On Ship Roll Damping: Analysis and Contributions on Experimental Techniques

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Abstract

Ship roll damping represents a key factor for a proper prediction of the ship behaviour in a seaway. However, accurately estimating the roll damping is a challenging task. The most accepted way to reliably estimate the ship roll damping is by experimental tests. The most typical means for roll damping determination are free roll decays. Other tests are either excited roll tests, where the model is freely floating and rolled by regular beam waves or internal mechanical devices, or forced roll tests, in which the ship model is rotated by mechanical means with a fixed axis. These different experimental approaches are associated with distinct hydrodynamic scenarios. Therefore, in principle, estimated roll damping may differ depending on the considered approach. In this Thesis, excited roll tests and decay tests are used to determine roll damping for a trawler fishing vessel, and the arising differences are analysed. Regarding excited roll tests, a technique is proposed, based on an internal shifting mass. Concerning decay tests, three methodologies for impressing the initial heel angle are introduced, with the aim to study heave response influence on ship roll motion and fluid memory effects. From the direct comparison of estimated nonlinear damping coefficients, it may be concluded that the three methodologies of decay tests present the same trend, which differs from internally excited roll tests. Then, to assess the differences found between decay and internally excited roll tests, numerical simulations of the ship rolling in regular beam waves using the different roll damping estimations are compared to experimental data. From these results, it is seen that damping estimated from internally excited roll tests is closer to the ship roll damping in beam waves. Finally, a sensitivity analysis of stability-related international regulations to roll damping is performed. From this analysis, it is derived that for one of the regulations considered, roll damping may become a major parameter.

Keywords

roll damping; nonlinear rolling; experimental techniques; decay test; forced roll tests; excited roll tests; internal excitation; regular beam waves; fishing vessels.
Resumen

El amortiguamiento en balance es un factor clave para la correcta predicción del comportamiento de un buque en la mar. No obstante, predecir de manera adecuada el coeficiente de amortiguamiento es una tarea compleja. La forma más aceptada para estimarlo es la realización de ensayos experimentales, siendo el más común el ensayo de extinción. Otras técnicas son el ensayo de excitación en balance, en el que el modelo está flotando libremente y es excitado mediante olas regulares de través o elementos mecánicos internos, o los ensayos de oscilaciones forzadas en balance, donde al modelo se le induce un movimiento de balance mediante elementos mecánicos, fijando el eje de rotación. Cada una de estas técnicas está asociada a un escenario hidrodinámico distinto, con lo que las estimaciones del amortiguamiento en balance pueden diferir. En esta Tesis, se han realizado ensayos de excitación en balance y de extinción para determinar el amortiguamiento en balance de un buque arrastrero. Con respecto los ensayos de excitación en balance, se ha propuesto una metodología basada en una masa interna móvil. Con respecto a los ensayos de extinción, se han considerado tres metodologías distintas para generar el ángulo inicial, con el objetivo de estudiar la influencia del movimiento vertical del buque y los efectos de memoria del fluido. De la comparación directa de los coeficientes no lineales estimados con las distintas técnicas, se puede concluir que las tres metodologías de los ensayos de extinción presentan la misma tendencia, que difiere de la de los ensayos de excitación en balance con la masa interna móvil. Luego, para evaluar las diferencias entre ellos, se han realizado simulaciones numéricas de balance en olas regulares considerando las distintas estimaciones del amortiguamiento, comparando los resultados obtenidos con los ensayos de excitación en balance usando olas de través. De estos resultados se puede observar que el amortiguamiento obtenido con los ensayos de excitación usando la masa interna móvil es más cercano al amortiguamiento del buque con olas de través. Finalmente, se ha llevado a cabo un análisis de sensibilidad del amortiguamiento en balance en las regulaciones internacionales relativas a la estabilidad de los buques. De este análisis se deduce que, para una de las regulaciones consideradas, el coeficiente de amortiguamiento en balance puede ser un parámetro determinante.

Palabras Clave

amortiguamiento en balance; balance no lineal; técnicas experimentales; ensayos de extinción; ensayos de oscilaciones forzadas en balance; ensayos de excitación en balance; excitación interna; olas regulares de través; buques pesqueros.
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Acronyms

CEHINAV  Canal de Ensayos Hidrodinámicos de la Escuela Técnica Superior de Ingenieros Navales.

CEHIPAR  Centro de Experiencias Hidrodinámicas de El Pardo.

CFD  Computational Fluid Dynamics.

DOF  Degree Of Freedom.

DSC  Dead Ship Condition.

DT  Decay Roll Tests.

EERT  Externally Excited Roll Tests.

ERT  Excited Roll Tests.

FC  Forcing Case.

FFT  Fast Fourier Transform.

FRT  Forced Roll Tests.

IERT  Internally Excited Roll Tests.

IMO  International Maritime Organization.
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<td>ITTC</td>
<td>International Towing Tank Conference.</td>
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<td>L1</td>
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<td>LC01</td>
<td>Loading Condition 1.</td>
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<td>PIT</td>
<td>Parameter Identification Technique.</td>
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<td>RMSE</td>
<td>Root Mean Square Error.</td>
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<td>SGISC</td>
<td>Second Generation Intact Stability Criteria.</td>
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<td>SOLAS</td>
<td>Safety of Life at Sea.</td>
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<td>THD</td>
<td>Total Harmonic Distortion.</td>
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<td>WeC</td>
<td>Weather Criterion.</td>
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Nomenclature

$A$ Roll amplitude.

$A_k$ Area of the lateral projection of the bar keel.

$A_{0,\text{mean}}$ Mean initial heel angle for a decay test case.

$A_0$ Initial heel angle for decay tests.

$A_1$ First peak angle considered for the analysis of decay tests.

$A_{FC}$ Nominal amplitude of forcing for internally excited roll tests.

$A_{FFT,f}$ Amplitude of the fundamental harmonic $f$ obtained through the Fast Fourier Transform analysis.

$A_{FFT,n}$ Amplitude of the harmonic $n$ obtained through the Fast Fourier Transform analysis.

$A_{\text{end}}$ Last peak angle considered for the analysis of decay tests.

$A_{\text{max}}$ Maximum peak resonance amplitude in the case of internally excited roll tests, and the initial heel angle in the case of roll decay tests.

$A_{\text{res}}$ Resonance peak of the roll response curve.

$A_{\text{roll, max}}$ Maximum rolling amplitude measured in the analysed time window from excited roll tests.

$A_{\text{roll, min}}$ Minimum rolling amplitude measured in the analysed time window from excited roll tests.

$A_{\text{roll}}$ Rolling amplitude measured in the analysed time window from excited roll tests.

$B$ Ship breadth overall.

$C$ Long-term probability failure index for the Level 2 criterion of Dead Ship Condition failure mode of the Second Generation Intact Stability Criterion.

$C_B$ Ship block coefficient.
Nomenclature

\( C_M \) Ship midship coefficient.

\( C_s \) Short-term probability failure index for the Level 2 criterion of Dead Ship Condition failure mode of the Second Generation Intact Stability Criterion.

\( D(\dot{\phi}) \) Damping moment function, dependent only on the instantaneous roll velocity.

\( G \) Center of gravity of the ship.

\( G_{Z_{res}}(\phi) \) Residual righting lever.

\( G_s \) Center of gravity of the ship without the moving mass for internally excited roll tests.

\( H \) Ship depth to upper deck.

\( H_s \) Significant wave height.

\( J_{xx}^T \) Total roll moment of inertia, including the hydrodynamic added inertia.

\( L_{OA} \) Ship length overall.

\( L_{pp} \) Ship length between perpendiculars.

\( L_{wl} \) Ship length at waterline.

\( M_{ext} \) Roll external moments applied to the ship.

\( Q \) Center of gravity of the ship with the moving mass, when the mass is on the centerplane, for internally excited roll tests.

\( R_{DS0} \) Standard value for the Level 2 criterion of Dead Ship Condition failure mode of the Second Generation Intact Stability Criterion.

\( S \) Spectrum of the relative roll motion.

\( S_v \) Spectrum of the wind gust.

\( S_{M\text{waves}} \) Spectrum of the waves moment excitation.

\( S_M \) Spectrum of the total moment excitation.

\( S_{\alpha\alpha,c} \) Spectrum of the effective sea wave slope.

\( S_{\alpha\alpha} \) Spectrum of the sea wave slope.

\( S_{\delta M_{\text{wind,ext}}} \) Spectrum of the wind gust moment excitation.
$S_x$ Spectrum of the absolute roll motion.

$S_{zz}$ Spectrum of the sea elevation.

$T$ Wave period.

$T_0$ Undamped ship roll natural period.

$T_{req}$ Required wave excitation period for externally excited roll tests.

$T_z$ Waves zero-crossing period.

$U_w$ Mean wind speed.

$X_1$ Parameter of the Weather Criterion, function of the beam-draught ratio.

$X_2$ Parameter of the Weather Criterion, function of the block coefficient.

$\Delta$ Ship displacement.

$\alpha_{FC}$ Static heel that the ship model reaches when the moving mass is placed at the extreme position in internally excited roll tests.

$\beta$ Quadratic damping coefficient.

$\delta$ Cubic damping coefficient.

$\delta\mu_{eq}$ Friction scale effects correction for the equivalent linear roll damping coefficient determined from model experiments.

$\delta h$ Simplified beating signal index for externally excited roll tests.

$\lambda$ Wave length.

$\lambda_{EA}$ Average time between two capsize events.

$\mu$ Linear damping coefficient.

$\mu_{eq}$ Equivalent linear damping coefficient.

$\omega$ Excitation (wave) frequency.

$\omega_0$ Undamped ship roll natural frequency.

$\omega_{0,eq}$ Undamped amplitude dependent ship roll natural frequency.

$\omega_{0,e}$ Modified ship roll natural frequency close to the steady heeling angle.
Nomenclature

$\omega_{req}$ Required wave excitation frequency for externally excited roll tests.

$\omega_{res}$ Resonance frequency of the roll response curve.

$GM$ Metacentric height with respect to the center of gravity $G$.

$GZ(\phi)$ Hydrostatic roll righting lever with respect to the center of gravity $G$.

$KG$ Vertical position of the center of gravity $G$.

$KG_m$ Vertical position of the center of gravity of the moving mass for internally excited roll tests.

$KG_s$ Vertical position of the center of gravity of the ship without the moving mass for internally excited roll tests.

$LCG$ Longitudinal position of the center of gravity $G$.

$LCG_s$ Longitudinal position of the center of gravity of the ship without the moving mass for internally excited roll tests.

$QM$ Metacentric height of the ship with the moving mass on its centerplane with respect to the center of gravity $Q$ for internally excited roll tests.

$QZ(\phi)$ Hydrostatic roll righting lever of the ship with the moving mass on its centerplane with respect to the center of gravity $Q$ for internally excited roll tests.

$l_{wind,tot}$ Lever of the mean wind heeling moment.

$\phi$ Roll angle (dots represent derivatives with respect to time).

$\phi_1$ Angle of roll to windward due to wave action (roll-back angle) of the Weather Criterion of the Intact Stability Code 2008.

$\phi S$ Steady heel angle only considering the action of mean beam wind.

$\phi_{1r}$ Angle of roll to windward due to regular beam waves action of the Weather Criterion of the Intact Stability Code 2008.

$\phi_{EA,+}$ Limiting heel angle to leeward.

$\phi_{EA,-}$ Limiting heel angle to windward.

$\phi_{VW,+}$ Angle of vanishing stability due to mean wind action to leeward.

$\phi_{VW,-}$ Angle of vanishing stability due to mean wind action to windward.

$\phi_f,+$ Progressive flooding angle to leeward.
Nomenclature

\( \phi_{f,-} \)  Progressive flooding angle to windward.

\( \phi_{fail,+} \)  Physical failure angle to leeward.

\( \phi_{fail,-} \)  Physical failure angle to windward.

\( \sigma_{C_s} \)  Standard deviation of relative roll motion.

\( \sigma_x \)  Standard deviation of absolute roll velocity.

b.f.r.s.  Body-fixed reference system.

d  Ship draught.

\( d(\dot{\phi}) \)  Non-dimensional damping moment function, dependent only on the instantaneous roll velocity.

e.f.r.s.  Earth-fixed reference system.

h  Wave height.

\( h_{\text{max}} \)  Maximum wave height measured in the wave time history for externally excited roll tests.

\( h_{\text{mean}} \)  Mean wave height measured in the wave time history for externally excited roll tests.

\( h_{\text{min}} \)  Minimum wave height measured in the wave time history for externally excited roll tests.

\( h_{\text{req}} \)  Required wave height for externally excited roll tests.

k  Parameter of the Weather Criterion, function of the bilge keel area and the lateral projection of the bar keel.

\( k_{xx} \)  Dry roll radii of inertia.

\( m_T \)  Total mass for internally excited roll tests.

\( m_m \)  Moving mass for internally excited roll tests.

\( m_{ext} \)  Normalised roll external moments applied to the ship.

r(\( \omega \))  Effective wave slope.

r(\( \phi \))  Non-dimensional righting arm, i.e. relation between the righting lever and the metacentric height.

\( s_\omega \)  Wave steepness.
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<th>Symbol</th>
<th>Definition</th>
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<td>$s_{\omega, \text{req}}$</td>
<td>Required wave steepness for externally excited roll tests.</td>
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<tr>
<td>w.r.t.</td>
<td>With respect to.</td>
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<tr>
<td>$y_{m, \text{max}}$</td>
<td>Maximum amplitude of motion of the moving mass used for internally excited roll tests and decay tests.</td>
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1.1 Background

This Thesis examines different experimental techniques to determine ship roll damping, making particular emphasis in large rolling amplitudes. This Chapter sets the general background of roll damping, describes the motivation and the objectives of the study and presents the outline of the Thesis. The publications derived from this Thesis are also introduced.

1.1 Background

Ship roll damping is the energy that the ship dissipates when rolling. From a seakeeping perspective, ship roll damping is a crucial factor for a proper estimation of the ship behaviour in a seaway. Moreover, from a regulatory, and hence design, perspective roll damping is a major parameter in some stability-related international regulations. It is the case of MSC.1/Circ.1200 (IMO, 2006), relative to the alternative experimental assessment of the Weather Criterion, where roll damping may be necessary to determine the regular waves roll-back angle when direct experimentation cannot be carried out. Damping is also fundamental in case of the majority of failure modes addressed by the, still under development, Second Generation Intact Stability Criteria (e.g., Peters et al., 2011; IMO, 2016b, 2017).

Roll damping was already considered by Froude (Froude et al., 1955) and, since his seminal contributions, this topic has continued gaining attention over the years. The main particularity of roll damping is that it may not be estimated accurately following theoretical approaches, because of its large non-linear behaviour. Therefore, despite the significant effort placed in this topic, still nowadays roll damping is estimated mainly by experiments or by semi-empirical methods. Research studies were and are still focused on different complementary aspects, such as mathematical modelling of roll damping and the corresponding analysis of experimental data, semi-empirical methods to predict roll damping, or the generation of experimental reference data sets, and experimental procedures to estimate roll damping. Other aspects have been less studied due to their complexity such as scale effects of roll damping (e.g., Valle-Cabezas and Pérez-Rojas, 1997; Bertaglia et al., 2004; Bulian et al., 2009; Söder et al., 2012; Handschel et al., 2014).

Considerable attention has been given to the analytical mathematical modelling of roll damping and the corresponding analysis of experimental data. The first mathematical model of roll damping is attributed to Froude (Froude et al., 1955) and constitutes the linear-quadratic damping representation, which seems to be based on the analogy of ship roll damping to a body drag force, as the drag of a body in a real fluid is proportional to the velocity squared. However, the quadratic damping term presents some shortcomings in the analytical treatment of the ship roll motion equation. To overcome this problem, in the 70’s, the quadratic term was replaced by a cubic term (e.g., Dalzell, 1978), constituting the linear-cubic damping representation. Furthermore, to consider the well-known dependence of the roll amplitude into the roll damping, other models were proposed (e.g., Cardo et al., 1982; Contento et al.,
CHAPTER 1. INTRODUCTION

and their suitability was tested extensively (e.g., Roberts, 1985; Haddara and Bennett, 1989; Francescutto and Contento, 1999; Spyrou and Thompson, 2000). Until now, it seems that the most suitable model is the linear-quadratic-cubic damping model, fixing, if necessary, either to zero the quadratic or cubic terms on a specific ship hull basis (e.g., Bulian, 2004).

Nonlinear roll damping prediction at the design stage has mostly been based on semi-empirical methods (e.g., Ikeda et al., 1978; Himeno, 1981; ITTC, 2011; Kawahara et al., 2012; Falzarano et al., 2015) and on the use of experimental reference data sets (e.g., Blume, 1979; Handschel et al., 2012). Recently, thanks to the increase in available computational resources and thanks to the improvement in numerical methods, also CFD are being applied for the estimation of roll damping (e.g., el Moctar et al., 2012; Handschel et al., 2014; Iral et al., 2016; Ommani et al., 2016; Piehl, 2016; Mancini et al., 2018).

While the effort on roll damping modelling and estimation was originally placed into small and medium ship roll angles, more focus is presently given to large angles of roll (e.g., Bačkalov et al., 2016). The need to have a proper estimation of roll damping at large rolling amplitudes is linked more to safety rather than to operability. It is therefore understandable that large amplitude roll damping may represent a relevant topic from a regulatory perspective when ship roll motion is directly addressed (e.g., IMO, 2006; Peters et al., 2011; IMO, 2016b, 2017). Current semi-empirical methods, such as the Ikeda’s Method (ITTC, 2011) or the Simplified Ikeda’s Method (Kawahara et al., 2012), are limited to cargo vessels and specific ranges of hull particulars (Falzarano et al., 2015), although some attempts have been made to apply semi-empirical methods to other types of ships such as fishing vessels (e.g., Kuroda et al., 2003; Ali et al., 2004; Pesman et al., 2007; Paroka and Umeda, 2007; Míguez-González et al., 2013; Aarsæther et al., 2015). However, such semi-empirical methods have been criticised for lacking accuracy at large roll angles (e.g., Bassler, 2013). Hence, as stated in the draft guidelines for direct stability assessment procedures in the framework of SGISC (IMO, 2017), the preferred source of the data to be used for the calibration of roll damping in motion prediction codes should be experimental roll decays or excited roll tests.

Extensive studies have been dedicated to the different experimental procedures which can be followed to determine the roll damping (e.g., Roberts, 1985; Spounge et al., 1986; Bertaglia et al., 2003, 2004; Oliveira, 2011; Handschel and Abdel-Maksoud, 2014; Wassermann et al., 2016; Oliveira et al., 2018). The most typical means for roll damping determination is based on the execution and processing of roll decays. One typical problem associated with roll damping estimation based on roll decays is the difficulty in gathering damping information at large rolling angles. Another problem that has recently gained importance is the fluid memory effects, although its consideration goes back to Roberts (1985) and Spounge et al. (1986). Memory effects are related to the concept that the ship motion, and hence the roll damping, depends on the previous ship motion history. According to Van’t Veer and Fathi (2011), memory effects in decay tests are considerable and should be avoided by pre-exciting the surround-
ing fluid before releasing the model. However other research studies suggest that this effect does not influence on the roll damping estimations (e.g., Oliveira, 2011; Söder et al., 2012; Zhao et al., 2016).

Other types of roll tests in calm water can be carried out for roll damping estimation, either with free model (excited tests) or with fixed axis (forced tests) (e.g., Blume, 1979; Spounge et al., 1986; Bertaglia et al., 2004; Grant et al., 2010; Oliveira, 2011; Handschel and Abdel-Maksoud, 2014; Wassermann et al., 2016; Park et al., 2018). Moreover, experimental tests in regular beam waves can also be used, which may be understood as a particular case of excited tests (e.g., Contento et al., 1996; Francescutto and Contento, 1998, 1999; Bertaglia et al., 2004; IMO, 2006; Oliveira, 2011).

Despite being experimental tests the most accepted way to reliably estimate ship roll damping, different alternative approaches still coexist in available international guidelines (e.g., ITTC, 2011; IMO, 2006). Since different experimental approaches are associated with distinct hydrodynamic scenarios, estimated roll damping may, in principle, differ depending on the considered approach. These possible differences are generally neglected, and not many extensive studies focusing on this aspect are available in the literature (e.g., Mathisen and Price, 1985; Bertaglia et al., 2004; Oliveira, 2011; Handschel and Abdel-Maksoud, 2014; Wassermann et al., 2016).

1.2 Motivation

Notwithstanding undergoing continuous research studies, many aspects of ship roll damping remain unsolved and/or present a significant uncertainty, such as estimating the nonlinear roll damping coefficients by experimental means. Moreover, now that focus has been placed on large rolling angles, the effectiveness of experimental procedures and semi-empirical methods usually used at small and medium roll amplitudes has to be verified for large rolling amplitudes. Therefore, it may be expected that some of the assumptions generally made are no longer valid.

This Thesis intends to analyse the goodness of the most common experimental approaches to estimate roll damping, also for large rolling amplitudes, and the possible associated differences between the approaches and testing methodologies. The aim is to address whether different experimental techniques and different procedures to carry out each experimental technique may affect the roll damping estimations and to which extent.

To this end, the experimental techniques considered have been free roll decay tests and excited roll tests. Forced roll tests are not considered because fixing the ship roll axis is a significant simplification for specific ship types, as for fishing vessels. Each technique has been performed using different experimental methodologies. Regarding excited roll tests, regular beam waves have been used to excite the ship model externally, and also a technique is proposed based on an internal shifting mass. With regard to decay tests, three methodologies for impressing the initial heel angle are introduced, with the
CHAPTER 1. INTRODUCTION

objective to study heave response influence on ship roll motion and fluid memory effects. From the damping coefficients determined for each experimental technique, an analysis is performed to assess the differences between roll damping estimations.

1.3 Objectives

The scope of this Thesis is to analyse different experimental methodologies to estimate ship roll damping and determine if the estimated roll damping coefficients differ depending on the hydrodynamic scenario involved and to which extent the possible discrepancies may affect the ship behaviour estimations, especially when applying stability-related international regulations.

The specific objectives of the Thesis are as follows:

- Develop methodologies to perform and analyse excited roll tests and decay roll tests to estimate nonlinear ship roll damping coefficients;
- Determine the variations in roll damping estimations when using the different experimental methodologies;
- Analyse the influence of fluid memory effects in decay tests;
- Analyse the influence of the heave response in decay tests;
- Evaluate the influence of roll damping estimations in stability-related international regulations (i.e., Weather Criterion, Second Generation Intact Stability Criteria).

1.4 Overview of the Thesis

The Thesis is organised into seven Chapters, comprising the present one.

Chapter 2 introduces general concepts of ship roll motion and roll damping, as well as the mathematical models typically used. Also, the main particularities of the ship hull studied throughout the Thesis and the experimental facilities used are reported.

Chapter 3 describes internally excited roll tests in calm water. The experimental set-up is explained as well as the nonlinear mathematical model for representing the ship roll dynamics. A procedure is proposed for determining roll damping coefficients and, following the procedure, roll damping coefficients for the reference ship hull are determined.

Chapter 4 introduces three techniques for performing roll decay tests in calm water. After describing the experimental set-up required for each decay test approach, a standard procedure to determine roll damping coefficients is reported, based on the logarithmic-decrement approach. From the different
techniques, roll damping estimations for the reference ship hull are obtained, and the determined values are compared regarding nonlinear roll damping coefficients and equivalent linear roll damping values.

Chapter 5 presents results from externally excited roll tests using regular beam waves. The methodology followed to analyse the experimental data is described in detail, and the resulting roll response curves are reported. However, due to limited experimental data, estimations of nonlinear roll damping coefficients are not obtained.

Chapter 6 compares and analyses roll damping estimations determined from internally excited roll tests and decay tests by directly comparing nonlinear damping coefficients estimations and by comparing numerical roll motion predictions using the different roll damping estimations to experimental data from externally excited roll tests. This Chapter also analyses the sensitivity of stability-related international regulations to roll damping by applying the Weather Criterion of current Intact Stability Code 2008 and the last available draft of the Level 2 Criterion for Dead Ship Condition failure mode of the, still under development, Second Generation Intact Stability Criteria.

Chapter 7 summarises the main outcomes of the Thesis and presents some concluding remarks. Some areas for future work are also identified.

Complementary information and additional aspects are reported in the Annexes.

1.5 Publications of the Thesis

Partial results of this Thesis have been published in:


and presented in:


2 General considerations

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This Chapter introduces general concepts of ship roll motion and the mathematical modelling of roll motion and roll damping. Also, in this Chapter, the main particularities of the ship hull studied throughout the Thesis and the tested loading conditions are described, as well as the experimental facilities used.

2.1 Ship roll motion

A ship sailing in a seaway constitutes a dynamical system with 6-Degrees of Freedom (DOF). The ship roll motion is the rotation of the ship around a longitudinal axis, which is non-stationary when considering 6-DOF.

When modelling the ship roll motion, a common approach is to describe it by a 1-DOF nonlinear differential equation, neglecting coupling with other degrees of freedom (e.g., Spyrou and Thompson, 2000; IMO, 2006; ITTC, 2011; IMO, 2016a). Coupling with other degrees of freedom is neglected because a good balance is achieved between simplicity and accuracy when using 1-DOF, although it should be decided on a specific ship hull basis (e.g., Bulian et al., 2009).

The 1-DOF nonlinear differential equation for roll motion may be described by (IMO, 2006):

\[ J_{xx}^{*} \ddot{\phi} + D \left( \dot{\phi} \right) + \Delta \cdot GZ(\phi) = M_{ext} \]  

(2.1)

where:

- \( \phi \): [rad] is the roll angle (dots represent derivatives with respect to time);

- \( J_{xx}^{*} \): [kg \cdot m^2] is the total roll moment of inertia including the hydrodynamic added inertia;

- \( D \left( \dot{\phi} \right) \): [N \cdot m] is the damping moment function, assumed to be dependent only on the instantaneous roll velocity (\( \dot{\phi} \));

- \( \Delta \): [N] is the ship displacement;

- \( GZ(\phi) \): [m] is the hydrostatic roll righting lever with respect to the centre of gravity of the ship (G);

- \( M_{ext} \): [N \cdot m] represent the roll external moments applied to the ship.

Equation 2.1 is commonly represented in its normalized form, by dividing each component by the
CHAPTER 2. GENERAL CONSIDERATIONS

total roll moment of inertia. The normalised roll motion equation is described by:

\[ \ddot{\phi} + d(\dot{\phi}) + \omega_0^2 \cdot \frac{GZ(\phi)}{GM} = m_{ext} \]

\[ \begin{cases} 
  d(\dot{\phi}) = \frac{D(\dot{\phi})}{J_{xx}} \quad \omega_0^2 = \Delta \cdot \frac{GM}{J_{xx}} \\
  m_{ext} = \frac{M_{ext}}{J_{xx}} \quad GM = \left. \frac{dGZ}{d\phi} \right|_{\phi=0}
\end{cases} \tag{2.2} \]

where:

- \( d(\dot{\phi}) \) [1/s²] is the normalised damping function, assumed to be dependent only on the instantaneous roll velocity (\( \dot{\phi} \));
- \( \omega_0 \) [rad/s] is the undamped ship roll natural frequency;
- \( GM \) [m] is the metacentric height with respect to \( G \), considering the vessel freely floating with displacement \( \Delta \);
- \( m_{ext} \) [1/s²] are the normalised external moments. In experimental roll decay tests, \( m_{ext} = 0 \). Instead, when considering ship rolling in regular beam waves, the normalised external moment can be modelled as (IMO, 2006):

\[ m_{ext} = \omega_0^2 \cdot r(\omega) \cdot \pi \cdot s_w \cdot \sin(\omega \cdot t) \tag{2.3} \]

where:

- \( \omega \) [rad/s] is the wave frequency;
- \( s_w \) [nd] is the wave steepness;
- \( r(\omega) \) [nd] is the effective wave slope.

Equations 2.1 and 2.2 are based on the absolute angular approach, which is the approach used generally when working with experimental roll time histories as they are related to the absolute angle. However, in specific events, such as water on deck or critical events such as capsizing, the relative angle approach may be more adequate (Francescutto and Contento, 1998; Bulian and Francescutto, 2011). An example of the application of the relative angle approach is found in the Level 2 Criterion of the Dead Ship Condition failure mode of the Second Generation Intact Stability Criteria (IMO, 2016a), whose explanation may be found in Appendix E.
2.2 Modelling of roll damping

The normalised damping function in Equation 2.2 has been and still is extensively studied (e.g., Dalzell, 1978; Cardo et al., 1982; Haddara and Bennett, 1989; Contento et al., 1996; Spyrou and Thompson, 2000; Fernandes and Oliveira, 2009). As a result, many mathematical models exist to define the normalised damping. Most of them are based on expressing the roll damping as a series of expansions of the ship roll angular velocity and/or ship roll angle.

Among all the mathematical models of roll damping, the most typical is the linear-quadratic-cubic damping model, assuming only the dependency of roll damping on the instantaneous roll velocity ($\dot{\phi}$) (e.g., Bulian, 2004; ITTC, 2011):

$$d(\dot{\phi}) = 2 \cdot \mu \cdot \dot{\phi} + \beta \cdot \dot{\phi} \cdot |\dot{\phi}| + \delta \cdot \dot{\phi}^3 \quad (2.4)$$

where $\mu \ [1/s]$, $\beta \ [1/rad]$ and $\delta \ [s/rad^2]$ are the linear, quadratic and cubic damping coefficients, respectively. Linear-quadratic ($\delta = 0$) or linear-cubic ($\beta = 0$) damping models may be considered as well, depending on the ship hull and on the presence of bilge keels.

