Correction of propagation errors in Wide Area Multilateration systems

Gonzalo de Miguel Vela#1, Juan Besada Portas#2, Jesús García Herrero*3

#GPDS-SSR, Universidad Politécnica de Madrid
ETSI Telecomunicación, Avda Complutense Nº 30, 28040-Madrid, Spain
1gonzalo@grpss.ssr.upm.es
2besada@grpss.ssr.upm.es

*GIA, Universidad Carlos III de Madrid
3jgherrer@inf.uc3m.es

***Abstract*— Methods to estimate and correct slow time-variant position errors due to bad synchronization and propagation in Wide Area Multilateration Systems (WAM) are presented. The procedure does not use emitters at known locations. Instead it uses opportunity traffic emissions (from the aircraft present in coverage). The need for these methods came from the difficulty to install reference beacons that can be seen simultaneously by all WAM base stations.**

I. INTRODUCTION

With the improvements in performance of multilateration systems their range of applications have been extended from short range applications (airport surveillance) to medium range surveillance (eg: surveillance in TMA) [1]. This system has been called Wide Area Multilateration Systems (WAM). The positive evolution of this technology converts WAM as a firm candidate to substitute secondary radars in the surveillance network for air traffic control (ATC).

Multilateration system uses the time of arrival (TOA) of a signal emitted from an aircraft to a network of fixed receiver stations (ground or base stations) to determine aircraft position. If the emitted signal has an especial characteristic that differentiate it from others it is easy to associate the TOAs from the same signal in different base station. This is the case when we employ communication emissions (ADS-B or TCAS), the typical case in ATC. When the system has the set of TOAs for all ground stations, it can determine position.

The accuracy of the multilateration position is determined by the errors in TOAs. These errors, from the point of view of processing, can be principally grouped in three categories [2]: I) Time synchronization among ground stations, II) propagation effects and III) white noise. The first and third categories are present in all multilateration system, also in the short range ones. Effects of white noise in the determination of position are not very important for the typical S/N used in these systems. The propagation errors only appear if the maximum range of the system is important. For this reason, propagation error has not been taken into account in short range systems.

One characteristic of WAM system is its high baseline length among ground stations (20 to 30 nm [1]). This is necessary to guaranty an adequate accuracy along the coverage (a low DOP, dilution of Preccission). This characteristic poses a problem to the calibration subsystem: it is difficult to install fixed beacons in direct line of sigh with all base stations. These beacons can serve to reduce both synchronization and propagation errors. To solve the synchronization among station we can use methods based in GPS, but this not reduce the error due to propagation. Also, it is necessary to have a backup synchronization subsystem to mitigate hypothetical failures of GPS. The solution to this problem can be to add a processing system which correct synchronization and propagation error using the measurements of the aircraft present simultaneously in the WAM coverage (opportunity traffic).

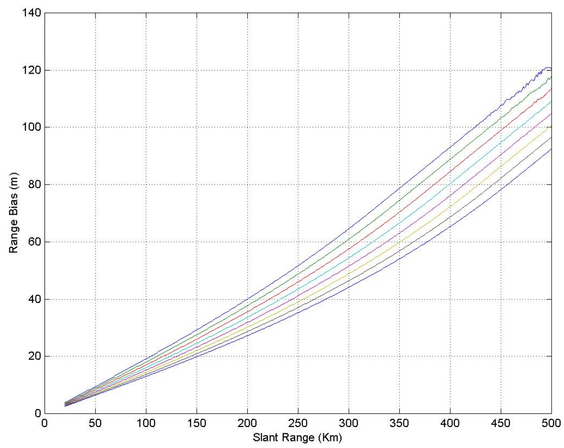

Fig. 1 Systematic error due to propagation for the standard atmosphere.

II. MULTILATERATION POSITION DETERMINATION

Each ground station measures the signal TOD and multiplied it by the standard speed of light (c). This quantity is called pseudorange (ρ) and can be represented by the following equation:

$$\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + cTe + c\Delta T_i + \Delta P_i + n_i = \rho_i \quad (1)$$

where subindex i indicates base station nº i, (x,y,z) are the aircraft coordinates, (x_i,y_i,z_i) are station coordinates, cTe refers to the uncertainty in the signal emission time, $c\Delta T_i$ is the error due to bad synchronism, ΔP_i represents the propagation error and n_i is the white random error. All the times in the previous equation are referenced to the same time scale for all the base station.

