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Doctoral dissertation

Dynamic phenomena in the design of new concepts of space systems

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A mi madre y a mis hermanos

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RESUMEN

El diseño estructural es un aspecto fundamental en el desarrollo de sistemas espaciales para garantizar su supervivencia frente al severo entorno mecánico a los que van a estar expuestos durante la misión, especialmente durante la fase de lanzamiento. Este entorno mecánico está compuesto por cargas dinámicas que interactúan con el comportamiento estructural de los sistemas espaciales, generando tensiones y deformaciones que pueden afectar a los componentes más sensibles. Esta tesis doctoral se centra en el estudio de dos aspectos importantes relacionados con el comportamiento estructural de los sistemas espaciales: el análisis de choque y el acoplamiento dinámico.

Una de las cargas mecánicas más importantes es el choque, que se caracteriza por su alta intensidad, corta duración y comportamiento transitorio. El primer bloque de esta tesis es el desarrollo de una nueva metodología para evaluar los efectos de las cargas de choque en las estructuras espaciales mediante simulaciones numéricas. El método elegido en este trabajo para evaluar la respuesta estructural frente a cargas de choque es el enfoque de elementos finitos, que se usa ampliamente en la industria espacial para analizar los efectos estructurales en otros entornos mecánicos como las aceleraciones inerciales estáticas y las vibraciones sinusoidales y aleatorias. El objetivo de esta investigación es la definición y la validación de una metodología para análisis de choque para ser implementada en el proceso de verificación de estructuras espaciales como instrumentos y satélites. Con este propósito, el estudio de las simulaciones numéricas de choque se aplica a los ejemplos de los instrumentos STEP y EPT-HET de la misión Solar Orbiter para demostrar la aplicabilidad de la metodología propuesta en casos reales de estructuras espaciales. La investigación comienza con una visión general de las diferentes opciones para las simulaciones de choque compatibles con el enfoque de elementos finitos, aplicándolas al modelo de elementos finitos del instrumento STEP para evaluar sus características y la precisión de los resultados obtenidos mediante la comparación con los datos del ensayo de choque. Entre las opciones estudiadas, el análisis transitorio modal es el más apropiado para calcular los efectos de las cargas de choque, destacando su mayor precisión, versatilidad y representatividad del entorno de choque, que supera sus aspectos negativos relacionados con su mayor tiempo computacional y su mayor cantidad de datos generados comparado con los otros métodos. Las diferencias entre resultados numéricos y experimentales en términos de valores de pico de las aceleraciones de respuesta encontradas en este trabajo con el ejemplo del instrumento STEP están dentro del rango aceptable de ± 3 dB para los análisis transitorios modales.

La investigación continúa con la evaluación de la precisión de los resultados numéricos calculados por los análisis transitorios modales que simulan los entornos de los ensayos de choque, estudiando la influencia de los parámetros de entrada más relevantes como el amortiguamiento y la definición del campo de aceleraciones de entrada, que puede ser no uniforme y con aceleraciones no despreciables en los tres ejes ortogonales. Este estudio revela que la técnica tradicional para representar la excitación dinámica de la base mediante un nodo conectado rígidamente a todos los nodos de la interfaz de la estructura analizada no es adecuada para representar el ambiente de entrada medido en los ensayos de choque por impacto cuando el campo de aceleración no es uniforme, así que se proponen nuevas técnicas en este trabajo para mejorar la definición del campo de aceleraciones de entrada. Dos de las técnicas propuestas consiguen una mejora considerable en la precisión de las aceleraciones de respuesta cuando se compara con los resultados experimentales extraídos de los ensayos de choque del instrumento STEP.

Este bloque finaliza con la descripción del procedimiento de verificación de choque aplicado a los instrumentos STEP y EPT-HET, donde la capacidad de ambas unidades para soportar las

cargas de choque especificadas se demuestra finalmente mediante análisis numéricos, siguiendo la metodología desarrollada en este trabajo. Este procedimiento es útil para ser adaptado a proyectos futuros con la misma necesidad de demostrar numéricamente la capacidad estructural para soportar las cargas de choque especificadas.

El segundo bloque de esta tesis doctoral se centra en el estudio de los efectos del acoplamiento dinámico o modal, que se produce cuando la frecuencia natural de una parte secundaria, como una carga de pago o un equipo, está cerca de la frecuencia natural de uno de los modos globales de la estructura primaria, que en este caso corresponde a un satélite. El acoplamiento dinámico se evita generalmente en las estructuras espaciales imponiendo limitaciones en las frecuencias naturales de los subsistemas secundarios para asegurar la separación necesaria de sus frecuencias locales con las frecuencias globales del sistema principal. Sin embargo, hay diseños particulares donde el acoplamiento dinámico entre una carga de pago y un satélite es inevitable debido a otros requisitos de la misión y, por tanto, se deben estudiar los efectos provocados por este fenómeno para predecir las consecuencias estructurales tanto para el satélite como para la carga de pago. Para empezar el estudio sobre el acoplamiento dinámico, se usa un sistema de dos grados de libertad para obtener expresiones analíticas sencillas que ayuden a comprender cómo cambian las frecuencias naturales, formas modales y masas modales efectivas con el grado de acoplamiento modal. Se muestra en este trabajo que la modificación de los valores de estos parámetros es notable, incluso para valores muy pequeños de la relación entre las masas secundaria y primaria. Las expresiones analíticas obtenidas se aplican posteriormente a dos casos de diseño para definir los límites de los valores de los parámetros de diseño para conseguir que el sistema global cumpla con los requisitos impuestos relativos a las frecuencias naturales.

La investigación del acoplamiento dinámico termina con el cálculo numérico de los efectos de este fenómeno con el ejemplo del satélite UPMSat-2 como estructura primaria y las antenas de comunicación como parte secundaria, mostrando la influencia del grado de acoplamiento modal en el comportamiento dinámico tanto del satélite como de las antenas. Uno de los resultados más interesantes es la reducción de un 57% de las fuerzas máximas generadas en los tornillos de interfaz del satélite cuando está sometido a una vibración sinusoidal como consecuencia del acoplamiento modal de las antenas, que es una consecuencia notable teniendo en cuenta que la masa de las antenas analizadas es sólo aproximadamente el 0.25% de la masa del resto del satélite. Sin embargo, las antenas sufren las consecuencias negativas del acoplamiento dinámico, donde las tensiones generadas en estos elementos aumentan más de dos veces con respecto al caso desacoplado, sobrepasando el límite permisible y provocando el colapso estructural de estas partes secundarias.

ABSTRACT

The structural design is a key aspect in the development of space systems to guarantee their survival against the severe mechanical environment to which they will be exposed during the mission, and especially during the launch phase. This mechanical environment is composed of dynamic loads that interact with the structural dynamic behaviour of the space systems, generating stresses and strains that can affect the most sensitive components. This doctoral dissertation focuses on the study of two important aspects related to the structural behaviour of space systems: shock analysis and dynamic coupling.

One of the most important mechanical loads is the shock, which is characterized by its high intensity, short duration and transient behaviour. The first block of this doctoral dissertation is the development of a new methodology to evaluate the effects of shock loads on the space structures by means of numerical simulations. The chosen method in this work to assess the structural response against shock loads is the finite element approach, which is widely used in the space industry to analyse the structural effects in other mechanical environments such as static inertial accelerations and sine and random vibrations. The objective of this investigation is the definition and the validation of a methodology for shock analysis to be implemented in the verification process of space structures such as instruments and satellites. With this purpose, the study of the numerical shock simulations is applied to the examples of the STEP and EPT-HET instruments of the Solar Orbiter mission to demonstrate the applicability of the proposed methodology in real cases of space structures. The investigation starts with an overview of the different options for shock simulations compatible with the finite element approach, applying them to the finite element model of STEP instrument to evaluate their characteristics and the precision of the obtained results by comparison with the shock test data. Among the studied options, the modal transient analysis is the most appropriate to calculate the effects of shock loads, highlighting its higher precision, versatility and representativeness of the shock environment, which overcomes its negative aspects related to its higher computational time and amount of output data compared to the other methods. The differences between the numerical and the experimental results in terms of peak values of response accelerations found in this work with the example of the STEP instrument are within the acceptable range of ± 3 dB for the modal transient analyses.

The investigation continues with the evaluation of the precision of the numerical results calculated by modal transient analyses that simulate the shock test environments, studying the influence of the most relevant input parameters such as the damping and the definition of the input acceleration field, which may be non-uniform and with non-negligible accelerations in the three orthogonal axes. This study reveals that the traditional technique to represent the dynamic base excitation by one node rigidly connected to all the interface nodes of the analysed structure is not adequate to represent the input environment measured in the impact shock tests when the acceleration field is not uniform, so that new techniques are proposed in this work to improve the definition of the input acceleration field. Two of the proposed techniques achieve a remarkable enhancement in the precision of the response accelerations when compared to the experimental results extracted from the shock tests of the STEP instrument.

This block ends with the description of the shock verification procedure applied to the STEP and EPT-HET instruments, where the capability of both units to withstand the specified shock loads is finally demonstrated by numerical analyses, following the methodology developed in this work. This procedure is useful to be adapted to future projects with the same need to demonstrate numerically the structural capability to withstand the specified shock loads.

The second block of this doctoral dissertation focuses on the study of the effects of the dynamic or modal coupling, which is produced when the natural frequency of a secondary part, such as a payload or an equipment, is close to the natural frequency of one of the global modes of the primary structure, which in this case corresponds to a satellite. The dynamic coupling is generally avoided in the space structures by imposing limitations on the natural frequencies of the secondary subsystems to ensure the appropriate separation of their local frequencies from the global frequencies of the main system. However, there are particular designs where the dynamic coupling between a payload and the satellite is unavoidable due to other mission requirements, and therefore, the effects provoked by this phenomenon must be studied to predict the structural consequences for both the satellite and the payload. To start the study about the dynamic coupling, a 2-degree-of-freedom system is used to obtain simple analytical expressions that help to understand how the natural frequencies, mode shapes and the modal effective masses change with the degree of coupling. It is shown in this work that the modification of the values of these parameters is remarkable, even for very small values of the ratio between the secondary and the primary masses. The obtained analytical expressions are then applied in two design cases to define the limits of the values of the design parameters to achieve that the entire system meets the imposed requirements related to the natural frequencies.

The investigation about dynamic coupling ends with the numerical calculation of the effects of this phenomenon with the example of the UPMSat-2 satellite as the primary structure and the communication antennas as the secondary part, showing the influence of the degree of modal coupling on the dynamic behaviour of both the satellite and the antennas. One of the most interesting results is the reduction by about 57% of the maximum forces generated on the interface bolts of the satellite when it is subjected to a sine vibration as a consequence of the modal coupling of the antennas, which is a remarkable consequence taking into account that the mass of the analysed antennas is only approximately the 0.25% of the mass of the rest of the satellite. However, the antennas suffer the negative consequences of the dynamic coupling, where the stresses generated on these elements increase more than twice with respect to the decoupled case, exceeding the allowable limit and provoking the structural collapse of these secondary parts.

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ACRONYMS

ASD	Acceleration Spectral Density
CDI	Cumulative Damage Index
CLA	Coupled Load Analysis
CQC	Complete Quadratic Combination
DOF	Degree of Freedom
DS	Damped Sinusoids
ECSS	European Cooperation for Space Standardization
EEPROM	Electrically Erasable Programmable Read Only Memory
EM	Engineering Model
EPD	Energetic Particle Detector
EPT-HET	Electron Proton Telescope – High Energy Telescope
EQS	Equivalent Quasi-Static
ERS	Extreme Response Spectrum
ESA	European Space Agency
ESTEC	European Space Research and Technology Centre
FDS	Fatigue Damage Spectrum
FE	Finite Element
FEA	Finite Element Analysis
FEM	Finite Element Model
FM	Flight Model
FPGA	Field Programmable Gate Array
IDR	Ignacio Da Riva
IEAP-CAU	Institute of Experimental and Applied Physics of the Christian Albrechts University of Kiel
IF	Interface
LVPS	Low Voltage Power Supply
MDOF	Multi Degrees of Freedom
MEM	Modal Effective Mass
MoS	Margin of Safety
NASA	National Aeronautics and Space Administration
NASTRAN	NASA Structural Analysis
NRL	Naval Research Laboratories
NSM	Non-Structural Mass
PCB	Printed Circuit Board
PFM	Proto-Flight Model
PSD	Power Spectral Density
PVSS	Pseudo-Velocity Shock Spectrum
QM	Qualification Model
RBE	Rigid Body Element
RDSD	Relative Displacement Spectral Density
RMS	Root Mean Square
RSA	Response Spectrum Analysis
SDOF	Single Degree of Freedom
SDRA	Shock Damage Risk Analysis
SEA	Statistical Energy Analysis
SRAM	Static Random Access Memory
SRS	Shock Response Spectrum

SRSS	Square Root of the Sum of Squares
STEP	Supra Thermal Electrons and Protons
STM	Structural and Thermal Model
TMD	Tuned Mass Damper
UPM	Universidad Politécnica de Madrid
WL	Wavelet

1 INTRODUCTION

The development of a space system is a complex and multidisciplinary process that involves different specialists working in various fields in an interrelated way to achieve the challenge of allowing the humanity to go beyond the limits of the planet Earth. To achieve this goal, it is necessary to contribute with effort and means in the design of new technologies by applying the most advanced methods.

One of the fields with the highest impact on the development of space systems is the structural design. It is impossible to achieve a design of a system with the objective of reaching the outer space without an optimal structure that, on the one hand, must support the intense mechanical environment generated during the mission and, on the other hand, must be light enough to allow the Earth escape. The structural evaluation of the most severe mechanical loads is a fundamental task since the early stages of the development of a space system, which involves the knowledge of the environment during all phases of the mission, the planning of different approaches for the structural design, the application of the most adequate verification methods and the execution of the necessary tests before starting the mission, to assure that the structure will survive the expected mechanical environment. This set of activities has been implemented in different ways in all space projects from the beginning of the Space Race and, nowadays, most of them have been standardized and compiled in guidelines and normative, as is the case of the European Cooperation for Space Standardization (ECSS) for European space projects. The guidelines and recommendations of the ECSS standards and handbooks related to the structural design and analysis [1,2] have been taken into account for most of the procedures and methods applied and developed in this investigation.

This doctoral dissertation focuses on two particular aspects of the structural design of space systems that are not fully studied and standardized: the shock numerical analysis and the dynamic coupling. Both topics are related to the effects caused by different dynamic phenomena on the space structures.

Shock is one of the loads established in the space industry to classify the mechanical events that happen along the life of a space system. The structural verification against shock is a fundamental activity within the verification plan of space projects. Its effects are potentially harmful for sensitive components such as electronics, optics and mechanisms, so its evaluation during the development phase is considered crucial. The problem concerning this load is the difficulty to define it with precision due to its complex nature, which includes non-stationary effects, high frequency content, transient behaviour and non-linearity. These characteristics have hindered the establishment of standard procedures of shock assessment in the space industry, from the definition of a representative specification to the implementation of verification actions. Compared to other mechanical loads, such as sine and random vibrations, the complexity of the mathematical treatment of shock loads has resulted in a lack of standard rules for the shock verification, especially for the numerical simulations necessary to evaluate by analysis the structural responses of systems subjected to this environment. This fact has slowed down the publication of the ECSS handbook with guidelines for shock verification, which was finally published in 2015 [2].

The most used verification approach for shock is by testing [3], which even with a lower degree of accuracy and repeatability compared to the sine and random vibration tests, is still nowadays the most reliable option to assure the capability of the tested structure to support adequately the

predicted shock environment. The existing methods for shock testing present deficiencies in the execution of representative environments, where some of them cannot reach the required high levels of intensity, while other methods cannot reproduce with sufficient precision the specified input. This lack of precision leads to the realization of tests that, with the aim of guaranteeing that the tested environment is at least as severe as the represented environment, could overpass the required limits, which put in risk the most sensitive components that may suffer unnecessary damages. In recent times, the demand of numerical analyses for shock evaluation is rising up due to the necessity of knowing in detail the effects of the intense shock environment on the tested components to acquire more confidence on their structural capability against this load. Furthermore, there is a growing wish of improving the design of space components by optimizing their structure against shocks, which can only be achieved with a higher precision of the numerical calculations. Therefore, one part of this doctoral dissertation is an in-depth investigation about the implementation and application of numerical shock analyses on space structures to know in detail the different aspects that influence on the precision of the results. This investigation includes an overview of the existing numerical methods with the finite element analysis (FEA) approach, the evaluation and comparison of their characteristics by their application on a real space structure, the proposal of new techniques to improve the definition of the numerical simulations and the description of the planned procedure for shock verification by analysis implemented in the project of a space instrument.

The second aspect dealt with in this doctoral dissertation is the dynamic coupling between a secondary structure and the primary system. This phenomenon has been studied for the application of the tuned mass dampers (TMD) in civil engineering since the beginning of the 20th century [4]. These devices are used in tall buildings to reduce the adverse effects of earthquakes. A TMD consists of a structural subsystem whose mass, damping and stiffness are tuned, so that its natural frequency is near the natural frequency of the main structure to be protected. The achieved effect is a significant reduction of the maximum amplitudes of accelerations, stresses and forces on the primary structure when it is subjected to a dynamic load, but at the expense of high displacements of the moving part of the TMD device [5]. This negative counter-effect found on TMDs can also be produced in complex systems such as a spacecraft with a secondary part whose fundamental frequency is close to one global mode of the main structure. In this situation, the exposure of the system to dynamic loads that excite by resonance these modes can provoke serious damages to sensitive secondary components caused by the high stresses induced by the amplification effect of the dynamic coupling. To avoid this problem, the generalized approach in the space industry is by imposing requirements to the designs of the different systems and subsystems in order to limit the fundamental natural frequency of each level of assembly within a separated frequency range [1,3]. In this way, the dynamic coupling between secondary components and primary systems is avoided by keeping the natural frequencies among the different levels of assembly sufficiently away from each other. The drawback arises for cases where the secondary part is a payload or a component whose structural design, defined according to the mission requirements or by other circumstances, leads to a situation where the dynamic coupling with the primary system is unavoidable. The standard option of imposing requirements that limit the natural frequency of such payload is not valid in this case. Therefore, the problem of dynamic coupling must be carefully studied from the early stage of the design to reduce the risk of damage when the entire system is subjected to the intense dynamic loads generated during its mission. To provide an insight on this phenomenon, the mathematical model of an undamped 2-degree-of-freedom (2-DOF) system is developed in detail in the second block of this doctoral dissertation. The obtained analytical equations allow a comprehensive evaluation of this phenomenon in a simple way. To complete the investigation in this field, a numerical assessment has been included with a case study by using the finite element model (FEM) of the UPMSat-2 satellite.

1.1 Objectives of the doctoral dissertation

The motivation of this doctoral dissertation is to investigate two topics belonging to the field of the structural design and analysis of space systems that have not been reached the full level of development in the space industry. These topics are the shock numerical analysis and the dynamic coupling of space structures.

The fundamental goal of the investigation about the shock response is to establish a methodology for the definition of numerical analyses to simulate with sufficient precision the effects of the mechanical shock loads on the space structures. This methodology should be in accordance with the existing approaches for the rest of load cases. This investigation has been planned to achieve the following objectives:

- Selection of the most suitable method from the existing numerical methods for shock analysis by evaluating and comparing their characteristics (precision, applicability, versatility and representativeness) by the application on a real case of space instrument.
- Evaluation of the influence of the different input parameters of the numerical shock simulations on the precision of the results by comparison with shock test data.
- Proposal of new techniques in the definition of the shock analyses to improve the precision of the calculated results, taking into account the complexity of the input acceleration field measured in the impact shock tests.
- Definition and application of a complete procedure of shock verification by analysis on the project of a space instrument.

The goal of the investigation about the dynamic coupling of space systems is to study the changes in the dynamic behaviour of the entire system caused by the mechanical design of the secondary subsystem. From the conclusions of this work, new design rules are established in order to avoid the negative consequences that can be provoked by this phenomenon. In this context, the following objectives are included in the doctoral dissertation:

- Analytical study of the effects caused by the dynamic coupling between a secondary component and the primary structure by a representative 2-DOF model.
- Numerical assessment of the structural consequences of the dynamic coupling by a case study with the finite element model of the UPMSat-2 satellite.