No analytical solution of the ship roll motion exist when using the nonlinear damping models, therefore, Parametric Identification Techniques (PIT) should be used or, alternatively, the nonlinear damping model may be replaced by a linear equivalent damping model in a limited time window, such as (Bulian et al., 2009):

$$d(\dot{\phi}) = 2 \cdot \mu_{eq} \cdot \dot{\phi} \quad (2.5)$$

where $\mu_{eq} \ [1/s]$ is the equivalent linear damping coefficient.

The relationship between the nonlinear damping coefficients of Equation 2.4 and the equivalent linear damping coefficient of Equation 2.5 is represented by the following parametric model:

$$\begin{cases} 
\mu_{eq} (A) = \mu + \frac{4}{3} \cdot \pi \cdot \beta \cdot (\tilde{\omega} (A) \cdot A) + \frac{3}{8} \cdot \delta \cdot (\tilde{\omega} (A) \cdot A)^2 \\
\tilde{\omega} (A) = \sqrt{\omega_{0,eq}^2 (A) + \mu_{eq}^2 (A)} \end{cases} \quad (2.6)$$

where $A$ represents the roll amplitude and $\omega_{0,eq} \ [rad/s]$ is the equivalent undamped roll natural frequency, which may be determined by:

$$\omega_{0,eq}^2 (A) = \frac{\omega_0^2}{GM} \cdot \frac{\int_0^{2\pi} GZ (\phi = A \cos (\alpha)) \cdot \cos (\alpha) \, d\alpha}{\pi \cdot A} \quad (2.7)$$
CHAPTER 2. GENERAL CONSIDERATIONS

The parametric model reported in Equation 2.6 is obtained by requiring, in a least squares sense, that the energy loss due to damping during one roll cycle using the equivalent linear roll damping model is equivalent to the energy loss considering the nonlinear roll damping model (ITTC, 2011):

\[
\begin{aligned}
\int_{0}^{2\pi/\tilde{\omega}} 2 \cdot \mu_{eq}(\dot{\phi}) \cdot \dot{\phi}^2 dt &= \int_{0}^{2\pi/\tilde{\omega}} d(\dot{\phi}) \cdot \dot{\phi} dt \\
\dot{\phi} &= A \cdot \tilde{\omega} \cdot \sin(\tilde{\omega} t)
\end{aligned}
\] (2.8)

A similar approach can be used for any generic parametrised nonlinear roll damping model, \(d(\dot{\phi}|p)\), where \(p\) is the set of parameters to be fitted.

2.3 Case study

To analyse the different experimental techniques, in this Thesis, a reference ship hull of a trawler fishing vessel of 47.1 m length has been considered.

The reference ship hull was selected considering the dimensions of the experimental facilities, described in Section 2.3.3, and taking into account the guidelines provided in MSC.1/Circ.1200 (IMO, 2006), paragraph §4.3.2, which specify that the overall model length should be at least of 2 m (or a scale of 1/75, whichever is greater).

As a consequence of the cited restrictions and limitations, a medium size vessel had to be used, which are mainly represented, in Spain, by fishing vessels. Furthermore, fishing vessels often experience significant rolling amplitudes and are generally fitted with anti-rolling devices, which make this type of ships interesting from a research point of view. However, no extensive literature exists concerning experimental estimations of roll damping for fishing vessels (Bass and Haddara, 1989; Chun et al., 2001; Ueno et al., 2003; Aarsæther et al., 2015). Therefore, studying a trawler fishing vessel was considered an opportunity to provide also experimental data sets for this kind of ships.

Hereafter, a description of the the hull form and the tested loading conditions is provided.

2.3.1 Hull form

The hull used in the present study is a model of a trawler fishing vessel at scale 1:20.667. The body plan and a profile view of the ship are illustrated in Figures 2.1 and 2.2. Table 2.1 summarises the main particulars of the ship. The model was tested in bare hull condition, i.e., without rudder and bilge keels.
2.3 Case study

Figure 2.1: Body plan of the tested hull form

Figure 2.2: Profile of the tested hull form. Bar keel highlighted in red

Table 2.1: Main ship particulars. Model scale 1:20.667.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Full scale</th>
<th>Model scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length between perpendiculars, $L_{pp}$ [m]</td>
<td>34.80</td>
<td>1.684</td>
</tr>
<tr>
<td>Length overall, $LOA$ [m]</td>
<td>41.70</td>
<td>2.018</td>
</tr>
<tr>
<td>Length at waterline (at draught $d$), $L_{wl}$ [m]</td>
<td>40.01</td>
<td>1.936</td>
</tr>
<tr>
<td>Breadth overall, $B$ [m]</td>
<td>11.50</td>
<td>0.556</td>
</tr>
<tr>
<td>Draught, $d$ [m]</td>
<td>4.07</td>
<td>0.197</td>
</tr>
<tr>
<td>Depth to upper deck, $H$ [m]</td>
<td>11.94</td>
<td>0.578</td>
</tr>
<tr>
<td>Area of the lateral projection of the bar keel, $A_k$ [m$^2$]</td>
<td>28.35</td>
<td>0.0664</td>
</tr>
<tr>
<td>Block coefficient (at draught $d$), $C_B$ [nd]</td>
<td>0.583</td>
<td>0.583</td>
</tr>
<tr>
<td>Midship coefficient (at draught $d$), $C_M$ [nd]</td>
<td>0.737</td>
<td>0.737</td>
</tr>
</tbody>
</table>
CHAPTER 2. GENERAL CONSIDERATIONS

2.3.2 Tested loading conditions

Two loading conditions (LC01 and LC02) have been considered for the present study. LC01 is an actual loading condition extracted from the Stability Booklet of the ship. LC02 is a variation of the previous loading condition, with the same displacement but a larger metacentric height. Table 2.2 reports the main particulars of the two considered loading conditions.

Table 2.2: Main characteristics of considered loading conditions. Model scale 1:20.667.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LC01</th>
<th>LC02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement, $\Delta$ [kg]</td>
<td>973,000</td>
<td>973,000</td>
</tr>
<tr>
<td>Vertical position of centre of gravity, $KG$ [m]</td>
<td>5.367</td>
<td>5.078</td>
</tr>
<tr>
<td>Metacentric height, $GM$ [m]</td>
<td>0.773</td>
<td>1.062</td>
</tr>
<tr>
<td>Roll natural frequency, $\omega_0$ [rad/s]</td>
<td>0.595</td>
<td>0.750</td>
</tr>
<tr>
<td>Roll natural period, $T_0$ [s]</td>
<td>10.57</td>
<td>8.38</td>
</tr>
<tr>
<td>Dry roll radii of inertia, $k_{xx}$ [m]</td>
<td>3.903</td>
<td>3.529</td>
</tr>
<tr>
<td>Non-dimensional dry roll radii of inertia, $k_{xx}/B$ [nd]</td>
<td>0.339</td>
<td>0.307</td>
</tr>
</tbody>
</table>

Figure 2.3: Picture of inclining experiments using the shifting mass system.

With respect to data reported in Table 2.2, the model displacement was determined from direct weighting. The metacentric heights ($GM$) in the two loading conditions have been determined from
inclining tests also considering large inclinations angles. The system used to perform the inclining tests is a shifting mass that is part of the loading condition. As the shifting mass is part of the loading condition and is only shifted transversally, the roll restoring moment can be determined directly by measuring the transversal distance and the static heel angle. In Figure 2.3, a picture of the actual inclining tests is illustrated. Comparisons between righting lever curves \((GZ(\phi))\) determined from free trim hydrostatic calculations and experimentally measured ones are shown in Figure 2.4 (for LC01) and Figure 2.5 (for LC02). It can be seen that the agreement is very good, which provide confidence on the determined metacentric heights and it also provided confidence regarding the agreement between the reference geometry for computations and the model as actually built.

The roll natural frequency reported in Table 2.2 was determined from the direct analysis of roll decays starting at small roll amplitudes and following the analysis procedure described in Section 4.3. Instead, the dry roll radius of inertia \(k_{xx}\) was not measured, but it was estimated indirectly from the measured roll natural frequency \(\omega_0\) and using linear seakeeping ship motion equations, considering coupled roll-sway-yaw.

Hydrodynamic coefficients and the effective wave slope, mentioned in Equation 2.3, were determined from strip theory calculations using the code from Bulian and Francescutto (2009). The effective wave slopes for each loading condition are reported in Figure 2.6. The points represented in Figure 2.6 correspond to the effective wave slopes calculated at the corresponding ship roll natural frequencies.

![Figure 2.4: Comparison between experimental and calculated GZ curve of the ship model: LC01](image-url)
2.3.3 Experimental facilities

Decay tests and internally excited roll tests have been carried out at "Canal de Ensayos Hidrodinámicos de la Escuela Técnica Superior de Ingenieros Navales" (CEHINAV). The towing tank having dimensions of 100 m in length, 3.8 m in breath and 2.2 m in depth.

Externally excited roll tests have been carried out in the Seakeeping Basin of the "Centro de Experiencias Hidrodinámicas de El Pardo" (CEHIPAR). The Seakeeping Basin having dimensions of 150 m in length, 30 m in breath and 5 m in depth.
2.3 Case study

In both facilities, the ship model was placed transversally at mid-length of the tank to minimise the effect of reflected waves and to maximise the usable test time.

In Figures 2.7 and 2.8 pictures of the experimental facilities used are reported.

![Figure 2.7: CEHINAV facilities. Towing tank.](image1)

![Figure 2.8: CEHIPAR facilities. Seakeeping basin.](image2)
3

Internally excited roll tests

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3.1 Introduction

Internally excited roll tests constitute one of the experimental procedures to estimate roll damping in calm water. In this technique, the ship is freely floating in calm water and is excited to roll by internal means, such as a gyro roll exciter (e.g., Spounge et al., 1986; Bertaglia et al., 2004), contra-rotating masses (e.g., Blume, 1979; Handschel and Abdel-Maksoud, 2014; Wassermann et al., 2016), or an internal shifting mass (e.g., Park et al., 2018).

This experimental approach has the benefit of potentially allowing the determination of roll damping at large rolling amplitudes, which are the amplitude ranges typically relevant for ship safety assessment. Moreover, internally excited roll tests without hard constraints on the model have the benefit of maintaining the natural coupling between roll motion and other relevant motions, particularly sway. This characteristic is lost when forced roll tests (FRT) are carried out with fixed axis (Bačkalov et al., 2016).

It is worth mentioning that this type of technique has been referred in some literature as "Harmonic Excited Roll Motion (HERM)" technique (Handschel and Abdel-Maksoud, 2014; Handschel et al., 2015; Wassermann et al., 2016). However, this nomenclature has not been used in the present work because the motion may not be harmonic. Therefore, to provide a more general nomenclature, merely Excited Roll Tests (or technique) (ERT) is considered, and Internally Excited Roll Tests (IERT) when exciting the ship model by internal devices.

In this Chapter, a technique for estimating roll damping from internally excited roll tests in calm water is described, which is based on exciting the model by an internal shifting mass. Roll damping parameters can then be determined from the analysis of the obtained roll response curves. In the following, the experimental technique set-up is described, as well as the nonlinear mathematical model for representing the system dynamics. A procedure is proposed for determining roll damping coefficients, using, as a basis for the analysis, the developed mathematical model. For model validation purposes, the experimental roll response curves are also compared with those simulated through the developed mathematical model.

It is important to note that, although it would seem appropriate to start the exposure of experimental techniques with decay tests as they constitute the most standard procedure, instead of internally excited roll tests, it has not been done because the experimental results from internally excited roll tests are used in one of the methodologies of decay tests.

3.2 Experimental set-up

In the proposed technique, the ship model is freely floating (or at most softly restrained) in calm water and it is excited to roll using an internal shifting mass that moves following a prescribed motion. The mass in the present experimental technique moves along a linear guide, and the prescribed motion of
the mass, which then generates the internal excitation, is sinusoidal.

The guide is fixed to the ship model near its centre of gravity, and the movement of the mass is obtained through a controllable electrical engine connected to an encoder, shown in Figure 3.1. The maximum amplitude of the moving mass is directly limited by the overall dimensions of the linear guide. For the case study reported hereinafter, the length of the guide corresponds to 206 mm. The moving mass is initially placed at the centre of the guide, and it is allowed to move from the centre up to 90 mm on each side, which therefore corresponds to the maximum amplitude of transversal motion of the mass \( y_{m,\text{max}} \). The oscillation frequency of the moving mass can be varied from 0.1 rad/s to 7.0 rad/s, corresponding to a range of forcing periods from 0.9 s to 62.8 s.

![Figure 3.1: Details of the linear guide used for internally excited roll tests.](image)

Different forcing cases \((FC)\) can be generated by different combinations of the moving mass \((m_m)\) and maximum motion amplitude \((y_{m,\text{max}})\). However, in the present tests, only the moving mass has been changed, always keeping the same maximum amplitude of motion \((y_{m,\text{max}} = 90 \text{ mm})\). Each forcing case can be associated with a nominal amplitude of forcing \((A_{FC})\), which is defined as follows:

\[
A_{FC} = m_m \cdot g \cdot y_{m,\text{max}}
\]  

(3.1)

In addition, it is also useful to associate to each forcing case the static heel \((\alpha_{FC})\) that the ship model reaches when the moving mass is placed at the extreme position.

Translations and rotations of the vessel during the tests are measured using the commercial optical motion capture system “Optitrack Flex 3” (Optitrack, 2017), whereas the actual internal mass motion is recorded from the encoder signal, mainly for checking purposes.

The positioning of the linear guide on the ship model is shown in Figures 3.2 and 3.3. Figure 3.3 also shows the positioning of the trackable markers of the optical system. Also, Figure 3.3 reports the shifting mass, and associated guide, which were used to experimentally determine the ship roll restoring moment for verification purposes, as described in Section 2.3.2.
3.3 Mathematical model

The scope of internally excited roll tests is to provide a set of data from which roll damping coefficients can be determined, with particular attention to cover a large range of rolling amplitudes. To this end, it is necessary to have at disposal a mathematical model of roll dynamics under such conditions. As the
CHAPTER 3. INTERNALLY EXCITED ROLL TESTS

internal moving mass generates forces and moments on the vessel, its effect, in principle, cannot be simplified as a quasi-static weight shift. Thus, the model presented in Section 2.1 cannot be used. The mathematical model has to describe the dynamics of the system in a general form, taking into account the actual excitation provided by the moving mass. It is therefore important to correctly model through first-principles mechanics the excitation from the mass. Moreover, the mathematical model shall embed damping in a parametric form suitable for parameters identification using available experimental data. Finally, it is necessary for the mathematical model to maintain a proper balance between simplicity and accuracy, for it to be practical in potentially routine applications.

This Section reports the development of a simplified mathematical model for describing the system dynamics under excited roll tests with internally moving mass. First, the dynamics of the system is described through 6-DOF nonlinear equations. Then, the equations of motion are reduced by simplifying the system to a 2-DOF nonlinear system accounting only for roll and sway motions. Finally, the 2-DOF equations are algebraically manipulated, and some further assumptions are introduced to arrive at a 1-DOF roll motion equation.

In the development of the modelling, two right-handed reference systems will be considered: an earth-fixed reference system (e.f.r.s.) \( \Omega XYZ \), and a body-fixed reference system (b.f.r.s.) \( OXYZ \). First and second derivatives as determined in the e.f.r.s. are indicated as single and double over dots, respectively, whereas the first and second derivatives in the b.f.r.s. are indicated as single and double prime, respectively.

In this Chapter, the ship without the moving mass itself and the moving mass are considered as different bodies. The centre of gravity of the ship without the moving mass is denoted as \( G_s \), and the shifting mass is considered as a point mass denoted as \( m \). In order to avoid confusions regarding nomenclature, the centre of gravity of the ship with the moving mass on its centerplane is denoted as \( Q \) (instead of \( G \)). Therefore, the metacentric height of the ship with the moving mass on its centerplane is denoted as \( QM \) and the righting lever restoring as \( QZ(\phi) \).

3.3.1 6-DOF equations of motion

The starting point for the development of the mathematical model is the general 6-DOF equations of motion of the ship. Herein, equations are written with reference to the coordinates of the origin \( O \) of the ship fixed reference system expressed in the earth-fixed reference system, and in terms of angular momentum vector. The moving mass is modelled as a point mass with prescribed motion in the b.f.r.s.
Equations of motion can therefore be written as follows:

\[
\begin{align*}
\begin{cases}
(m_s + m_m) \cdot \ddot{X}_O + \frac{\dot{R}_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} (m_s \cdot \delta_{G_S} + m_m \cdot \delta_m (t)) + \\
+ 2 \cdot m_m \cdot \frac{\dot{R}_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot \delta_m' (t) + m_m \cdot \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot \delta_m'' (t) = F_{ext}^s + F_{ext}^m \\
\frac{d}{dt} L_{G_{s,s}} + \left( \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot (m_s \cdot \delta_{G_S} + m_m \cdot \delta_m (t)) \right) \wedge \ddot{X}_O + \\
+ \left( \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot \delta_{G_S} \right) \wedge \left( \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot (m_s \cdot \delta_{G_S}) \right) + \left( \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot \delta_{m} (t) \right) \wedge \left( \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot (m_m \cdot \delta_{m} (t)) \right) + \\
\left( \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot \delta_{m} (t) \right) \wedge \left( 2 \cdot m_m \cdot \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot \delta_{m}' (t) + m_m \cdot \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot \delta_{m}'' (t) \right) = \\
= M_{ext}^s + \left( \frac{R_{S\rightarrow\Sigma}}{S\rightarrow\Sigma} \cdot \delta_{m} (t) \right) \wedge F_{ext}^m
\end{cases}
\end{align*}
\]

(3.2)

In (3.2), one underlining indicates vectors while two underlining indicates matrices, and the symbol “∧” indicates the cross product. The quantities appearing in the equations are defined as follows, together with their units:

- \( m_m \): [kg] mass of the moving mass (assumed to be a point mass);
- \( m_s \): [kg] mass of the ship (without \( m_m \));
- \( O \): reference point, corresponding to the origin of the b.f.r.s.;
- \( G_s \): centre of gravity of the ship (without \( m_m \));
- \( X_O \): position vector of the point \( O \) expressed in the e.f.r.s;
- \( R_{S\rightarrow\Sigma} \): transformation (rotation) matrix from b.f.r.s. to e.f.r.s.;
- \( \delta_{G_S} \): [m] relative position vector of point \( G_s \) with respect to \( O \), expressed in the b.f.r.s.;
- \( \delta_{m} (t) \): [m] relative position vector of (centre of gravity of) the moving mass with respect to \( O \), expressed in the b.f.r.s.;
- \( L_{G_{s,s}} \): [kg \cdot m^2/s] angular momentum of the ship (without \( m_m \)) with respect to \( G_s \), expressed in components with respect to the e.f.r.s.;
- \( F_{ext}^s \): [N] vector of external forces acting on the ship (weight and fluid-structure interaction, plus other external forces, if present), expressed in components with respect to the e.f.r.s.;
CHAPTER 3. INTERNALLY EXCITED ROLL TESTS

- $M_{\text{ext}}^O$: $[N \cdot m]$ moment of external forces acting on the ship (weight and fluid-structure interaction, plus others if present) calculated with respect to $O$, expressed in components with respect to the e.f.r.s.;

- $F_{\text{ext}}^O$: $[N]$ vector of external forces acting on the moving mass (weight), expressed in components with respect to the e.f.r.s.

3.3.2 2-DOF (roll and sway) equations of motion

The system of equations (3.2) is, in principle, appropriate for describing the global dynamics of the system. However, it is considered too complex for a practical use in the determination of roll damping parameters. Therefore, a series of assumptions and simplifications are considered to transform the 6-DOF equations in a simplified 2-DOF model, as follows:

- The motion of the vessel is assumed to be (mainly) two-dimensional, and it is assumed that the dynamics can be described by considering only the sway translation of the point plus a roll rotation. It is noted that the translation denoted herein as “sway” is a translation parallel to the calm water plane, i.e., a translation intended in the e.f.r.s. It is also noted that, as a consequence of this assumption, the effects of coupling with yaw motion are neglected. Although this assumption is reasonable in case of zero speed tests (as in the case tests presented herein), its applicability in case of experiments with forward speed needs to be considered with care, and possibly checked;

- The point $O$ is assumed to correspond to the intersection of the ship centreplane and the waterline of the vessel with zero heel and with the moving mass on-board;

- The vessel is assumed to be port/starboard symmetric and symmetrically loaded. As a result, the ship centre of gravity $G_s$ is on the ship centreplane, i.e.:

$$\delta G_s = (\delta_{G_s,x}, 0, \delta_{G_s,z})^T$$

- The tensor of inertia of the ship (without the mass $m_m$) w.r.t. $G_s$ is assumed to be (approximately) diagonal;

- The mass is assumed to translate only transversally in the b.f.r.s., which means that its position vector, in the b.f.r.s., can be expressed as:

$$\delta m(t) = (\delta_{m,x}, y_m(t), \delta_{m,z})^T$$

- For the rotated vessel, it is assumed that the buoyancy force vector ($\Delta$) and the weight vector of the ship plus the mass ($W_s + W_m$) approximately compensate during the motion, i.e.:

$$W_s + W_m + \Delta \approx 0$$
3.3 Mathematical model

- The force and the moment are split into two contributions:
  - One contribution due to the combination of weights (ship weight, moving mass) and buoyancy;
  - One contribution due to the fluid-structure interaction hydrodynamics.

- The hydrodynamic fluid-structure interaction force and moment are modelled using a combination of a linear hydrodynamic approach (for added mass terms) plus a nonlinear contribution. This latter contribution is assumed to comprise principally damping effects;

- The motion is assumed to be approximately harmonic, with a main forcing frequency $\omega \ [rad/s]$, in such a way that constant frequency dependent added mass and damping terms can be considered for describing the fluid-structure interaction at steady state for each forcing condition;

According to the above assumptions, the original 6-DOF equations of motion (3.2) simplify as follows:

\[
\begin{align*}
(m_s + m_m) \cdot \ddot{Y}_O + & \left[ - (m_s + m_m) \cdot \cos(\phi) \cdot \delta_{Q,z} - m_m \cdot \sin(\phi) \cdot y_m(t) \right] \cdot \phi' + \\
& + \left[ (m_s + m_m) \cdot \sin(\phi) \cdot \delta_{Q,z} - m_m \cdot \cos(\phi) \cdot y_m(t) \right] \cdot \phi'^2 + \\
& - 2 \cdot m_m \cdot \sin(\phi) \cdot y_m'(t) \cdot \phi + m_m \cdot \cos(\phi) \cdot y_m''(t) = \\
& = - A_{22}(\omega) \cdot \ddot{Y}_O - A_{24}^O(\omega) \cdot \phi + F_{D,Y}(t) \\
\end{align*}
\]

\[
\begin{align*}
[I_{G,s,xx} + m_s \cdot \delta_{G,s,z}^2 + m_m \cdot (\delta_{m,z}^2 + y_m'(t))] \cdot \phi' + \\
& + \left[ - (m_s + m_m) \cdot \cos(\phi) \cdot \delta_{Q,z} - m_m \cdot \sin(\phi) \cdot y_m(t) \right] \cdot \ddot{Y}_O + \\
& + 2 \cdot m_m \cdot y_m(t) \cdot y_m'(t) \cdot \phi - m_m \cdot \delta_{m,z} \cdot y_m''(t) = \\
& = - (m_s + m_m) \cdot g \cdot \overline{GZ}(\phi) - m_m \cdot g \cdot y_m(t) \cdot \cos(\phi) + \\
& - A_{44}^O(\omega) \cdot \phi - A_{42}^O(\omega) \cdot \ddot{Y}_O + M_{O,D}(t)
\end{align*}
\]

where:

- $Y_O: [m]$ is the horizontal lateral translation of point $O$ in the e.f.r.s;
- $\phi: [rad]$ is the roll angle;
- $Q: \ restarts$ is the centre of mass of the ship when the mass $m_m$ is on the centreplane. Accordingly, the coordinates of $Q$ with respect to $O$ in the b.f.r.s. can be determined as follows:
\[ Q = \left( \frac{m_s \cdot \delta_{G_s} + m_m \cdot \delta_{m,xz}}{m_s + m_m} \right) \]
\[ \delta_{m,xz} = (\delta_{m,x}, 0, \delta_{m,z})^T \]

- \( I_{G_s,x,xz} \): [kg\cdot m^2] is the (dry) moment of inertia of the ship (without moving mass) w.r.t. a longitudinal axis \( x \) passing through \( G_s \);
- \( y_m \): [m] is the instantaneous transversal coordinate of moving mass in the b.f.r.s.;
- \( \bar{Q}Z(\phi) \): [m] is the hydrostatic roll righting lever w.r.t. point \( O \) considering the vessel freely floating with displacement:
  \[ \Delta = (m_s + m_m) \cdot g \]
  This term is meant to represent the hydrostatic contribution to the fluid-structure interaction force;
- \( A_{22}(\omega) \): [N/(m/s^2)] is the frequency dependent sway added mass;
- \( A_{42}^O(\omega) \): [N/(rad/s^2)] is the frequency dependent roll-to-sway added mass coefficient, w.r.t. point \( O \);
- \( A_{44}^O(\omega) \): [N\cdot m/(rad/s^2)] is the frequency dependent roll added mass coefficient, w.r.t. point \( O \);
- \( A_{44}^O(\omega) \): [N\cdot m/( rad/s^2)] is the frequency dependent sway-to-roll added mass coefficient, w.r.t. point \( O \);
- \( F_{D,Y}(t) \): [N] is the sway force associated with, possibly nonlinear, sway damping;
- \( M_{O,D}(t) \): [N\cdot m] is the roll moment w.r.t. point \( O \) associated with, possibly nonlinear, roll damping.

### 3.3.3 1-DOF roll motion equation

The model (3.3) could in principle be used for modelling dissipation effects. It would require providing parametric models for the terms \( F_{D,Y}(t) \) and \( M_{O,D}(t) \), and it would require experimental measurement of both sway and roll. However, this is considered still too complex for routine practical roll damping determination. Therefore, some additional simplifications are introduced with the intention of further reducing the model complexity. The scope of the simplifications introduced at this stage is to arrive at a 1-DOF equation of roll motion, which can be more directly used for roll damping identification purposes, having available roll motion recordings.

As a first step, the system of equations (3.3) is manipulated in such a way to obtain a single equation of roll motion. \( \ddot{Y}_O \) is obtained from the first equation in (3.3) as a function of the remaining quantities,
and it is then substituted in the second equation in (3.3). The following roll equation is then obtained:

\[ J_T (\phi, t) \ddot{\phi} = M_{\phi, hs} (\phi) + M_{\phi, RB}^O (t) + M_{\phi,m}^O (\phi, t) - \frac{M_{42} (\phi, t) + A_{42}^O (\omega)}{M_{22} + A_{22} (\omega)} F_{Y, RB} (\phi, t) + M_{\phi,D} (\phi, t) \]

with:

\[
\begin{align*}
J_T (\phi, t) &= M_{44} (t) + A_{44}^O (\omega) - \frac{(M_{42} (\phi, t) + A_{42}^O (\omega)) (M_{24} (\phi, t) + A_{24}^O (\omega))}{M_{22} + A_{22} (\omega)} \\
M_{44} (t) &= I_{G,s,xz} + m_s \delta^2_{G,s,z} + m_m (\delta^2_{m,z} + y_m^2 (t)) \\
M_{24} (\phi, t) &= M_{42} (\phi, t) = - (m_s + m_m) \cos (\phi) \delta_{Q,z} - m_m \sin (\phi) y_m (t) \\
M_{22} &= m_s + m_m \\
M_{\phi, hs} (\phi) &= - (m_s + m_m) g QZ(\phi) \\
M_{\phi, RB}^O (t) &= - 2m_m y_m (t) y'_m (t) \\
M_{\phi,m}^O (\phi, t) &= - m_m y_m (t) \cos (\phi) + m_m \delta_{m,z} y''_m (t) - \frac{(M_{42} (\phi, t) + A_{42}^O (\omega))}{M_{22} + A_{22} (\omega)} F_{Y,m} (t) \\
F_{Y,m} (\phi, t) &= - m_m \cos (\phi) y''_m (t) \\
F_{Y, RB} (\phi, t) &= - [(m_s + m_m) \sin (\phi) \delta_{Q,z} - m_m \cos (\phi) y_m (t)] \phi^2 + 2m_m \sin (\phi) y'_m (t) \phi \\
M_{\phi,D} (\phi, t) &= - \left( \frac{M_{42} (\phi, t) + A_{42}^O (\omega)}{M_{22} + A_{22} (\omega)} \right) F_{Y,D} (t) + M_{O,D} (t)
\end{align*}
\]

The final simplification is to assume that the dissipative term \( M_{\phi,D} (\phi, t) \) can be approximated as being explicitly dependent only on the roll velocity, as follows:

\[ M_{\phi,D}(\phi, t) \approx M_{\phi,D}(\phi(t)) = - J_T (\phi, t) \cdot d(\dot{\phi}) \tag{3.5} \]

It is also to be reminded that, according to Equation (3.4), \( J_T \) is a frequency dependent term. Under the assumption (3.5), Equation (3.4) can be rewritten as follows:

\[
\ddot{\phi} + d(\dot{\phi}) + \left( \frac{m_s + m_m}{J_T (\phi, t)} \right) g QZ(\phi) = \\
= \frac{M_{\phi, RB}^O (t)}{J_T (\phi, t)} + \frac{M_{\phi,m}^O (\phi, t)}{J_T (\phi, t)} - \frac{M_{42} (\phi, t) + A_{42}^O (\omega)}{M_{22} + A_{22} (\omega)} \cdot \frac{F_{Y, RB} (\phi, t)}{J_T (\phi, t)} 
\]

(3.6)

For the roll damping term \( d(\dot{\phi}) \), the typical linear-quadratic-cubic roll damping model can be used, as well as any generic parametrised nonlinear roll damping model, as described in Section 2.2.
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3.4 Procedure for the determination of roll damping coefficients

The model (3.6) can be used for the determination of roll damping parameters, starting from the availability of experimental roll response curves from internally excited roll tests.

