Ageneric multi-attribute analysis system

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Abstract

This paper describes a generic decision support system based on an additive multiattribute utility model that is intended to allay many of the operational difficulties involved in the multicriteria decision-making process. The system accounts for uncertainty about the alternative consequences and admits incomplete information about the decision-makers' preferences, which leads to classes of utility functions and weight intervals. The additive model is used to assess, on the one hand, average overall utilities, on which the ranking of alternatives is based and, on the other, minimum and maximum overall utilities, which give further insight into the robustness of this ranking. When the information obtained is not meaningful enough so as to definitively recommend an alternative, an iteration process can be carried out by tightening the imprecise parameters and assessing the non-dominated and potentially optimal alternatives or using Monte Carlo simulation techniques to determine useful information about dominance among the alternatives.

1. Introduction

The generic multi-attribute analysis (GMAA) system¹ is a PC-based decision support system (DSS) based on an additive multi-attribute utility model that is intended to allay many of the operational difficulties involved in the decision analysis (DA) cycle [1–4]. This cycle can be divided into four steps:

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¹ <http://www.dia.fi.upm.es/~ajimenez/GMAA>.

structuring the problem (which includes specifying objectives, building a value hierarchy and establishing attributes for the lowest-level objectives); identifying the feasible alternatives, their impact and uncertainty (if necessary); quantifying preferences (which includes the assessment of the component attribute utilities as well as the value trade-offs); evaluating alternatives and performing sensitivity analysis (SA).

The system that we shall describe is an extension of the evaluation module developed for the European projects MOIRA (model-based computerized system for management support to identify optimal strategies for restoring radionuclide contaminated aquatic ecosystems and drainage areas) [5,6], and COMETES (implementing computerized methodologies to evaluate the effectiveness of countermeasures for restoring radionuclide contaminated fresh water ecosystems) [7]. It has been tested and validated in several real scenarios, like Lake Øvre Heimdalsvatn [8,9] or Lake Kozhanovskoe [10,11].

However, the GMAA system was not developed and implemented ad hoc for the above aquatic ecosystem restoration problem, it was designed to aid the DM in a range of complex decision-making problems where DA can be specially useful [12,13] like military systems acquisition processes [14], the analysis of alternatives for the disposition of surplus weapons-grade plutonium [15,16], etc.

In this sense, the DM can interactively build an objectives hierarchy that should include all the relevant aspects related to the complex decision-making problem under consideration.

The GMAA system accounts for uncertainty about the alternative consequences, as discussed in [17], which can be defined in terms of continuous uniformly distributed intervals instead of single values for each attribute. The system also admits incomplete information about the DM's preferences through value intervals as responses to the probability questions the DM is asked, which leads to classes of utility functions and weight intervals. This is less demanding for a single DM and also makes the system suitable for group decision support, where individual conflicting views in a group of DMs can be captured through imprecise answers.

The different alternatives under consideration can be evaluated by means of an additive multiattribute utility function. The additive model is used to assess, on the one hand, average overall utilities, on which the ranking of alternatives is based and, on the other, minimum and maximum overall utilities, which give further insight into the robustness of this ranking. Different displays of ranking results are provided, and it is also possible to select another objective to rank by.

Finally, the GMAA system provides several types of SA, like classical SA, which involves changing the parameters and observing their impact on the ranking of alternatives, or the assessment of weight stability intervals, in which an objective weight can vary maintaining a constant ratio among the other weights without affecting the overall alternative ranking.

Other types of SA are the determination of non-dominated and potentially optimal alternatives, which reduces the number of interesting alternatives for the DM [18–20], and the application of Monte Carlo simulation techniques, which allows simultaneous changes to the weights and generates results that can be easily analyzed statistically to provide more insights into the multiattribute model recommendations [21,22]. Both SAs take advantage of the useful imprecise information collected during the assignment of the component utilities and weights and the uncertain alternative consequences entered. In some cases, the information obtained from the alternatives evaluation is not meaningful enough so as to definitively recommend an alternative, i.e., we get very overlapped imprecise overall utilities. In these cases, the above techniques play a very important role. They may provide more meaningful information, and an interactive process can be carried out by tightening the respective imprecise alternative consequences, component utilities and weights and reassessing the non-dominated and potentially optimal alternatives or performing Monte Carlo simulation techniques again.

