An Approach to Static Performance Guarantees for Programs with Run-time Checks

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Abstract

Instrumenting programs for performing run-time checking of properties, such as regular shapes, is a common and useful technique that helps programmers detect incorrect program behaviors. This is specially true in dynamic languages such as Prolog. However, such run-time checks inevitably introduce run-time overhead (in execution time, memory, energy, etc.). Several approaches have been proposed for reducing such overhead, such as eliminating the checks that can statically be proved to always succeed, and/or optimizing the way in which the (remaining) checks are performed. However, there are cases in which it is not possible to remove all checks statically (e.g., open libraries which must check their interfaces, complex properties, unknown code, etc.) and in which, even after optimizations, these remaining checks still may introduce an unacceptable level of overhead. It is thus important for programmers to be able to determine the additional cost due to the run-time checks and compare it to some notion of admissible cost. The common practice used for estimating run-time checking overhead is profiling, which is not exhaustive by nature. Instead, we propose a method that uses static analysis to estimate such overhead, with the advantage that the estimations are functions parameterized by input data sizes. Unlike profiling, this approach can provide guarantees for all possible execution traces, and allows assessing how the overhead grows as the size of the input grows. Our method also extends an existing assertion verification framework to express “admissible” overheads, and statically and automatically checks whether the instrumented program conforms with such specifications. Finally, we present an experimental evaluation of our approach that suggests that our method is feasible and promising.

1 Introduction and Motivation

Dynamic programming languages are a popular programming tool for many applications, due to their flexibility. They are often the first choice for web programming, prototyping, and scripting. The lack of inherent mechanisms for ensuring program data manipulation correctness (e.g., via full static typing or other forms of full static built-in verification) has sparked the evolution of flexible solutions, including assertion-based approaches (Bueno et al.

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1997; Hermenegildo et al. 1999; Laï 2000; Pietrzak et al. 2006) in (constraint) logic languages, soft-
(Cartwright and Fagan 1991; Tobin-Hochstadt and Felleisen 2008) and gradual-
(Siek and Taha 2006) typing in functional languages, and contract-based approaches (Leavens et al. 2007; Lamport and Paulson 1999; Nguyen et al. 2014) in imperative languages. A
trait that many of these approaches share is that some parts of the specifications may be
the subject of run-time checking (e.g., those cannot be discharged statically in the case
of systems that support this functionality). However, such run-time checking comes at
the price of overhead during program execution, that can affect execution time, memory
use, energy consumption, etc., often in a significant way (Rastogi et al. 2015; Takikawa
et al. 2016). If these overheads become too high the whole program execution becomes
impractical and programmers may opt for sacrificing the checks to keep the required level
of performance.

Dealing with excessive run-time overhead is a challenging problem. Proposed approaches
in order to address this problem include discharging as many checks as possible via static
analysis (Puebla et al. 2000b; Puebla et al. 2000a; Fähndrich and Logozzo 2011; Hanus 2017;
Stulova et al. 2018), optimizing the dynamic checks themselves (Koukoutos and Kuncak
2014; Stulova et al. 2015; Ren and Foster 2016), or limiting run-time checking points (Mera
et al. 2009). Nevertheless, there are cases in which a number of checks cannot be optimized
away and must remain in place, because of software architecture choices (e.g., the case of
the external interfaces of reusable libraries or servers), the need to ensure a high level of
safety (e.g., in safety-critical systems), etc.

At the same time, run-time checks may not always be the culprit of low program
performance. A technique that can help in this context is profiling, often used to detect
performance “hot spots” and guide program optimization. Prior work on using profiling in
the context of optimizing the performance of programs with run-time checks (Furr et al.
2009; Mera et al. 2011; St-Amour et al. 2015) clearly demonstrates the benefits of this
approach. Still, profiling infers information that is valid only for some particular input data
values (and their execution traces). The profiling results thus obtained may not be valid
for other input data values. Since the technique is not exhaustive by nature, detecting the
worst cases can take a long time, and is impossible in general.

Our proposal is to use static cost analysis (Debray et al. 1990; Debray and Lin 1993;
Debray et al. 1997; Portillo et al. 2002; Albert et al. 2012; Serrano et al. 2014; Lopez-Garcia
et al. 2016) instead of (or as a complement to) dynamic profiling. Such analysis is aimed
at inferring statically safe upper and lower bounds on execution costs, i.e., bounds that
are guaranteed and will never be violated in actual executions. Since such costs are data-
dependent, these bounds take the form of functions that depend on certain characteristics
(generally, data sizes) of the inputs to the program. They show how the program costs
change as the size of the input grows. We propose a static cost analysis-based approach that
delivers guarantees on the costs introduced by the run-time checks in a program (run-time
checking overhead). Our method provides the programmer with feedback at compile-time
regarding the impact that run-time checking will have on the program costs. Furthermore,
we propose an assertion-based mechanism that allows programmers to specify bounds on
the admissible run-time checking overhead introduced in programs. The approach then
compares the inferred run-time checking overhead against the admissible one and provides
guarantees on whether such specifications are met or not. Such guarantees can be given
as constraints (e.g., intervals) on the size of the input data. We provide the formalization
of the method in the context of the Ciao assertion language and the CiaoPP verification
framework, and present also results from its implementation and experimental evaluation.
2 Assertions and Run-time Checking