To this end, a Parameters Identification Technique (PIT) could be set-up. Examples of PITs have been described by, e.g., Francescutto and Contento (1998), Francescutto and Contento (1999), Francescutto et al. (1998) and in MSC.1/Circ.1200 (IMO, 2006). The typical approach of PIT is, essentially, to define a set of free parameters in the mathematical model and then determine the optimum parameters by minimising the error between predictions and measurements. Such an approach can, in principle, be applied using the mathematical model as a black-box.

In the present case, however, a slightly different approach is proposed. The approach is tailored to the specific problem at hand, it is considered to be informative for the user who has to carry out the determination of the roll damping coefficients, and it is deemed to be robust.

3.4.1 Description of the procedure

In the procedure described in the following, it is assumed that a series of forcing cases \( FC \) are considered. Each \( FC \) corresponds to a specific combination of moving mass and maximum mass displacement, and for each \( FC \) a roll response curve is obtained by carrying out experimental tests at different forcing frequencies. The resonance peak of the roll response curve, \( A_{res} \), and the corresponding frequency, \( \omega_{res} \), are then obtained for each \( FC \).

The scope of the procedure is to fit roll damping coefficients in such a way that the mathematical model is able to reproduce approximately the same rolling amplitudes for the corresponding forcing cases.

In the procedure described herein it is assumed that the mathematical model can capture, in a sufficiently accurate way, the peak resonance frequency \( \omega_{res} \) for each \( FC \). If the matching of the resonance frequencies is not sufficiently accurate, then a tuning of the dry inertia may be carried out through the inertia term \( J_T(\phi, t) \). Added mass and inertia coefficients appearing in model (3.6) are assumed to be pre-calculated for the considered ship draught and trim using numerical calculations (typically potential linear hydrodynamics). It is worth noting here that the methodology for the calculation of added mass and inertia coefficients (e.g., strip-theory or three-dimensional calculations) may affect the final result of the damping identification procedure. This aspect has not been addressed in this study, where, as described in Section 2.3.2, strip-theory calculations have been used for the determination of necessary hydrodynamic coefficients.
3.4 Procedure for the determination of roll damping coefficients

The fitting of the experimental data is carried out in four main iterative steps, as follows:

- **Step 1.** For each FC a series of simulations of model (3.6) are carried out assuming a damping model characterised by an equivalent linear damping coefficient \( \mu_{eq} \):

\[
d(\dot{\phi}) = 2 \cdot \mu_{eq} \cdot \dot{\phi}
\]  

(3.7)

Different \( \mu_{eq} \) coefficients are tested to find the coefficient which provides the same peak response amplitude as that obtained from the experiments for the considered forcing case. The first estimation of \( \mu_{eq} \) is carried out by considering a simplified 1-DOF model, where the forcing from the moving mass is approximated by considering only the corresponding quasi-static heeling moment for small heeling angles, as follows:

\[
\ddot{\phi} + 2 \cdot \mu_{eq} \cdot \dot{\phi} + \frac{QZ(\phi)}{QM} = \frac{m_m \cdot g \cdot y_{m,\text{max}}}{\Delta \cdot QM} \cdot \sin(\omega t)
\]

(3.8)

where \( \omega_0 [\text{rad/s}] \) is the natural roll frequency, \( QM [\text{m}] \) is the metacentric height w.r.t. point \( Q \) considering the vessel freely floating with displacement \( \Delta \) \( (QM = QZ(\phi)/d\phi |_{\phi=0} ) \) and \( y_{m,\text{max}} [\text{m}] \) is the amplitude of mass translation.

The final result in (3.8) comes from the fact that, at resonance, the inertial part and the restoring part of the equation of motion (approximately) cancel out, and therefore the amplitude of the damping term can be directly related to the amplitude of the forcing term at the peak response frequency. For a linear system, there is a perfect cancelling between the inertial term and the restoring term when the forcing frequency is equal to the undamped roll natural frequency \( \omega_0 \), and this frequency is very close to the peak response frequency when the system, as in case of roll, is very lightly damped. In case of nonlinear restoring the same concept can be applied with good approximation. In such case, there is an approximate cancelling between the inertial part and the restoring part when the system oscillates at its amplitude-dependent roll resonance frequency, i.e., when the response is along the system backbone curve.

It is also worth noting that the determination of \( \mu_{eq} \) according to (3.8) can be considered as a generalisation of the Blume method (Blume, 1979; Handschel and Abdel-Maksoud, 2014). In fact, the Blume method assumes linear restoring, while \( \mu_{eq} \) obtained according to (3.8) takes into
account the possible shift of resonance frequency due to restoring nonlinearities, since, in general, it is $\omega_{res} \neq \omega_0$.

The equivalent linear damping coefficient $\mu_{eq}$ is then systematically varied around the first guess value obtained from (3.8), and the model (3.6) is simulated to obtain the peak amplitude and frequency of the roll response curve associated to each $\mu_{eq}$ value. The variation is specified in such a way that the range of peak amplitude values obtained from the simulations comprises the peak amplitude as obtained from the experiments. An example of a graphical representation of results from this step is shown in Figure 3.4, where different forcing cases (FC01 to FC07) are considered. It is noted that the Figure shows the dependence between $\mu_{eq}$ and the product $A_{res} \cdot \omega_{res}$ because this representation will allow the same graph to be directly used for the determination of the nonlinear damping coefficients, as specified at the fourth step of the procedure.

![Figure 3.4: Damping determination from internally excited roll tests: Step 1.](image)

- **Step 2.** Given the amplitude $A_{res}$ and frequency $\omega_{res}$ of the experimental peaks for each $FC$, the actual equivalent linear damping coefficient value can be obtained firstly by interpolation, as shown in Figure 3.5. Further simulations can be carried out to verify and, if necessary, refine, the value of $\mu_{eq}$ for each forcing case. However, considering the observed smooth behaviour of the dependence between $\mu_{eq}$ and $A_{res} \cdot \omega_{res}$, the interpolated value is expected to be sufficiently accurate. Eventually, from this step, the equivalent linear damping coefficient of the ship for the specific loading condition is obtained as a function of roll amplitude and a corresponding frequency. It is also noted that the determination of the equivalent linear damping is carried out through interpolation of $\mu_{eq}$ with respect to the rolling amplitude $A_{res}$, and not with respect to the product $A_{res} \cdot \omega_{res}$, as...
3.4 Procedure for the determination of roll damping coefficients

when the peak resonance frequencies are sufficiently well captured (see next step of the procedure), the interpolation based on $A_{\text{res}}$ produces mainly the same results as an interpolation based on $A_{\text{res}} \cdot \omega_{\text{res}}$. Nevertheless, results are still reported in Figure 3.5 as a function of the product $A_{\text{res}} \cdot \omega_{\text{res}}$ because this representation is more suitable for the final determination of nonlinear roll damping coefficients.

Figure 3.5: Damping determination from internally excited roll tests: Step 2.

- **Step 3.** This step is needed when peak frequencies of the roll response as obtained from simulations do not match the experimental peak frequencies. In such cases, the dry radius of inertia of the vessel is tuned to match the experimental results better. Steps 1 and 2 are then repeated, following an iterative process until the peak amplitudes and frequencies from the simulations match the experimental ones.

There are two main reasons requiring, typically, the tuning of the dry roll radius of inertia. One reason is associated with the fact that the dry roll radius of inertia may be unknown (as in the case presented herein) or it may be affected by measuring error which may be compensated by the tuning process. The second reason is associated with modelling aspects. In fact, the tuning of the dry roll radius of inertia can be considered as a practical way to compensate, to a certain extent, for the simplifications embedded in the mathematical model (particularly regarding coupling among motions) and for the approximations/assumptions associated with the numerical estimation of hydrodynamic coefficients (added mass).

- **Step 4.** The final step of the procedure is to use the obtained values of $\mu_{\text{eq}}$ for different combinations of amplitude and frequency, $A_{\text{res}}$ and $\omega_{\text{res}}$, to determine the characteristic coefficients for
the nonlinear roll damping model. If the nonlinear roll damping model is assumed to be the typical linear-quadratic-cubic model (reported in Equation 2.4), linear and nonlinear damping coefficients can be determined from fitting, using the following relation (e.g., Bulian et al., 2009):

$$\mu_{eq} (A_{res}, \omega_{res}) = \mu + \frac{4}{3} \cdot \frac{1}{\pi} \cdot \beta \cdot (A_{res} \cdot \omega_{res}) + \frac{3}{8} \cdot \delta \cdot (A_{res} \cdot \omega_{res})^2$$  \hspace{1cm} (3.9)

A similar approach can be used for any generic parametrised nonlinear roll damping model (see Equation 2.8).

An example is shown in Figure 3.6, where the quadratic damping coefficient was fixed to zero, and, therefore, the fitting was based on the apriori assumption of a linear-cubic damping model.

![Figure 3.6: Damping determination from internally excited roll tests: Step 4.](image)

### 3.4.2 Verification of the procedure

A consistency check has been done to verify the goodness of the proposed procedure to determine the roll damping coefficients from internally excited roll tests. In this case, roll response curves are synthetically generated by solving numerically model 3.6 with the 4th order Runge-Kutta integration scheme with known parameters and assuming a linear-cubic damping model. The synthetic response curves are then analysed according to the described procedure, and roll damping coefficients are determined.

Results are shown in Figure 3.7, where roll damping coefficients (target and fitted) are reported in the legend. It can be noticed that the agreement of roll damping coefficients is very good: a small overestimation of the linear roll damping coefficient is compensated by a slight underestimation of the
3.4 Procedure for the determination of roll damping coefficients

cubic damping coefficient. This compensation allows the equivalent linear roll damping coefficient to be very close to the target one. Figure 3.8 compares roll response curves obtained from simulation of model 3.6 using the fitted roll damping coefficients with the original synthetically generated roll response curves.

According to the obtained results, it appears that the used approach shows the expected consistency.

![Figure 3.7](image)

**Figure 3.7:** Verification of the internally excited roll test analysis procedure. Equivalent linear damping coefficient.

![Figure 3.8](image)

**Figure 3.8:** Verification of the internally excited roll test analysis procedure. Roll response curves.
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3.5 Experimental results

3.5.1 Test cases

For each loading condition described in Section 2.2, different forcing cases were tested, with characteristic parameters reported in Table 3.1 (LC01) and Table 3.2 (LC02).

Table 3.1: Forcing cases for internally excited roll tests: LC01.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>FC01</th>
<th>FC02</th>
<th>FC03</th>
<th>FC04</th>
<th>FC05</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_m$ [kg]</td>
<td>0.388</td>
<td>0.774</td>
<td>1.162</td>
<td>1.567</td>
<td>2.323</td>
</tr>
<tr>
<td>$A_{FC}$ [N·m]</td>
<td>0.343</td>
<td>0.683</td>
<td>1.026</td>
<td>1.384</td>
<td>2.051</td>
</tr>
<tr>
<td>$\alpha_{FC}$ [deg]</td>
<td>0.49</td>
<td>0.97</td>
<td>1.46</td>
<td>1.97</td>
<td>2.93</td>
</tr>
</tbody>
</table>

Table 3.2: Forcing cases for internally excited roll tests: LC02.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>FC01</th>
<th>FC02</th>
<th>FC03</th>
<th>FC04</th>
<th>FC05</th>
<th>FC06</th>
<th>FC07</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_m$ [kg]</td>
<td>0.539</td>
<td>1.076</td>
<td>1.618</td>
<td>2.133</td>
<td>3.228</td>
<td>4.283</td>
<td>5.378</td>
</tr>
<tr>
<td>$A_{FC}$ [N·m]</td>
<td>0.476</td>
<td>0.950</td>
<td>1.429</td>
<td>1.883</td>
<td>2.850</td>
<td>3.781</td>
<td>4.748</td>
</tr>
<tr>
<td>$\alpha_{FC}$ [deg]</td>
<td>0.49</td>
<td>0.98</td>
<td>1.47</td>
<td>1.95</td>
<td>2.95</td>
<td>3.93</td>
<td>4.97</td>
</tr>
</tbody>
</table>

Since the amplitude of moving mass motion is kept constant ($y_{m,\text{max}} = 0.09$ m), changing the forcing case corresponds to changing the moving mass. To keep the same vertical position of the point $Q$ from one $FC$ to the other, the difference between the moving mass for the strongest forcing case and the one under analysis was placed near the linear rail, at the centerplane and at the same height of the linear rail. With this experimental arrangement, for each forcing case and each loading condition, part of the mass conceptually moves from being considered as “moving mass” to being considered as part of the “ship without moving mass”. However, the total mass of the model as well as the total dry roll moment of inertia considering the moving mass on the centreplane remain constant (under the approximation of point moving masses). As a result, with reference to the mathematical model (3.6), the considered experimental arrangement is such that the following terms can be considered, at least with
3.5 Experimental results

good approximation, as constants:

\[
\begin{align*}
    m_s + m_m &= m_T = \text{constant} \\
    I_{G,s,xx} + m_s \cdot \delta_{G,s,z}^2 + m_m \cdot \delta_{m,z}^2 &= I_{Q,T,xx} + (m_s + m_m) \cdot \delta_{Q,z}^2 = \text{constant}
\end{align*}
\]  

(3.10)

The values of \( \alpha_{FC} \) for each forcing condition, as reported in Table 3.1 (LC01) and Table 3.2 (LC02), have been determined as solutions of the following equilibrium equation:

\[
(m_s + m_m) \cdot g \cdot QZ(\alpha_{FC}) = A_{FC} \cdot \cos(\alpha_{FC})
\]  

(3.11)

using \( QZ(\phi) \) from hydrostatic calculations.

The raw measured data was filtered using an 8th order Butterworth low-pass filter with a normalised cut-off frequency of 0.125.

3.5.2 Roll response curves

The measured roll-response curves, as a function of the excitation frequency, for each loading condition and each forcing case, are shown in Figure 3.9 (LC01) and Figure 3.10 (LC02). In Appendix A, the corresponding numeric values are reported.

The reported reference amplitudes for each test represent the average rolling amplitudes within the analysis time window based on the analysis of maxima and minima of roll. Considering that experimental roll motion, despite the nonlinearities of the system, was almost sinusoidal in all relevant steady-state conditions, the rolling amplitude based on the analysis of extremes is well representative of the actual magnitude of motion. An indication of the uncertainty level due to the variability of rolling amplitude is presented through bars corresponding to the maximum and minimum rolling amplitudes in the analysed time window. The analysed time window for each test was decided by trying to reduce the effect of the initial transient and by avoiding the analysis of portions of recorded time history which might have been affected by reflected waves from the tank ends (wavemaker side and beach side). Nevertheless, in some cases, oscillations of roll envelope were still present due to transient effects, and the magnitude of such oscillations is represented by the variability range shown in the Figures.
Figure 3.9: Experimental roll response curves from internally excited roll tests: LC01. Full results (top) and zoom close to the peak region (bottom). Black cross markers represent smoothed peaks which were used for the analysis.
3.5 Experimental results

Figure 3.10: Experimental roll response curves from internally excited roll tests: LC02. Full results (top) and zoom close to the peak region (bottom). Black cross markers represent smoothed peaks which were used for the analysis.
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From Figures 3.9 and 3.10, in both loading conditions, a noticeable secondary lower frequency peak can be observed, which becomes more evident as the roll angles increase. This secondary peak was an unexpected result, which the present modelling, as it will be shown later, cannot capture. The actual source of this secondary peak could not be fully clarified. However, it seems a parasitic peak which appears because, when the ship is excited to roll at large angles, pitch and heave motions are also excited due to nonlinear coupling, part of which can be explained from buoyancy effects. Due to the symmetry of the system, heave and pitch are excited with a frequency which is double the roll oscillation frequency. The excitation of longitudinal motions generates waves propagating in the longitudinal direction with respect to the vessel and reflecting on the tank walls. These waves, in some conditions, tend to generate a wave field around the model which eventually affects the roll motion of the ship. During the tests, this effect could be noticed visually for LC01, while this effect was less noticeable from visual observations for LC02. This difference in the magnitude of the visually observed wave reflection phenomenon between the two loading conditions appears to be in line with the relative magnitude of the secondary peak, which is more significant for LC01 compared to LC02.

To address the presence of this secondary peak, a smoothing polynomial was used to represent the peak region of each roll response curves and to determine the peak amplitudes $A_{res}$ and associated frequencies $\omega_{res}$. The corresponding points are reported in Figures 3.9 and 3.10 as black cross markers and are summarised in Table 3.3. It is important to highlight that the applied smoothing procedure may influence the peak points used for the determination of roll damping. For instance, in LC02 and FC03 (see Figure 3.10), the actual peak seems to be at a higher frequency compared to that determined by the polynomial smoothing. However, this effect appears to have limited importance considering the various assumptions of the overall damping assessment procedure.

Table 3.3: Peak amplitudes and corresponding normalised frequencies for each forcing case measured from internally excited roll tests: LC01 and LC02.

<table>
<thead>
<tr>
<th>Forcing Case</th>
<th>LC01 $\omega_{res}/\omega_0$ [rad]</th>
<th>$A_{res}$ [deg]</th>
<th>LC02 $\omega_{res}/\omega_0$ [rad]</th>
<th>$A_{res}$ [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC01</td>
<td>0.974</td>
<td>11.18</td>
<td>0.991</td>
<td>9.87</td>
</tr>
<tr>
<td>FC02</td>
<td>0.960</td>
<td>14.34</td>
<td>0.983</td>
<td>12.73</td>
</tr>
<tr>
<td>FC03</td>
<td>0.950</td>
<td>16.80</td>
<td>0.968</td>
<td>15.30</td>
</tr>
<tr>
<td>FC04</td>
<td>0.944</td>
<td>18.98</td>
<td>0.971</td>
<td>17.12</td>
</tr>
<tr>
<td>FC05</td>
<td>0.948</td>
<td>21.46</td>
<td>0.953</td>
<td>20.72</td>
</tr>
<tr>
<td>FC06</td>
<td>-</td>
<td>-</td>
<td>0.947</td>
<td>23.09</td>
</tr>
<tr>
<td>FC07</td>
<td>-</td>
<td>-</td>
<td>0.942</td>
<td>25.72</td>
</tr>
</tbody>
</table>
3.5 Experimental results

3.5.3 Repeatability analysis

An analysis of the repeatability of the tests was carried out to assess the precision level of roll measurements. The level of repeatability was assessed by repeating three times different run conditions, considering the resonance frequency region as well as the regions of low and high frequencies. It is noted that repeated tests are directly represented in Figure 3.9 and Figure 3.10. However, since the repeatability is very high, as described in the following, the different points mainly overlap in the graphs.

An example of the roll time histories of three repeated tests is shown in Figure 3.11, while Table 3.4 and Table 3.5 summarise the experimentally obtained mean amplitudes for each repeated test condition. Moreover, the relative percentage difference with respect to the first run of each test condition is also reported. From the reported results, it can be concluded that the repeatability level, hence the precision of the tests, is very good. In fact, the relative difference of amplitudes is typically below 1.5% with some cases presenting higher relative differences which, however, are always below 2.5% for the considered conditions.

![Figure 3.11: Example of repeatability assessment of internally excited roll tests: LC01.](image)
### 3.5.4 Determination of roll damping coefficients

Starting from experimental data as reported in Table 3.3, roll damping coefficients have been determined for the two loading conditions following the methodology described in Section 3.4. It is noted, in particular, that “Step 3” (i.e., the iterative tuning of the dry inertia) was also carried out. In both cases
a linear-cubic damping model was considered as a reference model, as the fitting of the other models (linear-quadratic or linear-quadratic-cubic) led to negative damping coefficients and/or to overall negative linear equivalent roll damping coefficient at small rolling amplitudes.

The reason why it was necessary to re-tune the dry roll radii of inertia is that the simulated roll-response curves showed a shifting of the peak frequencies compared to the experimental data when using nominal values based on the global dry radii of inertia estimated in Table 2.2. This shift was systematic, and simulated roll peak frequencies were lower than the ones observed in the experiments, for all forcing cases, as depicted in Figure 3.12 and Figure 3.13. The actual reasons for the observed shifts have not been fully clarified. However, the shifting may be, at least partially, a consequence of the fact that model (3.4), and, as a consequence, model (3.6), account only for roll and sway coupling, without actually considering the coupling with yaw. Lack of coupling with yaw is missing both from a hydrodynamic perspective as well as from rigid body dynamics. Instead, the global radii of inertia reported in Table 2.2 implicitly account also for the linear coupling with yaw. Moreover, calculations of hydrodynamic coefficients have been based on strip-theory, and they do not account, therefore, for three-dimensional effects. Three-dimensional effects might be non-negligible for the considered vessel, which has a relatively small length-beam ratio. Because of the various simplifications and assumptions in the derivation of the model (3.6), it may be expected that some physical phenomena are not properly accounted for. Since the most noticeable effect has been observed to be the underestimation of the resonance frequencies, this has therefore been addressed by a reduction of the dry roll radii of inertia used in the mathematical model, which proved to be an effective means to obtain a better matching.
Table 3.6 reports the roll damping coefficient as obtained from the fitting of model (3.6) on the experimental roll response peaks using a linear equivalent roll damping model after tuning of the dry inertia. The Table also reports initial guess values (i.g.) of linear equivalent roll damping coefficient as obtained from Equation 3.8. It can be noticed that initial guess values are systematically smaller than the values which are eventually obtained from the proposed procedure. Table 3.7 then shows the values of roll damping coefficients $\mu$ and $\delta$ as obtained from fitting (3.9), fixing $\beta$ to zero, starting from $\mu_{eq}$ in Table 3.6 and using the tuned dry roll inertia.

Figure 3.14 and Figure 3.15 provide a graphical representation of the fitting of the damping model for the determination of damping coefficients for LC01 and LC02, respectively. In addition to the fitting based on the reference linear-cubic model, the Figures also report the results of the fitting of the linear-quadratic-cubic roll damping model.

From the results in Figure 3.14, it can be noted that for LC01 the linear-cubic model is able to accurately fit the $\mu_{eq}$ data, with a relatively larger difference for FC04 which, however, seems to be outside the general trend from the other forcing cases. It can also be noticed that data are well represented, in principle, also by the linear-quadratic-cubic model which, however, is associated with a negative quadratic damping coefficient. It can also be noticed that, while both the linear-cubic and linear-quadratic-cubic models, thanks to the fitting constraint, show similar $\mu_{eq}$ within the experimentally tested range, they show significant differences in the region of small rolling amplitudes. These differences are due to the lack of experimental data from excited roll tests in the region of low amplitude rolling.
3.5 Experimental results

Table 3.6: Linear equivalent roll damping coefficient from internally excited roll tests for each loading condition and forcing case.

<table>
<thead>
<tr>
<th>Forcing Case</th>
<th>( \mu_{eq} [1/s] )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LC01</td>
</tr>
<tr>
<td>FC01</td>
<td>0.0632</td>
</tr>
<tr>
<td>(i.g. 0.0603)</td>
<td>(i.g. 0.0854)</td>
</tr>
<tr>
<td>FC02</td>
<td>0.0994</td>
</tr>
<tr>
<td>(i.g. 0.0914)</td>
<td>(i.g. 0.1334)</td>
</tr>
<tr>
<td>FC03</td>
<td>0.1282</td>
</tr>
<tr>
<td>(i.g. 0.1232)</td>
<td>(i.g. 0.1696)</td>
</tr>
<tr>
<td>FC04</td>
<td>0.1537</td>
</tr>
<tr>
<td>(i.g. 0.1479)</td>
<td>(i.g. 0.1991)</td>
</tr>
<tr>
<td>FC05</td>
<td>0.2022</td>
</tr>
<tr>
<td>(i.g. 0.1930)</td>
<td>(i.g. 0.2535)</td>
</tr>
<tr>
<td>FC06</td>
<td>-</td>
</tr>
<tr>
<td>(i.g. 0.3038)</td>
<td></td>
</tr>
<tr>
<td>FC07</td>
<td>-</td>
</tr>
<tr>
<td>(i.g. 0.3446)</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.7: Roll damping coefficients from internally excited roll tests for each loading condition.

<table>
<thead>
<tr>
<th>Damping Coefficients</th>
<th>LC01</th>
<th>LC02</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu [1/s] )</td>
<td>0.0065</td>
<td>0.0566</td>
</tr>
<tr>
<td>( \beta [1/\text{rad}] )</td>
<td>0.0000 (fixed)</td>
<td>0.0000 (fixed)</td>
</tr>
<tr>
<td>( \delta [s/\text{rad}^2] )</td>
<td>0.5702</td>
<td>0.4050</td>
</tr>
</tbody>
</table>

Results in Figure 3.15 for LC02 show that the linear-cubic model can provide a reasonable fit of the experimental data, although the fitting is worse compared to LC01. In particular, the linear-cubic model shows difficulties in fitting experimental damping at the smallest forcing, and the curvature which is implicit in the assumed linear-cubic model does not seem to be present in the experimental data. For LC02, the flexibility of the full linear-quadratic-cubic model provides a better fitting of data in the range of tested rolling amplitudes compared to the linear-cubic model. However, the linear-quadratic-cubic model leads to a negative linear damping coefficient, and it, therefore, provides an overall negative
CHAPTER 3. INTERNALLY EXCITED ROLL TESTS

linear equivalent roll damping coefficient for the range of small rolling amplitudes, which is distinctly non-
physical. Also, in this case, the differences between the two models are relatively small in the range of
tested rolling amplitudes, thanks to the constraint induced by the fitting of the data. However, the two
modelling show significant differences in the region of small rolling amplitudes and this, again, is due to
the lack of experimental data from excited roll tests in the region of low amplitude rolling.

Figure 3.14: Damping determination from internally excited roll tests: LC01.

Figure 3.15: Damping determination from internally excited roll tests: LC02.
3.5 Experimental results

3.5.5 Validation of the mathematical model

Once roll damping coefficients have been determined according to the described procedure, the mathematical model (3.6) can be used for simulating roll motion in the experimental forcing cases. Comparison of simulation results with experiments, both in terms of roll response curves and of roll time histories, can then be used to validate the used mathematical model.

Figures 3.16 (for LC01) and 3.17 (for LC02) show comparisons of experimental roll response curves and numerical simulations based on the mathematical model (3.6), as a function of the excitation frequency $\omega$. For the simulations, the damping coefficients specified in Table 3.7 have been used for the two considered loading conditions. As previously reported, linear hydrodynamic coefficients, for each excitation frequency, have been determined from linear strip-theory calculations. However, to match the experimental roll response curves, it was necessary to retune the dry roll radii of inertia of the model.

From the results in Figures 3.16 and 3.17, it can be observed that the matching between experiments and simulations is best around the peak of the response curves, and it worsens outside the resonance zone. One possible reason for this discrepancy may be that the mathematical model, due to the introduced simplifications, misses some relevant excitation forces and moments and/or some coupling effect among different motions.

![Figure 3.16: Roll response curves from internally excited roll tests: LC01. Comparison between experimental data and simulations (linear-cubic damping model, tuned dry roll inertia).](image.png)
CHAPTER 3. INTERNALLY EXCITED ROLL TESTS

Figure 3.17: Roll response curves from internally excited roll tests: LC02. Comparison between experimental data and simulations (linear-cubic damping model, tuned dry roll inertia).

Finally, for both loading conditions, the simulated and the experimental roll response curves for the milder forcing case (FC01) present a worse matching compared to larger forcing conditions. Part of this lack of matching can be associated with the fact that the linear-cubic roll damping model does not accurately represent roll dissipation at low rolling amplitudes, which are outside the fitting region (see Figures 3.14 and 3.15).