Throughout the paper we use an example of an application to illustrate the usefulness of the GMAA system. The example refers to the selection of a technology for the disposition of surplus weapons-grade plutonium by the Department of Energy in the USA. A major objective of this decision was to further US efforts to prevent the proliferation of nuclear weapons, but other concerns that were addressed include economic, technical, institutional, schedule, environmental, and health and safety issues. This problem has been studied in depth, see [15,16,23]. We have used the data reported in these papers, including, however, uncertainty about the alternative consequences and imprecision concerning the DM's preferences (5% deviation in both cases). As a consequence, average overall utilities and the consequent ranking of alternatives will match up, but more meaningful information will be output by taking advantage of the imprecision concerning the input parameters through the different SAs provided by the GMAA system.

We have divided the paper, according to DA stages, into the following five sections where we describe the steps mentioned above and their application to the above complex decision-making problem and, finally, provide some conclusions.

2. Structuring the problem

There are several benefits to be gained from using a hierarchy to model complex decision-making problems with multiple objectives. For instance, it helps to ensure that there will be no big gaps (missing objectives) at lower levels, situations where redundancy or double-counting could easily occur can be identified and it provides a basis upon which to develop and appraise screening criteria [24].

The objectives hierarchy must include all the relevant aspects related to the problem under consideration and should be structured in such a way that the objectives of similar importance are at the same level of the tree, improving the DMs' understanding of the decision.

In the GMAA system the user can interactively create or delete nodes and branches to build or modify the objectives hierarchy by means of a floating menu, which is displayed when the DM left-clicks a node of the tree. A name, label and description must be entered for each objective, as should the respective attribute units and ranges for the lowest-level objectives. Note that attributes are used to indicate to what extent the considered alternatives achieve the lowest-level objectives. For some basic properties related to the set of attributes see [1]. The system also accounts for attributes with a subjective scale and with discrete values.

The objectives hierarchy in the selection of a technology for the disposition of surplus weapons-grade plutonium is shown in Fig. 1. There are three main objectives at the highest level: *non-proliferation* (*Non-Prolifer*), *environmental, safety and health* (*ES&H*), and *operational effectiveness* (*Op. Effectiv*). *Non-proliferation* is made up of *theft, diversion, irreversibility* (*Irreversibil*), *international cooperation* (*Int'l Cooper*) and *timeless*; *ES&H* of *human health and safety* (*Human H&S*), *socio-economic* (*Socio Econom*) and *natural environment* (*Nat Environ*); and *cost for operational effectiveness*. Each of them is split into other sub-objectives and so on. Finally, 37 lowest level objectives are identified. A complete description of the objectives and attributes is given in [16].

Fig. 2 shows the objective, and its associated attribute information, for *H&S risk transportation* (*Transp Death*).

The lowest level objectives can have associated attributes with a continuous scale, like *H&S risk transportation* (*Transp Death*), discrete attribute value, like *IAEA attractiveness* (*IAEA Attract*), or a

Fig. 1. Objectives hierarchy.

constructed scale, like *Russian cooperation* (*Russian Coop*). In this case, the DM directly enters value between 0 and 1 just by thinking about how good or bad the alternative is with respect to the attribute in question, where 0 is the worst and 1 is the best.

3. Identifying feasible alternatives

Next, feasible alternatives must be identified, as well as should their consequences in terms of attributes associated with the lowest-level objectives and uncertainty (if necessary). As we have pointed out, the

The image shows a 'Node Information' dialog box with a title bar and a close button. It contains several tabs: 'Quantifying Preferences', 'Subjective Scale', 'Weight Stability Interval', 'Leaf Information', 'Viewing Component Utilities', and 'Viewing Alternative Consequences'. The 'Leaf Information' tab is selected. Inside, there are fields for 'Name:' (H&S Risks transportation), 'Label:' (Transp Death), and 'Attribute:' (Expected fatalities (transp)). A 'Description:' text area contains the text: 'Implementation of an alternative will require shipment of plutonium to various storage and disposition facilities. There is a risk of accidents during transport.' Below the attribute field, there are 'Units:' and 'Min. range' (0) and 'Max. range' (3) input boxes. At the bottom are 'OK', 'Cancel', 'Apply', and 'Help' buttons.

Fig. 2. Information about *H&S risk transportation*.