1.2 Scheme of the doctoral dissertation

The content of this doctoral dissertation have been divided into the following four chapters:

- In Chapter 1, the introduction of the doctoral dissertation is presented with a brief description about the frame of the investigation and the motivations behind it. The objectives and the scheme of the doctoral dissertation are summarized in this first chapter.
- In Chapter 2, the problem of the numerical simulations of the shock loads is discussed. The state of the art of this topic is described in the first subsection of this chapter. The existing numerical methods for shock analysis are summarized in an overview and their characteristics are compared by their application to the numerical models of the STEP and EPT-HET instruments of the Solar Orbiter mission. The influence of the input parameters of the transient analysis is evaluated. To improve the precision of the shock numerical analyses, different techniques are proposed in this chapter to define the input acceleration measured during the shock tests in a more realistic way by considering the effects of a non-uniform input acceleration field with accelerations in the three

orthogonal axes. The chapter concludes with the description of the planned procedure for the shock verification by analysis applied to the STEP and EPT-HET instruments.

- In Chapter 3, the dynamic coupling of space structures is investigated. The first subsection describes the state of art in this field. The analytical study of the effects of the modal coupling with a 2-DOF model is presented in the second subsection, while a case study using the FEM of the UPMSat-2 satellite is analysed in the third subsection to evaluate the numerical results due to the modal coupling.
- In Chapter 4, the conclusions derived from all the work described in this doctoral dissertation are summarized, together with future actions to continue these studies.

2 SHOCK NUMERICAL ANALYSIS

2.1 State of the art

From the initial phase of an engineering project that must deal with the evaluation of the effects provoked by the exposure of the designed structure to a variety of mechanical loads, it is necessary to first know and then classify the different mechanical environments. In space projects, the most severe mechanical environments appear during the ascent phase of the launcher, where the high accelerations of the launcher, the operation of the propulsion system, the aerodynamic effects, and the mechanical events such as the separation between the launcher stages and the release of the spacecraft, constitute a complex mechanical environment that exposes all the involved components to high dynamic and quasi-static loads. Therefore, the approach used in the space industry consists of classifying this complex mechanical environment into different types of loads to facilitate the evaluation of their effects separately in the development of space systems [3]. This classification (see Table 2-1) is established based on the differences between the types of loads, taking into account several aspects such as their variability with the time (static – dynamic), long-time effects (transient – permanent), deterministic or random character and frequency content (low – high frequency) [1,3].

Table 2-1: Classification of the load cases in the space industry.

Load Case	Time variability	Long-time effects	Mathematical treatment	Frequency range*
Quasi-static	Static	Stationary	Deterministic	-
Sine Vibration	Dynamic	Stationary	Deterministic	Low frequency (4 – 100 Hz)
Random Vibration	Dynamic	Stationary	Random	Medium frequency (20 – 2000 Hz)
Acoustic	Dynamic	Stationary	Random	Medium frequency (31.5 – 8000 Hz)
Shock	Dynamic	Transient	Deterministic	High frequency (100 – 10000 Hz)
*Typical values [3]				

The established classification of the main mechanical loads comprises the following load cases:

- Quasi-static loads, which involves the mechanical events whose variation with time is slow enough to consider negligible the dynamic response of the affected structure. The quasi-static specification is represented by a given value of constant acceleration that covers the maximum values of these loads. The quasi-static acceleration is typically defined in g levels for each orthogonal axis.
- Sine vibrations, which represent low-frequency dynamic events typically between 4 to 100 Hz. The standard representation of this load case is a deterministic acceleration of sinusoidal form, whose amplitude is defined as a function of the excitation frequency. In the structural evaluation of the effects on the analysed space systems subjected to this load case, it is established that the input acceleration is applied for each frequency with an infinite duration, where the transient effects are neglected.
- Random vibrations represent the dynamic events of random character produced among others by the operations of the turbo-pumps and the unsteady effects of the combustion,

where the generated dynamic acceleration field is then transmitted through the mechanical interfaces to all the connected parts. This environment is represented by a statistical power spectral density (PSD) function of the input acceleration defined in the frequency range typically between 20 and 2000 Hz. All the related parameters are defined in statistical terms by PSD functions and root mean square (RMS) global values. The transient effects are neglected in this load case.

- Acoustics are loads related to the variations of the sound pressure level provoked during the launch by the aerodynamic effects, which are then transmitted through the air to the affected structures. The specification is given by indicating the intensity of the acoustic load, defined in terms of decibels, as a function of the frequency bands defined by octaves. Typically, the acoustic loads cover a similar frequency range to that of random vibrations.
- Shocks are the mechanical loads related to the dynamic events that are intense and of very short duration compared to the dynamic response of the affected structures. These loads, with a strong transient character, are caused by the activation of pyrotechnic devices used to separate the launcher stages and the spacecraft. The shock specification is represented by an envelope defined in the frequency domain denominated shock response spectrum (SRS).

More load cases are defined in the standards of the space industry, such as the pressure level inside the fairing, the mechanical impact of micro-meteorites and orbital debris, and the thermoelastic effects due to the change of the external temperature, which are generally much less severe than the loads summarized above.

Shock load is one of the principal mechanical loads described in the standards of the space industry [1,2]. Its main characteristics, which make it different from other types of loads, are its intrinsic transient nature, short duration, high frequency content and high intensity. Precisely, these aspects make the mathematical treatment of shock loads more complicated than for the other load cases. This problem affects first the definition of the loads, which in turn makes it difficult to derive the shock specifications, the configuration of the representative tests, and the simulation of numerical analyses.

In space systems, one of the main causes of the generation of the shocks is the activation of pyrotechnic devices that allow the separation of different mechanical components in a sudden way [1,3]. During the ascent phase of the launch vehicle, the pyrotechnic devices are used to break the mechanical interfaces between the launcher stages, allowing the separation of each stage from the rest of the launcher after its propellant has been consumed. Furthermore, other similar devices are employed to open the fairing and to release the spacecraft when the last launcher stage reaches the space. The forces generated by these devices must be intense enough to break the interface pieces that join these systems, but not too much, so that they do not cause serious damages to the rest of the structure and components.

Once the shock is generated from the load source, a severe acceleration field is transmitted to the entire structure through the mechanical interfaces. Therefore, the shock load affects all the subsystems, but not with the same intensity due to the attenuation effects caused by the distance from the shock source, the transmission through the bolted connections and the damping. In the space industry, a classification of the shock loads is established depending on the distance between the analysed structure and the shock source [2]:

- The near-field shock environment is defined for the minimum distance between the shock source and the analysed structure. In this short distance, the shock load is barely attenuated and the transmitted acceleration is a signal that comprises a wide frequency

- range, which can reach up to 1 MHz, and with very large intensity with maximum peaks that can be above 5000 g. The shock wave is dominated mainly by the direct propagation.
- The mid-field shock environment is established for the medium distance from the shock source, with characteristics between the far and near shocks. The transmitted acceleration signal is the result of the combination of the direct shock wave and the excitation of the modes of the structural parts that are in the transmission path between shock source and measured point. The attenuation of the very high frequency waves due to the distance reduces the frequency bandwidth of the transmitted signal below 100 kHz. The intensity also decreases with respect to the near-field shock.
 - The far-field shock environment is defined for distances between analysed part and shock source that are above an established limit, which depends on each case. Generally, the far-field shock is characterized with a maximum frequency of 10 kHz, and with peaks of acceleration on the order of a few thousands of g levels, which have been decayed by the attenuation to about a fifth of the near-field peaks values. This is the most common case for most of the space equipment and components, which are located far from the shock source. The transmitted acceleration is fundamentally the result of the resonance excitations of the modes of the structural parts that conform the transmission path of the shock wave, where the high frequency content of the source signal above 10 kHz have been practically attenuated during the transmission process.

The investigation about shock in this doctoral dissertation corresponds to the far-field environment, which is the shock category that affects most of the equipment that are sufficiently far from the shock source, as it is the case of the analysed space instruments presented in this work.

The use of the mathematical tool “Shock Response Spectrum” (SRS) was adopted by the European space industry to characterize the shock [2,3], simplifying the definition of the input acceleration, where the most important information that is taken into account is the generated maximum peak of the response acceleration of the analysed structure. The SRS is mathematically defined by using a single-degree-of-freedom (SDOF) system that is excited by a transient base acceleration. The SRS curve is a plot that describes the maximum peak of acceleration in absolute terms of the response of the SDOF system as a function of its natural frequency considering a constant viscous damping of the SDOF system [5].

Let us consider a SDOF system with a mass m , a stiffness k and a viscous damping c as the represented in Figure 2-1.

Figure 2-1: SDOF system.

The input acceleration is a forced excitation applied to the base $\ddot{u}(t)$. The relative acceleration $\ddot{z}(t)$ of the system is the difference between the absolute acceleration of the mass $\ddot{x}(t)$ and the input acceleration $\ddot{u}(t)$

$$\ddot{z}(t) = \ddot{x}(t) - \ddot{u}(t). \quad (2.1)$$

Taking into account as main parameters the natural circular frequency $\omega_n = \sqrt{k/m}$, the damping ratio $\zeta = c/(2\sqrt{km})$, and the damped circular frequency $\omega_d = \omega_n\sqrt{1-\zeta^2}$, the equation of motion for the relative displacement $z(t)$ is given by

$$\ddot{z}(t) + 2\zeta\omega_n\dot{z}(t) + \omega_n^2 z(t) = -\ddot{u}(t). \quad (2.2)$$

The solution, assuming that the initial conditions of relative displacement and velocity are null, is obtained with the convolution integral [3]

$$z(t) = -\int_0^t e^{-\zeta\omega_n(t-\tau)} \frac{\sin \omega_d(t-\tau)}{\omega_d} \ddot{u}(\tau) d\tau, \quad (2.3)$$

and the relative velocity is obtained after the differentiation of Equation (2.3) [3]

$$\dot{z}(t) = -\int_0^t e^{-\zeta\omega_n(t-\tau)} \cos(\omega_d(t-\tau)) \ddot{u}(\tau) d\tau - \zeta\omega_n z(t). \quad (2.4)$$

The response absolute acceleration can be calculated from the solutions of the relative displacement and velocity using Equation (2.2)

$$\ddot{x}(t) = \ddot{z}(t) + \ddot{u}(t) = -2\zeta\omega_n\dot{z}(t) - \omega_n^2 z(t) \quad (2.5)$$

$$\ddot{x}(t) = 2\zeta\omega_n \int_0^t e^{-\zeta\omega_n(t-\tau)} \cos(\omega_d(t-\tau)) \ddot{u}(\tau) d\tau + \omega_n^2 (2\zeta^2 - 1) z(t). \quad (2.6)$$

The SRS curve for a given damping value is constructed (see Figure 2-2) by selecting the maximum absolute value of $\ddot{x}(t)$ as a function of the natural frequency f_n , which is included in the parameters of natural circular frequency $\omega_n = 2\pi f_n$ and damped circular frequency ω_d . Different numerical methods can be employed to perform this calculation, one of the most used being the Piece Wise Exact Method [3]. The approach of considering the maximum absolute value of the response during and after the application of the shock load is called the maximax SRS [5], which is the standard method in the space industry. Other types of SRS are described in detail in [5], where only the maximum positive or the minimum negative peaks are taken into account. Other definitions for the SRS [5] only consider the response in a determined period of time, such as the primary SRS, which only takes into account the response during the application of the input, and the residual SRS, which only takes into account the free response of the system after the application of the input load.

Figure 2-2: Construction of the shock response spectrum by a SDOF system [3].

Therefore, the SRS is a way to convert a time signal of acceleration, which is the input base excitation, into a spectrum defined in the frequency domain. In a more concise way, the SRS consists of a set of curves, each one associated with a different value of viscous damping, represented in a natural frequency vs. maximum peak of response acceleration graph, as the depicted in Figure 2-3. But the typical way of specifying the shock loads in the space industry is by using only one SRS curve, which is related to a damping value of 5% or $Q = 10$. Generally, each specification is created as an envelope of different SRS curves that represent different shock events. The resulting SRS curve is constituted in most cases with an initial ramp from 100 Hz to a cut-off frequency (between 1000 and 2000 Hz) in a log-log graph, followed by a section with a value up to the 10000 Hz [2].

Figure 2-3: Set of SRS curves referred to different values of the qualification factor.

The SRS representation is apparently similar to the specifications used for the other dynamic load cases such as sine and random vibrations, but two fundamental differences distinguish the SRS. The first one is the independent variable, which is not the frequency of excitation as for sine and random vibrations, but the natural frequency of the excited system. The other difference is the intensity represented by the ordinate axis, which is not related to the amplitude of the input acceleration, but to the expected response of the system. Both aspects cause that the SRS approach describes the shock in a more indefinite way compared to the rest of load cases, where the characteristics of the input acceleration are not directly provided in this representation, what generates the loss of this information as will be explained below.

The SRS concept was introduced in 1932 by M.A. Biot [6] in his thesis about the investigation of the effects of earthquakes on buildings. Therefore, despite the initially planned application of SRS is the definition of intense and long-term loads such as earthquakes in civil engineering, it was then implemented in other applications and for all types of loads. The mathematical properties of the SRS curves are described extensively by Christian Lalanne in [5], with examples of the most common types of shock pulses (half-sine, haversine, trapezoidal, saw-tooth) and oscillatory functions, including the mathematical algorithms to convert time functions into SRS curves and considering the influence of the most important parameters such as damping, frequency range, changes of the velocity and displacement on the generated SRS.

The SRS approach was established in the space industry to facilitate the definition of the shock specifications because the time history representation, despite being a more direct and natural way to describe a transient acceleration, presents difficulties to extrapolate the shock environments to be applied in representative tests and simulations. Furthermore, it is very complicated to compare the severity among different shocks directly by their time history functions due to the sharpness of these signals, with peaks that can appear in different instants of time and with either positive and negative values. While with the comparison with the SRS approach, the smoother curves allow a clearer identification of the severity among the compared shocks for the different frequency ranges. However, the shock representation by SRS has its disadvantages. The main limitation is that the SRS only takes into account the absolute value of the maximum peak of response acceleration as a function of the natural frequency of the SDOF system. Consequently, there is a loss of the information contained in the original time function, such as the duration of the signal, the instants when the maximum peak occurs, the frequency content of the whole signal and its shape. This lack of information provided by a SRS specification implies that in situations such as the configuration of a representative shock test or the definition of the shock simulation, where it is required to built-up an input time function of acceleration, this input acceleration has to be defined from the given SRS with arbitrary new information to complete its construction. It is in this point where the accuracy between the real and the simulated shocks in terms of time signals is completely lost. Nevertheless, it is important to highlight that the intention of using the SRS format to characterize shocks is not an exact definition of the input acceleration, but the representation of their effects on the analysed structure in a simpler way to facilitate its extrapolation. With this premise, it is supposed that two completely different time functions of acceleration that are related to the same SRS curve defined for a given damping will cause the same effects in terms of maximum peak values on the analysed structure. This is true for a SDOF system by definition, but it is not clearly demonstrated for multi-degrees-of-freedom (MDOF) structures. This aspect has motivated that one of the objectives of the doctoral dissertation is the evaluation of the differences found in the numerical results obtained by transient analyses using different time input accelerations functions from a given SRS.

In the space industry, the idea of describing the complex mechanical environment induced during the launch phase as a combination of different types of loads has as fundamental objective to

facilitate the evaluation of the structural behaviour of the space systems for each type of load, focusing on the particular problems that each load can cause, what finally leads to more efficient structural designs. In this context, the efforts made to simplify the mathematical definition of the mechanical loads were addressed to the establishment of standardized definitions for each mechanical load, trying to keep the loads descriptions as simple as possible, but without losing the representativeness of the real environment.

One of the most important load cases established in the space industry is the sine vibration (see Table 2-1). The real environment represented by this load case corresponds to the low frequency dynamic accelerations, which during the launch are caused by phenomena like the POGO effect, unsteady events of the combustion and other mechanical events provoked by the performance of the rockets [1,3]. This mechanical environment is complex and with a non-negligible transient character. But it was agreed [1] that the best way to characterize this environment is by assuming that is a long-term sinusoidal acceleration, which is represented by a plot that describes the amplitude of the input oscillation as a function of the excitation frequency. This way to characterize the low-frequency mechanical environment allows an easier mathematical treatment for both analytical and numerical calculations in the frequency domain, where the response is directly related to the modal behaviour of the analysed structure. It also constitutes an advantage in the configuration of representative tests, where this type of specification can be easily reproduced with sufficient accuracy in the electrodynamic shakers.

In the case of the shock representation by SRS, this approach gets a significant simplification of the description of the real environment, but without the expected improvement in the objective of achieving more efficient and accurate numerical simulations and representative tests. The drawback related to numerical shock simulations is that the analysed structure is generally a complex system, where the transient effects generated by the shock load cannot be ignored due precisely to that they are the main contribution of the shock. These aspects make it necessary to consider a higher level of detail in the simulations compared to the stationary analyses usually defined for sine and random vibrations, which conflicts with the lack of information provided by the specified SRS. This situation leads to the fact that a reliable calculation method for shock simulations has not been standardized yet. Different options, which are to be explained in detail in this doctoral dissertation, have been proposed and some of them used in the space industry. However, most of them present serious deficiencies in the accuracy of the results and in the representativeness of the real effects on the analysed structure that are a consequence of the limitations of the SRS approach.

In the field of the shock testing, the situation is not much more favourable. One of the obstacles is the high intensity level required to simulate the severity of the shock environment generated during the launch. The required input accelerations can reach values on the order of thousands of g levels, which is well above the capability of many of the tests techniques used for other mechanical environments, such as the electrodynamic shakers. To achieve the required input levels, different types of tests methods have been developed specifically for shock tests. Most of these facilities, which are nowadays widely used to qualify the space structures against shock, present diverse difficulties to reproduce with sufficient precision the required input SRS. The reason is that the state of art in shock testing has not reached yet a suitable level of maturity. Nowadays, the state of art in shock testing is represented by impact tests benches where the tested item is attached to a horizontal table that is hit by a falling piece or a pendulum, depending on the intended direction of the shock load, which, in the instant of the impact, generates the shock that is then transmitted to the mechanical interface of the tested item [7–10]. To reproduce the specified SRS, only a few input parameters can be configured such as the initial height and the mass of the hitting piece, the point of impact and the material of the impact zone, whose values

are finally selected after a trial-and-error process that may involve a great number of trials. In many cases, after this process the differences between the specified SRS and the achieved SRS are greater than the required quality limits, what generally leads to the realization of shock tests much more severe than the specification, putting the tested item at risk of structural damage.

This presentation summarizes the problems that the space industry has nowadays concerning the mechanical shock verification. Many of these problems have arisen as a direct consequence of the decision of establishing the SRS approach to represent the mechanical shock in the verification process. An efficient way to face these problems is to reconsider this decision and propose new alternatives to define the shock specifications, taking into account the trade-off between the representativeness of the real environment and the suitability and feasibility for the verification tests and simulations. The study about new ways of specifying the shock in the space engineering is beyond the scope of this work, but it would be an interesting subject for future works in this field. Other researchers have dealt with this problem by proposing new ways of defining the input shock. For instance, the pseudo-velocity shock spectrum (PVSS) proposed by H. A. Gaberson [11] is a plot very similar to the SRS, with the difference of using in the ordinate axis the pseudo-velocity parameter instead of the peak of absolute acceleration. The pseudo-velocity is the ratio between the maximum peak of absolute acceleration and the natural frequency of the SDOF system. The reason behind the utilization of this representation is that the pseudo-velocity $v_{\max}$ is directly proportional to the maximum generated stress $\sigma_{\max}$ on a material as given by [2,11]:

$$\sigma_{\max} = K\rho cv_{\max} \quad (2.7)$$

where ρ is the density of the material, $c = \sqrt{E/\rho}$ is the velocity of the traction-compression wave, which depends on the Young modulus E , and K is a shape factor applied when the bending waves are dominant in the shock transmission.

Therefore, the level of severity can be evaluated with the PVSS in a more direct way than with SRS. This way of defining the shock specifications is usual in the American industry [12], but is less common in European projects. The PVSS specification is defined by using a four-coordinate graph, as illustrated in the example in Figure 2-4. In this graph, the maximum absolute acceleration, which is the parameter used in the SRS, and the relative displacement are included together the pseudo-velocity to evaluate the level of severity with these three different criteria. Depending on the analysed item, the most determining parameter to evaluate the risk of failure under a shock load can be the relative displacement, the maximum stress related to the pseudo-velocity, or the maximum absolute acceleration.

Figure 2-4: Pseudo-velocity shock spectrum.

Another option, which is proposed in [13], is to replace the existing standard method of SRS by a representation with wavelets coefficients. In this report, after describing the limitations of the SRS approach, the proposed method is defined together with its applications and examples, concluding that the procedure of deriving shock specifications can be improved compared to the standard SRS approach.

Despite these investigations to improve the approach to define the shock loads, the SRS is currently the standard method in the European space projects. The user's guides of the different launchers specify the shock loads generated during the launch operations by a SRS, which is created to envelope the particular SRS curves obtained from the different shock events. When the team of engineers and designers of a spacecraft wants to demonstrate the capability of the structure to support the shock load specified by the launcher, the only available information is the provided SRS curve, which is usually defined for the qualification factor $Q = 10$. With this limited information, the spacecraft engineers must plan the verification activities, which include the derivation of the shock levels to the payloads and instruments, the configuration of the shock tests and the execution of numerical analyses.