Notwithstanding the previously discussed differences between simulations and experiments in terms of rolling amplitudes, the mathematical model (3.6), after the tuning of dry roll radii of inertia, can reproduce the behaviour of experimental roll time histories, both in the transient region as well as at steady state, with a very good accuracy. Figures 3.18 (LC01) and 3.19 (LC02) show two sets of example comparisons of simulated and experimentally measured roll time histories for forcing frequencies around the roll natural frequency. As can be seen from the Figures, although some differences can be observed in the amplitude of motion, the mathematical model (3.6), despite its simplicity, very well captures the general behaviour of measured roll for the whole recording period, i.e., both in the very initial transient as well as, later, in the steady-state regime. It is also to be highlighted that the capability of model (3.6) to reproduce the measured roll behaviour is not limited to frequencies which are (relatively) close to the roll natural resonance. This fact can be seen in Figure 3.20, which compares experimental and simulated roll time histories in a condition with high frequency forcing for LC01. From the time histories, it can be noted that, although the simulated roll motion tends to overestimate the experimental one, simulations are capable of very well reproducing the quite complex transient condition.
3.5 Experimental results

Figure 3.18: Comparison between experimentally measured (red solid lines) and numerically simulated (blue dashed lines) roll motion of internally excited roll tests: LC01. Different forcing cases with frequencies close to roll natural frequency. Black dashed lines define the time windows used for the analysis of rolling amplitude.
Figure 3.19: Comparison between experimentally measured (red solid lines) and numerically simulated (blue dashed lines) roll motion of internally excited roll tests: LC02. Different forcing cases with frequencies close to roll natural frequency. Black dashed lines define the time windows used for the analysis of rolling amplitude.
Figure 3.20: Comparison between experimentally measured (red solid lines) and numerically simulated (blue dashed lines) roll motion of internally excited roll tests: LC01. Example case with high frequency forcing. Black dashed lines define the time window used for the analysis of rolling amplitude.
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4 Roll decay tests

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4.1 Introduction

Roll decay tests are based on inducing an initial heel to the ship model, releasing it allowing to roll freely, and then recording and analysing the ephraim roll motion, i.e., the decaying oscillations. Free roll decays in calm water constitute the most common approach to estimate roll damping. Furthermore, it is the primary recommended technique in current and underdevelopment stability-related international regulations (IMO, 2006, 2016a). However, no proper guidelines to perform and analyse roll decays exist, being under disposal only the general recommendations in MSC. 1/Circ. 1200 (IMO, 2006).

Among other techniques, roll decays have the advantage of being simple and low time-consuming. However, this technique is the one that may lead to most substantial uncertainties in roll damping estimations, especially if no proper mechanical devices are used to produce the initial heel to the ship model. Moreover, the main challenge in decay tests is producing a large initial heel, especially for larger models, due to the significant moment required.

Extensive literature exists regarding the analysis of experimental data coming from roll decays (e.g., Roberts, 1985; Bass and Haddara, 1988; Haddara and Bennett, 1989; Ueno et al., 2003; Bulian, 2004; Bulian et al., 2009; Fernandes and Oliveira, 2009; Han and Kinoshita, 2013; Wassermann et al., 2016; Oliveira et al., 2018). However, in the majority of the studies, the initial heeling moment is impressed manually by pushing or pulling from one side of the ship model.

Concerns about manually heeling the ship model are related mainly on the impossibility to impress to the ship a pure roll moment and the impossibility to further fulfil the initial conditions required in roll decay tests, i.e., zero roll angular velocity and no excitation to other degrees of freedom. Furthermore, the tests repeatability is undoubtedly small. These disadvantages have been known for long. Haddara and Bennett (1989), being aware that pushing from one side of the model could produce a heave response as well as a roll response, proposed to push down from one side while pulling up the other but they did it manually. More recent studies have designed mechanical devices to avoid the human intervention in roll decays (e.g., Bulian et al., 2009; Irvine et al., 2013; Hashimoto et al., 2018).

Another aspect that has gained recent attention is the influence that fluid memory effects may have on roll damping estimations from roll decay tests in calm water. The fluid memory effect concept is based on the idea that, if a system is not in a steady state, as roll decays, the velocity field depends on one or more previous cycle as well as the roll motion itself. Van’t Veer and Fathi (2011) and Handschel et al. (2014) suggest that roll damping may be underestimated at large rolling amplitudes because the strength of the vortices in the first swings is reduced, while Söder et al. (2012) argument that roll damping may be overestimated. Spounge et al. (1986) quote that fluid memory effects especially distorted roll decay tests, making these tests more unreliable than other experimental techniques.

In this Chapter, three techniques for performing roll decay tests in calm water are introduced using mechanical devices. A standard procedure to determine roll damping coefficients is described, based
on the logarithmic-decrement approach. Afterwards, roll damping estimations from different decay tests techniques are compared in terms of the nonlinear roll damping coefficients and the equivalent linear roll damping for certain roll angles.

### 4.2 Experimental set-up

Existing decay test methodologies may be categorized into one of the following types:

1. Only a roll moment is applied, without changing the ship model displacement. Thus, without inducing any heave response;

2. A vertical force is applied, generating a roll moment, but changing the ship model displacement, which may induce a heave response;

3. Pre-exciting the ship rolling a certain number of cycles and then releasing it. The ship model displacement is maintained.

In this study, three different experimental techniques of roll decay tests in calm water are designed to analyse the heave response influence in roll damping estimations, as well as the fluid memory effects. Each technique is embedded in one of the above categories.

The main elements placed in the ship model are represented schematically in Figure 4.1. Translations and rotations of the vessel during the tests are measured using a commercial optical motion capture system “Optitrack Flex 3” (Optitrack, 2017). In all the proposed techniques, the ship model is freely floating (softly restrained) in calm water.

**Figure 4.1:** Perspective view of the experimental set-up for roll decay tests.
4.2 Experimental set-up

4.2.1 Technique 1

The first technique consists of exerting a pure transversal moment on the ship model, which corresponds to the theoretical concept of roll decays. Haddara and Bennett (1989) considered this approach and manually pushed down the ship model from one side and pulled up from the other side. A more recent approach was used by Bulian et al. (2009, 2010), using pulleys and pulling ropes.

The set-up proposed is based on the use of electromagnets, as Irvine et al. (2013) uses to perform roll decays pulling up the model from one side. The system is schematically represented in Figure 4.2, and a picture of real experiments is shown in Figure 4.3. Two electromagnets are attached to the ship model and the moment is created thanks to equal (or almost equal) and opposed vertical forces generated by the weights (see Figure 4.2: weight 1 and weight 2). One peculiarity is that weight 2 must be submerged to avoid disturbance of the water’s surface because of the weight falling once released. For this reason, both weights do not have the same value, so for weight 2 the buoyancy force should be accounted for. Additionally, the pulling rope of weight 1 should be directed vertically, to this end, the transversal platform of weight 1 is movable.

![Figure 4.2: Schematic representation of Technique 1 for decay tests.](image1)

![Figure 4.3: Experimental arrangement of Technique 1 for decay tests.](image2)
CHAPTER 4. ROLL DECAY TESTS

4.2.2 Technique 2

This technique may be the most straightforward as it only requires one electromagnet for pulling up the model (Irvine et al., 2013). The system is schematically represented in Figure 4.4. The actual experimental arrangement is equivalent to the one illustrated in Figure 4.3, being the only difference that the weight in the water is not present.

Variants of current set-up can be done to simplify it. The electromagnet could be placed in the deck of the model if the ship model is small enough and the heeling moment required is not large. Moreover, instead of using a weight, a tensor can be used.

![Figure 4.4: Schematic representation of Technique 2 for decay tests.](image)

4.2.3 Technique 3

The third technique was conceived to consider the so-called memory effects of the fluid. Since the initial work of Spounge et al. (1986), where a gyroscope roll moment generator was used to pre-excite the ship, memory effects have been studied in more simplified, and maybe less precise, ways. One approach has been to excite the ship a number of times manually pushing down the ship model from one side (Van’t Veer and Fathi, 2011). Another approach is based on carrying out a number of roll decay tests in calm water starting at different initial roll angles and analysing all the obtained decay curves for the same rolling amplitudes range, as the different decay curves present different previous time-histories (Söder et al., 2012; Zhao et al., 2016).

In this case, it is proposed to use the shifting mass of the internally excited roll tests (IERT) to pre-excite the fluid surrounding the ship model. The details of the shifting mass system and the linear guide characteristics may be found in Section 3.2. As for IERT, the moving mass is initially placed at the centre and is allowed to move from the centre up to 90 mm on each side. Therefore, different forcing moments are obtained by changing the moving mass weight, see Equation 3.1 in Section 3.2.
4.3 Procedure for the determination of roll damping coefficients

The experimental procedure mainly consists of pre-exciting the ship model at its resonance frequency a certain amount of cycles. The minimum number of cycles should be fixed to the number of cycles required to reach the resonance amplitude (i.e., the steady rolling amplitude). It must be noted that when the ship model presents a nonlinear restoring, the dependence of the frequency on the amplitude of rolling should be accounted for. The excitation frequency ($\omega$) should correspond to the undamped amplitude dependent roll oscillation frequency ($\omega_{0,eq}(A)$, see Section 2.2). The decay test initial heel angle corresponds in this technique to the achieved steady rolling amplitude.

To carry out decay tests using this technique it is mandatory to fix the mass moving weight and oscillation frequency, where the oscillation frequency can be fixed by using Equation 2.7 or either from internally excited roll tests results.

4.3 Procedure for the determination of roll damping coefficients

Roll decays have been analysed using the procedure described in detail in Appendix 1 of Bulian et al. (2009). The procedure is based on the logarithmic decrement approach. It considers nonlinearities in restoring and damping terms and explicitly takes into account the amplitude-dependence frequency.

The main advantages of this methodology are its simplicity, and also that allows to aggregate data from different decay tests if they represent the same test case, allowing a robust estimation of roll damping coefficients. It is important to mention that aggregating data from different decay tests before the least-square fitting to determine the nonlinear damping coefficients is more reliable than analysing each decay test separately and calculating the mean of the determined nonlinear damping coefficients.

4.3.1 Description of the procedure

The procedure is based on the logarithmic roll-decrement curve by approximating the nonlinear model described in Section 2.1 (Equation 2.2) by a linear equivalent model in a limited time window:

$$\ddot{\phi} + \mu_{eq} (A) \cdot \dot{\phi} + \omega_{0,eq}^2 (A) \cdot \phi = 0 \quad \left|_{t-\Delta t/2, t+\Delta t/2} \right. \tag{4.1}$$

where:

- $\mu_{eq}$: $[1/s]$ is the equivalent linear damping coefficient. Its parametric model may be represented as (e.g., Bulian et al., 2009):

$$\mu_{eq} (A) = \mu + \frac{4}{3} \cdot \pi \cdot \beta \cdot (\tilde{\omega} (A) \cdot A) + \frac{3}{8} \cdot \delta \cdot (\tilde{\omega} (A) \cdot A)^2 \tag{4.2}$$
being $\dot{\omega} (A)$:

$$\dot{\omega} (A) = \sqrt{\omega_{0, eq}^2 (A) + \mu_{eq}^2 (A)} \quad (4.3)$$

- $\omega_{0, eq}$: $[\text{rad/s}]$ is the equivalent undamped roll natural frequency. Its parametric model is as follows:

$$\omega_{0, eq}^2 (A) = \omega_0^2 \cdot \frac{2\pi}{\pi \cdot A} \int_0^{2\pi} r (\phi = A \cos (\alpha)) \cdot \cos (\alpha) \cdot d\alpha \quad (4.4)$$

which may be rewritten as:

$$\omega_{0, eq}^2 (A) = \frac{\omega_0^2}{GM} \cdot \frac{1}{\pi \cdot A} \int_0^{2\pi} GZ (\phi = A \cos (\alpha)) \cdot \cos (\alpha) \cdot d\alpha \quad (4.5)$$

If $GZ (\phi)$ is known from hydrostatic computations, as well as the corresponding $GM$, the frequency $\omega_0$ may be obtained directly from roll decay data as follows:

$$\omega_0 = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \frac{\omega_{0, eq}^2 (A_i) \cdot GM \cdot \pi \cdot A_i}{\int_0^{2\pi} GZ (\phi = A_i \cos (\alpha)) \cdot \cos (\alpha) \cdot d\alpha}} \quad (4.6)$$

where the variable $N$ represents the total number of available half cycles which were used for the analysis.

Before analysing the decays using the after-mentioned methodology, along with Roberts (1985), the roll decay envelope may be filtered to reduce spurious effects due to the presence of possible bias.

For each half roll cycle (see Figure 4.5), the corresponding reference roll amplitude $A_i$ and linear equivalent roll damping coefficient $\mu_{eq, i}$ are defined as:

$$\begin{align*}
A_i &= \frac{|C_i| + |C_{i+1}|}{2} \\
\mu_{eq} (A_i) &= \mu_{eq, i} = \frac{1}{t_{i+1} - t_i} \cdot \ln \left( \frac{|C_i|}{|C_{i+1}|} \right)
\end{align*} \quad (4.7)$$

The amplitude-dependent roll oscillation frequency may be determined from the peak time instants as follows:

$$\tilde{\omega}_{eq} (A_i) = \tilde{\omega}_{eq, i} = \frac{\pi}{t_{i+1} - t_i} \quad (4.8)$$

Since the system is lightly damped, it is assumed that the oscillation frequency $\tilde{\omega}_{eq} (A)$ is a good
4.3 Procedure for the determination of roll damping coefficients

approximation of the undamped amplitude dependent roll oscillation frequency $\omega_{0,eq}(A)$, thus:

$$\omega_{0,eq}(A_i) = \omega_{0,eq,i} \approx \tilde{\omega}_{eq,i}$$

(4.9)

Finally, once $A_i$, $\mu_{eq,i}$ and $\omega_{0,eq,i}$ for each half cycle are determined, the data obtained for all decay tests of the same case are aggregated, and the experimental data is fitted to the analytical models of $\mu_{eq}$ (Equation 4.2) and $\omega_{0,eq}$ (Equation 4.4) determining the nonlinear damping coefficients and the undamped roll natural frequency.

Figure 4.5: Example of roll decay curve.

4.3.2 Verification of the procedure

The analysis method for decay tests has been checked to verify its consistency. A synthetic roll decay may be created by solving numerically Equation 2.2 with Equation 2.4 using the 4th order Runge-Kutta integration scheme, with known roll natural frequency and roll damping coefficients. The procedure for roll decay analysis is then applied, and coefficients obtained from the fitting are compared with known target values.

Figure 4.6 shows the original synthetically generated roll decay, together with the roll decay obtained from the fitted roll damping coefficients. Original and fitted coefficients are reported in the legend. It can be noticed that the identification procedure works as expected. Figure 4.7 shows the equivalent linear roll damping coefficient, and Figure 4.8 shows the equivalent undamped roll oscillation frequency, as obtained using the original (target) roll damping coefficients and the fitted ones. It can be seen that differences are negligible, especially for the equivalent undamped roll natural frequency.
According to the obtained results, it appears that the used approach shows the expected consistency.

**Figure 4.6:** Verification of the decay test analysis procedure. Time histories.

**Figure 4.7:** Verification of the decay test analysis procedure. Equivalent linear damping coefficient.
4.4 Experimental results

4.4.1 Test cases

Decay tests for loading condition LC02, reported in Table 2.2, were carried out. The selected decay tests cases and their characteristic parameters are reported in Table 4.1.

Table 4.1: Roll decay tests cases: LC02.

<table>
<thead>
<tr>
<th>FC</th>
<th>$A_0$ [deg]</th>
<th>$m_m$ [kg]</th>
<th>$\omega/\omega_0$ [rad]</th>
<th>Min. Cycles</th>
<th>Max. Cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC01</td>
<td>9.87</td>
<td>0.539</td>
<td>0.991</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>FC03</td>
<td>15.30</td>
<td>1.618</td>
<td>0.968</td>
<td>8</td>
<td>18</td>
</tr>
<tr>
<td>FC05</td>
<td>20.72</td>
<td>3.228</td>
<td>0.953</td>
<td>4</td>
<td>18</td>
</tr>
<tr>
<td>FC07</td>
<td>25.72</td>
<td>5.378</td>
<td>0.942</td>
<td>4</td>
<td>18</td>
</tr>
</tbody>
</table>

The characteristic parameters have been selected considering results from internally excited roll tests, reported in Section 3.5. Four of the seventh forcing cases of internally excited roll tests were used to carry out the decay tests (i.e., FC01, FC03, FC05, FC07). The peak amplitudes and corresponding resonance frequencies from internally excited roll tests (see Table 3.3) are associated with the initial heel angle of decay tests and with the moving mass frequency of decay tests required for Technique 3. Decay test cases, hereafter referred as $FC$ to use a common nomenclature with respect to internally excited roll tests.
excited roll tests, are reported in Table 4.1, as well as the moving mass weight for Technique 3.

To consider the influence of the fluid memory effects in Technique 3, for each decay test case, two sets of mass cycles have been considered; the minimum number required to reach the steady roll amplitude (min. cyc) and a maximum number of 18 cycles (max. cyc).

Three repetitions of each decay test case have been performed for each technique to increase the precision level of roll damping coefficients, as by doing so the total experimental data points for each decay test case is greater than 100.

4.4.2 Roll decays histories

In Figure 4.9, measured roll time histories for FC07 are reported as an example. In Table 4.2, some characteristics of the roll decays are reported, where data has been aggregated from the repetitions of the same decay tests case and technique. The characteristics shown are the absolute value of the initial heel angle ($|A_0|$), the amplitude of the first and last peak considered for the analysis ($|A_1|$ and $|A_{end}|$, respectively) and the percentage difference between the initial heel angle for each decay test technique and the mean initial heel angle for the decay test case ($A_{0, mean}$). It should be noted that the first peak considered for the analysis is not equivalent to the initial heel angle, as the time instant when the ship model is released for Techniques 1 and 2 could not be known with enough confidence. Therefore, it was decided to start the analysis of decays from the second peak.

In all decay test cases, the minimum peak amplitudes considered were approximately 2 deg, except for Technique 3, FC05 (max) and FC07 (max), where the minimum amplitudes were of 4 deg and 5 deg, respectively. The reason is that, as in Technique 3 and FC05 (max) and FC07 (max) the initial heel angles and the number of cycles were large, the model reflected waves that, eventually, affect the ship roll motion, as seen in Figure 4.9.

The raw measured data was filtered using an 8th order Butterworth low-pass filter with a normalised cut-off frequency of 0.125.
4.4 Experimental results

Figure 4.9: Roll decay histories: LC02 - FC07.
CHAPTER 4. ROLL DECAY TESTS

Table 4.2: Measured roll decay tests characteristics for each case: LC02.

| FC   | Technique | $|A_0|\ [deg]$ | $|A_1|\ [deg]$ | $|A_{end}|\ [deg]$ | $|A_0| - A_{0,\text{mean}}\ [%]$ |
|------|-----------|----------------|----------------|-----------------|-------------------------------|
| FC01 | 1         | 10.67          | 9.87           | 1.91            | 4.03                          |
|      | 2         | 10.04          | 9.40           | 1.89            | -2.11                         |
|      | 3 (min)   | 10.09          | 9.22           | 1.91            | -1.65                         |
|      | 3 (max)   | 10.23          | 9.30           | 1.99            | -0.28                         |
| FC03 | 1         | 15.72          | 14.10          | 2.02            | 0.19                          |
|      | 2         | 15.32          | 13.52          | 1.98            | -2.36                         |
|      | 3 (min)   | 15.73          | 13.33          | 2.11            | 0.28                          |
|      | 3 (max)   | 15.99          | 13.49          | 2.33            | 1.89                          |
| FC05 | 1         | 20.56          | 17.63          | 1.90            | -1.63                         |
|      | 2         | 20.34          | 17.15          | 1.80            | -2.68                         |
|      | 3 (min)   | 21.16          | 16.44          | 1.62            | 1.23                          |
|      | 3 (max)   | 21.54          | 16.61          | 3.80            | 3.08                          |
| FC07 | 1         | 25.88          | 20.95          | 1.91            | -0.61                         |
|      | 2         | 25.19          | 20.05          | 1.91            | -3.26                         |
|      | 3 (min)   | 26.63          | 19.20          | 1.87            | 2.27                          |
|      | 3 (max)   | 26.46          | 18.96          | 5.10            | 1.61                          |

4.4.3 Determination of roll damping coefficients

Considering the experimental data and the analysis methodology shown in Section 4.3 and Bulian et al. (2009), roll damping coefficients have been determined for each decay test case. In all cases, the linear-cubic damping model was considered, as the fitting of the other models (linear-quadratic-cubic or linear-quadratic) led to negative damping coefficients.

In Table 4.3, linear and cubic damping coefficients are reported for each decay test case, as well as the fitted undamped ship roll natural frequency. In Figures 4.10, 4.11 and 4.12, the fitted coefficients for each forcing case are represented. Linear and cubic damping coefficients are reported non-dimensionalised by $\omega_0$. The error bars reported in Figure 4.10 represent the difference between the fitted natural frequency and the mean fitted natural frequency, determined as the mean of the $\omega_0$ values reported in Table 4.3. The error bars reported in Figures 4.11 and 4.12 represent the confidence inter-
4.4 Experimental results

vals with a confidence level of 95% and have been calculated from the least square fitting of coefficients \( \mu \) and \( \delta \), assuming a Gaussian distribution, and dividing (for \( \mu \)) or multiplying (for \( \delta \)) the confidence intervals obtained by the fitted value of \( \omega_0 \), neglecting uncertainties of \( \omega_0 \). The coefficients for Technique 3 (max. cyc) and FC05 and FC07 should be treated with care because of the reflected waves (see Figure 4.9). In Appendix B, the corresponding numerical values of Figures 4.10, 4.11 and 4.12 are reported. Also, in Appendix B, Figures showing the analysis of roll decays for each test case and technique are illustrated.

Table 4.3: Roll damping coefficients and fitted roll natural frequency from roll decay tests for each case: LC02.

<table>
<thead>
<tr>
<th>FC</th>
<th>Technique</th>
<th>( \omega_0 ) [rad/s]</th>
<th>( \mu ) [1/s]</th>
<th>( \delta ) [s/rad²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC01</td>
<td>1</td>
<td>3.414</td>
<td>0.0291</td>
<td>0.5829</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3.421</td>
<td>0.0263</td>
<td>0.6625</td>
</tr>
<tr>
<td></td>
<td>3 (min)</td>
<td>3.417</td>
<td>0.0261</td>
<td>0.5878</td>
</tr>
<tr>
<td></td>
<td>3 (max)</td>
<td>3.421</td>
<td>0.0294</td>
<td>0.5382</td>
</tr>
<tr>
<td>FC03</td>
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<td>0.0299</td>
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<td>0.0280</td>
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</tr>
<tr>
<td></td>
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<td>0.0270</td>
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<td>3 (max)</td>
<td>3.427</td>
<td>0.0229</td>
<td>0.6537</td>
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</table>
CHAPTER 4. ROLL DECAY TESTS

Figure 4.10: Damping determination from decay tests: LC02. Fitted roll natural frequency ($\omega_0$).

Figure 4.11: Damping determination from decay tests: LC02. Non-dimensional linear damping coefficient ($\mu/\omega_0$).
4.4 Experimental results

From the results in Table 4.3 and Figure 4.10, it can be noted that the fitted roll natural frequency is always slightly larger than the imposed value. Nevertheless, the differences are minimal, with an average reduction of 0.352%.

From the results in Figures 4.11 and 4.12, it may be concluded that differences of linear and cubic damping coefficients between the techniques are of the order of magnitude of the uncertainty of data for FC01, FC03 and FC05. As for FC07, the same happens for the linear damping coefficients if the results of Technique 3 (max. cyc) are omitted. However, concerning the cubic coefficients, it seems that the uncertainty of data is not the sole cause to explain the discrepancies between the techniques.

To consider a global quantity of the damping, the equivalent linear damping coefficient \( \mu_{eq}(A) \), see Equation 2.4, has been calculated for two rolling amplitudes, namely 10 deg and 20 deg. The coefficients are represented in Figures 4.13 and 4.14. In these Figures, the confidence intervals represent the ±1.96 Root Mean Square Error (RMSE), which is an approximation for 95% of the confidence interval of any new experimental value of \( \mu_{eq}(A) \). The RMSE is defined as:

\[
RMSE \approx \sqrt{\frac{\sum_{k=1}^{N_{exp}} (y_{exp,k} - y_{fit,k})^2}{N_{exp} - n}}
\] (4.10)

where \( N_{exp} \) is the total number of experimental data points, \( n \) is the number of free parameters (i.e., equal to 2, for \( \mu_{eq} \) when considering the linear-cubic damping model), \( y_{exp,k} \) is the experimental data point and \( y_{fit,k} \) is the fitted data point calculated using Equations 4.2 and 4.5.
CHAPTER 4. ROLL DECAY TESTS

Figure 4.13: Damping determination from decay tests: LC02. Dimensionless equivalent linear damping coefficient. Rolling amplitude: 10 deg.

Figure 4.14: Damping determination from decay tests: LC02. Dimensionless equivalent linear damping coefficient. Rolling amplitude: 20 deg.

From Figures 4.13 and 4.14, the significant influence that the rolling amplitude has on the equivalent linear damping coefficient can be seen. It can also be found that the difference of $\mu_{eq}(A)$ among the three techniques are of the order of magnitude of the uncertainty levels, except for FC07. In FC07, the equivalent linear damping coefficient is slightly different for all techniques, differing between them by approximately 15% (without considering Technique 3 (max. cyc)). Therefore, if damping coefficients for rolling amplitudes below 20 deg have to be determined, it may not be necessary to use complicated techniques to carry out decay tests; using Technique 2 would be sufficient. However, if rolling amplitudes larger than 20 deg need to be studied, the above affirmation may not be accurate and, indeed, how the initial heel angle is impressed into the ship model may influence the roll damping estimations.
In line with the previous paragraph, it can be noted that the uncertainty levels, i.e., 95% confidence intervals, significantly reduce with the rolling amplitude. For 10 $\text{deg}$ of roll amplitude, variations of linear equivalent damping reach 6%. Meanwhile, at 20 $\text{deg}$, the variations are around 2%.

The obtained results indicate that nonlinear damping coefficients determined using the different techniques are similar for small to medium roll angles, showing discrepancies of the order of magnitude of the uncertainty associated with the fitting procedure. The equivalent linear damping coefficient for medium rolling amplitude lead to the same conclusion. At large angles of rolling, as the uncertainty levels drop considerably, the discrepancies between the decay tests techniques may not be only attributed to statistical uncertainties, and further work is required in this respect to understanding the origin of these differences. In general, it seems that it may not be necessary to carry out complex decay tests, as pushing or pulling the ship model from one side, using a proper mechanical device, could be sufficient.
Externally excited roll tests

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5.1 Introduction

Externally excited roll tests, most commonly known as roll tests in regular beam waves, are used extensively to obtain the roll response of a ship under regular waves and to estimate the ship roll damping (e.g., Contento et al., 1996; Francescutto and Contento, 1999; Bertaglia et al., 2003, 2004; IMO, 2006; Lee et al., 2018).

This experimental technique has the same benefits of internally excited roll tests. It allows the determination of roll damping at large rolling amplitudes and, when soft (or none) constraints are applied to the model, the natural coupling between roll motion and other relevant motions are maintained. Furthermore, the scenario of regular beam waves is the closest to the ship navigating in a seaway, apart from the even closer scenario of irregular waves, therefore, roll damping estimations using this technique are supposed to be more realistic than the estimations from internally excited roll tests in calm water or from free roll decay tests. However, the excitation waves may require large waves steepness, depending on the ship model scale and loading condition, and roll time histories need to be long enough to reach the steady state and remain there a sufficiently large number of oscillations without being interfered by reflective waves. These requirements are usually not fulfilled in a towing tank of medium dimensions. Therefore, the main drawback of this approach is that usually requires a seakeeping basin.

In this Chapter, the experimental configuration used for the externally excited roll tests is described. The methodology followed to analyse the experimental data is reported in detail and a simplified procedure to determine roll damping coefficients is introduced. The resulting roll response curves are reported and illustrated. However, opposite to the other experimental approaches, roll damping estimations have not been possible due to limited experimental data. Although roll damping estimations have not been possible, externally excited roll tests are presented in this independent Chapter to highlight the importance of this experimental approach if differences on ship roll damping estimations arising using different experimental methodologies want to be studied. The results reported in this Chapter are used in Section 6.3 to evaluate roll damping estimations from roll decays and internally excited roll tests.

5.2 Experimental set-up

In this technique, the ship model is placed in the centre of the seakeeping basin and freely floating (softly restrained), being the ship excited by regular beam waves.

Translations and rotations of the vessel during the tests are measured using a commercial optical motion capture system “Rodym DMM 6D (Krypton)”. Incident waves were recorded using one wave probe, located sufficiently far away from the model to neglect the effect of ship motions but at enough distance from the wavemaker to correctly measure the generated wave. In Figure 5.1, the experimental arrangement is shown.
CHAPTER 5. EXTERNALLY EXCITED ROLL TESTS

Figure 5.1: Experimental arrangement of externally excited roll tests.

The main elements placed in the ship model are the trackable markers for the optical motion measurement system, represented schematically in Figure 5.2. In this case, the trackables and their position changed with respect to other experimental approaches. These new trackables increased the ship displacement slightly (by 0.15%) and the vertical position of the ship’s centre of gravity (by 0.25%). In the following, these variations have been considered negligible.

Figure 5.2: Perspective view of the experimental set-up for externally excited roll tests.
5.3 **Procedure for the determination of roll damping coefficients**

From available experimental roll response curves from tests in regular beam waves, roll damping parameters can be estimated considering the model described in Section 2.1 (Equation 2.2). The most common procedure is the Parameter Identification Technique (PIT), which has been briefly introduced in Section 3.4. However, a different method is proposed, inspired by the approach used for internally excited roll tests.

