Full-wave Coupling Matrix Description in Microwave Filters via Reduced-Basis Approximations in Electromagnetics

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Abstract—Current computational resources are leading electromagnetic computer aided design towards brute-force approaches, where global optimization techniques are taken into account and reduced knowledge of the actual electromagnetic behavior is considered. Within this strategy, the design development time increases dramatically.

In this work, on the contrary, we focus on obtaining an accurate and reliable physical-based description of the Electromagnetics we are dealing with in order to carry out a fast and accurate design. Appropriate Finite Element solution to time-harmonic Maxwell's equations in the frequency band of interest is involved and a reliable reduced-order model is obtained. Then, the electromagnetic behavior in the microwave device is rearranged and described in terms of circuit theory, giving rise to the coupling matrix representation in the actual coupling topology of the original Maxwell problem.

The ingredients of this methodology are: 1) the use of field solutions at given frequencies as basis functions to reduce the original Maxwell problem, 2) adaptively choosing the field solution frequency samples and 3) change of basis in the reduced-order model in order to get the full-wave coupling matrix description in microwave circuits.

Index Terms—Circuit theory, computer aided engineering, error analysis, finite element methods, Galerkin method, reduced basis methods, reduced order systems.

I. INTRODUCTION

Increasing deployment of telecommunication services is urging microwave industry to speed up product development time, where not only prototyping and final tuning steps in the production line have to be reduced, but also simulation time to achieve an actual electrical design by means of electromagnetic computer-aided design (EM-CAD). Performing robust electrical designs has thus become of paramount importance, and powerful numerical techniques capable of predicting electromagnetic behavior are at the heart of this robust design. However, current computational resources are leading EM-CAD design towards brute-force approaches, where global optimization techniques are considered and reduced physical insight of the actual electromagnetic behavior is taken into account. The design development time can by no means be decreased following this trend.

Finite Element Methods (FEM) have proven to be helpful in solving the time-harmonic Maxwell's equations for the analysis of microwave devices. As a result, microwave engineering not only stands upon full-wave analysis to accurately predict the electromagnetic behavior before construction of microwave circuits, but also to actually design their electromagnetic response. However, we are still lacking deep understanding of electromagnetic phenomena and only rather experienced microwave engineers are able to develop complicated electromagnetic designs.

In this last framework, reduced-order models (ROM) may be of help. Indeed, model order reduction techniques are as valuable as FEM analysis if the reduced-order model is a reliable surrogate of the original problem [1], [2], [3], [4], [5], [6], [7], [8]. Efforts should be placed in getting circuit information out of these reliable reduced-order models and in presenting this circuit information in a suitable form to provide physical insight about the electromagnetic behavior, namely, a coupling matrix representation of the physics [9], [10].

II. PROBLEM STATEMENT

Let us consider a two-port microwave circuit, namely, a microwave filter, where Maxwell's equations describe the physical behavior. We are interested in the filter transfer function representing the electromagnetic phenomena in this circuit, namely, an impedance matrix transfer function. After discretization of the time-harmonic Maxwell's equations by means of FEM, we get the following linear dynamical system representation of the microwave circuit:

$$\begin{pmatrix} v_1 \\ v_2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & jb_1^T \\ 0 & 0 & jb_2^T \\ sjb_1 & sjb_2 & S + s^2T \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ x \end{pmatrix}, \quad (1)$$

where S and T are the stiffness and mass matrices, x stands for the state-space variable and s is the complex frequency. Once the state-space variable, which represents the FEM discretization of the electric field, is eliminated from equation (1), the impedance matrix transfer function arises, thus,

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = -s \begin{pmatrix} jb_1^T \\ jb_2^T \end{pmatrix} (S + s^2T)^{-1} \begin{pmatrix} jb_1 & jb_2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}. \quad (2)$$

It should be noted that the dynamical system (1) has a rather large state-space, thus solving this system frequency by frequency cannot be afforded. In this regard, we propose a reduced-order model to carry out the frequency sweep in an efficient way and, ultimately, to get a circuit theory representation within Electromagnetics.