Assertion Language Assertions are linguistic constructions that allow expressing properties of programs. We recall the pred assertions of the Ciao assertion language (Hermenegildo et al. 1999; Puebla et al. 2000b; Hermenegildo et al. 2012), following the presentation of (Stulova et al. 2018). Such pred assertions allow defining the set of all admissible preconditions for a given predicate, and for each such pre-condition a corresponding post-condition. These pre- and post-conditions are formulas containing literals defined by predicates specially labeled as properties, to which we refer to as prop literals. A set of assertions for a predicate (identified by a normalized atom Head) is:

\[
\text{pre} \quad \text{post} \quad \text{prop} \quad \text{lit} \\
\begin{align*}
\text{Pre}_i \&= \text{Post}_i \\
\text{Pre}_n \&= \text{Post}_n
\end{align*}
\]

The Pre and Post fields are logic formulas (e.g., conjunctions) of prop literals that refer to the variables of Head. Informally, such a set of assertions states that in any execution state immediately before the call to Head at least one of the Pre conditions should hold, and that, given the (Pre, Post) pair(s) where Pre holds, then, if the predicate succeeds, the corresponding Post should hold upon its success.

Example 1 (Program with Assertions)
Consider the following implementation of a predicate for reversing a list and its assertions:

\[
\begin{align*}
\text{check pred rev(X,Y) : (list(X), var(Y)) => (list(X), list(Y)).} \\
\text{rev([], []).} \\
\text{rev([X|Xs], Y) : rev(Xs, Ys), app(Ys, X, Y).} \\
\text{check pred app(Y,X,Z) : (list(Y), term(X), var(Z)) => (list(Y), term(X), list(Z)).} \\
\text{app([],X,[X]).} \\
\text{app([E|Y],X,[E11]): app(Y,X,T).}
\end{align*}
\]

Assertion A1 states that if rev/2 is called with a list X and a free variable Y, on its success Y will also be a list. Assertion A2 says if app/3 is called with a list Y, a term X, and a free variable Z, on success Z will be a list. The algorithmic complexity of rev/2 is \(O(N^2)\) in the size (list length in this case) \(N\) of its input argument \(X\). While this implementation is obviously not optimal, we use it as a representative of the frequent case of nested loops with linear costs.

Every assertion also has a Status field which indicates whether the assertion refers to intended or actual properties. Programmer-provided assertions by default have status check, and only assertions with this status generate run-time checks. Static analysis can prove or disprove properties in assertions for a given class of input queries, statically verifying assertions (if all the prop literals proved to be true, in which case their status is changed to checked) or flagging errors (if any prop literal is proved to be false, and then the status is changed to false). Assertions can also be simplified by eliminating the prop literals proved to be true, so that only the remaining ones need to be checked. Other information inferred by static analysis is communicated by means of true assertions (e.g., see Ex. 5).

Example 2 (Assertions Processed by Static Analysis)
Consider the following implementation of a predicate for reversing a list and its assertions:

\[
\begin{align*}
\text{check calls rev(X,Y) : (list(X),var(Y)).} \\
\text{check pred rev(X,Y) : (list(X),var(Y)) => (list(X),list(Y)).} \\
\text{check pred app(Y,X,Z) : (list(Y),term(X),var(Z)) => (list(Y),term(X),list(Z)).} \\
\text{calls app(X,Y,Z).}
\end{align*}
\]

Result of static assertion checking for Example 1. We assume that the code is in a module, exporting only rev/2, and that it is analyzed in isolation, i.e., we have no information on the callers to rev/2. The interface assertion (calls) for the rev/2 predicate remains active.
and generates run-time checks (calls into the module are sanitized). This contrasts with the situation in Example 1, where all assertions generate run-time checks.

Run-time Check Instrumentation

We recall the definitional source transformation of (Stulova et al. 2015), that introduces wrapper predicates that check calls and success assertions, and also groups all assertions for the same predicate together to produce optimized checks. Given a program, for every predicate \( p \) the transformation replaces all clauses \( p(\bar{x}) \leftarrow \text{body} \) by \( p'(\bar{x}) \leftarrow \text{body} \), where \( p' \) is a new predicate symbol, and inserts the wrapper clauses given by \( \text{wrap}(p(\bar{x}), p') \):

\[
\text{wrap}(p(\bar{x}), p') = \begin{cases} 
  p(\bar{x}) & : \text{not ChecksC}. \\
  p_c(\bar{x}, r), p'(\bar{x}), p_s(\bar{x}, r). & \text{ChecksC}. \\
  p_s(\bar{x}, r) & : \text{ChecksS}. 
\end{cases}
\]

Here ChecksC and ChecksS are the optimized compilation of pre- and postconditions \( \bigwedge_{i=1}^{n} \text{Pre}_i \) and \( \bigwedge_{i=1}^{n} (\text{Pre}_i \rightarrow \text{Post}_i) \) respectively; and the additional status variables \( r \) are used to communicate the results of each \( \text{Pre}_i \) evaluation to the corresponding \( (\text{Pre}_i \rightarrow \text{Post}_i) \) check, thus avoiding double evaluation of preconditions.

The compilation of checks for assertions emits a series of calls to a \text{reify_check}(P, \text{Res}) \) predicate, which accepts as the first argument a property \( P \) and unifies \( \text{Res} \) with 1 or 0, depending on whether the property check succeeds or not. The results of those reified checks are then combined and evaluated as boolean algebra expressions using bitwise operations and the Prolog \text{is}/2 \) predicate. That is, the logical operators \( (A \lor B) \), \( (A \land B) \), and \( (A \rightarrow B) \) used in encoding assertions are replaced by their bitwise logic counterparts \( R \) is \( A \lor B \), \( R \) is \( A \land B \), \( R \) is \( (A \# 1) \lor B \), respectively.