GMAA accounts for uncertainty about the consequences or outcomes of the alternatives under consideration in such a way that they can be provided by means of uniformly distributed ranges instead of single values as an approach under certainty would demand. Thus, given an objectives hierarchy with attributes $X_i, i = 1, \dots, n$, the consequences of a decision alternative $S^j, j = 1, \dots, m$, can be described under uncertainty by a vector of ranges $[\mathbf{x}^{jL}, \mathbf{x}^{jU}] = ([x_1^{jL}, x_1^{jU}], \dots, [x_n^{jL}, x_n^{jU}])$, where x_i^{jL} and x_i^{jU} are the lower (L) and upper (U) level of attribute X_i for S^j , and both end-points being equal, i.e., $x_i^{jL} = x_i^{jU}$, would be equivalent to the case under certainty, where the policy effects for an alternative S^j in attribute X_i are precisely known.

Moreover, alternatives with missing consequences, that is, alternatives that do not provide values or consequences for some attributes in the hierarchy, could be represented by the respective attribute ranges, $[x_i^{jL}, x_i^{jU}] = [r_i^{\min}, r_i^{\max}]$, where $r_i^{\min}$ and $r_i^{\max}$ are the bounds of the i th attribute range. This means that all the respective attribute values are possible and with the same probability.

In the complex decision-making problem in question, 13 technologies for the disposition of surplus weapons-grade plutonium were considered, see Table 1. Briefly, *reactor alternatives* would use surplus plutonium to fabricate mixed oxide fuel for nuclear reactors that generate electric power; while *immobilization alternatives* would immobilize surplus plutonium by mixing it with different substances, like non-radioactive and radioactive glass, or ceramic, and pouring it in recipients, like cans, canisters; and *direct disposal alternatives* would place it in a borehole.

The technology consequences were entered in the GMAA system and some uncertainty about them was added. For instance, an original value equal to a in the i th attribute was translated into the value interval $[a - 0.05(r_i^{\max} - r_i^{\min}), a + 0.05(r_i^{\max} - r_i^{\min})]$, where $r_i^{\min}$ and $r_i^{\max}$ are the bounds of the i th attribute range, i.e., an attribute deviation of 5% was used. Note that out-of-range consequences are not allowed, and $r_{\min}$ or $r_{\max}$ are used as the respective bounds of the consequence interval. Fig. 3 shows the window displayed by the system where the technology consequences can be viewed.

Table 1
Technologies to be evaluated

Reactor alternatives

Existing light water reactors, existing facilities
Existing light water reactors, greenfield facilities
Partially completed light water reactors
Evolutionary light water reactors
CANDU reactors

Direct disposal alternatives

Deep borehole (immobilization)
Deep borehole (direct emplacement)

Immobilization alternatives

Vitrification greenfield
Vitrification can-in-canister
Vitrification adjunct melter
Ceramic greenfield
Ceramic can-in-canister
Electrometallurgical treatment

	Exist LWR, Ex. Fac	Part. Comp. LWR	Exist LWR, Green F.	Evol
DOE Attract.	0.000	0.000	0.000	
FM Separated	0.000	0.000	0.000	
Bulk Throug1	5.000	5.000	5.000	
N Proc Step1	22.000	22.000	22.000	
MAX Pu Inven	0.500	0.500	0.500	
No of Trips	4.000	4.000	3.000	
SST Tr Miles	5300.000	4700.000	3900.000	
M Uncertain1	0.800	0.800	0.800	
Type of MAS1	20.000	20.000	20.000	

☐ Minimum
 ☒ Average
 ☐ Maximum

OK Add Alternative

Fig. 3. Viewing technology consequences.

The DM can use the “minimum” and “maximum” radio buttons to view the lower and upper bounds of the consequence intervals, respectively. Note that new alternatives can be added, an existing alternative can be deleted/discarded or its consequences can be modified.

Fig. 4. Class of utility functions for *Type of nuclear accounting system*.

4. Quantifying preferences

Quantifying preferences involves assessing the DM's single attribute utilities, which represent the DM's preferences concerning the possible alternative consequences, and weights, which represent the relative importance of criteria in the objectives hierarchy. Both will be used later to evaluate the alternatives through a multi-attribute utility function.

In both cases, the system admits incomplete information through value intervals as responses to the probability questions the DM is asked, which leads to classes of utility functions and weight intervals, respectively. This is less stressful on experts, see [25–27]. Moreover, it makes the system suitable for group decision support, because individual conflicting views or judgements in a group of stakeholders can be captured through imprecise responses.

