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**CONTRIBUTIONS TO SENSOR
MODELING AND BIAS
ESTIMATION ALGORITHMS FOR
AIR TRAFFIC SURVEILLANCE**

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Contributions to Sensor Modeling and Bias Estimation Algorithms for Air Traffic Surveillance

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Abstract

This Ph. D thesis deals with the development of tracking algorithms in Air Traffic Control Scenarios (ATC). It focuses specifically in the modeling and correction of the systematic error (biases) of the measurements produced by the sensors and in the estimation methods for the correction of these biases. For this correction it is developed a theoretical model for the sensor biases and it is developed a simulator for the testing of the estimation algorithms. Apart from the theoretical model for the sensors, some improvements for the current estimation methods are proposed for the applications in real scenarios. These algorithms and models are tested with both simulated and real data from a real system.

The first chapter of the thesis is an introduction that describes an Air Traffic Management (ATM) system where the bias estimation topic is used and shows the goals and organization of this work.

In the second chapter the systematic errors of the sensors used in ATM systems are described and modeled. The parametric models used in the estimation methods are presented. This chapter is focused in the model for the Secondary Surveillance Radar (SSR) and in the target dependent biases.

In the third chapter the methods that are currently used are described the methods that are currently used for the estimation of the biases in the surveillance function of the ATM systems. In this chapter the validity of the model presented in the previous chapter is tested the validity of the model presented in the previous chapter using the estimation methods presented at the beginning this chapter.

The fourth chapter proposes improvements for the estimation methods described in the chapter 3. The first improvements is an algorithm to get the

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best accuracy in the permanent regimen including initialization techniques for the reduction of instabilities in the initial stages. The second improvement is a method for the online detection of the lack of observability in the scenarios. In this section is described both analytically and geometrically the observability.

The fifth chapter proposes new methods for the bias estimation. The first method is an extension for the meteorological model with the use of surfaces for the estimation of the pressure and temperature bias in the barometric altitude. The second method is a modification of an existent estimation method for the target dependent biases. The last proposed method is used for the estimation of the biases in the velocity measurement of the aircraft.

Resumen

En esta tesis doctoral se aborda el desarrollo de algoritmos de seguimiento de aeronaves en el ámbito del control del tráfico aéreo. En concreto, esta tesis se centra en el modelado en la corrección de los errores sistemáticos (sesgos) de las medidas producidos por los sensores y en los métodos de estimación para corregir estos errores. Para la corrección de estos sesgos se desarrolla un modelo teórico de los sesgos de los sensores y se desarrolla un simulador con el que poder probar los algoritmos de estimación. Aparte del modelo teórico de los sensores se desarrollan algoritmos de estimación partiendo de algoritmos existentes y se proponen mejoras para la aplicación en entornos reales. Estos algoritmos y modelos son probados con datos procedentes de un sistema real.

En el primer capítulo se realiza una introducción que describe un sistema de gestión del tráfico aéreo (ATM) donde se sitúa la estimación de sesgos y se describen los objetivos y la organización de la tesis.

En el segundo capítulo se describen y modelan los errores sistemáticos de los sensores. Se presenta un modelo paramétrico que será utilizado en los métodos de estimación. Este capítulo está centrado en el modelo de radares secundarios y en los errores dependientes de las aeronaves.

En el tercer capítulo se describen los métodos de estimación que se utilizan actualmente para la estimación de sesgos en la función de vigilancia de los sistemas ATM. En este capítulo se prueba la validez del modelo presentado en el capítulo anterior con los métodos actuales.

En el capítulo cuatro se proponen mejoras para los métodos descritos en el capítulo 3. La primera de las mejoras son técnicas de inicialización de algoritmos. La segunda de las mejoras es un método para detectar la falta de

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observabilidad de los escenarios en tiempo real. Este método va acompañado de una descripción tanto analítica como geométrica de la observabilidad.

En el quinto capítulo se proponen nuevos métodos para la estimación de sesgos. El primero de estos métodos es una extensión de modelo meteorológico con el uso de curvas de nivel para la estimación del sesgo de temperatura y presión utilizado en la medida de altura barométrica. El segundo es una modificación de un método de estimación de sesgos dependientes de la aeronave. El ultimo método propuesto es utilizado para la estimación de sesgos en la medida de velocidad de las aeronaves.

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Chapter 1

Introduction

Nowadays Air Traffic Management (ATM) (The group of systems that assists and manage the aircraft of the airspace) supports one of the biggest industries of the worlds. As it is indicated in [ICAO, 2005], the air transport industry plays a major role in world economic activity and remains one of the fastest growing sectors of the world economy. Just in Europe, the average daily traffic in 2017 was of 29057 flights. That is an increase of 4.4 % compared to previous years [EUROCONTROL, 2017].

An ATM system is composed by several subsystems: the air traffic flow management, the surveillance system and the aeronautical information systems. The basis of the present and future of ATM is the surveillance of the airspace. The control of the flow and the avoidance of conflicts needs from accurate and reliable surveillance information. An accurate surveillance allows to achieve the objectives of safety, capacity and efficiency that are indicated in the capacity and efficiency plan [ICAO, 2014].

This thesis focus on the improvement of the accuracy of the positions in multitarget and multisensor scenarios. The analyzed systems and proposals are designed to exploit the available information in the airspace using the measurements of the aircraft from different sensors.

1.1. ATM SYSTEM DESCRIPTION

1.1 ATM System Description

In the aviation world the term CNS/ATM describe the aeronautical communications, air navigation, surveillance and ATM. In the next paragraphs each one will be described. The definition of ATM [ICAO, 2005] in words of the International Civil Aviation Organization (ICAO) is:

”Air traffic management is the dynamic, integrated management of air traffic and airspace — safely, economically and efficiently — through the provision of facilities and seamless services in collaboration with all parties.”

As it is indicated by the European Organisation for the Safety of Air Navigation (EUROCONTROL), the ATM is divided in three different activities [EUROCONTROL, 2018b]. The first one is the Air Traffic Flow Management, the second one is the Air Traffic Control (ATC) and the third is the Aeronautical Information Systems.

The Air Traffic Flow Management is determined before the takeoff of the aircraft. An Air Traffic Controller (ATCO) can handle a limited number of aircraft simultaneously because of safety reasons. The Flow Management ensures that the effort of each ATCO do not compromise the safety of the airspace.

The ATC is the service provided by the ATCO for the ordering and separation of the aircraft in accordance with traffic and environment conditions. The Air traffic control is divided in three different areas:

- Air control service: ATCO manage the controlled flights in route and airways.
- Approach control service (APP): ATCO manage the arrival and departure of controlled flights in a control zone and in zones determined by the terminal control areas.
- Aerodrome control service (TWR): ATCO manage the aircraft at the airport and its environs.

This thesis deals with the traffic control of the air control service and the approach control service.

The Aeronautical Information Systems include information on: safety,

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navigation, technical, administrative or legal matters and their updates. For example the meteorological prediction information, changes in the structure of the airspace as restricted areas, QNH pressure information...

The ATM systems are helped by the aviation communications. These communications provide a voice link between the pilots or between pilot and ground ATCO and several data links between ATCO, sensors and pilots in order to increase the safety.

The evolution of the surveillance networks include the total collaboration between the sensors and the aircraft. Because of that the measures of the aircraft positions are made by the airborne equipment of the aircraft (more accurate than the ground based measures). The ATM includes system to support this evolution and the correct navigation of the aircraft.

The air navigation systems can be classified in different categories. The first one is the radio navigation [Clausing, 1997], the Inertial Navigational System (INS) and the Global Navigation Satellite System (GNSS). The radionavigation can be defined as the set of radio electric signals generated in the Earth surface that help to the guidance of the aircraft. The radionavigation used in ATM are the Not Directional Beacon (NDB), VHF Omnidirectional Radio range (VOR) [Winick and Brandewie, 1970], Tactical Air Navigation System (TACAN) [Prichard, 1965], Distance Measure Equipment (DME), Instrumental Landing System (ILS)... [Corbasi, 1998] All this systems are based in different principles that provided information to the airborne equipment as the distance or the azimuth to a fix point.

The INS [Britting, 1971] are based in the airborne equipment and they do not need reference point nor external signals. The INS uses the initial position of the flight and uses the information of the accelerometers and gyroscopes to calculate the current position. These kind of systems needs the correction of the position periodically with another navigation systems because the errors in the positions are accumulative.

The GNSS [Hofmann-Wellenhof et al., 2007, Forssell, 2008] as Global Position System (GPS) [Hofmann-Wellenhof et al., 2012], GLONASS [Stewart and Tsakiri, 1998] or GALILEO [Nurmi et al., 2015] have almost substituted [of Defense, 2008] the old navigation systems as main navigation source. The accuracy of these kind of sensors is much higher than the radio navi-

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gation systems. The use of GNSS is easier to implement in remote areas where the installation of another kind of equipment is no viable. Nowadays the use GNSS must be used with another navigation system because any of the satellite system provides the integrity to now if there is any fail in the position determination.

Typically the planning of the air routes was based in the positions of the radio navigation system due to the airborne determination of the positions was provided by these fixed point but nowadays the planning of the air routes is changing. The first Free Route Airspace initiatives started as early as 2009 [EUROCONTROL, 2011]. Free routes airspace initiatives has several advantages in terms of time and fuel consumption. This initiative is included in a collaborative project called Single European Sky ATM Research (SESAR).

The increasing number of aircraft using the Free Route Planning has increased the need of a precise trajectory prediction. The BADA model [EUROCONTROL, 2018a] a set of equation and parameters that model accurately the movement of the aircraft. This model is used to predict the 4D trajectories of the aircraft for the planning and conflict detections of the airspace. The flow management described in the previous paragraph uses the trajectories predicted by the BADA model.

In figure 1.1 it is shown a block diagram of an ATM system. There are a long term planning that is made previous to the takeoff of the aircraft. The information of this planning is given to the ATCO and to the short term planning. The short term planning predicts the trajectory of the aircraft accurately from the current position of the aircraft (provided by the surveillance system). With the accurate prediction of the trajectories the conflict are detected if two trajectories cross in the same point. The long term planning include the planning for scenarios for more than 20-30 minutes and short planning include the nearer conflicts. The ATCO and some other safety systems that are described in the next sections solve the immediate conflicts.

With the information provided by the surveillance system, the trajectory predictor and the conflict detector, taking in account the requested maneuver by the pilot of the aircraft, the ATCO indicates modification to the planned

Figure 1.1: Block Diagram of an ATM System

trajectory of the aircraft in order to ensure the safety of the airspace and increase as much as possible the efficiency of this airspace.

In figure 1.1 it is shown also the radionavigation systems and the aeronautical information systems that has been described in above.

In this thesis mainly the surveillance systems are studied and described. In the next section this system is introduced and the proposal are presented.

1.2 Surveillance Systems

In an ATM system all the systems are supported by the surveillance system. In this section the surveillance system is described in order to give the context to the proposals of this thesis.

The surveillance system is composed by the two different subsystems. The first one is the sensor network and the second one is the tracker for the

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information fusion of the sensor network.

The sensor network provides the positions (and for some sensors the velocity) of the aircraft. There are several kinds of sensor that provides the position measures but usually the information must be completed and corrected. In some cases two aircraft can have the same identification and the association of the plots with the tracks must be done. The measures are affected by different kind of errors, random (noise) and systematic (biases). The information that is incomplete is completed and corrected in the tracker subsystem.

The tracker subsystem must to estimate the positions of the aircraft accurately reducing the noise of the measurement and estimating the hidden parameters of the measures as the velocity, acceleration, rate of turn... This subsystem must associate the measures to the correct track and estimate the biases too. In the next section both subsystem are widely described.

1.2.1 ATM Sensor Network

In ATM Systems there are several kinds of sensors that are used currently. They are based in different working principles with different collaboration levels with the aircraft.

The most used sensor in current ATM surveillance networks is the radar. A radar is a device that transmit a radio electric pulse and receive the echo from the aircraft. Measuring the time span between the transmission of the pulse and the receiving of the echo the distance to the target can be determined. The azimuth of the aircraft is determined with the azimuth of antenna pointing. The described radar is a Primary Surveillance Radar (PSR) ([Skolnik, 1962], [Skolnik, 1970]) and It was in 1952 when the first civil radars started to be used in the proximity of the airports [Nolan, 2010]. Currently the radars used in ATM are the Secondary Surveillance Radar (SSR), an evolution of these PSR.

In this case the aircraft is not a passive element. The SSR transmit an interrogation and the aircraft respond to it sending some requested information. An SSR is a device that use a principle similar to a PSR, determining the distance with the time span between the interrogation and the response

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and the azimuth with the pointing of the antenna.

Figure 1.2: Image of a radar with two antennas. The big one is a PSR and the small one is a SSR. This image has been extracted from [INDRA, b]

The SSR works with different modes. The first developed mode (for civil traffic) was the Mode A. When the radar interrogates the target with this mode the aircraft must answer with its identification. The identification codes are assigned dynamically and there can be two aircraft with the same code in the same area. In some cases the flights that are uncontrolled by the ATCO receive the same codes. This problem is solved with the association [Bar-Shalom et al., 1990] in the tracker (process that assigns a measure to a trajectory based on distance).

The Mode C interrogation asks for the aircraft altitude. Using this mode the aircraft respond to the interrogation with the barometric altitude measured by the airborne equipment. Using this scheme the horizontal and vertical position are determined by independent devices.

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The Mode S [Bussolari and Bernays, 1995] is the most advanced mode used in ATM surveillance systems. Using this mode the radar uses a selective interrogation in order to solve the problems produced by the previous described modes [Harris, 1956]. The identification used in this case is a static and unique code that is different for each aircraft. The mode S was designed to solve the existent problems of the SSR but it was designed as a communication channel.

The PSR works currently as a backup system for the SSR. In civil application the collaboration of the aircraft is assumed for its identification but the transponders of the aircraft can fail and the PSR are still working under these conditions and because there are small and old aircraft that have not transponder. In figure 1.2 it is shown an image of a radar station. Usually these station have both systems SSR and PSR in the same position. In the image there are two different antennas. The lower and bigger with a reflector is the PSR. The smaller array antenna is a Monopulse Secondary Surveillance Radar (MSSR) [INDRA, a], [Jacovitti, 1983].

In modern scenarios more advanced sensors are used for the measurement of the aircraft positions. These sensors are the Automatic Dependent Surveillance - Broadcast (ADS-B) and Wide Area Multilateration (WAM). Both system requires the collaboration of the aircraft. The ADS-B uses the Mode S messages to communicate the aircraft, the ADS-B ground stations and the surrounding aircraft. In this system the aircraft broadcast its position, speed and identification periodically. These messages are received by the ADS-B ground stations (and other aircraft) and are sent to the fusion center. In figure 1.3 can be seen a scheme of the ADS-B system.

Figure shows messages ADS-B out and ADS-B in between two aircraft. The ADS-B messages are used in the Traffic alert and Collision Avoidance System (TCAS) onboard on surrounding aircraft [EUROCONTROL, 2016]. This airborne system is used to detect a possible collision and avoid it with a coordinated maneuver of both aircraft (without the intervention of the ATCO). The ADS-B system has coverage problems in remote areas as the oceans. Typically these areas are covered with the ADS-C system [De Haan

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Figure 1.3: Scheme of ADS-B system extracted from [Delille and Muller, 2010]

et al., 2013] [Mattos, 2010]. In the recent years the coverage of the ADS-B system is being augmented with satellites [AIREON, 2018].

The WAM system is composed by a network of antennas allocated in a coverage area. The aircraft sends a signal (some versions uses ADS-B messages and SSR Mode S [Galati et al., 2005]) that are received by several WAM stations. Using a Time Difference Of Arrival (TDOA) algorithm the positions of the aircraft are calculated with the multilateration of the signals [Galati et al., 2008], [Mathias et al., 2008]. In figure 1.4 it is shown a simplified working scheme of the WAM system.

Even with the introduction of more accurate and cheap sensor, the use of radar is still needed because of the security [Kim et al., 2017], [McCallie et al., 2011] and as a backup system in case of failure. The SSR is the collaborative backup system and the PSR is the backup system used in critical areas as

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Figure 1.4: Scheme of WAM system extracted from [Delille and Muller, 2010]

the airports.

In figure 1.5 a complete sensor network with radar, WAM and ADS-B is represented. The cooperative aircraft send its position and speed that is received by the ADS-B stations. The ADS-B measures are sent to a central station to filter the repeated measures. The time measures of WAM stations are sent to a central station in order to calculate the position of the aircraft. All the position measurements are sent to the fusion center in order to make the tracking function that is described in the next section.

1.2.2 Surveillance Tracker Architectures

The tracker subsystem is the part of the ATM surveillance that orders the received position measurements (plots), associates each plot to its estimated aircraft trajectory (track) and filters the positions in order to reduce the measurement noise and estimate the hidden parameters.

Figure 1.5: Scheme of a global integrated surveillance sensor network

One of the steps of the tracker is the association. The association is the process where it is determined the track of the plot. In PSR the association [Blackman, 2004] [Blackman et al., 1995] of the plots is a difficult phase because the plot are not identified with the aircraft ID. In the case of secondary radar, the aircraft usually sends the Mode A in the response. This ID can be not unique and in cases where there are garbling (interferences produced by the simultaneous responses from several aircraft) [Harris, 1956] the association of the plots must be done. Using the Mode S, ADS-B and WAM the association is much easier because the aircraft sends a unique ICAO 24-bit aircraft address (protected by a redundancy code).

There are several different schemes for the processing of the plots that can be used [Olivier et al., 2009]. In this section some of these schemes are

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Figure 1.6: Scheme of a plot selection tracking

described in order to compare them and introduce the problems studied in this thesis. In the first years of the tracker systems the airspace was divided in different areas (mosaic) where each cell of the mosaic had the sensors ordered with quality parameters. Using these quality parameters the best plot of each mosaic cell is selected and this plot is used for the tracking. In figure 1.6 is shown a block diagram with this scheme.

Figure 1.7: Scheme of a track selection tracking

Getting a little higher computational workload and using a similar division of the airspace is presented a track selection scheme. In figure 1.7 it is shown this scheme. In this case there is a monosensor track for each sensor and the best trajectory is selected. One of the methods to select the track is measure the difference between the measures and the predicted positions (residual) by the tracker and select the track with the lower one.

In the next paragraphs some schemes with information fusion are presented. The first one is the classic working scheme with a decentralized

Figure 1.8: Scheme of a distributed tracking where the fusion is made track to track

tracking of the aircraft and a track to track fusion [Bar-Shalom, 1981], [Chang et al., 1997]. In this scheme that is shown in figure 1.8 there are a track for each sensor. That implies that the association described above is made to the local tracks. When the monosensor tracks are processed they are sent to the fusion center where there are a track-to-track association and fusion (weighted average of the local tracks in function of the quality of each one). After the fusion of the monosensor tracks it is get the central track composed by all the monosensor tracks.

Figure 1.9: Scheme of a centralized tracking where the fusion is made plot to plot

The other scheme that is shown in figure 1.9. In this case the sensors send the plot without a track process. In the fusion center the plots are associated to the central track. In this scheme the tracking filter receives the plots from every sensor and process them jointly. Figure 1.10 shows a diagram comparing the different described schemes evaluating the accuracy and computation workload.

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Figure 1.10: Diagram comparing different scheme showing the performance versus the computational workload

In the ideal case the centralized scheme presents some advantages that are important in the ATM surveillance systems. As in the tracker the information comes from several sensors, the sampling rate of the trajectory is higher and the maneuvers are detected sooner. As in the same tracker the number of plots is higher, the reduction of the noise variance is higher than with the track-to-track fusion. The accuracy in position is about a 10% higher using the centralized scheme [Chen et al., 2003].

1.2.3 Need for sensor alignment

In real scenarios there are several considerations that must be taken in account. The measures are affected by random error but they are affected by systematic errors too [Nabaa and Bishop, 1999] for example a bad calibration of the north mark. Each sensor of the scenario has different properties that make it have biases. Some biases are produced by the need of maintenance, weather conditions, installation errors... These systematic errors are common to all the plots of the same radar but they produce misalignment between the plots from different radars.

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Figure 1.11: Trajectory with misaligned plots. The different colors indicate the measures from different radars.

Figure 1.12: Scheme of a hybrid tracking where the fusion is made plot to plot but the association uses information from the monosensor track

In figure 1.11 it is shown a situation where the plots from the different sensors are misaligned in position. In the aforementioned figure, the measures (uncorrected) are shown with asterisk and the different colors represent the different sensor. The red lines represent the velocity vector of the filtered plots using the uncorrected plots (asterisk) and the blue lines represent the velocity vector of the filtered plots using the corrected plots (not represented

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in this figure). Using the uncorrected plots, the velocity is unstable and it is far away from the ideal course. In some cases there is a zig-zag effect in the trajectory.

Using a centralized scheme there are two problems that must be solved. The first problem is the zig-zag track described in the previous paragraph. The second problem is the association. When the systematic errors are big, the distance between two plots from different radars can be assigned to different tracks.

The association problem can be solved using a hybrid scheme (figure 1.12). This scheme processes the plots in a global tracker using all the plots from all the sensor. In order to avoid the problem of associate a plot to a wrong track, the association is made with the monosensor track and this information is used to associate the plots to the central (multisensor) track.

The errors produced in the tracking filter are more difficult to solve and the correction of the biases is needed. The systematic errors are deterministic and usually they can be modeled. If the values of the biases are known or estimated, the measured plots can be corrected. The errors that are visible in figure 1.11 are position biases, but the measurements can be affected by another kind of errors (time stamp error or longitudinal errors) that are not visible just with the horizontal projection.

In the fusion center (represented in figure 1.13) the plots are processed in the receiving order. In cases where the delay of the communication lines is very big the, plots can be processed disordered. In figure 1.14 a disordered chain of plots is shown where delayed plots are processed after the plots with small delay.

In this case when the time of the plot is corrected, the delayed plot are processed after the nondelayed plots, but the corrected time stamp in this case is well indicated. For these scenarios a backward tracking filter ([Bar-Shalom, 2002], [Zhang et al., 2005], [Mallick et al., 2002]) must be used in order to filter plots which the time is lower than the last filtered plot.

One of the solutions to solve the misalignments between the plots is the use of a decentralized processing averaging the local tracks [Papageorgiou and

Figure 1.13: Simplified scheme of a fusion center showing the delays in the communication lines.

Figure 1.14: Representation of disordered plots.

Holender, 2009]. This method has a stable speed and course but the final positions of the central track can be far away from the ideal positions. The time bias using this scheme can be solved as is proposed in [Challa and Legg, 2002] or [Novoselsky et al., 2007] using the Bayesian solution to the out-of-sequence information or using the process noise covariance to retrodict, both the central state and its covariance matrix to time

Using a centralized scheme the biases of the sensors must be estimated. The problem of the bias estimation is also known as the sensor registration.

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The sensor registration is a topic that has been widely described in the literature [Cruz et al., 1992], [Okello and Ristic, 2003], [Vermaak et al., 2005]. In all the existent literature theoretical problem is described and studied but there are some practical aspects that are not addressed in the papers. This thesis deals with the sensor registration using a centralized scheme solving some practical aspects that are not studied in the classical literature.

1.3 Objectives of the Thesis

The main objective of this thesis is the alignment of the measurements through the estimation of the systematic errors of the sensors. As it has been indicated before, these systematic errors produces the misalignments between the measurements from different sensors. In order to solve this problem there are two aspects that must be studied: the parametric error models for the sensors and the estimation of the modeled parameters. The models are needed for the design of estimation methods and for the correction of the measures.

In the chapter 2 the models for the ATM sensors are deduced. The most often used sensor in current ATM systems are the SSR and because of that this chapter is mainly focused in the SSR modeling. In section 2.2 a complete and rigorous mathematical model for SSR systematic errors is developed. The model takes in account the physical effects that are systematically affecting the measurement processes. It takes in account deviations in the positions of the antenna, errors in the signal propagation, the difference between the barometric and geometric altitude and the delay of the communication lines.

In the same chapter, in section 2.3.2, the biases for the ADS-B sensors are described. The ADS-B has two kind of biases, the first kind is related with the inner working of the sensor (measure of the pseudo distances) and the second kind related with the communication of the measure. The errors modeled (of the second kind) in this section can be estimated using only the time and position measures. The errors dependent on the pseudodistances must be corrected in the aircraft.

In the ADS-B and WAM there are systematic errors that are out of the

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scope of this thesis. For example the position for the ADS-B system can be determined using the GPS. In this case there are systematic errors that change in time that depends on the time deviation of the clocks or depends on the atmospheric conditions. The WAM system have systematic errors that depends on the propagation of the signals that can be determined in the WAM stations but that cannot be estimated using just the position measures. For the estimation of these biases are needed the measures of pseudodistances that are not available in the tracker.

In the chapter 3 the estimation methods are described and studied. In this chapter, the estimation methods are focused in ATM applications. Finally the model for the SSR described in the chapter 2 is applied to these method with simulated and real data in order to see check the validity of the models.

In the estimation theory there are multitude of estimation methods but not all of them are useful for the estimation of systematic errors in the ATM surveillance. Even in the specific methods for biases there are some practical problems that are not solved in the literature. Two of these problems (initialization and observability) are studied in the chapter 4.

The first practical estimation problem studied in this thesis is the initialization of the values for the biases. Generally the biases are nonlinear functions and the estimation methods works accurately when the estimated values of the biases are near to the ideal values. In some cases the algorithms have a fast convergence but the final accuracy of the error is not the best reachable value. In other methods where the final error covariance is low, the convergence is slow and the methods are unstable. In section 4.1 there are proposed some initialization techniques in order to get the best performances in the speed of convergence and the accuracy of the permanent regimen getting a stable algorithm.

The second studied problem is the observability of the methods. The biases cannot be directly measured by the radar and they must be estimated as hidden parameters through the measures of the aircraft positions. As the errors in the position are projections of the biases over the horizontal plane, under some conditions, the projection from one or more biases can be similar. In the literature the observability is studied in a generic way. In section 4.2 the observability is studied applying the methods to the particular case of

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bias estimation in ATM scenarios.

In the aforementioned section the numerical problems derived from the estimation methods are described. In some cases there are numerical problems that can be similar to observability problems. In this section there is a proposal to reduce the numerical problems in order to detect correctly the observability problems. Finally, in this section there is a method to check the observability online and a way to avoid the instabilities produced by the lack observability.

In the chapter 5 there are some proposal for the improvement of the current methods and the presented model. In this section an extension of the atmospheric model using surfaces is proposed. In the chapter 2 a meteorological model has been proposed with a single pressure and temperature parameters. In real scenarios where the coverage area is bigger than a country the use of a single parameter can be not good enough to an accurate estimation of the biases.

In the aforementioned chapter there is presented a modification for a current estimation method. The current method estimates the target and sensor dependent biases in a three stages method. The modification presented in this chapter fuses the two last steps in a single one reducing the dependence of the estimation method with the mean of the estimated values. In the same chapter is proposed a method to estimate the biases of the velocity measures transmitted by the aircraft.

Chapter 2

Measurements Models for sensors used in ATM

In the estimation theory one of the most important points is the modeling of the data (measures). As it is explained in section 3.2.2 the models can be used to relate the measurement to the state vector of the estimation methods, to predict its next state or even to take in account the noise of the measurements. In the bias estimation theory is very important the parametric mathematical model of the sensors used in the ATM applications. The estimation methods minimize the errors in the estimations fixing the measures into a data model that is known a priori. If the models are erroneous or barely accurate, then the estimations have errors that cannot be corrected.

In this chapter the parametrical models for some of the sensor used in ATM are developed. The determination of the aircraft positions is different for each kind of sensor and the bias models for them will be different too. In the case of the radar, the position is determined by the propagation time of a signal that is reflected (in PSR) or transmitted (in SSR) by the aircraft. ADS-B sensors receive the position measured by the airborne navigational system of the aircraft (INS or GNSS [Kaplan and Hegarty, 2005]). WAM sensors have an antenna network that measure the difference between the time of arrival of the same signal transmitted by the aircraft.

After the development of the models an analysis of the errors produced by the approximations (made in the modelling process) is made in order to

2.1. RADAR DEPENDENT BIASES

shown that the errors of the model are negligible. These errors are described in the appendix A.

The biases described above consider only the macro (sensor-dependent) biases. Macro biases [Bar-Shalom and Blair, 1992] depends on the sensors and are common for all the aircraft of the scenario. For example, the radar biases that will be described below are macro biases. The micro (target-dependent) biases are those that are different for each aircraft but for one single target are always the same biases. For example the delay of each transponder or the delay in the transmission of the ADS-B messages to the receiver station.

2.1 Radar Dependent Biases

The estimation methods are widely studied in the literature. In the major part of the papers of the literature, the model of the sensor is too simple, just for illustrate the process of the estimation but rarely the behavior of the sensor is deeply studied.

In 1980 a model for the radar biases was introduced by [Fischer et al., 1980] in a self-registration procedure for military air surveillance. In this report the advantages are introduced of a netted air surveillance system over a stand-alone configurations. The major obstacle for this configuration was the registration of the radars in a common system of coordinates. In this report, the provided model was complete and rigorous for the radar of those years, but in the model were included parameters that currently have not sense. For example, in the report it is assumed that the position of the radars can be erroneous and it is included in the model. It is also included that the errors in the clock give an error in the azimuth of the measurement. Currently the position of the sensor is very accurate and the determination of the azimuth isn't time-dependent.

In 1987 a report for Deep Space Networks (DSN) antennas with measurements in azimuth and elevation provides a model for the systematic errors of these kinds of antennas [Guiar et al., 1987]. As in this case the application is different, the biases studied are not useful for ATM applications. For example, in this report, as the antennas have a very big size with huge reflectors,

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the structural flexure due to gravity is introduced. This kind of antennas is not used in ATM and the presented models in this paper are not useful.

In [Helmick and Rice, 1993] a model for 3D sensors is developed. In this case the range bias is approximated with a constant parameter and four angular parameters that modify the azimuth and elevation. In this work the biases are developed for a 3D non-rotatory sensor. This model cannot be used for any of the sensors used in ATM.

More modern works that address this topic are mainly focused in the estimation method and these works use a very simplified model. In [Lin et al., 2003, Lin et al., 2005] just an offset in azimuth and range, and a bias proportional to the range is used. In [Besada et al., 2012] the model used in the example is the same that in [Lin et al., 2003, Lin et al., 2005] but in this case authors assume that there can be different sensor with different configuration of parameters.

In [Besada et al., 2009] was proposed an offline bias estimator used for trajectory reconstruction. In this paper the models for the sensor used have more importance than in other works. The ADS-B and WAM sensor have bias parameters related with the time and the SSR sensor are modeled with the common bias parameters and the eccentricity is added.

In most of the estimation method, the measurement from all sensors are assumed synchronous. In several works, the papers address the topic of the synchronicity of the sensor and take different approaches to the solution of the problem. One of them is the weighted averaging of the plots [Bar-Shalom and Li, 1998] and other method is using the information of the tracking filter [Pu et al., 2017]. But even in these cases where the author are solving a real problem, the model used for the estimation is too simple to model all real biases of the sensors.

All the previous described papers only model (with more or less parameters) the biases produced by the sensor hardware. The physical disturbances of the propagation processes are not included in the models of the previous papers. In radar applications the range bias is modeled as constant term plus a range dependent term. In the real atmosphere the propagation of the signals and the bending of them due to the refractivity of the air make that the range bias must be modeled with a more complex model. In [Barton,

2.2. SYSTEMATIC ERROR MODEL OF SSR

2004] the bending of the ray paths is studied. In other works as [Adegoke and Onasanya, 2008] the effect of the atmosphere in the delay of the signals is studied in order to be able to determine the efficiency of the transmitting and receiving antenna system to be used. In [Abbud et al., 2011] [Abbud and De Miguel, 2014] the propagation phenomena is analyzed in order to develop a model for the WAM biases.

To all this must be added that one single position measurement is made by several independent sensors. In the case of the radars, the range and azimuth is determined by the radar but the altitude is provided by the airborne barometric altimeter of the aircraft. In the case of ADS-B and WAM, the sensors determine the 3D position of the aircraft but only the latitude and longitude are used in the tracking. The altitude used is the barometric altitude. The geometric altitude is used in precision applications as the landing

2.2 Systematic Error Model of SSR

In this section a complete and rigorous mathematical model for SSR systematic errors is developed. The model takes into account the physical effects affecting the measurement processes. The azimuth biases are calculated from the physical error of the antenna calibration and the errors of the angle determination device. Range bias is calculated from the delay of the signal produced by the refractivity index of the atmosphere and from clock errors, while the altitude bias is calculated taking into account the atmosphere conditions (pressure and temperature).

The objective of this section is to derive a complete systematic error model for ATC radars enabling an important improvement of the bias estimation processes. This model was presented in [Jarama et al., 2017] but in this section it is more detailed.

2.2.1 SSR

Nowadays radars are the basic element in ATC networks. Modern sensors as WAM [Neven et al., 2005] and ADS-B [Quitar, 2009] have equivalent (or even higher) performance than radars with lower cost, so in the future it's expected they partially take the place of radars. In any case, radars will still be used as a backup network for many years. The type of radar most used in ATC sensor network is the Secondary Surveillance Radar (SSR), being Mode S radar an enhanced version of this system [Stevens, 1988].

To enable surveillance of wide areas, radars with overlapped coverages are deployed. Therefore, aircraft measures (plots) from different sensors are sent to a control center where the data from all sensors is fused in order to get a unique trajectory estimation (track) for each aircraft. All the measures are transformed from radar polar local coordinates to a common coordinated system, usually Cartesian coordinates projected over the stereographic plane [Blackman and Popoli, 1999]. To make this data fusion process stable and accurate, the systematic errors affecting the measures from each radar must be corrected before the change of coordinates.

SSR is a rotating 2D radar sending an interrogation through an antenna with high directivity in azimuth and low directivity in elevation (2D antenna). In this kind of antennas the horizontal radiation pattern is very narrow and it is very wide in the vertical radiation pattern. ATC radars only can measure the azimuth and range of the aircraft. The range is measured with the time interval between the transmission and the receipt of the response to this pulse.

In PSR the response is the passive reflection of the aircraft and in SSR the response is a message transmitted by the aircraft. The azimuth of the target is the angle of the antenna boresight while the target altitude is measured by the aircraft navigation system and can be communicated to radar through a data link. PSR used in ATM only can determine the range and the azimuth of the target. Nowadays in ATC systems primary radars are used as backup system to secondary radar network. SSR is based in the same concept that PSR but in this case, the SSR has data link capability. SSR sends an interrogation message and the aircraft responds with its Mode A (identifying code) or mode C (barometric altitude).

2.2. SYSTEMATIC ERROR MODEL OF SSR

2.2.1.1 Measurement model for SSR

The measurement model of the SSR described in this section follows the scheme of the previous paragraphs. As it has been explained just the azimuth and the range is measured by the radar. In this section the range and azimuth are typically referenced to a local Cartesian coordinated system: y-axis pointing to north, x-axis pointing to East and z-axis pointing to up (ENU system). Azimuth reference has its origin in the y-axis and it grows in clockwise. Barometric altitude is measured by an airborne barometric altimeter with reference at mean sea level. The relation between radar coordinates (ρ_m, θ_m, h_m) and local Cartesian coordinates (x_m, y_m) is:

$$\begin{aligned} x_m &= \rho_m \sin(\theta_m) \cos(\varphi_m) \\ y_m &= \rho_m \cos(\theta_m) \cos(\varphi_m) \end{aligned} \quad (2.1)$$

Where ρ_m, θ_m, h_m are the range, azimuth and altitude of the measurement and φ_m is the target elevation with respect to i-th sensor horizontal plane, which can be calculated as:

$$\varphi_m = \arcsin \left(\frac{2R(h_m - h_i) + h_m^2 - h_i^2 - \rho_m^2}{2\rho_m(R + h_i)} \right) \quad (2.2)$$

being h_i the geometric altitude of the i-th sensor with reference at mean sea level. R is the local Earth radius at the radar position for the target azimuth, calculated as described in appendix C. The elevation is calculated modelling the earth as an ellipsoid. The altitude measured by the aircraft is perpendicular to the Earth surface and the Flat-Earth model is not accurate enough. Local Cartesian coordinates are used as middle step in the conversion between radar measures and common tracking coordinates for all sensors. The radar measurement model can be summarized as:

$$\begin{aligned} \rho_m &= \rho + n_\rho + \Delta\rho \\ \theta_m &= \theta + n_\theta + \Delta\theta \\ h_m &= h + n_h + \Delta h \end{aligned} \quad (2.3)$$

where n_ρ, n_θ and n_h are the noise measurement errors for range, azimuth and altitude respectively, $\Delta\rho, \Delta\theta$ and Δh are systematic error terms due to bad system calibration, and ρ, θ and h are the ideal range, azimuth and

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altitude of the target. Noise terms are modeled typically as uncorrelated additive white noises (usually considered Gaussian distributed), while systematic error terms can be considered constants or slow time variant (bias terms). Bias terms of range, azimuth and altitude depend on target position and will be deduced in next subsections for SSR sensors. They are composed by several environmental and equipment perturbations that are common to all the measurements of each sensor. The parameters of the mathematical model may change with time, due to variable weather conditions and hardware aging.