## 5.3.1 Description of the procedure

The scope of the procedure is to fit the roll damping coefficients in such a way that the mathematical model of 2.2 is able to reproduce approximately the same rolling amplitudes and frequencies. Given a series of experimental roll response curves, each of them associated to a wave steepness, the peak amplitudes and corresponding frequencies ($A_{res}$ and $\omega_{res}$) can be obtained. Then, the fitting of the experimental data is carried out in three main iterative steps, as follows:

- **Step 1.** For each wave steepness, simulations of model 2.2 are carried out assuming a damping model characterised by an equivalent linear damping coefficient, $\mu_{eq}$, defined as:

\[
\dot{\phi} = 2 \cdot \mu_{eq} \cdot \dot{\phi}
\]  

(5.1)

Different $\mu_{eq}$ coefficients are tested to find the coefficient which provides the same peak response amplitude as that obtained from experiments for the considered wave steepness. The first estimation of $\mu_{eq}$ is obtained by considering the linearised model of 2.2, from which an approximate equivalent linear damping coefficient can be determined as follows:

\[
\begin{align*}
\ddot{\phi} + 2 \cdot \mu_{eq} \cdot \dot{\phi} + \frac{GZ(\phi)}{GM} = \omega_0^2 \cdot r(\omega) \cdot \pi \cdot s_w \cdot \sin(\omega \cdot t)
\end{align*}
\]

(5.2)

At resonance: $2 \cdot \mu_{eq} \cdot A_{res} \cdot \omega_{res} \approx \omega_0^2 \cdot r(\omega) \cdot \pi \cdot s_w \rightarrow $  

$\rightarrow \mu_{eq} \approx \frac{\omega_0^2 \cdot r(\omega) \cdot \pi \cdot s_w}{2 \cdot A_{res} \cdot \omega_{res}}$

The final expression of $\mu_{eq}$ in Equation 5.2 is obtained assuming that, at resonance, the inertial and restoring part of the equation of motion approximately cancel out, which leads to the equalisation of the amplitude of the damping term and the amplitude of the forcing term at the peak response.

The equivalent linear damping coefficient $\mu_{eq}$ in each simulation is systematically varied around the first guess value obtained from Equation 5.2. The variation of $\mu_{eq}$ is defined to comprise the peak
amplitude obtained from experiments. In Figure 5.3, an example of a graphical representation of this step is shown.

![Graphical representation](image)

**Figure 5.3:** Damping determination from externally excited roll tests: Step 1.

- **Step 2.** Given the amplitude, $A_{\text{res}}$, and frequency, $\omega_{\text{res}}$, of the experimental peaks for each wave steepness, the actual equivalent linear damping coefficient can be obtained by interpolation with respect to the rolling amplitude $A_{\text{res}}$, as shown in Figure 5.4.

It is important to highlight that, generally, the equivalent linear damping coefficient will be almost equivalent to the initial guess value, as roll motion is a lightly damped system, and even with a nonlinear restoring, the inertial term and the restoring term approximately cancel out. However, this step is recommended to overcome possible discrepancies, especially when the restoring term is significantly nonlinear.

- **Step 3.** After obtaining the equivalent linear roll damping coefficients for different wave steepness (i.e., for different combinations of peak rolling amplitudes and frequencies), the nonlinear roll damping coefficients may be determined from fitting, using model 2.4:

$$
\mu_{eq}(A_{\text{res}}, \omega_{\text{res}}) = \mu + \frac{4}{3} \cdot \pi \cdot \beta \cdot (A_{\text{res}} \cdot \omega_{\text{res}}) + \frac{3}{8} \cdot \delta \cdot (A_{\text{res}} \cdot \omega_{\text{res}})^2
$$

(5.3)

In Figure 5.5, an example of a fitting is shown, where the fitting was based on the apriori assumption of a linear-cubic roll damping model.
5.3 Procedure for the determination of roll damping coefficients

In the following, a consistency check is performed by synthetically generating roll response curves according to model 2.2 and analysing them following the described procedure. The synthetic roll response curves were generated by solving numerically Equation 2.2 with Equation 2.4 using the 4th order Runge-Kutta integration scheme, with known natural frequency and roll damping coefficients, and
CHAPTER 5. EXTERNALLY EXCITED ROLL TESTS

deriving the average rolling amplitude in the steady state based on the analysis of maxima and minima
of roll.

The results are shown in the following Figures 5.6 and 5.7. According to the obtained results, it
seems that the used approach is robust and consistent if sufficient roll response curves are available,
which is not generally the case. If not enough roll response curves are under disposal, the preferred
procedure should be based on the PIT technique.

Figure 5.6: Verification of the externally excited roll test analysis procedure. Roll response curves.

Figure 5.7: Verification of the externally excited roll test analysis procedure. Equivalent linear damping coefficient.
5.4 Experimental results

5.4.1 Test cases

Externally excited roll tests have been carried out for loading condition LC02, described in Section 2.3.2, with the characteristic parameters reported in Table 5.1. In total, four wave steepness were tested: 1/77, 1/65, 1/57 and 1/46.

Table 5.1: Required wave characteristics for externally excited roll tests: LC02.

<table>
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<tr>
<th>Register ID</th>
<th>$s_{ω,req}$</th>
<th>$ω_{req}/ω_0$</th>
<th>$ω_{req}$</th>
<th>$T_{req}$</th>
<th>$h_{req}$</th>
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<td>0.970</td>
<td>3.307</td>
<td>1.90</td>
<td>0.123</td>
</tr>
</tbody>
</table>

5.4.2 Analysis of generated waves

The analysed time window for each test was decided by avoiding the effect of the initial transient and avoiding the reflected waves from the tank ends. The actual wave period and height for each test case
have been determined by a standard Fast Fourier Transform (FFT) analysis of the useful part of the time histories.

The quality of generated waves has been assessed by:

1. Comparing the required wave steepness ($s_{\omega,req}$) with the estimated wave steepness obtained from measured data. Following Bulian et al. (2009), a wave has been considered acceptable when the measured wave steepness differs less than 10% of the required one;

2. Comparing the Total Harmonic Distortion (THD), calculated following Equation 5.4, with the admissible limits, determined using Equation 5.5 (González-Álvarez-Campana, 1988). The THD has been calculated taking the root sum squares of the first five harmonics of the fundamental, i.e.:

\[
\%THD = 100 \cdot \sqrt{\sum_{n=2}^{N} (A_{FFT,n})^2 / A_{FFT,f}}
\]  

(5.4)

\[
\%THD_{lim} = \max \left( 160 \cdot \frac{h}{\lambda} + 5; 160 \cdot \frac{h}{\lambda} + \frac{\lambda}{1.5} \right)
\]  

(5.5)

where:

- $N$: [nd] is the number of harmonics considered in the analysis;
- $A_{FFT,n}$: [m] is the amplitude of the harmonic $n$;
- $A_{FFT,f}$: [m] is the amplitude of the fundamental harmonic $f$;
- $h$: [m] is the wave height, determined as the amplitude of the fundamental harmonic;
- $\lambda$: [m] is the wave length, determined from the wave period of the fundamental harmonic.

3. Comparing a simplified beating signal index (Equation 5.6) with an acceptable value. The acceptable value has been tentatively set to 10%.

\[
\%\delta h = 100 \cdot \frac{(h_{\text{max}} - h_{\text{min}})}{h_{\text{mean}}}
\]  

(5.6)

where:

- $h_{\text{max}}$: [m] is the maximum wave height measured in the wave time history;
- $h_{\text{min}}$: [m] is the minimum wave height measured in the wave time history;
- $h_{\text{mean}}$: [m] is the mean wave height of the wave time history.
5.4 Experimental results

In Table 5.2, the generated waves characteristics are reported. Two registers have been rejected, Registers 04 and 15. Register 04 has been rejected because the signal beating index exceeded the 10% limit, and Register 15 has also been rejected because of its large signal beating index and because its THD was above the allowable limit. As a result, three wave frequencies have been considered for wave steepness 1/77, 1/65 and 1/46 and four wave frequencies for wave steepness 1/57.

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<th>T [s]</th>
<th>h [m]</th>
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<th>δh [%]</th>
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<td>0.969</td>
<td>3.303</td>
<td>1.90</td>
<td>0.116</td>
<td>-5.93</td>
<td>13.43</td>
<td>8.90</td>
<td>8.28</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In Figure 5.8, the error of the estimated wave steepness with respect to the required one is reported. Only the acceptable waves have been plotted, neglecting Registers 04 and 15. Figures 5.9, 5.10, 5.11 and 5.12 show the wave elevation time histories for Registers 01, 04, 10 and 15. As it may be seen, Registers 04 and 15 show a significant beating. Figures 5.13, 5.14, 5.15 and 5.16 show the FFT analyses for the same registers. The FFT amplitudes and frequencies are normalised, respectively, by the amplitude and frequency of the fundamental harmonic. As expected from the results reported
in Table 5.2, the normalised amplitude of the second harmonic of Register 15 is higher than in other registers, which explain the value of THD derived from the data analysis.

Figure 5.8: Error of the estimated wave slope for the acceptable waves from externally excited roll tests: LC02.

Figure 5.9: Wave elevation time history from externally excited roll tests: LC02 - Register 01. Black dashed lines define the time window used for the analysis.
5.4 Experimental results

Figure 5.10: Wave elevation time history from externally excited roll tests: LC02 - Register 04. Black dashed lines define the time window used for the analysis.

Figure 5.11: Wave elevation time history from externally excited roll tests: LC02 - Register 10. Black dashed lines define the time window used for the analysis.
CHAPTER 5. EXTERNALLY EXCITED ROLL TESTS

Figure 5.12: Wave elevation time history from externally excited roll tests: LC02 - Register 15. Black dashed lines define the time window used for the analysis.

Figure 5.13: Wave Fast Fourier Transform analysis from externally excited roll tests: LC02 - Register 01.
5.4 Experimental results

**Figure 5.14:** Wave Fast Fourier Transform analysis from externally excited roll tests: LC02 - Register 04.

**Figure 5.15:** Wave Fast Fourier Transform analysis from externally excited roll tests: LC02 - Register 10.
CHAPTER 5. EXTERNALLY EXCITED ROLL TESTS

5.4.3 Roll response curves

Measured roll response curves for the LC02 loading condition are reported in Appendix C and illustrated in Figure 5.17, as a function of the normalised excitation frequency. Also, in Figure 5.17, the variability of rolling amplitudes is represented through bars, which corresponds to the maximum \( A_{\text{roll, max}} \) and minimum \( A_{\text{roll, max}} \) rolling amplitudes measured in the analysed time window.

The analysed time windows of the roll time histories correspond to the time windows considered for the waves analyses. The raw measured time histories were filtered using an 8th order Butterworth low-pass filter with a normalised cut-off frequency of 0.125 to avoid the low-frequency trends.

The reference amplitudes \( A_{\text{roll}} \) for each test have been determined as the average rolling amplitudes within the analysed time window, based on the analysis of maxima and minima of roll, considering that the experimental roll motion was almost sinusoidal in the steady-state conditions. Figures 5.18 and 5.19 show that the sinusoidal approximation is acceptable for the measured roll motion, in the considered tests, even at large rolling amplitudes.

Externally excited roll tests, provided that sufficient roll response curves are obtained and that the definition of the response curves near the resonance zone is accurate enough, may be used to determine the roll damping. In this study, due to time constraints, not a sufficient number of tests could be performed (i.e., only four different wave steepness and four tests per each wave steepness). This lead to insufficiently defined roll response curves to determine roll damping, as clearly seen in Figure 5.17. However, the obtained experimental results are going to be used in Section 6.3 to evaluate roll damping.
estimations from the other approaches, i.e., roll decay and in internally excited roll tests.

**Figure 5.17:** Experimental roll response curves from externally excited roll tests: LC02.

**Figure 5.18:** Roll time history from externally excited roll tests: LC02 - Register 01.
Figure 5.19: Roll time history from externally excited roll tests: LC02 - Register 10.
6

Analysis of roll damping estimations

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6.1 Introduction

Different experimental techniques can be used to estimate roll damping, and each technique involves a specific hydrodynamic scenario. Therefore, it is possible that damping coefficients determined from different experimental approaches differ.

From an application (design and/or regulatory) perspective, roll damping is determined (numerically or experimentally) in order to be eventually used for roll motion predictions in waves (see, e.g., Kuroda et al. (2003); Neves et al. (2003); Paroka and Umeda (2006, 2007) for the specific case of fishing vessels and IMO (2006, 2016b) as an example of current and underdevelopment International Regulations directly affected by roll damping estimations).

In this Chapter, nonlinear roll damping coefficients determined from different experimental techniques and methodologies are analysed. Firstly, a direct comparison of the estimated roll damping coefficients is performed. Secondly, numerical roll motion predictions considering the roll damping estimations from internally excited roll tests and decay tests are compared to experimental data from externally excited roll tests, which are used as a reference. Thirdly, the sensitivity of international regulations to roll damping is quantified and analysed by applying the Weather Criterion (IMO, 2009b, 2006), i.e., which corresponds to the Level 1 Criterion of the Dead Ship Condition failure mode, and the current draft of the Level 2 Criterion for the Dead Ship Condition failure mode of the Second Generation Intact Stability Criteria (IMO, 2016b).

6.2 Direct roll damping comparison

Roll damping values determined from the analysis of roll decays and internally excited roll tests for LC02 are compared in this Section. Table 6.1 summarises the nonlinear roll damping coefficients determined for LC02. In the Table, it is reported by means of $A_{\max}$ the maximum peak resonance amplitude in the case of internally excited roll tests and the initial heel angle in the case of roll decay tests. Figures 6.1, 6.2, 6.3 and 6.4 report graphically the amplitude dependent roll damping ($\mu_{eq}$) derived from the analyses of decay tests and internally excited roll tests, assuming a linear-cubic roll damping model.

The reported results indicate that, for the considered vessel and loading condition (LC02), the nonlinear roll damping estimated from the two techniques show clear differences. It is important to underline that the ranges of rolling amplitudes on which the two techniques have based the fitting of roll damping coefficients are different. Although the more meaningful comparisons should be based on data contained in the overlapping range of rolling amplitudes, the two ranges only partially overlap for all forcing cases. For FC01 and FC03, as shown in Figures 6.1 and 6.2, data from roll decays cover the range of smaller rolling amplitudes while peaks of the roll response curves from the internally excited roll exper-
iments cover, instead, the range of large rolling amplitudes. For FC05, see Figure 6.3, data from roll
decays covers from small to medium rolling amplitudes and, again, internally excited roll tests cover the
range of large rolling amplitudes. Finally, for FC07, see Figure 6.4, decay tests cover from small to large
rolling amplitudes but internally excited roll test only the range of large rolling amplitudes. As a result, to
perform a proper comparison, mainly FC07 should be considered.

Table 6.1: Roll damping coefficients from internally excited roll tests (IERT) and roll decay tests (DT): LC02.

<table>
<thead>
<tr>
<th>Technique</th>
<th>$A_{\text{max}}$ [deg]</th>
<th>$\mu$ [1/s]</th>
<th>$\delta$ [s/rad$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>IERT</td>
<td>25.72</td>
<td>0.0566</td>
<td>0.4050</td>
</tr>
<tr>
<td>DT - FC01 - Tech. 1</td>
<td>10.67</td>
<td>0.0291</td>
<td>0.5829</td>
</tr>
<tr>
<td>DT - FC01 - Tech. 2</td>
<td>10.04</td>
<td>0.0263</td>
<td>0.6625</td>
</tr>
<tr>
<td>DT - FC01 - Tech. 3 (min)</td>
<td>10.09</td>
<td>0.0261</td>
<td>0.5878</td>
</tr>
<tr>
<td>DT - FC01 - Tech. 3 (max)</td>
<td>10.23</td>
<td>0.0294</td>
<td>0.5382</td>
</tr>
<tr>
<td>DT - FC03 - Tech. 1</td>
<td>15.72</td>
<td>0.0280</td>
<td>0.5819</td>
</tr>
<tr>
<td>DT - FC03 - Tech. 2</td>
<td>15.32</td>
<td>0.0260</td>
<td>0.5976</td>
</tr>
<tr>
<td>DT - FC03 - Tech. 3 (min)</td>
<td>15.73</td>
<td>0.0278</td>
<td>0.6017</td>
</tr>
<tr>
<td>DT - FC03 - Tech. 3 (max)</td>
<td>15.99</td>
<td>0.0285</td>
<td>0.5733</td>
</tr>
<tr>
<td>DT - FC05 - Tech. 1</td>
<td>20.56</td>
<td>0.0285</td>
<td>0.5623</td>
</tr>
<tr>
<td>DT - FC05 - Tech. 2</td>
<td>20.34</td>
<td>0.0266</td>
<td>0.5845</td>
</tr>
<tr>
<td>DT - FC05 - Tech. 3 (min)</td>
<td>21.16</td>
<td>0.0299</td>
<td>0.5713</td>
</tr>
<tr>
<td>DT - FC05 - Tech. 3 (max)</td>
<td>21.54</td>
<td>0.0291</td>
<td>0.5877</td>
</tr>
<tr>
<td>DT - FC07 - Tech. 1</td>
<td>25.88</td>
<td>0.0303</td>
<td>0.5221</td>
</tr>
<tr>
<td>DT - FC07 - Tech. 2</td>
<td>25.19</td>
<td>0.0280</td>
<td>0.5668</td>
</tr>
<tr>
<td>DT - FC07 - Tech. 3 (min)</td>
<td>26.63</td>
<td>0.0270</td>
<td>0.6049</td>
</tr>
<tr>
<td>DT - FC07 - Tech. 3 (max)</td>
<td>26.46</td>
<td>0.0229</td>
<td>0.6537</td>
</tr>
</tbody>
</table>

It is also to be noted that, due to the lack of data from internally excited roll tests in the region of
smaller amplitudes, $\mu_{eq}$ in that region as predicted by the fitted roll damping model is strongly dependent
on the assumed analytical form of nonlinear roll damping (linear-cubic in this case). In fact, very different
results are obtained in that region depending on the selected form of roll damping model (see Figure 3.14 and 3.15). Instead, damping obtained from roll decays in the region of small rolling amplitudes is
constrained by actual data and not by analytical assumptions, and it, therefore, reflects the actual
physics at smaller angles in the corresponding experimental technique.
6.2 Direct roll damping comparison

**Figure 6.1:** Comparison of the amplitude dependent linear equivalent roll damping coefficient as obtained from roll decays and internally excited roll tests: LC02 - FC01. Red background (delimited by dashed lines) corresponds to experimental range of roll decay tests and blue background (delimited by dot-dashed lines) corresponds to experimental range of internally excited roll tests.

**Figure 6.2:** Comparison of the amplitude dependent linear equivalent roll damping coefficient as obtained from roll decays and internally excited roll tests: LC02 - FC03. Red background (delimited by dashed lines) corresponds to experimental range of roll decay tests and blue background (delimited by dot-dashed lines) corresponds to experimental range of internally excited roll tests.
CHAPTER 6. ANALYSIS OF ROLL DAMPING ESTIMATIONS

Figure 6.3: Comparison of the amplitude dependent linear equivalent roll damping coefficient as obtained from roll decays and internally excited roll tests: LC02 - FC05. Red background (delimited by dashed lines) corresponds to experimental range of roll decay tests and blue background (delimited by dot-dashed lines) corresponds to experimental range of internally excited roll tests.

Figure 6.4: Comparison of the amplitude dependent linear equivalent roll damping coefficient as obtained from roll decays and internally excited roll tests: LC02 - FC07. Red background (delimited by dashed lines) corresponds to experimental range of roll decay tests and blue background (delimited by dot-dashed lines) corresponds to experimental range of internally excited roll tests.
For the considered vessel, in the overlapping range of amplitudes and in the range of large roll amplitudes, roll damping estimated from roll decays tend to be larger than that obtained from internally excited roll tests. Part of the differences may be ascribed to the entirely different procedures for the analysis of the experimental data since decay tests deal with the transient motion and excited roll tests with the steady state. However, part of the discrepancy could also be associated with the different hydrodynamic scenarios.

Notwithstanding, the different decay tests techniques allows to consider different scenarios. In fact, in the case of Technique 1 and 2 of roll decays, the water is initially at rest, and roll dissipation, at least in the initial cycles, occurs in a transient condition. Instead, Technique 3 of roll decays is based on pre-exciting the water in order to create a hydrodynamic scenario more similar to that of internally excited roll tests where damping obtained is based on steady state rolling motion, where the initial transient has already completed (to a large extent), and the system is undergoing (almost) harmonic periodic motions. It is to be noted that, opposed to the expected results, Technique 3 of roll decays for FC07 present more substantial discrepancies to internally excited roll tests, in comparison with Technique 1 and 2. In the case of FC07, it can be argued that due to large rolling amplitudes, waves were radiated by the ship model and they may affect the subsequent decay curves. However, in other forcing cases the results for Technique 3 are apparently in line with other techniques of roll decays. Thus, memory effects are not relevant for the present case study and are not the reason why decays and internally excited roll tests present different trends in roll damping estimations. It should be highlighted that this ship model does not have bilge keels, in contrast to other research studies (e.g., Van’t Veer and Fathi, 2011; Oliveira, 2011). However, apart from the study presented by Van’t Veer and Fathi (2011), memory effects seem not to affect in a significant manner roll damping estimations (e.g., Oliveira, 2011; Söder et al., 2012; Zhao et al., 2016). Therefore, the effects of previous roll oscillations seem not to be significantly relevant even when bilge keels are present or in a ship hull with sharp bilges. Present tests, at least, demonstrate that they are not significant for the reference hull. Moreover, it has been seen as well that the effect of heave responses due to pushing or pulling the ship only from one side, without creating a counter force on the other side, are not significant. As a result, the most straightforward approach of roll decay tests (Technique 2) can be applied, although being strongly recommended (almost mandatory) to use proper mechanical devices to impress the initial heel to the ship model.

Comparisons between experimental approaches are not extensive in the literature. Handschel et al. (2014) compared nonlinear roll damping obtained from excited tests with contra-rotating masses and from roll decays with and without forward speed for the post panamax containership DTC (el Moctar et al., 2012). Their results indicated that decay tests underestimated ship roll damping for rolling amplitudes up to 17 deg. In future publications (Wassermann et al., 2016), distinct results were obtained. From the reported graphs, no significant differences between damping estimations from the two experimental
approaches seem to exist. However, it should be noted that in their study roll damping estimations from
decays tests at zero forward speed are slightly above estimations from exited tests with contra-rotating
masses for the ballast loading condition. Other results on this regard were published by Bertaglia et al.
(2003), where they compared damping estimations from decay and excited tests with a gyroscopic roll
moment generator for four different large passenger ship vessels. The reported results showed differ-
ces up to 14% on the quadratic roll damping coefficients, and, in some ships, roll decays showed
larger values than excited roll tests while smaller in others.

In this respect, it may be that differences among methodologies could depend on the characteristics
of the considered hull form. However, to decide on the most appropriate technique for roll damping
determination, roll experiments in waves should be used as a reference.

6.3 Numerical simulations of ship rolling in regular beam waves

Roll experiments in waves are used as a reference to evaluate the different experimental approaches
and decide on the most appropriate technique for roll damping determination. Numerical roll motion
predictions based on roll damping estimated by different approaches are compared to experimental data
from externally excited roll tests, in order to conclude on the roll damping values providing, eventually,
the best prediction capabilities.

From the determined nonlinear roll damping coefficients, simulations have been performed using
model 2.2 and the linear-cubic damping model, Equation 2.4, fixing $\beta$ to zero. Simulations have been
done considering roll damping coefficients determined from internally excited roll tests (IERT) and FC07
of decay tests (DT), as FC07 has the broadest experimental range. Only decay tests results from
Techniques 1 and 3 with minimum number of cycles are used to simplify the analysis.

Figures 6.5, 6.6 and 6.7 show the comparison between experimental and simulated roll response
curves of externally excited roll tests. From the Figures, it may be clearly seen that simulations con-
sidering nonlinear roll damping coefficients coming from internally excited roll tests are in line with ex-
perimental values, while simulations considering nonlinear roll damping coefficients coming from decay
tests under predict rolling amplitudes.

Despite not having enough experimental data points, the outcomes from the simulations have sig-
nificant importance. It may be concluded that, for the present ship model and loading condition, decay
tests are non-conservative. Also, it may be concluded that internally excited roll tests using a shifting
mass can be used to predict roll damping in waves properly.
6.3 Numerical simulations of ship rolling in regular beam waves

Figure 6.5: Externally excited roll response curves: LC02. Comparison between experimental data and simulations. Roll damping coefficients of simulations from decay tests (Technique 1).

Figure 6.6: Externally excited roll response curves: LC02. Comparison between experimental data and simulations. Roll damping coefficients of simulations from decay tests (Technique 3 (min. cyc)).
CHAPTER 6. ANALYSIS OF ROLL DAMPING ESTIMATIONS

6.4 Sensitivity analysis of roll damping to international intact ship stability regulations

Roll damping plays a fundamental role in the prediction of ship roll motion and this aspect may become particularly critical for ship safety assessment and when applying international regulations requiring roll damping as a parameter (e.g., IMO, 2006, 2016b). It is therefore important to further assess whether differences in roll damping estimations from different experimental techniques may potentially influence to a non-negligible extent ship motions predictions and/or regulatory assessments.

To this end, in this Section, the Severe Wind and Rolling Criterion of current Intact Stability Code 2008 (IMO, 2009b) and last available draft of the Dead Ship Condition vulnerability criteria of Second Generation Intact Stability Criteria (SGISC) (IMO, 2016a) are applied to the ship hull tested, considering the different estimations of roll damping.

The Severe Wind and Rolling Criterion, also known as the Weather Criterion (WeC), was developed to evaluate the ability of a ship to withstand the combined effect of beam wind and rolling. On the other hand, Dead Ship Condition is defined as the situation when the ship has no power, when the main propulsion plant and auxiliaries are not in operation (SOLAS Regulation II-1/3-8, IMO (2000)). Therefore, under dead ship condition, the ship is subject to wind and waves. In this situation, the worst possible scenario is when the ship is subject to beam wind and waves, as it is when the ship may suffer from harmonic resonance. The criteria developed for the Dead Ship Condition failure mode evaluates
6.4 Sensitivity analysis of roll damping to international intact ship stability regulations

this scenario, thus, the same situation as the Weather Criterion. Actually, the first criterion of Dead Ship Condition failure mode, i.e., Level 1 vulnerability criteria, is the Weather Criterion as reported in IMO (2009b), with the modified wave steepness table of MSC.1/Circ.1200 (IMO, 2006).

For the sensitivity analysis, the trawler fishing vessel under LC02 is used. As in Section 6.3, only estimated roll damping values determined from internally excited roll tests (IERT) and from the maximum initial roll angle of decay tests (DT) cases, i.e., FC07, Technique 1 (Tech. 1) and Technique 3 with minimum number of cycles (Tech. 3 (min. cyc)), are taken into account. The nonlinear roll damping values used are reported in Table 6.1.

6.4.1 Intact Stability Code 2008: Severe Wind and Rolling Criterion

The Severe Wind and Rolling Criterion (Weather Criterion) is established in the Intact Ship Stability Code 2008, in Section 2.3 of Part A (IMO, 2009b). An alternative assessment of Weather Criterion is defined in MSC.1/Circ.1200 (IMO, 2006), which provides guidelines for determining some of the characteristic parameters required to evaluate the criterion (i.e., wind heeling lever and/or angle of roll). In Appendix D, the physical background of the criterion about the determination of the roll-back angle is summarised. Other aspects of the Weather Criterion have not been introduced as they are well described in other publications (e.g., IMO, 2008, 2009b, 2006) and the roll-back angle is the only parameter dependent on the ship roll damping.

The roll-back angle is going to be calculated using the two procedures described in Appendix D:

• The semi-empirical formula defined in §2.3.4 of the IS Code 2008 (IMO, 2009b):

\[
\phi_1 = 109 \cdot k \cdot X_1 \cdot X_2 \cdot \sqrt{s_\omega \cdot r(\omega_0)} [\text{deg}] \tag{6.1}
\]

where the parameters of the formula are described in detail in Section D.3 of Appendix D;

• The three-step procedure described in the "Interim Guidelines for Alternative Assessment of the Weather Criterion" (IMO, 2006), considering the amplitude dependence of the roll oscillation frequency as described in Section D.4, i.e., applying the following Equation:

\[
\begin{align*}
\phi_{1r} &= \frac{\pi \cdot r(\omega_0) \cdot s_\omega}{2 \cdot (\mu_{eq}(\phi_{1r}) - \delta \mu_{eq}(\phi_{1r}))} \omega_{0,eq}(\phi_{1r}) [\text{rad}] \\
\delta \mu_{eq}(\phi_{1r}) &= \frac{1}{\pi} \cdot \frac{2.11 \cdot S \cdot r_s^2}{\Delta \cdot G_M \cdot (2 \cdot \pi)^{1.5}} \cdot \omega_{0,eq}^2(\phi_{1r})
\end{align*}
\tag{6.2}
\]

where the equivalent undamped roll natural frequency \(\omega_{0,eq}(\phi_{1r})\) is determined using Equation 2.7. As the hull used in the present case study is not fitted with bilge keels and the length overall
is slightly larger than 2 m (see Table 2.1), the measured roll damping should be corrected to consider scale effects \( (\delta \mu_{eq} (\phi_1) \neq 0) \) with the Formula §4.6.1.2.1-1 of IMO (2006), also reported in Equation 6.2.