The derivation of the shock load consists in estimating the shock levels at one location of interest of the spacecraft structure, generally at the mechanical interface of an instrument, by the application of calculations or empiric rules knowing the shock levels at another point, which is in most cases at the mechanical interface between spacecraft and launcher. Different approaches documented in the European standards [2] and other technical reports can be followed. In [14], a database was established with the information related to the transmission of shock levels through the spacecraft structures compiled from various space programs. The purpose is to use this information in future space projects to create empirical rules that help in the specification of the shock levels to the payloads and equipment. This procedure was applied in [15] with the shock test data extracted from the SPOT-5 mission. Another approach consists in using analyses to calculate these shock levels. In [16], various analysis methods were proposed with the aim of predicting the shock acceleration levels at the different parts of the INTEGRAL spacecraft generated by the Ariane 5 launcher. In [17,18], FEA simulations and tests data were employed to calculate the propagation of the shock levels through space structures to improve the estimations. All the techniques mentioned have in common that the calculated shock levels are defined in the SRS format. The precision of the results achieved in most of these techniques is very low due to both the intrinsic complexity of the shock loads and the limitations of the SRS approach.

Once the input shock levels have been specified for a space system, which can be a spacecraft or a component, the next step in the verification process is to evaluate the capacity of this structure to support these input levels. Different ways to perform the shock verification are defined in the space industry [2], which are not fully standardized unlike for other mechanical environments such as the static, sine and random vibrations. The most used verification method is by shock testing [19], where the item is subjected to a mechanical environment that shall be at least as intense as the flight environment. The integrity of the tested structure must be verified after the completion of the shock tests with low-level mechanical tests, functional tests and visual inspections. There exist different shock test techniques that can be employed to qualify space structures depending on the category of the tested structure (spacecraft or equipment), the maximum levels of intensity and the required precision.

The most representative test technique is by the use of the same type of pyrotechnic devices that will be employed during the launch events. Generally, this is applied in test facilities conducted by the launcher's engineers to test the spacecraft that will fly on board. Examples of such facilities are the SHOGUN (Shock Generator Unit) and VESTA (VEGA Shock Test Apparatus), which

can generate the shock events caused by the Ariane 5 and VEGA launchers respectively [2]. This shock test technique can simulate with precision the flight mechanical environment without the need of using the SRS approach or other assumptions that simplify the input levels, what confers it the maximum representativeness among the different shock test techniques. However, the utilization of this technique is limited due to its higher costs and the impossibility of its utilization to components and instruments.

Electrodynamic shakers, which are well-established tools for sine and random vibrations tests, can also be employed for shock tests, with the advantage of generating uniform shock acceleration fields with a sufficient precision to match the specified input levels, which can be defined either by SRS or by pulses. Its main drawback is the limitation of the maximum input levels that can be achieved, which in many cases make them unsuitable for qualification shocks of space structures [2,5].

Other shock test techniques are widely employed to qualify the space structures, which can reach the required levels, but with less precision in the generation of the specified input loads. One of these shock test methods consists in reproducing the same effects of the shock in terms of acceleration by the mechanical impact between a robust piece and the test bench where the tested item is joined. Depending on the required direction of the generated shock, the robust piece can fall freely in vertical direction to hit the upper side of the test bench (out-of-plane shock tests), or it can hit in the lateral edge of the test bench by a pendulum mechanism (in-plane shock test) [7–10,20]. In both cases, the initial height and the mass of the robust piece and the contact material are chosen after a trial-and-error process to achieve the reproduction of the specified input levels at the mechanical interface of the tested item [10]. Generally, the values of these parameters are found after several trials. To improve the current procedure, the influence of these input parameters on the generated shock with the support of numerical simulations and analytical estimations of the mechanical shock has been studied in following works. The shock test facility by mechanical impact developed by the European Space Research and Technology Centre (ESTEC) is employed in an investigation to find the effects of the drop height, the mass and dimensions of the hammer, the mechanical properties of the anvil and the location of the tested item on the generated SRS [10]. The numerical simulations with FEM of the test setup to predict the shock levels are included in the aforementioned work. In [7,8], the problem of the in-plane impact with a hammer pendulum is studied by means of analytical calculations with simple models of both the robust piece and the impacted plate. The impact generates in this case a non-uniform acceleration field due to the excitation of the in-plane normal modes of the impacted plate. The in-plane impact excitation is also investigated in [9] with numerical simulations using a detailed FEM of the test bench, which provides accurate results that help in the definition of the input parameters reducing the number of trials before the test.

An alternative option with the same approach of getting the required shock levels by impacts is with the use of projectiles that are shot against a plate that is mechanically connected to the tested item. Various works have treated this test method in their studies. In [21], an experimental setup consisting of a light gas gun to shoot projectiles is used to investigate the influence of the input test parameters, which in this type of test method are the mass and velocity of the projectiles, on the generated shock levels in SRS format. The same test facility was then used in [22] to analyse the transient response of different configurations of the GOCE satellite. The experimental results compiled in the two aforementioned works allowed the creation of a database to validate the numerical tools that help in the assessment of the mechanical environment generated by the impact of micrometeoroids and space debris. One of these tools is an estimation method proposed in [23] to predict the complex acceleration field transmitted in plates subjected to high-speed impacts.

The lack of precision on the generated acceleration field achieved by the described mechanical impact test methods is still nowadays their fundamental problem, which has as a direct consequence the risk of damage to the tested items due to the unavoidable over-testing presented in many situations. This problem of accuracy comes from the use of the SRS approach, which has already been commented below, and due to the difficulty of matching the specified SRS with only a few input parameters, which are usually defined after a trial-and-error process. Generally, more than one accelerometer is used to measure the input signals, showing in many situations the differences between the input accelerations, which indicate that the generated acceleration field in these test facilities is non-uniform, unlike the specified input. Furthermore, the cross input accelerations that are usually measured in the shock tests with tri-axial accelerometers show that these signals cannot be considered negligible compared to the component of the input acceleration in the main axis, indicating that the acceleration fields achieved during the shock tests are more complex than expected. All these aspects make it necessary the implementation of accurate and standardized numerical methods that take into account these phenomena with the aim to analyse the effects of the shocks in the space structures. Precisely, one of the objectives included in this doctoral dissertation is the proposal of new techniques to represent with higher fidelity these phenomena in the numerical simulations with the FEA approach, comparing the calculated results with the shock test data to evaluate the achieved precision.

In addition to the tests, other alternatives can be applied in the space projects to justify the capability of the structure of a spacecraft or component to withstand the specified shock loads. In the handbook of shock design and verification provided by ECSS [2], different approaches are described to demonstrate the shock verification. One of the proposed options is the justification of the capability to support the shock by the comparison with other specified mechanical loads. For example, if a structure has been successfully tested in an intense random vibration test, and if it can be demonstrated that the tested random vibration levels are more severe than the specified shock levels, there exist the possibility of justifying that the structure will support adequately the shock environment without the need to perform any shock test. The main difficulty of this approach is the comparison between the random and shock specifications, taking into account their conceptual differences in the mathematical definitions for both load cases, where random vibrations consist of steady mechanical oscillations described in statistical terms with a PSD function, while the shock is a deterministic load described with the SRS format, which represents the peak response of acceleration of a SDOF system subjected to a transient acceleration. To overcome this problem, different equations are provided in [2] to convert the PSD function into an equivalent Extreme Response Spectrum (ERS), which is defined in the same terms as the SRS to be easily compared with it. The justification that the shock verification can be covered by the random vibration tests is achieved if the equivalent ERS is higher than the SRS for the frequency range that contains the main modes of the analysed structure. In [24], the Fatigue Damage Spectrum (FDS) is used as an alternative mathematical tool to compare the shock levels with the random vibration levels, using as a common criterion the risk of damage due to fatigue caused by both environments on the analysed structure. Other analytical tools to find the equivalences between the rest of main mechanical load cases, such as static and sine vibrations, and the shock are described in [2].

Another approach consists of the verification by similarity [2]. The objective is to avoid the repetition of unnecessary shock tests in structures designed from previous systems that have successfully passed the qualification tests. This type of situations is given in programs that develop many spacecraft or equipment in a consecutive manner for the same type of missions, what leads to the repetition of most of the design characteristics of the initial product. Therefore, if the key structural characteristics of the design of the new system are the same or with minor

changes than those of the tested structure, and if the specified shock load is at least as severe as the shock load tested on the first system, the new product can be certified by similarity indicating that it can support the specified shock environment. This approach cannot be applied in most of the space systems, which are usually new designs completely different from any previous system.

The last verification approach commented in this review of the state of art is the numerical analysis. There is a growing demand to implement reliable numerical simulations in the verification plans of the space projects to evaluate the impact of the mechanical shocks on the space systems. Due to the existing limitations of shock testing and the rest of verification approaches, the interest to know better the effects of the shock with numerical calculations is increasing in these last years. The numerical results can provide much more information about the structural response of a system than the shock tests, which are usually limited to the accelerations measured by the sensors in a few locations. The simulations can calculate stresses, strains, forces, accelerations and other derived parameters that allow a complete evaluation of the structural integrity of the analysed model subjected to the input shock. Furthermore, if it can be demonstrated that the precision of a simulation method is adequate, the structural verification of a system can be justified by analysis without the need of performing tests. This situation is considered in the European standards [1] for the FEA approach used to simulate the mechanical load cases of inertial static acceleration and the dynamic sine and random vibrations. However, shock simulations have not reached yet the level of maturity necessary to consider this possibility at the moment [2]. The transient effects cannot be ignored in shocks as in the cases of sine and random vibrations, because these effects are the main contribution in the structural responses. The high frequency content of the input load, which covers at least up to 10000 Hz, makes it necessary to take into account all the modes of the affected structure over this wide frequency range when the modal approach is used for the simulations. This implies the consideration of a higher quantity of modes compared to the other dynamic loads, what increases the computational cost in the numerical simulations. Furthermore, the FEA approach is generally less precise at high frequency. This aspect can be improved by modelling in detail the structure, but it requires more nodes that increase even more the computational cost of the analysis. Finally, the high intensity of the shock together with its transient nature contribute to the generation of nonlinear effects on the structural responses that are difficult to take into account in the simulations, what can suppose a lack of precision of the numerical results.

Recent studies focus on the improvement of the modelling techniques and the definitions of the numerical simulations to achieve the required precision in the shock analysis [25]. Many of these investigations have worked with the FEA approach, which is the most employed and standardized method in the space industry for the structural analyses with the rest of load cases, and therefore, it constitutes the first candidate method for shock simulations. In [26], two different techniques to model a PCB, detailed with 3D elements and simplified with 2D planar elements, are employed in the shock simulations to evaluate the precision of the calculated results with shock test data. The modelling of the joints is assessed in [27] for a truss frame structure subjected to a shock environment and in [28] to evaluate the shock propagation, while the effect of a new damping definition for bolted joints of space structures is studied in [29]. In [30], the meshing characteristics as the type of elements and the modelling of the mesh transitions are investigated to know their influence on the results of transient analysis, while the modelling of the material interfaces is investigated in [31]. The response of the composite materials subjected to shock loads is investigated in [32] by FEA simulations that reproduce the test setup by impact of projectiles, where the numerical results are compared to the test data. FEA approach is also used in [33] to characterize a pyro-shock test method that generates the shock load by the detonation of an explosive charge. In [34], the FEM of the parabolic mirror assembly of the Sentinel-3 spacecraft is correlated with the shock test results to finally achieve more realistic input shock specifications

for their components. In [23], the influence on the results of the parameters that define an input load represented by a trapezoidal pulse force is studied to correlate with the measurements obtained from the shock tests. In [35,36], the effect of the nonlinear modelling for the clamp band separation system is evaluated in a coupled dynamic system composed of models of a launcher and a spacecraft subjected to the shocks generated by the activation of the pyrotechnic devices. Other studies [37–41] employ detailed FEM with 3D elements to enhance the precision of the calculated results, with the disadvantages of requiring more time of analysis and of obtaining a higher amount of output data. Apart from the FEA approach, other numerical alternative methods have been studied for shock simulations. The Statistical Energy Analysis (SEA) method can be appropriate for structural simulations with high frequency loads, but it presents deficiencies in providing precise results at particular points or elements of the structure [2]. To improve these limitations, the combined SEA-FEA method is evaluated in [42] for shock analysis, while in [43], the SEA is used in conjunction with Virtual Mode Synthesis and Simulation (VMSS). Other numerical methods investigated for shock simulations are the Spectral Element Method (SEM) [44] and the hydrocodes [41]. The drawback of these alternative numerical methods is that their use is not very extended in the space industry and they present particularities that hinder their implementation in the space projects. The advantage of the FEA approach is that it is an established method for the main load cases, and it would be easier to expand its use for shock simulations, where the same FEM used for static, sine and random vibrations analyses can also be used for the shock analysis. Furthermore, it provides an acceptable prediction for small structures, where it is less problematic to have a sufficiently fine mesh for accurate results without an excessive quantity of nodes.

The purpose of this doctoral dissertation is to continue the investigations carried out for shock simulations with the FEA approach to improve the existing techniques in the analysis of space systems with the goal of establishing a methodology that can be implemented in future space projects.

2.2 Investigation of the numerical methods for shock analysis

The investigation carried out in this doctoral dissertation in the framework of shock numerical analysis is motivated by the need to implement a methodology for the analysis of space systems. In particular, the work and the results presented in this document have been performed for the development of the instruments of the EPD project, which is a payload of the Solar Orbiter mission. The description of the instruments is included in section 2.2.1, while the shock tests performed on the STEP unit are detailed in section 2.2.2. The purpose of this investigation is to establish a procedure of modelling and analysis of mechanical shock to be applied in the verification of space instruments. The investigation starts with an overview of the existing types of analysis compatible with the FEA approach. These methods are applied to the FEMs of the EPD units, which are described in section 2.2.3, to study their characteristics and to evaluate their precision by the comparison with the data extracted from the shock tests. From this comparison presented in section 2.3, it will be concluded that the modal transient analysis is the best option to simulate the shock. The influence of the input parameters of this method, such as the damping and the definition of the input acceleration, is evaluated in section 2.4. To improve the precision of this method in the simulations of the shock environments measured in the tests, different techniques to represent the input base excitation are proposed in section 2.5. These new techniques achieve an important enhancement of the level of precision of the numerical results calculated in the simulations of the shock test environment. Finally, the complete set of activities planned and performed in the shock verification procedure of the EPD units is described in section 2.6, where the approach of verification by analysis was the option that allowed the certification of the structural capability of these instruments against the specified shock loads.

2.2.1 EPD project description

The Solar Orbiter is a scientific mission of the exploration programme of the European Space Agency (ESA) within the framework of the Cosmic Vision 2015-2025 Programme. The spacecraft and its instruments are being developed in collaboration with other partners such as NASA and the main companies of the space industry and it is scheduled to be launched in 2020 with an Atlas V launcher. It is conceived to study the Sun and the heliosphere in much more detail than the previous missions did. One of the payloads is the Energetic Particle Detector (EPD), which is composed of different instruments located in separated zones of the spacecraft structure [45–48]. This set of instruments has the objective of analysing the particles expelled from the Sun, where each instrument is specialized in the collection of particles within a selected energy range. Two of these instruments are the Supra Thermal Electrons and Protons (STEP) and the Electron Proton Telescope – High Energy Telescope (EPT-HET), which have been mechanically designed by the research team of the Institute of Experimental and Applied Physics (IEAP) that belongs to the Christian Albrechts University of Kiel (CAU), in Germany.

The STEP instrument (see Figure 2-5(a)) consists of two detectors that will collect the electrons and protons in the low energy range (3 – 100 keV), mounted on an electronic box that contains inside the three main printed circuit boards (PCB) located horizontally. These PCBs are, from bottom to top, the Low Voltage Power Supply (LVPS), the Digital and the Analogic boards. Two radiators, one located at the front and the other one at the top of the instrument, have the purpose of evacuating the excess of heat generated during the operations of the instrument. The mechanical interface of the instrument is constituted by 6 bolts with Ultem® standoffs, that join the unit via a mounting bracket to the structure of the Solar Orbiter spacecraft. The total mass of the instrument is approximately 2 kg and the structural parts are made of aluminium alloy EN AW 6061-T6 and titanium alloy Ti-6Al4V. The maximum dimensions of the unit are 127 x 188 x 137 mm.

The Solar Orbiter spacecraft has two identical EPT-HET units, which are very similar in design, materials and dimensions to the STEP instrument. Each EPT-HET unit (see Figure 2-5(b)) is an electronic box with two different telescopes attached on the top, the EPT and the HET. The electronic box is divided into two parts, the lower part that hosts the LVPS and Digital boards, and the upper part that contains the Analogic board, together with the EPT and HET boards. At the bottom, 4 bolts with Ultem® standoffs constitute the mechanical interface of the instrument that connects it to the structure of the Solar Orbiter. The main structural parts are made of the same aluminium alloy used for STEP instrument and the mass of each EPT-HET unit is approximately 1.6 kg.

Figure 2-5: Designs of EPD units.

The structural properties of the materials of the EPD instruments, including the copper and polyimide of the PCBs, are indicated in Table 2-2.

Table 2-2: Material properties of the EPD units.

Material	Density (kg/m ³)	Young's modulus (GPa)	Poisson ratio
EN AW 6061-T6	2710	68.9	0.330
Ti-6Al4V	4430	113.8	0.342
Polyimide	1850	24.8	0.180
Copper	8960	110.0	0.340
Ultem®	1270	3.2	0.300

Both STEP and EPT-HET instruments have followed the same development from the early design phases up to the qualification and acceptance tests. One of the most critical points of their development has been their structural qualification. The initial verification plans for both instruments are based on the prototype approach, where apart from the flight models (FM) that will be mounted on board the Solar Orbiter probe, qualification models (QM), structural-thermal models (STM) and engineering models (EM) are also manufactured to perform with them the qualification and development tests during the project. After passing successfully the structural tests with the STMs, the next step was the qualification tests with the QMs of both instruments. The first unit that was tested was the EPT-HET QM, where after passing the sine vibrations tests in each of the three orthogonal axes and the random vibrations tests in the lateral axes, it was not able to withstand the last random vibration test in the vertical axis (Z). By the functional tests and the visual inspections done after the vibration tests, it was found that the pins and electrical connections of some electronic components were irretrievably broken, provoking the malfunction of the operations of the unit [48,49]. This fact motivated the in-depth investigation about the causes of the failures, which influenced the decisions taken later about the structural design and verification strategy. The taken actions were addressed in two directions (see Appendix A). On one side, the input levels were revised by ESA to be reduced accordingly to more accurate estimations, while on the other hand, the mechanical designs of the instruments were improved by adding stiffeners attached to the centres of the PCBs to reduce the maximum amplitude of their oscillations during the mechanical vibrations. Due to the problem of the QM, the verification strategy was redefined to a proto-flight approach, where the qualification and acceptance objectives should be reached in the same test campaign (proto-flight tests) performed on the FMs, which became the proto-flight models (PFM). These actions were decisive to successfully pass the proto-flight vibration tests [48,49].

The pending task of the EPD units relative to the structural verification at the end of the development phase was the mechanical shock verification. Due to the that the shock qualification was not initially taken into account for these instruments in the verification plan and that the schedule constraints did not allow to include the shock in the qualification tests, the shock verification was finally postponed at the end of the development phase, when the PFMs of both units had already been manufactured, tested and delivered for the assembly on the Solar Orbiter probe. In this point, few options were available to demonstrate the structural capability of the EPD instruments against the specified shock input environments, taking into account that the design modifications were not possible at this stage. Therefore, different actions were proposed to achieve the shock verification of these instruments. The planned and performed actions of the shock verification procedure of the EPD units are detailed in section 2.6.

2.2.2 Shock tests of STEP instrument

The shock testing is not recommended for PFM units [2] due to the great risk of failure for the flight parts, especially for the most sensitive components, which for the EPD instruments correspond to the electronics components distributed on the PCBs. STM and QM are more adequate for shock tests, so that it was decided in this project to perform the shock tests on the STEP QM. The EPT-HET QM was inoperative due to the severe failures found after the random vibrations tests [48,49], where the pins of the most susceptible electronic components broke due to the high deflections of the PCBs generated as a consequence of the resonance of their main modes. With the lessons learned after this event, the shock tests were carefully planned to avoid the same type of failures on the STEP QM.