Example 3 (Run-time Checks (a))

The program transformation that introduces the run-time checking harness for the program fragment from Example 1 (assuming none of the assertions has been statically discharged by analysis) is essentially as follows:

\begin{verbatim}
rev(A,B) app(A,B,C) :-
  revC(A,B,C),
  rev'(A,B),
  revS(A,B,C).
revC(A,B,E) :-
  reify_check(list(A),C),
  reify_check(var(B), D),
  E is C\(\lor D\),
  warn_if_false(E,'calls').
revS(A,B,E) appS(A,B,C,G)
  reify_check(list(A),C),
  reify_check(list(B),D),
  F is C\(\lor D\),
  G is D\(\lor (E\(\land F\))\),
  warn_if_false(G,'success').
rewrC([],[]).
rewrC([X|Xs],Y) :-
  rewC(Xs,Ys),
  app(Ys,X,Y).
rewrS([],[]).
rewrS([X|Xs],Y) :-
  rewS(Xs,Ys),
  app(Ys,X,Y).
\end{verbatim}

The \text{warn_if_false}/2 \) predicates raise run-time errors terminating program execution if their first argument is 0, and succeed (with constant cost) otherwise. We will refer to this case as to the worst performance case in programs with run-time checking.
Example 4 (Run-time Checks (b))

```
rev(A,B)
  revC(A,B),
  rev'(A,B).
revC(A,B) :-
  reify_check(list(A),C),
  reify_check(var(A), D),
  E is C\{D,
  warn_if_false(E,'calls').
rev'(A,B) :- rev_i(A,B).
rev_i([],[]).
rev_i([X|Xs],Y) :-
  rev_i(Xs,Ys),app(Ys,X,Y).
```

This example represents the run-time checking generated for the scenario of Example 2, i.e., after applying static analysis to simplify the assertions. Run-time checks are generated only for the interface calls of the `rev/2` predicate. Note that `rev'/2` here is a point separating calls to `rev/2` coming outside the module from the internal calls (now made through `rev_i/2`). Note also that `app/3` is called directly (i.e., with no run-time checks). Clearly in this case there are fewer checks in the code and thus smaller overhead. We will refer to this case, where only interface checks remain, as the base performance case.

3 Static Cost Analysis

Static cost analysis automatically infers information about the resources that will be used by program executions, without actually running the program with concrete data. Unlike profiling, static analysis can provide guarantees (upper and lower bounds) on the resource usage of all possible execution traces, given as functions on input data sizes. In this paper we use the CiaoPP general cost analysis framework (Debray et al. 1990; Debray et al. 1997; Navas et al. 2007; Serrano et al. 2014), which is parametric with respect to resources, programming languages, and other aspects related to cost. It can be easily customized/instantiated by the user to infer a wide range of resources (Navas et al. 2007), including resolution steps, execution time, energy consumption, number of calls to a particular predicate, bits sent/received by an application over a socket, etc.

In order to perform such customization/instantiation, the Ciao assertion language is used (Navas et al. 2007; Puebla et al. 2000b; Hermenegildo et al. 2012). For cost analysis it allows defining different resources and how basic components of a program (and library predicates) affect their use. Such assertions constitute the cost model. This model is taken (trusted) by the static analysis engine, that propagates it during an abstract interpretation of the program (Serrano et al. 2014) through code segments, conditionals, loops, recursions, etc., mimicking the actual execution of the program with symbolic “abstract” data instead of concrete data. The engine is fully based on abstract interpretation, and defines the resource analysis itself as an abstract domain that is integrated into the PLAI abstract interpretation framework (Muthukumar and Hermenegildo 1992) of CiaoPP.

The engine infers cost functions (polynomial, exponential, logarithmic, etc.) for higher-level entities, such as procedures in the program. Such functions provide upper and lower bounds on resource usage that depend on input data sizes and possibly other (hardware) parameters that affect the particular resource. Typical size metrics include actual values of numbers, lengths of lists, term sizes (number of constant and function symbols), etc. (Navas et al. 2007; Serrano et al. 2014). The analysis of recursive procedures sets up recurrence equations (cost relations), which are solved (possibly safely approximated), obtaining upper- and lower-bound (closed form) cost functions. The setting up and solving of recurrence relations for inferring closed-form functions representing bounds on the sizes of output arguments and the resource usage of the predicates in the program are integrated into the PLAI framework as an abstract operation.
Example 5 (Static Cost Analysis Result)

The following assertion is part of the output of the resource usage analysis performed by CiaoPP for the \texttt{rev/2} predicate from Example 1:

\begin{verbatim}
:- true pred rev(X,Y) : (list(X),var(Y),length(X,L)) => (list(X),list(Y), length(X,L), length(Y,L)) + cost(exact(0.5*(L)**2+1.5*L+1), [steps]).
\end{verbatim}

It includes, in addition to the precondition (\texttt{:Pre}) and postcondition (\texttt{=>Post}) fields, a field for computational properties (\texttt{+Comp}), in this case \texttt{cost}. The assertion uses the \texttt{cost/2} property for expressing the \texttt{exact} cost (first argument of the property) in terms of resolution \texttt{steps} (second argument) of any call to \texttt{rev(X,Y)} with the \texttt{X} bound to a list and \texttt{Y} a free variable. Such cost is given by the function \(0.5L^2+1.5L+1\), which depends on \(L\), i.e., the length of the (input) argument \(X\), and is the argument of the \texttt{exact/1} qualifier. It means that such function is both a lower and an upper bound on the cost of the specified call. This aspect of the assertion language (including the \texttt{cost/2} property) and our proposed extensions are discussed in Section 4.