Component utility functions and attribute weights can be used to identify efficient candidate solutions or to stimulate negotiation strategies, see, e.g., [28–32].

4.1. Assessment of component utility functions

Component utilities can be assigned using four procedures depending on the level of knowledge and features of the attribute under consideration.

When there is in-depth and precise knowledge about the attribute, the DM can directly construct a piecewise linear utility function by providing the best and the worst attribute values and up to three intermediate values with their respective imprecise utilities.

The class of utility functions will be constructed in this case by joining up to four linear segments between the best and the worst values for upper and lower values of the assignments. So, the user is asked to provide intermediate intervals (up to three). If no intermediate points are specified, then the result will be a single linear function. Fig. 4 shows the entered values and the respective class of utility functions for *type of nuclear accounting system* (*Type of NAS*).

This measure is included in both *theft* (*Theft*) and *diversion* (*Diversion*). The scale is defined over the percentage of time that item accounting can be used for a kilogram of plutonium at the facility, ranging from 0 to 100% item accounting. The shape of the utility function can also be changed by mouse dragging the intermediate points.

Fig. 5. Constructing an imprecise utility function with the gamble-based method.

The second procedure is used when the DM has little knowledge about or experience with the topic. This assessment approach involves the DM answering indifference conditions between lotteries and sure amounts, and does not require the DM to indicate the form of the imprecise piecewise utility function. The procedure is based on the combination of two slightly modified standard procedures for utility assessment: the *fractile method* [33–36] and the *extreme gambles method* [37,38].

In the fractile method, the DM is asked to provide attribute value intervals that he/she considers equivalent to different gambles, whose results are the most and least preferred attribute values with certain probabilities and their complementaries, respectively. The extreme gambles method, on the other hand, demands imprecise probabilities for the lottery [9].

The system includes a routine implementing a wheel of fortune to output these probability intervals, see [39], which displays the probabilistic questions and provides guidance to the expert until an interval of indifference probabilities is obtained. A number of additional questions are included as consistency checks.

If the intersection of the classes of utility functions output by the above methods were to be empty for some attribute values, the DM would have provided inconsistent responses and he/she should reassess his/her preferences. Otherwise, the intersection will be the range for the DM's utility functions, see Fig. 5.

The third procedure is used when different discrete attribute values have been identified for the attribute instead of a continuous scale. The DM is asked to provide the possible discrete attribute values (up to eight), their description and the respective imprecise utilities. Fig. 6 shows imprecise utilities for discrete attribute values for *IAEA attractiveness (IAEA Attract)*. If a technology scores well on this attribute, the *International Atomic Energy Agency (IAEA)* will be less concerned about the threat of diversion by the host nation, because the material in process is difficult to reuse in a weapons program.

The discrete attribute values correspond to direct-use unirradiated, direct-use irradiated, indirect-use and eligible for termination, respectively.

Finally, the fourth procedure applies when the DM decides to use a subjective scale without defining a utility function, because it is difficult to ascertain what the value of each alternative is for the respective subjective attribute. This procedure is implemented in the system by means of a thermometer scale [40].

Fig. 6. Imprecise utilities for discrete attribute values.

Fig. 7. Subjective values for *Russian cooperation*.

The DM will enter utility intervals by hand using scrollbars, as shown in Fig. 7 for *Russian cooperation* (*Russian Coop*).

4.2. Weight elicitation

Quantifying preferences also involves assessing weights, which represent the relative importance of criteria in the objectives hierarchy. The attribute weights, used in the alternative evaluation, reflect the relative importance of the change in the attribute from the worst attribute level to the best attribute level. Attribute weights are hierarchically assessed, i.e., the objective weights are elicited by weighting

Weight Elicitation Based on Tradeoffs

Provide a probability interval for P such that you are indifferent between the lottery and the sure consequence in :

	P	1-P	
Mater. Form -->	(6.00000 ,	1.00000)	~ 4.00000
Location -->	(3.00000 ,	1.00000)	~ 1.00000

P min. : P max. :

<<Back Cancel Next>>

Fig. 8. First probability question for weight elicitation based on tradeoffs for *irreversibility*.

attributes and objectives along one branch at a time and multiplying the local weights through the objective hierarchy.