Among the different shock tests methods summarized in section 2.1, the selected one for STEP QM was by the mechanical impact of a hitting piece on a test bench. The shock tests were carried out at the Airbus Defence & Space Environmental Testing Laboratories in Portsmouth, England. For the in-plane tests, the instrument is attached horizontally to an adaptor plate located on the drop table, which receives on a lateral edge the impact of the hitting piece joined to a pendulum (see Figure 2-6). For the out-of-plane shock tests, the instrument is located in the same position, but the impact is performed by a piece that falls freely inside a hollow vertical duct to hit on the upper face of the drop table (see Figure 2-7).

Figure 2-6: Scheme of the shock test of the STEP QM for the in-plane (Y-axis) excitation [50].

Figure 2-7: Shock test setup of the STEP QM for out-of-plane (Z-axis) excitation [51].

The tests performed on the STEP QM were the in-plane shock test in Y-axis with the specified qualification levels and the out-of-plane test in the vertical Z-axis with low levels (-12 dB of the specified qualification levels). The objective of this second test was the correlation of the FEM of the STEP instrument for shock analysis with the experimental data to predict numerically with sufficient precision the response in terms of maximum stresses, forces and acceleration when the instrument will be subjected to the qualification full levels of shock. For both test configurations, the impact zone is constituted by an anvil that is joined to the drop table that receives the impact. The input parameters are the mass and the initial height of the hitting piece and the material of the anvil. The values of these parameters are configured to reach the required input levels of acceleration at the mechanical interface of the tested item by a trial-and-error process, which is performed with a dummy structure that substitutes the tested item during this process. The criterion used to determine whether a combination of the input test parameters are adequate to represent the specified environment is to achieve that the accelerations measured by the input accelerometers in SRS format yields within the required limits of +6 dB / -3 dB with respect to the shock specification. To measure the input acceleration, two reference tri-axial accelerometers (Ref1 and Ref2) are located on the adaptor plate near two diagonally opposite interface bolts (see Figure 2-8), as recommended the European standards [2] for shock testing. Both accelerometers obtain the input signals in each of the three orthogonal axes, allowing to know if the shock tests generate non-negligible accelerations in the directions perpendicular to the axis of the intended excitation. The response of the instruments under the shock load is measured by a set of accelerometers located in different points of the structure (see Figure 2-8), being mostly tri-axial accelerometers. Also, a one-directional accelerometer (A1-3Z) is located just on the head of one of the interface bolts (IF4), with the aim of measuring the level of attenuation of the acceleration in Z-axis when is transmitted through the interface bolts. With these accelerometers, the time acceleration functions and their corresponding SRS curves are experimentally obtained and are

used in the investigation documented in this doctoral dissertation for the comparison with the simulation results.

Figure 2-8: Shock tests instrumentation on the STEP QM with reference accelerometers (Ref1 and Ref2), tri-axial response accelerometers (TriA2 – TriA7) and one-directional response accelerometers (A1-1Z, A1-2Z and A1-3Z) [50,51].

The input accelerations measured during the in-plane (Y-axis) shock test recorded by the two reference accelerometers in each of the three orthogonal axes are shown in Figure 2-9. The corresponding SRS curves are shown in Figure 2-10, where they are compared to the shock specification and the tolerance range. It can be appreciated that in this test, the differences between the input signals measured by both reference accelerometers are small. Also, the levels of the cross acceleration in Z-axis are on the same order of magnitude of the levels in the main direction (Y-axis). The SRS curves of acceleration in the main direction do not match perfectly the specified SRS, but they meet the requirement of being within the tolerance limits in all frequency range.

Figure 2-9: Input time acceleration signals for in-plane shock test measured by Ref1 and Ref2 reference accelerometers [50].

Figure 2-10: Input SRS curves of the in-plane shock test measured by Ref1 and Ref2 accelerometers. Y-axis: main direction of excitation. X and Z axes: cross directions [50].

The input signals of acceleration extracted from the low-level shock test in Z-axis are shown in Figure 2-11 for each orthogonal axis, comparing the measurements between both reference accelerometers. For this case, where the performed test is out-of-plane, the differences between the input signals are greater than those of the in-plane test. These differences are also appreciable in the comparison between the corresponding SRS curves shown in Figure 2-12. The reason is that the impact of the out-of-plane test excites mainly the bending modes of the drop table, which generates a higher degree of non-uniformity in the acceleration field transmitted to the adaptor plate than the shear and traction-compression modes excited during the in-plane tests.

Furthermore, the experimental acceleration signals in the three axes are on the same order of magnitude, with peaks that are above 150 g (see Figure 2-11). This result is also appreciated in the comparison of the SRS curves (see Figure 2-12), where in the high frequency range (2000 – 10000 Hz), the SRS curves measured in the cross axes reach and even exceed the SRS curves in the main axis of excitation. The numerical simulations that take into account these effects are investigated in this document in section 2.4.2.

Figure 2-11: Input time acceleration signals for out-of-plane shock test measured by Ref1 and Ref2 reference accelerometers [51].

Figure 2-12: Input SRS curves of the out-of-plane shock test measured by Ref1 and Ref2 accelerometers. Z-axis: main direction of excitation. X and Y axes: cross directions [51].

Usually, an acceptable level of precision in the generated input levels compared to the specification is more difficult to achieve with the out-of-plane tests than with the in-plane tests. For the case of the STEP QM, it can be appreciated in Figure 2-12 how the input levels in the main axis (Z) are outside the required limits in the low frequency range (100 – 300 Hz). Considering also the high levels of the cross accelerations, the resulting shock can put at a serious risk of damage to the sensitive components due to undesirable over-testing. For this reason, it was decided in this project to stop in this point the shock qualification tests of the STEP instruments and to try other options for shock verification, which are detailed in section 2.6.

A positive aspect of this shock method is its repeatability, which is a requirement that shall be demonstrated in at least three consecutive trials. The time acceleration signals recorded by a reference and a response accelerometers during the three consecutive trials of the out-of-plane tests of STEP QM can be seen in Figure 2-13, demonstrating the excellent repeatability of the performed impact tests.

Figure 2-13: Comparison between the time acceleration signals in the three consecutive shock out-of-plane tests with the same conditions. (a) Signals measured by Ref2z reference accelerometer. (b) Signals measured by TriA2z response accelerometer [51].

2.2.3 Finite element models

The recommended method for structural calculations in the European space projects is the FEA [1], being the NASTRAN code one of the preferred and most used programs to perform these simulations. The initial purpose of the FEM of the STEP and EPT-HET units described in this section was the calculation of the main results (stresses, strains, forces) for the standard structural load cases such as inertial static acceleration, sine and random vibrations. With the need to perform shock simulations, both models are employed in this doctoral dissertation to investigate the application of the different types of shock analysis with the FEA approach in the verification of space instruments.

The STEP FEM (see Figure 2-14(a)) contains approximately 80000 nodes and 80000 elements, mainly planar elements of quadrilateral (QUAD4) and triangular (TRIA3) shapes. The QUAD4 elements are the most numerous, while the TRIA3 elements are used sporadically in geometrically complex zones, where an irregular mesh is needed. The EPT-HET FEM (see Figure 2-14(b)) contains about 30000 nodes and 30000 elements, which as for the STEP FEM, are mostly QUAD4 planar elements that represent the thin-walled structural parts.

Figure 2-14: Finite element models of EPD instruments.

The PCBs of both instruments are modelled with shell elements (QUAD4 and TRIA3), where the mass contribution of the electronic components is taken into account by a uniform distribution for each PCB of a non-structural mass (NSM) to add this mass to the structural mass of the PCBs. This approach, known as the globally smeared technique [52], was selected by its simplicity, which avoids the excessive amount of output data generated by the dynamic analyses when other options such as the detailed modelling of the components by 3D elements are used. The models fulfil the checks required for the FEMs used for European space projects [53], which among other aspects, affect the modelling assumptions, the topology of the quadrilateral elements (skew, taper, aspect ratio and warp) and the dynamic behaviour of the models related to the rigid body modes and the strain energy.

With the purpose of employing the same FEMs for shock analyses, it is important to check if they meet the recommendations indicated by ESA in [2], which are described below.

The first recommendation is related to the maximum length of the elements, which should be at least 8 times shorter than the flexural wavelength in thin-walled structures. In both models, the main parts are made of aluminium and titanium alloys with a minimum thickness of 1 mm. Using the following expression [2] and considering the material properties in Table 2-2 and the maximum frequency of 10000 Hz for shock analysis, the minimum wavelength is approximately 30 mm for both materials. This implies that the maximum recommended length for 2D elements is 3.75 mm, which is met in both instruments, where the maximum element length is 3 mm.

$$\lambda = \left(\frac{2\pi}{f} \right)^{1/2} \left(\frac{Et^2}{12\rho(1-\nu^2)} \right)^{1/4} \quad (2.8)$$

Another modelling recommendation indicated in [2] is the homogeneity of the mesh. At this regard, the preferred element for planar meshes is the quadrilateral with one node per vertex (QUAD4), where the size and the shape of the elements should be as regular as possible. For both STEP and EPT-HET FEMs, this recommendation is fulfilled as indicated above.

Regarding the representation of the non-structural parts, it is recommended for shock analysis to avoid the utilization of lumped masses joined to the structure by rigid elements, which may cause erroneous results in the adjacent elements. As alternatives, an increase of the NSM value for the closest structural elements or the modelling of the non-structural parts with 3D blocks elements

are the recommended options indicated in [2]. Precisely, the electronic components of both instruments are represented by NSM distributions on the elements of each PCB.

The next rule affects the transient analysis, where the time step for the numerical calculations should be chosen to have a sampling rate of at least 5 to 10 times the maximum frequency of interest, which is 10000 Hz. This gives an adequate value of sampling rate of 100000 Hz and a time step of 10^{-5} s. This requirement is not as severe for implicit codes (NASTRAN) as for explicit codes, where the time step is on the order of microseconds to achieve the required numerical stability [2].

Concerning the damping assumption, different options can be used in NASTRAN such as viscous and structural damping. The most recommended option for the dynamic analysis that uses the modal decomposition method is the modal viscous damping, which allows the utilization of uncoupled modal equations, achieving a significant reduction of the computational time with regard to other damping models.

2.3 Comparison between different types of shock analysis

In this section, a summary of the different types of analysis existing with the FEA approach to simulate the mechanical shock is presented, indicating the characteristics, requirements and limitations for each option. After the description, they are applied in the STEP FEM to obtain the results of acceleration in the formats of peak values, transient signals and SRS curves. The simulated environment for all analyses in section 2.3.2 is the in-plane (Y-Axis) shock test performed on the STEP QM (see section 2.2.2), where the acceleration signal measured by the reference accelerometer Ref2 in Y direction (or the corresponding SRS) is used as input for the different proposed numerical analyses. The precision of the analyses is evaluated by comparing the numerical results of response accelerations with the test measurements, where the differences are indicated in terms of decibels (dB). The input used for the analyses presented in section 2.3.3 corresponds to the SRS specification defined for the STEP instrument, where for the transient analyses it is necessary to first calculate different time functions of accelerations that meet the specified SRS.

2.3.1 Description of the numerical methods for shock analysis

The following numerical methods are compatible with NASTRAN code and represent different approaches to estimate the structural results of a system subjected to a shock load.

2.3.1.1 Transient analysis

Transient analysis is the most general way to compute the behaviour of a structure subjected to a time-varying load or forced transient acceleration. Two transient methods are available in NASTRAN code: Modal Transient Response Analysis (Sol 112) and Direct Transient Response Analysis (Sol 109).

The main advantage of the both transient methods is the possibility of obtaining time functions of any requested result (stresses, forces, accelerations). From these time functions, it is straightforward to calculate other forms to present the results used for shock evaluation, such as the peak values and the SRS curves. Therefore, the complete behaviour of the analysed structure can be obtained by transient analysis, but with the disadvantage of the high computational time and large amount of the output results, which can exceed the capability of the computers in some cases. The requirement of using the modal damping formulation for STEP instrument analyses is

incompatible with Direct Transient Response Analysis, where only viscous damping is allowed. For this reason, the transient analysis included in this study corresponds only to Modal Transient Response Analysis.

Two methods are widely used to configure the shock environment defined by a base excitation in a NASTRAN transient analysis. The first one is the direct enforced acceleration and consists in defining the acceleration via the SPCD command, which can be directly applied at the interface nodes or at a base node connected to the IF nodes by a rigid element. The second approach is the Large Mass method, which is similar to the previous configuration but with the difference of using a point mass element at the base node with a mass value higher than 10^6 times the mass of the analysed model. In this second method, a dynamic force, which results from multiplying the large mass value by the forced acceleration function, is applied with the NASTRAN command DAREA at the base node instead of the enforced acceleration function. The differences between the results obtained from these two techniques are negligible in most cases. The direct enforced acceleration with SPCD command has the advantage that the results do not depend on the value of the large mass, avoiding the numerical problems due to round-off errors and due to the influence of the pseudo rigid body modes. For this reason, all the FEM results in this study are obtained from the direct enforced acceleration method by SPCD NASTRAN command.

Simpler but less accurate alternative methods are summarized below with the advantage of being less time-consuming and of generating smaller amount of results.

2.3.1.2 *Response Spectrum Analysis*

The Response Spectrum Analysis (RSA) method consists in estimating directly the approximation of the peak responses of any type of result using a normal modes calculation of the structure (Sol 103) and defining as input the SRS specification applied to the base. This method gives the typical results of normal modes calculation (list of natural frequencies in the range of interest and modal participation factors for each mode) and the peak values of the requested results. These peak values are obtained directly from the combination of all the modal contributions of acceleration. Thus, a modal combination method must be selected in the analysis among the following available options in NASTRAN:

- ABS: This summation method gives the most conservative results (high values of accelerations, forces and stresses) and consists in considering that all modal peak values of acceleration for every node occur in the same instant and phase. The total peak acceleration is the summation of the absolute values of each modal contribution.
- SRSS: The Square Root of the Summation of Squared values (SRSS) method gives the total peak acceleration in a way similar to that of assuming that the modal contributions are randomly combined, i.e. that the peak accelerations of each mode appear at different instants and phases. This approach is usually very optimistic.
- NRL: This combinatory method was developed by the Naval Research Laboratories (NRL) as a compromise between the two previous options, which considers that, for a given excitation, there is only one important mode that contributes the most to the peak response. Therefore, the total peak value is the sum of the absolute peak response associated to this mode and the SRSS calculation with the contribution of rest of modes. The results are intermediate between the two previous methods.
- CQC: The Complete Quadratic Combination (CQC) method involves the calculation of the cross-modal (covariance) coefficients for pairs of modes (p_{ij}) to take into account the coupling between these modes in the SRSS formulation.

The SRSS and NRL options also provide the possibility of defining that consecutive modes with frequencies ratios below the threshold indicated by the parameter CLOSE are summed with ABS option, while the rest of modes are taken into account with SRSS or NRL methods respectively.

The main disadvantage of RSA method is the low accuracy of the results and their dependence with the chosen summation option.

2.3.1.3 Sine Transmissibility Method

An alternative method [2] to calculate SRS curves of response acceleration for selected nodes consists in multiplying the input SRS by the shock transmissibility (TF_i^{SHOCK}) between the base and the nodes of interest. The shock transmissibility can be estimated from the sine transmissibility (FRF_i^{SINE}) using the following expression

$$TF_i^{SHOCK} = \sqrt{K \times FRF_i^{SINE}} \quad (2.9)$$

where the parameter K can take a value between 1 and 2.

The sine transmissibility can be calculated by a Frequency Response Analysis (Sol 111) or from the sine sweep test data. The main disadvantage of this method is that it cannot calculate the peak values or the time functions of the response. Another drawback is the appreciable differences between the SRS curves calculated by this method and the SRS curves obtained from the tests results of this instrument, especially due to high peaks that appear in the natural frequencies when low damping values are considered.

2.3.1.4 Equivalent Quasi-Static Load Method

Finally, the last analysis method presented in this work is the calculation of the equivalent quasi-static (EQS) inertial acceleration, which approximately gives the same total interface force as the shock load. Two different equations [2] are proposed depending on the modal coupling of the analysed structure,

for decoupled main modes

$$EQS = \frac{1}{M_{Total}} \left\{ \sum_i [MEM_i \times SRS(f_i)] + M_{Res} \times SRS^{HF} \right\} \quad (2.10)$$

for coupled main modes

$$EQS = \frac{1}{M_{Total}} \sqrt{\sum_i [MEM_i \times SRS(f_i)]^2 + [M_{Res} \times SRS^{HF}]^2} \quad (2.11)$$

where i is the index for the natural frequencies of the model, $SRS(f_i)$ is the value of the input SRS at the natural frequency f_i , M_{Total} is the mass of the model, MEM_i is the modal effective mass of the mode i , M_{Res} is the residual mass of the model that is not taken into account in the considered modes summation and SRS^{HF} is the value of the input SRS at high frequencies (typically the value at the maximum frequency of 10000 Hz).

The interface forces obtained from the static analysis (Sol 101) by applying the equivalent static acceleration are estimations of the peak interface forces for the shock simulation.

2.3.2 FEA simulations of in-plane shock test of the STEP instrument

In this section, the peak values, transient signals and the corresponding SRS curves of acceleration of the points of the structure where the accelerometers were located in the shock test are calculated by the proposed analysis methods to be compared with test results with the aim of evaluating their precision. For transient analyses, the input time signal measured by Ref2 accelerometer in Y direction (see Figure 2-9(a)) is applied as input. For the rest of analysis methods, the SRS curve related to the signal measured by the Ref2 accelerometer in Y direction (see Figure 2-10) is used as input. For the SRSS and NRL options, 4 values of the CLOSE parameter (frequency ratio threshold) are considered: 1.001, 1.005, 1.01 and 1.10. The modal damping used for all analyses is the standard value of 0.05 [2] in the frequency range of interest (100 – 10000 Hz).

The main modes of STEP FEM (with modal effective mass fractions greater than 10%) are within 400 and 1000 Hz (see Table 2-3). The excitation of these modes is the main contribution on the numerical results. The frequency ratio is included in this table, which is used to classify the modes in the SRSS and NRL analyses with the CLOSE parameter activated.

Table 2-3: First natural frequencies and modal effective masses (MEM) for each axis obtained from normal modes analysis of the STEP FEM [50].

Natural frequency (Hz)	MEM fraction (%) X-axis	MEM fraction (%) Y-axis	MEM fraction (%) Z-axis	Frequency ratio (f_i/f_{i-1})
440.1	63.47	0.25	0.00	
551.6	0.51	37.61	2.00	1.253
553.0	0.06	25.46	8.73	1.003
632.1	0.14	0.23	0.00	1.143
638.8	0.38	0.00	0.00	1.011
658.9	0.00	0.06	0.11	1.031
668.4	0.29	0.33	0.02	1.014
685.8	0.98	0.00	0.09	1.026
755.1	0.02	1.14	28.90	1.101
781.1	0.16	1.30	4.09	1.034
809.9	0.11	12.96	3.79	1.037
833.5	1.26	0.00	1.73	1.029
912.6	0.00	0.00	0.08	1.095
931.5	0.02	0.27	3.47	1.021
977.3	0.25	9.25	2.60	1.049
990.5	0.03	2.67	10.58	1.013

The peak values of accelerations at the accelerometers locations calculated by the transient and RSA methods, considering different options for modal contribution summation, are compared in Table 2-4 with the test measurements. The shown values correspond to the Y component of acceleration (A_{peak}^{FEM}), which is parallel to the intended shock excitation. The differences (*Diff*) in dB between the calculated peak accelerations with respect to the acceleration peak values measured during the shock test (A_{peak}^{Test}) are indicated in Table 2-5, using the following expression

$$Diff = 20 \log_{10} \frac{A_{peak}^{FEM}}{A_{peak}^{Test}} \quad (2.12)$$

Table 2-4: Peak accelerations (g) from different analysis methods for the shock test simulation in Y direction [50].

Type of analysis	Peak accelerations (g) at accelerometers locations					
	TriA2y	TriA3y	TriA4y	TriA5y	TriA6y	TriA7y
Test	207.49	359.49	180.84	186.48	180.25	402.99
Transient Analysis	228.95	268.00	189.77	191.73	225.62	269.78
RSA ABS	498.76	572.93	325.45	348.45	429.27	894.17
RSA SRSS	130.34	188.02	85.56	95.94	153.85	160.54
RSA SRSS close 1.001	157.43	220.32	107.59	115.78	169.12	245.36
RSA SRSS close 1.005	300.97	381.48	183.37	190.65	273.91	507.02
RSA SRSS close 1.01	355.29	452.31	224.93	235.94	293.62	693.26
RSA SRSS close 1.10	494.02	569.72	322.62	345.38	425.67	892.89
RSA NRL	181.23	264.14	118.70	133.58	216.11	222.63
RSA NRL close 1.001	208.29	296.28	140.54	153.29	231.31	305.93
RSA NRL close 1.005	349.88	451.93	215.47	226.94	331.24	565.16
RSA NRL close 1.01	403.49	520.30	256.10	271.15	350.89	743.89
RSA NRL close 1.10	498.08	571.51	325.28	348.17	428.21	893.65
RSA CQC	187.03	232.11	133.45	147.07	192.79	251.21

Table 2-5: Differences of peak acceleration values for different analysis methods with respect to test results [50].