4 Analyzing and Verifying the Run-time Checking Overhead

Our approach to analysis and verification of run-time checking overhead consists of three basic components: using static cost analysis to infer upper and lower bounds on the cost of the program with and without the run-time checks; providing the programmer with a means for specifying the amount of overhead that is admissible; and comparing the inferred bounds to these specifications. The following three sections outline these components.

4.1 Computing the run-time checking overhead (Ovhd)

The first step of our approach is to infer upper and lower bounds on the cost of the program with and without the run-time checks, using cost analysis. The inference of the bounds for the program without run-time checks was illustrated in Example 5. The following two examples illustrate the inference of bounds for the program with the run-time checks. They cover the two scenarios discussed previously, i.e., with and without the use of static analysis to remove run-time checks.

Example 6 (Cost of Program with Run-time Checks (a))

The following is the result of cost analysis for the run-time checking harness of Example 3 for the \texttt{rev/2} predicate, together with a (stylized) version of the code analyzed, for reference. Note the jump in the execution cost of \texttt{rev/2} from quadratic to cubic in the size \(L\) of the input (the list length of \(A\)), which is most likely not admissible:

\begin{verbatim}
:- true pred rev(A,B) : (list(A),var(B)) => (list(A),list(B),length(A,L)) + cost(exact(0.5*L**3+7*L**2+14.5*L+8), [steps]).
revC(A,B,C) :- list(A),var(B),bit.ops. revS(A,B,C) :- list(A),list(B),bit.ops.
rev'([],[]). rev'([X|Xs],Y) :- rev(Xs,Ys),app(Ys,X,Y).
\end{verbatim}
Example 7 (Cost of Program with Run-time Checks (b))

true pred rev(A,B) : (list(A), var(B)) => (length(A,L))
+ cost(exact(0.5*L**2+2.5*L+7), [steps]).

rev(A,B) :- revC(A,B), rev'(A,B).
revC(A,B) :- list(A), var(B), bit_ops.

rev'(A,B) :- rev_i(A,B).

rev_i([],[]).
rev_i([X|Xs],Y) :- rev_i(Xs,Ys), app(Ys,X,Y).

This example shows the result of cost analysis for the base instrumentation case of Example 4: although there are still some run-time checks present for the interface, the overall cost of the rev/2 predicate remains quadratic, which is probably admissible.

4.2 Expressing the Admissible Run-time Checking Overhead (AOvhd)

We add now to our approach the possibility of expressing the admissible run-time checking overhead (AOvhd). This is done by means of an extension to the Ciao assertion language. As mentioned before, this language already allows expressing a wide range of properties, and this includes the properties related to resource usage. For example in order to tell the system to check whether an upper bound on the cost, in terms of number of resolution steps, of a call p(A, B) with A instantiated to a natural number and B a free variable, is a function in $O(A)$, we can write the following assertion:

:- check pred p(A, B): (nat(A), var(B)) + cost(o_ub(A), steps, std)).

The first argument of the cost/2 property is a cost function, which in turn appears as the argument of a qualifier expressing the kind of approximation. In this case, the qualifier o_ub/1 represents the complexity order of an upper bound function (i.e., the “big O”). Other qualifiers include ub/1 (an upper-bound cost function, not just a complexity order), lb/1 (a lower-bound cost function), and band/2 (a cost band given by both a lower and upper bound). The second argument of the cost/2 property is a list of qualifiers (identifiers). The first identifier expresses the resource, i.e., the cost metric used. The value steps represents the number of resolution steps. The second argument expresses the particular kind of cost used. The value std represents the standard cost (the value by default if it is omitted), the value acc the accumulated cost (Lopez-Garcia et al. 2016), etc.

The language also allows writing assertions that are universally quantified over the predicate domain (i.e., that are applicable to all calls to all predicates in a program), which is particularly useful in our application. An issue that appears in this context is that different predicates can have different numbers and types of arguments. To solve this problem we introduce a way to express complexity orders without requiring the specification of details about the arguments on which cost functions depend nor the size metric used, by means of identifiers without arguments, such as constant, linear, quadratic, exponential, logarithmic, etc. For example, in order to extend the previous assertion to all possible predicate calls in a program (independently of the number and type of arguments), we can write:

:- check pred * + cost(so_ub(linear), [steps]).

Alternatively, we also introduce complexity order expressions that do not specify the relation with the arguments of the predicates, i.e., we allow the use of free variables in the expressions:
In the context of the previous extensions, our objective is expressing and specifying limits on how the complexity/cost changes when run-time checks are performed, i.e., expressing and specifying limits on the run-time checking overhead. To this end we propose different ways to quantify this overhead. Let \( C_p(n) \) represent the standard cost function of predicate \( p \) without any run-time checks and \( C_{p,rtc}(n) \) the cost function for the transformed/instrumented version of \( p \) that performs run-time checks, \( p_{rtc} \). A good indicator of the relative overhead is the ratio:

\[
\frac{C_{p,rtc}(n)}{C_p(n)}
\]

We introduce the qualifier \texttt{rtc\_ratio} to express this type of ratios. For example, the assertion:

\[
\begin{align*}
\text{:- check pred } p(A, B) : (\text{nat}(A), \text{var}(B)) \\
\quad + \text{ cost}(\text{so\_ub}(\text{linear}), \text{steps}, \text{rtc\_ratio})
\end{align*}
\]

expresses that \( p/2 \) should be called with the first argument bound to a natural number and the second one a variable, and the relative overhead introduced by run-time checking in the calls to \( p/2 \) (the ratio between the cost of with and without run-time checks) should be at most a linear function. Similarly, using the universal quantification over predicates, the following assertion:

\[
\text{:- check pred } + \text{ cost}(\text{so\_ub}(\text{linear}), \text{steps}, \text{rtc\_ratio})
\]

expresses that, for all predicates in the program, the ratio between the cost of with and without run-time checks should be at most a linear function.