The effective wave slope coefficient \( (r(\omega_0)) \) have been determined using direct hydrodynamic approach, whose values are reported in Figure 2.6 and in Table 6.2. Although using the direct hydrodynamic approach, the effective wave slope calculated according to the Weather Criterion formulation, Equation D.12, have been reported as well in Table 6.2. It is important to highlight the large differences between both approaches. As pointed out by Francescutto and Contento (1998), the differences are in the right side as a larger effective wave slope leads to an overestimation of the rolling amplitude when applying the Weather Criterion. These differences exist because IMO formula is based on experimentally measured effective wave slopes, which were obtained by measuring the heeling moment at waves with the fixed hull, while the direct hydrodynamic approach used in this case study leave the ship freely floating.

**Table 6.2:** Main ship parameters for the estimation of the angle of roll to windward due to wave action for the Weather Criterion. Model scale 1:20.667. LC02

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LC02</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B/d ) [( nd )]</td>
<td>2.82</td>
</tr>
<tr>
<td>((\bar{K}G/d - 1) ) [( nd )]</td>
<td>0.25</td>
</tr>
<tr>
<td>( T_0 ) [s]</td>
<td>8.38</td>
</tr>
<tr>
<td>( k ) (without bar keel) [( nd )]</td>
<td>1.00</td>
</tr>
<tr>
<td>( k ) (with bar keel) [( nd )]</td>
<td>0.70</td>
</tr>
<tr>
<td>( X_1 ) [( nd )]</td>
<td>0.925</td>
</tr>
<tr>
<td>( X_2 ) [( nd )]</td>
<td>0.847</td>
</tr>
<tr>
<td>( r(\omega_0) ) (from Equation D.12) [( nd )]</td>
<td>0.8783</td>
</tr>
<tr>
<td>( r(\omega_0) ) (from direct calculations) [( nd )]</td>
<td>0.7003</td>
</tr>
<tr>
<td>( \omega_w ) [( rad )]</td>
<td>0.0903</td>
</tr>
</tbody>
</table>

In Table 6.2, the parameters required for Equation 6.1 are reported. As it may be seen, the requirements to use the semi-empirical formulation (reported in Section D.3) are fulfilled. Two values of the coefficient \( k \) are reported, one is calculated considering the area of the lateral projection of the bar keel, which is reported in Table 2.1 and illustrated in Figure 2.2, while the other one is calculated without considering the bar keel area. These two values of \( k \) have been considered because the definition of the lateral projection of the bar keel area that should be considered as \( A_k \) is not clearly defined in the IS
6.4 Sensitivity analysis of roll damping to international intact ship stability regulations

Code 2008 and it may affect the final value of $\phi_1$ considerably, leading to non-conservative results.

Numerical values of the angle of roll to windward and the regular waves roll-back angle for the wave steepness reported in Table 6.2 are shown in Table 6.3. The evaluation of the criterion using the three-step procedure for the roll-back angle has been performed for each method. In Figure 6.8, the regular waves roll-back angle as a function of the wave steepness is illustrated.

**Table 6.3:** Evaluation of the Weather Criterion: LC02.

<table>
<thead>
<tr>
<th>Method</th>
<th>$\phi_{1r}$ [deg]</th>
<th>$\phi_1$ [deg]</th>
<th>$b/a$ [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three-step procedure - IERT</td>
<td>33.85</td>
<td>23.70</td>
<td>1.66</td>
</tr>
<tr>
<td>Three-step procedure - DT (FC07) Tech. 1</td>
<td>31.54</td>
<td>22.08</td>
<td>1.87</td>
</tr>
<tr>
<td>Three-step procedure - DT (FC07) Tech. 3 (min. cyc)</td>
<td>30.03</td>
<td>21.02</td>
<td>2.03</td>
</tr>
<tr>
<td>§2.3.4 of the IS Code 2008 ($A_k = 0$)</td>
<td>37.70</td>
<td>26.39</td>
<td>1.38</td>
</tr>
<tr>
<td>§2.3.4 of the IS Code 2008 ($A_k \neq 0$)</td>
<td>26.39</td>
<td>18.47</td>
<td>2.52</td>
</tr>
</tbody>
</table>

**Figure 6.8:** Regular waves roll-back angle ($\phi_{1r}$) determined from different methods as a function of the wave steepness. LC02.

In Table 6.3, it may be seen that the Weather Criterion is fulfilled using any of the methodologies and roll damping estimations. However, the margins differ, especially when using the formula from IS Code 2008 (IMO, 2009b). It is to be noted that when using the semi-empirical formula from IS Code 2008 (IMO, 2009b), if the bar keel is not considered, the estimated roll-back angle is considerably large and considering the bar keel the roll-back angle is significantly diminished. Thus, it seems that not the overall area of the bar keel should be used when applying the Weather Criterion but a percentage of it.
Regarding the roll-back angle determined using different roll damping estimations, although differences exist, they may be considered negligible. Thus, in the Weather Criterion, the differences between experimental approaches to determine ship roll damping seem not significant to the outcome of the criterion, which is of pass-fail type. It should be highlighted that to perform this analysis, roll damping estimations have been extrapolated to larger roll angles, which may lead to inaccurate quantitative results but, qualitatively, the results from this analysis may be useful to understand the ship roll damping influence on the roll-back angle.

6.4.2 Second Generation Intact Stability Criteria: Dead Ship Condition failure mode

The under development Second Generation Intact Stability Criteria (SGISC) intend to improve the current Intact Stability Code 2008. The IS Code 2008 (IMO, 2009b) is based on empirical and statistical approach, except the Weather Criterion which is based on a mathematical model. However, as explained in Appendix D, some of the parameters of the Weather Criterion were tuned using a reduced sample of ships. Also, current criterion presents a deterministic approach, i.e., the Weather Criterion is of pass-fail type. The main difference between both criteria is that the SGISC evaluates the stability of the ship considering some dynamic phenomena that may lead to capsizing (Umeda, 2013). Also, the new criteria are based on the physics of the dynamic phenomena and intend to evaluate the ship stability using probability measures or likelihood of occurrence (Peters et al., 2011). Therefore, as the criteria are based on physics, their applicability is not limited to a population of ships.

The new criteria consider five dynamic phenomena that may lead to capsizing or critical roll angles or excessive accelerations, the so-called failure modes. The failure modes are pure loss of stability, surf-riding and broaching-to, parametric roll, dead ship condition and excessive acceleration (IMO, 2017, 2016a). The criteria are based on a multi-tiered approach to simplify its application. This multi-tiered approach consists of checking, firstly, the vulnerability of the ship to suffer the failure mode under analysis, applying the first two levels (Level 1 and Level 2). Then, if the ship is found vulnerable, the third level should be applied.

Damping is fundamental in three of the failure modes, namely parametric roll, dead ship condition and excessive acceleration. In this work, the sensitivity of dead ship condition failure mode to roll damping is analysed. The other two failure modes have not been studied because they consider the ship sailing at certain forward speed. Therefore, roll damping estimations of the present case study are not valid.

In the following, the Level 2 vulnerability Criteria is applied to analyse the influence of roll damping in the final output of the criterion. Level 1 vulnerability criteria of Dead Ship Condition has been already considered in Section 6.4.1, as Level 1 corresponds to the current Severe Wind and Rolling Criterion. The physical background of the Level 2 criterion of DSC is summarised in Appendix E.
6.4 Sensitivity analysis of roll damping to international intact ship stability regulations

No damping scale effects have been considered as it is not specified in the current draft of L2 DSC (IMO, 2016a). The effective wave slope coefficient has been determined using direct hydrodynamic approach, whose values are reported in Figure 2.6. Outside the frequency range shown in Figure 2.6, the effective wave slope coefficient has been set to zero. The wave scatter diagram used is the wave scatter from the North Atlantic sea from IACS (2001).

The mean wind speed \(U_w\), the spectrum of wind gustiness \(S_v\) and the spectrum of waves \(S_{zz}\), do not depend on the damping moment. Therefore, they are equal and independent on the nonlinear damping coefficients used. Figures 6.9, 6.10 and 6.11 illustrate the wind \(S_v\) and wave elevation spectra \(S_{zz}\), the wave slope spectra \(S_{\alpha\alpha}\) and \(S_{\alpha\alpha,c}\), Equations E.4 and E.28 and the resulting excitation spectra \(S_{M_{\text{wind,tot}}}\) and \(S_{M_{\text{waves}}},\) Equations E.27 and E.29, respectively, for the case of the significant wave height of 5.5 m and zero-crossing period of 8.5 s. As it may be seen, by not considering frequencies higher than 2 rad/s, no substantial errors are induced.

The steady heeling lever moment due to the action of steady wind and characteristic parameters and angles are reported in Table 6.4. For the maximum significant wave height, the mean wind heeling moment is larger than the maximum value of the righting lever curve. Therefore, the ship would capsize under this situation, and no static heel angle \(\phi_S\) and derived parameters can be found.

![Figure 6.9: Wave elevation spectrum and wind gust spectrum for the calculation of the dead ship failure index: LC02 and \(H_s = 5.5\text{m}\) and \(T_z = 8.5\text{s}\)](image)

Figure 6.9: Wave elevation spectrum and wind gust spectrum for the calculation of the dead ship failure index: LC02 and \(H_s = 5.5\text{m}\) and \(T_z = 8.5\text{s}\)
CHAPTER 6. ANALYSIS OF ROLL DAMPING ESTIMATIONS

Figure 6.10: Wave slope and effective wave slope spectra for the calculation of the dead ship failure index: LC02 and $H_s = 5.5m$ and $T_z = 8.5s$

Figure 6.11: Wave, wind and total moment excitation spectra for the calculation of the dead ship failure index: LC02 and $H_s = 5.5m$ and $T_z = 8.5s$
6.4 Sensitivity analysis of roll damping to international intact ship stability regulations

Table 6.4: Steady heeling lever moment, and related values, and relevant angles for the calculation of the dead ship stability failure index: LC02

<table>
<thead>
<tr>
<th>$H_s$</th>
<th>$U_w$</th>
<th>$l_{\text{wind,tot}}$</th>
<th>$\phi_S$</th>
<th>$\omega_{0,e}$</th>
<th>$\phi_{VW,+}$</th>
<th>$\phi_{EA,+}$</th>
<th>$\phi_{VW,-}$</th>
<th>$\phi_{EA,-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>[m]</td>
<td>[m/s]</td>
<td>[m]</td>
<td>[deg]</td>
<td>[m]</td>
<td>[rad/s]</td>
<td>[deg]</td>
<td>[deg]</td>
<td>[deg]</td>
</tr>
<tr>
<td>0.5</td>
<td>3.81</td>
<td>0.0026</td>
<td>0.14</td>
<td>1.062</td>
<td>0.750</td>
<td>142.59</td>
<td>50.00</td>
<td>45.26</td>
</tr>
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<td>0.0114</td>
<td>0.61</td>
<td>1.061</td>
<td>0.750</td>
<td>142.15</td>
<td>50.00</td>
<td>45.23</td>
</tr>
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<td>0.747</td>
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<td>7.5</td>
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<td>0.0971</td>
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<td>50.00</td>
<td>46.04</td>
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<td>8.5</td>
<td>25.21</td>
<td>0.1147</td>
<td>6.37</td>
<td>0.974</td>
<td>0.718</td>
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<tr>
<td>9.5</td>
<td>27.15</td>
<td>0.1330</td>
<td>7.46</td>
<td>0.953</td>
<td>0.710</td>
<td>136.45</td>
<td>50.00</td>
<td>46.75</td>
</tr>
<tr>
<td>10.5</td>
<td>29.02</td>
<td>0.1520</td>
<td>8.62</td>
<td>0.934</td>
<td>0.703</td>
<td>135.62</td>
<td>50.00</td>
<td>47.04</td>
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<td>11.5</td>
<td>30.83</td>
<td>0.1716</td>
<td>9.83</td>
<td>0.917</td>
<td>0.697</td>
<td>134.76</td>
<td>50.00</td>
<td>47.31</td>
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<tr>
<td>12.5</td>
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<td>0.1918</td>
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<td>0.691</td>
<td>133.88</td>
<td>50.00</td>
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<tr>
<td>13.5</td>
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<td>0.687</td>
<td>132.98</td>
<td>50.00</td>
<td>47.66</td>
</tr>
<tr>
<td>14.5</td>
<td>35.99</td>
<td>0.2337</td>
<td>13.80</td>
<td>0.880</td>
<td>0.683</td>
<td>132.05</td>
<td>50.00</td>
<td>47.78</td>
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<td>0.2555</td>
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<td>0.679</td>
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<td>50.00</td>
<td>47.85</td>
</tr>
<tr>
<td>16.5</td>
<td>39.22</td>
<td>0.2777</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

In Figures 6.12 and 6.13, a graphical example of the relevant angles are shown for the significant wave height of 5.5 m, and in Figure 6.14 for the significant wave height of 11.5 m. It is important to quote that, as illustrated in Figure 6.14, the negative angle of vanishing stability could not be found. Therefore, under these circumstances, the angle of vanishing stability is considered to be -50 deg. This situation happens for significant wave heights equal to or larger than 9.5 m.
CHAPTER 6. ANALYSIS OF ROLL DAMPING ESTIMATIONS

Figure 6.12: Graphical example of the relevant angles calculation for the calculation of the dead ship failure index considering the $GZ$ curve: LC02 and $H_s = 5.5 m$.

Figure 6.13: Graphical example of the relevant angles calculation for the calculation of the dead ship failure index considering the $GZ_{res}$ curve: LC02 and $H_s = 5.5 m$. 
6.4 Sensitivity analysis of roll damping to international intact ship stability regulations

Figure 6.14: Graphical example of the relevant angles calculation for the calculation of the dead ship failure index considering the $GZ$ curve: LC02 and $H_s = 11.5$ m.

Figures 6.15 and 6.16 show the absolute ($S$, Equation E.37) and relative roll spectra ($S_x$, Equation E.31) for the case of the significant wave height of 5.5 m and 8.5 s. Due to nonlinearities in the restoring term, the spectra are slightly bent towards low frequencies region. Also, small differences may be appreciated between absolute and relative resulting roll spectra, as it is more clearly illustrated in Figures 6.17 and 6.18. However, the most noticeable point is the huge differences between spectra by considering nonlinear damping coefficients from internally excited roll tests (IERT) and decay tests (DT). Differences between decay tests techniques are limited and do not influence in a great manner the final evaluation result.

To understand the large discrepancies between roll spectra by considering nonlinear damping coefficients from internally excited roll tests and decay tests, the equivalent linear roll damping coefficient and the standard deviation of absolute roll velocity are reported for each significant wave height and zero-cross period in Figures 6.19 and 6.20, respectively. For the sake of simplicity, values for Technique 3 (minimum cycles) of decay tests have not been plotted, mostly because they present almost the same trend and values than Technique 1.
CHAPTER 6. ANALYSIS OF ROLL DAMPING ESTIMATIONS

**Figure 6.15:** Absolute roll spectra considering different roll damping coefficients estimations for the calculation of the dead ship failure index: LC02 and $H_s = 5.5\, m$ and $T_z = 8.5\, s$.

**Figure 6.16:** Relative roll spectra considering different roll damping coefficients estimations for the calculation of the dead ship failure index: LC02 and $H_s = 5.5\, m$ and $T_z = 8.5\, s$. 
6.4 Sensitivity analysis of roll damping to international intact ship stability regulations

Figure 6.17: Absolute and relative roll spectra considering roll damping estimations from internally excited roll tests for the calculation of the dead ship failure index: LC02 and $H_s = 5.5\text{ m}$ and $T_z = 8.5\text{s}$.

Figure 6.18: Absolute and relative roll spectra considering roll damping estimations from decay tests (Technique 1 and FC07) for the calculation of the dead ship failure index: LC02 and $H_s = 5.5\text{m}$ and $T_z = 8.5\text{s}$.
CHAPTER 6. ANALYSIS OF ROLL DAMPING ESTIMATIONS

Figure 6.19: Equivalent linear roll damping coefficient ($\mu_{eq}(\sigma_x)$) for the calculation of the dead ship failure index: LC02.

Figure 6.20: Standard deviation of absolute roll velocity ($\sigma_x$) for the calculation of the dead ship failure index: LC02.
6.4 Sensitivity analysis of roll damping to international intact ship stability regulations

As it may be seen in Figure 6.19, large differences exist on the equivalent linear roll damping coefficient, depending on the experimental technique used. Instead, standard deviations of absolute roll velocity, represented in Figure 6.20, present at least the same order of magnitude for all experimental techniques, although differences exist as well. In Figure 6.4, it may be seen that, at low amplitudes (i.e., below $9 \text{ deg}$ aprox), the IERT equivalent linear roll damping coefficient is greater than DT. Noting that the equivalent linear roll damping coefficient reported in Figure 6.19 is function of the standard deviation of absolute roll velocity, if considering roughly the horizontal axis of Figure 6.4 to be the standard deviation of absolute roll velocity (for the present case study ranges approximately from 0.0 to 0.27) it may be seen that, in fact, the equivalent linear roll damping coefficient from IERT might be larger for almost all the standard deviation roll velocity values. Thus, the discrepancies on the equivalent linear roll damping coefficient as a function of the standard deviation of absolute roll velocity are mainly due to the large differences between linear damping coefficients from both techniques. Furthermore, the huge differences between roll spectra are mainly due to the differences in the nonlinear roll damping coefficients, which demonstrates that the L2 DSC criterion is significantly sensitive to the damping coefficients.

The long-term probability index obtained are reported in Table 6.5. As expected from the roll motion spectra, the $C$ index determined from IERT damping coefficients is more than three times smaller than the $C$ index from decay tests damping coefficients. Also, no substantial difference exists on the $C$ value when using Technique 1 or Technique 3 (min. cyc) of roll decay tests. Another important point is that, if the standard value $R_{D30}$, see Equation E.1, is set to 0.04, the ship will fail to pass the criterion when using damping coefficients from decay tests. As the ship fulfils with the Level 1 criterion, i.e. the Weather Criterion, following current results the standard should be fixed to 0.06 to avoid inconsistencies.

**Table 6.5:** Evaluation of the Level 2 Dead Ship Condition failure mode: LC02.

<table>
<thead>
<tr>
<th>Method</th>
<th>$C$ [nd]</th>
</tr>
</thead>
<tbody>
<tr>
<td>IERT</td>
<td>0.013</td>
</tr>
<tr>
<td>DT (FC07) Tech. 1</td>
<td>0.043</td>
</tr>
<tr>
<td>DT (FC07) Tech. 3 (min. cyc)</td>
<td>0.045</td>
</tr>
</tbody>
</table>

Figures 6.21, 6.22 and 6.23 report some of the parameters required to determine the short-term probability failure index, as well as the short-term index itself, as a function of the significant wave height and zero-crossing period.
CHAPTER 6. ANALYSIS OF ROLL DAMPING ESTIMATIONS

Figure 6.21: Standard deviation of relative roll motion ($\sigma_{C3}$) for the calculation of the dead ship failure index: LC02.

Figure 6.22: Average time between two capsize events ($\lambda_{EA}$) for the calculation of the dead ship failure index: LC02.
To conclude, following current results, the Level 2 criterion for the Dead Ship Condition failure mode is significantly sensitive to the estimation of nonlinear roll damping coefficients. Moreover, if the range of standard deviation of the absolute roll velocity is small, as in the current case, the emphasis should be placed to determine precisely the linear damping coefficient.
CHAPTER 6. ANALYSIS OF ROLL DAMPING ESTIMATIONS
Chapter 7

Conclusions

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7.2 Future work ................................................................. 123
This Chapter summarises the main aspects of the Thesis and the most relevant conclusions, and establishes areas for future work.

7.1 Summary and concluding remarks

This thesis has evaluated existing experimental approaches to estimate roll damping. The experimental approaches studied have been internally excited roll tests and decay roll tests, using different techniques to impress the initial heel angle. Roll experiments in regular beam waves have been used as a reference to decide on the most appropriate technique for roll damping determination. Furthermore, the influence of roll damping in stability-related international regulations depending on the experimental approach used has been addressed, by evaluating the Weather Criterion of Intact Stability Code 2008 and the last draft of the Level 2 vulnerability criteria of the Dead Ship Condition failure mode of the Second Generation Intact Stability Criteria.

Regarding internally excited roll tests, a technique has been proposed based on exciting the model by an internal shifting mass. The nonlinear mathematical model for representing the system dynamics has been developed and the procedure to derive roll damping coefficients, using as a basis the developed mathematical model has been introduced. The mathematical model has been validated using experimental data, by comparing simulation results with experiments, both in terms of roll response curves and of roll time histories. The matching between roll response curves from experiments and simulations have been better around the peak of the response curves than outside the resonance zone. One possible reason for the discrepancies may be the introduced simplifications in the mathematical model, which misses some important excitation forces and moments and/or some coupling effect among different motions. Despite these discrepancies, the behaviour of experimental roll time histories, both in the transient region as well as steady state, has shown a good accuracy, not being limited to frequencies close to the roll natural frequency. Overall, it may be concluded that the proposed technique, with the developed mathematical model and procedure to derive roll damping coefficients, is suitable to estimate ship roll damping.

Regarding roll decay tests, three techniques have been proposed and tested. The first technique is based on exerting a pure roll moment, by pulling and pushing the ship model, impressing equal vertical forces but opposed in sign in both sides of the vessel. The second technique merely impresses the initial heel angle by pulling the ship from one side. The third technique was conceived to address the so-called fluid memory effects. It consists of pre-exciting the surrounding fluid a certain number of cycles before releasing the model using the internal shifting mass from internally excited roll tests. In this technique, two sets of mass cycles have been considered to further study the memory effects, the minimum number of cycles required to reach the steady roll amplitude and a maximum number.
of eighteen cycles. The procedure followed to analyse all decay tests has been introduced, which is based on the logarithmic-decrement approach. Differences among the decay tests techniques exist, but these differences, in terms of the estimated nonlinear roll damping coefficients, are of the order of magnitude of the uncertainty of data. Regarding the equivalent linear roll damping at large amplitudes, discrepancies are higher than their associated uncertainty levels, but from a practical perspective, they may be considered negligible. As a result, by using proper mechanical devices to perform roll decay tests, the influence of memory effects and/or heave response can be considered as negligible. However, it should be noted that the ship hull used in the present work was not fitted with bilge keels and did not have sharp bilges.

Comparing roll damping estimations from internally excited roll tests and decay roll tests, for the considered vessel, in the overlapping range of amplitudes, roll damping estimated from roll decays tend to be larger than damping estimations from internally excited roll tests. Part of the differences may be ascribed to the entirely different procedures for the analysis of the experimental data, as decay tests deal with the transient ship roll response and excited roll tests with the steady rolling amplitudes. However, part of the discrepancy could also be associated with the different hydrodynamic scenarios. Concerning the differences associated with the different hydrodynamic scenarios, the pre-excited roll decays tests present a hydrodynamic scenario similar to the scenario of internally excited roll tests, as damping is obtained based (almost) on a steady state rolling motion. Instead of showing a damping trend similar to internally excited roll tests, this technique presents the most substantial discrepancies, although approximately all roll decays techniques present the same trend and (practically) the same damping estimations. On the view of these results, it may be concluded that the discrepancies between decay and internally excited roll tests are not due to the different hydrodynamic scenarios, and are mostly due to the procedures used to analyse the experimental data.

Some limitations have been found in the experimental data and the estimations of roll damping from internally excited roll tests. Experimental data for this technique cover only the range from medium to large rolling amplitudes. As a result, the equivalent linear damping coefficient predicted in the small rolling amplitudes is strongly dependent on the assumed analytical form of nonlinear roll damping and, therefore, it may not reflect the actual physics of roll damping in the excited roll tests technique. Instead, damping obtained from roll decay tests for the maximum initial heel angle is constrained by actual data in the whole range of rolling amplitudes and, therefore, it may be assumed that the actual physics of roll decays is well represented by the fitted roll damping model and its corresponding coefficients.

Numerical roll motion predictions based on roll damping estimated by different approaches are compared to experimental data to decide which technique is most suitable for roll damping determination in the situation of ship rolling in beam waves. Numerical simulations using roll damping estimations from internally excited roll tests show good accuracy with experimental results. Simulations from roll
damping estimations of roll decays under-predict the ship rolling amplitudes. By these results, it may be conjectured that roll dissipation from internally excited roll tests may be more appropriate for being used for predictions of roll motion in waves. However, at this moment, this conjecture cannot be generalised. Further tests should be carried out to verify whether the observed differences in roll damping estimations from the considered experimental approaches are also confirmed in other cases.

The sensitivity analysis of stability-related international regulations has shown that roll damping estimations may significantly influence the results of the Level 2 vulnerability criteria of Dead Ship Condition failure mode. Instead, the Weather Criterion, probably due to their deterministic approach and the use of the equivalent linear roll damping coefficient, is not affected by the roll damping differences obtained among the experimental techniques. As a result, due to the tendency towards developing probabilistic and physical-based regulations, there is indeed the need to further address the differences between experimental approaches and develop guidelines to carry out and analyse experimental tests to determine ship roll damping.

7.2 Future work

It could be said that this thesis has established a methodology to evaluate existing experimental approaches. With this in mind, future work in this regard may be just to use the experimental approaches considered in the Thesis to other ship hulls fitted with and without bilge keels. Some improvements can be introduced. Firstly, for internally excited roll tests, roll response curves at small rolling amplitudes should be obtained. Secondly, tests in regular beam waves should be performed to determine the whole roll response curve for different wave steepness, allowing the determination of roll damping. Thirdly, a complete uncertainty analysis of the experimental approaches should be performed. In this work, three repetitions have been done to assess the uncertainty and/or repeatability of the tests, however, a more systematic and comprehensive uncertainty analysis may be carried out. Finally, different loading conditions should be studied using all the experimental approaches. In this work it has not been possible because the experimental techniques were tested sequentially, i.e., first internally excited for the two loading conditions were done, and then roll decay tests and tests in regular waves were carried out. As it was thought that changing to the first loading condition to perform the remaining experimental techniques could let to substantial errors if internally excited roll tests were not repeated, due to time constraints, it was not possible to test the first loading condition for all the experimental techniques.

One aspect that has not been accounted for in the experimental tests is the effect of forward speed on roll damping. It should be addressed in future areas of work. It is thought that the implementation of the experimental methodologies to tests with forward speed is relatively easy except the re-evaluation of the mathematical model developed for internally excited roll tests.
CHAPTER 7. CONCLUSIONS

Other aspects to be studied, as a continuation of the present research topic, is the suitability of existing semi-empirical methods to estimate ship roll damping. Moreover, the use of CFD simulations should be addressed as well and, along with it, the scale effects of roll damping.
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without liquid cargo in spherical tanks. *Journal of Ocean Engineering and Science*, 1:84–91, DOI:
Experimental data from internally excited roll tests
Table A.1: Experimental results from internally excited roll tests: LC01 - FC01

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<th>(A_{roll,min})</th>
<th>(\omega/\omega_0)</th>
<th>(A_{roll})</th>
<th>(A_{roll,max})</th>
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## APPENDIX A. EXPERIMENTAL DATA FROM INTERNALLY EXCITED ROLL TESTS

### Table A.4: Experimental results from internally excited roll tests: LC01 - FC04

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</tr>
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Experimental data from roll decay tests
Table B.1: Fitted roll natural frequency from roll decay tests: LC02

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<tr>
<th>FC</th>
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<th>$\omega_0$</th>
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<th>$\omega_{0,\text{max}}$</th>
<th>RMSE $\omega_0$</th>
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<td></td>
<td>[rad/s]</td>
<td>[rad/s]</td>
<td>[rad/s]</td>
<td>[rad/s]</td>
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<tr>
<td></td>
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<td>3.403</td>
<td>3.438</td>
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</tr>
<tr>
<td></td>
<td>3 (min)</td>
<td>3.417</td>
<td>3.405</td>
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</tr>
<tr>
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<td>3.403</td>
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<tr>
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<td>3.406</td>
<td>3.436</td>
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<tr>
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<td>3 (min)</td>
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### APPENDIX B. EXPERIMENTAL DATA FROM ROLL DECAY TESTS

**Table B.2:** Fitted roll damping coefficients from roll decay tests: LC02

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<tr>
<th>FC</th>
<th>Technique</th>
<th>$\mu$</th>
<th>$\mu_{\text{min}}$</th>
<th>$\mu_{\text{max}}$</th>
<th>$\delta$</th>
<th>$\delta_{\text{min}}$</th>
<th>$\delta_{\text{max}}$</th>
<th>RSME $\mu_{eq}$</th>
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<tr>
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<td>[1/s]</td>
<td>[1/s]</td>
<td>[1/s]</td>
<td>[s/\text{rad}^2]</td>
<td>[s/\text{rad}^2]</td>
<td>[s/\text{rad}^2]</td>
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<tr>
<td></td>
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<tr>
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<td>0.6537</td>
<td>0.6301</td>
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Figure B.1: Damping determination from decay tests: LC02. Decay tests analysis of FC01 - Tech. 1.
APPENDIX B. EXPERIMENTAL DATA FROM ROLL DECAY TESTS

Figure B.2: Damping determination from decay tests: LC02. Decay tests analysis of FC01 - Tech. 2.
Figure B.3: Damping determination from decay tests: LC02. Decay tests analysis of FC01 - Tech. 3 (min. cyc).
Figure B.4: Damping determination from decay tests: LC02. Decay tests analysis of FC01 - Tech. 3 (max. cyc).
Figure B.5: Damping determination from decay tests: LC02. Decay tests analysis of FC03 - Tech. 1.
Figure B.6: Damping determination from decay tests: LC02. Decay tests analysis of FC03 - Tech. 2.
Figure B.7: Damping determination from decay tests: LC02. Decay tests analysis of FC03 - Tech. 3 (min. cyc).
Figure B.8: Damping determination from decay tests: LC02. Decay tests analysis of FC03 - Tech. 3 (max. cyc).
Figure B.9: Damping determination from decay tests: LC02. Decay tests analysis of FC05 - Tech. 1.
Figure B.10: Damping determination from decay tests: LC02. Decay tests analysis of FC05 - Tech. 2.
Figure B.11: Damping determination from decay tests: LC02. Decay tests analysis of FC05 - Tech. 3 (min. cyc).
Figure B.12: Damping determination from decay tests: LC02. Decay tests analysis of FC05 - Tech. 3 (max. cyc).
Figure B.13: Damping determination from decay tests: LC02. Decay tests analysis of FC07 - Tech. 1.
Figure B.14: Damping determination from decay tests: LC02. Decay tests analysis of FC07 - Tech. 2.
Figure B.15: Damping determination from decay tests: LC02. Decay tests analysis of FC07 - Tech. 3 (min. cyc).
Figure B.16: Damping determination from decay tests: LC02. Decay tests analysis of FC07 - Tech. 3 (max. cyc).
Experimental data from externally excited roll tests
Table C.1: Experimental results from externally excited roll roll tests: LC02.