Before solve pseudorange equations for aircraft position we will characterise the propagation term ΔP. For the standard atmosphere there are good approximations to calculate the error in distance between two fixed points due to propagation [3]. In figure 1 we represent the error due to propagation for aircraft flying at different heights over earth surface (between 7 and 14 Km). In this graph we can observe the two main characteristics of tropospheric propagation error: I) is quasi linear with distance and II) its slope depends on aircraft height. This has two consequences for the estimation procedure: we approximate the term ΔP by a line (passing by zero) with the slope dependent on height. In the estimation algorithm the opportunity traffic selected for systematic error correction is selected in height layers. Including this model in eq. (1) results in the complete pseudorange equation:

$$(1+K)\sqrt{(x_i-x)^2+(y_i-y)^2+(z_i-z)^2}+cTe+c\Delta T_i+n_i=\rho_i \tag{2}$$

where K is the constant which characterise the propagation error and depends on the atmospheric conditions but not on the ground station.

The problem of position determination consists in the solution of the set of equations (2) for all ground stations. Some comments must be done about the possibility of solve this equation system. The first is that the number of equations must be equal or greater than the number of unknowns. Due that for each aircraft we must determine four independent variables (coordinates plus emission time), we need a minimum of four ground stations to determine these variables. But, with the minimum number of stations it is impossible to determine the other calibration variables in eq. (2). This implies that the result is biased (has a non random error). There are two ways to solve this problem. The first is to determine the calibration variables using fixed emitters. In this case the system knows the position and uses eq. (2) directly to solve the calibration variables. The second solution is to add some ground stations and solve the system of eq. (2) simultaneously for a few aircraft, estimating simultaneously position, emission time and synchronisation variables.

A last remark with respect to system of eq. (2): it is not possible to determine the time of emission and the synchronization error for all station simultaneously. This is solved designating a base station as reference and synchronize the other respect to it. In the rest of the paper we assume the reference station are numbered with 1 ($c\Delta T_1=0$) and ΔT_i represents the time error of station i respect to 1. Summarizing, with this methods we can synchronize all stations with one marked as reference.

III. CALIBRATION WITH FIXED BEACONS

In the case the system has fixed emitters the propagation constant and time bias estimation is done directly on pseudorange (without determining position). This method is used in GPS [5]. We must solve the following system of equations:

$$\begin{aligned}
&(1+K)\cdot R_1+cTe+n_1=\rho_1\\
&(1+K)\cdot R_2+cTe+c\Delta T_2+n_2=\rho_2\\
&\dots\\
&(1+K)\cdot R_n+cTe+c\Delta T_n+n_n=\rho_n
\end{aligned} \tag{3}$$

where R_i represents the Euclidean distance between emitter and station i.

Constant cT_e is known from the emitter. Them, the number of equations is always equal to the number of unknows and the system only need a fixed emitter to do calibration. The system (3) can be solved using minimum square error techniques to diminish the effect of random noise (terms n_i).

This calibration technique has the problem that can be difficult to install a fixed emitter in the line of sigh of all base station in WAM. Besides, because the calibration emitters are on a very low height the estimation of propagation constant is done for low heights where the propagation effects are more important. To overcome this problem we must use a method based on opportunity traffic.

IV. CALIBRATION USING OPPORTUNITY TRAFFIC

When the system don´t have fixed emitters it have to auto calibrate using the emissions of aircraft flying in the coverage. In that case, we must have more stations (n) than the number of coordinates to be determined for each aircraft (x,y,z and T_e, four). As we have to determine n-1 synchronization constants ($c\Delta T_i$) and one propagation constant (K) we need n new independent pseudorange equations to do calibrations. As the determination of calibration constants must be done jointly with coordinates, we need to process jointly the pseudorange equations of m≥ceil(n/(n-4)) aircraft to solve coordinates and calibration. The joint system that permits to determine simultaneously calibration and aircraft variables is as follows (we have suppressed the random noise terms for clarity):

$$\begin{aligned}
&(1+K)\sqrt{(x_1^b-x_1)^2+(y_1^b-y_1)^2+(z_1^b-z_1)^2}+cTe_1\cong\rho_{11}\\
&\dots\\
&(1+K)\sqrt{(x_i^b-x_1)^2+(y_i^b-y_1)^2+(z_i^b-z_1)^2}+cTe_1+c\Delta T_i\cong\rho_{i1}\\
&\dots\\
&(1+K)\sqrt{(x_1^b-x_j)^2+(y_1^b-y_j)^2+(z_1^b-z_j)^2}+cTe_j\cong\rho_{1j}\\
&(1+K)\sqrt{(x_i^b-x_j)^2+(y_i^b-y_j)^2+(z_i^b-z_j)^2}+cTe_j+c\Delta T_2\cong\rho_{ij}\\
&\dots
\end{aligned} \tag{4}$$

where the subindex i represent number of station and j the number of aircraft.