In surveillance data fusion systems the measures obtained from different kind of sensor are fused in order to improve the information update ratio and the accuracy of the aircraft trajectory estimation. At the moment of the data fusion all measurement must be expressed in a common coordinated reference system. There are several coordinated systems that can be used in ATC. For small scenarios, flat Earth model can be assumed, therefore Cartesian coordinates are precise enough to track the trajectories with several sensors. When the scenario is larger and sensors are widely separated, the Earth curvature must be considered and a different coordinate system must be used. Afterwards, the multisensor tracking process will be performed in the stereographic reference system for horizontal track. Aircraft barometric altitude is usually tracked separately in ATC applications.

2.2.2 Azimuth Bias

Azimuth bias can be modeled as the superposition of several terms that can be separated in three different groups. The first one is the erroneous orientation of the antenna boresight due to misalignment between the antenna and the rotation axis. The second one is the non-orthogonality between the rotation axis and the Earth surface. And the last one is the calibration error of the device of angle determination (azimuth encoder). Some of these terms will have the same effect in the measures and they cannot be distinguished in the estimation process using measures from radar. For example, the azimuth offset of the antenna, the azimuth offset of the rotation axis and the azimuth offset of the encoder will have equivalent effects and only one of them will be considered. In this case, the model adds all the effects in a single parameter.

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In other cases, the superposition of these terms will depend on aircraft position. Antenna skew and rotation axis skew will be distinguishable if the aircraft are distributed in azimuth. For these two bias terms both the elevation and azimuth diversity of the aircraft are needed in order to estimate them. Generally, the aircraft will be distributed in the airspace (position and altitude) but sometimes there will be cases where all the aircraft will be concentrated in a small range of azimuth, altitude or range. In these cases, the bias estimation algorithms will not have the capability to separate the different azimuth and bias components (there would be an observability problem [Bar-Shalom et al., 2011, Fortunati et al., 2012]).

2.2.2.1 Antenna Deviation

The first factor that affects the azimuth bias is the deviation between the vertical axis of the antenna reference and the reference of the rotation axis. This deviation will be defined with three parameters describing the rotation in the three axes of the Cartesian coordinate system. In order to determine the antenna deviation, the antenna coordinated system is defined as: y-axis, parallel to the ideal antenna boresight; z-axis, orthogonal to the y-axis, resulting that both axes compose the ideal plane of the 2D antenna pattern (z-axis is the ideal antenna rotation axis); the x-axis, orthogonal to the y-axis and z-axis positive to the right.

In order to derive the model, the biased antenna coordinated system (x', y', z') is calculated. The axes of the coordinated system (x, y, z) are rotated to get the new system (x', y', z') and the measured azimuth (θ_0) is calculated in the new system. Under the assumption that the deviation angles of the antenna axes are small (below 1°), the order of the rotation around each axis has a negligible influence in the model. The three rotation angles (Figure 2.1) correspond to the following bias parameters components:

Azimuth offset (θ_0): it is generated with the rotation of the antenna in the horizontal plane. This offset is constant for all the targets of the sensor. This rotation is defined positive in counter-clockwise direction.

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Antenna squint (s_{ant}): it is the bias produced due to the rotation in the orthogonal plane to the antenna boresight. This component produces an azimuth bias dependent on the target elevation (angle of line of sight respect to horizontal plane). This rotation is defined positive in counter-clockwise direction.

Antenna tilt (t_{ant}): produced due to the rotation of the plane of the 2D antenna pattern around the x-axis. As radars used in ATC applications do not measure the elevation, this bias has not consequences in the first order terms of the azimuth bias. This rotation is defined positive in clockwise direction.

Figure 2.1: Representation of: (a) antenna azimuth offset, (b) antenna squint (c) and antenna tilt.

The coordinates are rotated using rotation matrix. These matrix are defined using the direction of the rotation indicated in the previous paragraphs. In this way, the rotation matrix for each one of the axes are:

$$R_z(\theta_0) = \begin{pmatrix} \cos(\theta_0) & \sin(\theta_0) & 0 \\ -\sin(\theta_0) & \cos(\theta_0) & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2.4)$$

$$R_x(t_{ant}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(t_{ant}) & -\sin(t_{ant}) \\ 0 & \sin(t_{ant}) & \cos(t_{ant}) \end{pmatrix} \quad (2.5)$$

$$R_y(s_{ant}) = \begin{pmatrix} \cos(s_{ant}) & 0 & -\sin(s_{ant}) \\ 0 & 1 & 0 \\ \sin(s_{ant}) & 0 & \cos(s_{ant}) \end{pmatrix} \quad (2.6)$$

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In order to transform the coordinates from (x, y, z) to (x', y', z') the next operation is applied:

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = R_z(\theta_0) R_x(t_{ant}) R_y(s_{ant}) \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad (2.7)$$

Due to the convention in geographical systems of the azimuth with the origin in the y-axis, positive in clockwise, the measured azimuth $(\theta_{m,ant})$ is:

$$\theta_{m,ant} = \arctan\left(\frac{x'}{y'}\right) \quad (2.8)$$

These calculations are made under two assumptions: the error is only caused by the deviations of the antenna and this antenna can only acquire the targets that are in the 2D antenna main lobe. A multidimensional Taylor series on the bias variables θ_0, s_{ant} and t_{ant} is made to derive the following linearized approximation:

$$\begin{aligned} \theta_{m,ant} \approx & \theta + \theta_0 - s_{ant} \tan(\varphi) \cos(\theta) + t_{ant} \tan(\varphi) \sin(\theta) + \\ & + \left(\frac{1}{2} \sin(\theta) \cos(\theta) - \frac{\sin(\theta) \cos(\theta)}{\cos^2(\varphi)} \right) s_{ant}^2 - \\ & - \left(\frac{1}{2} \sin(\theta) \cos(\theta) - \frac{\sin(\theta) \cos(\theta)}{\cos^2(\varphi)} \right) t_{ant}^2 + \\ & + \left(\cos^2(\theta) - \frac{2\cos^2(\theta)}{\cos^2(\varphi)} + \frac{1}{\cos^2(\varphi)} \right) t_{ant} s_{ant} + \dots \end{aligned} \quad (2.9)$$

where φ is the ideal elevation of the aircraft. The variable θ may be particularized to zero in this case as the antenna only measures the aircraft that are into the antenna main lobe ($\theta \approx 0$). Assuming that θ_0, s_{ant} and t_{ant} are small values then the higher order terms may be assumed to be negligible, resulting in:

$$\theta_{m,ant} \approx \theta_0 - s_{ant} \tan(\varphi) \quad (2.10)$$

The previous measure just contains the projection of the antenna deviation parameters in the azimuth component ($\theta \approx 0$; noise and other bias terms are neglected for this calculation), resulting in the following additive

azimuth bias term:

$$\Delta\theta_{ant} = \theta_0 - s_{ant}\tan(\varphi) \quad (2.11)$$

2.2.2.2 Axis Skew

The axis skew is the inclination of the rotation axis. It causes an effect similar to the antenna deviation but in this case the bias also depends on the azimuth. For this model we use the local Cartesian coordinated system in the radar position: the y-axis pointing to the north and the x-axis pointing to the east both in the Earth horizontal plane. As in the previous subsection the biases of the azimuth can be calculated with rotations of the system axis as is represented in Figure 2.2.

In order to be coherent with the names of the rotations, the rotation in y-axis will be named axis squint and the rotation in x-axis will be named axis tilt. In this case, the tilt gives errors on the measures because it is the rotation in the x-axis and not in the orthogonal plane to the antenna boresight.

In this case the rotation matrix are:

$$R_x(t_{axis}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(t_{axis}) & -\sin(t_{axis}) \\ 0 & \sin(t_{axis}) & \cos(t_{axis}) \end{pmatrix} \quad (2.12)$$

$$R_y(s_{axis}) = \begin{pmatrix} \cos(s_{axis}) & 0 & -\sin(s_{axis}) \\ 0 & 1 & 0 \\ \sin(s_{axis}) & 0 & \cos(s_{axis}) \end{pmatrix} \quad (2.13)$$

The new coordinates system (x', y', z') is:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = R_x(t_{axis}) R_y(s_{axis}) \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} \quad (2.14)$$

The measured azimuth is calculated again ($\theta_{m,axis} = \arctan\left(\frac{x'}{y'}\right)$) and the result is approximated linearizing with a multidimensional Taylor series

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Figure 2.2: Skew of the rotation axis of the radar.

on the bias variables s_{ant} and t_{ant} . In this case the azimuth is not particularized to zero ($\theta \approx 0$) because the reference axes are fixed to north and the ideal measurement can have any value of azimuth. The linear approximation, rejecting higher order terms in Taylor series is:

$$\theta_{m,axis} \approx \theta + t_{axis} \tan(\varphi) \sin(\theta) - s_{axis} \tan(\varphi) \cos(\theta) \quad (2.15)$$

Therefore, the azimuth bias due to the axis skew is:

$$\Delta\theta_{axis} = \theta_{m,axis} - \theta \approx t_{axis} \tan(\varphi) \sin(\theta) - s_{axis} \tan(\varphi) \cos(\theta) \quad (2.16)$$

An alternative equivalent model to the one in 2.16 can be derived as follows. Under the assumption that the inclination of the axis is small, the spherical triangles with components t_{axis} and s_{axis} can be approximated by the right triangle of Figure 2.3, expressed in terms of β_{axis} and α_{axis} where β_{axis} represents the total skew and α_{axis} the direction where the axis is skewed. The new relation is:

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Figure 2.3: (a) Right triangle of the axis skew and (b) relation between the rotation of the axis and the skew of the rotation axis and its direction.

$$s_{axis} = \beta_{axis} \sin(\alpha_{axis}) \quad (2.17)$$

$$t_{axis} = \beta_{axis} \cos(\alpha_{axis}) \quad (2.18)$$

Substituting 2.17 and 2.18 in 2.16 the final model could be written, after some minor algebra, as:

$$\Delta\theta_{axis} = \beta_{axis} \sin(\theta - \alpha_{axis}) \tan(\varphi) \quad (2.19)$$

The azimuth offset due to this effect at the direction of the skew is zero, and it follows a sinusoidal law with azimuth. The model with s_{axis} and t_{axis} parameters in 2.16 is equivalent to the model with β_{axis} and α_{axis} parameters in 2.19. The former model 2.16 results generally in better stability when used for the development for bias estimation algorithms.

In other hand, the model of the axis skew with β_{axis} and α_{axis} parameters

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is a very representative model. In real radars, the rotatory axis is skewed because the pole is inclined (β_{axis}) in a direction (α_{axis}). The model with the parameters s_{axis} and t_{axis} is less intuitive because the rotatory axis is not skewed in "two phases".

2.2.2.3 Optical Encoder

In old radars the determination of the azimuth was made with different devices to the used nowadays. One of the devices had a flag in the north and the azimuth was determined with the difference of time between the current measure and the last north mark. This model is described in [Fischer et al., 1980].

In modern radars the azimuth of the antenna boresight is determined with an optical encoder that is allocated in the rotating radar shaft. In the installation of this encoder, small calibration errors can be made and these are transformed in azimuth biases. One of the errors is the misalignment between azimuth reference of the encoder and the axis, this is an azimuth offset. This error is the same kind that the azimuth offset of the antenna and the azimuth offset of the rotating axis, described in 2.2.2.

Other error to be considered is the eccentricity bias. Usually this is the encoder error with the biggest impact in the measures [Hudgens et al., 2010]. In the installation, the rotation axis (O) is not exactly placed in the geometric center (O') of the encoder.

The resultant geometry is shown in figure 2.4, supposing that the difference of the centers is in the azimuth $\theta = 0$. Resolving the triangle shown in Figure 2.5 and making a deduction similar to [Qin et al., 2010, Zhao et al., 2011] we have the following relation:

$$\left. \begin{array}{l} A = \Delta R \sin(\theta) \\ A = R \sin(\alpha) \end{array} \right\} \Rightarrow \sin(\alpha) = \frac{\Delta R}{R} \sin(\theta) \quad (2.20)$$

where, ΔR is the eccentricity offset, R is radius of the encoder and θ and α are angles of the triangle shown in Figure 2.5.

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Figure 2.4: Optical encoder with eccentricity bias.

Figure 2.5: Resolution of the resultant triangle for eccentricity bias.

Using the relation between sine and cosine for α ($\cos(\alpha) = \sqrt{1 - \sin^2(\alpha)}$) we also have the following relation:

$$\cos(\alpha) = \sqrt{1 - \left(\frac{\Delta R}{R}\right)^2 \sin^2(\theta)} = \frac{1}{R} \sqrt{1 - \Delta R^2 \sin^2(\theta)} \quad (2.21)$$

On the other hand, from Figure 2.5 we can also see:

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$$r = R \cos(\alpha) - \frac{\Delta R \cos(\theta)}{\sqrt{R^2 - \Delta R^2 \sin^2(\theta)}} \quad (2.22)$$

$$r = -\Delta R \cos \theta + \sqrt{R^2 - \Delta R^2 \sin^2(\theta)} \quad (2.23)$$

Finally:

$$\left. \begin{array}{l} h = R \sin(\theta_m) \\ h = r \sin(\theta) \end{array} \right\} \Rightarrow \theta_m = \arcsin\left(\frac{r}{R} \sin(\theta)\right) \quad (2.24)$$

The measured azimuth is also linearized with a Taylor series on the bias variable ΔR in order to get an approximation to model. Assuming that $\frac{\Delta R}{R} \ll 1$ the approximation of $\left(\frac{\Delta R}{R}\right)^2 = 0$ can be made. With these simplifications, the azimuth measurement can be approximated as:

$$\theta_m \approx \theta - \frac{\Delta R}{R} \sin(\theta) \quad (2.25)$$

The second error produced by the optical encoder is due to the swash between the encoder and the horizontal plane. A small swash of the encoder produces that the projection of the circumference is an ellipse and the determination of the azimuth has errors when this azimuth does not point to the ellipse axis.

Figure 2.6: Swash of the optical encoder (a) Perspective where the circular encoder (rotated) is projected into an ellipse in the horizontal plane. (b) Representation of the circle and the ellipse in the same plane.

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In Figure 2.6a a schematic model of the optical encoder is represented. The circle line on the x-y' plane represents the mechanical encoder and the dotted line on the x-y plane represents the projection of the rotated mechanical encoder circumference on the horizontal plane. The ideal azimuth is represented on the horizontal plane but the measured azimuth is obtained by the rotated encoder. In Figure 2.6b both mechanical encoder circumference and its projection on the horizontal plane are represented in the same plane in order to represent the 3D projection in a 2D figure to see the difference between the real azimuth and the measured azimuth. Following the methods used in [Torge and Müller, 2012] to convert geocentric latitude in reduced latitude we can obtain the relation between θ_m and θ . Using the next variables:

$$x = r \sin(\theta_m) \quad (2.26)$$

$$y_m = r \cos(\theta_m) \quad (2.27)$$

$$y = y_m \cos(s_{enc}) \quad (2.28)$$

and solving the triangle for θ :

$$\theta = \text{atan}\left(\frac{x}{y}\right) = \text{atan}\left(\frac{\tan(\theta_m)}{\cos(s_{enc})}\right) \quad (2.29)$$

and then θ_m can be calculated as:

$$\theta_m = \text{atan}(\cos(s_{enc}) \tan(\theta)) = \text{atan}\left(\cos(s_{enc}) \frac{x}{y}\right) \quad (2.30)$$

A Taylor series of order 2 on the bias variable s_{enc} is made to get an approximation of the measure:

$$\theta_m \approx \theta - \frac{1}{2} \cos(\theta) \sin(\theta) s_{enc}^2 = \theta - \frac{s_{enc}^2}{4} \sin(2\theta) \quad (2.31)$$

In this Taylor series, the first order term is zero and the second order term is considered. As this term is raised to second power, with small values of s_{enc} the value of the bias will be negligible. In both terms related with the encoder the error has been calculated supposing that the difference of the centers and the swash are in the azimuth $\theta = 0$. Supposing that in reality the azimuth of the difference of centers is α_{ecc} and the azimuth of the swash

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is α_{enc} the resultant encoder bias, comprising both the effects described in 2.25 and 2.31, is:

$$\Delta\theta_{enc} = -\frac{s_{enc}^2}{4} \sin(2\theta - 2\alpha_{enc}) - \frac{\Delta R}{R} \sin(\theta - \alpha_{ecc}) \quad (2.32)$$

An equivalent model can be derived by using trigonometric relations, in order to express all the biases with a linear structure similar to the one in (16):

$$\Delta\theta_{enc} = s_{enc,s} \sin(2\theta) + s_{enc,c} \cos(2\theta) + \Delta R_y \sin(\theta) - \Delta R_x \cos(\theta) \quad (2.33)$$

where:

$$s_{enc,s} = \frac{s_{enc}^2 \sin^2(\alpha_{enc}) - s_{enc}^2 \cos^2(\alpha_{enc})}{4} \quad (2.34)$$

$$s_{enc,c} = s_{enc}^2 \cos(\alpha_{enc}) \sin(\alpha_{enc}) \quad (2.35)$$

and ΔR_y and ΔR_x are the terms of the center deviation in the y-axis and in the x-axis.

2.2.2.4 Azimuth Bias Composition

The global azimuth bias model is a composition of the three bias elements previously discussed. Neglecting noise effects, in a radar, the measured azimuth of the antenna is the azimuth where the rotation axis is pointing plus the antenna bias. The azimuth of the rotation axis is the azimuth indicated by the encoder plus the axis bias. Finally, the azimuth indicated by the encoder is the ideal azimuth plus the encoder bias. With this:

$$\theta_{enc} = \theta + \Delta\theta_{enc} \quad (2.36)$$

$$\theta_{axis} = \theta_{enc} + \Delta\theta_{axis} \quad (2.37)$$

$$\theta_m = \theta_{ant} = \theta_{axis} + \Delta\theta_{ant} \quad (2.38)$$

And then:

$$\theta_m = \theta + \Delta\theta = \theta + \Delta\theta_{ant} + \Delta\theta_{axis} + \Delta\theta_{enc} \quad (2.39)$$

With this, the complete model for azimuth bias is:

$$\begin{aligned}
 \Delta\theta &= \Delta\theta_{ant} + \Delta\theta_{axis} + \Delta\theta_{enc} = \\
 &= \theta_0 - s_{ant} \tan(\varphi) + \\
 &+ \beta_{axis} \sin(\theta - \alpha_{axis}) \tan(\varphi) - \\
 &- \frac{1}{4} \sin(2\theta - 2\alpha_{enc}) s_{enc}^2 - \frac{\Delta R}{R} \sin(\theta - \alpha_{exc})
 \end{aligned} \tag{2.40}$$

This model should not be used in general in the bias estimation algorithms due to instability problems due to lack of linearity of some of part of the model, although its physical meaning is clear. The alternative equivalent model (preferred for bias estimation due to its linearity with respect to bias parameters) should be obtained composing 2.11, 2.16 , 2.33 and 2.39, resulting:

$$\begin{aligned}
 \Delta\theta &= \theta_0 - s_{ant} \tan(\varphi) + [t_{axis} \sin(\theta) - s_{axis} \cos(\theta)] \tan(\varphi) + \\
 &+ s_{enc,s} \sin(2\theta) + s_{enc,c} \cos(2\theta) + \Delta R_y \sin(\theta) - \Delta R_x \cos(\theta)
 \end{aligned} \tag{2.41}$$

2.2.3 Range

Range biases are again due to several factors. The main factors that compose the biases are:

Transponder delay: every aircraft has a different transponder delay (typically between -75m and 75m, usually assumed to be uniformly distributed). This aircraft dependent bias must be estimated for each aircraft in the tracking phase. This delay is calibrated periodically or when the controllers observe unusual behavior in the measurements. The range model developed in this section is for biases due to sensor and environment. As the estimation will be made with many targets, the mean of the delays of all the aircraft tends to be near zero. For more details on this bias term and methods to estimate it, see [Besada et al., 2012]. This Transponder delay will be widely discussed in 2.3.1.

Temporal reference error: radars measure the difference of time between the interrogation and the arrival of the response. Usually there is a small constant error in the reference time and it produces a constant error

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$(\Delta\rho_0)$ in the range estimation.

Clock error: A small error in the clock calibration may produce a linear term in the range estimation [Fischer et al., 1980]. In modern radar this term can be considered negligible due to the precision of the clocks.

Propagation error: The conversion between time and distance is made using the speed of light in the International Standard Atmosphere (ISA) (International Standard Atmosphere) at mean sea level. However, the speed of light changes along the propagation path due to the change in the refraction index with altitude and weather conditions. Any change in the real speed of light will produce a range bias term $(\Delta\rho_{prop})$.

Considering the range biases terms previously explained, the range bias $(\Delta\rho)$, as defined 2.3) results:

$$\Delta\rho = \Delta\rho_0 + \Delta\rho_{prop} \quad (2.42)$$

In cases when the clock has a bad calibration, the bias has additionally a linear term dependent on the clock error, and parameterized through a range gain factor (α_{clk}) , being the complete model:

$$\Delta\rho = \Delta\rho_0 + \alpha_{clk}\rho + \Delta\rho_{prop} \quad (2.43)$$

Next, we will focus on the propagation term.

2.2.3.1 Propagation Error

The index of refraction of the atmosphere changes with the altitude and this affects to the local speed of light. As the conversion between time and range is made with a constant speed of light, the determination of the position of the aircraft from the radar generally has biases (due to change of speed of light with altitude and weather conditions). As the bias is produced by accumulated error in the speed of light along the propagation path, the range bias will be a function of the target range. In order to model the relation between the ideal range and the bias we will use a spherically stratified

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atmosphere. The exponential model of the refractivity [Bean, 1962] is used because it models the typical atmosphere variations in the rank of altitude used in ATC applications:

$$N(h) = N_s \exp\left(-\frac{h}{H}\right) \quad (2.44)$$

where $N(h)$ is the refractivity at a given altitude (h) over mean sea level, H is a reference altitude (equal to 6950 m), and $N(0)=N_s$. (with typical values between 300 and 350). The relation between the refractivity and the index of refraction ($n(h)$) is:

$$n(h) = 1 + N(h) \cdot 10^{-6} \quad (2.45)$$

The model in 2.44 can be used also redefining altitude (h) to be referred to radar altitude, which just results in a change of the N_s constant, to be now the refractivity at the radar altitude over mean sea level. This will be the altitude reference and refractivity model to be used for the rest of this section.

The height gradient of the index of refraction bends the ray, resulting in a slightly increased distance from that of the straight line used to model propagation on free space. Specifically, as it is deduced in [Barton, 2004] using a ray tracing procedure with the Snell's law for spherically stratified media [Zeng et al., 2014], the geometrical distance of the bent ray path may be calculated as:

$$s(h_t, \varphi_{0t}) = \int_0^{h_t} \frac{dh}{\sqrt{1 - \left\{ n_0 \cos(\varphi_{0t}) \left[\frac{n(h)}{1 + \frac{h}{R_e}} \right] \right\}^2}} \quad (2.46)$$

where φ_{0t} is the elevation ray angle observed by the radar, h_t is the target altitude relative to the radar, n_0 is the index of refraction at radar altitude, and R_e is the Earth radius plus radar height. Additionally, the speed of light varies as it traverses different heights with different associated index of refraction. The effective distance (measured distance assuming constant speed of light, is affected by the propagation error), and can be calculated as proposed in [Barton, 2004]:

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$$r(h_t, \varphi_{0t}) = \int_0^{h_t} \frac{n(h) dh}{\sqrt{1 - \left\{ n_0 \cos(\varphi_{0t}) \left[\frac{n(h)}{1 + \frac{h}{R_e}} \right] \right\}}} \quad (2.47)$$

In the following we will work with the approximation that the geometrical distance between the target and radar is almost equal to a straight-line (free-space) path length [Barton, 2004] ($\rho \approx s(h_t, \varphi_{0t})$). Then, the range bias due to propagation can be calculated as:

$$\Delta\rho(h_t, \varphi_{0t}) = r(h_t, \varphi_{0t}) - s(h_t, \varphi_{0t}) \quad (2.48)$$

Both distances (geometrical distance and effective distances), and their difference, can be numerically estimated for different values of h_t and φ_{0t} . With these pairs we can get a table of range biases ($\Delta\rho$) and geometrical distances (s). In these estimations we work with the approximation that the geometrical distance between the target and radar is equal to the ray path length [Barton, 2004]. Without this approximation the estimations are similar, but at distances lower than 20 Km the results of the biases are different. But in these places the total bias is very low (between 2m and 6m, depending on the altitude) and the approximation does not introduce significant error.

The resultant range error is represented in figure 2.7, where each line represents the range error for a different altitude (h_t) as a function of slant range. As it is seen [Abbud and De Miguel, 2014] two relevant characteristics can be observed. The first one is that the range error depends on the altitude of the aircraft. The second one is that a second-grade polynomial seems to be good enough to approximate the range error dependency with the slant range for every altitude [Abbud et al., 2011]. Considering only this polynomial, the range error can be expressed as:

$$\Delta\rho(\rho, h) = \beta_1(h) \rho + \beta_2(h) \rho^2 \quad (2.49)$$

where $\beta_1(h)$ and $\beta_2(h)$ are polynomial coefficients, different for each altitude. This model does not allow to make a good estimation of the parameters with opportunity traffic, because the number of aircraft in each altitude

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Figure 2.7: Range bias for aircraft at different altitudes.

layer is reduced. We must find a model which allows the simultaneous use of aircraft measures at all altitudes in the estimation process. A key idea in the derivation of such model is to search for decoupled range and altitude dependencies in $\Delta\rho(\rho, h)$:

$$\Delta\rho(\rho, h) = f(\rho) g(h) \quad (2.50)$$

To do so, the quotient between a parabola in an altitude and the parabola at an arbitrary reference altitude ($h_r=14000$ m in the following) for the same slant range is made:

$$g(\rho, h) = \frac{\Delta\rho(\rho, h)}{\Delta\rho(\rho, h_r)} \quad (2.51)$$

This quotient is shown as a function of the slant range in figure 8 and can be observed that for each altitude it is almost constant. The right end of the horizontal lines in this figure corresponds to the radio-wave propagation horizon. This result demonstrates that the range bias at a specific altitude is the range bias at the reference altitude multiplied by a constant, supporting 2.50, as the dependency of $g(\rho, h)$ with the slant-range is negligible (i.e. it could be substituted by $g(h)$), and $f(\rho)$ would be directly $\Delta\rho(\rho, h_r)$, a function not depending of h . From those ideas, we have that:

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Figure 2.8: Quotient between range bias at an altitude and bias at 14000m for the same slant range.

$$f(\rho) = \beta_1(h_r)\rho + \beta_2(h_r)\rho^2 = \alpha_1\rho + \alpha_2\rho^2 \quad (2.52)$$

In order to derive $g(h)$ we have represented in figure 2.8 the quotient between the range bias at an altitude and the bias at a reference altitude as a function of altitude. In this figure, it can be seen that a linear model is a good approximation. A second order polynomial would be an almost exact approximation but it implies the addition of another parameter that would make the convergence of the bias estimation processes slower with reduced accuracy gains.

Then $g(h)$ can be modelled with a linear function normalized to reference altitude (h_r):

$$g(h) = \frac{\Delta\rho(\rho, h)}{\rho(\rho, h_r)} \approx 1 + \alpha_3 \left(1 - \frac{h}{h_r}\right) \quad (2.53)$$

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Figure 2.9: Ratio between range bias at an altitude and bias at reference altitude (14000m) for the same slant range as a function of altitude.

Finally, the contribution of the tropospheric propagation to the range bias can be modelled, combining 2.50, 2.52 and 2.53:

$$\Delta\rho_{prop} = (\alpha_1\rho + \alpha_2\rho^2) \left[1 + \alpha_3 \left(1 - \frac{h}{h_r} \right) \right] \quad (2.54)$$

2.2.3.2 Range Error Composition

The final range bias is the addition on the one hand of the propagation bias due to the atmosphere and on the other hand to the constant bias produced by the transponder delay and the error of reference time:

$$\Delta\rho = \Delta\rho_0 + (\alpha_1\rho + \alpha_2\rho^2) \left[1 + \alpha_3 \left(1 - \frac{h}{h_r} \right) \right] \quad (2.55)$$

The term produced by the bad calibration of the clock (α_{clk}) is ignored in this equation.

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2.2.4 Altitude

Civil surveillance radars used in ATC do not measure elevation of the targets. The reply of the aircraft transponder to the SSR interrogation codifies the altitude of the aircraft in the message (Mode C), completing the 3D position determination by the radar sensor. Aircraft altitudes may be of two different types:

Geometric altitude: it is the Euclidean distance between the mean sea level (MSL) and the position of the aircraft.

Barometric altitude: this altitude is calculated from the air pressure of the atmosphere at aircraft location. The conversion between pressure and altitude is made assuming ISA model (temperature 288.15 K and pressure 101325 Pa, at Mean Sea Level (MSL)). Barometric altitude is the same that geometric altitude only when the atmospheric conditions are the ISA conditions.

Barometric altitude is used in ATC systems, because all aircraft must always measure the pressure to fly and its flight performance depends critically on this pressure, barometers are easy to be calibrated and vertical separation can be done easily using this magnitude. On board certified sensors for geometric altitude usable in all phases of flight (such as GPS) have only recently become widely available, and this information is not always available in the ground. All the surveillance systems send barometric altitude but only some of them can send the geometric altitude. Using the barometric altitude usually implies systematic errors in altitude determination that are almost equal for every aircraft and those depends on the pressure and temperature.

This altitude bias term, when the polar measure is transformed to the horizontal plane, introduces a systematic error in the 2D position. The projection of this bias will be negligible when the elevation of the aircraft is close to zero, but it will be an important factor when the aircraft is near to the sensor and the elevation is big. This situation is typical in some air traffic control systems, as that of a radar close to an airport with several others far away of that airport. Then the horizontal projection of the measures from

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the near radar are heavily distorted due to the systematic altitude errors.

The projection in the horizontal plane is implemented using the target elevation, as calculated in 2.2. This equation is equivalent to search the cross point between the circle defined with radius ρ and the line defining the constant measured altitude. It is shown in figure 2.10. In this figure ρ is the measured range, h and h_m are the ideal and measured altitude respectively. ρ_p is the projected range calculated with the ideal altitude and ρ_{mp} is the projected range calculated with the measured altitude.

Figure 2.10: Difference between the projection in the slant range for the same range measure and different altitude.

In [Poles, 2010] a model of the atmosphere for its use in ATM trajectory prediction, based on ISA model, is defined. The ranges of altitudes of civil aircraft cover the troposphere and the lower stratosphere. The tropopause is the boundary between both layers. Hence there are two different models for different altitudes. In this model, the tropopause is allocated where the barometric altitude is 11000m. Below the tropopause the atmospheric model of barometric altitude is a non-linear function that depends on the temperature and the pressure at MSL. Above the tropopause, the model is a linear function because the gradient of temperature is zero.

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In the model, the barometric altitude (h_p) is the measured altitude and the geometric altitude (h_g) will be the ideal altitude. The difference between both altitudes will be considered the altitude bias. The model proposed in [Poles, 2010] is:

$$\text{Below tropopause:} \quad h_g = h_p - \Delta H_P + \frac{\Delta T}{\beta_t} \ln \left(\frac{T_0 + \beta_t h_p}{T_0 + \beta_t \Delta H_P} \right) \quad (2.56)$$

$$\text{Above tropopause:} \quad h_g = h_{g,trop} + \frac{T_0 + \Delta T + \beta_t h_{p,trop}}{T_0 + \beta_t h_{p,trop}} (h_p - h_{p,trop}) \quad (2.57)$$

where ΔH_P is the barometric altitude offset with respect to ISA (which depends on the difference between current pressure and standard pressure at mean sea level), ΔT is the difference between current temperature and standard temperature at mean sea level, T_0 is the temperature of the standard atmosphere at mean sea level, β_t is the temperature gradient and $h_{g,trop}$ is the geometric altitude where the barometric altitude is $h_{p,trop} = 11000\text{m}$ (which should be calculated with 2.56 to guarantee geometric height continuity).

To reduce the impact of altitude biases in measurement projection in the horizontal plane ΔT and ΔH_P must be estimated and corrected. Next we will derive a parametric model for these terms. Note the altitude bias effect is important when the elevation of the aircraft is big (high altitude and short range). We will next fit and show the consistency of a linear model between the geometric altitude and the barometric altitude based on the previously presented model, resulting in:

Below tropopause: we may approximate linearly $\ln(1+x) \approx x$ when $x \approx 0$. Using this approximation in $\ln \left(1 + \frac{T_0 + \beta_t h_p}{T_0 + \beta_t \Delta H_P} - 1 \right)$ from 2.56 we can derive:

$$h_g = h_p - \Delta H_P + \frac{\Delta T}{\beta_t} \left(\frac{T_0 + \beta_t h_p - T_0 + \beta_t \Delta H_P}{T_0 + \beta_t \Delta H_P} \right) \quad (2.58)$$

After some algebra we have:

$$h_g = (h_p - \Delta H_P) \left(1 + \frac{\Delta T}{T_0 + \beta_t \Delta H_P} \right) \quad (2.59)$$

And then we can solve for h_p , resulting:

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$$h_p = \frac{h_g}{1 + \frac{\Delta T}{T_0 + \beta_t \Delta H_p}} + \Delta H_p \quad (2.60)$$

Above tropopause: Regrouping the terms in 2.57, and approximating $h_{g,trop}$ using 2.59 with $h_p = h_{p,trop}$, the following relation is obtained:

$$h_g = h_{g,trop} + \left(1 + \frac{\Delta T}{T_0 + \beta_t h_{p,trop}}\right) (h_p - h_{p,trop}) \quad (2.61)$$

where

$$h_{g,trop} = (h_{p,trop} - \Delta H_p) \left(1 + \frac{\Delta T}{T_0 + \beta_t \Delta H_p}\right) \quad (2.62)$$

Then, we can solve again for h_p , resulting:

$$h_p = \frac{h_g - h_{g,trop}}{1 + \frac{\Delta T}{T_0 + \beta_t h_{p,trop}}} + h_{p,trop} \quad (2.63)$$

In this section h_g is always the ideal geometric altitude (called h in 2.2) and h_p is the barometric altitude (measured by the barometer, called h_m in 2.2, assuming negligible altitude noise). Changing the notation to that of 2.2 the resultant model is:

Below tropopause:

$$h_m = \frac{h}{1 + \frac{\Delta T}{T_0 + \beta_t \Delta H_p}} + \Delta H_p \quad (2.64)$$

Above tropopause:

$$h_m = \frac{h - h_{g,trop}}{1 + \frac{\Delta T}{T_0 + \beta_t h_{p,trop}}} + h_{p,trop} \quad (2.65)$$

In both cases, the model has an offset height and a height gain. In Figure 2.11 the bias associated to the more exact EUROCONTROL model [Poles, 2010] in 2.56 and 2.57, and the linearized model are represented (in 2.64 and 2.65) vs the geometric altitude (h). Those two models use the same values for the parameters $\Delta H_p=300\text{m}$ and $\Delta T=15^\circ$, and the linearization has very low error at low altitudes but at high altitudes the errors between both models can be around 100 meters. Meanwhile, the black line (labelled expected

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Figure 2.11: Representation of altitude bias for EUROCONTROL model, linearized model and expected estimation for $\Delta H_p = 300\text{m}$ and $\Delta T = 15^\circ$

estimation) represents a least squares regression of the EUROCONTROL model using the model in 2.64 and 2.65 with parameters ΔH_p and ΔT . The optimal values of the estimated values, assuming uniform distribution of heights, are $\Delta H_p = 298.72\text{m}$ and $\Delta T = 17.06^\circ\text{C}$. Although those are not the actual values used for the EUROCONTROL model, the simplified model is able to adjust quite finely to it, and therefore seems a promising model for altitude bias error estimation. It should be noted the purpose of this model is not obtaining a very accurate estimate of ΔH_p and ΔT , but to be able to approximate correctly the altitude bias relation with geometric height.