A. Model Order Reduction

In order to propose an effective reduced-order model to obtain physical insight from Electromagnetics in the frequency band of interest, we had better make explicit the different state-space variables that come up in place in order to get the impedance matrix transfer function of the two-port microwave circuit, namely, x_1 and x_2 , each of these arising whenever we are solving the two unitary problems needed to characterize the electromagnetic behavior in the microwave device for any excitation. As a result, the following linear dynamical system arises:

$$\begin{pmatrix} v_1 \\ v_2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & j b_1^T & j b_2^T \\ 0 & 0 & j b_2^T & j b_1^T \\ s j b_1 & s j b_2 & S + s^2 T & 0 \\ s j b_2 & s j b_1 & 0 & S + s^2 T \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ x_1 \\ x_2 \end{pmatrix}. \quad (3)$$

The reduced-order model that we propose draws upon using field solutions for each unitary problem at specific frequencies as basis functions to represent the evolution of the electromagnetic field as a function of frequency. Thus, we carry out Galerkin projection onto the reduced-basis space $\mathcal{W}^N = \text{span}\{\zeta_1 \equiv \begin{pmatrix} x_1(s_1) \\ x_2(s_1) \end{pmatrix}, \dots, \zeta_N \equiv \begin{pmatrix} x_1(s_N) \\ x_2(s_N) \end{pmatrix}\}$, where $s_1, \dots, s_N$ are complex frequencies in the frequency band of interest, to obtain the reduced-order linear dynamical system:

$$\begin{pmatrix} v_1 \\ v_2 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & j \tilde{w}_{11} & j \tilde{w}_{12} \\ 0 & 0 & j \tilde{w}_{21} & j \tilde{w}_{22} \\ s j \tilde{b} & 0 & \tilde{S} + s^2 \tilde{T} & 0 \\ 0 & s j \tilde{b} & 0 & \tilde{S} + s^2 \tilde{T} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix}. \quad (4)$$

In here, the solution vector $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ is approximated by a linear combination of the elements in the reduced-basis space $\mathcal{W}^N$, thus, $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \approx P \tilde{x}$, where $P = \text{cols}\{\zeta_1, \dots, \zeta_N\}$. As a result, $\tilde{S} = P^T \begin{pmatrix} S & 0 \\ 0 & S \end{pmatrix} P$, $\tilde{T} = P^T \begin{pmatrix} T & 0 \\ 0 & T \end{pmatrix} P$, $\tilde{b} = P^T \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, and

$$\begin{aligned} \tilde{w}_{11} &= b_1^T (x_1(s_1) \dots x_1(s_N)) \\ \tilde{w}_{12} &= b_1^T (x_2(s_1) \dots x_2(s_N)) \\ \tilde{w}_{21} &= b_2^T (x_1(s_1) \dots x_1(s_N)) \\ \tilde{w}_{22} &= b_2^T (x_2(s_1) \dots x_2(s_N)). \end{aligned} \quad (5)$$

We refer to [8] for a detailed explanation on the actual automatic adaptive selection of the frequency samples $s_1, \dots, s_N$ in the frequency band of interest while certifying the accuracy

of the reduced order model. As a result, a fully automatic and reliable model order reduction is carried out.

B. Circuit Theory Representation in Electromagnetics

Once this reduced-order model is obtained, which is a reliable surrogate of the original Maxwell problem in the frequency band of interest, further manipulations need to be carried out to get some physical insight in the Maxwell problem. In particular, the linear dynamical system (4) describes the physics in the microwave filter within a reduced-order state-space, namely, the reduced-basis space. The state-space variable $\tilde{x}$ stands for the actual state of the microwave filter with respect to the basis describing the reduced-basis space. In order to have further insight about the actual behavior of the microwave circuit, a suitable representation basis should be taken into account instead. We prefer the representation basis describing the local resonators in the microwave filter, rather than the basis obtained by means of a greedy algorithm as detailed in [8]. Once this more suitable representation basis is obtained, the reduced-order linear dynamical system (4) will automatically convert into a coupling matrix description [9] with the actual coupling topology among the local resonators in the microwave filter in Electromagnetics. This serves for electromagnetic design and diagnosis purposes since all electromagnetic phenomena are taking into account.

Most electromagnetic devices are designed based on coupled resonating structures, for instance, this is the case in antennas, microwave filters and multiplexers. In this setting, it is known that the input and output ports do not see the individual resonators in the microwave devices but their corresponding global resonances, namely, eigenresonances or eigenmodes. For example, cf. [11] in microwave filter theory. For the sake of convenient circuit understanding, it is then important to force this global eigenresonance representation basis in the Maxwell problem.

In the reduced-order linear dynamical system (4), an eigen-decomposition is carried out to write down the reduced-order electromagnetic problem in terms of global eigenresonances or eigenmodes. As a result of this change of basis, reduced-basis stiffness and mass matrices, $\tilde{S}$ and $\tilde{T}$, convert into diagonal matrices, giving rise to a linear dynamical system representation of the microwave filter which results into a transversal topology coupling matrix representation of the Electromagnetics in the microwave circuit. Further postprocessing can now be applied to change the representation basis of global eigenmodes into local resonators. As a result, the full-wave coupling matrix in the actual coupling topology among the local resonators in the microwave circuit is obtained.