This is a system of nonlinear equations similar to that used to determine coordinates in multilateration systems. For its solution we will use a gradient method (linearization of order one) [8]. To solve the system we start from an initial solution and iterate using the linearized function up to a satisfactory convergence has been reached. The unknown vector is as follows:

$$\mathbf{x} = \begin{bmatrix} x_1 & y_1 & z_1 & cTe_1 & x_2 & y_2 & \cdots & K & c\Delta T_2 & \cdots & c\Delta T_n \end{bmatrix}^T \tag{5}$$

where the target coordinates are (x_j, y_j, z_j, cTe_j).

The system in (4) can be solved with the following iteration:

$$\mathbf{x}(k) = \mathbf{x}(k-1) + \Delta\mathbf{x}(k) \tag{6}$$

The initial value $\mathbf{x}(0)$ must be set around the true solution. This is done using previous estimations of calibration constants and the position determined for each aircraft by the WAM system before calibration process. The innovation of the process $\Delta\mathbf{x}(k)$ is obtained solving the first derivative terms of the linearized version of system (4). The system is:

$$\mathbf{b}(k) = \mathbf{A}(k)\cdot\Delta\mathbf{x}(k) \tag{7}$$

where the matrix **A** contains the gradient of eq. (4) and vector b have the corrected pseudoranges in iteration k. They have the following expressions:

$$\mathbf{b}(k) = \begin{bmatrix} \rho_{11} - \left\{(1+K(k))\hat{R}_{11}(k) + cTe_1(k)\right\} \\ \rho_{21} - \left\{(1+K(k))\hat{R}_{21}(k) + cTe_1(k) + c\Delta Te_2(k)\right\} \\ \vdots \\ \rho_{12} - \left\{(1+K(k))\hat{R}_{12}(k) + cTe_2(k)\right\} \\ \rho_{22} - \left\{(1+K(k))\hat{R}_{22}(k) + cTe_2(k) + c\Delta Te_2(k)\right\} \\ \vdots \end{bmatrix} \tag{8}$$

where $\hat{R}_{ij}(k)$ represents the estimated Euclidean distance between target j and station i using the distance determined in previous iteration.

$$\mathbf{A}(k) = \begin{bmatrix} A_1 & 0_{nx4} & 0_{nx4} & \cdots & B_{nx(n-1)} \\ 0_{nx4} & A_2 & 0_{nx4} & \cdots & B_{nx(n-1)} \\ 0_{nx4} & 0_{nx4} & A_3 & \cdots & B_{nx(n-1)} \\ \vdots & \vdots & \vdots & \ddots & B_{nx(n-1)} \end{bmatrix} \tag{9}$$

where $\boldsymbol{A_i}$ is a matrix of size (nx4) that contains the pseudorange gradients of all base station for target $\boldsymbol{i}$ and $B_{nx(n-1)} = [0_{(n-1)x1}\ I_{(n-1)x(n-1)}]^T$. $\boldsymbol{A_i}$ follows the expresion:

$$A_i = \begin{bmatrix} (1+K)\dfrac{(x_i - x_1^b)}{\hat{R}_{1i}} & (1+K)\dfrac{(y_i - y_1^b)}{\hat{R}_{1i}} & (1+K)\dfrac{(z_i - z_1^b)}{\hat{R}_{1i}} & \hat{R}_{1i} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Normally, system (7) has more equations that unknowns and must be solved using the minimum mean square error equation:

$$\Delta\mathbf{x}(k) = inv\left(\mathbf{A}(k)^T \cdot \mathbf{S}(k)^{-1} \cdot \mathbf{A}(k)\right)\cdot \mathbf{A}(k)^T \cdot \mathbf{b}(k) \tag{10}$$

where **S**(k) represents the pseudorange covariance matrix and can be estimated from the S/N in each receiver.