\subsection*{4.3 Verifying the Admissible Run-time Checking Overhead (A\text{Ovhd})}

We now turn to the third component of our approach: verifying the admissible run-time checking overhead (A\text{Ovhd}). To this end, we leverage the general framework for resource usage analysis and verification of (López-García et al. 2010; Lopez-Garcia et al. 2012), and adapt it for our purposes, using the assertions introduced in Section 4.2. The verification process compares the (approximated) intended semantics of a program (i.e., the specification) with approximated semantics inferred by static analysis. These operations include the comparison of arithmetic functions (e.g., polynomial, exponential, or logarithmic functions) that may come from the specifications or from the analysis results. The possible outcomes of this process are the following:

1. The status of the original (specification) assertion (i.e., \texttt{check}) is changed to \texttt{checked} (resp. \texttt{false}), meaning that the assertion is correct (resp. incorrect) for all input data meeting the precondition of the assertion,
2. the assertion is “split” into two or three assertions with different status (\texttt{checked}, \texttt{false}, or \texttt{check}) whose preconditions include a conjunct expressing that the size of the input data belongs to the interval(s) for which the assertion is correct (status \texttt{checked}), incorrect (status \texttt{false}), or the tool is not able to determine whether the assertion is correct or incorrect (status \texttt{check}), or
3. in the worst case, the assertion remains with status \texttt{check}, meaning that the tool is not able to prove nor to disprove (any part of) it.
In our case, the specifications express a band for the \( \text{AOvhd} \), defined by a lower- and an upper-bound cost function (or complexity orders). If the lower (resp. upper) bound is omitted, then the lower (resp. upper) limit of the band is assumed to be zero (resp. \( \infty \)).

This implies that we need to perform some adaptations with respect to the verification of resource usage specifications for predicates described in (López-García et al. 2010; Lopez-Garcia et al. 2012). Assume for example that the user wants the system to check the following assertion:

\[
\text{\texttt{p(A, B)} : (nat(A), var(B)) + cost(ub(2*A), [steps, rtc\_ratio])}
\]

which expresses that the ratio defined in Section 4.2 (with \( n = A \)) \( \frac{C_{\text{p,rtc}}(n)}{C_{\text{p}}(n)} \) must be in the band \([0, 2*A]\) for a given predicate \( p \). The approach in (López-García et al. 2010; Lopez-Garcia et al. 2012) uses static analysis to infer both lower and upper bounds on \( C_{\text{p}}(n) \), denoted \( C_{l}^{p}(n) \) and \( C_{u}^{p}(n) \) respectively. In addition, in our application, the static analysis needs to infer, both lower and upper bounds on \( C_{\text{p,rtc}}(n) \), denoted \( C_{l}^{p,rtc}(n) \) and \( C_{u}^{p,rtc}(n) \), and use all of these bounds to compute bounds on the ratio. A lower (resp. upper) bound on the ratio is given by \( \frac{C_{l}^{p,rtc}(n)}{C_{l}^{p}(n)} \) (resp. \( \frac{C_{u}^{p,rtc}(n)}{C_{l}^{p}(n)} \)). Both bounds define an inferred (safely approximated) band for the actual ratio, which is compared with the (intended) ratio given in the specification (the band \([0, 2*A]\)) to produce the verification outcome as explained above.

### 4.4 Using the accumulated cost for detecting hot spots

So far, we have used the standard notion of cost in the examples for simplicity. However, in our approach we also use the accumulated cost (Lopez-García et al. 2016), inferred by CiaoPP, to detect which of the run-time check predicates (properties) have a higher impact on the overall run-time checking overhead, and are thus promising targets for optimization. Given space restrictions we provide just the main idea and an example. The accumulated cost is based on the notion of cost centers, which in our approach are predicates to which execution costs are assigned during the execution of a program. The programmer can declare which predicates will be cost centers. Consider again a predicate \( p \), and its instrumented version \( p_{\text{rtc}} \) that performs run-time checks, and let \( C_{p}^{\text{p,rtc}}(n) \) and \( C_{p,rtc}^{p}(n) \) be their corresponding standard cost functions. Let \( \text{ck} \) represent a run-time check predicate (e.g., \texttt{list/1}, \texttt{num/1}, \texttt{var/1}, etc.). Let \( \Omega_{p,rtc} \) be the set of run-time check predicates used by \( p_{\text{rtc}} \). Assume that we declare that the set of cost centers to be used by the analysis, \( \Omega \), is \( \Omega_{p,rtc} \cup \{p_{\text{rtc}}\} \). In this case, the cost of a (single) call to \( p_{\text{rtc}} \) accumulated in cost center \( \text{ck} \), denoted \( C_{p,rtc,ck}^{\text{p,rtc}}(n) \), expresses how much of the standard cost \( C_{p,rtc}(n) \) is attributed to run-time check \( \text{ck} \) predicate (taking into account all the generated calls to \( \text{ck} \)). The \( \text{ck} \) predicate with the highest \( C_{p,rtc,ck}^{\text{p,rtc}}(n) \) is a hot spot, and thus, its optimization can be more profitable to reduce the overall run-time checking overhead. The predicate \( \text{ck} \) with the highest \( C_{p,rtc,ck}^{\text{p,rtc}}(n) \) is not necessarily the most costly by itself, i.e., the one with the highest standard cost. For example, a high \( C_{p,rtc,ck}^{\text{p,rtc}}(n) \) can be caused because \( \text{ck} \) is called very often. We create a ranking of run-time check predicates according to their accumulated cost. This can help in deciding which assertions and properties to simplify/optimize first to meet an overhead target.