Note that when the system is opened, the starting point is equally weighted objectives. If the DM disagrees with the objective weights, they can be modified by using one of the two weight elicitation methods provided by the GMAA system: weight elicitation based on trade-offs and direct assignment. Note that imprecision concerning the DM's responses is allowed in both methods by means of ranges of responses to the probability question that the DM is asked. A normalization process is automatically performed from the DM's responses, leading to an average normalized weight and a normalized weight interval for each sub-objective under consideration.

Weight elicitation begins with the attributes and then continues in ascending order through the hierarchy. The first method, perhaps more suitable for the low-level objectives in the hierarchy because it involves a more specific area of knowledge, is based on trade-offs among the respective attributes of the lowest-level objectives stemming from the same objective [1]. The DM is asked to give an interval of probabilities such that he/she is indifferent with respect to a gamble and sure consequences.

Figs. 8 and 9 show an example for the objective *irreversibility* (*Irreversibil*).

In both probability questions, the DM faces a lottery, whose outputs are the best values for the attributes stemming from *irreversibility* (*Irreversibil*) with the demanded probability and the worst values for the same attributes with the complementary probability, and some given sure attribute values. But, the mid-value of the attribute range and the worst attribute value for *material form* (*Mater. Form*) and *material location* (*Location*), are the given sure amounts, respectively, in the first question while, in the second, they are the worst attribute value and the mid-value of the attribute range, respectively.

On the other hand, direct assignment is perhaps more suitable for the possibly more political upper level objectives. The DM has to directly provide a weight interval for each sub-objective under consideration.

Once the relative importance of the objective and attributes has been rated along the branches of the hierarchy, the attribute weight can be assessed by multiplying the respective average weights and normalized weight interval bounds of the objectives in the path from the root (global objective) to each leaf (attribute), see Fig. 10.

Weight Elicitation Based on Tradeoffs

Provide a probability interval for P such that you are indifferent between the lottery and the sure consequence in :

	P	1-P	
Mater. Form --->	(6.00000 ,	1.00000)	~ 1.00000
Location --->	(3.00000 ,	1.00000)	~ 1.00000

P min. : P max. :

<<Back Cancel Finish

Fig. 9. Second probability question for weight elicitation based on tradeoffs for *irreversibility*.

Attribute weights over the decision

	low.	avq.	upp.	0	0.25	0.5	0.75	1
Int'l Class	0.002	0.003	0.003					
Russian Cooperation	0.074	0.098	0.124					
Civil use of Pu	0.011	0.015	0.019					
Material Form	0.042	0.056	0.071					
Location	0.042	0.056	0.071					
Time to Start	0.112	0.137	0.162					
Time to Complete	0.026	0.032	0.038					
Investment Cost	0.270	0.303	0.338					
Life-cycle Costs	0.145	0.173	0.204					
Pub. Rad Deaths	0.000	0.001	0.002					

<<Back Next>> OK

Fig. 10. Some attribute weights.

5. Evaluating alternatives

Once the DM's preferences have been quantified, the different alternatives under consideration can be evaluated by means of an additive multiattribute utility function, whose appearance is

$$u(S^j) = \sum_{i=1}^n w_i u_i(x_i^j), \quad (1)$$

Fig. 11. Overall utilities and ranking.

where w_i is the i th attribute weight, x_i^j is the consequence for alternative S^j in the i th attribute and $u_i(x_i^j)$ is the utility associated with the above consequence. For the reasons described in [41,42], we consider (1) to be a valid approach.

The additive model is used to assess, on the one hand, average overall utilities, on which the ranking of alternatives is based and, on the other hand, minimum and maximum overall utilities, which give further insight into the robustness of this ranking. Average overall utilities are obtained by taking into account the mid-points of the uniformly distributed consequence intervals in the respective attributes, their respective average component utilities and the average normalized attribute weights. To assess the minimum overall utilities, the system takes the lower end-points of the imprecise attribute weights, the lower end-point of the consequence intervals if the respective component utility function is increasing, or the upper end-point if it is decreasing, and the lower utilities in the imprecise utilities corresponding to the above consequences.

The system provides a graphical representation with bars, which includes their overall utilities and ranking, see Fig. 11. The vertical lines on each bar represent the average utilities, while the ends of the rectangles are the minimum and maximum utilities.