Type of analysis	Differences of peak acceleration values (dB)					
	TriA2y	TriA3y	TriA4y	TriA5y	TriA6y	TriA7y
Transient Analysis	0.86	-2.55	0.42	0.24	1.95	-3.49
RSA ABS	7.62	4.05	5.10	5.43	7.54	6.92
RSA SRSS	-4.04	-5.63	-6.50	-5.77	-1.38	-7.99
RSA SRSS close 1.001	-2.40	-4.25	-4.51	-4.14	-0.55	-4.31
RSA SRSS close 1.005	3.23	0.52	0.12	0.19	3.63	1.99
RSA SRSS close 1.01	4.67	2.00	1.89	2.04	4.24	4.71
RSA SRSS close 1.10	7.53	4.00	5.03	5.35	7.46	6.91
RSA NRL	-1.18	-2.68	-3.66	-2.90	1.58	-5.15
RSA NRL close 1.001	0.03	-1.68	-2.19	-1.70	2.17	-2.39
RSA NRL close 1.005	4.54	1.99	1.52	1.71	5.29	2.94
RSA NRL close 1.01	5.78	3.21	3.02	3.25	5.79	5.32
RSA NRL close 1.10	7.61	4.03	5.10	5.42	7.52	6.92
RSA CQC	-0.90	-3.80	-2.64	-2.06	0.58	-4.11

The results indicate that the analysis method with smaller average error is the transient analysis, where the differences are below the limits of ± 3 dB, except for TriA7y peak acceleration, where the difference is due to high frequency waves detected by this accelerometer in the shock test, which cannot be reproduced with sufficient accuracy by the transient analysis. This discrepancy is discussed below, after the calculation and comparison of the SRS curves. Among the proposed options for RSA method, the most similar to test results correspond to the SRSS and NRL, both with CLOSE parameter of 1.001. But in general, this method presents high differences and variability that depend on the combination option with differences that oscillate between -8 dB and +8 dB approximately with respect to test results, where the most conservative is the ABS option and the most optimistic is the SRSS option. The EQS method has not been considered in this comparison, because the only results that should be calculated by this method are the forces in the interface bolts, which have not been measured during the shock tests.

The SRS curves of acceleration can be calculated with the modal transient analysis and with sine transmissibility approach. For this last method, Equation (2.9) with $K = 1$ is applied to estimate

the shock transmissibility from the sine transmissibility, which is obtained from both sine sweep test and frequency response analysis. The modal damping of 0.05 for all frequency range is defined for the transient and the frequency response analyses. To compute the response SRS curves from the time functions of acceleration for the transient analysis and shock test results, the standard quality factor $Q = 10$, related to a damping constant of 0.05, is used for all results presented in this section. This damping parameter for SRS calculation has a different meaning and is independent of the damping factor defined in FEM analyses. The comparison of SRS curves obtained from these methods with the SRS curves from shock test is shown in Figure 2-15. The locations where the accelerometers of the shock test coincide with the accelerometers of the sine sweep test are TriA2y, TriA3y, TriA5y and TriA7y.

Figure 2-15: Comparison of SRS curves at the locations of TriA2y (a), TriA3y (b), TriA5y (c) and TriA7y (d) accelerometers for shock test, transient analysis and sine transmissibility method with frequency response analysis and with sine sweep test results [50].

The first conclusion is that transient analysis provides an acceptable prediction of the shock test results until 2000 Hz, which is the frequency range that contains the main natural frequencies of STEP instrument. But this analysis underestimates the response for the higher frequency range. These discrepancies are due to the generation of high frequency waves during the shock test, which are generated at the end of the first cycle of the main wave, as can be better appreciated in the comparison between time functions shown in the next section. The causes of these high frequency waves during test are unknown, but are not due to the input neither to any malfunction of the measuring equipment. The SRS curves calculated by the sine transmissibility functions obtained from low sine test cover only up to 2000 Hz. The SRS curves obtained with this method present very high peaks and deep valleys due to that the damping factors for sine vibration are lower than for the shock test, resulting in too sharp SRS curves. The SRS curves obtained from the sine transmissibility functions calculated by frequency response analysis with a modal damping value of 0.05 are smoother than the previous SRS curves, but from 2000 Hz to 10000 Hz decay considerably. The explanation is that these SRS curves have the same trend as the sine transmissibility, which decreases to zero for frequencies higher than the main natural frequencies

of the system, unlike the SRS curves computed from time functions, which tend asymptotically to the peak values of these functions for frequencies higher than the maximum frequency content of the signal. For this reason, the end section of the SRS curves computed by sine transmissibility method presents high differences with respect to the shock test results. A way to correct this defect, which is proposed in [2], consists in substituting this end section by a constant SRS line.

2.3.3 FEA simulations using a SRS specification

In this section, the shock environment is specified by a SRS curve instead of a transient acceleration function. This is the usual way to specify the shock load in most of the space projects, where much information of the original time function is lost. For the transient analysis, the calculation of a transient acceleration function that meets with the specified SRS to be used as input in the analysis is needed as a previous step. In this section, different numerical algorithms are employed to provide different time functions of acceleration from the given input SRS. For RSA method, the SRS is used directly as input. For EQS method, the input SRS is introduced in Equations (2.10) and (2.11) to calculate the equivalent static accelerations with these two options.

For these analyses, the given input SRS correspond to the shock specification defined for STEP instrument in Y-axis shown in Figure 2-10.

2.3.3.1 *Determination of the input time functions of acceleration from a given SRS*

To perform a transient analysis that simulates a shock environment specified by a SRS curve, it is necessary to have a time function of input acceleration that meets the SRS specification. The drawback is that infinite options are possible, and therefore, one of the objectives of this study is to find out if the results (peak values and SRS curves) in transient analyses are similar among them, considering different input functions from the same SRS.

Among different numerical calculations to obtain a time function of acceleration from a given SRS, the most used methods are the damped sinusoids decomposition and the wavelet synthesis [2,3], because they give oscillatory acceleration functions that match the specified SRS with acceptable accuracy (see Figure 2-16). One of the parameters that define the time functions is the duration. In order to evaluate its influence on the results, three different durations of 20, 40 and 100 ms have been proposed for both methods (see Figure 2-17). In addition, a rectangular pulse (see Figure 2-18) has been considered for the comparison, optimizing its two parameters (amplitude and duration) to better match the specified SRS, resulting in a rectangular pulse of 268.46 g and 0.2 ms. The maximum absolute peak values of acceleration and the durations of the calculated input time functions are summarized in Table 2-6.

Figure 2-16: SRS curves for different input time acceleration functions: oscillatory functions obtained by damped sinusoids decomposition (DS) and by wavelet synthesis (WL) for three different signal durations (20, 40 and 100 ms) and a rectangular pulse of 268.46g [50].

Table 2-6: Maximum peak values and durations for input time acceleration functions for transient analyses [50].

Input	Maximum peak acceleration (g)	Duration of the signal (ms)
Damped sinusoids DS 20 ms	217	20
Damped sinusoids DS 40 ms	176	40
Damped sinusoids DS 100 ms	149	100
Wavelet synthesis WL 20 ms	196	20
Wavelet synthesis WL 40 ms	189	40
Wavelet synthesis WL 100 ms	152	100
Rectangular pulse	268	0.2

Figure 2-17: Input time functions of acceleration for transient analyses obtained by damped sinusoids summation method (left) and by wavelet synthesis (right) [50].

Figure 2-18: Rectangular pulse of 268.46 g and 0.2 ms for input time function of acceleration for transient analysis [50].

2.3.3.2 *Comparison of peak results for different numerical shock analyses*

The results that are typically calculated to evaluate the structural behaviour of a space instrument are the maximum stresses, interface forces and accelerations. For the shock analyses, the maximum absolute values for these magnitudes, i.e. the peak values, are used to evaluate the structure. In this section, a comparison of these results for the different analysis methods is established. The modal damping factor for all analyses corresponds to the standard value of 0.05 for the shock. For the transient analyses, seven different acceleration time functions (see Figure 2-17 and Figure 2-18) are applied as input. For RSA method, different modal summation options are applied (ABS, SRSS, NRL and CQC), where for the SRSS and NRL options, the values of 1.001, 1.005, 1.01 and 1.10 for the CLOSE parameter are included. For the calculation of interface forces, the EQS method is included in the comparison, considering the options for coupled and decoupled normal modes. The peak values of acceleration calculated using the proposed set of numerical analyses are shown in Table 2-7. With the aim to determine the differences, the transient analysis with an input of 20 ms has been chosen as the reference case because this input load is the most similar in aspect, intensity and duration to the input test environment (see Figure 2-9(a)). The differences of the results of the rest of methods with the reference case are indicated in Table 2-8.

The differences between the peak values calculated by the transient analyses with the proposed inputs are within the limits of ± 3 dB, except for the transient analysis with a rectangular pulse as input, which shows higher peak values. The dispersion of results calculated by all the options for RSA method is high, with differences out of the range of ± 3 dB. The NRL option with CLOSE parameter of 1.001 is the most similar to transient analyses.

The forces calculated for each interface bolt (IF1 – IF6) of STEP instrument are shown in Table 2-9. The differences between results with respect to the reference case (transient analysis with 20 ms of acceleration input) are shown in Table 2-10. Similar to the acceleration results, the differences between the transient analyses results are within the limits of ± 3 dB. The NRL option for RSA method gives the results more similar to those of transient analyses. The results of EQS method for decoupled modes and coupled modes are quite similar to those of the RSA ABS option and RSA SRSS option respectively.

Table 2-7: Peak values of acceleration calculated by different numerical analysis [50].

Type of analysis	Peak accelerations (g) at accelerometers locations					
	TriA2y	TriA3y	TriA4y	TriA5y	TriA6y	TriA7y
Transient DS 20 ms	145.11	159.51	131.98	125.25	143.67	215.77
Transient DS 40 ms	175.87	206.61	143.87	148.41	174.01	300.77
Transient DS 100 ms	125.31	181.39	99.66	100.48	151.35	228.20
Transient WL 20 ms	143.39	200.80	125.75	120.48	152.36	268.80
Transient WL 40 ms	140.46	207.81	97.40	105.47	161.25	258.31
Transient WL 100 ms	143.35	178.27	113.15	115.36	153.14	266.03
Transient Rectangular pulse	257.76	265.31	221.04	220.48	227.95	394.30
RSA ABS	442.23	501.42	282.56	297.06	344.83	946.94
RSA SRSS	90.02	134.31	59.85	67.31	107.17	147.38
RSA SRSS close 1.001	122.37	171.21	83.90	89.15	123.95	243.03
RSA SRSS close 1.005	276.42	335.56	163.50	165.92	224.74	543.00
RSA SRSS close 1.01	339.86	412.50	209.64	216.30	249.06	765.94
RSA SRSS close 1.10	439.31	499.15	280.86	295.20	342.47	946.07
RSA NRL	125.09	189.46	80.08	91.25	151.51	198.78
RSA NRL close 1.001	157.40	226.09	103.99	112.95	168.14	293.33
RSA NRL close 1.005	309.58	384.48	183.10	189.00	262.45	591.54
RSA NRL close 1.01	371.44	456.45	228.16	237.93	286.57	798.51
RSA NRL close 1.10	441.71	500.34	282.43	296.84	344.06	946.56
RSA CQC	113.73	170.24	77.02	86.35	136.49	192.42

Table 2-8: Differences between peak values of acceleration calculated by different numerical analyses [50].

Type of analysis	Differences of peak acceleration values (dB)					
	TriA2y	TriA3y	TriA4y	TriA5y	TriA6y	TriA7y
Transient DS 40 ms	1.67	2.25	0.75	1.47	1.66	2.88
Transient DS 100 ms	-1.27	1.12	-2.44	-1.91	0.45	0.49
Transient WL 20 ms	-0.10	2.00	-0.42	-0.34	0.51	1.91
Transient WL 40 ms	-0.28	2.30	-2.64	-1.49	1.00	1.56
Transient WL 100 ms	-0.11	0.97	-1.34	-0.71	0.55	1.82
Transient Rectangular pulse	4.99	4.42	4.48	4.91	4.01	5.24
RSA ABS	9.68	9.95	6.61	7.50	7.60	12.85
RSA SRSS	-4.15	-1.49	-6.87	-5.39	-2.55	-3.31
RSA SRSS close 1.001	-1.48	0.61	-3.94	-2.95	-1.28	1.03
RSA SRSS close 1.005	5.60	6.46	1.86	2.44	3.89	8.02
RSA SRSS close 1.01	7.39	8.25	4.02	4.75	4.78	11.00
RSA SRSS close 1.10	9.62	9.91	6.56	7.45	7.55	12.84
RSA NRL	-1.29	1.49	-4.34	-2.75	0.46	-0.71
RSA NRL close 1.001	0.71	3.03	-2.07	-0.90	1.37	2.67
RSA NRL close 1.005	6.58	7.64	2.84	3.57	5.23	8.76
RSA NRL close 1.01	8.16	9.13	4.75	5.57	6.00	11.37
RSA NRL close 1.10	9.67	9.93	6.61	7.50	7.59	12.84
RSA CQC	-2.12	0.57	-4.68	-3.23	-0.45	-1.00

Table 2-9: Peak values of interface lateral forces calculated by different numerical analyses [50].

Type of analysis	Lateral peak forces for interface bolts (N)					
	IF1	IF2	IF3	IF4	IF5	IF6
Transient DS 20 ms	331	311	316	320	306	321
Transient DS 40 ms	426	405	420	384	378	385
Transient DS 100 ms	319	302	312	310	296	308
Transient WL 20 ms	301	280	291	307	284	307
Transient WL 40 ms	329	307	307	310	295	307
Transient WL 100 ms	324	308	315	315	303	315
Transient Rectangular pulse	531	504	522	467	462	465
RSA ABS	576	545	565	530	518	533
RSA SRSS	235	219	225	223	212	225
RSA SRSS close 1.001	247	229	239	231	220	233
RSA SRSS close 1.005	354	326	339	335	318	338
RSA SRSS close 1.01	386	359	371	372	356	379
RSA SRSS close 1.10	567	535	552	523	511	526
RSA NRL	331	309	318	315	299	317
RSA NRL close 1.001	344	319	332	323	307	325
RSA NRL close 1.005	440	406	422	404	389	408
RSA NRL close 1.01	470	438	453	439	425	447
RSA NRL close 1.10	573	542	561	528	517	533
RSA CQC	301	281	289	301	283	301
EQS Decoupled Modes	557	530	550	538	518	536
EQS Coupled Modes	228	217	225	220	212	219

Table 2-10: Differences between peak values of interface lateral forces calculated by different numerical analyses [50].

Type of analysis	Differences for peak force values (dB)					
	IF1	IF2	IF3	IF4	IF5	IF6
Transient DS 40 ms	2.18	2.30	2.47	1.59	1.84	1.58
Transient DS 100 ms	-0.34	-0.26	-0.11	-0.29	-0.28	-0.35
Transient WL 20 ms	-0.84	-0.89	-0.74	-0.35	-0.64	-0.38
Transient WL 40 ms	-0.06	-0.11	-0.25	-0.28	-0.31	-0.39
Transient WL 100 ms	-0.19	-0.07	-0.04	-0.13	-0.08	-0.17
Transient Rectangular pulse	4.09	4.21	4.35	3.27	3.58	3.22
RSA ABS	4.80	4.87	5.04	4.38	4.57	4.41
RSA SRSS	-3.00	-3.06	-2.94	-3.14	-3.20	-3.10
RSA SRSS close 1.001	-2.55	-2.64	-2.44	-2.85	-2.87	-2.80
RSA SRSS close 1.005	0.57	0.42	0.60	0.40	0.34	0.43
RSA SRSS close 1.01	1.31	1.25	1.39	1.30	1.32	1.45
RSA SRSS close 1.10	4.66	4.72	4.83	4.27	4.45	4.28
RSA NRL	-0.01	-0.06	0.05	-0.14	-0.19	-0.11
RSA NRL close 1.001	0.31	0.23	0.41	0.07	0.04	0.10
RSA NRL close 1.005	2.45	2.33	2.50	2.03	2.07	2.08
RSA NRL close 1.01	3.04	2.99	3.13	2.75	2.85	2.88
RSA NRL close 1.10	4.75	4.84	4.98	4.35	4.55	4.40
RSA CQC	-0.83	-0.86	-0.80	-0.53	-0.68	-0.57
EQS Decoupled Modes	4.51	4.65	4.80	4.50	4.57	4.45
EQS Coupled Modes	-3.24	-3.11	-2.95	-3.25	-3.19	-3.31

Figure 2-19: Stress distribution (Pa) on the titanium cover calculated by transient analysis (DS 20 ms) [50].

Figure 2-20: Stress distribution (Pa) on the titanium cover calculated by RSA method with ABS option [50].

Figure 2-21: Stress distribution (Pa) on the titanium cover calculated by RSA method with SRSS option [50].

The most stressed part of STEP instrument is the titanium cover, which is on top of the Ebox and under the detectors (see Figure 2-5(a)). The maximum peak stress values are compared in Table 2-11 and the peak stress distributions are shown in Figure 2-19 for transient analysis (input acceleration function of 20 ms), in Figure 2-20 for RSA with ABS option and in Figure 2-21 for RSA with SRSS option. The stress distributions are very similar between the different methods, where the maximum stresses are located in the same zone, near the hole of one of the flanges. However, the maximum values of stresses are very different among the analysis methods. The biggest differences of the peak stress values with respect to the reference case (transient analysis DS 20 ms) correspond to RSA ABS (+5 dB) and to RSA SRSS (-4 dB). The RSA option with less difference with respect to transient analysis is SRSS option with a CLOSE threshold of 1.005.

Table 2-11: Maximum peak stresses for titanium cover of STEP instrument calculated by different numerical analyses [50].

Type of analysis	Maximum stresses (MPa)	Differences (dB)
Transient DS 20 ms	89.2	-
RSA ABS	158.0	4.97
RSA SRSS	56.9	-3.91
RSA SRSS close 1.001	60.4	-3.39
RSA SRSS close 1.005	86.9	-0.23
RSA SRSS close 1.01	101.0	1.08
RSA SRSS close 1.10	146.0	4.28
RSA NRL	80.3	-0.91
RSA NRL close 1.001	82.7	-0.66
RSA NRL close 1.005	107.0	1.58
RSA NRL close 1.01	118.0	2.43
RSA NRL close 1.10	155.0	4.80
RSA CQC	76.2	-1.37

2.3.3.3 Comparison of SRS curves obtained from different input functions in modal transient analyses

The transient analyses using the different input acceleration functions give different time functions of response accelerations, but their corresponding SRS curves are quite similar among them, as can be seen in Figure 2-22, with differences within ± 3 dB, except for the rectangular pulse, with differences as much as +6 dB with respect to the other analyses. The same oscillatory functions have been applied in the study performed with a FEM of a vertical cantilevered beam shown in Appendix B. In this study, the differences found among the response SRS curves are higher due to the lower damping of the analysed structure, but within ± 10 dB.

Figure 2-22: Comparison of response SRS curves calculated at various locations by transient analyses considering different acceleration inputs that match the specified SRS: oscillatory functions by damped sinusoids decomposition (DS), wavelet synthesis (WL) methods (with durations of 20, 40 and 100 ms) and with a rectangular pulse [50].

2.3.3.4 Comparison of computational time and output data size

The comparison in terms of the computational time and output data size among the different analyses methods is established in this section to offer an insight about the advantages and disadvantages in these aspects for the different options. All the numerical analyses have been done in the same computer and with the same STEP FEM. The requested results are indicated in

Table 2-12, which correspond to the accelerations and displacements for all nodes and stresses and forces for all elements. There are important differences in the format of the calculated results between the proposed methods. All the requested results can be calculated by the modal transient analysis as time functions, where each function has 2000 time steps of 10^{-5} s. The peak values can be obtained from these functions. The RSA method can calculate the peak values directly for all the requested results. For the sine transmissibility method, only the output SRS curves for the displacements and accelerations are calculated, considering a total of 5132 steps for each frequency function. For EQS method, the calculated displacements, stresses and forces are static values that represent the equivalent peak values.

Table 2-12: Requested results for comparison of computational time and output data size among the numerical shock analyses.