Since \( p_{\text{rtc}} \) is declared as a cost center, the overall, absolute run-time checking overhead can be computed as \( \sum_{\text{ck} \in \Omega_{p,rtc}} C_{p,rtc,ck}^{\text{p,rtc}}(n) \). In addition \( C_{p,rtc}(n) = \sum_{q \in \Omega} C_{p,rtc}^{q}(n) \), and
app(A,B,_) | list concatenation
cons(E,L,_) | insertion into an ordered list
mmtx(A,B,_) | matrix multiplication
nrev(L,_) | list reversal
ldiff(A,B,_) | 2 lists difference
sift(A,_) | sieve of Eratosthenes
pfxsum(A,_) | sum of prefixes of a list of numbers
bsts(N,T) | membership checks in a binary search tree

Table 1. Description of the benchmarks.

\[ C_p(\bar{n}) = C_{p_{rtc}}^{\mathbb{R}^{rtc}}(\bar{n}). \] Thus, we only need to infer accumulated costs and combine them to both detect hot spots and compute the \textit{rtc ratio} described in Section 4.2.

**Example 8 (Detecting hot spots)**

Let \texttt{app rtc/3} denote the instrumented version for run-time checking of predicate \texttt{app/3} in Example 1. The following table shows the cost centers automatically declared by the system, which are the predicate \texttt{app rtc/3} itself and the run-time checking properties it uses (first column), as well as the accumulated costs of a call to \texttt{app rtc(A,B,_)} in each of those cost centers, where \( l_X \) represents the length of list \( X \) (second column):

<table>
<thead>
<tr>
<th>Cost center ((ck))</th>
<th>( C_{\texttt{app rtc}}(l_A,l_B) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{app rtc/3}</td>
<td>( l_A + 1 )</td>
</tr>
<tr>
<td>\texttt{list/1}</td>
<td>( 3 \times (l_A - 1)^2 + 6 \times (l_A + 1) \times (l_B + 1) + 8 \times (l_A + 1) - 12 )</td>
</tr>
<tr>
<td>\texttt{var/1}</td>
<td>( l_A + 1 )</td>
</tr>
<tr>
<td>\texttt{bit ops/1}</td>
<td>( 3 \times (l_A + 1) )</td>
</tr>
</tbody>
</table>

It is clear that the hot spot is the \texttt{list/1} property, which is the responsible of the change in complexity order of the instrumented version \texttt{app rtc/3} from linear to quadratic.

## 5 Implementation and Experimental Evaluation

We have implemented a prototype of our approach within the Ciao system, using CiaoPP’s abstract interpretation-based resource usage analysis and its combined static and dynamic verification framework. Table 1 contains a list of the benchmarks that we have used in our experiments.\(^1\) Each benchmark has assertions with properties related to shapes, instantiation state, variable freeness, and variable sharing, as well as in some cases more complex properties such as, for examples, sortedness. The benchmarks and assertions were chosen to be simple enough to have easily understandable costs but at the same time produce interesting cost functions and overhead ratios.

As stated throughout the paper, our objective is to exploit static cost analysis to obtain guarantees on program performance and detect cases where adding run-time checks introduces overhead that is not admissible. To this end, we have considered the code instrumentation scenarios discussed previously, i.e. (cf. Examples 3 and 4):

<table>
<thead>
<tr>
<th>performance</th>
<th>static assertion checking</th>
<th>run-time checking instrumentation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original</td>
<td>no</td>
<td>no {off}</td>
</tr>
<tr>
<td>Worst</td>
<td>no</td>
<td>yes {full}</td>
</tr>
<tr>
<td>Base</td>
<td>\texttt{shif + eterns}</td>
<td>yes {opt}</td>
</tr>
</tbody>
</table>

and we have performed for each benchmark and each scenario run-time checking overhead analysis and verification, following the proposed approach. The optimization in the \texttt{opt}

\(^1\)Sources available at \url{http://cliplab.org/papers/rtchecks-cost/}.
Table 2. Experimental results (benchmarks for which analysis infers exact cost functions).