The iteration in (6), for the typical levels of pseudorange random noise in this kind of systems (typically a few meters), converges very quickly. In the experiments we have stopped iteration when all elements in $\Delta\mathbf{x}(k)$ are less than 0.5 meters. For this condition the number of steps was 3 or 4.

Figure 2 presents the block diagram of the calibration mechanism which uses opportunity traffic. The TODs in each station are associated and sent to the central processor which computes position. These can be a master station in the multilateration system or a remote fusion centre that fuses information of many sensors (this can be the case for ADS-B technology [3]). The first operation is to apply calibration constants ($c\Delta T_i$) to pseudoranges. After calibration, the coordinates of target are determined as if calibration was perfect, using to represent propagation the previously estimated K constant. The output of this block is delivered as position determined by WAM system. To compensate for slow time variations in the real propagation and calibration conditions, the system has an open control loop to modify the estimated propagation constants and maintain the system calibrated. The system select targets around some height level and well distributed around the coverage. Then, only on these aircraft, system determines jointly position and calibration constants, using as initial guess for iterative algorithm the target coordinates delivered by WAM system and the calibration constants at the output of averaging filter. Calibration constants are introduced in an averaging filter to reduce their variance and, after that, they are used for the correction of future pseudoranges.

The position determination is done using the proposed mechanism in [6]. It uses an iterative algorithm with the linearized multilateration equation. In our case the linearized system corresponding to equations (4) assuming K and $c\Delta T_i$ are known (we use the output of the averaging filter). This algorithm needs an initial position which is determined using the closed form algorithm of [7]. The system can be implemented in TOA form (determining cT_e) or in DTOA form (determining cT_e). If cT_e is not necessary, the DTOA form is more accurate due to the complete ignorance about the time of emission in WAM. The same method applies to the determination of calibration constants plus position.

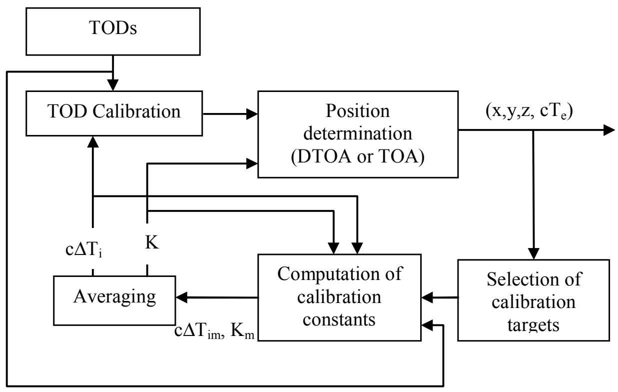

Fig. 2 Block diagram of calibration mechanism using opportunity traffic.

V. ILLUSTRATIVE EXPERIMENTS AND CONCLUSIONS

To demonstrate the performance of the propagation error compensation system we simulate the correction algorithm for a hypothetical WAM configuration. We consider a system with nine ground stations, eight distributed around a square of 100 Km side length (stations are approximately in the corners and in the middle of each side) and one in the centre. All stations have zero height respect to local plane. It is expected that performance in geometrical height determination is very bad due all the stations are at the same level. The station in the centre is designated as reference. The system uses Cartesian coordinates centred in the reference station, with X axes pointing to east, Y axes to north and Z up. The averaging filter of figure 2 is a sliding window of length 15 samples (approximately 3 s for WAM based on Mode S squitter).

For calibration we have selected four target (3 is the minimum number) situated in the corners of a 200 Km side square, centred in the reference station. The height of the targets is 7 Km over the ellipsoid. We have generated the pseudorange of stations distinct of reference with a systematic error of 15, -10, -5, -25, 10, -30, 10 meters. The propagation constant used to generate data has been 10^{-4} (in the range of values usual in standard atmosphere). The number of trials in Monte Carlo simulation has been enough to consider exact the result for our purposes.

TABLE I
Mean error of WAM error for x coordinate

Targ. Nº	σ_ρ=2 m		σ_ρ=5 m		σ_ρ=10 m	
	No	Yes	No	Yes	No	Yes
1	8.50	0.17	8.57	-3.06	7.4413	4.17
2	-36.77	-0.47	-36.46	-4.36	-37.90	-4.06
3	56.05	0.18	54.76	1.54	55.66	-7.15
4	49.74	-0.02	50.18	3.30	48.48	-5.77

