GAMIFICATION ACTIONS IN THE TEACHING OF MATHEMATICS AT EVERY EDUCATIONAL LEVEL

Sagrario Lantarón¹, Mariló López¹, Susana Merchán¹, Javier Rodrigo²

¹Departamento de Matemática e Informática Aplicadas a la Ingeniería Civil y Naval de la Universidad Politécnica de Madrid (SPAIN)

²Departamento de Matemática Aplicada de la Universidad Pontificia Comillas de Madrid. (SPAIN)

Abstract

The incorporation of game elements on behalf of the teachers in order to improve the interest and the motivation of the students is having a quick rise in every educational level. This work is a reflection about gamification as a strategy of enrichment and approach to mathematic topics. Through the paper there are described some of the activities which the Educational Innovation Group “Pensamiento Matemático” of the Universidad Politécnica de Madrid (UPM), is carrying out in the context of the Aula Taller Museo de las Matemáticas π -ensa. The gamification and the impact this technique is having on the students of the Civil and Territorial Engineering degree of the UPM, as well as on the students of Secondary Education and Bachillerato who visit π -ensa, are analysed. The paper tries to be an essential basis for starting processes of research and application of the gamification in the teaching of Mathematics at every level, especially in the students of new entry in the different university degrees.

Keywords: Gamification and Mathematics, learning dynamics, educational projects, serious games.

1 INTRODUCTION

Throughout the ages, mathematics has been a problematic course for virtually all education levels. It is loved by very few and hated by many. Who is responsible for this? Is there a solution to this problem? What can be done to potentially change this situation?

In today's society, wherever we look, science and technology abound, an especially significant result of our mathematics culture. Recently, however, reports published by the Conference of Rectors of Spanish Universities (CRUE) and by the Ministry of Education, Culture and Sports, suggest that the number of students opting for studies of the Technology branch has been on the decline for at least the past decade. We believe that this decline may be due, in large part, to the student's fear of mathematics, a course that is very present in this type of studies.

Where can we look for explanations to this situation? Clearly, at some point in time, the passion that primary school students tend to feel towards mathematics is lost. These young students are attracted to this class in which they can actively participate and which is often viewed as a game.

We believe that the key lies in this last word: “game”. It is vital to make mathematics the visual, approachable, applied and fun subject that it truly is. There is a clear need to implement actions intended to make math and its recreational form more approachable by all. There are very few places and activities related to this subject that might make it more present in our society's day to day life, and above all, that offer it as a fun science.

The recreational aspect of this science has not been sufficiently nurtured and therefore those who are not specialists are unaware of its beauty and do not feel inclined for this discipline. Therefore, it is necessary to attempt to teach and educate, while also surprising and delighting. It is all about communicating, improving the public's perception of math, disseminating, advertising and popularizing it. To do so, diverse means should be used, including interactivity, experimentation, competition and play, implementing them in multi-disciplinary environments that highlight the recreational aspects. The areas of reality perception and analysis, the use of real objectives and multi-media tools should also be included.

Socially speaking, throughout time, disciplines have not all been treated equally. No one has ever questioned the importance of reading or the relevance of Quixote. Typically, parents read to their children from a very young age and there are numerous campaigns that support and reinforce the reading habit (something that is clearly also quite important). But the fact is, there has been no such

push to promote the development of mathematics or the love of this field. Therefore, it would be useful to design campaigns that promote such aspects as: problem solving with your child, getting involved in geometry, playing with shapes, etc.

Ultimately, we can survive (not that it would be a good idea to attempt it) without knowing who wrote Romeo and Juliet, but we would be in a lot of trouble if we could not understand interest rates and other things when requesting a loan from our bank.

We need to take on a new attitude regarding math. This should come as no surprise given that for years now, the phrase “I’m bad at math” has abounded, and this is assumed to be something normal. An effort should be made to produce good materials, new activities and approaches to this area of study. Recreational spaces should be designed that work with it and above all, top notch, enthusiastic professionals should be trained to teach its content.

In order to work on this, the authors of this article (members of the Group of Educational Innovation of the UPM “Pensamiento Matemático”) have developed the π -ensa Mathematics Workshop-Museum Hall. Here, people of all ages and education levels can visit a space where they can come, not to study mathematics in the traditional sense, but rather, where they can “see” math, “play” with it and appreciate its beauty [1]. In this Workshop-Museum, we don’t want to demonstrate theorems, but rather, to interpret concepts and to demystify this science through an attractive and fun presentation with minimum technical equipment. More information may be found at: <http://innovacioneducativa.upm.es/museomatematicas/>

This article (section 3) presents some of the activities that the group developed with regards to topics of gamification.