2.2.5 Time Stamp Bias

In the previous sections the position of the aircraft has been determined by the radar with the cooperation of the aircraft. The analyzed biases have been described assuming nonidealities in the physical processes as the propagation or calibration errors in the antenna installation. In this section the time stamp bias is described. This is a multisensor error and it appears when

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the plot data is sent to the fusion center.

Once the position of the aircraft is measured, the information is codified to the ASTERIX format. The ASTERIX messages Cat 48 [EUROCONTROL, 2005a] are used for the transmission of monoradar target reports. In this messages there are several data items that must be transmitted and other that are optional. The more basic data item are the position of the aircraft and the time stamp of the measure.

Range and azimuth of the measurement is codified to a binary code. With this codification there are a small error due to the quantification. With the number of bits used the quantification error is approximately of 8 meters in range and $5.5 \cdot 10^{-3}$ degrees in azimuth. These quantification errors are much smaller than the expected noise.

The other basic data item is the time stamp. The quantification error is approximately of 7.8 millisecond. This is a negligible error too, but in this case there can be another source error.

For modern radars the time of the radars are very accurate and the error in the time stamp will be negligible, but in countries where the sensor network is very old, the equipment usually have defects in the installation and calibration. In this networks usually the time stamp is ignored. The receiving time in the fusion center is used in the multisensor tracking.

In this case the measurement time is different to the time stamp of the tracking filter. In figure 1.13 a simplified scheme of a fusion center with several sensors is shown. The delays produced by the communication lines in the WAM and ADS-B sensors are not considered because the measures are usually marked with the measurement time. In big countries the sensors can be allocated about 400-500 km from the information fusion and the communication lines can have a delay. If the sensor uses a dedicated line the delay will be small and constant, but if the information is transmitted in a public network the delay of the information depends on the load of the network. In figure different delays are shown for each sensor. The delays of the communications lines are modeled as a constant that varies slowly with the time.

2.3. TARGET DEPENDENT BIASES

2.3 Target Dependent Biases

In the previous section macro biases for SSR is widely described but in this case the target dependent biases is not considered. In ATM for civil application the position measure need a response from the target. SSR sends an interrogation and the aircraft transponder replied with an answer. This reply usually have an error in the standard transponder delay that is translated in a range bias.

In the ADS-B system the measure time is determined by the receiving station. Sometimes the receiving time of the message and the measuring time of the position (or velocity) are not the same. This time bias, different for each aircraft produced an error in position that depends on the speed of the aircraft.

Finally, the speed of the aircraft is measured by the airborne equipment and transmitted to the station, this speed can be biased but the indicated deviation can be small. This produces errors in the tracking systems that can be corrected adding more parameter to the estimation. Usually it is modeled as an offset in the course and in the module of the speed.

2.3.1 Transponder Delay

As it has been explained above the SSR sensors send an interrogation and waits until the response is received. As it is indicated in [ICAO, 2007a] the SSR sensor has some different working modes. In this section only the SSR mode will be described. Radar Mode-S are different, but works with the same principle and the transponder delay will be determined in the same way.

SSR send the interrogation in the frequency band of 1030 MHz and receive the response in 1090 MHz. The interrogation is sent through 2 antennas, one directional and one omnidirectional. The directional antenna sends two pulses (P_1 and P_3) with a determined time interval. The omnidirectional antenna sends a pulse (P_2) just after the first pulse (P_1). The power of the pulse P_2 is lower than the power of the pulses P_1 and P_3 in the main lobe, but higher than the pulses in the side lobes.

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The time interval between the pulses P_1 and P_3 indicates the interrogation mode of the radar.

- Mode A: The aircraft sends its identification. Time interval $8 \pm 0.2 \mu s$.
- Mode C: The aircraft sends its barometric altitude. $21 \pm 0.2 \mu s$.

The aircraft transponder receives the interrogation and sends its response. This response contains the requested information by the radar. In figure 2.12 a scheme with the interrogation and response is shown.

Figure 2.12: Scheme of a transponder interrogation and response.

After receiving the pulse P_3 the aircraft transponder must send the response with a delay of $3 \pm 0.5 \mu s$ in the case of SSR and $128 \pm 0.25 \mu s$ in the case of mode S. The SSR and Mode-S radar determines the range of the aircraft (ρ_m) measuring the time (τ_T) between the transmission of the interrogation and the receiving of the response, in this calculations the delay of the transponder (τ_d) is considered.

$$\rho = c \frac{\tau_T - \tau_d}{2} \quad (2.66)$$

where c is the speed of light.

In the range estimation the delay is calculated, but the tolerance margin can introduce an error in the determination of the range. This margin of $0.5 \mu s$ is translated to a bias of $\pm 75 m$ for SSR and the margin of $0.25 \mu s$ is translated to a bias of $\pm 37.5 m$ for Mode-S sensors. These measurement error due to the transponder delay are constant in time because of the error in the

2.3. TARGET DEPENDENT BIASES

transponder delay is a calibration error in the configuration of the equipment but is not a random error.

Using the equation 2.66 and adding the error term in the delay of the transponder ϵ_τ , the complete range determination is:

$$\rho_m = c \frac{\tau_T - (\tau_d + \epsilon_\tau)}{2} \quad (2.67)$$

resulting in the transponder delay bias ($\Delta\rho_\tau$):

$$\Delta\rho_\tau = \rho_m - \rho = c \frac{\epsilon_\tau}{2} \quad (2.68)$$

In the literature several estimation methods for this problem can be find [Besada et al., 2012, Bar-Shalom and Li, 1998]. An estimation method for this kind of errors will be developed in section 5.2.1.2.

Forwarding the equations 2.55 and 2.68 the resulting range bias is:

$$\Delta\rho = \Delta\rho_0 + (\alpha_1\rho + \alpha_2\rho^2) \left[1 + \alpha_3 \left(1 - \frac{h}{h_r} \right) \right] + \Delta\rho_\tau \quad (2.69)$$

2.3.2 ADS-B Biases

In the ADS-B systems the aircraft sends its position and velocity through extended squitter ADS-B messages. In [ICAO, 2007b] the different format and fields are explained. In this section we will focus only in some of these fields. The first field is the time synchronization and the second one is the airborne velocity.

2.3.2.1 ADS-B delay

The aircraft determine its own position using the airborne equipment (satellite or inertial) and it is sent through the communication channel. The measurement time is marked by the ADS-B station with the receiving time of the message. In modern airborne equipment the position is measured and charged in the transponder in a little time interval, but in old equipment the time interval between the measurement and the transmission can be big

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enough to get a significant error. For example, an aircraft flying at 250 m/s with a delay of 0.4s in the transmission gives an error of 100 meters when the accuracy of the GPS can be better than 10 meters.

As it can be read in the document ASTERIX Cat 21 [Quitar, 2009]:

”With ADS-B information on time can be provided by two different instances: the aircraft or the Ground Station (GS). If the avionics of the aircraft are synchronised to a high precision time-source (such as GPS), it is able to downlink the position and velocity information synchronised to a precise moment in time, the “Time of Applicability”. In this case, items I021/071 (Time of Applicability for Position) or I021/072 (Time of Applicability for Velocity) shall be used to transmit the time-stamp for the respective information. If the avionics are not synchronised to a high precision time-source, the information downlinked from the aircraft is not synchronised in time. In this case, the only precise time available is the time of reception of the respective message in the GS.”

In the extended squitter message top two horizontal position precision categories (format TYPE Codes 9, 10, 20 and 21) uses a field that indicates the time synchronization. This field indicates if the position measurement and the transmission have been realized in the same 0.2 second epoch.

TYPE codes 9 and 20 give a 95% containment error lower than 3 meters and TYPE codes 10 and 21 give a 95% containment error between 3 and 10 meters. For lower precision codes, the synchronization field is not used.

In the cases where the synchronization field is not used or the field indicates $T = 0$ we can assume that the position is not synchronized and a time bias can be estimated. In figure 2.13 can be seen the horizontal projection of the time bias.

The parametrical model of the error can be obtained using the speed of the aircraft ($\bar{V} = [V_x, V_y]$) as it can be seen in figure 2.13:

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} -V_x \Delta t \\ -V_y \Delta t \end{bmatrix} \quad (2.70)$$

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Figure 2.13: Representation of the position error due to ADS-B time bias.

where Δt is the bias time produced by the delay in the transmission of the position by the transponder of the aircraft. The cancellation of this biases are studied in [Besada et al., 2000].

2.3.2.2 Velocity Bias

In modern aircraft where the position and the velocity is determined by very accurate systems as ADS-B the velocity of the aircraft can be supposed unbiased, but for some aircraft we can observe a bias in the velocity measurement.

As it is indicated in [Quitar, 2010] the heading for the ADS-B messages can be obtained from magnetic and not-magnetic sources. In this document it is indicated that "heading does not need to come from the same source as the position and velocity" and "True heading is preferred, but magnetic heading is acceptable".

In the ADS-B messages, there is a field where the accuracy of the measurement is indicated. In the table 2.1 is indicated the speed categories and there are the next categories:

As it can be seen in the previous table, the accuracy indicated in the message can be 0.15 m/s in the highest category. In some cases the heading of the velocity can be determined by the magnetic compass. The magnetic heading must be corrected with the magnetic dip and this correction value can be badly determined.

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NUCr/NACv	σ_{ADSV}
0	30
1	7.6
2	2.3
3	0.5
4	0.15

Table 2.1: Speed accuracy as function of the category of the equipment.

In this case the accuracy indicated in the message is very good, but the speed can be biased about 1-2 degrees. In the tracking filter, as the indicated speed measurement is very accurate the biased speed will be very important and there will be serious error in the estimation. In 5.2.2 a filter will be developed for the estimation of these microbiases.

If we assume that the velocity biases are caused only by the magnetic dip, they will be correlated by allocation for all the aircraft. Then the biases can be jointly estimated. This assumption is not valid because the source error of each aircraft is different and not all of them are biased due to the magnetic dip.

The current solution for this error is the use of an integrity test. In the tracking system if the speed value and direction received is coherent with the estimated speed calculated by the system, the velocity measurement is used. If this test is negative the velocity measurement is discard. With this the errors are lower but the information of the velocity is lost.

2.4 Negligible Biases

The biases described above (both macro and microbiases) are developed considering the accuracy and availability of sensor used in ATM surveillance applications. These sensor are ADS-B, WAM, SSR and Mode-S. In this section some negligible biases are described in order to shown that in other applications, the development of specific model is needed but they are not developed in this document.

2.4. NEGLIGIBLE BIASES

2.4.1 Surface bias

In other application that are far away from the scope of this thesis there are another biases that can be modeled. In airport surface surveillance the accuracy of the sensor must much higher than in route surveillance. In this case the main sensor used in almost all airports is a Surface Movement Radar (SMR). A SMR is a primary radar with a very high angular speed (60 rpm) with a range resolution lower than 5 meters and an azimuth accuracy lower than 0.05 degrees.

In the case of big aircraft the SMR resolution is much higher than the aircraft size and its position is made calculating the center of the all radar detection of the same target. This position is the geometrical center of the aircraft.

In airports where modern systems has been installed, the surface positions of the aircraft are measured also with WAM systems. The WAM system acquire the target position using the multilateration of a signal transmitted by the aircraft with an antenna network installed in the airport. The measured position is not the geometrical center of the aircraft as with the SMR, in this case the measured position is the transmitting antenna.

Figure 2.14: Scheme with the antenna allocation in a Boeing 737 copied from [Brady, 2017].

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In the case of the Boeing 737 [Brady, 2017] the antenna used for the WAM system is allocated some meter away from the center in the front half of the aircraft as it can be seen in figure 2.14 . This difference on position measurement results in a position bias. The position of these antennas is indicated with ATC in figure.

The difference in position of the antenna and the geometrical center of the aircraft is about 5-10 meters (higher than the surface resolution). In route surveillance this bias is not applicable because of the position acquisition is made with a SSR or a Mode-S sensor. In both cases the position measurement is made with the antenna and not with the geometrical center.

2.4.2 Position Sensor Bias

One of the negligible errors is the error in the allocation of the sensor. In old networks the positions of the radars could be inaccurate and it affects to all the positions.

In [Fischer et al., 1980] the position errors of the sensors are modeled but in this case the date of the research must be taken in account. Currently the position of the sensors is acquire very accurately with satellite systems and the biases produced by this errors are negligible.

Just as an observation in this topic, we will assume that the sensors allocation has error in their positions. Then their positions are:

$$Pos_1 = \begin{pmatrix} x_1 + \Delta x_1 \\ y_1 + \Delta y_1 \end{pmatrix} \quad Pos_2 = \begin{pmatrix} x_2 + \Delta x_2 \\ y_2 + \Delta y_2 \end{pmatrix} \quad (2.71)$$

The measured position obtained with this bias has a constant error in Cartesian coordinates. Then the position measured by the radar i-th is:

$$X_m = \begin{pmatrix} x + \Delta x_i \\ y + \Delta y_i \end{pmatrix} \quad (2.72)$$

As the error in Cartesian coordinates (In the radar biases the errors are constant in polar coordinates) the estimated biases must satisfy the next equation:

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$$\begin{aligned}\Delta x_1 + \Delta x_2 &= \hat{\Delta x}_1 + \hat{\Delta x}_2 \\ \Delta y_1 + \Delta y_2 &= \hat{\Delta y}_1 + \hat{\Delta y}_2\end{aligned}\tag{2.73}$$

There are infinity pairs of $(\hat{\Delta x}_1, \hat{\Delta x}_2)$ and $(\hat{\Delta y}_1, \hat{\Delta y}_2)$ that satisfy the previous equation and the solution is fix some parameters. One of the radar locations must be assumed unbiased and all the estimated error will be relative to the fixed radar. In section 4.2.4 an analytic method to determine the observability of a model will be described.

Chapter 3

Estimation Methods

In this chapter the estimation methods used in ATM will be described. In the previous chapter the models for the sensor biases have been described. The models used in the estimation method are as important in the process as the selected method.

The bias parameters of the sensor are estimated using only the measurements of the aircraft made by several sensors and comparing between them. We do not have in any moment direct measurements of the parameters that must be estimated. In order to get the relationship between the position measurement and the bias parameters we have defined the models of the previous chapter.

The estimation methods generally use linearized models for the estimations. Usually the models has strong nonlinearities and the linearization produces errors in the estimations and they slow the convergence of the algorithms. In this chapter some initialization techniques will be introduced and discussed.

After the linearization of the model, the estimation of the biases presents another problem. The biases of the measures cannot be measured directly by the sensor. In estimation theory these kind of parameters are called hidden parameters [Bahl et al., 1988]. In this case the measured parameter is the horizontal position of the aircraft and the biases are projected over the horizontal plane. In one single position, the projection of two different

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biases is mixed and they cannot be estimated. The estimation methods solve an equation system resultant from several distributed measurements. In the case of a single position measure from several sensors the system is underdetermined. The diversity in the space can ensure the observability of the scenario. This topic will be widely described in section 4.2.

3.1 Dynamical Models

An estimation method is a statistical process that estimate the value of a set of parameter with the measurement of another (usually different) set of parameters with uncertainties in the measurements. The relationship between the estimation set and the measurement set of parameters is established with the measurement model and with the state-transition model.

In ATM the classic estimation problem is the tracking of a moving target. The dynamical model of the target must be established in order to make the prediction of the state as a function of the previous step. The movement of the aircraft and its measurements are modeled as a continuous process:

$$\begin{aligned}\dot{X}(t) &= f(X(t), u(t), t) + w(t) \\ z(t) &= h(X(t), t) + v(t)\end{aligned}\tag{3.1}$$

where $X(t)$ is the state vector (position, velocity, etc.) at time t , $u(t)$ is the control input that model the maneuver of the target, $z(t)$ is the measurement vector and $w(t)$ and $v(t)$ are process and measurement noises respectively. The process noise represent the unpredictability of the aircraft maneuver. Finally $f(\cdot)$ and $h(\cdot)$ are the transition and measurement functions (possibly time-varying) respectively. Following [Li and Jilkov, 2003] and [Bar-Shalom et al., 2004] this continuous model can be discretized sampling where k represent the measurement time:

$$\begin{aligned}X_{k+1} &= f_k(X_k, u_k) + w_k \\ z_k &= h(X_k) + v_k\end{aligned}\tag{3.2}$$

This approximation is not totally exact because $w_k \neq w(t_k)$ and because the motion of the target is not determined by the sampling of the process. It is more appropriate use a mixed time model. The measurement is determined

CHAPTER 3. ESTIMATION METHODS

by the sampling instant and the state must be calculated integrating the continuous process from the previous sample.

$$\begin{aligned}\dot{X}(t) &= f(X(t), u(t), t) + w(t) \\ z_k &= h(X_k) + v_k\end{aligned}\tag{3.3}$$

As it is described in [Singer, 1970] the aircraft dynamic equations in a straight movement can be expressed with the next linear equations:

$$\dot{X}(t) = FX(t) + Gw(t)\tag{3.4}$$

where G is the projection of the process noise over the state vector. Usually the process noise in dynamic systems is modeled with an acceleration that is projected to the speed and position with the matrix G .

The appropriate (discrete time) aircraft equation integrating from the previous sample time is:

$$X(k+1) = \Phi(T)X(k) + W(t)\tag{3.5}$$

Integrating the equation 3.4 and using the property (the differential equation $\frac{d}{dt}y(t) = Ay(t)$ with the initial conditions $y(0) = y_0$ have the solution $y(t) = e^{At}y_0$) then:

$$X(k+1) = e^{FT}X(k) + \int_t^{t+T} e^{F(t+T+\tau)}Gw(\tau)d\tau\tag{3.6}$$

For an unidimensional uniform accelerated movement the state x is composed by (x, v, a) that are position, velocity and acceleration the formulation is:

$$\dot{X}(t) = \begin{bmatrix} \dot{x} \\ \dot{v} \\ \dot{a} \end{bmatrix} = \begin{bmatrix} v \\ a \\ 0 \end{bmatrix} \Rightarrow F = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\tag{3.7}$$

and it can be verified that [Singer, 1970]:

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$$\Phi(T) = e^{FT} = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix} \quad (3.8)$$

In the previous dynamical model the acceleration has a modification term. This term is called plant noise and it is used to model nonlinearities in the movement of the estimated targets. Generally only the measurements of position are available in the systems and because of that the measurement model has the next structure:

$$x_m = x + n_x \quad (3.9)$$

In the equation 3.9 n_x is the noise term. In this chapter and usually in all the estimation methods used in ATM the distribution of this noise will be White Gaussian noise with zero mean. When the biases are presented in the measurement the mean of the noise is not zero mean. In these cases the number of parameters (for the trajectories or sensors) is increased in order to estimate the systematic errors. The standard deviation of the noise distribution must be known in order to estimate the set of parameters with accuracy.

Once the estimation problem has been identified and the state-transition and measurement models have been established, the estimation method used in ATM will be describe showing the advantages and disadvantages of each one.

3.2 Bias Estimation Methods

In this section there is a revision of the most used algorithms and methods with a brief description of each one. The sensor registration is a topic that has been widely studied. Systematic errors affecting to the measurement can produce error in the association of the plots to its central trajectory or introduce a zig-zag error in the estimation of the velocities and position. Because of that in the literature is easy to find some examples that tackle this topic with different approximations.

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The scenario where the biases are going to be estimated in this thesis is an ATM scenario with a sensor network. The sensors can be of the same kind or different and the accuracy of the measurements will be different. The position of the sensors is arbitrary. In the scenario there is a big number of aircraft. The positions of a trajectory are usually measured by different sets of sensor.

In the previous section it has been described that the biases affect to the noise as a different mean to zero. The optimal method is the joint estimation of the biases and the trajectories of all the aircraft because it estimates the noises and their means jointly. This solution implies a very high computational workload and a lot of authors have studied different solutions for the reduction of this work load.

[Friedland, 1969] introduced a Kalman-like estimation concept when the state vector is composed of a set of dynamic states of the targets and constant biases parameters. The decoupling of the dynamic state vector and the bias vector improved the computational efficiency and the stability. This solution has been the base of future researches related with the estimation of systematic errors in variable measurements.

[Bierman, 1975] proposed an alternative derivation of the Friedland using a square-root information filter. A square-root information filter is numerically more stable (in section 4.2.3 the numerical problems produced in the estimation methods are discussed). In 1978 [Mendel and Washburn, 1978] provided another alternative derivation for linear systems and extended the method to the case where more bias parameters must be added or deleted providing a solution for scenarios where the number of biases parameters can change in real time.

[Ignagni, 1981] derived a simplified version of the Friedland's generalizing it and giving to the method a more practical interest. Ignagni continued his work in this topic with [Ignagni, 1990] where a decoupled estimator structure is presented where the tracker of the aircraft and the biases are filtered separately. In this kind of two-stage Kalman estimators another example is [Alouani et al., 1991]. In 2000 Ignagni continued his research with optimal and suboptimal separate Kalman filter for stochastic biases.

The algorithms based on [Friedland, 1969] are an approximation of the

3.2. BIAS ESTIMATION METHODS

state-augmented Kalman filter (filter that estimate jointly the bias parameters and aircraft trajectories) but due to the decoupling of the tracker and the bias filter, they are more computational efficient loosing accuracy in the estimation of the covariances and making the convergence slower.

In [Bar-Shalom and Blair, 1992] is included an algorithm that estimate the biases with the resultant information from the tracking filter making an accumulation of the residual of the predictions. This algorithm makes the assumption that the biases are well estimated. This assumption is fulfilled in a permanent regimen but not in the initialization. This algorithm is widely described in section 3.3.1. In 1993 the same authors developed a Kalman-like filter with changeable structure for different sensor models with a similar structure to [Bar-Shalom and Blair, 1992].

In 2003 a method based in pseudomeasurement is presented in [Lin et al., 2003] for the case of synchronous sensors. With this scheme the biases are estimated without the tracker information. The pseudomeasurments are constructed with the difference of two raw measures at the same time from different sensors. This method admits the use of sensors with different models of the biases. One year later was presented a research in [Lin et al., 2004b] where the biases are estimated with the information of local tracks without a priori association. The generalization for synchronous sensor with the same sample rate is developed in 2004 in [Lin et al., 2004a] and the research was completed in 2005 in [Lin et al., 2005]. A similar scheme is used in [Rafati et al., 2006] based in the pseudomeasurement. This work assumes the biases are constant and the algorithm is generalized to dynamical biases in [Rafati et al., 2007]. and [Qi et al., 2008] for asynchronous systems. This kind of algorithm do not use the information of the tracker and assumes that the measures are close enough to approximate the trajectories as straight movements.

Another improvements using pseudomeasurements In [Portas et al., 2004] a new approach to the bias estimation was presented. In this approach the biases are estimated with pairs of sensor. After this, the "pair-biases" are combined in order to calculate the global biases. In 2007 in [Herrero et al., 2007] the previous work was continued. These papers are designed to be used in online applications.

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In [Jaramillo et al., 2005] was proposed an estimator in two phases for airport surfaces. The first phase estimates the SMR biases using the information of the map. And the second phase uses the SMR corrected measures. The advantage of this work is that in the airport surface the aircraft are moving in the routes. This is not an advantage in the systems of this thesis because the aircraft in route do not flight exactly in the route, the can be displaced.

In [Besada et al., 2009] an offline bias estimation procedure is proposed to trajectory reconstruction (as the proposed in [Besada et al., 2008]). In this method models for ADS-B, WAM and secondary radar are included. The model for the secondary radar is more complete than the used in other works as it has been commented in 2.1. These methods are not applicable to the methods of this thesis because they are focused in online procedures.

In 2013 in [Besada et al., 2012] a structure for the estimation of macro and microbiases is presented. This structure admits different kinds of sensor with different models. The algorithm of this paper is a reordering of the complete Kalman equation matrix. This structure estimate the sensor-target dependent biases jointly for each trajectory. With an average of the biases of all the trajectories the global sensor dependent are calculated and finally the target-dependent biases.

In different years more approximations modifying the pseudomeasurements can be found in the literature. One of the examples applied to generic context is [Zhou et al., 1997]. Another examples applied to 3D radar can be found in [Helmick and Rice, 1993], [Nageswara Rao, 1996], [Belfadel et al., 2014], [Chen et al., 2014] ... In the literature can be found bias estimations based in particle filters [Ristic and Clark, 2012] or in unscented Kalman filters [Bando et al., 2012]. In the application of this thesis, the increment of the computation workload is not justified with the low increment of the performance using unscented Kalman filters or particle filters.

In the next sections, some of the algorithms that has been cited previously and are used in the context of bias estimation in this thesis are described.

3.2. BIAS ESTIMATION METHODS

3.2.1 Weighted Least Squares (WLS)

The Weighted Least Squares (WLS) [Gelb, 1974] is a generalization of the standard Least Squares method. In the basic Least Squares methods all the measurement of the estimation has the same variance and have the same weight on the final estimation. In the WLS method the weighted Mean Square Error (MSE) of a set of measurement is minimized. In this method the variance of the measurement is used and the weight of the measurement is proportional to the inverse of the covariance. With this, the more accurate measurement will have more weight in the estimation and the approximation will fit better in these measurements.

This method has a very important disadvantage: The estimation is made with a set of measure and it cannot be made with online measurement. The input can be divided in sets of measures but the estimation has a delay. The general solution of this method [Kay, 1993] minimizing the weighted MSE using the weighted pseudoinverse matrix is:

$$X = \frac{HR^{-1}}{H^T R^{-1} H} r \quad (3.10)$$

where R is the covariance matrix of the measurement vector r . H is the measurement matrix that projects the measurement vector over the state vector.

With just one measurement this method cannot be applied. A set of measurement must be acquired in order to build complete matrix to get consistent solutions. As it will be explained in section 4.1 this method can be used as the initialization of another recursive method or it can be used in a method that estimate in temporal windows.

The covariance of the solution (P) using this method is

$$P = E[(X - \bar{X})(X - \bar{X})^T] = (H^T R^{-1} H)^{-1} \quad (3.11)$$

3.2.2 Kalman Filter

One of the most important filters used in online applications in ATM is the Kalman filter. This is a recursive implementation of the previous WLS assuming that the measurements at different times are uncorrelated taking account all the measures from the initial measure to the current time. The equations of the algorithm are in the appendix B.1. One of the advantages of this filter is that it estimates the state and the covariances of the state.

The Kalman filter is an optimal estimator when operates over a linear dynamical system with white Gaussian noise uncorrelated (a measure in the same time can have correlated terms). This filter is designed to minimize the variance of the last estimation. The gains calculated are optimal for the estimation variance minimization when the variances of the measurements are known.

The estimation of the Kalman filter is based in the model of the temporal evolution of the dynamical state and the model of the projections of the measures over the state. The optimization is based in the selection of the parameter to minimize the difference between the measures and the modeled state. This is the importance of the correct and accurate modeling of the processes as in the bias modeled in chapter 2. If the model is inaccurate then the estimation of the method will be incorrect.

For each new measure this algorithm estimates the state in two phases: the state prediction and the update of the predicted state with the measure. The previous state estimation (X_{k-1}) is extrapolated to the current time using the dynamical model and it is compared with the current measurement. The previous covariance (P_{k-1}) of the estimation is extrapolated to the current time too.

In the second phase the predicted is updated proportionally with the difference of the measure and the prediction. The weight that is given to this difference is called gain. The gain of the filter is calculated using the predicted covariance of the state and the covariance of the measurement. If the covariance of the measurement is high the prediction will have more weight and vice versa. With the calculated gain the predictions of the state and covariance matrix are updated.

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In the algorithm there are some information needed. The dynamical model of the state vector and the measurement model must be known. The variances of the measurement must be known too.

Dynamical model: The Kalman filter can be fit to different stochastic processes. These processes must respond to a linear dynamical model (if the model is not linear, a modification of the algorithm must be used). This model is used to the calculation of the prediction matrix Φ_k . Using the easy example of section 3.1 of a straight movement with constant acceleration with the prediction matrix is:

$$\Phi_k = \begin{pmatrix} 1 & \Delta T & \frac{\Delta T^2}{2} \\ 0 & 1 & \Delta T \\ 0 & 0 & 1 \end{pmatrix} \quad (3.12)$$

The parameters estimated by the Kalman filter are modeled as a Markov process.

Measurement Model The measurement model is the relationship between the measurement vector and the state vector. Usually there are "hide parameters" that are not measured by the sensor but that must be estimated. Using the easy model for the velocity and acceleration estimation the measurement matrix (H) of the model indicated in equation 3.9 is:

$$H = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} \quad (3.13)$$

As it can be seen in the previous equation the velocity and acceleration are not measured by the sensors, because of that the values of the second and the third column of the matrix are zeros.

The selection of the models, both dynamical and measurement usually are related. In section 5.2.2 there are an example of a filter with velocity bias estimation. This filter can be designed with two different models. As the models are not linear the estimation is not optimal and the selection of the model will change the results. In the mentioned section the results for two different dynamical and measurement models are shown.

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Noise Matrix: The variance of the measurement is used in the calculation of the gain. In the Kalman filter the distribution of the noise must be additive white Gaussian Noise with zero mean. When the noise is not Gaussian the filter is not optimal. When the mean of the noise is not zero or when the noise is correlated the estimation of state vector will be biased. In the previous easy problem (equation 3.9) the measurement is affected with a noise term w_x with a deviation σ_x . In this case the covariance matrix R is a scalar ($R = \sigma_x^2$).

Plant Noise Matrix: The Kalman filter admit the model of nonidealities in the dynamical models of the state vectors. In the dynamical movement of an object with constant acceleration there can be small differences between ideal acceleration and real acceleration. The plant noise models these non-idealities. Another example of plant noise is in the tracking of aircraft in ATM. The dynamic model of an aircraft is design as a straight movement, but in real systems the aircraft are affected by the winds and errors in the heading. Because of that a straight trajectory is not totally straight.

In the easy example the acceleration is affected by a plant noise with variance σ_a . In this section the structure of the plant noise matrix (Q) is not explained. In the Kalman filter the distribution of the plant noise can be different to Gaussian and it can be correlated. In [Bogler, 1990] there are a wide explanation about the distribution of the plant noise.

The plant noise can be modeled as a white noise acceleration. In [Bogler, 1990] the acceleration is modeled as:

$$\begin{aligned} E \{a_x(k)\} &= 0 \\ E \{a_x^2(k)\} &= \sigma_a^2 \\ E \{a_x(k)a_x(j)\} &= 0 \quad k \neq j \end{aligned} \tag{3.14}$$

And the related noise vector is:

$$q(k) = a_x(k) \begin{bmatrix} \frac{\Delta T^2}{2} \\ \Delta T \\ 1 \end{bmatrix} \tag{3.15}$$

And then, the matrix Q is calculated as:

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$$Q = E \{q_x(k)q_x^T(k)\} = \sigma_a^2 \begin{bmatrix} \frac{\Delta T^4}{4} & \frac{\Delta T^3}{2} & \frac{\Delta T^2}{2} \\ \frac{\Delta T^3}{2} & \Delta T^2 & \Delta T \\ \frac{\Delta T^2}{2} & \Delta T & 1 \end{bmatrix} \quad (3.16)$$

In [Singer, 1970] it is described the method to determine the plant noise matrix when the acceleration is modeled as a Markov process with autocorrelation different to zero.

As it has been explained above the Kalman filter is optimal in terms of variance minimization when the dynamical model and the measurement models are linear. In other cases a sub optimal solution must be used. The Extended Kalman Filter (EKF) is a modification of the standard Kalman filter. In this case the prediction of the current state or the measurement are nonlinear functions. The estate prediction and the projection of the measurement over the state vector (Z) will be:

$$\begin{aligned} X_k &= f(X_{k-1}, u_k) \\ Z_k &= h(X_k) + w_k \end{aligned} \quad (3.17)$$

where u_k is the plant noise term of the dynamical model and w_k is the noise term of the measurement. The equation for the gain calculation and for the covariance prediction expressed in appendix B.1 are the same that for the standard Kalman filter. In this case the prediction matrix Φ_k and the measurement matrix H are the Jacobian (matrix of partial derivatives) of functions f and h respectively (f and h are the functions expressed in equation 3.17).

In ATM application the model are rarely linear and the EKF must be applied. Unfortunately the EKF is not the optimal solution in terms of the variance minimization. In [Kay, 1993] is indicated that the EKF has no optimality properties and the performance of the filter will depend on the linearization. As the linearization is dynamical, it is impossible determine its performance beforehand. When the estimation has converge to the real value we can expect that the linearization is very precise and the results will be near to the optimal solution.

3.2.3 Optimal Estimation for Joint Trajectory and Bias Estimation

In this section is described the optimal estimation method in ATM surveillance systems used in the estimation of the biases of the sensors and the positions of the aircraft. In order to illustrate this optimal solution (in the linear case) a simple scenario with a reduced number of n trajectories and sensors is used.

As it has been explained in the previous section the models used for the estimation methods must be established. For the design of the model more details of the scenario must be known. The kind of sensors, the estimated biases, the kinematic model of the targets, etc...

First the kinematic model of the targets will be described. As this is an example, the altitude of the aircraft will not be considered and only the horizontal movement is modeled. The movement of each aircraft can be modeled with position and velocity.

$$\begin{aligned} x_k &= x_{k-1} + Vx_{k-1}\Delta T \\ y_k &= y_{k-1} + Vy_{k-1}\Delta T \\ Vx_k &= Vx_{k-1} + u_v \\ Vy_k &= Vy_{k-1} + u_v \end{aligned} \tag{3.18}$$

where u_v is the plant noise applied to the velocity of the aircraft. The sensors used in this scenario are radars that acquire the position in polar coordinates (the altitude is not considered here) and the position x and y measured by the radar are (measurement model):

$$\begin{aligned} x_m &= \rho_m \sin \theta_m = (\rho + \Delta\rho + w_\rho) \sin(\theta + \Delta\theta + w_\theta) \\ y_m &= \rho_m \cos \theta_m = (\rho + \Delta\rho + w_\rho) \cos(\theta + \Delta\theta + w_\theta) \end{aligned} \tag{3.19}$$

In this equation the azimuth are measured with the origin in the north and clockwise positive. w_ρ and w_θ are the additive white Gaussian noise term with variances σ_ρ and σ_θ respectively.

With the models described above there are several parameter that must be estimated. From the dynamical model the position (x and y) and the velocity (Vx and Vy) will be estimated. From the measurement model the

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biases of the sensor ($\Delta\rho$ and $\Delta\theta$) are selected to the estimation. This model for the biases is a simplified model, in section 2.2 a complete model for SSR biases is described but in this section the simple model is good enough for showing the working of this method.

The parameters that will be estimated are the same for all the trajectories and for all the radars. For one single i-th trajectory that is measured by the j-th radar the state vector is:

$$\hat{X} = \begin{pmatrix} x_{k,i} \\ y_{k,i} \\ Vx_{k,i} \\ Vy_{k,i} \\ \Delta\rho_j \\ \Delta\theta_j \end{pmatrix} \quad (3.20)$$

With one single trajectory and radar the biases cannot be estimated due to the observability (see 4.2). The optimal solution is get when all the trajectories and sensor biases are estimated jointly. In this case the resultant state vector is:

$$\hat{X} = \begin{pmatrix} x_{k,1} \\ y_{k,1} \\ \vdots \\ Vx_{k,n} \\ Vy_{k,n} \\ \Delta\rho_1 \\ \Delta\theta_1 \\ \Delta\rho_2 \\ \Delta\theta_2 \end{pmatrix} \quad (3.21)$$

For example, in a scenario with only 15 ($n = 15$) trajectories and 2 sensors the number of parameters in the state vector is $(15 \times 4 + 2 \times 2)$ 64 and consequently the number of elements of the covariance matrix is 64×64 . This is a very elevated number of parameters even when the scenario is as simple as this one. In real scenario with several hundreds of trajectories and between 5 and 10 radars the size of the matrix and the workload can be unmanageable.