TABLE II
Standard deviation of WAM error for x coordinate

Targ. Nº	σ_ρ=2 m		σ_ρ=5 m		σ_ρ=10 m	
	No	Yes	No	Yes	No	Yes
1	16.84	18.85	42.25	47.66	81.70	90.12
2	20.37	22.47	47.60	51.63	94.44	103.36
3	17.68	19.64	50.30	56.01	98.41	109.53
4	19.13	21.25	47.02	51.69	92.21	100.39

TABLE III
Mean error of WAM error for y coordinate

Targ. Nº	σ_ρ=2 m		σ_ρ=5 m		σ_ρ=10 m	
	No	Yes	No	Yes	No	Yes
1	-54.51	0.04	-54.31	-2.73	-55.76	3.16
2	31.64	0.48	31.73	3.55	32.68	-4.21
3	-50.75	-0.19	-49.58	-2.79	-50.33	-0.29
4	-16.11	-0.11	-15.53	3.71	-17.70	-6.83

TABLE IV
Standard deviation of WAM error for y coordinate

Targ. Nº	σ_ρ=2 m		σ_ρ=5 m		σ_ρ=10 m	
	No	Yes	No	Yes	No	Yes
1	16.84	18.85	42.11	47.10	81.8945	89.72
2	20.37	22.47	51.70	56.71	102.18	113.01
3	17.68	19.13	43.63	48.41	85.02	94.22
4	19.64	21.25	48.63	53.66	95.21	104.06

For each calibration target we have measured the mean and variance of the position at the output of the system in figure 2, with and without calibration, for different values in the pseudorange accuracy. The comparative results are presented in tables below for the four targets using DTOA techniques. We can observe how for typical random errors in WAM (below 5 meters) the reduction in systematic error is very important: practically disappears in x and y coordinates and is reduced in z coordinate. For worse accuracies (10 m) the system work well for x and y but do not work for z. This is an expected result because the geometry is not good for height determination due to distance and relative low height over the plane of stations (the target have 7000 m over the ellipsoid, not over x-y plane). In the last case we have to increase the sliding window length of the averaging filter or the number of calibration targets. This action influences the system velocity of response.

Finally, we can observe that for typical cases (random error below 5 meters) the increment in output standard deviation due to calibration system is very low and is greatly compensated by the reduction in position bias.

TABLE V
Mean error of WAM error for z coordinate

Targ. Nº	σ_ρ=2 m		σ_ρ=5 m		σ_ρ=10 m	
	No	Yes	No	Yes	No	Yes
1	-54.51	0.04	-251.9	-25.6	-257.3	80.48
2	31.64	0.48	337.9	-17.36	329.3	169.50
3	-50.75	-0.19	-859.9	-13.02	-871.1	146.84
4	-16.11	-0.11	-360.8	-7.43	-361.6	271.60

TABLE VI
Standard deviation of WAM error for z coordinate

Targ. Nº	σ_ρ=2 m		σ_ρ=5 m		σ_ρ=10 m	
	No	Yes	No	Yes	No	Yes
1	49.48	99.29	124.06	233.75	243.82	428.94
2	56.17	124.76	140.81	293.35	283.25	534.12
3	64.45	113.39	162.18	268.36	325.54	485.68
4	78.99	147.61	201.52	363.01	403.84	631.93

ACKNOWLEDGMENT

The work has been financed by Spanish science and technology office under projects TEC-2008-06732 y TIN-2008-06742

REFERENCES

[1] W. H. L. Neven et al, *Wide Area Multilateration*, EUROCONTROL, 2004.

[2] M. J. Leeson, *Error Analysis for a Wide Area Multilateration System*, QinetiQ/C&IS/ADC/520896/7/19, 2006.

[3] *EUROCONTROL standard document for surveillance data echange. Part 12: Category 21, ADS-B messages* , EUROCONTROL, ed 1.1 September 2008.

[4] D. K. Barton, *Radar System Analysis and Modeling*, Artech House, 2005

[5] B. W. Parkinson, J. J. Spilker ed., Global Positioning System: Theory and Applications, AIAA publishers 1996.

[6] G. Galati et al, "Wide Area Surveillance using SSR Mode S Multilateration: advantages and limitations", European Radar Conference 2005, EURAD 2005.

[7] W. H. Foy, "Position-Location Solutions by Taylor-Series Estimation", *IEEE Trans. on Aerospace and Electronic Systems*, vol. AES-12, pp. 187-194, March 1976.

[8] G. Strang, K. Borre, Linear Algebra, Geodesy, and GPS. Wellesley-Cambridge Press, 1997.