The technologies are ranked as in [16,23]. However, the average overall utilities do not match up due to some mistakes in the above papers, where erroneous component utilities are associated with the technology consequences for some attributes.

As shown in Fig. 11, the best ranked technologies are *ceramic can-in-can* and *vitrification can-in-can*, with equal overall utilities. It is clear that, taking into account the average utilities, one of these strategies should be recommended. Looking at the overlapped utility intervals (robustness of the ranking of alternatives), however, we can conclude that the information obtained by this evaluation is not meaningful enough to definitively recommend a technology. In the next section, several SAs in the GMAA system aimed at outputting more meaningful results will be introduced.

The system provides different displays of ranking results. The *stacked bar ranking* is similar to the alternatives classification but provides more detail on how the alternative's average utilities for the attributes affect the average utility of the overall objective. The *measure utilities for alternatives* displays a bar graph showing performance of a single alternative for the attributes, taking into account average

Fig. 12. Compare alternatives graph.

Fig. 13. Normalized average weights and imprecise alternative consequences for *ceramic can-in-can*.

consequences and individual utilities, and where the width of an attribute is proportional to its weight. The *compare alternatives graph* provides a detailed comparison of the differences between two alternatives, see Fig. 12. Finally, the *paired attributes correlation display* evaluates/compares alternative component utilities with respect to pairs of selected attributes.

Another display that can be useful for the DM is shown in Fig. 13. It includes the assigned average normalized weights in the objectives hierarchy and the imprecise consequences of a selected (*Ceramic*

Fig. 14. Ranking of alternatives for *human health and safety*.

Can-in-can) alternative in the different attributes. The system reminds the DM which attributes are rated on a subjective scale (*Sub. Scale*) or using discrete values (*Discrete Value*).

It is also possible to select another objective to rank by. Fig. 14 shows the ranking of alternatives for *human health and safety* (*Human H&S*).

6. Sensitivity analysis

DA is typically an iterative process. Once the model has been built SA is performed. This step should be considered as a means of stimulation that makes the DM think about the problem in more depth and can give further insight into the robustness of the recommendations. Ríos Insua [18] and Ríos Insua and French [19] introduce a framework for SA in multi-objective decision-making.

Several types of SA are provided by the GMAA System. Following Ríos Insua and French [19] they can be classed as *pure SA*, where the critical judgements are identified for closer consideration, and *decision making with partial information*, which takes advantage of the imprecise inputs (alternative consequences, component utilities and weights).

Classical SA and the assessment of stability weight intervals can be considered as pure SA. Classical SA essentially involves examining changes in the ranking of alternatives as a function of the input parameters (weights, component utilities or alternative consequences) varying within a reasonable range. Any alternative consequence, component utility or weight can be changed and the system takes charge of how this change is propagated through the objectives hierarchy and automatically recalculates the overall utilities for each alternative and the resulting ranking.

Furthermore, the stability weight interval for any objective at any level in the hierarchy can be assessed. In this case average normalized weight for the considered objective can vary without affecting the overall ranking of alternatives. For instance, the current average normalized weight and the stability interval for *diversion* is shown in Fig. 15.

The stability weight interval is [0.093, 0.41], i.e., the average normalized weight for *diversion* could be changed within the specified interval without leading to changes in the ranking of alternatives. However,

Fig. 15. Weight stability interval for *diversion*.

if the new value is outside the interval, for instance 0.42, a new alternatives ranking is output. Thus, this SA can be useful for identifying weight objectives to which the ranking of alternatives is sensitive. For instance, in the disposition of surplus plutonium problem, the objectives with the narrowest weight stability intervals are *environmental*, *safety and health (ES&H)*, *investment costs (Invest. Cost)* and *life-cycle costs (L-cycle Cost)*, whose intervals are $[0, 0.029]$, $[0.564, 0.647]$ and $[0.353, 0.436]$, respectively. So, the DM should think about them more closely.

The other two SAs provided by the system, the assessment of non-dominated and potentially optimal alternatives and the application of Monte Carlo simulation techniques, can be considered as *decision making with partial information*.

As mentioned above, the information obtained from the alternatives evaluation, by means of the additive multiattribute utility model, is not always meaningful enough to definitively recommend an alternative, i.e., we get very overlapped imprecise overall utilities. In these cases, the above techniques play a very important role. They may provide more meaningful information, and an interactive process can be carried out by tightening the respective imprecise alternative consequences, component utilities and weights and reassessing the non-dominated and potentially optimal alternatives or performing the Monte Carlo simulation techniques again, see [43].