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Using this filter configuration the covariance matrix will be composed by all the trajectory and bias covariances, but it will be composed by all the cross-covariances between trajectories, between sensors and cross-covariances between sensors and trajectories. In the next equation the covariance matrix of a trajectory is denoted P_{X_i} and the matrix of the sensors is denoted P_{B_i} . The matrix with two sub index are the cross-covariance matrix between different trajectories states, between radar biases and trajectories states or between different radar biases.

$$P = \begin{pmatrix} P_{X_1} & \cdots & P_{X_1, X_M} & P_{X_1, B_1} & \cdots & P_{X_1, B_N} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ P_{X_1, X_M} & \cdots & P_{X_M} & P_{X_M, B_1} & \cdots & P_{X_M, B_N} \\ P_{X_1, B_1} & \cdots & P_{X_M, B_1} & P_{B_1} & \cdots & P_{B_N, B_1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ P_{X_1, B_N} & \cdots & P_{X_M, B_N} & P_{B_N, B_1} & \cdots & P_{B_N} \end{pmatrix} \quad (3.22)$$

This matrix is expressed for a generic scenario with M trajectories and N sensors. The size of this matrix is very high and there is a lot of cross-covariances that can be approximated to zero. The different real implementations simplifies the optimal solution approaching some of the covariances terms to zeros, for example the cross-covariances matrix between the radar biases and the trajectories.

3.3 Current Bias Estimation Filters

In order to get a feasible workload some approximations can be made. In the next sections some method with different approaches to the optimal solution are described and compared.

3.3.1 Bias Estimation with Temporal Windows

In [Bar-Shalom and Blair, 1992] an approach to the optimal solution is presented. This algorithm estimate the biases of the sensors dividing the process in several phases. The estimation of the biases is made dynamically

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due to the possible changes on the biases of the sensors and the update of the estimation is made accumulating the data in temporal windows.

In this algorithm it is separated the estimation of the biases and the tracking of the trajectory reducing drastically the number of operations. This separation is made developing the equations of the EKF (for the joint estimation of the biases and aircraft states) and approximating the cross-covariance matrix between the radars and trajectories denoted in the previous section P_{X_j, B_i} with the projection of the radar covariances (P_B) over the trajectories.

Using this approximation it is demonstrated that the single sensor case is unobservable but with the introduction of the multisensor scenario the observability is get. The equations of the EKF include the terms of the covariances of the trajectories, the covariances of the radars and the cross-covariances of the radars and the trajectories. After the approximation of the previous paragraph, the gain calculation of the EKF is expressed using just the covariances of the radars and trajectories removing the cross-covariance matrices.

Finally the authors make a very important assumption for the next equation. In the reference the assumption made is $\|F_{l,i,t}F_{\Delta,i,t}\bar{P}_{\Delta,t}F_{l,i,t}^TF_{\Delta,i,t}^T\| \ll \epsilon_{min}(\bar{P}_{s,t})$. This approximation means that the covariance of the biases projected in the Cartesian coordinates is much lower than lowest covariance matrix of any filtered trajectory. In the initialization of the algorithm when the biases are totally unknown the cross variances must be introduced in the calculation. The approximations of this method makes it unstable in the initial states. In the appendix B.3 this expression is explained with more detail.

After making this assumption the biases can be calculated with temporal windows. The scheme of this algorithm is shown in figure 3.1. The algorithm uses all the measurement inside of a temporal window. The residual of the tracking filter is used for the calculation of an accumulated bias residual and an accumulated residual covariance (denoted v and V in the appendix B.3). The duration of the temporal windows is called ΔT and $k = 1, 2, 3...$ The residual is accumulated during all the time of the windows and at the end of this window the residual is reset. When the temporal window ends the

Figure 3.1: Block diagram of the algorithm described in section 3.3.1

biases are updated with the accumulated residual and covariances and the process starts again.

The variance of this algorithm is very low in the permanent state when the assumption of the previous paragraphs ($\|F_{l,i,t}F_{\Delta,i,t}\bar{P}_{\Delta,t}F_{l,i,t}^TF_{\Delta,i,t}^T\| \ll \epsilon_{min}(\bar{P}_{s,t})$) is fulfilled. This algorithm is used and tested in chapter 4 for the improvement of the initialization.

3.3.2 Bias Estimation with Pseudomeasurements

In this algorithm proposed in [Lin et al., 2003] the bias filter is totally decoupled from the tracking filter. In this case the estimation of the biases is not based in the residual of the tracking filter.

The authors propose in this paper the possibility of extract the bias information from the bias ignorant measurements. Summarizing the process of the paper, the measurements in Cartesian coordinates are the addition of the ideal position of the aircraft and the projection of the biases over the local Cartesian coordinated plane (and the noise).

$$z_i(k) = Hx(k) + B_i(k)b_i + w_i(k) \quad (3.23)$$

where z is the measurement vector, x the ideal state vector, H the measurement matrix, b_i the biases of the i -th sensor, B_i the projection matrix of

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the biases over the Cartesian coordinated and w_i the noise of the measurement.

With a single measurement the terms $x(k)$ and b_i are indistinguishable but with two measurements from different sensors of the same time k a pseudomeasurement of the biases can be constructed removing the position term ($Hx(k)$). The method eliminates the dependencies between the trajectories and the biases. The characteristic of two measurement taken at the same time instant is that the ideal position is the same and that result in:

$$\begin{aligned} Z(k) &= z_i(k) - z_j(k) \\ &= HX(k) - HX(k) + B_i(k)b_i + w_i(k) - B_j(k)b_j - w_j = \\ &= B_i(k)b_i - B_j(k)b_j + w_i(k) - w_j(k) \end{aligned} \quad (3.24)$$

Getting the pseudomeasurement Z as the subtraction of two measurement at the same instant time it is get the subtraction of the biases projection over the coordinated plane. As it can be seen in figure 3.2 the subtraction of two measurement is the difference in position produced by the biases. This pseudomeasurements are filtered with an EKF where the state is composed by the biases of the radars and the terms of the trajectories are not included.

The presented method is developed in this paper for the case with synchronous sensor. In more recent papers the authors have improved the method in [Lin et al., 2005] with the generalization for asynchronous systems. This generalization includes a technique to calculate the pseudomeasurements when the time of the raw measurements is not synchronous.

When a set of measurement is not synchronized some different structures must be found. In these structures there are more than a measurement of a sensor and a single measurement of another sensor. The measurements of the first sensor are interpolated (or extrapolated) to the time of the measurement of the second sensor. This is based in the fact that the trajectories of the aircraft in ATM scenarios are rectilinear and between samples the maneuver can be approximated as rectilinear.

This is the algorithm that will be use to validate the model presented in the previous chapter.

Figure 3.2: Representation of two biased measurement with the difference due to the projection of the biases.

3.4 SSR Bias Model Validation

In section 2.2 the bias model for the SSR has been developed. In this section we include results to show the performance improvement due to the use of our improved bias models both using simulated data (in Section 3.4.3) and real data (in Section 3.4.2). Also, the consistency of the estimator using this model will be assessed. The evaluation of the results will be made evaluating the Root Mean Square (RMS) value of the difference between the positions of the bias corrected measurements from two different radars at the same time for the same target:

$$error_{RMS} = \sqrt{\frac{1}{N} ((x_{i,1} - x_{i,2})^2 + (y_{i,1} - y_{i,2})^2)} \quad (3.25)$$

where $(x_{i,1}, y_{i,1})$ and $(x_{i,2}, y_{i,2})$ are the i -th corrected measures from first and second radar for the same target at the same time. This method is used because the evaluation of this model is done principally with real data and the true aircraft positions are unknown. Another way to test the results for real data is to have access to ADS-B or WAM data position that can be

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supposed unbiased compared with radar measurements.

As the only available information are the measurements of the aircraft, just the differences of the corrected plots (equation 3.25) can be evaluated but it is almost impossible know the real value of the biases. In route tracking in ATC it is more important the alignment between plots from different sensors than the accuracy in position of the measurement. For simulated data the value of the biases is known and the results are presented in convergence graphics (as a function of the number of samples used in the simulation).

3.4.1 Validation Method

The validation of the model is done with two different scenarios. A scenario with simulated data and a scenario with real data. In the two cases the algorithm used is different in order exploit all the information of each scenario and focus in the application of each data set.

In the simulated scenario the measures are uniformly distributed in the airspace and all of them are synchronous measured by two radars. This data is used to check the observability of the model and the estimation method used in this case is not valid for online applications because the biases are estimated using an iterative algorithm. For each point simulated in the scenario there are two measures (one from each radar). A pseudomeasurement is calculated subtracting both measures as it is explained in section 3.3.2). The pseudomeasures are considered the measurement vector. With the biased positions are calculated the measurement matrices of each measure. The biases are calculated using all the measures at the same time with a WLS method described in section 3.2.1.

An approximated value of the biases has been calculated and the biased measures are corrected with these estimated biases. In a next iteration the corrected measures are used as biased measures and the "residual biases" (the biases of the corrected measures) are calculated. This is done because it is assumed that the corrected positions produce lower linearization errors. The biases are updated with the "residual biases" and this process is repeated until the stabilization of the bias values. This iterative method is described in section 4.1.3.1.

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In the real scenario there are several aspects that must be taken in account. The first aspect is that the position measures are not distributed uniformly in the airspace. They are distributed in trajectories (with areas more concentrated than other as it can be seen in figure 3.7). In this case the algorithm used is designed to work in online application and an iterative scheme cannot be used. In this estimation method there is an initial phase where the biases are estimated using a WLS estimator and finally the method for asynchronous measures described in the appendix B.4. The initialization of this method is justified in chapter 4 where there are proposed the initialization techniques for the estimation methods in ATM. The initial phase of this algorithm is done with the measures that are taken in the two initial scans of the radars. When the initial value of the biases is calculated it is used to the recursive estimation of the biases with the measures ordered in time.

Another aspect to take in account in the real scenarios are the changes of coordinates. The real data is processed projected with the stereographic projection (the complete change of coordinates from radar local to the stereographic projection is described in the appendix C). The measurement matrix is linearized using the expression:

$$B = \begin{pmatrix} \frac{\delta x}{\delta \Delta \rho} & \cdots & \frac{\delta x}{\delta \Delta T} \\ \frac{\delta y}{\delta \Delta \rho} & \cdots & \frac{\delta y}{\delta \Delta T} \end{pmatrix} \quad (3.26)$$

It is important to denote that in the previous equations the variables x and y are expressed in the stereographic projection and the biases terms $(\Delta \rho \dots \Delta T)$ are added to the measures in radar local coordinates). The complete projection matrix must be calculated taking in account all the changes of coordinates of the appendix C using a linearization of each step of the change.

$$B = B_{geo \rightarrow ste} B_{ECEF \rightarrow geo} B_{Cart \rightarrow ECEF} B_{radar \rightarrow Cart} \quad (3.27)$$

where $B_{radar \rightarrow Cart}$ is the linearized projection matrix of the radar coordinates to the local Cartesian:

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$$B_{radar \rightarrow Cart} = \begin{pmatrix} \frac{\delta x_l}{\delta \Delta \rho} & \dots & \frac{\delta x_l}{\delta \Delta T} \\ \frac{\delta y_l}{\delta \Delta \rho} & \dots & \frac{\delta y_l}{\delta \Delta T} \\ \frac{\delta z_l}{\delta \Delta \rho} & \dots & \frac{\delta z_l}{\delta \Delta T} \end{pmatrix} \quad (3.28)$$

$B_{Cart \rightarrow ECEF}$ is the linearized change of coordinates from local Cartesian to ECEF:

$$B_{Cart \rightarrow ECEF} = \begin{pmatrix} \frac{\delta x_{ECEF}}{\delta x_l} & \frac{\delta x_{ECEF}}{\delta y_l} & \frac{\delta x_{ECEF}}{\delta z_l} \\ \frac{\delta y_{ECEF}}{\delta x_l} & \frac{\delta y_{ECEF}}{\delta y_l} & \frac{\delta y_{ECEF}}{\delta z_l} \\ \frac{\delta z_{ECEF}}{\delta x_l} & \frac{\delta z_{ECEF}}{\delta y_l} & \frac{\delta z_{ECEF}}{\delta z_l} \end{pmatrix} \quad (3.29)$$

$B_{ECEF \rightarrow geo}$ is the linearized change of coordinates from ECEF to geodetic:

$$B_{ECEF \rightarrow geo} = \begin{pmatrix} \frac{\delta lat}{\delta x_{ECEF}} & \frac{\delta lat}{\delta y_{ECEF}} & \frac{\delta lat}{\delta z_{ECEF}} \\ \frac{\delta lon}{\delta x_{ECEF}} & \frac{\delta lon}{\delta y_{ECEF}} & \frac{\delta lon}{\delta z_{ECEF}} \end{pmatrix} \quad (3.30)$$

and finally $B_{geo \rightarrow ste}$ is the linearized projection from geodetic to stereographic.

$$B_{geo \rightarrow ste} = \begin{pmatrix} \frac{\delta x}{\delta lat} & \frac{\delta x}{\delta lon} \\ \frac{\delta y}{\delta lat} & \frac{\delta y}{\delta lon} \end{pmatrix} \quad (3.31)$$

In the real data scenario the biases can change slowly (in the simulated scenario the biases are constant). This slow change in the value of the biases is modeled with the plant noise matrix (Q) adjusting it with a very low value of the covariance.

3.4.2 Simulated Scenario Results

In order to test the observability of the different biases modeled in the previous chapter, the estimation method will be tested with a simulated scenario. In this scenario there are two radars in position (0, 0) and (50, 0) (in NM). The scenario has 1000 point uniformly distributed in a cubic volume with a side of 400 NM centered at (0, 0). The maximum altitude of these points is 15,000 m.

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Figure 3.3: Estimated bias values for Radar 1 using only the basic model ($\Delta\rho_0$, α_1 and $\Delta\theta_0$). (a) Range offset; (b) Azimuth offset; (c) Range gain; (d) Temperature bias.

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Figure 3.4: Estimated bias values for Radar 1 using only the model with the parameters $\Delta\rho_0$, α_1 , $\Delta\theta_0$, s_{ant} , β_{axis} and α_{axis} . (a) Range offset; (b) Azimuth offset; (c) Range gain; (d) Temperature bias.

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Parameters	Bias Value
$\Delta\rho_0$	$100m$
α_1	$10^{-3}m/m$
α_2	$10^{-9}m/m^2$
α_3	1.15
$\Delta\theta_0$	0.04°
s_{ant}	0.5°
β_{axis}	0.4°
α_{axis}	45°
β_{enc}	0.5°
α_{enc}	90°
$\frac{\Delta R}{R}$	10^{-4}
α_{exc}	45°
ΔH_p	$-500m$
ΔT	$15^\circ C$

Table 3.1: Bias values used for the simulated scenario

In this case the simulated measurements are synchronized and the noise of the measurements are white Gaussian noise with zero mean with standard deviation of 75 m in range and 0.05° in azimuth. This test is made using Cartesian coordinates with the flat Earth model.

The measures obtained by the radars are biased using the previously developed models with all the parameters. The evaluation of the results is made using the RMS error in position of all the corrected measurement in Equation 3.25. As the ideal position of the points is known, the error will be calculated between the corrected measurement and the ideal position.

In this simulated scenario, the RMS error of the measured plots is 541.64 m and the RMS error of the corrected plots is 122.99 m. As in the simulated data the ideal position of the aircraft is known, we can measure the errors produced only by the noise. In this case, the RMS error produced just by the noise is 120.08 m. As the RMS error of the corrected plots is almost the RMS error produced by the noise of the measurements, it is proved the observability of the model when the plots are well distributed. With a bigger number of samples, the RMS error of the estimation is nearer to the RMS error produced by the noise.

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Figure 3.5: Estimated bias values for Radar 1 using only the model with the parameters $\Delta\rho_0$, α_1 , $\Delta\theta_0$, β_{enc} , α_{enc} , $\frac{\Delta R}{R}$ and α_{ecc} . (a) Range offset; (b) Azimuth offset; (c) Range gain; (d) Temperature bias.

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Figure 3.6: Estimated bias values for Radar 1 using the complete model. (a) Range offset; (b) Azimuth offset; (c) Range gain; (d) Temperature bias.

3.4. SSR BIAS MODEL VALIDATION

In this simulated scenario the bias parameter used in the generation of the simulated position are shown in table 3.1. The next paragraphs will show the estimation of some parameters using different models but always generating the simulated plots with the complete model. The next figures shows the mean estimated value of the biases with a blue line and the mean plus/minus the standard deviation of the estimation with a dotted red line.

The basic model used classically in the bias estimation literature uses only $\Delta\rho_0$, α_1 and $\Delta\theta_0$. The obtained results are shown in figure 3.3. The temperature bias is not estimated with this model and in figure corresponding to this bias the value and the deviation is zero. The RMS error get with the corrected plots is 277.03m. Using this simple model the errors in the positions are reduced compared with the uncorrected plots but this values is far away from the noise level. In order to check the model different sets of parameters are included in the estimation.

With a more complex model where the parameters used in the estimation are $\Delta\rho_0$, α_1 , $\Delta\theta_0$, s_{ant} , β_{axis} and α_{axis} the estimated values are shown in figure 3.4. In this estimation the biases produced by the position of the antenna are estimated (and the basic model). This error reduces the RMS error to 259.03. In this simulation the error is reduced compared with the simple model but the effect is not too noticeable. The errors of the antenna are increased with the elevation of the targets and the high elevations are get near to the radar. The correction of this bias in the measures that are far from the radar are not corrected because they are not affected by this bias.

Estimating different parameters than in the previous simulation ($\Delta\rho_0$, α_1 , $\Delta\theta_0$, β_{enc} , α_{enc} , $\frac{\Delta R}{R}$ and α_{ecc}) the results are slightly different as it can be seen in Figure 3.5. In this case the parameters included are related with the optical encoder of the radar. These biases are not conditioned by the elevation and the measures that are far from the radar are more affected than the near measures. In this case, estimating these biases the RMS error is reduced to 172.31m. The error is reduced but the biases estimated are different to the simulated biases.

In these three simulations it can be seen that the range and azimuth estimations are different from the real values. This is due to the fact that there are parameters that affect range and azimuth that are not included

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in the estimation model and the included parameters values compensate the effect of the not included parameters.

Finally Figure 3.6 shows the estimated values for the complete model used in the generation. In this case all the parameters are estimated and the result are near to the real values (shown with a black dotted line). In the table 3.2 the RMS error for each experiment are shown.

Parameters	RMS Error
Without correction	541.64m
$[\Delta\rho_0, \alpha_1, \Delta\theta_0]$	277.03m
$[\Delta\rho_0, \alpha_1, \Delta\theta_0, s_{ant}, \beta_{axis}, \alpha_{axis}]$	259.03m
$[\Delta\rho_0, \alpha_1, \Delta\theta_0, \beta_{enc}, \alpha_{enc}, \frac{\Delta R}{R}, \alpha_{ecc}]$	172.31m
$[\Delta\rho_0, \alpha_1, \alpha_2, \alpha_3, \Delta\theta_0, s_{ant}, \beta_{axis}, \alpha_{axis}, \beta_{enc}, \alpha_{enc}, \frac{\Delta R}{R}, \alpha_{ecc}, \Delta H_p, \Delta T]$	122.99m

Table 3.2: Bias error deviation for different configuration of the model parameters in the simulated scenario

This simulation has been done to validate the observability of the model and to demonstrate that the including of a bigger number of parameters make the alignment of the biases more accurate. In the real scenario the same models parameters are used to check the reduction of the RMS error.

3.4.3 Validation with Real Data

The models have been tested with data from two real radars. The maximum range of both radars is bigger than 150 NM and the separation between both radars is 52 NM. This configuration gives enough common coverage and avoids observability problems due to the sensor separation. Radar measurements have been transformed to local Cartesian coordinates and projected to a common stereographic plane. The center of the stereographic projection (point (0, 0) in Figure 3.7) is far away from the radars in order to evaluate if the models work with the rotation, translation and scaling produced by the stereographic projection.

The real scenario has 227 aircraft distributed in the airspace in different flight phases. As it can be seen in 3.7 tracks are not uniformly distributed in

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Figure 3.7: Real scenario with 227 tracks measured by two radars

the airspace. The major part of the measures is concentrated in the bottom-left corner of the common radar coverage. As the algorithm minimizes the mean square error the measurement of this corner will be better aligned than the measurements from other parts of the scenario.

As the biases values are not known, the performance of the model will be evaluated estimating the RMS value of the deviation in measured position of the same track at the same time from two different radars (equation 3.25). The evaluation will compare the deviation of the positions for all the measures in the common coverage. The RMS error of the measurement noise of the radar is 160.95 m for the first radar and 88.92 m for the second radar. The RMS value of the difference of uncorrected measured positions is 532.45 m.

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3.4.3.1 Simple Model

The biases used in this estimation are the range offset ($\Delta\rho_0$), the range gain (α_1) and the azimuth offset ($\Delta\theta_0$). With these three parameters the majority of the error produced by the biases is corrected. With these models, the deviation of the bias errors σ_b is reduced up to 286.60 m obtained with equation 3.25. In figure 3.8 a,b two illustrative trajectories are represented. The asterisks mark the measured position and the circles mark the corrected position. Both trajectories are allocated in the bottom left corner of the scenario and the raw plots of radar 2 are displaced in azimuth in counter-clockwise direction.

The algorithm corrects the plots and displaces the plots towards the correct azimuth. In figure 3.8 c several trajectories placed in top-right corner (low density traffic area) are represented. Raw plots are aligned and the corrected plots are displaced clockwise. In this scenario is easy to see that the simple model is not good enough to estimate and align the plots of the radars for the whole radar coverage. In this scenario, the trajectories of figure 3.8 a,b are separated almost 180° in azimuth from trajectories of figure 3.8 c. The azimuth dependence of the biases is clearly shown in these figures. This dependence is included in the complete azimuth bias model of equation 2.40.

3.4.3.2 Complete Model

Next, it will be shown the results with variations of the complete model developed in this paper. Several configurations of the model have been used with the objective to test the final correction contribution of each parameter of the model. In Table 3.3 the error deviation due to the biases is expressed for different configurations of the models (in the first column the bias parameters used in each configuration are indicated). The first row is the basic model from the previous section. Second row estimates the biases due to the deviation of the antenna and the rotation axis. Third row estimates the biases produced by the encoder. Finally the fourth row estimates all the biases presented in the model.

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Figure 3.8: (a) Straight trajectory; (b) Take off trajectory corrected with basic model; (c) Several trajectories allocated at the upper-right corner of the scenario corrected with the basic model.

Parameters	RMS Error
Without correction	532.45m
$[\Delta\rho_0, \alpha_1, \Delta\theta_0]$	286.60m
$[\Delta\rho_0, \alpha_1, \Delta\theta_0, s_{ant}, \beta_{axis}, \alpha_{axis}]$	260.65m
$[\Delta\rho_0, \alpha_1, \Delta\theta_0, \beta_{enc}, \alpha_{enc}, \frac{\Delta R}{R}, \alpha_{ecc}]$	248.22m
$[\Delta\rho_0, \alpha_1, \alpha_2, \alpha_3, \Delta\theta_0, s_{ant}, \beta_{axis}, \alpha_{axis}, \beta_{enc}, \alpha_{enc}, \frac{\Delta R}{R}, \alpha_{ecc}, \Delta H_p, \Delta T]$	242.19m

Table 3.3: Bias error deviation for different configuration of the model parameters

The encoder eccentricity seems to be a very important parameter in the bias estimation. For example if the encoder has a diameter of 25 cm and

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an error in the rotation center of 0.25 mm produces a maximum bias of 200 m at a range of 200 km. As could be expected, the estimation with all the parameters is the best one and in figure 3.9 (equivalent to figure 3.8) the corrected tracks are shown. In this case, as the complete model has azimuth dependent biases, every track in every azimuth is better corrected in comparison with the simple model. But even reduced parameters bias models result in relevant accuracy gains with respect to the usual basic model.

Figure 3.9: (a) Straight trajectory; (b) Take off trajectory corrected with basic model; (c) Several trajectories allocated at the upper-right corner of the scenario corrected with the complete model.

The algorithm determines bias parameters for both radars to minimize the mean squared error between the ideal position and the corrected position.

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As consequence of results we should conclude that (locally in the in the shown areas) the radar 1 is better aligned to ideal position than radar 2, although measures from both radars are corrected.

Chapter 4

Initialization and Observability Analysis

In this chapter two important topics are studied: the initialization of the algorithms and the observability of the scenarios. Both effects are not widely studied and the problems produced by these topic are ignored because it is assumed that they are previously solved.

The initialization of the algorithms is a sensitive problem in the estimation field. For example in the tracking filters the initial velocity of the aircraft is estimated using the two first position measures but the initialization of the biases is more difficult. The initial estimation of the values usually is set to zero and this leads to linearization errors that produce instabilities in the estimation methods. In the next section there are proposed some initialization techniques in order to reduce the instabilities with a higher speed of convergence.

The observability of the scenarios is studied in this chapter too. In the papers and books that study the estimation methods the observability is a topic that is mentioned but in all of them the scenario is assume observable. In the real scenarios the observability conditions change dynamically with the position of the aircraft or with the availability of the sensors. In this section the geometrical and analytic meaning of the observability is described. Once the observability problems are identified, a technique to deal with the scenario with lack of observability is proposed.

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4.1 Initialization Techniques for Bias Estimation

In this section are studied the problems and the behavior of the initialization are studied for the estimation methods applied to the biases of the sensors. In this section some initialization techniques are proposed describing the advantages and disadvantages of each one. These techniques are designed to solve the instabilities of the initialization without losing the speed of convergence.

The initialization problems are caused by the linearization error in the initial stages of the estimation. The linearized functions are particularized with the estimated values of the parameters. The models used for the sensor biases in ATM are non linear and the partial derivatives of the models are used to get the linearized equations. If the parameters are badly estimated, the linearization errors are big. In the next section the errors produced by the linearization are widely described.

4.1.1 Linearization Errors

Usually the partial derivative of the model with respect to a parameter depends on the values of the function variables. As these values are unknown they are substituted by their estimated values. For example, in the equations presented in section 2.2.4 the linearized model for the altitude bias is described. The function of the altitude bias depends on the altitude and pressure offset. The partial derivatives with respect to the temperature bias depends on both pressure and temperature bias. In general, with the models used in this thesis the partial derivatives depends on several parameters.

$$\frac{\delta x}{\delta \Delta T} = f(\Delta H_p, \Delta T) \quad (4.1)$$

The model is linearized particularizing the function f with the estimated values of the biases. In the previous equation the partial derivative is calculated using the estimated value for ΔH_p and ΔT . At the beginning of the estimation the bias values are usually erroneous and the linearization of

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the models will have errors. In other hand, when the biased measure is far away from the real position, the linearization is not a valid approximation for the models. In this case the measures must be corrected with the estimated values before the new estimation.

In the measurement model of chapter 2 with a simple bias model (range and azimuth offset):

$$x_m = \rho_m \sin \theta_m = (\rho + \Delta\rho) \sin (\theta + \Delta\theta) \quad (4.2)$$

the partial derivatives used in the model are:

$$\begin{aligned} \frac{\delta x_m}{\delta \Delta\rho} &= \sin \theta_m \\ \frac{\delta x_m}{\delta \Delta\theta} &= \rho_m \cos \theta_m \end{aligned} \quad (4.3)$$

Figure 4.1: Detail of a sine showing the error produced by the linearization.

As it can be seen the value of the partial derivatives depends on the value of the measurement. As the value of the biases is not known at the beginning these values are erroneous. In figure 4.1 it is shown the error produced by the linearization of a sinusoidal function when the estimated values of the biases is not known. In this example the measured value of the angle is 71 degrees but the real value is 85 degrees. The sine is linearized in the measured value and the error produced in the real position is too big.

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4.1.2 Initialization Remarks

The estimation method with temporal windows (explained in section 3.3.1) is very sensitive to the linearization errors. The method during the initialization is unstable and the covariance matrix can become non positive definite matrix. This produces a reset of the filter and the values of the biases are reset too. In order to solve this erroneous update, the values of covariance matrix used in the initialization must be low and the speed of convergence of the results are slow.

In the other hand, when the temporal windows method enters in the permanent regimen, the values of the estimations are very accurate. This method decouples the tracking filter and the bias filter but the information from both filters is shared in order to get better performances.

Finally this methods has the advantage that the biases of all the sensors are estimated jointly and the projection (measurement) matrix used in the update step is totally constructed considering all the cross covariances between all the sensors. As all the information is available in this matrix, the observability can be tested (see section 4.2).

The WLS method described in section 3.2.1 is less sensitive to the linearization errors than the temporal windows method. The errors in the linearization (for the models used in ATM) do not make the estimation method unstable. In order to get initial values of the bias estimation this method has good performances but it has disadvantages too.

In order to get good initial values several measurements must be used. The complete measurement matrix (H in section 3.2.1) is built with a composition of all the projection matrix from all the measures. The built measurement matrix of temporal windows method has always the same size $M \times N$ where M is the number of biases multiplied by the number of sensors and N is the dimension of the measurement (usually 2) and it is such as an averaged measurement matrix (this is explained with more detail in the appendix B.3). The built measurement matrix of the WLS method has a variable size. In this case the size is $M \times N'$ M is the number of biases multiplied by the number of sensor but in this case N' is the dimension of the measurement (usually 2) multiplied by the number of measurements.

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The measurements used in the WLS method are the pseudomeasurements get with the method explained in 3.3.2. And in order to avoid the big size of the generated measurement matrix the recursive WLS method can be used [Kay, 1993]. The recursive formulation of the method has several advantages comparing it with the classical formulation. Some of the advantages are that the workload is lower than with the classical WLS and that the estimations are get online. As a disadvantage, the building of the complete measurement matrix can be used to make an observability test (section 4.2) but this matrix is not built with the recursive formulation.

The convergence the WLS method using pseudomeasurement is fast and stable, but in the permanent regimen the performance of the filter is lower than using the temporal windows method.

All the papers mentioned in section 3.2 consider that the radars measure the position of the aircraft with the range and azimuth (and the altitude provided by the airborne equipment) and this measure is converted to Cartesian coordinates. This change of coordinates is made using a Flat-Earth model. As it has been explained previously the reference of the radar is pointing the y-axis to the geographical north. If the Earth is modeled as an ellipsoid (WGS84) the geographical north is not parallel for all the radars when they are allocated in different longitudes and the Flat-Earth model is not correct.

Figure 4.2: Representation of 2 radar in different locations. The horizontal plane of each radar is not parallel to the other one.

As it can be seen in figure 4.2 the horizontal planes of two radar are not parallel due to the curvature of the Earth surface. The altitude measured by the aircraft is the distance perpendicular to the Earth and not perpendicular to the horizontal plane of the radar.

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The Cartesian coordinates used in the major part of the papers assumes that the altitude is equivalent to the z-axis and that the local Cartesian coordinates of each radar are parallel between them.

The assumption described above are good enough for the test of the algorithms but for real scenarios more accurate changes of coordinates must be done. The initialization techniques presented in the following sections are designed for real scenario where a stereographic projection is used. This projection introduces some disturbances (appendix C) in the shapes of the trajectories that will make the linearizations even more unstable.

Figure 3.1 shows the diagram of the algorithm described in section 3.3.1 (temporal windows). In this figure it can be seen that the measures are corrected in the step previous to the tracking filter (as it is done usually) and in this case the data for the bias estimator is extracted from the corrected measurements. The correction of the measures can be done in two ways. With the first way, the measurement model can be represented as:

$$X_{m,k} = H_k X_k + H_{b,i} b_i + W_k \quad (4.4)$$

where $X_{m,k}$ is the measure, X_k is the ideal state, H_k the measurement matrix, $H_{b,i}$ the projection matrix of the biases from the i-th sensor, b_i the biases of the i-th sensor and W_k the vector with the noise terms of the measurement. With that model, the corrected measure $X_{c,k}$ can be expressed as:

$$X_{c,k} = X_{m,k} - H_{b,i} b'_i \quad (4.5)$$

In this case the biases projected are the estimated biases (b'_i). This way to correct the biases includes a linearization of the model and in this case there are two error sources, the first one is the error in the bias estimation ($b_i \neq b'_i$) and the second one is the linearization of the functions. In order to get a higher accuracy in the correction the linearization must be avoided. The exact model of the biases must be used to correct the measures, with the second way of the correction of the measures, the measurement model is represented as:

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$$\begin{aligned}x_{m,k} &= x_k + fx(b_i) + w_{x,k} \\y_{m,k} &= y_k + fy(b_i) + w_{y,k} \\z_{m,k} &= z_k + fz(b_i) + w_{z,k}\end{aligned}\tag{4.6}$$

With this model the corrected measure is:

$$\begin{aligned}x_{c,k} &= x_{m,k} - fx(b'_i) \\y_{c,k} &= y_{m,k} - fy(b'_i) \\z_{c,k} &= z_{m,k} - fz(b'_i)\end{aligned}\tag{4.7}$$

where fx, fy, fz are the ideal projection function for each component of the coordinates and the terms w are the noise terms. In this case the error sources of the corrected measures is just the error in the estimation.

In the pseudomeasurement method described in section 3.3.2 the estimation is made over the raw measurement and because of that the linearization of the equations are particularized with wrong bias values. In the next section a proposal for the initialization of the bias values is presented.

4.1.3 Initialization Techniques

In the initialization of the estimation method applied to biases there are several effects that degrade the accuracy of the linearized equations. Firstly the difference between the estimations and the real values introduces error. These errors are degraded by the linearizations in the changes of coordinates. In some algorithms the initialization is very sensitive because it is designed using some approximations to the optimal scheme as in the temporal windows method.

There are multitude of algorithms that have different properties in the first steps. In one hand we have an algorithm (temporal windows) that is very unstable in the initialization but with good performance in the permanent regimen. In the other hand we have an algorithm which behavior in the initialization is fast and stable but the permanent regiment is not very accurate due to the linearization of the projections. The proposals included in this section is the use of one algorithm in the initialization and when it is estimated that the biases have converge near to the ideal value starts using

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the permanent method.

The point of this chapter is the design of a complete algorithm combining some existent estimation methods. The initial values of the biases are zeros and the algorithms present different behaviors in the initial stages. In the next sections the advantages and disadvantages of the estimation methods applied to biases estimation used for the initialization algorithm are described. A modification for the WLS is introduced and the initialization of the pseudomeasurement and temporal windows methods are described. Finally the proposed combined algorithm (from the initialization to the permanent regimen) is developed.

4.1.3.1 Iterated Weighted Least Squares

There is an implementation of the WLS that increases the accuracy of the estimation sacrificing the computational work load. This modification consists in estimate recursively the biases of a set of measures, correct the set of measures with the estimated values of the biases, estimate the residual biases of the corrected measured and recompute the biases estimation.

This approximation reduces the error due to the linearization of the projection matrix. The first estimation must be done with the raw measures and the linearization error is the same that using the standard WLS. Assuming that the scenario fulfills the observability conditions, the measures corrected with the estimated biases are nearer to the ideal positions than the raw measures. For the second iteration (with the same set of measurements), the linearization of the biases is more accurate than the first iteration and the biases are better estimated. In figure 4.3 there is a block diagram representing this algorithm.

As it can be seen in figure 4.3 the estimated values calculate with the method are the difference between the real biases and the previously estimated biases. The estimated values in this iteration must be added to the previous estimation to get the global biases.

With this technique the initial value of the biases is expected to be more precise than using the standard WLS. In both cases the covariance of the

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Figure 4.3: Block diagram of the Iterated WLS method showing the correction of the biases with the residual estimated biases.

estimation has the same expression but the value will be slightly different due to the projection matrix (H). In the Iterated WLS and in the standard WLS the calculated covariance is:

$$P = (H^T R^{-1} H)^{-1} \quad (4.8)$$

This iterative initialization has a very important advantage that has not been aforementioned. In order to understand this advantage the formulation of the bias measurement matrix must be in mind. A pseudomeasurement is composed by the partial derivatives of the measures with respect to the bias parameters (Jacobian). In the ideal case the functions are linear and independent but in the real scenario that does not happens.

When the derivatives depends on more than one bias parameter they can be zero in some cases. In this example will be used the model presented in section 2.2 for SSR sensors. For this examples only a reduced set of the parameters will be used but the complete matrix include all the parameters described in the SSR model section.