<table>
<thead>
<tr>
<th>Bench</th>
<th>RTC</th>
<th>Bound Inferred</th>
<th>%DT(\text{A}) (ms)</th>
<th>Ovhd</th>
<th>Verif.</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{app}(A,_B,_L)</td>
<td>off</td>
<td>(l_A + 1)</td>
<td>0.0</td>
<td>98.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>(l_A^2 + 6 \cdot l_A \cdot l_B + 17 \cdot l_A + 6 \cdot l_B + 8)</td>
<td>0.0</td>
<td>521.18</td>
<td>(l_A + l_B) false</td>
</tr>
<tr>
<td></td>
<td>opt</td>
<td>(3 \cdot l_A + 2 \cdot l_B + 8)</td>
<td>0.0</td>
<td>311.98</td>
<td>(l_A + l_B) false</td>
</tr>
<tr>
<td>\text{rev}(_L,_A)</td>
<td>off</td>
<td>(\frac{1}{2} \cdot l_L^2 + \frac{5}{2} \cdot l_L + 1)</td>
<td>0.0</td>
<td>218.15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>(\frac{1}{2} \cdot l_L^3 + 7 \cdot l_L^2 + \frac{35}{4} \cdot l_L + 8)</td>
<td>0.0</td>
<td>885.08</td>
<td>(l_L) false</td>
</tr>
<tr>
<td></td>
<td>opt</td>
<td>(\frac{1}{2} \cdot l_L^2 + \frac{5}{2} \cdot l_L + 7)</td>
<td>0.0</td>
<td>756.82</td>
<td>1 checked</td>
</tr>
<tr>
<td>\text{shif}(_A,_L)</td>
<td>off</td>
<td>(l_A + 1)</td>
<td>0.0</td>
<td>255.55</td>
<td></td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>(\frac{1}{2} \cdot l_A^3 + \frac{15}{8} \cdot l_A^2 + \frac{25}{6} \cdot l_A + 7)</td>
<td>0.0</td>
<td>980.63</td>
<td>(l_A) false</td>
</tr>
<tr>
<td></td>
<td>opt</td>
<td>(\frac{1}{2} \cdot l_A^2 + \frac{5}{2} \cdot l_A + 7)</td>
<td>0.0</td>
<td>521.65</td>
<td>1 checked</td>
</tr>
<tr>
<td>\text{pfxsum}(_A,_L)</td>
<td>off</td>
<td>(l_A + 2)</td>
<td>0.0</td>
<td>146.98</td>
<td></td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>(2 \cdot l_A^2 + 12 \cdot l_A + 14)</td>
<td>0.0</td>
<td>749.94</td>
<td>(l_A) false</td>
</tr>
<tr>
<td></td>
<td>opt</td>
<td>(3 \cdot l_A + 10)</td>
<td>0.0</td>
<td>469.71</td>
<td>1 checked</td>
</tr>
</tbody>
</table>

Case has been performed using the \textit{eterms+shfr} domains. The resource inferred in these experiments is the number of resolution steps (i.e., each clause body is assumed to have unitary cost). The experiments were performed on a MacBook Pro with 2.5GHz Intel Core i5 CPU, 10 GB 1333 MHz DDR3 memory, running macOS Sierra 10.2.6.

Tables 2 and 3 show the results that our prototype obtains for the different benchmarks. In Table 2 we group the benchmarks for which the analysis is able to infer the exact cost function, while in Table 3 we have the benchmarks for which the analysis infers a safe upper-bound of their actual resource consumption. The analysis also infers lower bounds, but we do not show them and concentrate instead on the upper bounds for conciseness. Note that in those cases where the analysis infers exact bounds (Table 2), the inferred lower and upper bounds are of course the same. Column \textbf{Bench.} shows the name of the entry predicate for each benchmark. Column \textbf{RTC} indicates the scenario, as defined before, i.e., no run-time checks (off); full run-time checks (full); or only those left after optimizing via static verification (opt).

Column \textbf{Bound Inferred} shows the resource usage functions inferred by our resource analysis, for each of the cases. These functions depend on the input data sizes of the entry predicate (as before, \(l_X\) represents the length of list \(X\)). In order to measure the precision of the functions inferred, in Column \%\textbf{DT} we show the average deviation of the bounds obtained by evaluating the functions in Column \textbf{Bound Inferred}, with respect to the costs measured with dynamic profiling. The input data for dynamic profiling was selected to exhibit worst case executions. In those cases where the inferred bounds is exact, the deviation is always 0.0%. In Column \textbf{Ovhd} we show the relative run-time checking overhead as the ratio \(\text{rtc ratio}\) between the complexity order of the cost of the instrumented code (for full or opt), and the complexity order of the cost corresponding to the original code (off). Finally, in Column \textbf{T}\(\text{A}(\text{ms})\) we list the \textit{cost} analysis time for each of the three cases.\(^2\)

From the results shown in Column \textbf{Ovhd} we see that the analysis correctly detects that

\(^2\) This time does not include the static analysis and verification time in the \textit{opt} case, performed with the \textit{eterms+shfr} domains, since the process of simplifying at compile-time the assertions is orthogonal to this paper. Recent experiments and results on this topic can be found in (Stulova et al. 2018).
Table 3. Experimental results (rest of the benchmarks; we show the upper bounds).