6.1. Non-dominated and potentially optimal alternatives

The concepts of dominance and potential optimality have been widely studied by several authors. A number of methods have been proposed to test dominance and potential optimality considering both weight and utility imprecision, see, e.g., [44,45].

In this SA, we intend to take advantage of the imprecise information collected during the assignment of component utilities and weights and the entered uncertain alternative consequences to definitely reject bad alternatives, mainly by discarding dominated and/or non-potentially optimal alternatives. Thus, the consequence of any alternative S^j in the i th attribute, x_i^j , belongs to a certain interval $[x_i^{jL}, x_i^{jU}]$, the utility associated with any alternative consequence x_i^j is imprecise $u_i(x_i^j) \in [u_i^L(x_i^j), u_i^U(x_i^j)]$ and the i th attribute weight is imprecise $k_i \in [k_i^L, k_i^U]$.

According to these considerations, given two alternatives S^r and S^t , the alternatives S^r dominates S^t if $f_{rt}^* > 0$, where f_{rt}^* is the optimal value of the optimization problem (2), see [18,19]

$$\begin{aligned}
\min \quad & f_{rt} = u(S^r) - u(S^t) = \sum_{i=1}^n k_i u_i(x_i^r) - \sum_{i=1}^n k_i u_i(x_i^t) \\
\text{s.t.} \quad & x_i^{rL} \leq x_i^r \leq x_i^{rU}, \quad i = 1, 2, \dots, n, \\
& x_i^{tL} \leq x_i^t \leq x_i^{tU}, \quad i = 1, 2, \dots, n, \\
& k_i^L \leq k_i \leq k_i^U, \quad i = 1, 2, \dots, n, \\
& u_i^L(x_i^r) \leq u_i(x_i^r) \leq u_i^U(x_i^r), \quad i = 1, 2, \dots, n, \\
& u_i^L(x_i^t) \leq u_i(x_i^t) \leq u_i^U(x_i^t), \quad i = 1, 2, \dots, n.
\end{aligned} \tag{2}$$

On the other hand, from a mathematical point of view, the alternative S_r is potentially optimal, if the optimum value of the optimization problem (3), f_r^* , is less than or equal to 0, see [18,19],

$$\begin{aligned}
\min \quad & f_r \\
\text{s.t.} \quad & \sum_{i=1}^n k_i u_i(x_i^r) - \sum_{i=1}^n k_i u_i(x_i^t) + f_r \geq 0 \quad \forall t \neq r, \\
& x_i^{jL} \leq x_i^j \leq x_i^{jU} \quad \forall i, j, \\
& k_i^L \leq k_i \leq k_i^U \quad \forall i, \\
& u_i^L(x_i^j) \leq u_i(x_i^j) \leq u_i^U(x_i^j) \quad \forall i, j.
\end{aligned} \tag{3}$$

Optimization problems (2) and (3) are non-linear. However, they can be easily transformed into linear optimization problems, as shown in [20].

In the disposition of surplus plutonium problem, only four out of the 13 technologies are non-dominated and potentially optimal, see Fig. 16. Consequently, the DM should confine his/her attention to these technologies and the remainder should be discarded from further consideration.

Further details about the dominance between alternatives can be also viewed, see Fig. 17. The position of the alternative in the rows and columns depends on the alternatives ranking in the table shown in Fig. 17, and only the necessary optimization problems are solved, i.e., if the result of the dominance problem is greater than zero, the alternative placed in this column is dominated so it can be discarded.

6.2. Monte Carlo simulation techniques for SA

This kind of sensitivity analysis uses Monte Carlo simulation, see [21,22], allows simultaneous changes of the weights and generates results that can be easily analyzed statistically to provide more insight into the multi-attribute model recommendations.

The attribute weights will be selected at random using a computer simulation program so that the results of many combinations of weights can be explored efficiently. The system uses a multiplicative linear congruential generator based on Schrage's method, first published in 1979, and later refined in 1983, see [46]. It provides a virtually infinite sequence of statistically independent random numbers, uniformly distributed between 0 and 1.

Fig. 16. Non-dominated and potentially optimal alternatives.