As it is indicated in section 2.2.3.1 the range bias can be expressed as:

$$\Delta \rho = \Delta \rho_0 + (\alpha_1 \rho + \alpha_2 \rho^2) \left[1 + \alpha_3 \left(1 - \frac{h}{h_r} \right) \right] \quad (4.9)$$

And the measurement model is:

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$$\begin{aligned} x &= \rho_m \sin \theta = (\rho + \Delta\rho) \sin \theta \\ y &= \rho_m \cos \theta = (\rho + \Delta\rho) \cos \theta \end{aligned} \quad (4.10)$$

For this example we have to take in account that a reduced model is been using with only the range bias. Using only these parameters the bias measurement matrix results in:

$$H_b = \begin{bmatrix} \frac{\delta x}{\delta \Delta\rho_0} & \frac{\delta y}{\delta \Delta\rho_0} \\ \frac{\delta x}{\delta \alpha_1} & \frac{\delta y}{\delta \alpha_1} \\ \frac{\delta x}{\delta \alpha_2} & \frac{\delta y}{\delta \alpha_2} \\ \frac{\delta x}{\delta \alpha_3} & \frac{\delta y}{\delta \alpha_3} \end{bmatrix}^T \quad (4.11)$$

The more illustrative derivative for this example is $\frac{\delta x}{\delta \alpha_3}$ ($\frac{\delta y}{\delta \alpha_3}$ is almost the same equation and the description is the same for both equations). The result of this derivative is:

$$\frac{\delta x}{\delta \alpha_3} = \frac{\delta x}{\delta \rho_m} \frac{\delta \rho_m}{\delta \alpha_3} = (\alpha_1 \rho + \alpha_2 \rho^2) \left(1 - \frac{h}{h_r} \right) \frac{\delta x}{\delta \rho_m} \quad (4.12)$$

In the propagation model the parameter α_3 modifies the bias produced by the propagation in function of the altitude. And because of that it is expected that the derivative depends on the altitude (h) and on the propagation parameters (α_1 and α_2).

As we are studying the initialization of the estimation the initial values of the biases are not known a priori and the values $\alpha_1 = 0$ and $\alpha_2 = 0$ are set (all the parameters are set to zero). This initialization produces that the terms of the measurement matrix corresponding to the parameter α_3 are zeros.

Taking that in account we have that one of the columns of H_b are zeros. When the matrix H used in the WLS method is used for the first time there is a column that equal to zero:

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$$H = \begin{bmatrix} H_{b,1} \\ \vdots \\ H_{b,i} \\ \vdots \\ H_{b,N} \end{bmatrix} \quad (4.13)$$

The solution of the WLS is expressed in the equation 3.10. The matrix R is composed with all the covariance matrix in diagonal and the r vector is the measurement matrix. As it can be seen in the mentioned equation the solution needs to solve the inverse matrix of $inv(H^T R H)$. When one or more of the columns of the matrix H are zeros, the matrix is singular and the WLS has not solution.

In this example α_3 has been used to illustrate the problem of the estimation when the dependent parameter has not been well estimated but in the SSR model bias the parameters of the equation 2.40 (complete azimuth bias of SSR) the problem is the same. In this equation α_{axis} depends on β_{axis} , α_{enc} depends on β_{enc} and α_{exc} depends on β_{exc} . In section 2.2.2.4 is mentioned that a reformulation of the model to convert this pairs of parameters α and β into rectangular parameters in order to get more stability. The reasons for this have been explained in this section.

Returning to the advantage of the iterated WLS the first iteration of the biases can be made with a reduced set of parameter. Using the aforementioned example, the first iteration can be made using the parameters $\Delta\rho_0$, α_1 and α_2 . This first iteration will estimate a coarse estimation of the bias parameters. In the second estimation the partial derivative with respect to α_3 will be calculated with the coarse estimation of α_1 and α_2 .

Assuming that the method is convergent the estimations of the biases will be more accurate with the iterations. In the other hand, more iterations are translated to more computational workload. This iterative process is stopped when the correction of the biases is below a threshold defined by the user of the method.

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4.1.3.2 Initialization of Pseudomeasurement Method

Another approximation to use as the first step in the complete algorithm is the pseudomeasurement algorithm describe in section 3.3.2. In the next paragraph the advantages and disadvantage of using this method as the first stage of the first algorithm are described. The initial values of the biases in this case are zeros.

The first advantage of this method is that the computational workload admits the online processing of the measurements. As it is a recursive formulation of the problem, measurement are processed one by one. With the recursive formulation the first estimation is get when the first measurement is processed (in the Iterated WLS a set of N measurements is needed to the first estimation).

In old systems where the available computational workload was much lower than in modern system, the bias estimation could be made using pairs of radars and make an estimation in a very big temporal window (several minutes). When the time of the temporal windows ends the measures are corrected with the estimated values and the operators can see a very big jumps in all the measures. In the tracker the measures previous to the first correction are the raw measures and the tracking filter can have a zig-zag effect.

The second advantage is that using the recursive Kalman filter the estimations are available since the first measure and the measures used in the tracker have less influence from the biases.

One of the disadvantages of this method is related with the problem described in the previous section. Some partial derivatives of the parameter can depend on other parameters. In the first estimation the parameters used in the calculation of the derivatives are zeros and the results can be zeros (the described case of α_3 that depends on α_1 and α_2) and the estimation can be unstable.

The second disadvantage is that the complete measurement matrix is not built and the observability test cannot be done. In the other described methods where all the measures are processed jointly the complete matrix is built.

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Finally another disadvantage is that the linearizations are made in the points of the raw measures. As the raw measures can be far from the ideal positions, the linearization value will be erroneous. This error in the linearization has the same effect that the explained in the previous paragraph.

4.1.3.3 Initialization of Temporal Window Method

For the initial stage of the complete algorithm, the temporal windows algorithm described in section 3.3.1 presents serious disadvantages compared with previous methods. In this section is important to remember that this method estimates the bias values using the residual of the measurement in the central track (the central track is the filtered track using the measurements from all the sensors). Because of that, this method is actually using measures corrected with the estimated values of the biases.

One of the advantages of this method (as it can be read in the previous paragraph) is that the biases are estimated with corrected measurements and the linearizations are more accurate. This is translated to a higher accuracy in the final estimations.

The most important disadvantage is that this method is very sensitive to the linearization errors and it makes it very unstable. When the bias values are initialized with a far value of the real biases the linearization are bad calculated and when the update of the biases and their covariances is made the covariance matrix can be definite negative.

One of the disadvantages of this method is that the biases are processed in temporal windows and the effect of sudden corrections can be observed in the screen of the operator.

In order to avoid that there are several solutions. The first one is the reduction of the temporal windows duration. When the temporal window duration is reduced, the sensitivity of the method is reduced too. The reduction of the temporal window duration reduces the accuracy and increase the computational workload too. Another solution is the reduction of the values of the initial covariances. This reduction increases the stability of the method but reduces the speed of convergence for the estimation.

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The instabilities are produced because the assumption of convergence described in section 3.3.2 is not fulfilled. A priori this instability can seem only a disadvantage, but the side effect of this is that a detector for abrupt changes in the sensors is available online. When the algorithm has reached the permanent regimen an instability in the method is produced because a new sensor has been added to the scenario or because a sensor has change the value of the biases.

In the next section is proposed a complete algorithm from the initialization to the permanent regimen. The described methods are used in order to exploit all the advantages and disadvantages in the initialization to get the best speed of convergence with the higher accuracy. This algorithm is focused principally in the initialization.

4.1.3.4 Proposal for Initialization Techniques

In this section a proposal for a complete bias estimation algorithm is presented. As it has been explained in the previous sections, each estimation method has different advantages and disadvantages. For the permanent regimen it is important the final covariance of the biases. Because of that the selected method for the permanent regimen is the temporal window method. With this method the final expected error is the lowest one between the presented algorithms.

This method has important instabilities in the initialization that can be solved reducing the speed of convergence or increasing the computational workload. None of these solution seems to be the best one and then it is proposed the initialization of the biases using another different method. The first approximation in order to reduce the instabilities of the initialization for this method is set the initial covariances of the filter with a low value. This produces a drastic reduction in the speed of convergence of the biases.

The second approximation is reduce the duration of the temporal windows. With this reduction the accuracy of the method is slightly reduced but the computational workload is increased. The solutions to this problems is the gradual increment of the temporal windows duration. In the first steps the temporal window used includes few plots (0.5 - 1 seconds). When

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the value of the biases have converge to a value (inaccurate) the temporal windows is increased (for example 5 seconds). This increment of the duration increases the accuracy and reduces the computational workload. Finally when the values have converge the temporal window is increased to a final value (30 seconds). This is a first technique for the initialization of the algorithm but even with that the instabilities of the algorithm can appear in the changes of the duration. In the next paragraph there is another technique that increases the stability of the algorithm without losing accuracy.

The pseudomeasurement filter give us the availability of the estimation in real time and this is an important feature for the scenarios used in ATM. If the correction due to the estimation is very big, the positions represented on the screen show a big discontinuity when they are estimated using temporal windows. The recursive filter estimates progressively the biases and the discontinuities are not noticeable.

The pseudomeasurement filter is a very good method when the values of the biases are not known, but in the first steps of the initialization some partial derivatives can be zero in the initials steps of the estimation and can appear some instabilities (section 4.1.3.1).

In order to solve this problem of the first estimation steps it is proposed to start the estimation using the Iterated WLS. Using this method with a set of the initial measurement a first estimation is get (if the biases are observable) and the instabilities of the next phases are avoided. The set of measurement cannot be big because the computational workload increases with the number of measurement and this algorithm is focused in online estimation methods.

Summarizing, a complete estimation methods is composed by three phases:

- The first phase uses the Iterated WLS to exploit the capabilities of this method. In this phase the instabilities due to the partial derivatives are avoided and an approximation to the real values is get. With this algorithm the observability can be tested using the complete measurement matrix. This algorithm is used with one pseudomeasurement from each trajectory.

- The second phase uses the pseudomeasurement method. In this phase a coarse value for the biases has been estimated and it is expected that the value of the estimation continue changing. The pseudomeasurement filter

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provides updates in real time and the discontinuities are not noticeable in the screen. This algorithm is used during 1-5 minutes.

- The final phase uses the temporal window method. This method gets very good covariance values and give us information about the observability. In this final phase is expected that the biases change slowly and the corrections at the ends of each temporal window are not noticeable in the operator screen.

In the real scenarios there are several causes that produces abrupt changes in the bias values. One of these causes is the maintenance of the radar and another one can be the strong gusts of wind in radar without radome [Cady, 2008]. When this abrupt changes are produced the covariance matrix in the third phase gets definite negative. This is an indicator of this abrupt change in the biases and a reinitialization can be done.

In the next section the results obtained using this procedure will be shown. The scenario used will be composed by several radars and the coverage will be big enough to consider the curvature of the Earth surface in the changes of coordinates.

4.1.3.5 Algorithm Performance

In this results section the algorithm presented in the previous chapter is evaluated. The data used for the presentation of the results obtained in this section is simulated using a scenario with real-like properties. The used scenario models the Earth Surface with the ellipsoid WGS84 and the coordinated projected over a horizontal plane cannot be approximated using Cartesian coordinates. As in other results sections, the coordinates change used are the explained in the appendix C.

The scenario is composed by 4 radars. The position of the sensors are arbitrary but with enough distance to ensure the observability of the biases (see section 4.2). In the scenario there will be always around 20 trajectories with arbitrary initial positions (uniformly distributed inside of the common coverage area). The radars and trajectories are allocated as it can be seen in figure 4.4. In the aforementioned figure the small black circles represent the position of radars and the big blue circles represent the coverage of each

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radar. The straight lines represent the position of the 20 initial trajectories and the cross from each trajectory represents the first point.

Figure 4.4: Scenario used in the simulation for the initialization techniques section

The radars simulated are SSR with a max range of 250nm. The values of biases are 100m for the range offset, 1m/km for the range gain and 0.5° for the azimuth offset. The standard deviation of the measurement noises is 75m for range and 0.1° for azimuth.

The generation of the scenarios has several particularities. The 20 trajectories are initialized at the same time in the positions (arbitrary) that are shown in figure 4.4. The trajectories are straight (great circles over the ellipsoid with constant altitude) with a duration of 300 seconds and a speed of 250 m/s. In some cases for the performance evaluation of the initialization 300seconds are enough but for the evaluation of the permanent convergence larger scenarios are required.

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In this scenario there are trajectories only in the common coverage areas. The trajectories measured just for one radar are not used in the bias estimation because the biases are not observable with a single sensor.

In order to get larger scenario the same trajectories will be used but in this case the initial position will be random in the proximity of the original trajectory and the initial time will be random around the ending time of the original trajectory.

The results shown in figures of this section represent the RMS error. In some figures the RMS error of the bias estimation is represented and in other figures the RMS error of the trajectories is represented.

In all the simulations the basic model of the biases is used. The basic model includes modeling of range offset and gain and azimuth offset ($\Delta\rho$, α_1 and $\Delta\theta$). The scope of this section is showing the accuracy of the initialization with different method but the accuracy of the estimation using a complete model as the presented in chapter 2 have been already evaluated.

The first presented results is the comparison between the pseudomeasurement filter and the temporal windows method. The temporal windows method has a very slow convergence in the beginning using big windows. In order to get reasonable values of convergence speed using this method from the beginning, the first steps will use short temporal windows (1 second) during several seconds and after that medium windows (5 seconds). Finally the standard temporal windows of 30 seconds will be used.

This first comparison is shown in figure 4.5. In the view of the results, the temporal windows method represented with the red line seems to be a worse method to estimate the biases always. The duration of this simulation is 300 seconds and in this time interval the biases have not converged. When the biases are not accurate the speed of convergence of this algorithm is very slow because of the instabilities of this method.

In the aforementioned figure, the pseudomeasurement filter seems to have better performance in all the phases of the simulations. In later results will be shown that in the permanent phase, the Root Mean Square (RMS) of the estimation is lower using the temporal windows method.

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(a)

(b)

(c)

Figure 4.5: Comparison of the initialization using the pseudomeasurement filter (blue line) and the temporal windows method (red line).

The performance is much better using the pseudomeasurement filter in

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Figure 4.6: Comparison of the track initialization in velocity and heading using the pseudomeasurement filter (blue line) and the Temporal Windows (red line).

the bias estimation and in the tracking accuracy performance. In figure 4.6 it can be seen that the errors produced by the biases in the initialization of the track are higher using the temporal windows method in the initialization. Using the pseudomeasurement filter in the initialization, the values of the positions and velocity are near to the ideal ones. Because of that seems clear that the temporal windows method is not valid to the initialization of the biases.

Viewing the previous results seems that the temporal windows method is less accurate than the pseudomeasurement method. As it has been described in previous sections this is produced because some of the assumptions made in the development of the filter are not fulfilled. In figure 4.7 there are represented two lines. One represents the bias estimation using always the

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(a)

(b)

(c)

Figure 4.7: Comparison of the initialization using the pseudomeasurement filter (blue line) and a hybrid configuration (red line). This hybrid configuration starts with the pseudomeasurement filter and ends with the temporal windows method.

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(a)

(b)

(c)

Figure 4.8: Comparison of the initialization using the pseudomeasurement filter (blue line) and the Iterated WLS (red line).

pseudomeasurement filter and the other one represents the bias estimation initializing the biases during 400 seconds and then using the temporal win-

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dows biases with windows of 30 seconds. In the view of the results presented in figure the accuracy in the permanent value of the biases is slightly better using the temporal windows method.

Figure 4.8 compares the initialization of the first steps using the Iterated WLS filter during the first seconds of the estimation and the same pseudomeasurement filter used in the previous results. In this figure it is shown that the accuracy in both algorithms is similar in the first steps.

The use of the Iterated WLS has not important advantages in the proposed scenario because in this case the speed of convergence is similar. Nevertheless the computational workload is a bit higher using the Iterated WLS. The main advantage of this method cannot be shown in this scenario because the used biases do not produce instabilities with the initialization of the values. The parameters as α_3 produces instabilities when the values of α_1 and α_2 are zeros. Using the basic model there is not any parameters with instabilities but using the complete model from chapter 2 the advantages of the Iterated WLS are noticeable.

4.2 Observability

In this section the observability of the scenarios is described from two different points of view. Firstly the observability is described from a geometrical point of view and after that the mathematical expression is described.

In the control theory the observability is a measure of the "uniqueness" of the estimation of the system internal parameters through the measures of their external outputs. This concept was introduced in [Kalman, 1959] and [Kalman, 1963]. In these research an explicit criteria for the complete controllability and observability was introduced.

From this research multitude of papers have been published studying the observability of the linear systems. Few years after this first research the same author published [Kalman, 1965] where he described the minimal realization. In a system controllable and observable, the minimal realization is the realization with the lower number of states that describes the system.

The research developed in [Moore, 1981] is a classical work where the

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controllability and observability based on Kalman minimal realization are described. In these years a lot of bibliography can be found as [Cobb, 1984], [Lin and Wonham, 1988], [Poirier, 1980]... In the classical literature, the observability for nonlinear systems has been studied in works as [Hermann and Krener, 1977] or [Lee et al., 1982].

In the view of the presented works the observability is a widely discussed topic studied since the 60s but it is not an old topic. In more recent years the observability has been studied in works as [Goshen-Meskin and Bar-Itzhack, 1992], [Bemporad et al., 2000], [Xu and Abur, 2004] and much more. The research about observability is focused in the estimation method applied to multitude of applications as the analysis of digital circuits [Goldstein, 1979], power systems [Baldwin et al., 1993] or bias estimation [Lin et al., 2003].

In more recent years, the observability of the estimation methods applied to different study field is being researched. In [Hesch et al., 2014] the observability is applied to a Vision-aided Inertial Navigation System and in [Bai and Awan, 2017] is applied to the estimation of relative position with bearing-only measurements.

In this thesis the observability for the ATM scenario is studied because the uniqueness in this kind of scenarios is not studied from the point of view of the sensor positions or the trajectory spatial distribution. In the next section the observability is applied to the ATM surveillance systems.

4.2.1 Geometric Meaning

The observability is a measure of the "uniqueness" of the estimation. In the next section this definition will be developed mathematically but in this section it will be described in a more visual way applied to ATM applications.

The bias of an ATM scenario can be estimated because of the diversity of the characteristics and locations of the sensors and the trajectories of the aircraft. One of the sensors with biases used in ATM are the radars that measure the positions of the aircraft in slant range, azimuth and altitude, this characteristic and the diversity in position of two radars allows the observability of the scenarios because the projection of the biases are not parallel

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in all the space of the scenario.

When the positions are measured directly in Cartesian coordinates (modeling the biases as an offset in Cartesian coordinates) the biases cannot be estimated. In figure 4.9 is represented a scenario with 2 aircraft in 2 different positions (measured by two different sensor in Cartesian coordinates). The position of the aircraft are unknown but the measures for the first aircraft are $Z_{11}(k)$ and $Z_{12}(k)$ from the first and the second sensor respectively ($Z_{21}(k)$ and $Z_{22}(k)$ are the measures of the second aircraft).

Figure 4.9: Representation of the observability in a scenario with Cartesian coordinates.

In figure are shown three corrected positions (X_{c1}, X_{c2} , X'_{c1}, X'_{c2} and X''_{c1}, X''_{c2}) for each aircraft using three different pairs of biases sets (B_1, B_2 , B'_1, B'_2 and B''_1, B''_2). The corrected positions with each pair of bias sets are coincident for each aircraft and there is not any way to know which is the correct solution because there are infinity pairs of biases that make equal the corrected positions. This is shown to illustrate that when a scenario is not observable, the uniqueness of the solution is not ensured. The introduction of fixed transponders could ensure observability in the scenarios.

Using sensors that measure the positions using radar coordinates make possible the observability of the scenarios. In figure 4.10 is represented a

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scenario with two radar and two aircraft. The examples will be made assuming that the measures are only affected by a range offset and by an azimuth offset. There are infinity pairs of bias sets that make equal the corrected position for one single aircraft because it is a underdetermined system.

Figure 4.10: Representation of the observability in a scenario with polar coordinates.

Using the scheme of figure 4.10, the corrected position $(x_{c,i,j}, y_{c,i,j})$ for the i -th measure from the j -th radar is:

$$\begin{aligned} x_{c,i,j} &= (\rho_{i,j} - \Delta\rho_j) \sin \theta_{i,j} - \Delta\theta_j + Xr_j \\ y_{c,i,j} &= (\rho_{i,j} - \Delta\rho_j) \cos \theta_{i,j} - \Delta\theta_j + Yr_j \end{aligned} \quad (4.14)$$

where Xr_j and Yr_j are the coordinates of the j -th radar. In order to get the values of the biases the corrected measures from 2 radars must be equal ($x_{c,i,1} = x_{c,i,2}$ and $y_{c,i,1} = y_{c,i,2}$), then:

$$\begin{aligned} (\rho_{1,1} - \Delta\rho_1) \sin \theta_{1,1} - \Delta\theta_1 + Xr_1 &= (\rho_{1,2} - \Delta\rho_2) \sin \theta_{1,2} - \Delta\theta_2 + Xr_2 \\ (\rho_{1,1} - \Delta\rho_1) \cos \theta_{1,1} - \Delta\theta_1 + Yr_1 &= (\rho_{1,2} - \Delta\rho_2) \cos \theta_{1,2} - \Delta\theta_2 + Yr_2 \end{aligned} \quad (4.15)$$

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In this equation there are four unknowns and two equations. In figure 4.11 it is represented the scenario corrected with arbitrary biases estimated for the first aircraft ignoring the second aircraft. For the sake of clarity in figure, just the azimuth offset labels are represented.

Figure 4.11: Representation of the scenario from figure 4.10 with the biases bad estimated (corrected positions in red) and with the good correction (corrected positions in black)

In order to get a unique solution, the equation system must be expanded adding more equations without expand the number of unknowns. With a second aircraft allocated in a different position, the equation system is:

$$\begin{aligned}
 (\rho_{1,1} - \Delta\rho_1) \sin(\theta_{1,1} - \Delta\theta_1) + Xr_1 &= (\rho_{1,2} - \Delta\rho_2) \sin(\theta_{1,2} - \Delta\theta_2) + Xr_2 \\
 (\rho_{1,1} - \Delta\rho_1) \cos(\theta_{1,1} - \Delta\theta_1) + Yr_1 &= (\rho_{1,2} - \Delta\rho_2) \cos(\theta_{1,2} - \Delta\theta_2) + Yr_2 \\
 (\rho_{2,1} - \Delta\rho_1) \sin(\theta_{2,1} - \Delta\theta_1) + Xr_1 &= (\rho_{2,2} - \Delta\rho_2) \sin(\theta_{2,2} - \Delta\theta_2) + Xr_2 \\
 (\rho_{2,1} - \Delta\rho_1) \cos(\theta_{2,1} - \Delta\theta_1) + Yr_1 &= (\rho_{2,2} - \Delta\rho_2) \cos(\theta_{2,2} - \Delta\theta_2) + Yr_2
 \end{aligned}
 \tag{4.16}$$

Adding a second position (with range and azimuth different the first measure) the system has four equation with four unknowns. And then, the corrected positions of all the measurements will be coincident and the biases will be well estimated.

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4.2.2 Analytic Expression for the Observability

In this section the observability will be described in a more general and analytic way. In the literature described in section 4.2 the observability is studied in linear systems described with the state and measure models:

$$\begin{aligned} X_{k+1} &= \phi X_k + u \\ Y_k &= H_k X_k + w \end{aligned} \quad (4.17)$$

where u and w are independent noise processes with zero mean. In order to simplify the description of the next equations the noise term (w) and plant noise term (u) will be ignored. The previous system can be particularized for each measurement time instant:

$$\begin{aligned} Y_0 &= H_0 X_0 \\ Y_1 &= H_1 X_1 = H_1 \phi X_0 \\ Y_2 &= H_2 X_2 = H_2 \phi^2 X_0 \\ &\vdots \\ Y_n &= H_n X_n = H_n \phi^{n-1} X_0 \end{aligned} \quad (4.18)$$

Writing the previous equation as a matrix:

$$\begin{bmatrix} Y_0 \\ Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} H_0 \\ H_1 \phi \\ H_2 \phi^2 \\ \vdots \\ H_n \phi^{n-1} \end{bmatrix} X_0 \quad (4.19)$$

In the bias estimation, the biases are considered static and the transition matrix is the identity matrix ($\phi_k = I$). Resulting this expression in:

$$Y = H X \quad (4.20)$$

As the matrix H is not square, the solution for this system is solved using the pseudoinverse matrix:

$$X = (H^T H)^{-1} H^T Y \quad (4.21)$$

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This system is observable if the matrix $(H^T H)$ is regular (has inverse). The measurement vector (Y) is a vector composed by all the pseudomeasurement ($Z(k)$) calculated using the equation 3.24. The state vector (X) is a column vector composed by the biases of the sensors and the measurement matrix (H) is the matrix composed by the projection matrix of the biases ($B_i(k)$).

The system formed with the measures of the scenario presented in figure 4.10 is:

$$\begin{aligned}
 \Delta\rho_1 \frac{\delta x_1}{\delta\Delta\rho_1} + \Delta\theta_1 \frac{\delta x_1}{\delta\Delta\theta_1} - \Delta\rho_2 \frac{\delta x_1}{\delta\Delta\rho_2} - \Delta\theta_2 \frac{\delta x_1}{\delta\Delta\theta_2} &= x_{1,1} - x_{1,2} \\
 \Delta\rho_1 \frac{\delta y_1}{\delta\Delta\rho_1} + \Delta\theta_1 \frac{\delta y_1}{\delta\Delta\theta_1} - \Delta\rho_2 \frac{\delta y_1}{\delta\Delta\rho_2} - \Delta\theta_2 \frac{\delta y_1}{\delta\Delta\theta_2} &= y_{1,1} - y_{1,2} \\
 \Delta\rho_1 \frac{\delta x_2}{\delta\Delta\rho_1} + \Delta\theta_1 \frac{\delta x_2}{\delta\Delta\theta_1} - \Delta\rho_2 \frac{\delta x_2}{\delta\Delta\rho_2} - \Delta\theta_2 \frac{\delta x_2}{\delta\Delta\theta_2} &= x_{2,1} - x_{2,2} \\
 \Delta\rho_1 \frac{\delta y_2}{\delta\Delta\rho_1} + \Delta\theta_1 \frac{\delta y_2}{\delta\Delta\theta_1} - \Delta\rho_2 \frac{\delta y_2}{\delta\Delta\rho_2} - \Delta\theta_2 \frac{\delta y_2}{\delta\Delta\theta_2} &= y_{2,1} - y_{2,2}
 \end{aligned} \tag{4.22}$$

This equations can be expressed as a matrix:

$$\begin{bmatrix} x_{1,1} - x_{1,2} \\ y_{1,1} - y_{1,2} \\ x_{2,1} - x_{2,2} \\ y_{2,1} - y_{2,2} \end{bmatrix} = \begin{bmatrix} \frac{\delta x_1}{\delta\Delta\rho_1} & \frac{\delta x_1}{\delta\Delta\theta_1} & -\frac{\delta x_1}{\delta\Delta\rho_2} & -\frac{\delta x_1}{\delta\Delta\theta_2} \\ \frac{\delta y_1}{\delta\Delta\rho_1} & \frac{\delta y_1}{\delta\Delta\theta_1} & -\frac{\delta y_1}{\delta\Delta\rho_2} & -\frac{\delta y_1}{\delta\Delta\theta_2} \\ \frac{\delta x_2}{\delta\Delta\rho_1} & \frac{\delta x_2}{\delta\Delta\theta_1} & -\frac{\delta x_2}{\delta\Delta\rho_2} & -\frac{\delta x_2}{\delta\Delta\theta_2} \\ \frac{\delta y_2}{\delta\Delta\rho_1} & \frac{\delta y_2}{\delta\Delta\theta_1} & -\frac{\delta y_2}{\delta\Delta\rho_2} & -\frac{\delta y_2}{\delta\Delta\theta_2} \end{bmatrix} \begin{bmatrix} \Delta\rho_1 \\ \Delta\theta_1 \\ \Delta\rho_2 \\ \Delta\theta_2 \end{bmatrix} \tag{4.23}$$

The problem of the bias estimation is nonlinear and the measurement matrix (H) defined in the classical estimation problem is composed by the partial derivatives of the bias model in the measured position (Jacobian matrix). In the simple example of figure 4.10 the inverse of the matrix is ensured when the radar and the trajectories are allocated in different positions but for more complex scenario with all the biases parameters presented in chapter 2 the conditions for the analysis of the singularity of the matrix are more critical.

4.2.3 Numerical Consideration for Observability

In theory the singularity of the matrix $(H^T H)$ is easy to determine. If the determinant of a matrix is equal to zero, the matrix is singular, it has not inverse and the system is not observable. When the measures are not affected by the noise the determinant of the matrix can be evaluated as zero

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(not observable scenario) or different to zero (observable scenario).

In real application where the noise is present, there will never be several equal measures and the determinant will never be exactly zero in non observable situations. In these cases the condition number [Golub and Van Loan, 2012] is very high, but the determinant is not zero.

As it is indicated in [Cline et al., 1979] the sensitivity of the solution of a singular system is directly related to the condition number. In [Vu and Tao, 2007] is studied the condition number in matrix in presence of random terms.

The condition number of a matrix is the coefficient between the maximum and the minimum eigenvalue (in absolute value). When one of the eigenvalues is zero, the determinant of the matrix ($H^T H$) is zero and the matrix has not inverse. In bias estimation the analyzed matrix depends on the measured range and azimuth. As the range and azimuth are affected by noise, is almost impossible that one eigenvalue is zero. In not observable scenarios the condition number is not infinity but it is a very big number.

One of the solution to this problem is to set a threshold to decide if a matrix is regular or singular but in some cases with complex models there are problems with the numerical precision. In an ideal scenario all the parameter are in the same magnitude order but this is not always accomplished. In order to explain the numerical problem will be used the model of the range bias described in section 2.2.3 in the equation 2.55:

$$\Delta\rho = \Delta\rho_0 + (\alpha_1\rho + \alpha_2\rho^2) \left[1 + \alpha_3 \left(1 - \frac{h}{h_r} \right) \right] \quad (4.24)$$

In this equation there are two parameters that have a very different magnitude order. Let's assume that in a scenario the bias parameters have the next values:

$$\begin{aligned} \Delta\rho_0 &\approx 100 \text{ m} \\ \alpha_1 &\approx 10^{-3} \text{ m/m} \\ \alpha_2 &\approx 10^{-9} \text{ m/m}^2 \\ \alpha_3 &\approx 1 \end{aligned} \quad (4.25)$$

In this example the attention will be focused in the parameters $\Delta\rho_0$ and

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α_2 . A synthetic projection matrix (H') formed just with this two parameters is:

$$H'_i = \begin{pmatrix} \frac{\delta x_i}{\delta \Delta \rho_{0,1}} & \frac{\delta x_i}{\delta \Delta \alpha_{2,1}} & \frac{\delta x_i}{\delta \Delta \rho_{0,2}} & \frac{\delta x_i}{\delta \Delta \alpha_{2,2}} \\ \frac{\delta y_i}{\delta \Delta \rho_{0,1}} & \frac{\delta y_i}{\delta \Delta \alpha_{2,1}} & \frac{\delta y_i}{\delta \Delta \rho_{0,2}} & \frac{\delta y_i}{\delta \Delta \alpha_{2,2}} \end{pmatrix} \quad (4.26)$$

For the sake of simplicity the altitude is equal to the reference altitude and then the α_3 term does not affect to the derivatives, then the partial derivatives are:

$$\begin{aligned} \frac{\delta x}{\delta \Delta \rho_0} &= \cos(\theta_m) \\ \frac{\delta y}{\delta \Delta \rho_0} &= -\sin(\theta_m) \\ \frac{\delta x}{\delta \Delta \alpha_2} &= \rho_m^2 \cos(\theta_m) \\ \frac{\delta y}{\delta \Delta \alpha_2} &= -\rho_m^2 \sin(\theta_m) \end{aligned} \quad (4.27)$$

As there are four parameter to determine, we need the measures of two aircraft in two different positions. In this synthetic scenario the sensors are far enough and the trajectories are in different locations. The measures for both aircraft from both sensors are:

<i>aircraft number</i>	(ρ_{m1}, θ_{m1})	(ρ_{m2}, θ_{m2})	
1	$(101.98km, 78.68^\circ)$	$(20km, 0^\circ)$	(4.28)
2	$(70.71km, 135^\circ)$	$(70.71km, -135^\circ)$	

With these values, the composed projection matrix with both measures is:

$$H' = \begin{pmatrix} 0.1963 & 2.04 \cdot 10^9 & 1 & 4 \cdot 10^8 \\ 0.9805 & 1.01 \cdot 10^{10} & 0 & 0 \\ -0.707 & -3.53 \cdot 10^9 & -0.707 & -3.53 \cdot 10^9 \\ 0.707 & 3.53 \cdot 10^9 & -0.707 & -3.53 \cdot 10^9 \end{pmatrix} \quad (4.29)$$

The determinant of the matrix ($H'^T H'$) is $-5.71 \cdot 10^{38}$ and the matrix has inverse. But as the difference between some values is very different the condition number of this matrix is $6.16 \cdot 10^{20}$. As this condition number is much bigger than 1 then this matrix is bad conditioned and the solutions are unstable and inaccurate.

The solution to solve this problem is the estimation of alternative param-

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eters. As the parameters $\Delta\rho_0$ and α_2 are too different, the parameters $\Delta\rho'_0$ and α'_2 are proposed where:

$$\begin{aligned}\Delta\rho'_0 &= \Delta\rho_0 \cdot 10^{-3} \\ \alpha'_2 &= \alpha_2 \cdot 10^6\end{aligned}\tag{4.30}$$

The parameters has been escalated to get more similar values. In this case the partial derivatives with respect to the previous values are:

$$\begin{aligned}\frac{\delta x}{\delta \Delta\rho'_0} &= \frac{\delta x}{\delta \Delta\rho_0} \cdot 10^3 \\ \frac{\delta y}{\delta \Delta\rho'_0} &= \frac{\delta y}{\delta \Delta\rho_0} \cdot 10^3 \\ \frac{\delta x}{\delta \Delta\alpha'_2} &= \frac{\delta x}{\delta \Delta\alpha_2} \cdot 10^{-6} \\ \frac{\delta y}{\delta \Delta\alpha'_2} &= \frac{\delta y}{\delta \Delta\alpha_2} \cdot 10^{-6}\end{aligned}\tag{4.31}$$

And the matrix expressed in the equation 4.29 is now:

$$H' = \begin{pmatrix} 0.1963 \cdot 10^3 & 2.04 \cdot 10^3 & 1 \cdot 10^3 & 4 \cdot 10^2 \\ 0.9805 \cdot 10^3 & 1.01 \cdot 10^4 & 0 & 0 \\ -0.707 \cdot 10^3 & -3.53 \cdot 10^3 & -0.707 \cdot 10^3 & -3.53 \cdot 10^3 \\ 0.707 \cdot 10^3 & 3.53 \cdot 10^3 & -0.707 \cdot 10^3 & -3.53 \cdot 10^3 \end{pmatrix}\tag{4.32}$$

Using this technique the condition number is reduced to $6.33 \cdot 10^2$. Comparing the condition number with the parameter (without the escalate) with this condition number, matrix with the parameters escalated is much more well-conditioned.

In [Mohinder and Angus, 2001] there are described the effects of the roundoff errors and some solutions are presented to deal with these numerical problems. The "Square-Root" filtering [Bellantoni and Dodge, 1967], [Kaminski et al., 1971] reduces the condition number due to the square root of the covariances is used. The numerical considerations are studied too in [Verhaegen and Van Dooren, 1986] and it is described the Square-Root Information filter. In [Grewal and Kain, 2010] is described a sigma-rho implementation for the Kalman filter that increases the stability over the traditional implementations.

4.2.4 Proposal for Online Observability Test

In the previous section the observability of a system has been described. As it has been described above, the observability depends on the number and the position of the aircraft and on the positions of the sensors.

There are a lot of factors that affect to the observability of a scenario. The scenarios used in the previous examples are composed by a pair of radar, but the real scenarios are composed by a bigger number of sensors.

As in real scenarios the aircraft are changing the positions and the number of them is changing too, in this section is proposed a method to determine the observability of a scenario in real time. Firstly the problem will be described, secondly a test to get the observability information is proposed and finally a solution to deal with unobservable scenarios is proposed. This test is designed to detect if two radars are too near and a solution in this case is proposed.