III. NUMERICAL RESULTS

In this section, we apply the proposed reliable circuit model for electromagnetic diagnosis in a $N = 10$ order 1800MHz ceramic TM mode filter with two transmission zeros. See Fig. 1, where the intended coupling topology is highlighted. The purpose in here is to understand the actual electromagnetic coupling behavior among the different local

dielectric resonators in the structure. The 1.6-1.9 GHz band is taken into account, which gives rise to a reduced-order model of dimension 14, namely 14 eigenmodes are present. Fig. 2 details the S parameter response of this filter. Among these 14 eigenmodes described in the reduced-order model, $N = 10$ correspond to dominant global resonances (the order of the filter), whereas the remaining 4 stand for parasitics behavior in Electromagnetics. The latter will be taken into account as a direct source to load coupling in the coupling matrix representation. Once this technique is applied, the N-order coupling matrix (6) is obtained for this filter, which

reveals that all local resonators actually couple to each other in a greater or lesser extent. In the N-order coupling matrix (6), the intended coupling topology from circuit synthesis is highlighted. These additional couplings are the result of Electromagnetics. However, not only this technique is able to predict the existence of additional unwanted couplings, but also their actual location within the circuit layout and their corresponding coupling values. This information is extremely helpful not only for diagnosis but also in the design process. Having a reliable circuit representation of electromagnetic behavior allows a better understanding of the physics in microwave engineering.

$$\begin{pmatrix} \mathbf{1.0645} & \mathbf{0.0379} & -0.0001 & -0.0011 & -0.0005 & -0.0004 & 0.0009 & 0.0021 & -0.0004 & -0.0009 \\ \mathbf{0.0379} & \mathbf{1.1926} & \mathbf{0.0324} & 0.0005 & 0.0002 & -0.0006 & 0.0006 & 0.0001 & -0.0012 & -0.0013 \\ -0.0001 & \mathbf{0.0324} & \mathbf{1.1824} & \mathbf{-0.0207} & \mathbf{0.0034} & 0.0106 & -0.0096 & 0.0003 & -0.0105 & -0.0096 \\ -0.0011 & 0.0005 & \mathbf{-0.0207} & \mathbf{1.1781} & \mathbf{-0.0106} & 0.0162 & -0.0110 & -0.0050 & 0.0019 & 0.0036 \\ -0.0005 & 0.0002 & \mathbf{0.0034} & \mathbf{-0.0106} & \mathbf{1.1951} & \mathbf{-0.0286} & 0.0043 & -0.0015 & 0.0151 & 0.0148 \\ -0.0004 & -0.0006 & 0.0106 & 0.0162 & \mathbf{-0.0286} & \mathbf{1.2100} & \mathbf{0.0208} & \mathbf{0.0078} & 0.0026 & 0.0028 \\ 0.0009 & 0.0006 & -0.0096 & -0.0110 & 0.0043 & \mathbf{0.0208} & \mathbf{1.2124} & \mathbf{-0.0228} & -0.0017 & -0.0030 \\ 0.0021 & 0.0001 & 0.0003 & -0.0050 & -0.0015 & \mathbf{0.0078} & \mathbf{-0.0228} & \mathbf{1.2006} & \mathbf{0.0315} & 0.0014 \\ -0.0004 & -0.0012 & -0.0105 & 0.0019 & 0.0151 & 0.0026 & -0.0017 & \mathbf{0.0315} & \mathbf{1.1144} & \mathbf{0.0532} \\ -0.0009 & -0.0013 & -0.0096 & 0.0036 & 0.0148 & 0.0028 & -0.0030 & 0.0014 & \mathbf{0.0532} & \mathbf{1.1040} \end{pmatrix} \quad (6)$$

Fig. 1. 10th order 1800MHz ceramic TM mode filter. Intended coupling topology.

Fig. 2. 10th order 1800MHz ceramic TM mode filter response.

IV. CONCLUSION

A reduced-order model in FEM analysis has been detailed. Special emphasis has been placed in determining a circuit theory representation within the reduced-order model in FEM analysis. The proposed technique has been applied for electromagnetic diagnosis in microwave devices. A full-

wave coupling matrix representation, namely, a circuit theory representation of electromagnetic phenomena with the actual coupling topology among the local resonators has given rise to a better understanding for CAD in Electromagnetics.

This procedure represents the frequency behavior of the field solution as a linear combination of field solutions at properly chosen frequency points, which stand for a basis for the reduced-basis space. A proper change of basis has been carried to determine first, the eigenbasis which converts the reduced-order linear dynamical system into a transversal topology coupling matrix and, second, the basis which brings the actual coupling topology among the local resonators up in Electromagnetics.

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