysis at *Saturn* requires models for both magnetic field and plasma density N_e , in the limited radial distance domain above, $r/R = 1$ –1.5, of interest for tethers, covering the ionosphere (reaching to about 10,000 km or 1.16 R) and the two inner, weaker rings D and C . The first is represented by its well-characterized dipole term. This is not the case of N_e , however.

First, because the Coulomb cross section is much larger than the cross section for collisions of atoms and molecules, the plasma exobase lies higher than the exobase of the neutral atmosphere (Chamberlain, 1963). Yet the electron distribution function may be not quite Maxwellian at the altitude range of interest here. It apparently might present a high-energy tail, corresponding to a *Lorentzian* distribution function, characterized by a *kappa* value as low as 2, resulting, at given temperature, in higher N_e . Well outside the radial

range of interest, there may also exist complex evolution in dominant ion species, *hot-cold* electron temperatures, and plasma corotation conditions (Schippers et al., 2008; Thomsen et al., 2010).

At *Orbit Insertion*, *Cassini* reached as close as 1.3 R , well within the domain of interest. Unfortunately, however, the plasma it met at Saturn might not be considered particularly representative because a variety of Saturn conditions favor surprisingly large changes in density, temperature, and ion composition. Its obliquity, about 26°, make for strong seasonal variations, while its sidereal period, 29.5 years, mean Saturn takes almost three 11-year solar cycles to go through the full seasonal variation.

On the other hand, the short, 10.6-hr sidereal day lead to fast plasma changes. The *optically thick* outer rings, B and A , reaching beyond 2 R , lie between Saturn and moon *Enceladus*, at about 4 R from Saturn. Day/night plasma parameters correspond to quite different conditions. Observations and modeling suggest that the UV sunlight on the rings might dissociate molecules and atoms from the icy surfaces, and these low-energy exospheric neutrals then ionize to form a *ring exoionosphere* in that side of the rings. Under unlit conditions the plasma content is anomalously low; on unlit faces, the electron-absorbing rings determine the plasma content. The result might be a sensible increase of plasma density from dawn to dusk.

Cassini Orbital Insertion took place past midnight, in mid-2004, when the spacecraft was in the northern hemisphere, which had recently passed its winter solstice, and the solar cycle was approaching a minimum in 2008. The mission *Grand Finale*, however, took place last September 2017, at northern solstice, though again near a solar cycle minimum in 2019.

At present, after an abundance of experimental data and model attempts, there is no generally acceptable definite model for plasma density, as tether analysis, in principle, requires. *Cassini*, now at end-of-life mission, may round up measurements at distances of interest and make basic contribution to modeling (André et al., 2008; Moore & Mendillo, 2005).

To escape all this *variability* and achieve efficient capture, ensuring the highest electron density possible, the need is evident of our tethered spacecraft arrival on solar maximum conditions, during one of the solstices (and to its respective hemisphere), and with the capture operation occurring on the diurnal side of Saturn.

There are a number of Saturn missions proposed in the past: on Saturn itself: *Kronos*, *Saturn Atmospheric Entry Probe*, and *Saturn Ring Observer*; on Titan: *AVIATR*, *Dragonfly*, *TALISE*, and *Titan Mare Explorer*; on Enceladus: *Enceladus Life Finder*, *Enceladus Explorer*, and *Life Investigation for Enceladus*; and on mixed objectives: *Journey to Enceladus and Titan* and *Titan Saturn System Mission*.

A new proposal is just sketched here, hoping that data on plasma density from the *Grand Finale* will allow completing it. Emphasis is in the capture and operations approach.

4. Gross Estimates for Concept of Tether Use at Saturn

Drag work $|W_d|$ only involves plasma density through the dimensionless length-averaged current inside the normalized work integral W_d^* , i_{av} (L/L_{ch}) varying with the characteristic length L_{ch}

$$\frac{4}{3} e N_e \frac{2wL_{ch}}{\pi} \sqrt{\frac{2eE_m L_{ch}}{m_e}} = \sigma_c E_m w h, \quad (14)$$

as discussed above.

Using the magnetic dipole law, $B = B_s a_s^3/r^3$ ($B_s = 3.37 \times 10^{-6}$ Vs/m²), $v' = (v^2 + \Omega^2 r^2 + 2\Omega r v_p)^{1/2}$ ($v_s = 26.2$ km/s) yields

$$E_m = v' B = \frac{r_M^{*4/3}}{2^{7/6}} v_s B_s \times E_m^*(r^*), \quad E_m^* = \left(\frac{r^{*3} + 2r_M^* r^{*2} + r_M^{*2}}{r^{*7}} \right)^{1/2} \quad (15)$$

which, when used in (14), leads to a radial dependence $L_{ch}(r) \propto E_m^{*1/3}/N_e^{2/3}$, with numerator continuously decreasing with increasing r , from $(1 + r_M^*)^{1/3} \approx 1.65$ at r_p . Writing $N_e = N_e^* \times 10^2$ cm⁻³, and setting $h = 10$ μ m, $\sigma_t = 3.5 \times 10^7$ /ohm $\times$ meter for Al , there results

$$L_{ch}(r^*) \approx 74 \text{ km} \times E_m^{*1/3}/N_e^{*2/3}.$$

Consider a reasonable density in the lower ionosphere, $N_e(r \approx r_p) = 10^3 \text{ cm}^{-3}$ (Kliore et al., 2009; Nagy et al., 2009), leading to

$$L_{\text{ch}}(r^* = 1) \approx 26 \text{ km}$$

and select $L = 2 L_{\text{ch}}(r_p) = 52 \text{ km}$, yielding $i_{\text{av}}[L/L_{\text{ch}}(r_p) \approx 2] \approx 0.5$, in (10a) and (10b). Away from the lower ionosphere, drops in density, resulting in larger L_{ch} and thus lower i_{av} at selected L , can be partially compensated by the drop in $E_m^{*1/3}$. Independently, contributions to the radial average $\langle i_{\text{av}} \rangle_r$, become less relevant the farther away from r_p , as i_{av} approaches unity as $L_{\text{ch}}(r)$ decreases.

Further, note that the effect of increases in L_{ch} , and thus in i_{av} decreased values, from density drops, is smaller the larger L . One may just select a conveniently longer tape, with no penalty on the captured mass ratio, tape mass m_t being balanced by a reduction in width w , which has no other effect in the analysis.

5. Conclusions

Tether capture and apoapsis-down operations with mass ratio (M_{SC}/m_t) comparable to the value derived for a Jupiter application appear possible for a broad range of plasma density profiles in the approximate range $1 < r/R < 1.5$, dominant as regards tether efficiency at Saturn missions.

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