2 GAMIFICATION

Gamification attempts to use elements of play to increase the interest and motivation of the participants. The concept, defined by Deterding, Dixon, Khaled and Nacke in [2], refers to the use of elements of game design in non-play contexts. This is a relatively new field but one that is experiencing rapid growth.

Over recent years, gamification has been incorporated into all sorts of business activities [3], as well as marketing and, of course, education. Some educational centers are using games to promote student motivation and participation, based on the idea that learning can take place in a different and fun way.

Universities are not to be left out of this effort. Some teaching universities use elements of play to encourage student involvement in the courses ([4], [5] and [6]) to improve skills [7] or promote competencies such as social or communicative interaction. And of course, it has also been used to improve knowledge acquisition [8].

As teachers, we understand that gamification is not a passing fad, but something that is going to be set in our vocabulary and in our lives. Mathematics as a field is better suited for the application of gamification as compared to other fields.

In education, motivation is fundamental and it is used to draw in the student’s attention so that they will devote time to specific activities [9]. But, it is not enough to only apply elements of play, it is also necessary for participants to be involved and to use the educational content as real goals that they wish to achieve.

There should be a focus on the benefits provided by play. According to [10], play is capable of motivating as it impacts the cognitive, emotional and social areas of the players. It can also be used to assist in improvement of the areas of success and failure. When a player overcomes problems or difficulties, he/she experiences positive emotions. Any type of game may increase our feelings of success through compensation systems (obtaining points, recognition, awards, etc.).

Below we present some of the activities that are being put into practice along these lines of mathematics gamification, as well as those being carried out with the students from the early study years of Civil and Territorial Engineering at the UPM, and with high school and baccalaureate students visiting π -ensa.

3 OUR MATHEMATICS GAMIFICATION ACTIVITIES

This work developed out of the authors' desire to offer the educational community new tools for mathematics courses that are based on two pillars: gamification and manipulative tools. We believe that it is necessary to incorporate techniques that are attractive to students of these complex and challenging courses.

The proposal objectives are:

- To awaken an interest in mathematics learning given that it is considered to be useful and beneficial in everyday life, and linking this learning to positive emotions.
- To have the student relate mathematics with the real life and with other areas and to know how to apply the acquired knowledge.
- To strengthen the problem solving capacity.
- To strengthen skills based on strategic mathematics reasoning: strategy, planning, decision making, etc.

In addition to the proposed objectives, work was also carried out on other capacities in a transverse manner, including social interaction, communicative competency and learning to learn.

The following were pursued:

- To apply the dynamics and principles of play in order to improve motivation, interest and implication of degree-seeking students of the UPM, as well as in secondary and baccalaureate students, in those subjects including mathematics in their content.
- To develop techniques related to games in order to reinforce the learning process in subjects related to mathematics from the first degree courses.
- To create a series of recreational proposals, challenges, gymkanas, logic boxes, escape shows, etc. that help increase knowledge of mathematics and bring recreational enjoyment to this science.
- To strengthen logical thought and reasoning through the implementation of challenges.
- To stimulate mathematical creativity and knowledge through the creation, by students, of new proposals in which the knowledge of mathematic subjects studied in their degrees is presented. Learning based on games applied to the mathematics content of the UPM technical degree programs is pursued through games that are adapted by professors and the students themselves.

Generally speaking, the proposals are based on the intent to include manipulative gamification in typical procedures such as consolidation activities of the math courses to different levels and to offer students the opportunity to play as a means of advancing the learning experienced in their daily classroom work. An attempt is made to promote the discovery of the importance of this knowledge as a tool for problem solving.

Based on these premises, the goal is to learn math through play. To do so, game environments are being created, some of which are inspired by escape games, taking advantage of the current popularity of these games in today's society. Some of these actions are described below.

3.1 Logic and strategy games: help in the development of mathematical thinking

As mentioned in the introduction, today's society does not tend to promote logical thinking in general and much less so at early ages. But, why work on logic? Often, educators complain that children in secondary education and even university levels are unable to apply logic to resolve problems or they have great difficulty in following logical lines of reasoning. The problem is that this type of logical thought is never worked on or promoted.