<table>
<thead>
<tr>
<th>Bench</th>
<th>RTC</th>
<th>Bound Inferred</th>
<th>%DT_A(ms)</th>
<th>Ovhd</th>
<th>Verif.</th>
</tr>
</thead>
<tbody>
<tr>
<td>oins(E,L,...)</td>
<td>off</td>
<td>( l_L + 2 )</td>
<td>0.09</td>
<td>142.55</td>
<td>1 checked</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>( \frac{1}{2} \cdot l_L + \frac{3}{2} \cdot l_L^2 - \frac{3}{2} \cdot l_L + \frac{1}{2} )</td>
<td>99.95</td>
<td>917.33</td>
<td>( l_L^2 ) false</td>
</tr>
<tr>
<td></td>
<td>opt*</td>
<td>( \frac{1}{2} \cdot l_L + 6 )</td>
<td>50.14</td>
<td>340.15</td>
<td>1 checked</td>
</tr>
<tr>
<td>mtx(A,B,...)</td>
<td>off</td>
<td>( r_A \cdot c_A \cdot c_B + 3 \cdot r_A \cdot c_B + 2 \cdot r_A - 2 \cdot c_B )</td>
<td>7.58</td>
<td>460.21</td>
<td>1 checked</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>( 4 \cdot r_A^2 \cdot c_A + 4 \cdot r_A \cdot c_B + 4 \cdot r_A \cdot c_A \cdot c_B + 4 \cdot r_A \cdot c_A \cdot c_B + 2 \cdot r_A \cdot c_A \cdot c_B + 11 \cdot r_A \cdot c_A \cdot c_B + 20 \cdot r_A \cdot c_A + 15 \cdot r_A + 7 )</td>
<td>0.0</td>
<td>1082.54</td>
<td>( N^1 ) false</td>
</tr>
<tr>
<td></td>
<td>opt*</td>
<td>( r_A \cdot c_A \cdot c_B + 2 \cdot c_A \cdot c_B + 2 \cdot r_A \cdot c_A + 4 \cdot r_A \cdot c_A + 6 \cdot r_A + 2 \cdot c_A + 11 )</td>
<td>0.0</td>
<td>1120.23</td>
<td>1 checked</td>
</tr>
<tr>
<td>ldiff(A,B,...)</td>
<td>off</td>
<td>( l_A \cdot l_B + 2 \cdot l_A + 1 )</td>
<td>2.06</td>
<td>786.22</td>
<td>1 checked</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>( l_A^2 + 3 \cdot l_A \cdot l_B + 10 \cdot l_A + 2 \cdot l_B + 7 )</td>
<td>0.27</td>
<td>1769.22</td>
<td>( l_B^1 ) false</td>
</tr>
<tr>
<td></td>
<td>opt*</td>
<td>( l_A \cdot l_B + 5 \cdot l_A + 2 \cdot l_B + 8 )</td>
<td>0.0</td>
<td>1226.15</td>
<td>1 checked</td>
</tr>
<tr>
<td>bsts(U,T)</td>
<td>off</td>
<td>( d_T + 3 )</td>
<td>0.1</td>
<td>714.83</td>
<td>( d_T^1 ) false</td>
</tr>
<tr>
<td></td>
<td>full</td>
<td>( 2 \cdot d_T + 2 )</td>
<td>1.19</td>
<td>438.72</td>
<td>( d_T^1 ) false</td>
</tr>
<tr>
<td></td>
<td>opt*</td>
<td>( 2 \cdot d_T + 4 \cdot d_T + 14 )</td>
<td>4.01</td>
<td>245.00</td>
<td>( d_T^1 ) false</td>
</tr>
</tbody>
</table>

\[ N = \max(r_A, c_A, c_B) \]

the full run-time checking versions of the benchmarks (full case) are asymptotically worse than the original program, showing for example a quadratic asymptotic ratio (run-time checking overhead) for oins/3, or even exponential for bsts/2. In the case of app/3, we can see that the asymptotic relative overhead is linear, but the instrumented versions become dependent on the size of both arguments, while originally the cost was only depending on the size of the first list (though probably it is worth it since the list check on the second argument should be performed anyway). On the other hand, for all the benchmarks, except for app/3 and bsts/2, the resulting asymptotic relative overhead of the optimized run-time checking version (opt case), is null, i.e., \( Ovhd = 1 \).

In the case of bsts/2, the overhead is still exponential because the type analysis is not able to statically prove the property binary search tree. Thus, it is still necessary to traverse the input binary tree at run-time in order to verify it. However, the optimized version traverses the input tree only once, while the full version traverses it on each call, which is reflected in the resulting cost function. In any case, note that the exponential functions are on the depth of the tree \( d_T \), not on the number of nodes. Analogously, in oins/3 the static analysis is not able to prove the sorted property for the input list, although in that case the complexity order does not change for the optimized version, only increasing the constant coefficients of the cost functions. We have included optimized versions of these two cases (marking them with *) to show the change in the overhead if the properties involved were verified; however, the eterms+shfr domains used cannot prove these complex properties.

Column Verif. shows the result of verification (i.e., checked/false/check) assuming a global assertion for all predicates in all the benchmarks stating that the relative run-time checking overhead should not be larger than 1 \( (Ovhd \leq 1) \). Finally, Column \( T_A(ms) \) shows that the analysis time is \( \approx 4 \) times slower on versions with full instrumentation, and \( \approx 2 \) times slower on versions instrumented with run-time checks after static analysis, respectively, but in any case all analysis times are small.

We believe that these results are encouraging and strongly suggest that our approach can
provide information that can help the programmer understand statically, at the algorithmic level whether the overheads introduced by the run-time checking required by the assertions in the program are acceptable or not.

6 Conclusions

We have proposed a method that uses static analysis to infer bounds on the overhead that run-time checking introduces in programs. The bounds are functions parameterized by input data sizes. Unlike profiling, this approach can provide guarantees for all possible execution traces, and allows assessing how the overhead grows as the size of the input grows. We have also extended the Ciao assertion verification framework to express “admissible” overheads, and statically and automatically check whether the instrumented program conforms with such specifications. Our experimental evaluation suggests that our method is feasible and also promising in providing bounds that help the programmer understand at the algorithmic level the overheads introduced by the run-time checking required for the assertions in the program, in different scenarios, such as performing full run-time checking or checking only the module interfaces.

References