Analysis	Accelerations	Displacements	Stresses	Forces
Modal transient	Time functions	Time functions	Time functions	Time functions
RSA	Peak values	Peak values	Peak values	Peak values
Sine transmissibility	SRS curves	SRS curves	-	-
EQS	-	Peak values	Peak values	Peak values

The comparison of the computational time and output data size are shown in Table 2-13. With respect to the computational time, the EQS method is clearly the most quick with only 23 s. It is due to that is a static analysis where the normal modes are not calculated, unlike the rest of methods. The second quicker method is the RSA, which takes approximately 1 h 46 min to complete the calculations. The duration of the modal transient analysis and sine analysis (frequency response analysis) are 4 h 41 min and 6 h 40 min respectively. These two last methods need more time to calculate all the requested output results for each time step (transient analysis) or for each frequency step (sine transmissibility). With respect to the amount of output data, it is related to both the quantity of requested results and their format. Both RSA and EQS methods generate between 20 and 30 MB of data, which correspond to the equivalent peak values, where there is one value for each result (displacement, force, stress or acceleration) and for each node or element. The size of output data increases for transient analysis, because it generates a function with 2000 values for each requested result. The same problem occurs for the sine transmissibility method, where despite stresses and forces are not calculated, it generates about 60 GB of output data that correspond to the frequency functions of accelerations and displacements, with 5132 values for each function. It is important to highlight that typically the response SRS curves are calculated not for all the nodes of the FEM, but only for few nodes that represent points of interest such as the mechanical interfaces of sensitive components. In this case, the computational time and the amount of output data calculated by the transmissibility method are reduced considerably. As a conclusion, with respect to the computational resources, the order of the numerical methods from the best to the worst is: EQS, RSA, modal transient and sine transmissibility.

Table 2-13: Comparison in terms of computational time and output data size among the different shock analyses.

Analysis	Output data size (bytes)	Computational time (s)
Modal transient	42291560	16868
RSA	23687	6350
Sine transmissibility	63896551	24005
EQS	29028	23

2.3.4 Conclusions of the FEA methods comparison

Therefore, after the comparison of the peak values and SRS curves of acceleration computed by the different types of FEA simulations, the conclusion is that modal transient analysis is the best option to represent the environment measured during an impact test, taking into account the smaller average differences of the results with respect to shock test measures, both in peak values and SRS curves. Furthermore, another advantage of the modal transient analysis is that it can calculate the time functions of response acceleration, which can be directly compared with the corresponding functions extracted from the shock test. The lower computational time and smaller amount of output data presented in the other options do not compensate their lack of precision in the generated results. Therefore, modal transient analysis is the best of the proposed methods to simulate the mechanical shocks. In the next section, different input parameters of the modal transient analysis are considered to study their influence on the results and on the acquired level of precision by comparing both the time signals and SRS curves with the shock test data.

2.4 Transient analysis

Among the different numerical methods of analysis studied in this investigation, the modal transient analysis is the most precise and the most versatile, providing the results in any of the proposed formats (peak values, SRS and time functions). In the previous section, the results calculated from the transient analysis, as well as from the other methods, correspond to a given value of 0.05 for the modal viscous damping. Additionally, the input forced acceleration was defined in only one direction, which corresponds to the main direction of excitation. In this section, both input parameters will be modified separately to evaluate their influence on the results. As for section 2.3.2, the simulated environment correspond to the in-plane shock test performed on the STEP QM, where the input acceleration is the signal measured in Y direction by the reference accelerometer Ref2y (see Figure 2-9(a)) for the one-directional excitations. To evaluate the influence of the cross input accelerations, the signals measured by the same accelerometer in the other axes (see Figure 2-9(b) and (c)) are added to the acceleration in the main direction to configure an excitation applied in the three orthogonal axes simultaneously.

2.4.1 Influence of the model damping

In this section, the influence of the damping value used for the transient analysis on the results is evaluated. Three different values are proposed (0.01, 0.05 and 0.15), each one being constant for all the frequency range. The excitation is a forced acceleration applied along the Y-axis direction with the time function signal measured by the reference accelerometer Ref2 (see Figure 2-9(a)) during the in-plane shock test of STEP QM. The qualification factor used to compute the SRS curves from the time functions is $Q = 10$ for all of the results.

As can be appreciable in the results shown in Figure 2-23 and in Figure 2-24, the higher the damping used in the transient analysis, the lower the acceleration peaks in time functions are. The damping affects mostly in the frequency range where the main natural frequencies are located (400 – 2000 Hz). When the damping value increases, the SRS curves decrease and the peaks in the natural frequencies are hidden. The damping of 0.15 provides the SRS curves more similar to the test results for low and medium frequency bands, achieving a very good correlation with the shock tests results measured by some accelerometers such as TriA5y and TriA6y. However, a damping value of 0.01 seems to be more adequate to correlate the response measured by TriA3y accelerometer for the high frequency range. The largest discrepancies between FEA and test results are found for the TriA7y results due to the high frequency oscillations obtained in the test,

which are not reproduced in the FEA simulations. The influence of the damping in this high frequency band is practically negligible, as can be appreciated in the SRS curves comparison. The same problem also occurs in the TriA2y and TriA4y results.

(a) TriA2y location – time functions

(b) TriA2y location – SRS curves

(c) TriA3y location – time functions

(d) TriA3y location – SRS curves

(e) TriA4y location – time functions

(f) TriA4y location – SRS curves

Figure 2-23: Time functions and SRS curves of acceleration calculated by transient analysis at the locations of TriA2y, TriA3y and TriA4y for different values of damping: 0.01, 0.05 and 0.15 [50].

(a) TriA5y location – time functions

(b) TriA5y location – SRS curves

(c) TriA6y location – time functions

(d) TriA6y location – SRS curves

(e) TriA7y location – time functions

(f) TriA7y location – SRS curves

Figure 2-24: Time functions and SRS curves of acceleration calculated by transient analysis at the locations of TriA5y, TriA6y and TriA7y for different values of damping: 0.01, 0.05 and 0.15 [50].

2.4.2 Influence of the cross input acceleration

A comparison of the results of the transient analyses considering the excitation only in the main axis (Y-axis) measured in the test (see Figure 2-9(a)) and an excitation that combines the input signal in the 3 orthogonal axes (see Figure 2-9) is shown in Figure 2-25 and in Figure 2-26, using the modal damping factor of 0.05 for the analyses. Slight differences are observed in the Y component of the acceleration results, concluding that the influence of the cross input accelerations on these results is negligible in this instrument.

Figure 2-25: Comparison of SRS curves calculated by transient analysis at TriA2y, TriA3y and TriA4y between applying an excitation in the main axis and an excitation in the 3 orthogonal axes simultaneously [50].

(a) TriA5y location – time functions

(b) TriA5y location – SRS curves

(c) TriA6y location – time functions

(d) TriA6y location – SRS curves

(e) TriA7y location – time functions

(f) TriA7y location – SRS curves

Figure 2-26: Comparison of SRS curves calculated by transient analysis at TriA5y, TriA6y and TriA7y between applying an excitation in the main axis and an excitation in the 3 orthogonal axes simultaneously [50].

2.5 Proposed techniques to improve the precision of shock numerical analyses

The evaluation of the precision achieved by the proposed numerical methods was presented in the previous sections, where the modal transient analysis has proven to be the best option to characterize the shock environment measured during the impact test. One of the hypothesis applied in the numerical analyses performed in the previous sections is the uniformity of the input acceleration field, where all the interface points are subjected to the same signal of forced acceleration. This hypothesis can be considered appropriate to simulate the in-plane shock tests, where the difference between the input signals measured by the reference accelerometers in each of the orthogonal axes can be considered negligible, as is demonstrated in Figure 2-9 and Figure 2-10 for the in-plane test of the STEP QM. However, this hypothesis cannot be considered adequate for the out-of-plane shock tests performed on the same instrument, where the differences between the time signals (see Figure 2-11) and their corresponding SRS curves (see Figure 2-12) are more evident than for the in-plane test. This aspect has also been found in many studies [21,32,33] that have been working with impact tests data, where the tested items are mainly plates. In [54], the complex acceleration fields generated by the impact of micrometeoroids and space debris on composite panels are investigated by numerical simulations, where the excitation of longitudinal, shear and flexural waves causes the non-uniformity of the acceleration measured on the panel. In [55], a comparison is made between the responses obtained by both uniform and non-uniform shock excitations of a building structure. The main causes are the excitation of the normal modes and, with less influence, the attenuation of the shock wave with the distance. The excitation of the bending normal modes of the tested bench when is impacted by the hitting piece, especially for the out-of-plane tests, are the main contribution of the non-uniformity of the acceleration field transmitted to the mechanical interface of the tested item.

The work detailed in this section focuses on the definition of the input shock levels at the interface nodes of the STEP FEM in the modal transient analyses, working with the acceleration signals extracted from the out-of-plane shock tests performed on the STEP QM (see section 2.2.2). The objective of this study is to find the way to simulate with precision the same shock environment measured in the test taking into account the non-uniformity of the input acceleration field. Five different techniques are proposed in section 2.5.1 to define in the FEA simulations the shock input environment measured in the test. The simplest technique assumes a uniform input shock acceleration field only in the main direction of excitation that is directly applied at one base node rigidly connected to all interface points of the model. The rest of techniques are different options that take into account aspects such as the measured cross input accelerations, which have also been considered for the in-plane shock simulations in section 2.4.2, and the differences between the input signals in the definition of non-uniform input acceleration fields in the transient analyses. These techniques require complex modelling assumptions to represent with higher precision the shock test environment. The results with the different FEM techniques and their comparison with shock test responses are exposed in section 2.5.2.

2.5.1 Proposed techniques to represent the non-uniformity of the input acceleration fields in the shock simulations

To represent the same input acceleration field generated during an impact test, the time function of the acceleration measured by the reference accelerometer can be applied directly in the transient analysis. The problem arises when the two or more reference accelerometers used in the same test record different input signals. This is indicative of the complexity and non-uniformity of the acceleration field transmitted to the interface points of the tested item, which cannot be

reproduced adequately in the FEA simulations with the standard technique of using one base node attached to all the interface nodes of the analysed model. To achieve a better representation of the shock test environment with the aim to improve the precision of the numerical results, it is necessary to use a different way to represent the input acceleration. In this section, different techniques are proposed and explained, each one with different assumptions and simplifications to represent the shock test environment. The proposed techniques are summarized in Table 2-14 and the different configurations to define the base nodes are depicted in Figure 2-27.

Table 2-14: Summary of different techniques to simulate the shock environment in transient analyses [51].

Technique	Description	Conf.
Uniform acceleration in the main direction	The acceleration is applied to one base node in the main translational direction. The input acceleration can be the signal measured by any of the reference accelerometers or an average signal.	1
Uniform acceleration in the three orthogonal directions	The acceleration is applied to one base node in the three orthogonal directions simultaneously. The input accelerations can be the signals measured by any of the reference accelerometers or an average signal for each axis.	1
Rigid base motion with rotations	The motion of the base node combines translational and rotational accelerations, which are calculated from test data by considering the base (adaptor plate) as a rigid solid. The resulting motion provides a non-uniform acceleration field.	1
Non-uniform acceleration with test input signals	There are as many base nodes as reference accelerometers, typically two. Each base node represents one reference accelerometer and is connected to the closest set of interface nodes. The input accelerations (in the main direction or in the three orthogonal directions) correspond to the measured signals for each reference accelerometer.	2
Non-uniform acceleration with calculated input signals	There can be one or more base nodes. Some of the input acceleration functions are the test signals directly applied, while other input functions are calculated from the test signals measured by the response accelerometers and with the transmissibility obtained from FEM analysis between the response accelerometer location and the interface point.	Any

Figure 2-27: Different configurations for shock FEA simulations. (a) Configuration 1: uniform base excitation or rigid base motion with rotations with one base node A. (b) Configuration 2: non-uniform base excitation with two base nodes B and C. (c) Configuration 3: Non-uniform base excitation with three base nodes B, D and E [51].

2.5.1.1 Uniform acceleration in the main direction

The most used technique, not only for shock analysis, but also for sine and random vibration base excitations, is with a single base node connected by a rigid element, usually a RBE2 rigid element in NASTRAN code, to all the interface nodes. This configuration is represented in Figure 2-27(a), where the enforced acceleration is applied to the base node (A) and transmitted equally to all interface points. This technique assumes that the input acceleration field is uniform for all interface points, which is only applied in the main direction of the shock test. For these reasons, this technique is the simplest of the proposed techniques, where the measured cross input accelerations and the differences between the input signals in the main direction of excitation are ignored in the analysis. This technique has been applied for the analyses presented in section 2.3.

2.5.1.2 Uniform acceleration in the three orthogonal directions

This technique is a variation of the previous one with the addition of the cross input accelerations applied to the base node. As in the previous case, Configuration 1 (see Figure 2-27(a)) is used, where the rigid element must transmit the linear accelerations in the three orthogonal axes to all interface nodes. Therefore, this technique does take into account the cross input signals, but does not consider the non-uniformity of the input acceleration field, which is the cause of the differences between the signals measured by the reference accelerometers. This technique has also been applied in the study of the influence of the input parameters of the transient analysis in section 2.4.2, with the test data of the in-plane shock tests.

2.5.1.3 Rigid base motion with rotations

With this technique, it is assumed that the support or adaptor plate is sufficiently rigid to be considered as a solid rigid base, which can rotate and translate during shock test induced by the modal response of the more flexible drop table where it is attached. To calculate the rotational acceleration functions to be applied in the shock transient analysis that simulates the test environment, it is necessary to have the measured signals from at least three non-collinear accelerometers located on the support plate. If only two reference accelerometers are available in the shock test, the necessary data to use this technique can be obtained from the response accelerometers located on the tested instrument by the method explained in section 2.5.1.5.

The scheme shown in Figure 2-28 represents a rigid base plate, where the point O located on the centre represents the position of the base node used to simulate the shock environment in the FEA transient analysis. In this configuration, the points A, B and C are the positions with known acceleration functions, which can be the locations of either the reference accelerometers or the nearest interface points of the instrument. The vectors $\bar{u}$ and $\bar{v}$ represent an orthonormal coordinate system linked to the rigid base with origin at the point O and initially parallel to the global reference axes X and Y respectively.

Figure 2-28: Solid rigid base scheme. Centre point O is the base node of the analysis and the points A, B and C are the locations with known accelerations. Orthonormal coordinate system is represented by $\bar{u}$ and $\bar{v}$ [51].

The coordinates of the points A, B and C with respect to the global reference system is defined for $i = A, B, C$ in

$$\bar{r}_i = \bar{r}_O + \alpha_i \bar{u} + \beta_i \bar{v} \quad (2.13)$$

where $\bar{r}_O$ is the position vector of the centre point O of the rigid base with respect to the global reference coordinate system, and α_i and β_i are the coordinates of the points A, B and C with regard to the reference system with origin O and linked to the base.

This study focuses on the out-of-plane motion of the rigid base during the shock test, that is to say, in the vertical acceleration and in the rotations around the two horizontal axes. Additionally, the in-plane translational motion is neglected in the following equations. The angles of rotation around the global axes X and Y are respectively φ_x and φ_y with the sign indicated in Figure 2-29. It is assumed that these angles oscillate around zero with small values during shock excitation. With all these assumptions, the equations of the vertical position (Z) of the points of the rigid base are given by

$$Z_i = Z_O - \alpha_i \varphi_y + \beta_i \varphi_x \quad (2.14)$$

Figure 2-29: Definition of the angles used for the rigid base technique [51].

From this system of equations, the expressions for Z_O , and the angles φ_x and φ_y as functions of the coordinates Z_A , Z_B and Z_C are given by

$$Z_O = K_{ZA}Z_A + K_{ZB}Z_B + K_{ZC}Z_C \quad (2.15)$$

$$\varphi_x = K_{XA}Z_A + K_{XB}Z_B + K_{XC}Z_C \quad (2.16)$$

$$\varphi_y = K_{YA}Z_A + K_{YB}Z_B + K_{YC}Z_C \quad (2.17)$$

where the constants are defined as

$$K_{ZA} = \frac{(\alpha_C\beta_B - \alpha_B\beta_C)}{(\alpha_C\beta_B - \alpha_B\beta_C) + (\alpha_A\beta_C - \alpha_C\beta_A) + (\alpha_B\beta_A - \alpha_A\beta_B)}$$

$$K_{ZB} = \frac{(\alpha_A\beta_C - \alpha_C\beta_A)}{(\alpha_C\beta_B - \alpha_B\beta_C) + (\alpha_A\beta_C - \alpha_C\beta_A) + (\alpha_B\beta_A - \alpha_A\beta_B)} \quad (2.18)$$

$$K_{ZC} = \frac{(\alpha_B\beta_A - \alpha_A\beta_B)}{(\alpha_C\beta_B - \alpha_B\beta_C) + (\alpha_A\beta_C - \alpha_C\beta_A) + (\alpha_B\beta_A - \alpha_A\beta_B)}$$

$$K_{XA} = \frac{\alpha_B - \alpha_C}{(\alpha_C\beta_B - \alpha_B\beta_C) + (\alpha_A\beta_C - \alpha_C\beta_A) + (\alpha_B\beta_A - \alpha_A\beta_B)}$$

$$K_{XB} = \frac{\alpha_C - \alpha_A}{(\alpha_C\beta_B - \alpha_B\beta_C) + (\alpha_A\beta_C - \alpha_C\beta_A) + (\alpha_B\beta_A - \alpha_A\beta_B)} \quad (2.19)$$

$$K_{XC} = \frac{\alpha_A - \alpha_B}{(\alpha_C\beta_B - \alpha_B\beta_C) + (\alpha_A\beta_C - \alpha_C\beta_A) + (\alpha_B\beta_A - \alpha_A\beta_B)}$$

$$K_{YA} = \frac{\beta_B - \beta_C}{(\alpha_C\beta_B - \alpha_B\beta_C) + (\alpha_A\beta_C - \alpha_C\beta_A) + (\alpha_B\beta_A - \alpha_A\beta_B)}$$

$$K_{YB} = \frac{\beta_C - \beta_A}{(\alpha_C\beta_B - \alpha_B\beta_C) + (\alpha_A\beta_C - \alpha_C\beta_A) + (\alpha_B\beta_A - \alpha_A\beta_B)} \quad (2.20)$$

$$K_{YC} = \frac{\beta_A - \beta_B}{(\alpha_C\beta_B - \alpha_B\beta_C) + (\alpha_A\beta_C - \alpha_C\beta_A) + (\alpha_B\beta_A - \alpha_A\beta_B)}$$

The accelerations are given by the following expressions

$$\ddot{Z}_O = K_{ZA}\ddot{Z}_A + K_{ZB}\ddot{Z}_B + K_{ZC}\ddot{Z}_C \quad (2.21)$$

$$\ddot{\varphi}_x = K_{XA}\ddot{Z}_A + K_{XB}\ddot{Z}_B + K_{XC}\ddot{Z}_C \quad (2.22)$$

$$\ddot{\varphi}_y = K_{YA}\ddot{Z}_A + K_{YB}\ddot{Z}_B + K_{YC}\ddot{Z}_C \quad (2.23)$$

With these equations, the vertical and the out-of-plane rotational accelerations to be applied to the base node in the transient analysis are determined from the known vertical acceleration functions of three points A, B and C. The rigid element that connects the base node with all the interface nodes must be created to transmit adequately both translational and rotational accelerations. This technique considers the differences between the vertical accelerations detected by the reference accelerometers and assumes that are related to the rigid solid motion of the base plate. Therefore, it is a way to analyse the model with the assumption of a non-uniform input acceleration field, which may be more realistic than the previous techniques. Furthermore, the configuration of the base node and the rigid connection with the interface nodes is the same as the two previous techniques, with the only difference of including the calculated enforced rotational accelerations in the shock simulation.

2.5.1.4 *Non-uniform acceleration with test input signals*

This technique consists of applying the measured signals of more than one reference accelerometer in the FEA transient analysis. In a general case, it is necessary to include in the model as many base nodes as reference accelerometers. Each base node should be connected by a rigid element to the interface nodes closest to the corresponding accelerometer. In the usual case with two reference accelerometers located near two opposite corners of the mechanical interface of the tested item, Configuration 2 shown in Figure 2-27(b) can be used as an example to define the base nodes and their connections with the interface nodes in the shock analysis. The analysis performed with this configuration simulates a non-uniform input acceleration field, where one half of the interface nodes has an acceleration motion with the signal measured by one reference accelerometer, whereas the other half of interface nodes with a different signal measured by the other reference accelerometer. Therefore, it is an alternative approach of considering the differences of the measured input signals to generate a non-uniform input acceleration environment. Its implementation involves the use of more than one base node, which can be excited with the input signals in the main axis only or in the three orthogonal directions simultaneously.

2.5.1.5 *Non-uniform acceleration with calculated input signals*

This last technique employs the acceleration signals measured by the response accelerometers located on the tested instrument to estimate some or all the input accelerations to be applied in the shock simulations. This approach makes use of the complex transmissibility functions between the response locations and the input locations (interface points of the instrument), which are determined by the Fourier transform method to convert time signals into the frequency domain. The purpose of this technique is to compute one or more input signals when the enforced accelerations of some of the interface nodes are unknown and their estimations are necessary for a more realistic non-uniform shock analysis.