	Vitrif. Can-in-Can	Exist LWR, Ex. Fac	Borehole (Direct)	CANDU	Borehole (li)
Ceramic Can-in-Can	-0.0525	-0.0388	-0.0013	0.0086	0.025
Vitrif. Can-in-Can		-0.0406	-0.0023	...	...
Exist LWR, Ex. Fac			-0.1135	...	...
Borehole (Direct)				...	...
CANDU					...
Borehole (Immob.)					
Vitrif. Adj. Melter					
Exist LWR, Green F.					
ElectroMetalurgical					
Ceramic Greenfield					
Part. Comp. LWR					

Fig. 17. Dominance details.

While the simulation is running, the system computes several statistics about the rankings of each alternative, like mode, minimum, maximum, mean, standard deviation and the 25th, 50th and 75th percentiles. This information can be useful for discarding some available alternatives, aided by a display that presents a multiple boxplot for the alternatives.

Three general classes of simulation are provided by the GMAA system: *random weights*, *rank order weights* and *response distribution weights*. In the random weights option, weights for the attributes are generated completely at random, which implies no knowledge whatsoever of the relative importance of the attributes. In the rank order weights option, attribute weights are randomly generated preserving a total or partial attribute rank order, which places substantial restrictions on the domain of possible weights that are consistent with the DM's judgement of criteria importance, leading to more meaningful results. Finally, the response distribution weights option recognizes that the weight elicitation procedures

Fig. 18. Resulting multiple boxplot.

	Mode	Min.	25th percentile	50th percentile	75th percentile	Max.	Mean	Std. Deviat
Exist LWR, Ex. Fac	3	3	3.000	3.000	3.000	3	3.000	0.000
Part. Comp. LWR	11	10	11.000	11.000	12.000	12	11.386	0.500
Exist LWR, Green F.	8	7	8.000	8.000	8.000	9	8.060	0.304
Evolutionary LWR	13	13	13.000	13.000	13.000	13	13.000	0.000
CANDU	5	4	5.000	5.000	5.000	6	4.869	0.339
Vitrif. Greenfield	12	11	11.000	12.000	12.000	12	11.607	0.488
Vitrif. Can-in-Can	2	1	2.000	2.000	2.000	2	1.797	0.402
Vitrif. Adj. Melter	7	7	7.000	7.000	7.000	8	7.018	0.133

OK

Fig. 19. Associated statistic measures.

are subject to variation and attribute weights are now randomly assigned values taking into account the normalized attribute weight intervals provided by the DM in the weight elicitation methods.

Fig. 18 shows the resulting boxplot for the disposition of surplus plutonium problem for the response distribution weights option, i.e., attribute weights have been randomly generated from the normalized weight intervals shown in Fig. 10.

It is clear that more meaningful information for the DM is output than in the overall utility intervals, obtained by the alternative evaluation. Only two technologies are best ranked, *vitrification can-in-can* and *ceramic can-in-can*, and the worst classification for both is second. Thus, one of these two technologies should be recommended. The DM can also view the associated statistics measures for this simulation, see Fig. 19. Taking into account the mean classifications, we arrive at the conclusion that *ceramic can-in-can* is the technology to be recommended.

7. Conclusion

This paper presents a comprehensive DSS based on an additive multi-attribute utility model. The system is very user friendly and makes provision for all the stages of the DA cycle, from construction of the objectives hierarchy to evaluation of the set of alternatives for ranking. It accounts for uncertainty about the alternative consequences (real problems are usually plagued with uncertainty), and admits incomplete information about the DM's preferences, which is less demanding for a single DM and also makes the system suitable for group decision support.

A ranking of alternatives based on average overall utilities is output. Moreover, imprecise overall utilities are also assessed. These utilities give further insight into the robustness of this ranking. However, in some cases, the information obtained from the alternatives evaluation is not meaningful enough to definitively recommend an alternative. In these cases, some of the SAs provided by the system, the assessment of non-dominated and potentially optimal alternatives and the application of Monte Carlo simulation techniques, play a very important role. They may provide more meaningful information, and an iteration process can be carried out by tightening the respective imprecise alternative consequences, component utilities and/or weights.

The GMAA system is not developed and implemented ad hoc for one complex decision-making problem, it is designed to aid the DM in a range of problems where DA can be especially useful. The system is illustrated throughout the paper using a well-known application example, the selection of a technology for the disposition of surplus weapons-grade plutonium. This example illustrates the main features and usefulness of the system.

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