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<th>$\omega_r / \omega_0$</th>
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<th>$A_{\text{roll, max}}$</th>
<th>$A_{\text{roll, min}}$</th>
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</thead>
<tbody>
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<td>[rad]</td>
<td>[rad]</td>
<td>[deg]</td>
<td>[deg]</td>
<td>[deg]</td>
</tr>
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Physical background of the Weather Criterion
The Weather Criterion considers that a ship, under dead ship condition and subject to a heeling angle due to steady beam wind, is subject to a resonant roll motion in beam seas. Then, the ship suffers a sudden gusty wind when is rolled at the maximum windward angle. The gusty wind lasts long enough to allow the ship to roll to the other side completely. When the ship is under the resonant roll, it may be assumed that roll damping and exciting wave moment cancel out. Therefore, the stability of the oscillating ship may be estimated by its ability to absorb and dissipate the kinetic energy transmitted by external forces and/or moments, and it may be considered that the main portion of the energy of the rolling ship is transferred into the work of the righting moment. This is the reason why the stability of rolling is considered to depend in the current criterion, mainly, on the characteristics of the righting moment curves. The Weather Criterion evaluates the ship stability by the energy balance between restoring and wind heeling energy. If the heeling energy is larger than the restoring energy, it is considered that the ship would capsize and, thus, the criterion is not fulfilled.

In this Appendix, the concept of the energy balance approach is described briefly for introduction purposes and, then, the physical background of the angle of roll to windward due to wave action is described in detail. In the explanations, firstly the mathematical model of roll motion used in the Weather Criterion is described. Then, two procedures for determining the roll-back angle in waves are introduced, i.e., the empirical formulae in IMO (2009b) and the three-step procedure described in IMO (2006). No further explanations are given concerning this criterion as the roll damping only affects the roll-back angle in waves and a detailed description may be found in Section 3.5 of the “Explanatory Notes to the International Code on Intact Stability, 2018”, MSC.1/Circ 1281 (IMO, 2008).

### D.1 Energy balance approach

The energy balance approach applied to the Weather Criterion imposes that the ship should withstand the wind heeling moment when the ship is in a position transversely to the direction of wind and wave. It means that heeling moments, whether acting statically or dynamically, have to be balanced with the righting moment and the resulting heeling angle must be kept within safe limits.

When a heeling moment is acting dynamically in a ship (i.e., gusty wind lever $I_{w_2}$), the angle of heel is determined when the heeling moment work due to the gusty wind ($E_{w_2}$) is equal to the righting moment work ($E_R$). It may be expressed by:

\[
\begin{align*}
E_{w_2} &= E_R \Leftrightarrow e_{w_2} = e \\
E_{w_2} &= \int_{\phi_1}^{\phi_2} M_{w_2} d\phi = g \cdot \Delta \cdot \int_{\phi_1}^{\phi_2} I_{w_2} d\phi = g \cdot \Delta \cdot e_{w_2} \\
E_R &= \int_{\phi_1}^{\phi_2} M_R d\phi = g \cdot \Delta \cdot \int_{\phi_1}^{\phi_2} GZ(\phi) d\phi = g \cdot \Delta \cdot e
\end{align*}
\]  

(D.1)
where:

- $e_{w_2}$: [m] is the dynamic heeling moment arm due to the gusty wind;
- $e$: [m] is the dynamic stability arm;
- $\phi_1$: [rad] is the roll angle in waves. In the Weather Criterion scenario, when the ship suffers the gusty wind, the ship is assumed to roll at the maximum windward angle due to the resonant roll motion in beam waves. Thus, the roll angle in waves correspond to the maximum windward angle. It is important to highlight that being heeled to the maximum windward angle right before the gusty wind comes is the most dangerous case because, when the ship is returning to the upright position, both the righting and the heeling moment are working in the same direction, and the ship returning to the upright position possesses additional kinetic energy developed by the righting moment;
- $\phi_c$: [rad] is the heeling angle derived from the energy balance and corresponds to the capsizing angle. The capsize angle determined from the energy balance is limited to the down-flooding angle ($\phi_f$) or an arbitrary angle of 50 deg, whichever is less. The resulting heeling angle is so-called $\phi_2$.

In Figure D.1, the conceptual scheme of the Weather Criterion is shown. As it may be seen in the Figure, there is a common area below the righting moment curve ($GZ(\phi)$) and the lever of the gusty wind ($l_{w2}$). Taking this into account, a positive energy balance is defined by Equation D.2:

$$b \geq a$$  \hspace{1cm} (D.2)

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{figureD1.png}
\caption{Conceptual scheme of the Weather Criterion (IMO, 2009b).}
\end{figure}
The reserve stability in the energy balance approach is considered proportional to the difference between the limiting value of work of the righting moment and the total kinetic energy in rolling under the influence of external moments. Therefore, if Equation D.2 is fulfilled, the external moments are considered not dangerous. It is to be noted that, accordingly to Equation D.2, the output of the Weather Criterion is of pass-fail type.

The angle of roll to windward due to wave action ($\phi_1$) is the only parameter affected by roll damping. In the following, the physical background of $\phi_1$ formulation and possible means to determine it are explained.

### D.2 Roll motion model

The amplitude of roll to the windward side $\phi_1$ is calculated from the static angle of heel to leeward side $\phi_0$ (see IMO (2009b) and Figure D.1), modelling the ship motion in regular beam waves as a nonlinear 1-DOF equation:

$$\ddot{\phi} + d(\dot{\phi}) + \omega_0^2 \cdot \frac{GZ(\phi)}{GM} = \omega_0^2 \cdot \pi \cdot r(\omega) \cdot s_\omega \cdot \cos(\omega \cdot t) \tag{D.3}$$

where:

- $\omega_0$: [rad/s] is the undamped ship roll natural frequency;
- $\omega$: [rad/s] is the wave frequency;
- $r(\omega)$: [nd] is the effective wave slope coefficient;
- $s_\omega$: [nd] is the wave steepness.

Linearising the damping term of Equation D.3 by considering the equivalent linear roll damping coefficient $\mu_{eq}$, the following equation is derived:

$$\ddot{\phi} + 2 \cdot \mu_{eq} \cdot \dot{\phi} + \omega_0^2 \cdot \frac{GZ(\phi)}{GM} = \omega_0^2 \cdot \pi \cdot s_\omega \cdot r(\omega) \cdot \cos(\omega \cdot t) \tag{D.4}$$

From Equation D.4, an approximate peak rolling amplitude, i.e., the roll angle in regular beam waves $\phi_{1r}$, can be determined, by assuming that the wave frequency is equal to the undamped ship roll natural frequency, thus, the ship rolls with the undamped ship roll natural frequency and suffers a resonant harmonic motion. Therefore, the damping and external moment terms are in phase and cancel out:

$$2 \cdot \mu_{eq} \cdot \phi_{1r} \cdot \omega_0 = \omega_0^2 \cdot \pi \cdot s_\omega \cdot r(\omega_0) \Rightarrow \phi_{1r} = \frac{\omega_0 \cdot \pi \cdot s_\omega \cdot r(\omega_0)}{2 \cdot \mu_{eq}} \tag{D.5}$$
Equation D.5 may be written regarding Bertin’s roll damping coefficient $N(\phi)$ as follows:

$$
\begin{align*}
\phi_{1r} &= \frac{\omega_0 \cdot \pi \cdot s \cdot r(\omega_0)}{2 \cdot \mu_{eq}} [\text{rad}] \\
\mu_{eq} &= \nu_{eq} \cdot \omega_0 = \frac{1}{\pi} \cdot N(\phi_{1r}) \cdot \phi_{1r} \cdot \omega_0
\end{align*}
\Rightarrow \phi_{1r} = \sqrt{\frac{90 \cdot \pi \cdot s \cdot r(\omega_0)}{N(\phi_{1r})}} [\text{deg}] \quad (D.6)
$$

The Weather Criterion considers the roll angle in irregular seas $\phi_1$. However, Equation D.3, and the derived Equation D.6, are formulated for regular waves. To account for the effects of the irregular waves, a reduction factor of 0.7 to the roll angle in regular beam waves is applied such as:

$$
\phi_1 = 0.7 \cdot \phi_{1r} \quad (D.7)
$$

This factor was determined from a study made by Watanabe et al. (1956), see Figure D.2. The study compared the roll amplitude in irregular waves, with significant wave height and mean wave period equal to height and period of regular waves, with the resonant roll amplitude in regular waves. Considering the maximum amplitude obtained after 20 to 50 roll cycles, the corresponding reduction factor was approximately fixed to 0.7.

Consequently, the roll angle in waves may be determined from the following equation:

$$
\phi_1 = 0.7 \cdot \left( \frac{\omega_0 \cdot \pi \cdot s \cdot r(\omega_0)}{2 \cdot \mu_{eq}(\phi_{1r})} \right) [\text{rad}] = 0.7 \cdot \sqrt{\frac{90 \cdot \pi}{N(\phi_{1r})}} \cdot \sqrt{s \cdot r(\omega_0)} [\text{deg}] \quad (D.8)
$$

Where the wave steepness should be fixed and the roll damping coefficient (Bertin’s coefficient, $N(\phi_{1r})$ or equivalent linear roll damping coefficient, $\mu_{eq}(\phi_{1r})$) and the effective wave slope coefficient should be determined.

\[ Figure D.2: \] Comparison of roll amplitude in regular and irregular waves by Watanabe et al. (1956).
D.2 Roll motion model

To use Equation D.8 for regulatory purposes, two procedures were developed. The first procedure is based on empirical values to develop a theoretical formulation, and it is defined in IMO (2009b). The second procedure is based on determining the roll damping coefficient and the effective wave slope by experiments, whose guidelines are found in IMO (2006).

In both procedures, the wave steepness is defined as a factor dependent on the undamped ship roll natural period. The wave steepness $s_\omega$ was defined as the relationship between wave age ($\beta$) and wave steepness observed at sea by Sverdrup and Munk (1947), shown in Figure D.3. In this Figure, the wave age is defined as the ratio between the wave phase velocity ($u$) and the wind velocity ($v$, in this work, denoted as $U_w$).

\[ u = \frac{g}{2 \cdot \pi} \cdot \frac{T_0}{U_w} \]  

The wind velocity in the current Weather Criterion is fixed to 26 m/s. Therefore, from Figure D.4, a direct relationship between the wave steepness and the undamped ship roll natural period is established. This relation is the one used in the current Weather Criterion to determine the factor $s_\omega$ once the undamped ship roll natural period is known.

Figure D.3: Relation between wave age and wave steepness by Sverdrup and Munk (1947).
D.3 Determining the roll angle in waves: Empirical procedure

The first procedure establishes empirical values to transform Equation D.8 into Equation D.10, allowing the determination of the roll angle in waves ($\phi_1$) only considering main ship particulars and ship centre of gravity of the loading condition under analysis.

$$\phi_1 = 109 \cdot k \cdot X_1 \cdot X_2 \cdot \sqrt{s_\omega \cdot r(\omega_0)} \ [\text{deg}]$$  

(D.10)

where:

- 109: corresponds to a tuning factor which was included to keep the safety level of Japanese domestic standard, from which Equation D.10 was derived. This factor was determined by sample calculations of 58 Japanese ships in 1982;

- $k$: [nd] is a function of the bilge keel area and the lateral projection area of the bar keel ($A_k$);

- $X_1$: [nd] is a function of the beam-draught ratio ($B/d$);

- $X_2$: [nd] is a function of the block coefficient ($C_B$);

- $s_\omega$: [nd] is the wave steepness. The wave steepness is fixed according to the ship roll natural period, which may be estimated applying an empirical formula. The empirical formula to determine the ship roll natural frequency was derived statistically from data measured from 71 ships in 1982.
D.4 Determining the roll angle in waves: The three-step procedure

and is as follows:

\[ T_0 = \frac{2 \cdot C \cdot B}{\sqrt{GM}} \]

with:

\[ C = 0.737 + 0.023 \cdot \frac{B}{d} - 0.043 \cdot \frac{L_{WL}}{100} \]

- \( r(\omega_0) \) is the effective wave slope coefficient at the undamped ship roll natural frequency. It is estimated from an empirical formula. This formula is a function of the vertical centre of gravity and the draught of the ship, and is as follows:

\[ r(\omega_0) = 0.73 + 0.60 \cdot \frac{(KG - d)}{d} \]

It was derived from sample calculations of 60 ships, where the effective wave slope coefficient was theoretically calculated integrating undisturbed water pressure over the hull under the calm water surface, neglecting the wave diffraction due to the ship (IMO, 2008).

As this procedure is based on empirical values, Equation D.10 can be used if the following requirements are fulfilled:

- \( B/d \) smaller than 3.5;

- \( (KG/d - 1) \) between -0.3 and 0.5; and

- \( T_0 \) smaller than 20 s.

D.4 Determining the roll angle in waves: The three-step procedure

If the ship parameters are outside of the limits reported, the procedure described in the “Interim Guidelines for Alternative Assessment of the Weather Criterion” (IMO, 2006) should be used. This procedure proposes to measure directly from experiments the regular waves roll-back angle \( (\phi_1_r) \) and then, obtain the angle of roll to windward due to wave action \( (\phi_1) \) using Equation D.5.

However, when it is not feasible to perform the experiments with the required wave steepness, two alternative procedures are proposed. The first alternative is based on a three-step procedure and is the procedure that is going to be considered. The second alternative is based on the Parameter Identification Technique (PIT) approach and may be used if at least two response curves for two different wave
steepness are available. The description of the second alternative based on PIT may be found in IMO (2006).

The three-step procedure consists of determining:

1. **Roll damping at full scale**:

   The procedure allows to use roll damping from roll decay tests, or, alternatively, from forced or excited roll tests using internal or external systems. In any case, the ship model scale should be not less than 2 m or a scale 1:75, whichever is greater.

   Brief indications for executing roll decay tests are reported in §4.6.1.1.1. of IMO (2006). According to §4.6.1.1.1., roll decays should be performed without generating any vertical force and with an initial roll angle large enough, in such a way that the mean roll angle of the first oscillation is not smaller than 20°. It also says that recording should continue up until roll angles smaller than 0.5°.

   However, no guidelines are established to analyse roll decay tests. Furthermore, no guidelines or indications are given for forced and excited roll tests.

   The non-dimensional equivalent linear roll damping coefficient, at full scale, corresponds to the measured one at model scale corrected by the frictional effect of roll damping, if scale effects are not avoided. It is stated that scale effects exist for a ship not having bilge keels nor sharp bilges, if the ship model length is less than 4 m.

   The frictional effect on roll damping should be corrected with theoretical methods. The formulation proposed for the correction of the damping is based on the assumption that only exist a laminar boundary layer at model scale. Thus, a frictional component of roll damping is produced at model scale, but it does not exist at full scale. Therefore, this frictional component of roll damping at model scale is estimated and then deducted from the experimentally measured roll damping, following the formula reported in Equation D.13, where all the dimensional variables are at model scale and where $T_\phi$ is the roll period.

   \[
   \delta \nu_{eq} = \frac{1}{\pi} \cdot \frac{2.11 \cdot S \cdot r_s^2}{\Delta \cdot GM \cdot T_\phi^{1.5}}
   \]

   \[
   S = L_{wl} \cdot (1.7 \cdot d + C_B \cdot B)
   \]

   \[
   r_s = \frac{1}{\pi} \left[ (0.877 + 0.145 \cdot C_B) \cdot (1.7 \cdot d + C_B \cdot B) + 2 \cdot (KG - d) \right]
   \]  

2. **Effective wave slope coefficient ($r(\omega_0)$)**:

   The effective wave slope coefficient should be determined with the methods provided in §4.6.1.2.2. of IMO (2006). However, another approach not considered in the Interim Guidelines is to calculate the effective wave slope coefficient using the direct hydrodynamic approach.
3. Regular waves roll-back angle ($\phi_{1r}$):

The regular waves roll-back angle and, consequently, the angle of roll to windward due to wave action ($\phi_1$), should be determined from Equation D.6.

Equation D.6 and D.13 do not consider the amplitude-dependence of the roll oscillation frequency explicitly. The amplitude dependence of the roll oscillation frequency may have an influence on ships with significant nonlinear righting lever curves, as in the present case study. As in previous analyses of this study the dependence of roll oscillation frequency and roll amplitude has been considered, in order to be consistent, Equations D.6 and D.13 have been expressed in such a way that the amplitude dependence of the roll oscillation frequency is explicitly considered, as follows:

\[
\begin{align*}
T_\phi &= \frac{2 \cdot \pi}{\tilde{\omega}(A)} \\
\nu_{eq}(A) &= \frac{\mu_{eq}(A)}{\tilde{\omega}(A)} \\
\tilde{\omega}(A) &= \sqrt{\omega_{0,eq}^2(A) + \mu_{eq}^2(A)} \approx \omega_{0,eq}(A)
\end{align*}
\]

\[\Rightarrow \quad \phi_{1r} = \frac{\pi \cdot r(\omega_0) \cdot s_\omega}{2 \cdot \nu_{eq}(\phi_{1r})} = \frac{\pi \cdot r(\omega_0) \cdot s_\omega}{2 \cdot \left(\frac{\mu_{eq}(\phi_{1r})}{\omega_{0,eq}(\phi_{1r})}\right)} [rad]
\]

\[
\delta \nu_{eq}(\phi_{1r}) = \frac{\delta \left(\frac{\mu_{eq}(\phi_{1r})}{\omega_{0,eq}(\phi_{1r})}\right)}{\omega_{0,eq}(\phi_{1r})} = \frac{1}{\pi} \cdot \frac{2.11 \cdot S \cdot r_s^2}{\Delta \cdot GM \cdot \left(\frac{2 \cdot \pi}{\omega_{0,eq}(\phi_{1r})}\right)^{1.5}}
\]

Therefore:

\[
\phi_{1r} = \frac{\pi \cdot r(\omega_0) \cdot s_\omega}{2 \cdot \left(\mu_{eq}(\phi_{1r}) - \delta \mu_{eq}(\phi_{1r})\right) \omega_{0,eq}(\phi_{1r})} [rad]
\]

\[
\delta \mu_{eq}(\phi_{1r}) = \frac{1}{\pi} \cdot \frac{2.11 \cdot S \cdot r_s^2}{\Delta \cdot GM \cdot \left(2 \cdot \pi\right)^{1.5} \cdot \omega_{0,eq}^2(A)}
\]

where the equivalent undamped roll natural frequency ($\omega_{0,eq}(\phi_{1r})$) is determined using Equation 4.5.
Physical background of the Level 2 Dead Ship Condition failure mode
APPENDIX E. PHYSICAL BACKGROUND OF THE LEVEL 2 DEAD SHIP CONDITION FAILURE MODE
E.1 Determination of the long-term dead ship stability failure index

The Dead Ship Condition Level 2 criterion considers the dead ship condition scenario with the ship at zero speed subject to beam irregular waves and gusty wind for a specified exposure time. It evaluates the long-term probability of capsizing from the probability to capsize in the short-term approach. The short-term approach is based on an estimation of the roll motion of the ship under dead ship condition failure mode, considering an uncoupled roll model and an environmental condition defined by a stationary sea state.

In this Appendix, the current draft of the Level 2 criterion of the Dead Ship Condition (IMO, 2016a), as well as its physical background are described. The explanations reported hereafter are a summary of the reference article of DSC criterion (Bulian and Francescutto, 2004) and on information available in the current draft Explanatory Notes (IMO, 2016c) and on information submitted to the Intersessional Correspondence Group on Intact Stability (e.g., IMO, 2009a).

The Appendix starts with the description of the long-term index. Then, the mathematical model of the roll motion for the short-term assessment is described in detail, and its analytical solution is derived. Finally, the determination of the short-term index is explained.

E.1 Determination of the long-term dead ship stability failure index

A ship is considered not to be vulnerable to the dead ship condition failure mode if the estimated long-term probability of capsizing is smaller than the acceptable safety level, such as:

\[ C \leq R_{DS0} \]  

(E.1)

where:

- \( C \) is the probabilistic measure, which is a long-term failure index ranging from 0.0 to 1.0. It is obtained as a weighted average of short-term failure indices \( C_{S,i} \) of each short-term environmental condition under consideration (significant wave height \( H_s \) and zero-crossing period \( T_z \)), also ranging from 0.0 to 1.0. The value of \( C \) is calculated as:

\[ C = \sum_{i=1}^{N} W_i (H_s, T_z) \cdot C_{S,i} \]  

(E.2)

where \( W_i (H_s, T_z) \) is the weighting factor for the short-term environmental condition and represents the probability of occurrence of the short-term environmental condition, i.e., of each short-term stationary sea state condition given by the reference wave scatter diagram (e.g., IACS, 2001); and \( N \) is the number of considered short-term environmental conditions;
APPENDIX E. PHYSICAL BACKGROUND OF THE LEVEL 2 DEAD SHIP CONDITION FAILURE MODE

• $R_{DS0}$: [nd] is the standard value for Level 2 dead ship condition vulnerability criterion, which may be understood as the acceptable safety level. It has been tentatively set to 0.06 or 0.04, based on sample calculations of existing and designed ships.

The environmental conditions for the short-term failure index ($C_{S,i}$) are defined by the mean wind speed ($U_w$), the spectrum of wind gust ($S_v(\omega)$) and the spectrum of sea elevation ($S_{zz}(\omega)$). For the long-term failure index, the environmental conditions are defined by the wave scatter diagram (e.g., IACS, 2001). Wind and waves are linked by the significant wave height ($H_s$), while environmental conditions for the short-term and long-term failure index are related by the wave scatter diagram, as the short-term environmental condition considers the stationary sea states of the reference wave scatter diagram.

E.2 Roll motion model for the short term assessment

In the short-term approach, a short-term failure index $C_{S,i}$ is defined as a measure of the probability that the ship has to exceed a specified heel angle at least once in the exposure time considered and under a specific environmental condition. If the ship exceeds the specified heel angle, it is considered to be a capsizing event.

The roll motion of the ship under dead ship condition is modelled as a nonlinear 1-DOF system. The 1-DOF equation takes into account the mean heeling moment due to mean wind and the irregular roll moment due to the combined effects of wind gust and waves. The conceptual scheme of the physical modelling is shown in Figure E.1.

To determine the roll model, the ship is assumed to be in dead ship condition in irregular beam waves and gusty wind for a specified exposure time, having the ship zero speed and being restrained in yaw, remaining beam to wind and waves.

The sea state is characterized by a wave elevation spectrum, assuming long-crested waves and determining the significant wave height, $H_s$ [m], and the zero-crossing period, $T_z$ [s], from the reference wave scatter diagram. The wave elevation spectrum ($S_{zz}$ [m$^2$/(rad/s)]) is of the Bretschneider/Two-parameters Pierson-Moskowitz type, being defined as:

$$S_{zz}(\omega) = \frac{4}{\pi^3} \frac{H_s^2}{T_z^4} \cdot \omega^{-5} \cdot \exp \left( -\frac{16}{T_z^4} \cdot \omega^{-4} \right) \quad (E.3)$$

Forcing due to waves is calculated using the sea wave slope spectrum ($S_{\alpha\alpha}$ [rad$^2$/(rad/s)]), which is derived from the wave elevation spectrum and is defined as:

$$S_{\alpha\alpha}(\omega) = \frac{\omega^4}{g^2} \cdot S_{zz}(\omega) \quad (E.4)$$

where $g$ [m/s$^2$] is the gravitational acceleration, considered to be equal to 9.81 m/s$^2$. 

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E.2 Roll motion model for the short term assessment

Figure E.1: Conceptual scheme of the roll motion model for the short-term assessment of Dead Ship Condition failure mode of the Second Generation Intact Stability Criterion (IMO, 2016a).

The effect of wind is split into the action of the mean wind speed and the action of the wind speed fluctuations, i.e. gustiness wind. The effect of the mean wind speed is a constant drift motion to the ship, which may develop a hydrodynamic sway force equal in magnitude and opposite in direction of the mean wind force. The mean wind speed \(U_w\) is defined as:

\[
U_w = \left( \frac{H_s}{0.06717} \right)^{2/3} \quad \text{(E.5)}
\]

Assuming that the wind speed field can be modelled as a non-zero mean Gaussian process, if the mean wind speed is removed from the wind speed field, the gustiness process is obtained, which corresponds to the wind speed fluctuation around the mean wind speed. Gustiness can be characterised by a spectrum, as gustiness is a zero-mean Gaussian process. The spectrum of wind gust considered, \(S_v(\omega)\) [(m/s)^2/(rad/s)], is of a Davenport horizontal type. The spectrum depends on the mean wind speed and is defined as:

\[
S_v(\omega) = 4 \cdot K \cdot \frac{U_w}{\omega} \cdot \frac{X_D^2}{(1 + X_D^2)^{4/3}} \quad \text{(E.6)}
\]

with \(K = 0.003\) and \(X_D = 600 \cdot \omega / (\pi \cdot U_w)\).

The 1-DOF equation for the roll motion in the absolute form, taking into account previous stated...
assumptions, may be written as:

\[ J_{xx}^{*} \ddot{\phi} + D \dot{\phi} + \Delta GZ (\phi - \alpha_{eff}(t)) = M_{wind,tot} + \delta M_{wind,tot}(t) \]  

(E.7)

where:

- \( J_{xx}^{*} \): [kg \cdot m^2] is the roll mass moment of inertia plus the added mass moment of inertia;
- \( D \cdot \dot{\phi} \): [N \cdot m] is the damping moment, assumed to depend on the absolute roll velocity;
- \( \Delta \): [N] is the ship displacement of the loading condition under consideration;
- \( GZ \): [m] is the righting lever arm of the loading condition under consideration;
- \( \alpha_{eff}(t) \): [rad] is the instantaneous effective wave slope. It is assumed to be related to the instantaneous wave slope, \( \alpha(t) \) as follows:

\[ \alpha(t) = \sum_i \alpha_i(t) \rightarrow \alpha_{eff}(t) = \sum_i r(\omega_i) \cdot \alpha_i(t) \]  

(E.8)

where \( r(\omega_i) \) [rad] is the effective wave slope coefficient and the subscript \( i \) indicates the \( i^{th} \) harmonic component;

- \( M_{wind,tot} \) and \( \delta M_{wind,tot}(t) \): [N \cdot m] are the moment due to steady beam wind and the instantaneous moment due to wind gust, respectively, considering the corresponding hydrodynamic reaction.

The mean wind heeling moment \( M_{wind,tot} \) and the instantaneous moment due to wind gust \( \delta M_{wind,tot}(t) \) are defined from the pressure load of an undisturbed wind speed \( q_{tot}(t) \), as follows (Bulian and Francescutto, 2004):

\[
\begin{align*}
q_{tot}(t) &= \frac{1}{2} \cdot \rho_{air} \cdot \left( U_w^2 + 2 \cdot U_w \cdot u(t) + u^2(t) \right) \\
q_{tot}(t) &= \frac{1}{2} \cdot \rho_{air} \cdot U_w^2(t) \\
U_w(t) &= U_w(t) + u(t) 
\end{align*}
\]  

(E.9)

where \( \rho_{air} \) [kg/m^3] is the air density (and is considered equal to 1.222 kg/m^3); and \( u(t) \) [m/s] is the fluctuating wind speed.