4.2.4.1 Problem description

The biases and the scenarios change in function of time. The conditions that are fulfilled in a time instant can be unfulfilled at the next time instant. The bias estimation need some conditions for the functioning of the algorithms. The most important condition is the diversity in position of the aircraft and the diversity in position of the sensors to get absolute values for the biases.

The biases of two sensors are estimated with the aircraft that are in the overlapped coverage area between them. If two radars have not overlapped coverage, their biases cannot be estimated but this statement is not always true. In a scenario where the radar A share coverage with radar B and radar B share coverage with radar C, but radars A and C do not share coverage, the biases of all radars can be estimated (if the observability conditions are fulfilled with the positions of the aircraft). In figure 4.12 is shown a scenario with these characteristics.

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Figure 4.12: Representation of a scenario where the radar A and C without overlapped coverage are observable

As it has been described in the previous sections, the biases are not observable when the sensors are allocated in the same position (even when the measurement process is made in polar coordinates). This statement is true when the scenario is composed by 2 radars, but when the scenario has another radar with common coverage the observability can be ensured. This scenario is shown in figure 4.13. When there is another kind of sensors as ADS-B or WAM or there is fix transponders the observability can be ensured too.

In a big scenario, and under certain conditions, the radars can be divided in groups with separate coverages. In this conditions each group of radars must be evaluated separately to get the observability of each group and give different solutions. In figure 4.14 is shown a scenario with theoretical overlapped coverages that links all the radar in the same group, but if there

Figure 4.13: The radar pair A and B is not observable because they are allocated almost in the same position, but the radar C ensure the observability conditions

are not aircraft between the radar C and D this scenario is divided in two separated groups.

In this section the problem has been described showing that the observability conditions of a scenario can change in function of time due to the aircraft positions or due to the radars maintenance. In the next section there will be presented a method to determine the groups of radars and the observability of each group.

4.2.4.2 Groups Determination

The first step to determine the radar that belong to the same group is to determine the radars pairs. There are several ways to determine these pairs. The observability is determined in temporal windows. And then the radar pairs must be determined dynamically. The groups of radar can be determined with clustering methods as the described in [Mirkin, 1998]. In

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Figure 4.14: This scenario can be divided in two group if there are not aircraft with common coverage between radars C and D

this section a simple algorithm will be described.

The easiest way to determine the radar pairs is take the measurements received inside the last time window and evaluate each trajectory separately. If a trajectory is measured by two radar then both radars have overlapped coverage and they form a pair of radars. This method can be computational inefficient due to the number of trajectories can be big. Because of that another method based in covariance matrix is proposed. This method is more efficient because it is based in matrix that are already calculated.

The permanent regimen estimation of the biases is made with the temporal windows method (described in section 3.3.1). In this method the accumulated covariance matrix of the residual V (equation B.18) is estimated. This matrix has the structure:

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$$V = \begin{pmatrix} V_{1,1} & \cdots & V_{1,i} & \cdots & V_{1,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ V_{i,1} & \cdots & V_{i,i} & \cdots & V_{i,N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ V_{N,1} & \cdots & V_{N,i} & \cdots & V_{N,N} \end{pmatrix} \quad (4.33)$$

In this equation the sub-matrix $V_{i,j}$ are the cross-covariances between radars when $i \neq j$. If the sub-matrix $V_{i,j}$ is zero, then that means that both radars have not overlapped coverage. Comparing the sub-matrix that are different to zero in the row i -th, it is get the radar pairs of the i -th radar. In figure 4.14 it is represented a scenario with all the overlapped coverages. In this scenario there are multitude of possibilities. The first described case is when there are trajectories uniformly distributed and then, the matrix V has the next structure:

$$V = \begin{pmatrix} V_{A,A} & V_{A,B} & V_{A,C} & 0 & 0 & 0 \\ V_{A,B} & V_{B,B} & V_{B,C} & 0 & 0 & 0 \\ V_{A,C} & V_{B,C} & V_{C,C} & V_{C,D} & 0 & 0 \\ 0 & 0 & V_{C,D} & V_{D,D} & V_{D,E} & V_{D,F} \\ 0 & 0 & 0 & V_{D,E} & V_{E,E} & V_{E,F} \\ 0 & 0 & 0 & V_{D,F} & V_{E,F} & V_{F,F} \end{pmatrix} \quad (4.34)$$

In this matrix it can be seen that all the radar are in the same group. If the conditions of the scenario change and the trajectories that are in the common coverage between radar C and D disappear, the resultant matrix is:

$$V = \begin{pmatrix} V_{A,A} & V_{A,B} & V_{A,C} & 0 & 0 & 0 \\ V_{A,B} & V_{B,B} & V_{B,C} & 0 & 0 & 0 \\ V_{A,C} & V_{B,C} & V_{C,C} & 0 & 0 & 0 \\ 0 & 0 & 0 & V_{D,D} & V_{D,E} & V_{D,F} \\ 0 & 0 & 0 & V_{D,E} & V_{E,E} & V_{E,F} \\ 0 & 0 & 0 & V_{D,F} & V_{E,F} & V_{F,F} \end{pmatrix} \quad (4.35)$$

In this case there are two separated groups. The following is an algorithm to determine the groups of radars using the pairs obtained from the matrix V .

For this algorithm just the right triangular matrix of V because this

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matrix is symmetrical. The first step is create a list with the group of each radar. The initialization of this list is determine that every radar is not in any group.

Each row of the matrix represent a radar, and each column represent its pairs. The pseudo code of this algorithm is described in algorithm 1. Using the algorithm presented in 1 the radars are divided in groups. The first matrix V of the examples gives one single group and the second matrix gives two groups.

Algorithm 1 Grouping Pairs Algorithm

```

1: procedure GROUPINGPAIRS
2:   Create void list with radar groups
3:   for  $i$  From 1 To rows number do
4:     for  $j$  From i To columns number do
5:       if  $group(i) == group(j) == 0$  then
6:         Create new group
7:         Add radar i and j to this new group
8:       else if  $group(i) == 0$  then
9:         Add radar i to the group of radar j
10:      else if  $group(j) == 0$  then
11:        Add radar j to the group of radar i
12:      else if  $group(j) \neq group(i)$  then
13:        Join group of radar i with group of radar j

```

In the matrix of the second example, the groups are compact. In figure 4.15 is shown a scenario with two different groups. The first one composed by three radars in different locations and the second one composed by two radar almost in the same location. The resultant matrix V for this scenario is:

$$V = \begin{pmatrix} V_{A,A} & V_{A,B} & 0 & V_{A,D} & 0 \\ V_{A,B} & V_{B,B} & 0 & V_{B,D} & 0 \\ 0 & 0 & V_{C,C} & 0 & V_{C,E} \\ V_{A,D} & V_{B,D} & 0 & V_{D,D} & 0 \\ 0 & 0 & V_{C,E} & 0 & V_{E,E} \end{pmatrix} \quad (4.36)$$

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Figure 4.15: Scenario with two groups of radar one observable group and one unobservable group.

The observability of this scenario is determined when the determinant of this matrix is different to zero. In the matrix of the example the determinant is very near to zero, because the group of radars C and E is not observable. In order to determine the observability of the matrix V is rearrange to V' :

$$V' = \begin{pmatrix} V_{A,A} & V_{A,B} & V_{A,D} & 0 & 0 \\ V_{A,B} & V_{B,B} & V_{B,D} & 0 & 0 \\ V_{A,D} & V_{B,D} & V_{D,D} & 0 & 0 \\ 0 & 0 & 0 & V_{C,C} & V_{C,E} \\ 0 & 0 & 0 & V_{C,E} & V_{E,E} \end{pmatrix} = \begin{pmatrix} V_{G1} & 0 \\ 0 & V_{G2} \end{pmatrix} \quad (4.37)$$

The observability of the scenario can be determined evaluating the block matrix resultant from the rearranging of the original matrix. Using the Leibniz formula [Trefethen and Bau III, 1997] can be demonstrated that:

$$\det(V') = \det(V_{G1})\det(V_{G2}) \quad (4.38)$$

and then, the global observability does not change. In order to determine the observability of the groups, the determinant of each group must be

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evaluated independently.

4.2.4.3 Observability Test

The windows method proposed in section 3.3.1 updates the covariance using the matrix V (equation B.18) that is used to determine the groups of radars. In parallel with this method another matrix test must be calculated. In the appendix B the calculated matrices are:

$$V_{i,k} = [H^* B_i - \bar{F}_t]^T E_{i,k}^{-1} [H^* B_i - \bar{F}_t] = M^t E^{-1} M \quad (4.39)$$

The dimension of the matrix M is 2 rows and the columns are the number of biases multiplied by the number of radars and it is composed by the sub matrices of each radar where N_{fuente} is the sensor that take the last measure:

$$M = \begin{bmatrix} M_1 & \cdots & M_{N_{fuente}} & \cdots & M_{N_{TotalSensors}} \end{bmatrix} \quad (4.40)$$

The submatrices are calculated within the algorithm of the temporal windows. In an unobservable group the contributions from each radars to the matrix M are similar but the trajectories that measure can be different and the contribution for the covariance matrix will be bigger with more matrix. Because of that it is proposed a modification for the matrix M . The radar that has made the last measure contributes more than the others to the matrix V . In order to get a uniform contribution the submatrix correspondent to the measure is normalized by the times that is added to the average. For the observability test, the number of radars ($N_{Sensors}$) that measure a trajectory must be known. A modification for the matrix M is proposed in order to calculate the test matrix:

$$M' = \begin{bmatrix} M_1 & \cdots & \frac{M_{N_{fuente}}}{N_{Sensors}-1} & \cdots & M_{N_{TotalSensors}} \end{bmatrix} \quad (4.41)$$

Using this modified matrix the matrix W is calculated with a similar equation B.18 that has been written above:

$$W = \sum_{i=1}^n \sum_{k \in (t-1, t]} W_{i,k} \quad (4.42)$$

$$W_{i,k} = |M'^T E^{-1} M'|$$

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where the operator $|\cdot|$ represent the absolute value of each term of the matrix. The absolute value is calculated in order to avoid that the contributions positive and negative produced by the trajectories make null (or with very little values) the final submatrices. When the modified matrix W has been calculated the test is done using the information of the groups. The structure for the groups is equal to the matrix V and both can be used to determine the groups of radars. Then in this case the resultant matrix of the previous example has the structure:

$$W' = \begin{pmatrix} W_{A,A} & W_{A,B} & W_{A,D} & 0 & 0 \\ W_{A,B} & W_{B,B} & W_{B,D} & 0 & 0 \\ W_{A,D} & W_{B,D} & W_{D,D} & 0 & 0 \\ 0 & 0 & 0 & W_{C,C} & W_{C,E} \\ 0 & 0 & 0 & W_{C,E} & W_{E,E} \end{pmatrix} = \begin{pmatrix} W_{G1} & 0 \\ 0 & W_{G2} \end{pmatrix} \quad (4.43)$$

Algorithm 2 Observability Test

```

1: procedure OBSERVABILITYTEST
2:   for  $i$  From 1 To Number groups do
3:      $n = \text{First sensor of the group}$ 
4:     for  $j$  From 2 To number of sensor of the group do
5:       Calculate  $D$  as the division term to term between  $W_{n,n}$  and  $W_{n,j}$ 
6:       Calculate  $\hat{D}$  as the mean of the values of  $D$ 
7:       if  $\hat{D} > 1.1 || \hat{D} < 0.9$  then
8:         The group is observable
9:       if All the means of the divisions are approx 1 then
10:        The group is unobservable

```

In this case there are two different groups the test must be done two times. For each group the first submatrix of the principal diagonal of the group ($W_{n,n}$) is selected as reference. This matrix is divided term to term with the cross matrices of the same group ($W_{n,j}$). The mean of the calculated values is calculated. If all the means of the values is around 1 ($\pm 10\%$) then the group is not observable. If one of the matrices is different to 1 then the group is observable. The pseudocode for this algorithm is in Algorithm 2.

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4.2.4.4 Dealing with Unobservable Groups

This test is designed to detect if two radars are near and it produces instabilities in the observation conditions of the scenario. In the previous section the observability test has been described. The way to deal with the scenarios where two radars are near is to establish one of the sensor as the reference for the others.

In a sensor network the sensor used has different performances and there are some of them with higher accuracy. The sensors must be ordered in order of preference for the use one before the others.

In an iteration of the bias filter the biases of the sensors are estimated. In the previous example shown in figure 4.15 the radars C and E from an unobservable group. In this case the radar C has higher performance than the radar E and it is taken as the reference. In the iteration the biases (b_t) has been calculated. As the radar C is the reference, the modification of the biases in this last is calculated as:

$$\Delta b_C = b_{C,t} - b_{C,t-1} \quad (4.44)$$

With the calculated modification for the reference sensor, the estimated biases of the sensors that belong to the group are corrected. In this case the sensor C and E.

$$\begin{aligned} b_{C,t} &= b_{C,t} - \Delta b_C \\ b_{E,t} &= b_{E,t} - \Delta b_C \end{aligned} \quad (4.45)$$

Chapter 5

Extension of the Meteorological Model and Estimation Methods for Target Dependent Biases

In this chapter there are proposed three improvements to the current methods and models used in ATM: The first is an extension to the meteorological model proposed in chapter 2. This extension uses surfaces for the interpolation of the temperature and pressure biases in a specific position based on a grid of fixed points. The second proposal is a modification for a method that estimates the target-dependent biases in offline applications. The last proposal includes a method to the estimation of the velocity (target dependent) biases sent by the aircraft.

Currently one single parameter for all the covered area is used for each bias of the meteorological model (one for pressure and one for temperature). This approximation is good enough for reduced scenarios but for wide scenarios the pressure and temperature are not constants. The change of the conditions with the positions are modeled with surfaces constrained to a grid of fixed points. The values of the biases are estimated in the points of the grid and they are used for the interpolation in the measurement point.

In section 5.2 the modification for the method proposed in [Besada et al., 2012] is described. In this method the estimation of the target-dependent and sensor dependent biases is done in a three stages procedure. The pro-

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posed modification fuses the last two steps in order to estimated jointly both the sensor and target dependent biases. This modification reduces the dependence of the estimation with the mean of the biases. In the previous chapters the target dependent biases have been neglected.

Finally a method to determine the velocity biases of the aircraft is presented. This biases affect to the estimation of the trajectory and are different for each aircraft. This estimation is done augmenting the parameters of the state vector.

5.1 Extension of the Meteorological Model

Tracking systems cover wide areas, usually the extension is wider than a country. In these cases the assumption of constant atmosphere conditions is not enough for the atmosphere modeling. In previous chapters the altitude bias is composed by two terms. The first one is a constant altitude bias that depends on the pressure of the atmosphere. The second one is an altitude dependent term that depends on the temperature of the atmosphere. In the estimation methods used previously, the pressure and temperature terms is estimated as a constant but in larges scenarios this approximation can be improved.

In this section it is proposed an approximation with two surfaces, one for the pressure and one for the temperature at Mean Sea Level (MSL). Each surface will be defined by the values of pressure and temperature in a grid of fixed points. The improvement of this approximation will be evaluated comparing the estimation with the meteorological model using a single pressure and temperature. For the evaluation of this problem a typical pressure and temperature map will be used for the region of Spain. Both maps (pressure and temperature) correspond to December of 2017, have been obtained from the Agencia Española de METeorología (AEMET) and are shown in figure 5.1.

In the pressure map shown in figure 5.1 (a) the pressure is indicated in mBar. For ATM applications, the pressure bias is used for the correction of

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(a)

(b)

Figure 5.1: (a) Pressure map (b) Temperature map. Both get from AEMET.

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the aircraft altitude in the proximity of the airport but is not a visual parameter. Using the formula indicated in [Poles, 2010] the pressure is converted to meters.

$$H_{P,MSL} = \frac{T_0}{\beta_T} \left[\left(\frac{p_{MSL}}{p_0} \right)^{-\left(\frac{\beta_T R}{g_0} - 1 \right)} \right] \quad (5.1)$$

For the presented scenario over Spain the altitude measure have a pressure bias between 30 and 85 meters at MSL. This scenario has typical values in this area, with small differences between the maximum and minimum values. Pressure bias is the barometric altitude that is indicated when the geometric altitude of the aircraft is at the mean sea level. In different areas of the same scenario as the UK area, the pressure bias is between -80 and -260 meters at MSL. In the UK area using a single value is not accurate enough to the estimation of the biases and the use of surfaces seems to be justified.

Using only one value for the estimation of all the biases, the pressure can be well estimated if the pressure gradient is low in all the coverage of the scenario. For the rest of the cases the use of a surface will fit more accurately than with a single value.

In the presented scenario (Spain), if the estimation is the median of the pressure values, the error of the estimation is bigger than 25 meters and this is an acceptable error (the projection of 25 meters over the horizontal is low). In scenarios where there are strong low-pressure (as in UK in figure 5.1) or high-pressure areas the difference between the maximum and minimum values will be bigger. In this cases the error will have important differences in the projection.

In the next section some methods are analyzed to the calculation of the surfaces used for the pressure and temperature interpolation.

5.1.1 Interpolation methods

The proposed method in this section is based in the interpolation of the surface for the atmospheric conditions in any position of the scenario using a grid of values in fixed positions. In order to get the values in any position of the scenario, the pressure and temperature is interpolated using the values

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of the fixed points.

There are multitude of interpolation methods for the creation of 2D surfaces. In the next sections the convenience of some of these methods will be discussed.

Nearest-Neighbor This approximation is the easiest and most computational efficient method. The interpolation is made approximating the value in the selected point equaling it to the nearer fixed point. For the estimation of the pressure and the temperature, this is equivalent to divide the scenario in N different areas and where N is the number of points of the grid. With this method the error in the altitude will be lower than using just one value. But in figure 5.2 it can be seen that this approximation does not fit with a realistic scenario.

Figure 5.2: Example of Nearest-Neighbor interpolation

Bilinear interpolation In this interpolation the fixed points must be distributed in a regular grid. Figure 5.3 shows the points used in the calculation of the approximation. The bilinear interpolation is made doing two linear interpolations, first in one axis and finally in the other axis. In this

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approximation the abrupt discontinuities of the Nearest-Neighbor approximation has disappeared but the shape of the interpolation does not fit with the curvature of the models as it can be seen in figure 5.4.

Figure 5.3: Calculation method for bilinear interpolation

Bicubic interpolation In this interpolation the fixed points must be distributed in a regular grid as in the previous case. In terms of accuracy the bicubic interpolation seems to be the better approximation for our needs (between the more simple interpolations). As it can be seen in figure 5.5 the shape of the surface fits very well with an expected pressure curve.

This interpolation method has several disadvantages. The first disadvantage is the number of points used for the interpolation. In the bilinear interpolation just the four points that surround the estimated point are used. But in this case the number of points used in the interpolation is 9, three

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Figure 5.4: Example for bilinear interpolation

Figure 5.5: Example for bicubic interpolation

in each axis. This effect causes an increment in the computational load of the algorithm and the decrement of the method accuracy, because with one single measure the algorithm updates 9 parameters.

The second disadvantage is the lack of freedom in the grid setup. Both

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bilinear and bicubic interpolation use a regular grid and it is an advantage for the complexity of the algorithm and for the computational load. That is because in a rectangular grid, it is trivial to get the points that surround another point. The disadvantage in this case is that with this interpolation we cannot introduce data in known points. For example, in the airports, the pressure at mean sea level is indicated and it can be used as a fixed transponder in the tracking systems.

B-Splines There are multitude of interpolation methods more complex than the previously presented as the Basic splines (B-splines) but these methods are developed for different application as the 3D modeling of objects. These methods usually use a lot of surrounding point and the projection over the horizontal plane for the estimation is very complex. For some of the applications this spline generation fits very well because of its characteristics, but for our application this has some disadvantages. One of them is that for some control points, the value of the spline can be different to the value of the point. Other disadvantage is using the same control points the surface can have several different shapes changing a configuration parameter in the control points. The introduction of new parameters makes more complex the estimation of the biases.

Inverse Distance Weighting (IDW) The Inverse Distance Weighting (IDW) is a multivariate interpolation method that use a grid of known points. In this case we will use a 2D interpolation to calculate the pressure and the temperature in any point using a weighted average of the control points based on the distance.

This is the selected interpolation method for the pressure and temperature estimation. This interpolation method has some configurable parameter as the number of points used in the interpolation or the exponent of the weights of each points. In the equations corresponding to this interpolation (equations 5.3 and 5.5) the number of points is the variable N used in the summation and the exponent is the n that modifies the weights of each control point. In figure 5.6 the interpolated surface is interpolated using several exponent are used to the surface representation. For low exponents (*exponent* ≈ 0) the interpolation is a linear interpolation and for high expo-

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nents ($exponent = \infty$) the interpolation is similar to the nearest-neighbor. Lower is the exponent, softer is the slope of the curves.

Figure 5.6: Example for IDW interpolation using different exponents in the calculation of the weights

The pros and cons of this interpolation method will be discussed in the next sections.

5.1.2 Modifications to the Bias Estimation Method

In the next sections the interpolation method will be described and after that the adaptation of this interpolation for the estimation method will be introduced. Finally some results applying this modification are shown.

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5.1.2.1 Interpolation

The IDW interpolation calculates the value in any point using a weighted average from a set of control points. The set of control points has known values and positions. The weights of each control point can be modified using an exponential parameter that will be explained below.

Figure 5.7: Calculation example for IDW showing the control points and distances.

In figure 5.7 is shown a scenario with a grid of control points. In this figure P_i are the control points of the grid for $i = 1 \dots M$ where M is the number of control points of the scenario. $\hat{p}$ is the position where the interpolation is calculated. For each control point, d_i describes the distance between the i -th control point (P_i) and the interpolated point $\hat{p}$. For this grid, each control point has a known value (for example pressure bias) $\Delta H p_i$ and the interpolated value in $\hat{p}$ is $\hat{\Delta h}$

In this example can be seen that the number of points used in the interpolation and the position of these points can be arbitrary. This is the characteristic that will allow to introduce some additional points ("fixed

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transponders”) without change the structure of the estimator. These ”fixed transponders” are for example the pressure measurement in the airports.

The algorithm to calculate the interpolation is described below. The position of the interpolated point is $\hat{p} = (x_{\hat{p}}, y_{\hat{p}})$ and the position for each control point is $P_i = (x_{P_i}, y_{P_i})$. The distance d_i in a 2D plane is calculated as follows:

$$d_i = \sqrt{(x_{P_i} - x_{\hat{p}})^2 + (y_{P_i} - y_{\hat{p}})^2} \quad (5.2)$$

The weights used for the average is inverse to the distance. As it has been introduced above the weights are controlled with an exponent that can variate the shape of the surfaces. Then the weights for each control point can be calculated as:

$$w_i = 1/d_i^n \quad (5.3)$$

where n is the grade of weights. High values for the exponent n gives more weights to the nearest control points. Low values for n give more weight to far control points up to the extreme case (n=0) where the interpolation is the arithmetic mean of the control points values.

Once the distances has been calculated the number of control points used for the interpolation must be selected. One criterion for select the control points is chose the N nearest points to the interpolated points. Other criterion is set a distance threshold and chose the control points which are nearer that this threshold. Another criterion that can be used in regular grid is chose the surrounding control points.

In the previous paragraphs P_i was the M points of the grid. Choosing the N nearest control points to the interpolated point we get Q_j where $j = 1...N$. The values of these points are now $\tilde{\Delta H}_j$ and the calculated weights are $\tilde{w}_j$. The interpolated value is calculated as follow:

$$w_{sum} = \sum_{j=1}^N \tilde{w}_j \quad (5.4)$$

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$$\hat{\Delta h} = \sum_{j=1}^N \frac{\tilde{w}_j \tilde{\Delta H}_j}{w_{sum}} \quad (5.5)$$

As it can be seen the algorithm to interpolate (in this case) the altitude of a point with some given control points is very simple and with low computational load. The versatility of this algorithm give us the control for a lot of parameter. In the design phase it can be controlled the shape of the surfaces, the number of the point used for the interpolation and the position of the control points. And finally in this method is not necessary that the control points surround the interpolated point.

This algorithm has a very important disadvantage. If the number of the points used in the interpolation is the number of the control points ($N=M$) then the surface will be continuous but when the number of the point selected is lower than the control points ($N<M$) then the continuity is not warranted as it can be seen in figure 5.8

Figure 5.8: Detail of a discontinuity in a frontier area

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These frontiers appears when the set of control point selected Q_j changes. If the number of selected points (N) is big, the discontinuity of the frontier will be small because the weight of the less important point will be small, but if N is small, then the less important weight will be high and the discontinuity will be significant.

In the estimation of the pressure and the temperature, these discontinuities do not seem to be very important but in the results section will be shown that this is an error source.

5.1.2.2 Application to estimation method

In this section the interpolation described above is applied to the estimation methods. For the description of this method the IDW interpolation will be made in a very simple way. For this description a regular grid for the control points is used. In a general case with an irregular grid the determination of the control points used in the interpolation is more complex than in this description.

In figure 5.9 an example with a regular grid is shown. This grid has $M_x \cdot M_y = M$ points allocated in a squared shape. The set of points named Q_j in previous sections will be chosen using the surrounding criterion. As the control point form squared areas, the interpolated point always will be surrounded by the four corner of the smaller surrounding square, in this case the number points used in the interpolation (N) is 4.

Using the atmospheric model presented in chapter 2 the pressure and temperature is estimated with a single value and because of that the last rows of the matrix H_b is composed by the partial derivatives with respect to ΔT and ΔH_P ($\frac{\delta x}{\delta \Delta H_P}$, $\frac{\delta y}{\delta \Delta H_P}$, $\frac{\delta x}{\delta \Delta T}$ and $\frac{\delta y}{\delta \Delta T}$).

In the case of estimate the atmospheric parameters using the interpolation we will not estimate a single value for pressure and another for temperature. In this case the pressure parameter will be converted to a set of M parameters ($\Delta H_P \rightarrow [\Delta H_{P1}, \dots, \Delta H_{PM}]$) and the equivalent conversion will be made for the temperature ($\Delta T \rightarrow [\Delta T_1, \dots, \Delta T_M]$).

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Figure 5.9: Example of a regular grid for IDW estimation

With that the resulting measurement matrix for the biases (substituting the equation 3.26) is:

$$B = \begin{bmatrix} \frac{\delta x}{\delta \Delta \rho} & \frac{\delta y}{\delta \Delta \rho} \\ \vdots & \vdots \\ \frac{\delta x}{\delta \Delta H_{P1}} & \frac{\delta y}{\delta \Delta H_{P1}} \\ \vdots & \vdots \\ \frac{\delta x}{\delta \Delta H_{PM}} & \frac{\delta y}{\delta \Delta H_{PM}} \\ \frac{\delta x}{\delta \Delta T_1} & \frac{\delta y}{\delta \Delta T_1} \\ \vdots & \vdots \\ \frac{\delta x}{\delta \Delta T_M} & \frac{\delta y}{\delta \Delta T_M} \end{bmatrix}^T \quad (5.6)$$

The matrix described in equation 5.6 is valid when every control point affects to the interpolation for any point. In figure 5.9 can be seen that only 4 points affect to the interpolation then the projection matrix must be modified. For that only the rows corresponding to the affecting control point have a value different to zero.

In figure 5.9 the points that affect to the interpolation are:

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1. $Q_1 = P_{M_x(s-1)+k}$
2. $Q_2 = P_{M_x(s-1)+k+1}$
3. $Q_3 = P_{M_x(s)+k}$
4. $Q_4 = P_{M_x(s)+k+1}$

As the interpolation is made using only the N points that compose the set Q_j then the estimation algorithm only must update these parameter. Then the partial derivatives are zero for each control point that are not included in Q_j . Doing that the estimation algorithm has to estimate $2N$ parameters (atmospheric) with each pseudo-measurement.

Using the flat Earth model for the sake of simplicity, the horizontal Cartesian coordinates of a measurement is:

$$\begin{aligned} x &= \rho_m \sin(\theta_m) \cos(\phi_m(\rho, h)) \\ y &= \rho_m \cos(\theta_m) \cos(\phi_m(\rho, h)) \end{aligned} \quad (5.7)$$

This model is used only for describing the method and for the synthetic simulation (in the results section there are two different test, the first one is made to test the observability of the model with Cartesian coordinates).

As it can be seen in previous equations the elevation (ϕ_m) depends on the range of the measurement and the altitude of the aircraft. The altitude of the aircraft is a function that depends on the pressure and the temperature biases. In a general way the aircraft is not allocated just in a control point, then the biases that affect to its altitude ($\Delta\hat{H}_P$ and $\Delta\hat{T}$) are interpolated with the control point biases (ΔH_{Pj} and ΔT_j). The interpolated values of the aircraft positions are calculated using the equation 5.5 and doing the partial derivatives:

$$\frac{\delta x}{\delta \Delta H_{Pj}} = \frac{\delta x}{\delta \Delta \hat{H}_P} \frac{\delta \Delta \hat{H}_P}{\delta \Delta H_{Pj}} \quad (5.8)$$

where:

$$\frac{\delta \Delta \hat{H}_P}{\delta \Delta H_{Pj}} = \frac{\tilde{w}_j}{w_{sum}} \quad (5.9)$$

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Combining equations 5.8 and 5.9:

$$\frac{\delta x}{\delta \Delta H_{Pj}} = \frac{\delta x}{\delta \Delta \hat{H}_P} \frac{\tilde{w}_j}{w_{sum}} \quad (5.10)$$

In the previous equations the partial derivative $\frac{\delta x}{\delta \Delta \hat{H}_P}$ is not shown in this section because it is different for each scenario. For the stereographic projection this expression is calculate as is indicated in section 3.4.1.

5.1.3 Simulation and Validation of the Model Extension

Two experiments will be performed for the evaluation of this estimation method. First a synthetic scenario with unrealistic conditions is used in order to check the convergence and stability of the algorithm. After that, the algorithm will be tested with simulated data in a realistic scenario.

5.1.3.1 Synthetic simulation

In this section the simulation made is unrealistic but it is illustrative to see if the estimated interpolated surfaces fits with a real pressure and temperature map. The scenario covers a squared area measuring 50km in length. In this reduced scenario the pressure and temperature maps (Figure 5.1) are scaled in the horizontal plane in order to fit them in the reduced scenario. In the center of this area there is allocated a single radar with unbiased measurement in range and azimuth. The standard deviation in range (σ_ρ) and azimuth (σ_θ) of the measurement are 70 m and 0.1° respectively. For this scenario a variant number of random points are generated uniformly inside of the squared area distributed in altitudes between 1000 m and 20000 m. The altitude of the generated points is simulated with the altitude biased described in equations 2.56 and 2.57 proposed in [Poles, 2010].

The estimation method used is the pseudomeasurement method described in section 3.3.2. In real scenarios with one single radar the bias estimation is not possible but in this synthetic scenario the ideal position of the points is

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(a)

(b)

Figure 5.10: (a) Altitude bias map over Spain (b) Altitude bias map over UK. Extracted from figure 5.1

known and the pseudo-measurements used in the estimation method can be calculated. The error in the estimation will be lower than using measurement from more than a radar due to the lower uncertainty of the pseudomeasurement.

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Figure 5.11: Example of temperature map get from figure 5.1 used in the simulation

For this evaluation 2 different pressure surfaces are used. In figure 5.1(a) the first pressure map will be extracted over Spain (figure 5.10(a)) area and the second pressure map is extracted from the United Kingdom area (figure 5.10(b)). The size of these cells are adapted to the size of the scenarios.

The numerical simulation has been made with a Monte Carlo simulation. The presented results measures the RMS error of the pressure and temperature bias. In figures (from 5.12 to 5.14) the horizontal axis represent the number of random points used in the simulation and the vertical axis the RMS error. In each figure there are several lines. Each line has been calculated with a different number of control points for the grid. For the simulation the control points for pressure and temperature are allocated in the same positions.

In both simulation the results are the expected ones. The estimation with a single value is the worst case and the RMS error in temperature is lower when the number of control points is higher. This can be seen in figures 5.12(b) and 5.13(b).

In the pressure estimation this behavior is slightly different and the best estimation is get with the grid with $N = 4$ as it can be seen in figures 5.12(a)

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and 5.13(a). In the simulation with bigger grids the error in the pressure estimation is worse than using 4 points but it has an erratic behavior because with 36 points the estimation is worse than a grid with 25 points but better than with 16. This behavior seems to be produced by the discontinuities of the surfaces. In the next section it is shown how this effect modifies the estimations.

Only with this data is difficult to say which is the best approximation for this problem, because for the pressure bias seems clear that with 4 control points the best approximation is get, but for the temperature the behavior is different.

In order to determine which is the best approximation, the error in the altitude of the points is measured. In the next figures the RMS error of the altitude will be represented. In figure 5.14 (a and b) can be seen that the RMS Value is better when the number of control points is bigger. For the grids with more than 4 points the determination of the temperature parameter compensates the error in the pressure determination.

For real scenarios the number of points must be selected for the best compromise between accuracy and computational load. During the estimation the inverse of a matrix must be computed. In this scenario just for pressure and temperature the difference between a grid with 4 point and a grid of 9 point is very important. For $N = 4$ the matrix dimension is 8×8 and for $N = 9$ is 18×18 . In this scenario the best option in terms of accuracy is the use of the biggest possible grid. In the next section it is shown the behavior of the algorithm using a realistic scenario where the practical use of this algorithm is described.

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(a)

(b)

Figure 5.12: (a) RMS pressure and (b) RMS temperature error for Spain pressure Map

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(a)

(b)

Figure 5.13: (a) RMS pressure and (b) RMS temperature error for UK pressure Map

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(a)

(b)

Figure 5.14: (a) RMS altitude error for Spain map and (b) RMS altitude error for UK pressure Map

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5.1.3.2 Realistic simulation

In this section the extended atmospheric model is tested using simulated data but in a more realistic scenario than in the previous section. The ideal positions of the aircraft are not known. A big number of trajectories are simulated. The aircraft follow a straight trajectories with constant velocity and leveled altitude. In the scenario are 9 radars distributed and all of them measure all the aircraft trajectories. The atmospheric biases are projected in the horizontal plane proportionally to the elevation of the measure. In the major part of the measures of this scenario the biases are not noticeable and because of that the results of this test are expected to be worse than in the synthetic scenario. The measures of the radars are affected just by the atmospheric biases. The other biases of the radars described in chapter 2 are not simulated.

In a first test the atmospheric bias has been simulated using a synthetic surface and after that using the real maps of pressure and temperature. Figures of the results of the estimation using this simulated bias and after that using real maps are shown at the end of this section for the sake of clarity. In the tables 5.1 and 5.2 are indicated the RMS error of the corrected plots using the different estimations grids. The RMS error has been calculated as in the result section 3.25.

In figure 5.15 are shown the result of the simulation using a simulated atmospheric biases map. In the first row of figure there are shown the surfaces that generates the simulated pressure bias and temperature bias. In the next rows the estimated values of the biases are shown using a grid of 4, 9 and 16 points in the second, third and fourth row respectively.

In the aforementioned figure can be seen that the estimation with four point is similar to the simulated surface but when the estimation is done using 9 or 16 points the estimations are inaccurate. In the table 5.1 the different values of the RMS error are compared using these estimations. As it can be seen in this table the estimation using 4 point for pressure and temperature improves the estimation using one single point. Using a grid with a bigger number of points the estimation is getting worse.

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Parameters	RMS error
Uncorrected	574.34 m
1 Point	159.48 m
4 Points	37.63 m
9 Points	121.56 m
16 Points	179.72 m

Table 5.1: RMS error of the plots using simulated atmospheric biases

This behavior is due to the discontinuities of the surfaces. These discontinuities introduces instabilities that affect to the observability of the scenario. Using a grid with a low number of points make that all the measures modifies the estimation but when the number of fixed points is increased not all the measures modifies all the points and there are the discontinuities.

When the observability of the temperature bias is not fulfilled there is a phenomenon that can be observed. The temperature bias and the pressure biases are modified proportionally in order to compensate the horizontal projection of the biases. For example in figure 5.1 in the third row corresponding to the grid of 9 points, the temperature in the left corner is very low in order to compensate the high pressure of the same corner. The same effect is observed in the right corner of the fourth row from the same figure.