Logical capacities are innate in human beings and these capacities should be promoted in children and students of all ages. One of the paths that is available for this, includes the proposal of activities through strategic and logic games so that children may exercise these skills. The objective of working with this type of games is to develop the capacities of observing, relating, discovering, verbalizing, generalizing, finding strategies, reasoning, etc. which are the foundations of mathematical thinking.

Just as a football player cannot practice his/her skills if he/she is not in good physical shape, our students cannot tackle new mathematical concepts if they do not have a good foundation in logic. The activities that we are developing with logic games, in the classroom and outside of it, permit students to develop their logic capacities.

At all educational levels, table games, ingenuity games, puzzles and other situations created specifically for work on logic are of great interest. The foundation of the activities carried out in the Mathematics Museum π -ensa includes these types of games. Using them has allowed us to stimulate problem solving capacity, improve spatial vision, reasoning, structured thought, etc.

We use a variety of games, some of which we have created ourselves, others which are available commercially and others which are computer-based. The results that we are obtaining are truly positive. Participants are very motivated and are surprised by their own abilities, often times losing the fear of taking on challenges. See Figures 1 and 2.

Figure 1. Workshops of student-directed logic games

Figure 2. Museum visit made by the general public

3.1.1 Some of the proposals

The range of materials that may be used to carry out this type of activities is quite extensive. We can find and design games that are adapted to all ages and have the purpose of promoting different facets and capacities.

Below are some specific examples.

A. With the objective of introducing students to programming and algorithm creation.

1 A simple game is “Leap frog” (Figure 3)

Goal: To exchange the frogs of the same color with others of another color, based on the following rules:

- A frog can move over an adjacent empty space.
- A frog can jump over another frog of an opposing color if there is an adjacent empty space.
- The frogs cannot move backwards
- It is not possible to jump over same colored frogs or over more than one frog.

Figure 3. The "leap frog" game

Students should design an algorithm in which each step taken does not close the door to the following step.

- 2 Along this same line, there are other classic games such as the Towers of Hanoi (Figure 4):

Place all discs on the left-hand pivot from largest to smallest size. Pass all the discs to the right-hand pivot with the least possible number of movements and respecting two rules:

- In each movement, only one piece can be moved from one pivot to another.
- It is not possible to put a piece on top of a smaller piece.

Figure 4. "Towers of Hanoi"

B. Logic puzzles:

- 1 Classic problems such as the following promote the development of student logic:

A man, his dog, his hen and a carrot must cross a river in a boat. In the boat, he can only go by himself or with any of the other elements (only one). Take into account that:

- the dog eats the hen
- the hen eats the carrot

Therefore, none of these pairings should be left together on the river shores.

How should they cross the river?

If we present this logic puzzle with a doll, a dog, a hen and a carrot, a boat in which these characters can be placed to cross the river that is painted on a sheet of paper, students may experiment and have an easier time in reaching the solution.

- 2 There are many other proposals to develop mathematical and logical thought. Below is another of our challenges (Figure 5):

In a 16-position board, place 16 buildings of 4 different heights, according to these rules: the number in the margin indicates the buildings that can be seen from this position in the row or column, in each row or column, there may only be one building of each height.

Figure 5. "Buildings Challenge"

C. Geometric challenges:

- 1 Puzzles must be included as well in this chapter. They help to develop spatial skills, promote logical reasoning, may introduce geometric concepts such as perimeter, area, flat figures and volumes and they can familiarize the user with the construction of models.

All types of puzzles, both bi-dimensional and tri-dimensional ones can help in the development of spatial and geometric vision. Some of the proposals that we offer in π -ensa include the re-composition of letters, crosses, hearts or the forming of a rectangle by assembling 9 square pieces. Other proposals are also based on classic challenges such as the Soma cube, among others.

- 2 Other very useful tools included in this section are mirror games. They allow the user to work on symmetries, among many other things. For example, using a base figure, the user must attempt to achieve others through a game with mirrors, improving their geometric vision.
- 3 Classic games:

Well-known proposals such as Tantrix (Figure 6) are great contributions to these learning strategies. This game proposes the creation of distinct closed circuits using the pieces indicated in each case. It should be kept in mind that, in all challenges, the lines must always correspond with others of the same color.