This technique is based on the principle of superposition of linear systems. Therefore, it is only valid for linear models and for linear analyses, like the case presented in this work. The results of a transient analysis with different enforced accelerations applied simultaneously at the different interface nodes can also be obtained by the summation of the results calculated from individual transient analyses. In each individual analysis, only one of the interface nodes is excited by the corresponding enforced acceleration time function, while the rest of interface nodes remain constrained.

The response acceleration $\ddot{z}_{RT}(t)$ in the time domain at one location R caused by the simultaneous excitation of all interface nodes (1, 2, etc.) is the result of the summation of the responses on the same location calculated from the individual excitations of each interface node $\ddot{z}_{Ri}(t)$

$$\ddot{z}_{RT}(t) = \sum_i \ddot{z}_{Ri}(t) \quad (2.24)$$

By applying the Fourier transform to Equation (2.24), the linearity is preserved and leads to the following expression in the frequency domain

$$\ddot{Z}_{RT}(\omega) = \sum_i \ddot{Z}_{Ri}(\omega) \quad (2.25)$$

By considering the complex transmissibility functions between the response location and each interface node T_{Ri} , and the Fourier transform of each input acceleration signal $\ddot{Z}_i$, the expression is given by

$$\ddot{Z}_{RT}(\omega) = \sum_i T_{Ri}(\omega) \ddot{Z}_i(\omega) \quad (2.26)$$

The transmissibility functions and the total response are supposed to be known. The transmissibility functions can be calculated by FEM analysis, while the total response is the Fourier transform of the test signal on the considered location. If some of the input signals are also known, as for example when an interface node is close to a reference accelerometer, the only unknown variables are the rest of the input accelerations in the frequency domain. In the example shown in the following expressions, the known input signals correspond to IF points 5 and 6, while the unknown input signals to be estimated correspond to the points 1 to 4.

$$\begin{aligned} \ddot{Z}_{RD}(\omega) &= \ddot{Z}_{RT}(\omega) - T_{R5}(\omega) \ddot{Z}_5(\omega) - T_{R6}(\omega) \ddot{Z}_6(\omega) \\ \ddot{Z}_{RD}(\omega) &= T_{R1}(\omega) \ddot{Z}_1(\omega) + T_{R2}(\omega) \ddot{Z}_2(\omega) + T_{R3}(\omega) \ddot{Z}_3(\omega) + T_{R4}(\omega) \ddot{Z}_4(\omega) \end{aligned} \quad (2.27)$$

Therefore, if a unique solution is required for a given shock environment, it is necessary to consider as many known response functions as different unknown input accelerations to be calculated. In this way, the system of linear equations is determinate compatible, where the coefficients and the variables are complex functions in the frequency domain. An example of this system is

$$\begin{bmatrix} T_{R1} & T_{R2} & T_{R3} & T_{R4} \\ T_{S1} & T_{S2} & T_{S3} & T_{S4} \\ T_{T1} & T_{T2} & T_{T3} & T_{T4} \\ T_{U1} & T_{U2} & T_{U3} & T_{U4} \end{bmatrix} \begin{Bmatrix} \ddot{Z}_1 \\ \ddot{Z}_2 \\ \ddot{Z}_3 \\ \ddot{Z}_4 \end{Bmatrix} = \begin{Bmatrix} \ddot{Z}_{RD} \\ \ddot{Z}_{SD} \\ \ddot{Z}_{TD} \\ \ddot{Z}_{UD} \end{Bmatrix} \quad (2.28)$$

where four response points are considered (R, S, T and U) and four input accelerations are to be solved (1, 2, 3 and 4).

The complex acceleration functions $\ddot{Z}_{jD}$ are the desired solutions of the selected response nodes, which can be calculated as the example shown in the first expression of Equation (2.27). The transmissibility functions T_{ji} are calculated from the FEM of the tested instrument, so it is recommended to have the model well correlated with test data. The results of sine sweep tests are usually employed to correlate the numerical models, where the calculated eigenfrequencies should match the experimental eigenfrequencies. The calculation of the transmissibility function from experimental data is very complicated due to the difficulty of performing a test where only one interface point is excited while the rest remain constrained.

This technique has the versatility to be applied in different cases. For example, if it is considered that a set of n interface nodes is excited by the same input acceleration, the transmissibility functions between a response node and each of these interface nodes can be added to calculate a single transmissibility function, as stated in by

$$\text{For } \ddot{Z}_1(\omega) = \ddot{Z}_2(\omega) = \dots = \ddot{Z}_n(\omega) = \ddot{Z}_T(\omega) \quad (2.29)$$

$$\ddot{Z}_{RT}(\omega) = \sum_{i=1}^n [T_{Ri}(\omega) \ddot{Z}_i(\omega)] = \left[\sum_{i=1}^n T_{Ri}(\omega) \right] \ddot{Z}_T(\omega) = T_{RT}(\omega) \ddot{Z}_T(\omega) \quad (2.30)$$

In this way, the number of variables to be solved are reduced and the number of equations as well. Therefore, it can also be easily used for the case of uniform acceleration field, where only a base node is defined. In this case, only one response acceleration function is needed to solve this problem. The procedure of applying this technique to the FEM of the instrument is indicated in Figure 2-30.

Figure 2-30: Flow diagram for applying the technique of considering a non-uniform input acceleration field with calculated input signals [51].

As can be expected from the definition of this technique, the final FEA results at the response nodes employed for the estimation of the input functions are identical (or with small differences due to numerical errors) to the accelerations measured in the shock test at these locations. It is recommended to check if the input acceleration functions estimated with this technique represent a realistic shock environment compared to the input signals directly measured in the test. The most relevant characteristics such as the frequency content, the maximum peak values and the corresponding SRS curves should not be very different between both types of input signals.

2.5.2 Numerical results of the new proposed techniques for shock analysis

A summary of all proposed analyses performed with the different techniques explained in this study and applied to the FE model of the STEP instrument is shown in Table 2-15.

Table 2-15: Summary of performed shock analyses with the FEM of STEP instrument [51].

Analysis name	Technique	Input function	Excited node	Direction of excitation
UZ1	Uniform acceleration in the main direction	Ref1	1 base node (Conf. 1)	Z
UZ2		Ref2		Z
UZ3		Average test input		Z
UT1	Uniform acceleration in the three orthogonal directions	Ref2	1 base node (Conf. 1)	X, Y and Z
RB1	Rigid base motion with rotations	Calculated translational and rotational accelerations	1 base node (Conf. 1)	Z and rotations around X and Y
NZ1	Non-uniform acceleration with test input signals	Ref1 and Ref2	2 base nodes (Conf. 2)	Z
NT1		Ref1 and Ref2		X, Y and Z
NC1	Non-uniform acceleration with calculated input signals	Ref1, A1-3Z and a calculated input signal from TriA5 response	3 bases node (Conf. 3)	Z

The first three shock analyses are performed with the first technique (explained in section 2.5.1.1), where a uniform acceleration field is applied in the main direction of the shock test (Z-axis). The only variation between these simulations is the input acceleration function to be applied to the base node. The vertical accelerations measured by the reference accelerometers Ref1 and Ref2 are applied as input functions for the UZ1 and UZ2 shock analyses respectively, while in the third analysis UZ3, an average input acceleration from the two accelerometers signals is considered.

The FEA analysis UT1 is a variation of the analysis UZ2, where the cross input accelerations measured by the reference accelerometer Ref2 in the shock test are included to generate an acceleration field composed of the simultaneous application of the acceleration functions in each of the three orthogonal axes to the base node. This analysis corresponds to the technique of uniform acceleration in the three orthogonal directions explained in section 2.5.1.2. Like in the previous analysis, Configuration 1 shown in Figure 2-27(a) is used.

For the technique of considering the support plate as a rigid solid with rotations, the analysis RB1 is proposed. The test signals measured by the reference accelerometers Ref1 and Ref2 in the vertical direction, together with the signal measured by the A1-3Z accelerometer located on IF4, are used to calculate the translational vertical acceleration and the out-of-plane rotational

accelerations to be applied at the base node. It is considered that the positions of these accelerometers correspond to the locations of the interface nodes IF6, IF1 and IF4 respectively (see Figure 2-8) for this calculation with the equations explained in section 2.5.1.3. The base node created in the FEA analysis is located on the geometric centre of the interface nodes.

The FEA analyses NZ1 and NT1 employ the technique of applying a non-uniform input acceleration field using more than one base node. Configuration 2 shown in Figure 2-27(b) is used to define these analyses. The signals measured by Ref1 are applied to the base node B connected to the interface nodes IF3, IF5 and IF6 (the interface nodes closest to this accelerometer), while the signals obtained from Ref2 are applied to the base node C linked to IF1, IF2 and IF4. The only difference between both analyses is that in NZ1, only the vertical accelerations are applied, whereas in NT1 analysis, the cross accelerations signals are also included in the input enforced acceleration.

The technique NC1 is based on calculating some of the input accelerations from the shock test data measured by the response accelerometers. Configuration 3 shown in Figure 2-27(c) is defined for this analysis. It is assumed that the known input accelerations are the vertical signals measured by the reference accelerometer Ref2 and by the accelerometer A1-3Z. The input signal measured by Ref2 in vertical direction is directly applied to the base node D, which is connected to the closest interface nodes IF1 and IF2, while the signal measured by A1-3Z accelerometer is directly applied to the interface node (IF4) where this accelerometer was situated in the shock test. To calculate the unknown input acceleration in vertical direction to be applied in the base node B connected to the rest of interface nodes, the test signal measured by the response accelerometer TriA5 in Z-axis is considered due to the close position of this sensor to the three interface nodes. The procedure explained in section 2.5.1.5 is applied to obtain the unknown time acceleration function.

For all modal transient analyses presented in this section, the normal modes of the STEP FEM are calculated in the frequency range of 0 – 10000 Hz, while the used modal damping factor is shown in Table 2-16, which has been chosen to correlate with the measurements of the out-of-plane shock test.

Table 2-16: Modal damping factor used for the transient analyses of the out-of-plane shock test simulations [51].

Frequency (Hz)	Modal damping factor
0 – 2000 Hz	0.15
3000 – 10000 Hz	0.05

The results in the SRS format obtained from the proposed transient analyses at the positions of the response accelerometers are compared with the test data from Figure 2-31 to Figure 2-39. The results at each location are shown in separate graphs for more clarity, plotting in the same graph the most similar analyses to facilitate the visualization of their differences.

Figure 2-31: Comparison of the response SRS at A1-1Z accelerometer location between the proposed FEA techniques and the test result. (a) UZ1, UZ2, UZ2. (b) UZ2 and UT1. (c) NZ1 and NT1. (d) RB1 and NC1 [51].

Figure 2-32: Comparison of the response SRS at A1-2Z accelerometer location between the proposed FEA techniques and the test result. (a) UZ1, UZ2, UZ2. (b) UZ2 and UT1. (c) NZ1 and NT1. (d) RB1 and NC1 [51].

Figure 2-33: Comparison of the response SRS at A1-3Z accelerometer location between the proposed FEA techniques and the test result. (a) UZ1, UZ2, UZ2. (b) UZ2 and UT1. (c) NZ1 and NT1. (d) RB1 and NC1 [51].

Figure 2-34: Comparison of the response SRS at TriA2 accelerometer location in Z-axis between the proposed FEA techniques and the test result. (a) UZ1, UZ2, UZ2. (b) UZ2 and UT1. (c) NZ1 and NT1. (d) RB1 and NC1 [51].

Figure 2-35: Comparison of the response SRS at TriA3 accelerometer location in Z-axis between the proposed FEA techniques and the test result. (a) UZ1, UZ2, UZ2. (b) UZ2 and UT1. (c) NZ1 and NT1. (d) RB1 and NC1 [51].

Figure 2-36: Comparison of the response SRS at TriA4 accelerometer location in Z-axis between the proposed FEA techniques and the test result. (a) UZ1, UZ2, UZ2. (b) UZ2 and UT1. (c) NZ1 and NT1. (d) RB1 and NC1 [51].

Figure 2-37: Comparison of the response SRS at TriA5 accelerometer location in Z-axis between the proposed FEA techniques and the test result. (a) UZ1, UZ2, UZ2. (b) UZ2 and UT1. (c) NZ1 and NT1. (d) RB1 and NC1 [51].

Figure 2-38: Comparison of the response SRS at TriA6 accelerometer location in Z-axis between the proposed FEA techniques and the test result. (a) UZ1, UZ2, UZ2. (b) UZ2 and UT1. (c) NZ1 and NT1. (d) RB1 and NC1 [51].

Figure 2-39: Comparison of the response SRS at TriA7 accelerometer location in Z-axis between the proposed FEA techniques and the test result. (a) UZ1, UZ2, UZ3. (b) UZ2 and UT1. (c) NZ1 and NT1. (d) RB1 and NC1 [51].

The SRS curves obtained from the different proposed methods present similar trends and are on the same order of magnitude than the results obtained from the shock test, but with some noticeable differences in various cases.

The comparison between the analyses with uniform vertical acceleration (UZ1, UZ2 and UZ3) shown from Figure 2-31 to Figure 2-39 reveals that the differences between the response SRS curves are a consequence of the differences between the input SRS curves (see Figure 2-12), where the average input (UZ3) provides intermediate responses between the results obtained from the excitations with the signals of each reference accelerometer. As a first conclusion of the comparison between these three analyses is that the differences of the results with the test data are significant.

The addition of the cross input accelerations has small influence on the presented FEA results, as can be appreciated in the comparisons between UZ2 and UT1 and between NZ1 and NT1 from Figure 2-31 to Figure 2-39. The response accelerations in the main excitation axis are practically the same whether the cross input accelerations are included or not, except for some cases such as for TriA2 and TriA3 locations, where small differences between the considered numerical results are appreciated above 500 Hz. The conclusion of this aspect is that the two first techniques studied in this work offer very similar results for the response accelerations in the main axis, which is the same conclusion extracted from the simulations of the in-plane shock test indicated in section 2.4.2.

The analyses NZ1 and NT1, which consider a non-uniform input acceleration field by applying the Ref1 signals to one half of the interface nodes and the Ref2 signals to the other half, provide results that, for some locations, are similar to the results calculated from UZ1 analysis, while for other locations, the results are similar to those obtained from the UZ2 analysis. The reason is the

greater influence of the excitation of each set of interface nodes on the nearest response locations. The results of NZ1 at TriA5, TriA6 and TriA7 locations are very similar to the results of UZ1 analysis at the same locations because these locations are closer to the set of interface nodes IF3, IF5 and IF6 (see Figure 2-8), which are excited in both analyses by the same enforced acceleration, the signal measured by the reference accelerometer Ref1. This fact indicates that the influence of the excitation of the rest of interface nodes on the responses of these locations is small. The same effect is observed at the locations (A1-3Z, TriA2), where the results are similar between NZ1 and UZ2 because the set of interface nodes (IF1, IF2 and IF4) closer to both response locations is excited in both FEA analyses by the same acceleration function measured by Ref2.

Finally, the techniques of a rigid solid base with rotations (RB1) and of considering a non-uniform acceleration field with calculated inputs (NC1) provide better results in various locations. This improvement is due to a greater complexity in the definition of these analyses, where more test signals such as A1-3Z and TriA5 have been taken into account for the definition of the enforced input accelerations. Both analyses present results at A1-3Z location (see Figure 2-33(d)) that match the test response. For the case of RB1 is due to the use of this test signal for the calculation of the functions of translational and rotational acceleration with the equations explained in section 2.5.1.3. The coincidence between the FEA results with this technique and the test results at this location demonstrates that the assumption of small values of rotational angles indicated in section 2.5.1.3 for this method is valid. For the NC1 analysis, the signal measured by A1-3Z is directly applied to IF4 node and, for this reason, this result must be practically exact. The SRS at TriA5 location calculated in the NC1 shock analysis is practically the same as the test result (see Figure 2-37(d)), because the test time signal in this location has been selected for the calculation of the input acceleration to be applied at base node B with Configuration 3 (see Figure 2-27(c)). The coincidence of this numerical result with the test data demonstrates the validity of the procedure indicated in section 2.5.1.5 for this technique.

In general, the results have been improved with these two last techniques in comparison with the rest of proposed analysis methods. The SRS from both analyses at some locations (A1-2Z, A1-3Z, TriA3, TriA4, TriA5 and TriA6) are very similar to the test results, especially at low frequency range (below 1000 Hz). This similarity of results can also be appreciated in the comparison between the time acceleration functions obtained from test and analysis shown in Figure 2-40. It is also shown the identical time signals at A1-3Z location between test and both analyses results and at TriA5z between test and NC1 analysis results.

Figure 2-40: Comparison of the time acceleration functions at the response accelerometers locations between the results calculated by the FEA techniques of Rigid Body base (RB1) and Non-uniform acceleration with calculated input (NC1) and the shock test results [51].

2.6 Verification plan for STEP and EPT-HET instruments

The detailed investigation about the numerical analysis of the shock environment with the FEA approach carried out in this doctoral dissertation has as a fundamental objective the establishment of modelling and analysis rules to be applied in the structural verification of space systems. In the previous sections of this doctoral dissertation, the different numerical methods have been studied by their application in the STEP FEM, where the precision of the results have been evaluated by comparison with the shock test data extracted from both in-plane and out-of-plane impact tests performed in the QM of the same instrument. In this section, the complete process carried out to demonstrate the structural capability of the STEP and EPT-HET instruments against the specified shock environment is detailed, where the verification by analysis plays a crucial role in the achievement of this certification.

The main objective of the structural verification of the space systems is to assure that these systems can withstand adequately the severe mechanical environment to which they will be subjected during their missions. The first task is to evaluate if the main structure will not be collapsed under the different mechanical loads. It is also important to study how the structures can protect the sensitive equipment, avoiding that the transmitted levels of accelerations can break these components. In the case of shock loads, the evaluation of the risk of damage to the sensitive parts is a key task to know if they can work correctly after and during the application of the intense shocks, taking into account the criticality of these components for the mission success. The sensitive parts are classified into three main categories by the European standard for shock design and verification [2]: electronic components, optical parts and mechanisms.

The process of the evaluation of these types of elements can be made with the so-called Shock Damage Risk Analysis (SDRA), which includes the creation of a list of sensitive components, the derivation of the shock accelerations from the specification provided by the launcher to the mechanical interfaces of the analysed components, the identification of the failure modes of the components under shocks and the determination of the capability of these components to resist the transmitted shock levels without degradation of their functionality or mechanical ruptures. The problem of this process is that some of the tasks, such as the derivation of the shock levels and the demonstration of the capability of the sensitive components to withstand the shock environment, are not fully-standardized due to the deficiencies of the existing methods, which have been explained in the review of the state-of-art described in section 2.1 of this document.

The derivation of the shock loads from the spacecraft level to the components level can be performed by analytical rules, using a database with information of previous space structures, by test data and by simulation results [2]. With respect to the verification of the capability of the sensitive components to withstand the specified shock load, the shock testing is the most used way. Nonetheless, it presents the risk of causing serious damages to the tested items due to the difficulty of defining with precision the required input levels. This was one of the reasons why STEP and EPT-HET instruments could not be completely tested. Therefore, alternative options, which are detailed in this section, had to be proposed to demonstrate that these units can withstand their specified shock environments. Among the proposed options, the verification by analysis was the approach that achieved the final acceptance of the shock qualification of the EPD units.

The level of precision demonstrated in the performed simulations by the transient analysis method with the STEP FEM has helped to enhance the confidence in the numerical results for this purpose. The advantage of the FEA simulations is that a complete set of responses, which includes the maximum stresses, forces, accelerations and displacements, is calculated to provide a detailed evaluation of the effects of the shocks on the structure and on the sensitive components.

As discussed in section 2.2.1, the shock verification of the EPD units was postponed until the end of the development phase, when the designs of both instruments had already been completed and tested in the vibration tests. The complete set of actions carried out in the shock verification procedure of the EPD units is depicted in Figure 2-41 and is described in the next sections. The solid lines in Figure 2-41 show the path followed in this project, whereas the dashed lines indicate other possible solutions for the shock verification that unfortunately were not met for this project.

The first task was the creation of the list of parts susceptible to shock, which essentially correspond to electronic components mounted on the PCBs of both instruments (see section 2.6.1). The survival of these components against the shock environment must be verified to guarantee their functionality after the mechanical events that will be generated during the launch phase.

The first shock verification method planned for EPD units was the justification by comparing the input random vibration loads with the specified shock levels in the frequency band that contains the main modes of the PCBs. In the case of the random vibration environment is more severe for the sensitive components than the shock, the survival of these components during the shock can be justified if they have successfully passed the random vibration tests [2]. However, as will be indicated in section 2.6.2, the resulting comparison for EPD units reveals that the levels of the random vibration tests performed on both units cannot cover the specified shock loads.