Following the previous experiment, this scenario has been tested using the temperature and pressure map that has been read from real data from figure 5.1. In this case the results are shown in figure 5.2 and the RMS error is shown in the table 5.2.

Parameters	RMS error
Uncorrected	801.88 m
1 Point	206.01 m
4 Points	84.82 m
9 Points	178.74 m
16 Points	261.95 m

Table 5.2: RMS error of the plots using real atmospheric biases over Spain

As it can be seen using real data for the generation of the biases, the effect is similar to the simulated one. When the number grid points is increased,

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the accuracy of the estimation is lower, and even worse than using a single value.

Using simulation with a bigger number of trajectories in the same scenario the results are similar to the results using the previous number of trajectories. The effect of the discontinuities affects to more trajectories and the observability is not solved adding more spatial diversity. In the view of these results it is shown that the best number of points in the grid for realistic scenarios is $N=4$ points. With that results the search of more accurate interpolations is proposed as future work. An interpolation without discontinuities in the border can be searched in order to get more stability losing in computational workload.

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Figure 5.15: Different estimations for simulated atmospheric data. The first row is the generated atmospheric data. The other three rows are the estimations using a grid with 4, 9 and 16 points.

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Figure 5.16: Different estimations for real atmospheric data. The first row is the generated atmospheric data. The other three rows are the estimations using a grid with 4, 9 and 16 points.

5.2 Target Dependent Biases Estimation

In this section two estimation methods of target dependent biases are described. Both methods are used to estimate biases described in section 2.3. In some cases the target dependent biases are mixed with the sensor dependent biases and they must be estimated in the same process. In other cases the microbiases are mixed with the parameters of the trajectory and they must be estimated in the tracker. In this section there are one method for each situation.

5.2.1 Transponder Delay

The transponder delay is a constant range error that depends on the time margins of the response. These margins are indicated in [ICAO, 2007a]. As it has been explained in 2.3.1, the range bias introduced by the transponder delay can be between ± 75 meters.

In this section we assume that the transponder delay is a random value (constant for each aircraft but random for every aircraft) uniformly distributed with zero mean. In scenarios with a low number of trajectories the mean of the delays is not zero, but for real scenario with a great number of aircraft (>150) the zero mean assumption will be good enough to the estimation process.

In the next sections the current method presented in [Besada et al., 2012] is described and after that the modification fusing the two last stages is presented

5.2.1.1 Description of the Basic Estimation Method

In this section the method that will be modified in the next section is described in order to estimate the target biases and sensor biases jointly. In [Besada et al., 2012] a generic multisensor multitarget bias estimation architecture is developed. The biases are estimated in three steps and it is assumed that there are sensor dependent (macrobiases) and target dependent (microbiases) biases. With this assumption the stages of the estimation

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process are:

1. Sensor-target biases estimation: In this step, for each trajectory, a set of sensor-target dependent biases is estimated. Using a simple scenario with 2 radar and N trajectories, N different sets of biases will be estimated. For the radar biases a simple model with range offset, range gain and azimuth offset is used ($\Delta\rho$, K and $\Delta\theta$) in this section.

The transponder delay for each aircraft is $\Delta\rho t_i$ and each set of bias estimation is named ξ_i . With this nomenclature, the parameter set to estimate is:

$$\xi_i = \begin{bmatrix} \Delta\rho_1 + \Delta\rho t_i \\ K_1 \\ \Delta\theta_1 \\ \Delta\rho_2 + \Delta\rho t_i \\ K_2 \\ \Delta\theta_2 \end{bmatrix} \quad (5.11)$$

The range offset and the transponder delay are additive and with a single trajectory they are not observable. In the next steps the estimations are mixed in order to separate both biases. These estimations can be obtained with any estimation method as [Bar-Shalom et al., 2011, Lin et al., 2003, Pu et al., 2017] or any other method presented in the literature.

2. Sensor dependent biases: In this step the sensor dependent biases are estimated. Assuming that the target dependent biases are zero mean distributed, the sensor-target dependent biases are mixed in order to remove the effect of the target dependent biases.

In the step 1, as the range gain and azimuth offset are just sensor dependent the estimated values are the final values. For the range offset the estimated values depend on the transponder delay too. Each estimation is associated to a covariance and the sensor dependent biases are calculated as the mean of the estimated values (equation 5.11) weighted with the inverse of the covariance for each trajectory.

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The expected value of the estimation is:

$$\hat{\Delta\rho_i} = \Delta\rho_i + \mu_{\Delta\rho t} \quad (5.12)$$

where $\mu_{\Delta\rho t}$ is the mean of all transponders delay of the aircraft. For a big enough number of trajectories the assumption of zero mean is usually valid but if the number of trajectories is small or the mean of their delays is not zero, the estimation of the bias will be "biased" to the mean of the transponders.

Once the sensor dependent biases are estimated, the ambiguity of the equation 5.11 is solved and the range biases can be separated.

3. Target dependent biases: Once the Sensor dependent biases are calculated, the target dependent biases are determined using the set of biases calculated in the first step and the value of the macrobiases obtained in the second step. If the value of the macrobiases is contaminated with the mean of the transponders ($\mu_{\Delta\rho t}$), the value of the transponders delays will be bad estimated with the same value. The target dependent biases are calculated subtracting the radar dependent biases from the joint estimation of equation 5.11.

5.2.1.2 Joint estimation for micro and macro biases

In this section is developed a modification for the method described in the previous section. This modification is proposed using the architecture of the method developed in [Besada et al., 2012]. The first step will be the same used in [Besada et al., 2012] but the other step will be modified and both kind of biases (macro and micro) will be estimated jointly.

This method is used for an offline processing of the data. This algorithm is a first stage in order to remove the target dependent biases. Because of that just the basic model (range gain and offset and azimuth offset) is used. Once the target dependent biases has been estimated the algorithms proposed in chapter 4 using the models proposed in chapter 2 is used. The complete model cannot be used in this stage because with a single trajectory the observability for some SSR biases is not ensured.

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This modification based in the algorithm of [Besada et al., 2012] fuse the last two step in a single step. The macro and micro biases are estimated jointly reducing the error when the mean of the transponder delays is not zero. This modification includes a recursive method that processes all the trajectories when received.

1. Sensor-Target dependent biases estimation: This step is maintained equal to the first step of the previous section. For this application we will assume that the trajectories are long enough to guarantee the observability of the biases. (The observability is widely discussed in section 4.2). In a scenario with M radars and N trajectories, N different bias sets with $3M$ parameters are get.

2. Sensor dependent and target dependent biases estimation: In the architecture developed in [Besada et al., 2012] the sensor dependent (macrobias) and target dependent (microbias) biases are estimated in different steps. In this modification both kinds of biases are estimated jointly in the same step.

For this estimation we have all the set of bias estimated in the previous step. For each trajectory the set of parameters is:

$$\xi_i = \begin{bmatrix} \Delta\rho_1 + \Delta\rho t_i \\ K_1 \\ \Delta\theta_1 \\ \vdots \\ \Delta\rho_M + \Delta\rho t_i \\ K_M \\ \Delta\theta_M \end{bmatrix} \quad (5.13)$$

It is assumed that in the estimation method used in the first step, the estimation variance is calculated. With that we have the vectors of estimations (ξ_i) and their associated variances matrix (P_{ξ_i}). The structure of the covariance matrix with dimensions $3M \times 3M$ is:

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$$P_{\xi_i} = \begin{bmatrix} \sigma_{\Delta\rho_1}^2 & \sigma_{\Delta\rho_1 K_1} & \cdots & \sigma_{\Delta\rho_1 K_M} & \sigma_{\Delta\rho_1 \Delta\theta_M} \\ \sigma_{\Delta\rho_1 K_1} & \sigma_{K_1}^2 & \cdots & \sigma_{K_1 K_M} & \sigma_{K_1 \Delta\theta_M} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sigma_{\Delta\rho_1 K_M} & \sigma_{\Delta K_M K_1} & \cdots & \sigma_{K_M}^2 & \sigma_{K_M \Delta\theta_M} \\ \sigma_{\Delta\rho_1 \Delta\theta_M} & \sigma_{K_1 \Delta\theta_M} & \cdots & \sigma_{K_M \Delta\theta_M} & \sigma_{\Delta\theta_M}^2 \end{bmatrix} \quad (5.14)$$

In order to estimate the sensor dependent biases, the bias sets estimated for each trajectory are filtered using a Kalman filter. The biases estimated in the first steps are used as the measurements of the filter and the bias estimation covariances estimated are used as the measurement noise matrices. The Kalman filter is configured with the next definitions:

State Vector: The state vector (X_ξ) is built with all the bias parameters of the radars and the bias parameters of the aircraft. It will be defined as:

$$X_\xi = \begin{bmatrix} \Delta\rho_1 \\ K_1 \\ \Delta\theta_1 \\ \vdots \\ \Delta\rho_M \\ K_M \\ \Delta\theta_M \\ \Delta\rho t_1 \\ \vdots \\ \Delta\rho t_N \end{bmatrix} \quad (5.15)$$

Measurement Matrix: The measurement matrix (H_ξ) used in the Kalman filter is a matrix that project the measure vector (ξ_i) to the state vector (X_ξ). In this section we will call $\Delta\rho_{\xi k}$ to the measurement corresponding to the k-th sensor that is composed by the radar bias and the aircraft bias. In the same way we will use $K_{\xi k}$ and $\Delta\theta_{\xi k}$. The measurement model is described as:

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$$\begin{aligned}\Delta\rho_{\xi k} &= \Delta\rho_k + \Delta\rho t_i + w_{\Delta\rho} \\ K_{\xi k} &= K_k + w_K \\ \Delta\theta_{\xi k} &= \Delta\theta_k + w_{\Delta\theta}\end{aligned}\tag{5.16}$$

Using the measurement model described above, the measurement matrix is:

$$H_{\xi,i} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 & \cdots & 0 & \cdots & 0 \end{bmatrix}\tag{5.17}$$

In the measurement matrix defined above the some different areas can be described. Each row correspond to a parameter of the measurement vector and each column correspond to each parameters of the state vector. The rows corresponding to the $\Delta\rho$ measurements have two columns different to zero. The first column is the corresponding to the k-th radar, and the other column is the corresponding to the i-th trajectory. In the same measurement, the column corresponding to the trajectory is the same, and there are more than one row in the trajectory column that is different to zero. If a radar does not measure a trajectory, the rows that correspond with this radar are zeros.

Measurement noise matrix: In this estimation method the measurement are not affected by noise as in the standard case of the position measurement of an aircraft. In this case the measurement vector will be affected by the bias estimation errors and their variances are the indicated by the covariance matrix (P_{ξ_i}) described above in equation 5.14.

Plant noise matrix (Q): In a Kalman filter usually is used a plant noise matrix in order to model the possible changes in the state estimated values with time. In this method this matrix is not used (a null matrix) due to the estimation is made in the same time and the values are assume static in the

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time of the estimation. The filter is processed at the end of the time window, when all the plots have been received.

Finally there are some considerations about the computation load that must be done. The estimation method presented in this section can have a very high computational load. The number of state variables for the simple macrobias model is $3M + N$ and the number of variables in the covariance matrix is $(3M + N)^2$. Using the simple model with only three macrobias parameters, in reduced scenarios with 3 radars and 12 trajectories the state vector will have 20 parameter. In real scenarios with 10 radars and 150 trajectories the number of parameters in the state vector is 180.

The number of values affects both to the memory used and to the number of operations. In the Kalman filter several matrix multiplication are done with matrix which dimensions are equal to the state vector dimensions. The operation with the highest computational load in the Kalman filter is the inverse of the residual covariance matrix. The inverse of the matrix shown in the appendix B.1 is used to calculate the gain:

$$K_k = \frac{\hat{P}_k H_k^T}{H_k^T \hat{P}_k H_k^T + R} \quad (5.18)$$

the matrix $H_k^T \hat{P}_k H_k^T + R$ that must be inverted is a squared matrix which dimensions is the same that the measurement vector (equation 5.13). Then for this algorithm the inverse of the matrix is determined by the number of radars and bias parameters. As it has been described above the number of rows of the measurement vector is $3M$ (assuming that sensors are modeled with 3 bias parameters) and the inverse matrix of the Kalman filter is of dimensions $3M \times 3M$. The computation for the inversion of this matrix is conditioned by the number of radars that measure a trajectory. When not all the radars measure the trajectory the matrix to invert is sparse and the calculation can be simplified.

5.2.1.3 Evaluation of the modification

For the evaluation of the improvement of this modification a scenario with three sensor is simulated. As this method is designed to the estimation of the

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microbiases, they are analyzed in this section (the macrobiases results are not analyzed). This estimation is used to remove the delay of the transponder before the estimation of the sensors biases with the complete model presented in chapter 2.

The simulated radars have the biases described in previous section (range and azimuth offset and range gain). In the scenario there are 30 trajectories and all them are measured by all the radars. The trajectories are rectilinear trajectories and the biases for each one are estimated using the pseudomeasurement method (section 3.3.2). For the evaluation of this modification several means for the transponder delays are simulated. When the number of trajectories is high the assumption of zero mean in the distribution of the delays is fulfilled but for a low number of trajectories the mean can be different to zero. In the next results the effect of this mean is shown in the error of the estimation.

Figure 5.17: RMS error of the transponder delay of the trajectories. The red line represents [Besada et al., 2012], the blue line represents the modification presented in section and the black line represent the generated mean of the transponder delays

In figure 5.17 the RMS error with the algorithm from [Besada et al., 2012] (red line) and with the proposed modification (blue line) are represented. The

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red line represent the mean of the generated transponder delay in order to show that the Besada algorithm converges to the mean value. Using the proposed algorithm in this section the RMS error is slightly lower than the mean of the transponders delay.

The results has been obtained using a Monte Carlo simulation with 1000 iterations. In each iteration the transponder delays are simulated with the mean indicated in the horizontal axis of figure 5.17. As the number of trajectories is low the sample mean is not exactly equal to the theoretical mean. When the value of the mean is near to zero, the RMS error is not near to zero because it is limited by the RMS value of the sample mean of the realizations.

When this method is used for the evaluation of the biases and the computational workload is not an important factor and this modification reduces the error committed in the estimation when the mean of the delays is different to zero. When the mean of the delays is zero, the performance from both algorithms is similar.

5.2.2 Velocity Bias

The WAM, ADS-B and radar Mode-S can use the communication channel with the aircraft to determine its velocity. The velocity of an aircraft (as the barometric altitude) cannot be determined by the sensor and they need the cooperation of the target to get this information. In modern equipment the position will be acquired by the airborne GNSS systems as the GPS and the velocity will be unbiased, but in some cases the velocity of the aircraft can be send with constant error to the stations as it has been explained in 2.3.2.2.

For the tracking filter one of the assumption made is that the measures are unbiased and affected with a zero mean white Gaussian noise. The position measured by the radars are corrected with the estimated bias parameters using the bias estimation methods described in chapter 3 and the radars bias model presented in 2.2.

The velocity measurement can be biased and it cannot be corrected with the positions errors in a global estimator because this bias is target dependent. The distribution of the measurement is assumed Gaussian with a covariance indicated by the measurement noise matrix (usually named R). The

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velocity biases can be corrected using the measurements of the positions. This must be done in the tracker because the biases are local to each aircraft and the biases are mixed with the velocities that are used in the tracker. In the previous section the transponder delay are estimated jointly with the sensor dependent biases because the local biases are mixed with the global biases.

In the global tracker it is important the correction of the biased velocity measurement due to several reasons. The first and more visual is the introduction of errors in the tracker that makes the heading of the state vector be changing randomly. The second one is the introduction of an error in the estimation of the tracker residual. This residual is used in the bias estimation when using the temporal windows method and the biases are estimated worse. This section is developed in order to get the best performance in the complete tracker.

5.2.2.1 Velocity Bias Estimation

For this microbias correction an augmented state vector is defined. A classical state vector used in ATM where there are maneuvers with coordinated turns [Gustafsson and Isaksson, 1996] is:

$$X_K = \begin{bmatrix} x_k \\ y_k \\ |V| \\ \theta \end{bmatrix} \quad (5.19)$$

The augmented state vector proposed for this state vector is composed by the previous state vector and two more terms that models the velocity bias (ΔV and $\Delta\theta$):

$$X_K = \begin{bmatrix} x_k \\ y_k \\ |V| \\ \theta \\ \Delta V \\ \Delta\theta \end{bmatrix} \quad (5.20)$$

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The measurement model in this filter has four parameters, two for position and two for velocity:

$$\begin{aligned}x_m &= x + n_p \\y_m &= y + n_p \\|V|_m &= |V| + \Delta V + n_v \\\theta_m &= \theta + \Delta\theta + n_\theta\end{aligned}\tag{5.21}$$

where n_p , n_v and n_θ are term of Gaussian white noise with zero mean with variances σ_p^2 , σ_v^2 and σ_θ^2 respectively. The velocity can be measured in local Cartesian coordinated but the conversion from V_x and V_y to speed and azimuth is trivial.

The prediction of the next state (in a rectilinear movement) affected by the velocity and the velocity bias:

$$\begin{aligned}\hat{x}_{k+1} &= x_k + \Delta T |V| \sin(\theta) \\\hat{y}_{k+1} &= y_k + \Delta T |V| \cos(\theta) \\\hat{|V|}_{k+1} &= |V|_k \\\hat{\theta}_{k+1} &= \theta_k \\\Delta \hat{V}_{k+1} &= \Delta V_k \\\Delta \hat{\theta}_{k+1} &= \Delta \theta_k\end{aligned}\tag{5.22}$$

In this model the azimuth is defined clockwise positive with the origin in the north. The speed parameters estimated in the state vector (Vx_k and Vy_k) are the unbiased speeds. The measurements and because of that in the prediction matrix (Φ) the terms that relates the bias parameters with the positions are zeros.

$$\Phi_k = \begin{pmatrix} 1 & 0 & \Delta T \sin(\theta) & \Delta T |V| \cos(\theta) & 0 & 0 \\ 0 & 1 & \Delta T \cos(\theta) & -\Delta T |V| \sin(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}\tag{5.23}$$

The velocity bias is estimated through the measurement matrix (H). In the equation 5.21 the velocity parameters of the measurement are the esti-

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mated speed (ideal speed) with the bias term.

In this section the measurement model relationship is described. Because of that the measurement matrix using the model described in equation 5.21 is:

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix} \quad (5.24)$$

With this model the matrix H modifies the Kalman gain of the filter and with the residual of the measurement, the velocity biases are estimated. In the Kalman gain calculation with the expression:

$$K_k = \hat{P}_k H_k^T \left(H_k \hat{P}_k H_k^T + R \right)^{-1} \quad (5.25)$$

In this case the rows corresponding to the velocity bias (rows 5 and 6) are modified because the columns of the measurements matrix are different to zero and the covariances of the measurements affect to the gain calculation.

5.2.2.2 Velocity Bias Estimation Results

The estimation of the velocity bias can be very useful when the speed is badly indicated in the measurement but when the speed is no biased the performance of the filter can be altered.

Firstly will be described the scenario where the test has been done and the performance of the filter will be shown. The scenario is composed by a single aircraft flying with constant speed. The trajectory has three maneuvers with a straight segment before. The maneuvers are three turn, one to the left and two to the right.

The maneuvers with the time in the simulation is described below and the trajectory is shown in figure 5.18.

- Straight trajectory flying at 300 m/s during 200 seconds.
- Coordinated turn maneuver to the right with a normal acceleration of

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5 m/s^2 from second 200 to 250.

- Straight trajectory flying at 300 m/s during 200 seconds from second 250 to 450.

- Coordinated turn maneuver to the left with a normal acceleration of 5 m/s^2 from second 450 to 500.

- Straight trajectory flying at 300 m/s during 200 seconds from second 500 to 700.

- Coordinated turn maneuver to the right with a normal acceleration of 5 m/s^2 from second 700 to 750.

Figure 5.18: Trajectory used in the evaluation of the speed bias estimation.

The tests have been made with a Monte Carlo simulation with 1000 runs of the same trajectory. The sampling time of the trajectory is random and the noise of the measurement is simulated with additive white Gaussian noise with zero mean.

The measurements are get simulating the ADS-B system without time bias (see section 2.3.2.1). The deviation of the measurements ($\sigma_p = 5m$) is

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equal and independent for both coordinates axis. The speed measurements are simulated using a deviation of 2.5 m/s

The results shown in the next figures are get calculating the RMS error in position and in speed. The RMS error is calculated using the 1000 runs of the Monte Carlo simulation.

In figures 5.19 and 5.20 two different filters are compared. The first one with the results shown with a continuous line is a filter with just position measurement and the second one is a filter with position and velocity measurements. There are two dotted lines, the thin one is generated with unbiased measurements and the gross one is generated with biased (in velocity) measurements.

Figure 5.19: RMS error for the filter without velocity (continuous line), the filter with unbiased velocity measurements (dotted line) and for the filter with biased velocity measurements (dotted gross line)

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Figure 5.20: RMS error for the filter without velocity (continuous line), the filter with unbiased velocity measurements (dotted line) and for the filter with biased velocity measurements (dotted gross line)

As it can be seen in the mentioned figures the performances in terms of accuracy are improved when the velocity measurements are introduced in the filter. In this configuration the perpendicular error is worse in the filter that incorporates velocity than in the filter with only position measured. In the other parameters (lengthwise error, velocity and heading error) the accuracy is better in when the velocity is incorporated.

When the velocity measurements are biased it can be seen that the performances of the filter are totally degraded. As it can be seen in figures 5.19 and 5.20 the position and velocity estimations are worse than in the first filter. Although in the second filter, more information is added, the assumption of zero mean on the velocity noise is not fulfilled. When the measurements

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have biases the mean of the noise is displaced to the value of the biases and the filters will not work as they must work, giving bad results.

In figures 5.19 and 5.20 it can be seen that the error in the right turns the error is low but this is due to the coincidence that the velocity is biased to the same direction of the turn. In all the others maneuvers (straight segments and left turns) the results are worse than in the others filters configurations.

In real system the advantages of the speed incorporation are very significative. In the control screens the speed vector of the aircraft has a smooth movement and that make easier the work for the ATCO. With the real measurements is very difficult the detection of the velocity biases. With the previous measurements an integrity detector can be made, but if the biases are small the detector will not notice them.

When the biases are present the errors are noticeable in the ATCO screens and the previously described filter correct these errors. In figures 5.21 and 5.22 there are two different graphics. The blue line represents the proposed filter generated with biased measurements and the dotted line represents the standard filter with velocity measurements generated without biases. The dotted line is the same line that the dotted line represented in figures 5.19 and 5.20.

The error of the filter with bias estimation is slightly worse in the maneuvers than in the previous filters. This is because the uncertainty of the estimation affected by the velocity and its biases. Just for the perpendicular error the estimation of the new filter is better than in the previous one.

The real advantage of this filter is that the maneuvers are well estimated when the velocity measurements are biased, because with the standard filter the results are erroneous as it can be seen in figures 5.19 and 5.20 with the gross dotted line.

The price of getting better results when the biases are estimated is that the computational load is drastically incremented. In the standard filter the state vector has four parameters and the covariance matrix has 4x4 parameters. In the proposed filter the state vector has six parameters and the

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Figure 5.21: RMS error for the filter with unbiased velocity measurements (dotted line) and for the filter with biased velocity measurements and velocity bias estimation (continuous line)

covariance matrix has 6x6 parameters. This increment in the number of parameters derives in an increment in the number of operations.

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Figure 5.22: RMS error for the filter with unbiased velocity measurements (dotted line) and for the filter with biased velocity measurements and velocity bias estimation (continuous line)

Chapter 6

Conclusions

As the conclusion of the presented Ph. D. thesis chapters and contributions are summarized. In the first chapter the introduction for this thesis has been redacted. The topics of the thesis have been described focusing in the surveillance systems of ATM. The needs of these surveillance systems have been described. An overview of the ATM systems and the sensor used in the surveillance has been given.

The second chapter is focused in the development of the models for the systematic errors of the sensors. There are two kinds of systematic error that appear in the measurements. The first kind are the sensor dependent biases and the second kind is the target dependent biases. The sensor dependent biases are generally produced by the sensors (they can be produced by physical phenomena as the propagation of the signals) and they are common to all the aircraft of the airspace. The target dependent biases are usually produced by the interaction between the target and the sensor.

In the literature the biases are widely discussed but the works are focused in the estimation but not in the modeling. In this thesis a complete and rigorous mathematical model for secondary surveillance radar biases is developed. The model takes into account the physical effects systematically affecting the measurement processes. The azimuth biases are calculated from the physical error of the antenna calibration and the errors of the angle determination device. Distance bias is calculated from the delay of the signal produced by the refractivity index of the atmosphere, and from clock er-

rors, while the altitude bias is calculated taking into account the atmosphere conditions (pressure and temperature).

This model considers the mechanical devices of the radars that include the antenna, the rotatory pole and the optical encoder (azimuth determination device). The improvement of the model is an important advance over the current model used typically in the literature that only considers a constant term for the azimuth offset and sometimes an azimuth gain without any justification. In the same way the range bias is improved with the propagation terms that are not included in the usual models. The use of the atmospheric model is proposed for the use in ATM surveillance systems. Finally, the errors produced by the delay of the communication lines are modeled and included in the estimation of the biases.

In the same chapter some target dependent biases are modeled. The first bias is the transponder delay of the aircraft. Each aircraft responds to the SSR interrogation with a determined delay. The standard norm impose that this delay can has a tolerance that can produce a bias between -75m and 75m over the true range. The second target dependent bias is produced by the time stamp of the measures in ADS-B surveillance. Finally the bias produced by the velocity measurement is modeled for ADS-B surveillance or enhanced radar surveillance. In some cases the velocity measurement made by the airborne device of the aircraft can be slightly deviated. When the tracking filter uses the velocity measures the tracking performance is affected when the velocity measure is erroneous.

In the third chapter the estimation methods are described, first in a general way and finally particularized to bias estimation in ATM systems. The current estimation methods that are present in the literature have been described showing their advantages and disadvantages. In this chapter one of the methods is used to test the model presented in the second chapter. The model presented is tested using both simulated and real data. The evaluation using simulated data is used to determine if the model is observable and the parameters can be estimated in a realistic scenario. The complete model improves the estimation over the use of the simple model presented in the literature. It has been demonstrated that including more parameters in the estimation is translated into a better alignment of the plots.

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Once has been proved that the modeled parameter can be estimated using only the information from the radars the model is tested using real data. In this case if the difficult the measurement of the performance because the ideal positions of the aircraft are not known. For this evaluation is calculated the RMS value of the difference of the corrected plots and the estimated trajectory of the aircraft. This measure give us a value of how near are the trajectories from the different sensors. Several different tests are done in order to prove the performance of the model. As it is shown in the results section of this chapter, the same data is tested with the same model but using a different number of model parameters. The bigger reduction of the error is achieved when the complete model is used.

The fourth chapter introduces some proposals for the improvement of the estimation methods described in chapter 3 and describes widely the observability of these estimation methods. One of the existent problems in the estimation theory applied to the bias estimation is the initialization of the algorithms. In a tracker the initialization can be done analytically because with two measures is easy estimated the speed of the aircraft but for the biases is very difficult estimate an initial value for the state vector of the algorithm.

In the literature this problem is not discussed in the major part of the works and usually the converge is assumed for the deductions of the algorithms. In real systems the initial value of the biases is unknown. In this chapter some initialization techniques has been proposed for online applications. The studied initialization techniques include the combination of algorithms or the dynamic change of the temporal windows for the performance optimization. As it is shown in the result section, the proposed techniques increase the convergence speed and the stability of the algorithms. The disadvantages of these techniques are a moderate increment of the algorithms complexity and workload.

The observability is a topic that is mentioned in the works of the bias estimation but usually the observability of the problem is assumed. In this chapter a global description of the observability has been given focusing in the difference between the observability problems produced by sensor and aircraft geometrical distribution and produced by the numerical problems. An easy technique has been provided in order to deal with the numerical

problems. This solutions need of the configuration for each scenario and the known of an approximated value (order of magnitude) of the biases.

Focusing on the observability due to geometrical aspects of the scenario a test has been proposed. This test analyzes the covariance matrix resultant of the estimation and deduces the scenario observability. It is important to remark that this test is done online and can detect dynamically the changes of the scenarios. In the case that the test indicates lack of observability, a technique is proposed to deal with the groups of radars with observability problems. This technique is the alignment with the best radar of the area estimating the relative biases between the different sensors, assuming that the observability is produced because there are two sensors in the same location.

Finally in chapter 5 several contributions to the estimation methods are proposed. The first topic of this chapter is the use of an extension for the atmospheric model using surfaces for the interpolation of the pressure and temperature in the space. In a wide area the atmospheric conditions are not constant and the estimation using a single parameter can be not enough.

For the extension of the meteorological model some surfaces are studied. In this chapter some algorithms are described for the interpolation of a grid of fixed points. Evaluating the advantages and disadvantages from each algorithm, the Inverse Distance Weighting is used. For the test of this extension two scenarios with simulated data are used. The first scenario has a huge number of point uniformly distributed with known positions and a single sensor centered in the scenario. Using this synthetic scenario it is shown that the model can be estimated and that the performance is better when the number of fixed points is increased.

The second experiment is done with realistic simulated data, with distributed sensors and 150 trajectories. In this case is shown that the performance of the model is limited with the number of plots. In this case the performance of the algorithm is degraded when the number of fixed points of the grid is increased. In the view of these results it is demonstrated that using a four points grid allocated in the extremes of the scenario. This reduces the RMS error of the altitude estimation. Using a grid with a higher number of parameters the error introduces is growing with the size of the grid. The improvement of this extension is leaved for the future work.

A modification for a current estimation method is proposed. The current method estimates the biases in three phases. The first phase estimate the mixed sensor-target dependent biases for each trajectories. The second phase assumes zero mean in the target dependent biases and estimate the sensor dependent biases averaging the biases from all the trajectories. Finally the target dependent biases are calculated subtracting the sensor biases from the sensor-target biases. In the modification both sensor and target biases are estimated jointly in the second phase.

The estimation method for the determination of the velocity bias is useful for the use of velocity measure in the tracking filter. When the measures are biased, the use of these measures is worse than using just the position measures. Incorporating the estimation method the accuracy of the filters is increased paying it with an increment of the computation workload.

6.1 Contributions

The main contributions to the state-of-art on bias estimation applied to ATM scenarios are listed below:

- A complete parametric model for the SSR systematic errors has been developed. This model includes the systematic errors in azimuth, range, altitude and time produced by the installation error but also produced by the physical phenomena of the measures as the propagation or the atmospheric conditions. This model gives an improvement over the current model of the literature. In the test of the model using real data, the complete model gets a significant improvement about 35-40% of reduction of the error compared with the basic model used in the literature.
- Some initialization techniques has been proposed for the current bias estimation methods. In these methods the linearization errors produces instabilities that make slow the convergence of the algorithms. With the proposed techniques, the stability and speed of convergence are improved compared with the current estimation methods

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- An online algorithm is proposed to check the observability of the scenarios. In real scenarios the allocation of the trajectories change dynamically and the observability conditions can be unfulfilled. This algorithm is executed in parallel with the bias estimation method and gives a solution in the cases of observability lack.
- A horizontal extension of the meteorological model is proposed. This extension gives a grid of fixed points with values of pressure and temperature biases. The temperature and pressure biases in the aircraft positions are estimated using surfaces to interpolate the values from the grid of the fixed points.
- The modification included for the estimation of the target dependent biases gets a low improvement compared to the number of calculation and the changes in the structure that must be done.
- Finally a method to estimate the biases in the velocity of the aircraft is proposed. This method improves the accuracy of the trackers when the velocity measurements are biased. And reduced the error of the residual when these measures are received.

6.2 Future Work

In this thesis some of the current problems that are present in the ATM scenarios has been solved but there are multitude of existent problems that must be solved. In this section some of the problems related with the study field of this are identified and briefly described.

The complete and systematic error for SSR sensor has been developed in this thesis. In future works can be studied a way to estimate the biases of the pseudodistances used by the ADS-B and WAM sensor using just the position measurements. It is difficult to deal with problem because in the tracker of the measures the information of which WAM station make the measures or the visible satellite for the navigation of the aircraft is not available.

In this thesis the estimation methods have used just the information from the SSR sensors for the evaluation of the bias estimation algorithms performances. The use of more advanced sensors as ADS-B or WAM can improve

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the accuracy of the methods but they must be designed for the exploitation of all the available information. The future work is the adaptation of the algorithms to these kinds of sensors.

In this thesis has been presented an extension of the meteorological model. The parameters of pressure and temperature are estimated using the information of the SSR sensors. The future work related with this extension of the meteorological model is the use of the geometric altitude (when it is available) in order to improve the observability of the scenarios. Using only the SSR measures, the vertical error must be estimated with the horizontal projection. Using the geometric altitude (unbiased) measures (usually available in the aircraft but they are not used) the temperature and pressure biases can be estimated more accurately than using just horizontal position measures. Another future work related with this extension is the use of interpolations without discontinuities in frontiers of the points grid.

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Acronyms

ADS-B Automatic Dependent Surveillance - Broadcast

ATC Air Traffic Control

ATCO Air Traffic Controller

ATM Air Traffic Management

DME Distance Measure Equipment

DSN Deep Space Networks

ECEF Earth-Centered Earth-Fixed

EKF Extended Kalman Filter

GNSS Global Navigation Satellite System

GPS Global Position System

ICAO International Civil Aviation Organization

IDW Inverse Distance Weighting

ILS Instrumental Landing System

INS Inertial Navigational System

ISA International Standard Atmosphere

MSL Mean Sea Level

MSE Mean Square Error

MSSR Monopulse Secondary Surveillance Radar

NDB Not Directional Beacon

ACRONYMS

PSR Primary Surveillance Radar

RMS Root Mean Square

SESAR Single European Sky ATM Research

SMR Surface Movement Radar

SSR Secondary Surveillance Radar

TACAN Tactical Air Navigation System

TCAS Traffic alert and Collision Avoidance System

TDOA Time Difference Of Arrival

VOR VHF Omnidirectional Radio range

WAM Wide Area Multilateration

WLS Weighted Least Squares

Appendix A

Appendix: Radar Model Error Analysis

In the chapter 2 a complete and rigorous model have been developed for the systematic error of the SSR sensor. In this chapter the macrobias (sensor dependent) has been developed for radars and some microbiases (target dependent) has been developed for different kind of sensors as ADS-B. The complete parametric model has been developed and it has been adapted for ATM applications.

In the installation of the ATM equipment a very rigorous calibration is applied and installation error usually are small, but with the age and the periodic maintenance the mechanical pieces of the sensor can have small modifications.

For example in the installation of the antenna, it can be installed rotated a small angle. As the calibration can be very accurate we can assume that the error in the angle will be smaller than 1 degree (This error is almost 2cm of error with each meter).

Assuming that some of the errors are very low, some approximations have been made in order to get a linear model that can be used to in the estimation methods presented in chapter 2. Some of this linearizations introduces error that will be quantified and evaluated in this appendix.

In the next sections the approximations will be explained and evaluated

A.1. ANTENNA DEVIATION ERROR

comparing the error with the real value mainly for the SSR sensor.

In the chapter 2 the source of the mechanical and physical errors has been explained and both the exact models and their approximations have been widely discussed.

In the chapter 2 the approximation errors have been described as negligible but without any justification. In the next sections the error for the approximations will be calculated and the negligibility assumption will be justified.

In the next section will be evaluated the error of the Taylor Series for the antenna squint (2.2.2.1), the Axis Skew (2.2.2.2) and for both biases of the optical encoder (2.2.2.3). The error in the approximation of the spherical triangle as a right triangle will be described and the linearization of the curve for the altitude bias will be evaluated too.

A.1 Antenna Deviation Error

The antenna deviation describes the bias produced by the rotations of the antenna widely discussed in 2.2.2.1. The analytic error has been developed using the rotation of the coordinated axes and linearizing the equations using a first order Tailor series.