Following the procedure of pressure linearisation, assuming that the coefficient of variation of the wind field is small, the second order terms may be neglected (\( u^2(t) = 0 \)). Equation E.9 may be
rewritten as:

\[
q_{\text{tot}}(t) = \bar{q} + \delta q(t)
\]

\[
\begin{align*}
\bar{q} &= \frac{1}{2} \cdot \rho_{\text{air}} \cdot U_w^2 \\
\delta q(t) &\approx \rho_{\text{air}} \cdot U_w \cdot u(t)
\end{align*}
\] (E.10)

The instantaneous horizontal force due to wind is defined as:

\[
F_{\text{wind,tot}}(t) = q_{\text{tot}}(t) \cdot C_m \cdot A_L = F_{\text{wind,tot}} + \delta F_{\text{wind,tot}}(t)
\]

\[
\begin{align*}
F_{\text{wind,tot}} &= \frac{1}{2} \cdot \rho_{\text{air}} \cdot U_w^2 \cdot C_m \cdot A_L \\
\delta F_{\text{wind,tot}}(t) &= \rho_{\text{air}} \cdot U_w \cdot C_m \cdot A_L \cdot u(t)
\end{align*}
\] (E.11)

where \(C_m \text{ [nd]}\) is the heeling moment coefficient, which may be considered equal to 1.22 if there is not enough information (see IMO, 2006); and \(A_L \text{ [m}^2\) is the lateral windage area.

As the hydrodynamic reaction due to steady drift motion is assumed to be fully developed, the heeling lever of the mean horizontal force due to wind, \(F_{\text{wind,tot}} \text{ [N]}\) is equal to the vertical distance between the hydrodynamic centre of pressure and the aerodynamic centre of pressure, \(Z \text{ [m]}\). The vertical distance \(Z\) may be considered as the vertical distance from the centre of \(A_L\) to the centre of the underwater lateral area or, approximately, to a point at one half the mean draught, as it is considered in the current Weather Criterion (see IMO, 2009b, 2006).

The mean wind heeling moment and its corresponding moment lever, \(l_{\text{wind,tot}} \text{ [m]}\), are defined as follows, assuming that the mean wind heeling moment is independent on the instantaneous roll angle:

\[
\bar{M}_{\text{wind,tot}} = \frac{1}{2} \cdot \rho_{\text{air}} \cdot U_w^2 \cdot C_m \cdot A_L \cdot Z
\]

\[
l_{\text{wind,tot}} = \frac{\bar{M}_{\text{wind,tot}}}{\Delta}
\] (E.12)

The fluctuation of the heeling moment is derived from the forces due to the effects of gustiness, \(\delta F_{\text{wind,tot}}(t) \text{ [N]}\). Although the effects of gustiness are assumed to occur on fast time scales, it is considered that the hydrodynamic reactions due to waves have enough time to develop and, thus, the heeling lever for the wind force fluctuation is, as the heeling lever for the mean wind force, the vertical distance between the hydrodynamic centre of pressure and the aerodynamic centre of
APPENDIX E. PHYSICAL BACKGROUND OF THE LEVEL 2 DEAD SHIP CONDITION FAILURE MODE

The fluctuation of the heeling moment is of the form:

\[
\delta M_{\text{wind,tot}} (t) = \delta F_{\text{wind,tot}} (t) \cdot Z = \rho_{\text{air}} \cdot U_w \cdot C_m \cdot A_L \cdot Z \cdot u(t) \tag{E.13}
\]

To solve Equation E.7 analytically in the frequency domain, it has to be linearised locally close to the static heeling angle due to the action of mean wind speed.

The heeling angle, \(\phi_S\), as it is a steady heel angle, may be obtained from Equation E.7 by considering only the action of mean beam wind, as follows:

\[
\Delta GZ (\phi_S) = M_{\text{wind,tot}} \to GZ (\phi_S) = \frac{M_{\text{wind,tot}}}{\Delta} = l_{\text{wind,tot}} \tag{E.14}
\]

The fluctuation of the absolute roll angle, \(x(t)\), with respect to the heeling angle \(\phi_S\) may be defined as:

\[
x (t) = \phi (t) - \phi_S \tag{E.15}
\]

Considering that the mean value of \(x(t)\) is small compared to \(\phi_S\) and compared to the standard roll deviation, \(\sigma_x\), Equation E.7 may be rewritten as:

\[
\ddot{x} + d (\dot{x}) + \Delta J_{xx} \cdot GZ (\phi_S + x - \alpha_{\text{eff}} (t)) = \frac{M_{\text{wind,tot}}}{J_{xx}} + \frac{\delta M_{\text{wind,tot}} (t)}{J_{xx}} \tag{E.16}
\]

Under the assumption that the nonlinearities of \(GZ\) are not extremely strong, linearisation of the restoring moment close to \(\phi_S\) can be done as:

\[
\overline{GZ} (\phi_S + x - \alpha_{\text{eff}} (t)) \approx \overline{GZ} (\phi_S) + \overline{GM_{\text{res}}} (\phi_S) \cdot (x - \alpha_{\text{eff}} (t)) \tag{E.17}
\]

where \(\overline{GM_{\text{res}}} (\phi_S) [m]\) is the derivative of the residual righting lever curve at \(\phi_S\). Since the heeling moment of the steady beam wave is constant, \(\overline{GM_{\text{res}}} (\phi_S)\) is defined as:

\[
\overline{GM_{\text{res}}} (\phi_S) = \left. \frac{d (GZ - l_{\text{wind,tot}})}{d\phi} \right|_{\phi=\phi_S} = \left. \frac{dGZ}{d\phi} \right|_{\phi=\phi_S} \tag{E.18}
\]
Therefore, Equation E.16 may be written, considering Equation E.17 as:

\[
\ddot{x} + d(\dot{x}) + \omega_{0,e}^2(\phi_S) \cdot x = \frac{M_{w\text{aves}}(t)}{J_{xx}} + \frac{\delta M_{\text{wind,tot}}(t)}{J_{xx}}
\]

\[
\omega_{0,e}^2(\phi_S) = \frac{\Delta \cdot GM_{\text{res}}(\phi_S)}{J_{xx}} = \omega_0^2 \cdot \sqrt{\frac{GM_{\text{res}}(\phi_S)}{GM}}
\]

\[
M_{\text{waves}}(t) = \Delta \cdot GM_{\text{res}}(\phi_S) \cdot \alpha_{\text{eff}}(t)
\]

(E.19)

where:

- \(\omega_{0,e}(\phi_S): [\text{rad/s}]\) is the modified roll natural frequency close to the steady heeling angle \(\phi_S\);
- \(M_{\text{waves}}: [\text{N} \cdot \text{m}]\) is the moment due to the action of waves. Under the assumption of long waves, the forcing moment due to waves is always in phase with the wave slope at the ship position.

The damping term of Equation E.19, \(d(\dot{x})\), is linearised considering the nonlinear damping terms being dependent on the roll velocity as follows:

\[
d(\dot{x}) = 2 \cdot \mu \cdot \dot{x} + \beta \cdot \dot{x} \cdot |\dot{x}| + \delta \cdot \dot{x}^3 \approx 2 \cdot \mu_e(\sigma_\dot{x}) \cdot \dot{x}
\]

(E.20)

where:

- \(\mu, \beta\) and \(\delta\): \([1/\text{s}], [1/\text{rad}]\) and \([\text{s/ rad}^2]\), respectively, are the linear, quadratic and cubic damping coefficients;
- \(\mu_e(\sigma_\dot{x}): [1/\text{s}]\) is the equivalent linear damping coefficient, which depends on the standard deviation of the absolute roll velocity \(\sigma_\dot{x} \text{ [rad/s]}\). The relation between \(\mu_e(\sigma_\dot{x})\) and the nonlinear damping coefficients terms is defined as:

\[
\mu_e(\sigma_\dot{x}) = \mu + \sqrt{\frac{2}{\pi}} \cdot \beta \cdot \sigma_\dot{x} + \frac{3}{2} \cdot \delta \cdot \sigma_\dot{x}^2
\]

(E.21)

The standard deviation of the absolute roll velocity, \(\sigma_\dot{x} \text{ [rad/s]}\), may be obtained from the spectrum of the absolute roll fluctuation \(x(t)\) as follows:

\[
\sigma_\dot{x} = \sqrt{\int_0^\infty \omega^2 \cdot S_\dot{x}(\omega) \cdot d\omega}
\]

(E.22)

Then, the linear 1-DOF Roll motion model, from the linearised Equation E.19, is defined as:

\[
\ddot{x} + 2 \cdot \mu_e(\sigma_\dot{x}) \cdot \dot{x} + \omega_{0,e}^2(\phi_S) \cdot x = \frac{M_{w\text{aves}}(t)}{J_{xx}} + \frac{\delta M_{\text{wind,tot}}(t)}{J_{xx}}
\]

(E.23)
APPENDIX E. PHYSICAL BACKGROUND OF THE LEVEL 2 DEAD SHIP CONDITION FAILURE MODE

E.3 Analytical solution of the linearised 1-DOF Roll motion model

The short-term assessment is based on a relative angle approach, to consider the effect of the metacentric height on the ship natural frequency. Usually, a low metacentric height implies also a low ship roll natural frequency, which is far from typical wave frequencies. Therefore, the absolute roll motion is limited due to the absence of resonance, as the reduction of static restoring is less strong than the reduction of the absolute roll motion. This situation leads to a global reduction of the short-term failure capsize index. However, the relative roll angle could be large since it is dominated by the spectrum of wave slope. For this reason, the determination of the short-term failure index is based on the relative angle, instead of the absolute angle framework. However, the absolute frequency domain solution of Equation E.23 is required to determine the equivalent linear roll damping coefficient \( \mu_e(\sigma_x) \), as it depends directly on the standard deviation of the absolute roll velocity \( \sigma_x \) and, thus, on the spectrum of the absolute roll fluctuation, \( S_x(\omega) \), as stated in Equation E.22.

E.3.1 Absolute frequency domain solution

The solution of the linear Equation E.23 in the absolute frequency domain may be obtained by splitting \( x(t) \) in the response due to waves and the response due to wind gust, under the hypothesis that fluctuating wind and wave moments are locally uncorrelated, although their spectra are globally linked through the wave generation mechanism by the action of wind, i.e., the mean wind speed. Thus:

\[
\begin{align*}
\ddot{x}_\text{waves} + 2 \cdot \mu_e(\sigma_x) \cdot \dot{x}_\text{waves} + \omega_0^2, e(\phi_S) \cdot x_\text{waves} &= \frac{M_\text{waves}(t)}{J_{xx}} \\
\ddot{x}_\text{wind} + 2 \cdot \mu_e(\sigma_x) \cdot \dot{x}_\text{wind} + \omega_0^2, e(\phi_S) \cdot x_\text{wind} &= \frac{\delta M_{\text{wind,tot}}(t)}{J_{xx}}
\end{align*}
\]

(E.24)

The complex transfer function for Equations E.24 is:

\[
\hat{H}_{abs}(\omega) = \frac{1}{(\omega_0^2, e(\phi_S) - \omega^2) + j \cdot 2 \cdot \mu_e(\sigma_x) \cdot \omega}
\]

(E.25)

From which the spectra of \( x_{\text{waves}}(t) \) and \( x_{\text{wind}}(t) \) can be calculated as:

\[
\begin{align*}
S_{x_{\text{waves}}}(\omega) &= \left| \hat{H}_{abs}(\omega) \right|^2 \cdot \frac{S_{M_{\text{waves}}}(\omega)}{(J_{xx})^2} \\
S_{x_{\text{wind}}}(\omega) &= \left| \hat{H}_{abs}(\omega) \right|^2 \cdot \frac{S_{\delta M_{\text{wind,tot}}}(\omega)}{(J_{xx})^2}
\end{align*}
\]

(E.26)

where:
• \( S_{\text{Mwaves}} (\omega) \): \([ (N \cdot m)^2/(rad/s)]\) is the spectrum of the moment due to the action of the waves. Under the assumption that the forcing moment due to waves is in phase with the wave slope at the ship position, it may be defined from the wave elevation spectrum as:

\[
S_{\text{Mwaves}} (\omega) = (\Delta \cdot GM_{res} (\phi_S))^2 \cdot S_{\alpha\alpha,c} (\omega)
\] 

(E.27)

where \( S_{\alpha\alpha,c} (\omega) \) \([rad^2/(rad/s)]\) is the spectrum of the effective wave slope. It is calculated by taking into account the sea wave slope spectrum \( S_{\alpha\alpha} (\omega) \), defined in Equation E.4, and the effective wave slope coefficient \( r (\omega) \):

\[
S_{\alpha\alpha,c} (\omega) = r^2 (\omega) \cdot S_{\alpha\alpha} (\omega)
\] 

(E.28)

The effective wave slope coefficient can be estimated by direct hydrodynamic approach or with the Froude-Krylov approach, assuming rectangular hull sections as described in IMO (2016a);

• \( S_{\delta M_{\text{wind,tot}}} (\omega) \): \([ (N \cdot m)^2/(rad/s)]\) is the spectrum of the moment due to fluctuating wind. Taking into account Equation E.13 and the spectrum of wind gust defined in Equation E.6, it may be calculated as:

\[
S_{\delta M_{\text{wind,tot}}} (\omega) = \left( \rho_{\text{air}} \cdot U_w \cdot C_m \cdot A_L \cdot Z \right)^2 \cdot \chi^2 (\omega) \cdot S_{v} (\omega)
\] 

(E.29)

where \( \chi (\omega) \) \([nd]\) is the aerodynamic admittance function. It allows considering the spatial variability of wind and the effects of the departure from the quasi-steady theory. However, by default, \( \chi (\omega) \) is equal to 1.

Consequently, the spectrum of the absolute roll fluctuation is determined as the sum of both wind and wind gust spectra as follows:

\[
S_x (\omega) = S_{x_{\text{waves}}} (\omega) + S_{x_{\text{wind}}} (\omega) = \frac{\hat{H}_{abs} (\omega)^2}{(J_{zz})^2} \cdot (S_{M_{\text{waves}}} (\omega) + S_{\delta M_{\text{wind,tot}}} (\omega)) = \omega_0^4 \cdot \left| \hat{H}_{abs} (\omega) \right|^2 \cdot \frac{S_{M_{\text{waves}}} (\omega) + S_{\delta M_{\text{wind,tot}}} (\omega)}{(\Delta \cdot GM)^2}
\] 

(E.30)
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which may be rewritten as:

\[
S_x (\omega) = H^2 (\omega) \cdot S_m (\omega)
\]

\[
\begin{align*}
H^2 (\omega) &= \omega_0^4 \cdot |\bar{H}_{abs} (\omega)|^2 = \frac{\omega_0^4}{(\omega_0^2 (\phi_S) - \omega^2)^2 + (2 \cdot \mu_e (\sigma_x) \cdot \omega)^2} \\
S_m (\omega) &= \frac{S_M (\omega)}{(\Delta \cdot GM)^2} = \frac{S_{Mwaves} (\omega) + S_{Mwind,tot} (\omega)}{(\Delta \cdot GM)^2}
\end{align*}
\]  
(E.31)

where \( S_M (\omega) [(N \cdot m)^2/(rad/s)] \) is the spectrum of the total moment due to the action of waves and wind gust.

To solve Equation E.31 and obtain the standard deviation of the absolute roll velocity, \( \sigma_x \), the following equation needs to be solved:

\[
F (\sigma_x) = 0
\]

with

\[
F (\sigma_x) = \sigma_x - \int_0^\infty \frac{\omega^2 \cdot \omega_0^4}{(\omega_0^2 (\phi_S) - \omega^2)^2 + (2 \cdot \mu_e (\sigma_x) \cdot \omega)^2} \cdot \frac{S_M (\omega)}{(\Delta \cdot GM)^2} \cdot d\omega
\]  
(E.32)

where the expression of \( \mu_e (\sigma_x) \) corresponds to Equation E.21.

E.3.2 Relative frequency domain solution

The solution of the linear Equation E.23 in the relative angle approach is described hereafter. The effective relative roll angle is defined as:

\[
\phi_{rel,eff} (t) = \phi_S + x (t) - \alpha_{eff} (t)
\]  
(E.33)

And the fluctuation of \( \phi_{rel,eff} (t) \) with respect to \( \phi_S \) is given by:

\[
x_{rel,eff} (t) = \phi_{rel,eff} (t) - \phi_S = x (t) - \alpha_{eff} (t) = x_{waves} (t) - \alpha_{eff} (t) + x_{wind} (t)
\]  
(E.34)

As in the absolute approach, it is assumed that fluctuating wind and wave moments are locally uncorrelated. Moreover, it is assumed that fluctuating wind and the effective wave slope are also uncorrelated. However, wave moments are influenced by the effective wave slope. Therefore, the spectrum of \( x_{rel,eff} (t) \) is defined as the sum of the spectrum of \( x_{waves} (t) - \alpha_{eff} (t) \), from now on called \( x_{rel,eff,waves} (t) \), and the spectrum of \( x_{wind} (t) \).

The spectrum of \( x_{wind} (t) \) corresponds to the one defined in Equation E.29, whereas the spectrum
E.4 Determination of the short-term dead ship stability failure index

do of $x_{\text{rel,eff,waves}}(t)$ may be obtained from the complex form of $x_{\text{rel,eff,waves}}(\omega)$. It is derived as follows:

$$
\dot{x}_{\text{rel,eff,waves}} = \dot{x}_{\text{waves}} - \dot{\alpha}_{\text{eff}} =
\sum_i \left[ \frac{\Delta GM_{\text{res}}}{J_{xx}} \cdot \hat{H}_{\text{abs}}(\omega_i) - 1 \right] \cdot r(\omega_i) \cdot \hat{\alpha}_i \cdot e^{j\omega_i t}
$$

(E.35)

Considering the expression between brackets the relative angle complex transfer function, the spectrum of $x_{\text{rel,eff,waves}}(t)$ can be deducted from Equation E.35 as:

$$
S_{x_{\text{rel,eff,waves}}}(\omega) = \left| \frac{\Delta GM_{\text{res}}}{J_{xx}} \cdot \hat{H}_{\text{abs}}(\omega) - 1 \right|^2 \cdot r(\omega) \cdot S_{\alpha\alpha}(\omega) =
\left| \omega_{0,e}^2 \cdot \hat{H}_{\text{abs}}(\omega) - 1 \right|^2 \cdot \frac{S_{\alpha\alpha,c}(\omega)}{(\Delta \cdot GM_{\text{res}}(\phi_S))^2}
$$

(E.36)

Therefore, the spectrum of the effective relative roll angle $S(\omega)$ presents the following expression:

$$
S(\omega) = S_{x_{\text{rel,eff,waves}}}(\omega) + S_{\text{wind}}(\omega) =
H^2_{\text{rel}}(\omega) \cdot \frac{S_{M_{\text{waves}}}(\omega)}{(\Delta \cdot GM_{\text{res}}(\phi_S))^2} + H^2(\omega) \cdot \frac{S_{\delta M_{\text{wind,tot}}}(\omega)}{(\Delta \cdot GM)^2}
$$

(E.37)

$$
\left\{ 
\begin{align*}
H^2_{\text{rel}}(\omega) &= \left| \omega_{0,e}^2 \cdot \hat{H}_{\text{abs}}(\omega) - 1 \right|^2 = \frac{\omega^4 + (2 \cdot \mu_e \cdot \omega)^2}{(\omega_{0,e}^2(\phi_S) - \omega^2)^2 + (2 \cdot \mu_e \cdot \omega)^2} \\
H^2(\omega) &= \omega^2 \cdot \left| \hat{H}_{\text{abs}}(\omega) \right|^2 = \frac{\omega_{0,e}^4}{(\omega_{0,e}^2(\phi_S) - \omega^2)^2 + (2 \cdot \mu_e \cdot \omega)^2}
\end{align*}
\right.
$$

E.4 Determination of the short-term dead ship stability failure index

To determine the short-term failure index $C_{S,i}$, a short-term prediction of the capsize probability should be defined. It is important to highlight that a capsizing event, or a ship failure, is considered to happen when the ship exceeds a specified heel angle at least once in the exposure time considered.
The limiting heel angles (or the equivalent critical heel angles), $\phi_{EA+}$ and $\phi_{EA-}$, where (+) and (−) signs account for leeward and windward sides, are defined using the equivalent area concept, to take into account the nonlinear restoring by considering the actual shape of the righting lever curve. Given physical failure angles, $\phi_{fail,+}$ and $\phi_{fail,-}$, the equivalent critical heel angles are determined to keep the area under the residual righting lever up to the physical failure angles, by considering the linearised residual righting lever to be the same as the linearised residual righting lever, i.e., to fulfil the following conditions:

\[
\begin{align*}
(1) & \quad \int_{\phi_S}^{\phi_{fail,+}} GZ_{res}(\phi)d\phi = \frac{GM_{res} \cdot (\phi_{EA+} - \phi_S)^2}{2} \\
(2) & \quad \int_{\phi_{fail,-}}^{\phi_S} GZ_{res}(\phi)d\phi = \frac{GM_{res} \cdot (\phi_S - \phi_{EA-})^2}{2}
\end{align*}
\] (E.38)

From E.38, the equivalent critical heel angles expression may be derived:

\[
\begin{align*}
\phi_{EA+} &= \phi_S + \sqrt{\frac{2}{GM_{res}} \cdot \int_{\phi_S}^{\phi_{fail,+}} GZ_{res}(\phi)d\phi} \\
\phi_{EA-} &= \phi_S - \sqrt{\frac{2}{GM_{res}} \cdot \int_{\phi_{fail,-}}^{\phi_S} GZ_{res}(\phi)d\phi}
\end{align*}
\] (E.39)

where:

- $GZ_{res}(\phi)$: [m] is the residual righting lever and is defined as:

\[
GZ_{res}(\phi) = GZ(\phi) - l_{wind,tot}
\] (E.40)

- $\phi_{fail,+}$ and $\phi_{fail,-}$: [rad] are the physical failure angles to leeward and windward, respectively. They are defined as the minimum/maximum between the angle of vanishing stability due to mean wind action ($\phi_{VW,+}$ and $\phi_{VW,-}$) and a critical angle ($\phi_{crit,+}$ and $\phi_{crit,-}$), respectively.

\[
\begin{align*}
\phi_{fail,+} &= \min \{ \phi_{VW,+}, \phi_{crit,+} \} \\
\phi_{fail,-} &= \max \{ \phi_{VW,-}, \phi_{crit,-} \}
\end{align*}
\] (E.41)

where the critical angles are defined as the minimum/maximum between the progressive flooding angle ($\phi_f$) or an arbitrary limiting angle of ±50 deg.

\[
\begin{align*}
\phi_{crit,+} &= \min \{ \phi_f, 50 \} \\
\phi_{crit,-} &= \max \{ -\phi_f, -50 \}
\end{align*}
\] (E.42)
Once the limiting heel angles have been defined, the short-term probability index equation may be derived. It is important to quote that the short-term probability index gives an approximate measure of the probability of capsizing, this is the reason why it is called an index. It does not give an actual probability because of the assumptions made through the whole process.

The probability of capsizing is estimated on the basis that the roll motion is a Gaussian, stationary and ergodic process and using the hypothesis that capsizes is a Poisson process. It is assumed that capsize may occur on both sides of the ship, considering capsize to leeward and windward as independent Poisson processes.

Theoretically, the probability of capsizing depends on the exposure time $T_{\text{exp}}$, which is defined as the time interval during which the ship is exposed to the given environmental conditions. The probability of capsizing in a specific exposure time $T_{\text{exp}}$, neglecting the probability of capsizing at time close to zero, is related to the probability of not capsize in a time $T_{\text{exp}}$ as follows:

$$P\{\text{capsize in } T_{\text{exp}}\} = 1 - P\{\text{NOT capsize in } T_{\text{exp}}\} \quad (E.43)$$

The probability of not capsizing in a time $T_{\text{exp}}$ is equivalent to the probability of having zero capsize events in $T_{\text{exp}}$. It may be obtained considering a Poisson probability mass function for the capsize event as follows:

$$P\{\text{having exactly } n \text{ capsize in } T_{\text{exp}}\} = \frac{1}{n!} \left( \frac{T_{\text{exp}}}{T_{\text{cap}}} \right)^n \cdot \exp \left( -\frac{T_{\text{exp}}}{T_{\text{cap}}} \right) \quad (E.44)$$

Thus, if considering zero capsize events ($n = 0$):

$$P\{\text{capsize in } T_{\text{exp}}\} = 1 - \exp \left( -\frac{T_{\text{exp}}}{T_{\text{cap}}} \right) \quad (E.45)$$

where $T_{\text{cap}} [s]$ is the characteristic parameter of the distribution and represents the mean time to capsize. $T_{\text{cap}}$ is estimated, considering a Gaussian process, as the inverse of the mean up-crossing frequency of the limiting heel angles $\phi_{EA^+}$ and $\phi_{EA^-}$:

$$\frac{1}{T_{\text{cap}}} = \frac{1}{2 \cdot \pi} \cdot \frac{\sigma_{x_{rel,eff}}}{\sigma_{x_{rel,eff}}} \cdot \left[ \exp \left( -\frac{1}{2} \left( \frac{\phi_{EA^+} - \phi_S}{\sigma_{x_{rel,eff}}} \right)^2 \right) + \exp \left( -\frac{1}{2} \left( \frac{\phi_S - \phi_{EA^-}}{\sigma_{x_{rel,eff}}} \right)^2 \right) \right] \quad (E.46)$$

Therefore, substituting Equation E.46 into Equation E.45, the probability to capsize in a time $T_{\text{exp}}$ can
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be determined as:

\[
P \{ \text{capsize in } T_{\text{exp}} \} = 
1 - \exp \left( -\frac{T_{\text{exp}}}{2 \cdot \pi} \cdot \frac{\sigma_{x_{\text{rel,eff}}}}{\sigma_{x_{\text{rel,eff}}}^2} \cdot \left[ \exp \left( -\frac{1}{2} \cdot \left( \frac{\phi_{EA^+} - \phi_{S}}{\sigma_{x_{\text{rel,eff}}}} \right)^2 \right) + \exp \left( -\frac{1}{2} \cdot \left( \frac{\phi_{S} - \phi_{EA^-}}{\sigma_{x_{\text{rel,eff}}}} \right)^2 \right) \right] \right)
\]

(E.47)

where:

- \( \sigma_{x_{\text{rel,eff}}} \): [rad] is the standard deviation of the relative roll angle. It may be obtained from the spectrum of the effective relative roll angle, defined in Equation E.37, as:

\[
\sigma_{x_{\text{rel,eff}}} = \sqrt{\int_{0}^{\infty} S(\omega) d\omega}
\]

(E.48)

- \( \sigma_{x_{\text{rel,eff}}} \): [rad/s] is the standard deviation of the relative roll velocity, which may be obtained from the spectrum of the effective relative roll angle, in Equation E.37, as:

\[
\sigma_{x_{\text{rel,eff}}} = \sqrt{\int_{0}^{\infty} \omega^2 \cdot S(\omega) d\omega}
\]

(E.49)

Equation E.47, considering that the expression of the probability of capsizing is not accurately a probability, may be rewritten as:

\[
C_{S} = 1 - \exp (-\lambda_{EA} \cdot T_{\text{exp}})
\]

(E.50)

with:

\[
\lambda_{EA} = \frac{1}{T_{x_{C_{S}}} \cdot \left[ \exp \left( -\frac{1}{2} \cdot \frac{1}{RI_{EA^+}^2} \right) + \exp \left( -\frac{1}{2} \cdot \frac{1}{RI_{EA^-}^2} \right) \right]}
\]

(E.51)

where:

- \( \lambda_{EA} \): [1/s] is the average time between two capsize events. Thus, the mean time to capsize can be estimated as \( 1/\lambda_{EA} \);

- \( T_{\text{exp}} \): [s], is the short-term exposure time and it is considered to be equal to 3600 s;
E.4 Determination of the short-term dead ship stability failure index

- \( T_{z,CS} \): [s] is the zero-crossing period of the roll motion process. It is defined as:

\[
T_{z,CS} = 2 \cdot \frac{\pi}{x_{rel,eff}}
\]

(E.52)

- \( RI_{EA^+} \) and \( RI_{EA^-} \): \([nd]\) are the risk indexes to leeward and windward, respectively. They are defined as:

\[
RI_{EA^+} = \frac{C_S}{\Delta \phi_{res,EA^+}}
\]

(E.53)

\[
RI_{EA^-} = \frac{C_S}{\Delta \phi_{res,EA^-}}
\]

- \( C_S \): \([rad]\) is the standard deviation of the roll motion, which corresponds to \( x_{rel,eff} \);

- \( \Delta \phi_{res,EA^+} \) and \( \Delta \phi_{res,EA^-} \): \([rad]\) are the residual margins to leeward and windward, respectively. They are defined as:

\[
\Delta \phi_{res,EA^+} = \phi_{EA^+} - \phi_S
\]

(E.54)

\[
\Delta \phi_{res,EA^-} = \phi_S - \phi_{EA^-}
\]

The short-term probability index is obtained, for a specific environmental condition, applying a simple methodology that analyse the roll behaviour of a ship under dead ship condition scenario for a specific exposure time. The value \( C_{S,i} \) depends on the ship characteristics, on the short-term environmental conditions and on the assumed short-term exposure time. As it may be seen, the short-term probability index equation, Equation E.50, takes into account static aspects, such as the effect of the mean wind action considered in the residual margins, and dynamic aspects in their approximate way, such as the variability of the roll motion given by the standard deviation \( C_S \) or the zero-crossing period of the roll motion process \( T_{z,CS} \).