Figure 6. "Tantrix"

This is only a small sampling of the options that may be found.

3.1.2 Logic boxes

Over recent years, escape games have become very popular across the globe. Typically, they consist of a room (or various rooms) from which one must leave within a limited amount of time. The room contains tests and clues that are hidden throughout, by which, little by little, the situation may be resolved and the way of escape from the room is provided. Along these lines, a gamification tool has been created, integrated in a problem solving methodology, which includes a challenge to be resolved through use of acquired mathematics knowledge and while constructing some new knowledge. It consists of a set of boxes having a specific number of drawers, closed with locks, in which different challenges and clues are contained (Figure 7). Only the solving of the previous clue will provide access to the following one. To solve the challenge, students must work together, thereby promoting cooperative learning. The game ends when one of the groups manages to open the box.

The content of the challenges varies, adapting to the concepts that are studied in the distinct topic areas. In this way, this content is reinforced and may be practiced from distinct approaches. Furthermore, in order to stimulate creativity and mathematical knowledge, the students themselves can make new proposals in which they highlight the knowledge of the mathematical areas that they study in their classes.

In this way, game play, something that should never be abandoned, is rediscovered, proposing a fun way to learn math. When playing, the use of unpredicted situations is one of the fundamental factors that assure this enjoyment, allowing for the search for creative solutions to the problems.

Figure 7. "Logic boxes"

Below are some examples of the challenges faced by the first year degree seeking students of the UPM technical schools, when working with the logic boxes. In this case, these are general subject boxes that stimulate logical reasoning, mathematics culture and team work.

The first challenge is to find the number that will open one of the drawers (Figure 8):

Last Fermat theorem + Fibonacci sequence * Second world war - Geometry founder / <u>Cartesian Geometry</u>	<table border="0" style="width: 100%;"> <tr> <td>Andrew <u>Wiles</u>=3</td> <td>Euclides=8</td> </tr> <tr> <td>Alan Turing=200</td> <td>Leonardo Pisano <u>Blgollo</u>=2</td> </tr> <tr> <td>Descartes=4</td> <td></td> </tr> </table>	Andrew <u>Wiles</u> =3	Euclides=8	Alan Turing=200	Leonardo Pisano <u>Blgollo</u> =2	Descartes=4	
Andrew <u>Wiles</u> =3	Euclides=8						
Alan Turing=200	Leonardo Pisano <u>Blgollo</u> =2						
Descartes=4							

Figure 8. Challenge that may be found in a general logic box

Another of the challenges leads to the following combination:

In the board, (Figure 9) place the first 7 natural numbers so that no two consecutive numbers are together, either horizontally, vertically or diagonally. For example, the number 4 cannot have either a 3 or a 5 in the boxes surrounding it.

Also, the sum of the three numbers you have placed in the upper row must be greater than 13.

The numbers that you seek are the figures that you have placed in the upper row of the board.

	8		

Figure 9. Board for one of the games of the general logic box

Another challenge included in the box:

Fill out the grid (Figure 10) such that the consecutive numbers are in horizontal, vertical or diagonal contact. The first and the last number of the series are in black. Add together the numbers situated in the red boxes.

			60					81
65	64		1			9		
		72	3		75	8	11	
		57		4				
					20	19	17	
	49	50	54		22			
48				53				
	44			36	33		27	28
42	41	39	38	35		31		

Figure 10. Challenge that may be found in a general logic box

4 CONCLUSIONS

Without a doubt, gamification is currently in style. It clearly can have many applications, but there is one that is currently generating great interest: its application in the world of education. It is a powerful mechanism that educators have in order to motivate and attract students.

Little by little, gamification applied to education is managing to ensure the improved performance of students, increasing their motivation to learn and allowing them to learn in an easier manner, given that it contains tools that allow them to try, simulate, explore and put into practice the knowledge, in a much more approachable manner.

The results of the use of gamification have been demonstrated by the real life experiences of teachers at all education levels, who, through gamification and games, are able to redirect their students towards a path of knowledge and learning.

This article shares some of the mathematical gamification proposals offered by the authors in order to bring the general public closer to this science, and to improve the math performance of university students seeking technical degrees as well as that of high school and baccalaureate students.

Gamification is here to stay and we should take advantage of all that it has to offer.

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