In this section the super index i means that the approximation is made with a i-th grade Taylor series. Following the equations described in equations 2.8 and 2.10 the error in the estimation can be defined as ϵ_{ant}^i .

$$\epsilon_{ant}^1 = \theta_{m,ant} - \theta_0 - s_{ant} \tan(\varphi) \quad (A.1)$$

The exact value of the angle measured by the antenna described in the equation 2.8 is calculated without approximations:

$$\theta_{m,ant} = \arctan\left(\frac{x'}{y'}\right) \quad (A.2)$$

where x' and y' can be calculated as:

APPENDIX A. APPENDIX: RADAR MODEL ERROR ANALYSIS

$$\begin{aligned}
 x' &= [\cos \theta_0 \cos S - \sin t \sin S \sin \theta_0] x - \\
 &\quad - \cos t \sin \theta_0 y + [\cos \theta_0 \sin S + \sin t \cos S \sin \theta_0] z \\
 y' &= [\sin \theta_0 \cos S - \sin t \sin S \cos \theta_0] x - \\
 &\quad - \cos t \cos \theta_0 y + [\sin \theta_0 \sin S + \sin t \cos S \sin \theta_0] z
 \end{aligned} \tag{A.3}$$

and x, y, z is the ideal measurement expressed in Cartesian coordinates and θ_0, S and t are the azimuth offset, the antenna squint and the tilt of the antenna respectively.

As it has been shown in the section 2.2.2.1 the Taylor series is:

$$\begin{aligned}
 \theta_{m,ant} &\approx \theta + \theta_0 - s_{ant} \tan(\varphi) \cos(\theta) + t_{ant} \tan(\varphi) \sin(\theta) + \\
 &\quad + \left(\frac{1}{2} \sin(\theta) \cos(\theta) - \frac{\sin(\theta) \cos(\theta)}{\cos^2(\varphi)} \right) s_{ant}^2 - \\
 &\quad - \left(\frac{1}{2} \sin(\theta) \cos(\theta) - \frac{\sin(\theta) \cos(\theta)}{\cos^2(\varphi)} \right) t_{ant}^2 + \\
 &\quad + \left(\cos^2(\theta) - \frac{2\cos^2(\theta)}{\cos^2(\varphi)} + \frac{1}{\cos^2(\varphi)} \right) t_{ant}s_{ant} + \dots
 \end{aligned} \tag{A.4}$$

In this equation the second order Taylor series is shown. As the antennas used in ATM for surveillance applications have very high directivity the measuring azimuth is approximately zero ($\theta = 0$) the Taylor series is:

$$\theta_{m,ant} \approx \theta_0 - s_{ant} \tan(\varphi) + \left(1 - \frac{1}{\cos^2(\varphi)} \right) t_{ant}s_{ant} + \dots \tag{A.5}$$

In the figures A.1 and A.2 the error produced by the approximation is shown. In the model developed in the chapter 2 the antenna deviation is modeled using only the squint parameter. The tilt parameter is not used because the antenna only can acquire measurement from azimuth zero and a rotation around an axes perpendicular to the vertical plane of the antenna will not affect to the measurement.

In the figures A.1 and A.2 the legend indicates the different Squint angles used in the simulation. The figure A.1 has been simulated using a tilt of 0 degrees and the figure A.2 has been simulated using a tilt of 1 degree.

A.1. ANTENNA DEVIATION ERROR

Figure A.1: Error of the Taylor order 1 approximation to antenna deviation with the antenna tilt of 0 degrees.

In the worst case when the elevation is 35° and the antenna squint is 1.5° the error produced due to the approximation is $2 \cdot 10^{-4}$ when the bias produced by the antenna with this values is near to 1° .

In the previous paragraph we can remark two important topics. The first one is that the bigger error is produced when the antenna squint is bigger than expected. With a squint of 1.5° the deviation of the position of the antenna with respect to its ideal position is 2.62 cm for each meter length of the antenna. This value is out of the limits of tolerance in the installation of this kind of devices. The expected values for this kind of deviations is lower than 1 degree. The second one is that the errors are increased with the elevation. The higher is the elevation the lower is the range of the measurement. The projection of the azimuth errors is proportional to the range of the measurement and then the projection of the bigger errors is small because of the low range.

APPENDIX A. APPENDIX: RADAR MODEL ERROR ANALYSIS

Figure A.2: Error of the Taylor order 1 approximation to antenna deviation with the antenna tilt of 1 degrees.

A.2 Axis Skew Error

In section 2.2.2.2 the azimuth bias produced by the inclination of the rotation axis is described. In the previous section where the approximation error of the antenna deviation has been analyzed just one parameter has been used for the modeling of the bias. In this case the axes are fixed and there are two parameter modeling the azimuth bias.

Unlike the previous case, in the axis skew the assumption of azimuth zero ($\theta = 0$) cannot be done, because in this case the targets are measured from all the directions (the azimuth reference is the north and is not the pointing of the antenna). In this case the shown figures will be simulated with a fixed elevation and the different axis tilt and axis squint will be used.

As in the previous case the error of the approximation can be calculated as:

A.2. AXIS SKEW ERROR

$$\epsilon_{ant}^1 = \theta_{m,axis} - t_{axis} \tan(\varphi) \sin(\theta) + s_{axis} \tan(\varphi) \cos(\theta) \quad (A.6)$$

where $\theta_{m,axis}$ is the measured azimuth that is calculated as it has been developed in section 2.2.2.2 following the same procedure than in the previous section A.1 $\left(\theta_{m,axis} = \arctan\left(\frac{x'}{y'}\right)\right)$.

In this case the order 2 Taylor series is:

$$\begin{aligned} \theta_{m,axis} \approx & \theta + \theta_0 - s_{axis} \tan(\varphi) \cos(\theta) + t_{axis} \tan(\varphi) \sin(\theta) + \\ & + \left(\frac{1}{2} \sin(\theta) \cos(\theta) - \frac{\sin(\theta) \cos(\theta)}{\cos^2(\varphi)}\right) s_{axis}^2 - \\ & - \left(\frac{1}{2} \sin(\theta) \cos(\theta) - \frac{\sin(\theta) \cos(\theta)}{\cos^2(\varphi)}\right) t_{axis}^2 + \\ & + \left(\cos^2(\theta) - \frac{2\cos^2(\theta)}{\cos^2(\varphi)} + \frac{1}{\cos^2(\varphi)}\right) t_{axis}s_{axis} + \dots \end{aligned} \quad (A.7)$$

In this case there are first order terms with the tilt parameter that can be ignored ($t_{axis} \tan(\varphi) \sin(\theta)$). The second order terms has been ignored in the model because of the tilt and the squint are assumed reduced (tilt < 1° and squint < 1°).

In the figure A.3 it is shown several curves. Each curve represent the bias error produced by several squint-tilt pairs. In this figure the tilt and the squint used are equals (tilt = squint) because this is the extreme case. If in this simulation is used the tilt zero the values of the error are similar to values calculated in the previous section and shown in the figure A.1. For intermediate values of the tilt ($0 < \text{tilt} < \text{squint}$) the value of the bias is lower than the values presented in the figure.

In the worst case shown in the figure A.3 the error produced by the approximation is 0.04°. This error cannot be ignored but in this case we can make an important remark. This extreme error has been simulated using and squint and a tilt of 1.5 degrees. In the section 2.2.2.2 the rotation axis of the radar is not inclined in two different directions. It is inclined just one time in one direction. The squint and the tilt are used in the modeling. Following the equations 2.17 and 2.18 (sum of the triangle of the figure A.6) of the chapter 2 the inclination of the rotation axis is 2.12°.

APPENDIX A. APPENDIX: RADAR MODEL ERROR ANALYSIS

Figure A.3: Error of the Taylor order 1 approximation of the axis skew with an elevation of 15° .

Using reasonable and realistic values of the axis skew with an inclination lower than 1° the max error is reduced to $5 \cdot 10^{-3}$.

The solution to reduce much more the produced error is the use of a second order Taylor series. As it can be seen in the figure A.4 the error of the approximation is reduced in the worst case to $3 \cdot 10^{-4}$.

A.3 Right Triangle

In the chapter 2 the axis skew is an inclination (β_{axis}) in one direction (α_{axis}). This bias has been modeled using two rotations in two perpendicular planes as it can be seen in the figure A.5 the conversion between both models can be done using a spherical triangle with one angle of 90° .

Using spherical trigonometry the triangle of the figure A.5 can be resolved using the nomenclature of the figure:

A.3. RIGHT TRIANGLE

Figure A.4: Error of the Taylor order 2 approximation of the axis skew with an elevation of 15°.

Figure A.5: Equivalence between the inclination of the axis (β_{axis}) and its direction (α_{axis}) and the inclinations in two perpendicular planes (squint (s_{axis}) and tilt (t_{axis}))

$$\begin{aligned} \cos \beta &= \cos t \cos s \\ \tan(\alpha) &= \frac{\tan(s)}{\sin(t)} \end{aligned} \tag{A.8}$$

APPENDIX A. APPENDIX: RADAR MODEL ERROR ANALYSIS

and then, the axis inclination (β_{axis}) and its direction (α_{axis}) are:

$$\begin{aligned}\beta &= \arccos(\cos t \cos s) \\ \alpha &= \arctan\left(\frac{\tan(s)}{\sin(t)}\right)\end{aligned}\tag{A.9}$$

Figure A.6: Right triangle equivalent to the spherical triangle.

The approximation made for this triangle is shown in the figure A.6. The equations of the right triangle are known with elemental trigonometry and the resultant equations are:

$$\begin{aligned}\tilde{\beta}_{axis} &= \sqrt{t_{axis}^2 + s_{axis}^2} \\ \tilde{\alpha}_{axis} &= \arctan\left(\frac{s_{axis}}{t_{axis}}\right)\end{aligned}\tag{A.10}$$

The maximum error of this approximation is made when the absolute value of the squint and the tilt are equal. When the squint or the tilt are zero, the error will be zero because the equation are the same.

In the figure A.7 in the horizontal axis is represented the squint of the axis and the legend indicates the different tilts used for the curves. In the vertical axis the error of the inclination axis approximation is indicated. As it can be seen, the maximum error is $1.2 \cdot 10^{-4}$ degrees with very high values of squint and tilt. Even with this unrealistic values of the inclination, the error in the inclination angle is negligible.

The figure A.8 represent the direction of the inclination of the axis. In this case the error of the approximation of the extreme values is 0.01° . This error can be assumed because small changes in the direction of the inclination are not very significant in the projection of the error.

A.4. OPTICAL ENCODER: ECCENTRICITY

Figure A.7: Error of axis inclination (β_{axis}) approximation

A.4 Optical Encoder: Eccentricity

The equations optical encoder error model are similar to the axis skew, but in this case the deduction to get those equation has been very different. In this case the azimuth error do not depend on the elevation angle and the deduction is get through the solving of triangles on a circle. The complete deduction of this model is described in section 2.2.2.3. In this section the focus is on the exact measured azimuth:

$$\theta_m = \arcsin\left(\frac{r}{R} \sin(\theta)\right) \quad (\text{A.11})$$

and the approximated measured azimuth:

$$\tilde{\theta}_m \approx \theta - \frac{\Delta R}{R} \sin(\theta) \quad (\text{A.12})$$

APPENDIX A. APPENDIX: RADAR MODEL ERROR ANALYSIS

Figure A.8: Error of axis inclination direction (α_{axis}) approximation

In the figure A.9 it is shown the bias in azimuth produced by encoder eccentricity. In the horizontal axis is shown the ideal azimuth, in the vertical axis the azimuth bias and the legend indicate the different values of $\frac{\Delta R}{R}$ used in the calculations.

In the figure A.10 it is shown the error produced with the approximation made for the calculation of the approximated bias. The function shown in this figure is $\epsilon = \tilde{\theta}_m - \theta_m$. In this figure the horizontal axis represents the ideal azimuth, the vertical axis represents the error produced and the legend indicate the different values of $\frac{\Delta R}{R}$ used in the calculations as in the previous figure. As it can be seen in this figure the error of the approximation is very low but the error rise exponentially with the growth of $\frac{\Delta R}{R}$. That is because the approximation of $\frac{\Delta R}{R} \ll 1$ is not true with the growth of this value. In [Hudgens et al., 2010] is indicated that in a encoder of 52 mm the error in the encoder is about 1.3 μm that makes $\frac{\Delta R}{R} \approx 2.5 \cdot 10^{-5}$.

A.4. OPTICAL ENCODER: ECCENTRICITY

Figure A.9: Values of the azimuth bias due to the encoder eccentricity

Figure A.10: Error of the Encoder Eccentricity approximation

A.5 Optical Encoder: Swash

The encoder Swash is slightly different to the previous biases analyzed in this chapter. The other biases are approximated using a Taylor series of order 1. The encoder swash is approximated using a Taylor series of order two as it has been described in the section 2.2.2.3.

In this section the error using a Taylor series of order one is not shown, because it is the value of bias. As in previous section, in the figure A.11 it is shown the azimuth bias produced by the encoder swash. In the horizontal axis is shown the ideal azimuth, in the vertical axis the azimuth bias and in the legend are indicated the different angles of swash of the encoder. In this figure only one line is seen for each swash angle. That is because both painted lines are similar and one is hide due to the other.

Figure A.11: Value of the azimuth bias due to the Encoder Swash approximation

The function show in the aforementioned figure is:

A.5. OPTICAL ENCODER: SWASH

Figure A.12: Error of the Encoder Swash approximation

$$\theta_m = \text{atan}(\cos(s_{enc}) \tan(\theta)) \quad (\text{A.13})$$

and its approximation:

$$\tilde{\theta}_m = \theta - \frac{s_{enc}^2}{4} \sin(2\theta) \quad (\text{A.14})$$

In the figure A.12 the error produced in the approximation is shown. It is shown the second order Taylor series and as it can be seen the error produced is much lower than the bias. With an error of 10^{-7} , a measured aircraft at 200km produces errors lower than 1mm.

The total error produced by the approximations is difficult to estimate. As the value of the biases depends on the azimuth of the measure in some areas the biases will be null and in other areas the biases are added. In the worst case the azimuth error is near to $5 \cdot 10^{-3^\circ}$. This error in azimuth implies about 17 meters at 200 km of range. The main source of error is the axis skew but this bias depends on the elevation of the measure. At 200 km the elevation of the measure is very low. The elevations that produces the aforementioned error is present when the range of the measure is about 30

APPENDIX A. APPENDIX: RADAR MODEL ERROR ANALYSIS

km to the radar. In this case the error produced by the approximation is about 2 meters.

Appendix B

Appendix: Bias Estimation Methods

In this appendix the estimation methods used in this thesis are described. The equations of all the methods are indicated here in order to give a reference to the lecture without consult these method in external documents.

B.1 Kalman filter

The Kalman filter is an optimal estimator of linear systems. This method estimates the state in two phases: prediction and update. The estimation is based in a linear model for the prediction of the state and another one for the measurement with additive noises:

$$\begin{aligned} X(k+1) &= \phi(k)X(k) + u(k) \\ z(k) &= H(k)X(k) + w(k) \end{aligned} \tag{B.1}$$

where X is the state vector, ϕ the state-transition matrix, H the measurement matrix, z the measurement vector, u the plant noise and w the measurement noise. The state vector has an associated covariance matrix ($P(k)$) that is defined as:

$$P(k) = E [(X(k) - \bar{X}(k))(X(k) - \bar{X}(k))^T] \tag{B.2}$$

B.1. KALMAN FILTER

The first phase of the Kalman filter is the prediction phase. In this phase it is known the previous state vector and the previous covariance matrix calculated with the previous measures and the state-transition matrix designed with the dynamical model of the system. The predicted state and predicted covariance matrix are:

$$\begin{aligned}\hat{X}_k &= \phi_{k-1} X_{k-1} \\ \hat{P}_k &= \phi_{k-1} P_{k-1} \phi_{k-1}^T + Q\end{aligned}\tag{B.3}$$

In this expression the matrix Q is the modeling of the plant noise term. With the estimated state, the update phase can be made. The difference between the current measure and the current prediction is called residual (r). For the update phase the residual and the covariance of the residual (S) are calculated:

$$\begin{aligned}r_k &= z_k - H_k \hat{X}_k \\ S_k &= H \hat{P}_k H^T + R\end{aligned}\tag{B.4}$$

The covariance of the residual is calculated using the noise matrix R and the resultant value depends on the statistics of the measure and the previous covariances. It does not depends on the current value of the measure. Using the covariance of the residual the Kalman gain (K) is calculated. This gain is used to give more weight to the measure or to the prediction.

$$K_k = \hat{P}_k H^T S_k^{-1}\tag{B.5}$$

Finally the updated state vector and state covariance matrix are calculated using the terms that have been calculated previously:

$$\begin{aligned}X_k &= \hat{X}_k + K_k r_k \\ P_k &= (I - K_k H) \hat{P}_k\end{aligned}\tag{B.6}$$

Straight Kalman filter example: In order to follow a rectilinear uniform movement in the horizontal plane the state vector must have four component, two for the position and two for the velocity. The state covariance matrix has dimensions 4x4:

APPENDIX B. APPENDIX: BIAS ESTIMATION METHODS

$$X = \begin{pmatrix} x \\ y \\ v_x \\ v_y \end{pmatrix} \quad P = \begin{pmatrix} \sigma_x^2 & \sigma_{x,y} & \sigma_{x,v_x} & \sigma_{x,v_y} \\ \sigma_{x,y} & \sigma_y^2 & \sigma_{y,v_x} & \sigma_{y,v_y} \\ \sigma_{x,v_x} & \sigma_{y,v_x} & \sigma_{v_x}^2 & \sigma_{v_x,v_y} \\ \sigma_{x,v_y} & \sigma_{y,v_y} & \sigma_{v_x,v_y} & \sigma_{v_y}^2 \end{pmatrix} \quad (\text{B.7})$$

The state transition matrix is calculate with the equation of the uniform movement:

$$\phi_k = \begin{pmatrix} 1 & 0 & \Delta T & 0 \\ 0 & 1 & 0 & \Delta T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (\text{B.8})$$

In this point there are two different filters that can be designed. If the sensor has only position measures and if the sensor has also velocity measures. For the first case, the measurement matrix H_1 is described and for the second case with velocity measure is described the matrix H_2 .

$$H_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad H_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (\text{B.9})$$

The covariance of the noise matrix has the same dimensions of the measurement vector. If the measure is only in position, the matrix is 2x2 (R_1) and if the measures has also velocity measurements, the matrix is 4x4 (R_2)

$$R_1 = \begin{pmatrix} \sigma_x^2 & \sigma_{x,y} \\ \sigma_{x,y} & \sigma_y^2 \end{pmatrix} \quad R_2 = \begin{pmatrix} \sigma_x^2 & \sigma_{x,y} & \sigma_{x,v_x} & \sigma_{x,v_y} \\ \sigma_{x,y} & \sigma_y^2 & \sigma_{y,v_x} & \sigma_{y,v_y} \\ \sigma_{x,v_x} & \sigma_{y,v_x} & \sigma_{v_x}^2 & \sigma_{v_x,v_y} \\ \sigma_{x,v_y} & \sigma_{y,v_y} & \sigma_{v_x,v_y} & \sigma_{v_y}^2 \end{pmatrix} \quad (\text{B.10})$$

The noise plant matrix depends on the model considered for the noise plant of the dynamic model. In [Bogler, 1990] is explained how to calculate the plant noise for different distributions of the acceleration noise. In this appendix it is assumed that the movement of the aircraft has an uncorrelated random noise in the acceleration (or that the correlation is much lower than the sampling rate). With that uncorrelated model, the expression of the Q

B.2. EXTENDED KALMAN FILTER

matrix is:

$$Q = \begin{pmatrix} \frac{\Delta T^4}{4} & 0 & \frac{\Delta T^3}{2} & 0 \\ 0 & \frac{\Delta T^4}{4} & 0 & \frac{\Delta T^3}{2} \\ \frac{\Delta T^3}{2} & 0 & \Delta T^2 & 0 \\ 0 & \frac{\Delta T^3}{2} & 0 & \Delta T^2 \end{pmatrix} \quad (\text{B.11})$$

The complete deduction for these equation can be found in references as [Bell, 2010], [Zarchan, 2005], [Brookner, 1998] or [Simon, 2006].

B.2 Extended Kalman filter

The EKF [Julier and Uhlmann, 2004] is a modification of the Kalman filter in order to estimate state in systems with nonlinear dynamic or measurement models. The extended Kalman filter is not an optimal estimator but the behavior of this filter is similar to the linear Kalman filter. The dynamical and measurement model are described below where f and h are nonlinear functions.

$$\begin{aligned} X(k+1) &= f(X(k)) + u(k) \\ z(k) &= h(X(k)) + w(k) \end{aligned} \quad (\text{B.12})$$

Using a linearization process the prediction of the estate can be approximated with the next equations:

$$\begin{aligned} \hat{X}_k &= f(X_{k-1}) \\ \hat{P}_k &= \tilde{\phi}_{k-1} P_{k-1} \tilde{\phi}_{k-1}^T + Q \end{aligned} \quad (\text{B.13})$$

where $\tilde{\phi}_{k-1}$ is the Jacobian matrix of the state transition function $f(\cdot)$. The prediction of the state is done with the exact function f but the prediction of the covariance is done with a linearized matrix.

The residual and its covariance are calculated as below (the projection of the measure is done with the exact function and the projection of the covariance is done with the linearized matrix).

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$$\begin{aligned} r_k &= z_k - h(\hat{X}_k) \\ S_k &= \tilde{H} \hat{P}_k \tilde{H}^T + R \end{aligned} \quad (\text{B.14})$$

where $\tilde{H}$ is the Jacobian matrix of the measurement function $h(\cdot)$. The next steps are equal to the Kalman filter equation changing the measurement matrix (H) by the Jacobian of the measurement function ($\tilde{H}$).

$$\begin{aligned} K_k &= \hat{P}_k \tilde{H}^T S_k^{-1} \\ \hat{X}_k &= \hat{X}_k + K_k r_k \\ \hat{P}_k &= (I - K_k \tilde{H}) \hat{P}_k \end{aligned} \quad (\text{B.15})$$

This method works properly and it is unstable when the non linearity of the functions are not too strong. When linearization errors of the functions are not too big, this method presents a very good performances.

B.3 Bias Estimation with Temporal Windows

In this section is described the method based in temporal windows described in section 3.3.1 developed by Henk A.P. Blom, René A. Hogendoorn and Bas A. Van Doorn in [Bar-Shalom and Blair, 1992].

In this section the nomenclature of the variables is coherent with this document and it is different to the nomenclature used in the original document.

This method assumes that there is a tracking filter that estimates the state of each aircraft in the airspace for the defined temporal window based on the corrected plots with the previous estimations. Each tracking filter estimates the residual of each measure and the covariance of the residual.

The prediction model of this bias filter is described as:

$$\begin{aligned} \hat{b}_k &= E\{b_0\} + a[b_{k-1} - E\{b_0\}] \\ P\hat{b}_k &= E\{Pb_0\} + a[Pb_{k-1} - E\{Pb_0\}] \end{aligned} \quad (\text{B.16})$$

In this prediction equation is assumed that the value of the biases are changing due to changing wind direction or technical maintenance. Because of that the values of the biases are "tied" to the initial values (b_0, Pb_0). If the value of the biases can change freely, then the value of a is one.

B.3. BIAS ESTIMATION WITH TEMPORAL WINDOWS

The update equations of this filters are:

$$\begin{aligned} b_k &\approx \hat{b}_k + P\hat{b}_k \left[\sum_{i=1}^n \sum_{k \in (t-1, t]} v_{i,k} \right] \\ Pb_k &\approx P\hat{b}_k - P\hat{b}_k \left[\sum_{i=1}^n \sum_{k \in (t-1, t]} V_{i,k} \right] P\hat{b}_k^T \end{aligned} \quad (\text{B.17})$$

In this equation the terms v_{i,X_k} and V_{i,X_k} are the accumulated projected residual and the accumulated projected covariance of the measures. Each value is calculated as follows

$$\begin{aligned} v_{i,k} &= [H^* B_i - \bar{F}_t]^T E_{i,k}^{-1} H^* (z - \hat{X}_k) \\ V_{i,k} &= [H^* B_i - \bar{F}_t]^T E_{i,k}^{-1} [H^* B_i - \bar{F}_t] \end{aligned} \quad (\text{B.18})$$

Where H^* is the left inverse of H ($H^* = (H^T H)^{-1} H^T$) and $\bar{F}_t$ is defined as:

$$\bar{F}_t = \left[\sum_{i=1}^n \Phi_i S_{i,t}^{-1} \right]^{-1} \sum_{i=1}^n [\Phi_i S_{i,t}^{-1} H^* B_i] \quad (\text{B.19})$$

The matrix S is the noise matrix (R) of the measurement vector projected to the state vector ($S = H^* R H^{*T}$) and the matrix E is the sum of this noise projected to the biases plus the predicted covariance of the tracking filter. Φ_i indicates the refresh rate of the radar. In order to give the same weight to all the radars, each measure from radars with higher rate have less weight.

The computation of $\bar{F}_t$ includes a summatory with the projection matrix B_i . The computation of this matrix can be problematic in terms of computation efficiency when the number of radars is high. The measurement is made by the radar j , in the computation of $\bar{F}_t$ a "synthetic" measure must be calculated transforming the measure of the radar j to a measure of the radar i .

In the figure B.1 it is represented an scenario with two radars and a single aircraft. The current position of the aircraft ($X(k)$) is measured by the radar 1. The measure $z(k)$ is used in the tracker. For the computation of the matrix $\bar{F}_t$ the pair $(\rho'_{m,2}$ and $\theta'_{m,2})$ must be calculated. This measure is virtual and it is used only to know how the biases of the first radar are projected over the

APPENDIX B. APPENDIX: BIAS ESTIMATION METHODS

biases of the second through the measures. With the virtual measure ($\rho'_{m,2}$ and $\theta'_{m,2}$) of the second radar the matrix B_2 is calculated.

Figure B.1: Synthetic measure for the radar 2 from a measure get from radar 1

B.4 Multisensor-Multitarget Bias Estimation for General Asynchronous Sensors

In the section 3.3.2 is described the way to get the pseudomeasurement for a bias filter using synchronous sensor as it is described in [Lin et al., 2003]. In [Lin et al., 2005] it is described a method to get the pseudomeasurements with asynchronous sensors.

In the aforementioned paper the plots are grouped in slots that have a common characteristic. In a proper slot there is two or more measures of a sensor and one measure of another sensor. In the figure B.2 are shown some example that accomplish this condition. This figure has been extracted from [Lin et al., 2005]

B.4. MULTISENSOR-MULTITARGET BIAS ESTIMATION FOR GENERAL ASYNCHRONOUS SENSORS

Figure B.2: Synthetic measure for the radar 2 from a measure get from radar 1

The easiest example is obtained with the first slot where the first and the last measure is from the first radar and the measure of the middle is from the second radar. As the three measures have different time stamp all of them must be interpolated (or extrapolated) to the same time. As the proper slots just have one measure from the second radar this measure will be fixed in time. The measures from the other radar will be interpolated to the time of the measure of the second radar.

The next equations are made assuming that the aircraft has a straight uniform movement. If the aircraft is maneuvering (turn or acceleration) this assumption is valid because in the time of a proper slot the classic maneuvers in route are imperceptible. The interpolation between two measures ($z_b^1(t_1)$ and $z_b^1(t_3)$) is:

$$\begin{aligned} z_b^1(t_2) &= \alpha_1 z_b^1(t_1) + \alpha_3 z_b^1(t_3) \\ \alpha_1 &= \frac{t_3 - t_2}{t_3 - t_1} \\ \alpha_3 &= \frac{t_2 - t_1}{t_3 - t_1} \end{aligned} \quad (\text{B.20})$$

Following this expression, the pseudomeasurement used for the bias estimation is:

$$z_b = z_b^2(t_2) - z_b^1(t_2) = z_b^2(t_2) - [\alpha_1 z_b^1(t_1) + \alpha_3 z_b^1(t_3)] \quad (\text{B.21})$$

and the measurement matrix (B_i are the projection matrix of the biases from each measure):

$$H_b = [-B_1(t_2) \quad B_2(t_2)] = [-\alpha_1 B_1(t_1) - \alpha_3 B_1(t_3) \quad B_2(t_2)] \quad (\text{B.22})$$

APPENDIX B. APPENDIX: BIAS ESTIMATION METHODS

finally, the noise matrix is calculated as:

$$R = R_2(t_2) + R_1(t_2) = R_2(t_2) + \alpha_1^2 R_1(t_1) + \alpha_3^2 R_1(t_3) \quad (\text{B.23})$$

The bias can be estimated with the resultant pseudomeasurement using a Kalman filter as the described in section B.1.

Appendix C

Appendix: Changes of Coordinates

In this appendix the changes of coordinates are described. One of the basic steps in the multisensor tracking is the use of a common coordinated system for all the aircraft.

In order to have all the measures in the same coordinated system, there are several changes that must be done. The measures are taken in the local coordinates of the radar. This coordinates includes the azimuth and the distance from the radar to the aircraft and the altitude over the mean sea level of the aircraft normal to the Earth Surface. This local coordinates are converted to local Cartesian coordinates. In a multisensor scenario where the sensors are allocated in different positions of the Earth surface, the local Cartesian are referred to different point and all of them must be converted to a common system. Due to the aircraft are moving in a spherical surface, the positions must be projected over a horizontal plane in order to represent the trajectories in a screen. The tracking of the trajectories is done with this projection (stereographic projection).

In this appendix is described the complete sequence of changes of coordinates from local radar to stereographic projection. The next step to the local Cartesian coordinates is the translation to the Earth-Centered Earth-Fixed (ECEF) coordinates. From these coordinates the measures are con-

C.1. RADAR COORDINATES TO LOCAL CARTESIAN

Figure C.1: Block diagram of the changes of coordinates

verted to latitude, longitude and altitude coordinates and finally they are projected in the stereographic plane. The sequence of changes of coordinates is shown in the figure C.1.

C.1 Radar Coordinates to Local Cartesian

The radar measures the distance between the aircraft and the sensor, the azimuth of the aircraft (with the origin pointing to the north) and the altitude is measured by the airborne barometric altimeter. Assuming that the barometric altitude is the geometric altitude (this usually is not true, but this difference is estimated using the bias models provided in the chapter 2), is the distance between the aircraft and the mean sea level normal to the Earth surface in the position of the aircraft.

The curvature of the Earth must be considered in order to transform from the radar coordinates to local Cartesian coordinates. In the figure C.2 it is

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Figure C.2: Scheme of the radar measurement showing the Earth surface.

shown the scheme of the measurement. Solving the resultant triangle, the elevation is calculated as

$$\varphi = \arcsin \left(\frac{2R_l(h - h_R) + h^2 - h_r^2 - \rho^2}{2\rho(R_l + h_R)} \right) \quad (\text{C.1})$$

where h is the altitude of the aircraft, h_R is the altitude of the radar, ρ is the range of the measure and R_l is the local Earth Radius looking to the azimuth of the target calculated as:

$$R_l = \frac{1}{\frac{\cos(\theta)}{R_N} + \frac{\sin(\theta)}{R}} \quad (\text{C.2})$$

where R is the radius of the Earth calculated as in the equation C.6 and R_N is the Radius of the Earth looking to the North in the point of the radar:

$$R_N = \frac{R_0(1 + e^2)}{(\sqrt{1 - e^2 \sin^2(\Phi_R)})^3} \quad (\text{C.3})$$

Using this data, the conversion to local Cartesian (x_l, y_l, z_l) is;

C.2. LOCAL CARTESIAN TO ECEF

$$\begin{aligned}x_l &= \rho \sin(\theta) \cos(\varphi) \\y_l &= \rho \cos(\theta) \cos(\varphi) \\y_l &= \rho \sin(\varphi)\end{aligned}\tag{C.4}$$

C.2 Local Cartesian to ECEF

This conversion of coordinates depends on the sensor that make the measure. In this case the position of the sensor is known. This change of coordinates is calculated as a rotation of the local Cartesian coordinates and a translation to the center of the Earth. The position of the radar in latitude, longitude and altitude is Φ_R, λ_R, h_R .

The translation matrix is calculated as:

$$Tr = \begin{bmatrix} (R + h_R)\cos(\Phi_R)\cos(\lambda_R) \\ (R + h_R)\cos(\Phi_R)\sin(\lambda_R) \\ (R(1 - e^2) + h_R)\sin(\Phi_R) \end{bmatrix}\tag{C.5}$$

where e is the eccentricity of the Earth ellipsoid and R is the Earth radius calculated using the Ecuator radius (R_0) as:

$$R = \frac{R_0}{\sqrt{1 - e^2 \sin^2(\Phi_R)}}\tag{C.6}$$

Using the translation matrix, the coordinates are calculated as

$$\begin{bmatrix} x_{ECEF} \\ y_{ECEF} \\ z_{ECEF} \end{bmatrix} = \begin{bmatrix} -\sin(\lambda_R) & -\sin(\Phi_R)\cos(\lambda_R) & \cos(\Phi_R)\cos(\lambda_R) \\ \cos(\lambda_R) & -\sin(\Phi_R)\sin(\lambda_R) & \cos(\Phi_R)\sin(\lambda_R) \\ 0 & \cos(\Phi_R) & \sin(\Phi_R) \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \end{bmatrix} + Tr\tag{C.7}$$

C.3 ECEF to geodetic

In this section the conversion from the ECEF coordinates to geodetic is described. In this case all the measures are converted with the same parameters. The ECEF coordinated are common to all the measures and

APPENDIX C. APPENDIX: CHANGES OF COORDINATES

the conversion does not depends on the sensor that make the measure.

The altitude of the aircraft (h) is the same of the original measure and the latitude and longitude are calculated as:

$$\begin{aligned}\Phi &= \text{atan}\left(\frac{y_{ECEF}}{x_{ECEF}}\right) \\ \lambda &= \text{atan}\left(\frac{1+h/R}{1-e^2+h/R} \frac{z_{ECEF}}{\sqrt{x_{ECEF}^2+y_{ECEF}^2}}\right)\end{aligned}\tag{C.8}$$

In this equation the R is the local radius of the Earth at the radar latitude. The exact solution for this change of coordinated is an iterative process. In [EUROCONTROL, 2005b] is indicated this approximation assuming that the Earth radius is equal in the aircraft position and in the radar position. The error produced by this approximation is lower than a meter in the stereographic plane.

C.4 Stereographic projection

This projection is the proposed to use in all the thesis. In [Snyder, 1987] it is described widely this projection and the figures and equations are extracted from this book. This projection is centered in a point of the Earth. The projection of any aircraft is the cross point between the stereographic plane (orthogonal to the center point) and the line that cross the aircraft point (in the Earth surface) and the opposite point (Antipodes) of the center point. This is shown in the figure C.3 where the center point is the North Pole and the Antipodes is the South Pole.

This projection cannot show both hemispheres at the same time but in the real scenarios the coverage scenarios is not so big to cover half Earth. In this projection the great circles are not projected as straight lines when the distance between the aircraft and the projection center is big. In scenario where the distances from the center to the furthest aircraft is about 2000Km the distortion in the trajectories is small. In this cases the straight filter can track the trajectories without detect maneuvers.

C.4. STEREOGRAPHIC PROJECTION

Figure C.3: Representation of the stereographic projection

In order to make the change of coordinates, are needed the latitude and longitude of the projection center (Φ_0, λ_0) , the latitude and longitude of the aircraft (Φ, λ) and the radius and eccentricity of the ellipsoid (a, e) . The equations for the change of coordinates is:

$$\begin{aligned} x_s &= A \cos(\Phi) \sin(\lambda - \lambda_0) \\ y_s &= A [\cos(\chi_0) \sin(\chi) - \sin(\chi_0) \cos(\chi) \cos(\lambda - \lambda_0)] \end{aligned} \quad (C.9)$$

In this equation χ is the conformal latitude of Φ (calculated bellow), χ_0 is the conformal latitude of Φ_0 and the parameter A is calculated as follows:

$$\begin{aligned} \chi &= 2 \operatorname{atan} \left[\left(\frac{1 + \sin(\Phi)}{1 - \sin(\Phi)} \right) \left(\frac{1 - e \sin(\Phi)}{1 + e \sin(\Phi)} \right) \right]^{\frac{1}{2}} - \frac{\pi}{2} \\ A &= \frac{2ak_0m_0}{[\chi_0[1 + \sin(\chi_0)\sin(\chi) + \cos(\chi_0)\cos(\chi)\cos(\lambda - \lambda_0)]]} \end{aligned} \quad (C.10)$$

where k_0 is a scale parameter (usually 1), m is calculated as follows and m_0 is calculated as m but using Φ_0 instead of Φ :

$$m = \frac{\cos(\Phi)}{(1 - e^2 \sin^2(\Phi))^{\frac{1}{2}}} \quad (C.11)$$