The next verification method proposed was the shock tests, but due to the risk of damage to the sensitive electronic components and the restricted availability of the tests facilities, only a subset of the shock tests was completed. The shock tests could not be performed with the PFMs due to the risk of damage, nor with the EPT-HET QM, which suffered irreparable damages after the random vibrations test (see Appendix A). Therefore, the only available unit to be tested was the STEP QM, which was prepared for shock testing. The performed shock tests on STEP QM were the shock test with qualification levels in the lateral Y-axis and the low level test (-12 dB of qualification levels) in the vertical Z-axis (see section 2.2.2). Despite the shock test campaign was not completed, the results of these tests were very useful for the validation of the FEM analyses in shock simulations (see sections 2.3, 2.4 and 2.5), which led to the verification by analysis explained in section 2.6.3.

The verification method that finally achieved the shock verification of the EPD units was by FEM analysis. After the exhaustive study described in this document (section 2.3) to evaluate and compare the results of different FEA methods for shock analyses, the selected analysis method was the modal transient analysis in NASTRAN code. This analysis method has been applied for the models of both EPD units to calculate the results such as stresses, accelerations and relative displacements to determine if the units and, especially their sensitive electronic components, can withstand without any damage the specified shock loads.

The main tasks of this process are detailed in the next sections, highlighting the most determinant results and conclusions.

Figure 2-41: Flowchart of the shock verification procedure followed for the EPD units indicated with the solid lines, where the main verification approaches are shown in black boxes. Dashed lines indicate alternative paths that finally could not be followed.

2.6.1 List of electronic components susceptible to shock damage

The lists of electronic components of STEP and EPT-HET instruments that can be in potential risk of damage under shock loads include large components, microcircuits, transformers and tantalum capacitors. The most critical components are indicated in Table 2-17. The most probable damage for large components such as Field Programmable Gate Array (FPGA), Static Random Access Memory (SRAM) and Electrically Erasable Programmable Read-Only Memory (EEPROM) is the mechanical failure of their pins and lead-wires due to the maximum deflections of the PCBs under dynamic loads. The risk is higher for the electric components located at the centre of the PCBs, as is the case for the FPGA components on the Digital boards of both instruments (see Figure 2-42), where the maximum curvature of the deflected PCB provokes the elongation of the lead wires generating stresses that can overpass the maximum strength capability of these joints [56]. This phenomenon was the cause of the failures of some pins and solder joints of the large electronic components of the EPT-HET QM during the random vibration test in Z-axis (see Appendix A). Other components such as crystal oscillators, quartz components, transformers and tantalum capacitors have fragile parts susceptible to suffer fragile ruptures under an intense shock acceleration. The first failure mode that can appear on the transformers is the crack initiation in the ferrite core, which can derive in the fragile rupture and malfunction of this component, while the tantalum capacitors can be damaged due to the bending of the PCB, which can lead to the destruction of the dielectric [2].

Table 2-17: Main sensitive electronic components of STEP and EPT-HET instruments.

EPD unit	PCB	Component
STEP	LVPS	Crystal
	Digital	FPGA
		SRAM
		EEPROM
		Crystal
EPT-HET	LVPS	Crystal
	Digital	FPGA
		SRAM
		EEPROM
		Crystal
	Analogic	FPGA

(a) Digital PCB of STEP instrument

(b) Digital PCB of the EPT-HET instrument

Figure 2-42: Sensitive electronic components on the Digital PCBs for EPD units (Courtesy of the Institute of Experimental and Applied Physics of the University of Kiel).

2.6.2 Shock justification by equivalent random response spectrum

The first shock verification approach planned for EPD units was by the method of shock justification by the comparison with the equivalent random response spectrum, which is recommended in the European standards [2] for electronic units when the shock tests cannot be performed. This method consists of the comparison of the input levels of the random vibration tests with the specified shock environment. To perform this comparison, the input random vibration specification, which is typically defined as an acceleration spectral density (ASD), must be transformed into an equivalent response spectrum in the same terms as the shock SRS. It is intended to demonstrate with this comparison that the random vibration levels of the qualification tests were more severe for the sensitive components than the specified shock SRS, and therefore, to justify that the shock verification can be covered by the random vibration tests. This method was the first option because it only involves a simple analytical calculation, taking into account the input specifications of random and shock loads, and the list of natural frequencies of each unit, which can be calculated from the FEM analyses.

The dynamic excitation of the fundamental mode (main flexural mode) of the PCB generates the greatest board deflection, which in turn causes the highest stresses on the pins and joints of the large and heavy electronic components, especially if they are located at the centre [56]. Therefore, the risk of failure of the pins of the electronic components is focused on the intensity of the excitation of the main mode of the PCB, which depends on the severity of the input load at this frequency. In this way, if the fundamental mode of the PCB lies within the frequency range where the random vibration is more severe than the shock, the shock verification of the sensitive electronic components mounted on this PCB can be covered by the random vibration tests. Also, the excitation of the main modes of the PCBs is more intense with the loads applied in the perpendicular direction of these boards, which for both EPD units corresponds to the Z-axis (see Figure 2-5).

The random specifications for the proto-flight tests in Z-axis for EPD units is shown in Figure 2-43 [48,49]. The duration of the proto flight random vibration test for each axis was 60 seconds. The notches are appreciable for both curves between 500 and 900 Hz for STEP and between 600 and 1000 Hz for EPT-HET. These frequency ranges include the main modes of the units and also the fundamental modes of their PCBs. The natural frequencies of the PCBs were calculated by FEM analysis, because the experimental determination of these modes could not be measured directly during the vibration tests due to the impossibility of locating accelerometers inside the units.

Figure 2-43: Random vibration specifications in Z-axis for STEP and EPT-HET instruments.

As the functional performance of STEP and EPT-HET units after the proto-flight random vibration tests was successful [48,49], the shock verification can be justified if the shock specification is less severe than the random vibration tested levels at the frequencies of the fundamental modes of the PCBs. Different equations are proposed in [2] to define the equivalent response spectrum of the random vibration load with respect to the structural effect. The difficulty lies in the fact that the random vibration specification is based on the statistical aspects of this dynamic load, while the shock is defined as a deterministic excitation. The selected option for this project was the approximation of the Extreme Response Spectrum (ERS) defined as

$$ERS(f_k) = z_{RMS}(f_k) \sqrt{2 \ln(n_0^+ T)} \quad (2.31)$$

where the ERS represents an estimation of the maximum response acceleration of a single-degree-of-freedom (SDOF) system as a function of its natural frequency f_k subjected to a random vibration environment, where T is the duration of random test in seconds, n_0^+ is the mean number of zero crossing with positive slope and it is assumed to be approximately equal to the natural frequency, f_k , and z_{RMS} is the value of the root mean square (RMS) of the absolute input acceleration of the SDOF system. z_{RMS} can be estimated with the Miles Formula defined as

$$z_{RMS}(f_k) = \sqrt{\frac{\pi}{2} f_k Q W(f_k)} \quad (2.32)$$

where Q is the amplification factor related to the damping, which for shock is typically $Q = 10$, and $W(f_k)$ is the value in g^2/Hz of the ASD random specification at the excitation frequency f_k .

With all these assumptions, the equivalent spectrum is finally given by

$$ERS(f_k) = \sqrt{\pi f_k Q W(f_k) \ln(f_k T)} \quad (2.33)$$

The comparisons between the equivalent random vibration spectrum (ERS) and the shock spectrum (SRS) in Z-axis for STEP and EPT-HET units are shown in Figure 2-44. For EPT-HET unit, the ERS are calculated for both the PFM and QM random vibration levels.

Figure 2-44: ERS-SRS comparison in Z-axis for shock justification for (a) STEP and (b) EPT-HET.

The random vibration ERS curves are above the shock SRS curves in the low frequency range until 332 Hz for STEP and 375 Hz for EPT-HET PFM. However, the fundamental natural frequencies of the PCBs for both instruments are higher than these frequencies, where SRS curves are not covered by random vibration loads. The conclusion of this comparison is that the severity of the input shock environment is higher than the random vibration tests for the electronic

components mounted on the PCBs and therefore, the shock verification could not be justified with this approach.

On the other hand, the initial random vibration specification planned to qualify the EPT-HET instrument was higher than the final PFM levels (see Appendix A). The EPT-HET QM was tested with this severe random vibration input and, consequently, various pins of its electronic components were broken during this test. The ERS-SRS comparison can also be used to evaluate if the shock can cause the same failures on the joints of the electronic components in case of it is more severe than the QM ERS. The shock SRS is higher than the QM random vibration ERS from approximately 700 Hz (see Figure 2-44(b)), a frequency that is higher than the main modes of the PCBs for the QM design, which are between 620 and 665 Hz (see Appendix A). Therefore, there is no conclusion with this method about the potential damage to the sensitive electronic components caused by the shock loads.

2.6.3 Shock verification by tests

The shock test of STEP QM with qualification levels in the lateral Y-axis was performed satisfactorily, achieving that the input SRS in the main direction was within the required limits (see Figure 2-10). Before testing with qualification levels in the most critical direction (Z-axis), low-level shock tests were performed on STEP QM to measure the shock transmissibility at different external locations of the unit. Because of the attenuation of the signals from the interface to the measurement points was not significant in the frequency band up to 1000 Hz (see Figure 2-45), the shock test campaign was stopped until having reliable predictions about the effects on the sensitive parts with the qualification shock levels. Due to the difficulty of locating accelerometers on the PCBs during the shock test, which are inside the unit, the only way to have predictions with sufficient accuracy of the shock response of the PCBs is by FEM analysis. The results of the FEM analyses can determine the risk of damage to the electronic components.

Figure 2-45: Input SRS (Ref1z) and response SRS curves (TriA5z and TriA6z) measured from the low level shock test of STEP QM in Z-axis.

2.6.4 Shock verification by analysis

The verification by analysis is one of the possible options to demonstrate that a system meets the imposed requirement in a certain aspect on the design or performance. According to the European standards [2], the verification by analysis shall be agreed among the participants of a space project. The structural verification by analysis of space systems can be accepted when the simulation method is validated. After the study carried out in this doctoral dissertation, it was demonstrated the capability of modal transient analysis method to provide reliable structural

results of the STEP instrument under the shock environment measured during the tests. Therefore, this numerical method is employed to provide the structural results that allow the verification of the EPD instruments against the shock loads.

To perform a shock simulation by transient analysis when the input shock specification is given in a SRS format, it is necessary to calculate previously a time function of input acceleration that meets the specified SRS. It is usually recommended that this time function has an oscillatory form and a short duration, on the order of tens of milliseconds, because this is the typical aspect of the pyroshock environment that appears during the launch operations. Different numerical algorithms can be used to calculate this function from a given SRS, being the sinusoids decomposition and the wavelet synthesis the most employed techniques [3]. The study presented in section 2.3.3 has demonstrated that the variability of the results, such as response SRS curves and peak values, calculated from the different techniques is small. The selected algorithm to obtain the time acceleration functions (see Figure 2-46) to be used as inputs in the shock transient analyses for STEP and EPT-HET models is the sinusoids decomposition method, where the chosen duration of the signals is 20 ms.

Figure 2-46: Input acceleration time functions for shock transient analyses in Z-axis calculated from the specified SRS curves by the sinusoids decomposition algorithm.

The input accelerations calculated from the specified SRS are included in the transient analyses to simulate the shock environment. The results of the transient analyses are the time functions of the requested responses such as stresses, interface forces, accelerations, velocities and displacements. In the next subsections, the main aspects from a structural point of view are evaluated to complete the shock verification of the EPD units.

2.6.4.1 Maximum peak stresses on the structural parts

To evaluate the shock effects on the structural parts of the instruments, it is necessary to compare the maximum peak values obtained from the transient analyses with the maximum allowable limits of the materials. The distribution of peak values of the stresses on the aluminium structural

parts of the EPT-HET instrument calculated from the shock transient analysis in Z-axis is shown in Figure 2-47, where the global maximum value is 117 MPa in this case.

Figure 2-47: Maximum peak stress values (Pa) on the main aluminium structural parts of EPT-HET unit calculated from a transient analysis in Z-axis.

To verify in a conservative way if the structure can withstand this load, the maximum peak value for each part (σ_{peak}) is increased with the required safety factor (K_y or K_u). These values are then compared with the yield strength (σ_y) and ultimate strength (σ_u) values of the corresponding material by the calculation of the margins of safety (MoS) defined in the following equations,

$$MoS_y = \frac{\sigma_y}{K_y \sigma_{peak}} - 1 \quad (2.34)$$

$$MoS_u = \frac{\sigma_u}{K_u \sigma_{peak}} - 1 \quad (2.35)$$

If all the margins are positive, it is demonstrated by analysis that the structure will not suffer plastic deformation or rupture under the applied shock environment. The margins of safety for the maximum stresses on the aluminium parts of STEP and EPT-HET units are indicated in Table 2-18 for the shock analyses in Z-axis. The resulting margins of safety are positive, which evidences that the structures of both units will support adequately the predicted shock levels during the launch.

Table 2-18: Margins of safety of the maximum stresses calculated on the aluminium parts from the shock transient analyses in Z-axis for STEP and EPT-HET instruments.

Unit	Material	σ_y (MPa)	σ_u (MPa)	K_y	K_u	σ_{peak} (MPa)	MoS_y	MoS_u
STEP	Al6061-T6	276	310	1.21	1.38	107	1.1	1.1
EPT-HET	Al6061-T6	276	310	1.21	1.38	117	0.9	0.9

2.6.4.2 Shock verification of the sensitive electronic components

It is also necessary to confirm if the sensitive electronic components can survive subjected to these intense shock loads. The way to perform this verification is by evaluating the accelerations and displacements on the PCBs calculated from the transient analyses, and considering the maximum allowable limits of the electronic components. Many electronic components have maximum allowable limits of shock acceleration, which can be specified as a maximum peak value or as a SRS envelope. These limits depend on the type of electronic component and are usually established by the manufacturer. Some electronic components have fragile parts, which are the most susceptible to be damaged under high accelerations. Examples of such components are the quartz, transformers with ferrite cores, tantalum capacitors or crystal oscillators. The standard limit for these components in terms of a SRS is 2000 g [2], but it should be checked for specific cases. Other sensitive components to high accelerations are the relays, where the first failure mode is the electric bouncing that can appear with shocks higher than 200 g of SRS [2].

The maximum accelerations transmitted to the electronic components under a shock load are generated by the excitation of the fundamental modes of the PCBs, which is more intense if the input shock is perpendicular to the boards. In the case of the EPD units, this direction corresponds to the Z-axis (see Figure 2-5). The plots with the peak values of the absolute acceleration on the power and Digital PCBs of EPT-HET unit are shown in Figure 2-48, where the maximum values are located at the centre of each PCB. In addition to the peak values, the acceleration levels for each electronic component under the shock environment can be indicated in a SRS format, by converting the time functions of the response accelerations obtained at the nodes located close to the attachment points between the components and the PCB. The SRS curves for the crystal oscillators mounted on the LVPS and Digital PCBs of the EPT-HET are shown in Figure 2-49. The maximum peak accelerations and the maximum values of the SRS curves at the nodes corresponding to locations of the sensitive components of EPT-HET unit are summarized in Table 2-19. These maximum response accelerations are below the allowable limits of these components, so that in this regard, the shock levels are within the tolerance margins.

Figure 2-48: Distributions of maximum peak accelerations (m/s^2) calculated from FEA transient analyses in Z-axis on the LVPS and Digital PCBs of the EPT-HET instrument.

Figure 2-49: Response SRS curves calculated from FEA transient analysis in Z-axis for crystal oscillators of the LVPS (a) and Digital (b) PCBs of the EPT-HET instrument.

Table 2-19: Maximum acceleration results for the most critical electronic components of EPT-HET unit: Peak accelerations and maximum values of the response SRS curves.

PCB	Component	Maximum peak acceleration (g)	Maximum SRS level (g)
LVPS	Crystal oscillator	316.2	1243.5
Digital	FPGA	717.9	1938.8
Digital	SRAM	357.1	1360.1
Digital	EEPROM	357.1	1360.1
Digital	Crystal oscillator	352.3	1219.6
Analogic	FPGA	637.1	1926.7

There is another aspect that must be taken into account to conclude the shock verification of the electronic components and corresponds to the risk of failure of joints and pins caused by the maximum deflections of the PCBs during the shock environment. The chosen approach to analyse this phenomenon is the Steinberg's method [56], which establishes a relationship between the maximum relative displacements of a PCB and the fatigue damage that can be caused to the joints of the electronic components. This approach provides empirical equations to determine the fatigue life of solder joints of electronic components mounted on a simply supported PCB subjected to sine and random vibration environments. With the aim to provide a similar equation for shock analysis, when the number of excited cycles is less than a few thousand, the Steinberg's method establishes the equation

$$Z_{peak}^{Stein} = \frac{0.16896 B}{C h r \sqrt{L}} \quad (2.36)$$

This equation determines the maximum allowable peak value of relative displacement of the PCB (Z_{peak}^{Stein}) as a function of parameters indicated in Table 2-20.

Table 2-20: Parameters of the Steinberg's fatigue equation for shock [56].

Parameter	Description	Unit
Z_{peak}^{Stein}	Peak value of the maximum allowable relative displacement of the PCB	mm
B	Length of the PCB edge parallel to the electronic component	mm
C	Constant that depends on the type of electronic component ($0.75 \leq C \leq 2.25$)	-
h	Thickness of the PCB	mm
r	Relative position factor of the electronic component on the board =1 (centre), =0.707 (side), =0.5 (corner)	-
L	Length of the electronic component	mm

This expression indicates that the risk of cracks on the joints caused by shocks is higher for large components and increases if they are mounted at the centre of the PCB and with rigid mechanical connections. For the EPT-HET instrument, the most susceptible components of such characteristics are two large FPGAs, each one located at the centre of the Digital and Analogic boards (see Figure 2-42(b)). Other critical components are a SRAM, an EEPROM and a crystal oscillator on the Digital PCB and a second crystal oscillator on the LVPS PCB (see Table 2-21). The distributions of the absolute peak values of the displacements on the Digital and Analogic boards of EPT-HET obtained from the FEA transient analysis in Z-axis are shown in Figure 2-50, while the time functions of the relative displacements between the centre and the edge of each PCB are shown in Figure 2-51.

Table 2-21: Values of the parameters of the Steinberg's equation for the EPT-HET critical components.

PCB	Component	B (mm)	h (mm)	L (mm)	C	r
LVPS	Crystal oscillator	100	2.1	10	1.26	0.707
Digital	FPGA	100	1.6	40	1.26	1.0
Digital	SRAM	100	1.6	35	1.26	0.707
Digital	EEPROM	100	1.6	25	1.26	0.707
Digital	Crystal oscillator	100	1.6	10	1.26	0.707
Analogic	FPGA	100	1.6	40	1.26	1.0

Figure 2-50: Distributions of absolute peak displacements perpendicular to the PCBs (m) calculated from FEA transient analyses in Z-axis on the Digital and Analogic PCBs of the EPT-HET instrument.

Figure 2-51: Time functions of the relative displacements between the centre of each PCB and its edge in perpendicular direction for the Digital and Analogic PCBs of the EPT-HET instrument.

The maximum values of the relative displacement for each PCB are compared with the allowable limits obtained by the Steinberg's method for each sensitive component to calculate the margins of safety (MoS_{RD}) by using

$$MoS_{RD} = \frac{Z_{peak}^{Stein}}{Z_{peak}^{FEM}} - 1 \quad (2.37)$$

The relative displacements obtained from FEM analysis, the allowable Steinberg's limits and the calculated margins of safety for the sensitive electronic components of EPT-HET are shown in Table 2-22. The margins of safety for all these sensitive electronic components for EPT-HET calculated by the Steinberg's method are positive and much higher than unity, indicating that the risk of damage to the solder joints and pines under the specified shock loads are low.

Table 2-22: Margins of safety calculated by the Steinberg's method for the most critical electronic components of EPT-HET instrument.

PCB	Component	Z_{peak}^{FEM} (mm)	Z_{peak}^{Stein} (mm)	MoS_{RD}
LVPS	Crystal oscillator	0.23	2.86	11.4
Digital	FPGA	0.22	1.33	5.0
Digital	SRAM	0.22	2.00	8.1
Digital	EEPROM	0.22	2.37	9.8
Digital	Crystal oscillator	0.22	3.75	16.0
Analogic	FPGA	0.14	1.33	8.5

This process was also applied to STEP instrument getting also positive margins. Therefore, with these last results, the shock verification for both EPD units has been completed satisfactorily.

2.7 Conclusions

An exhaustive study about the numerical simulations of structures subjected to shocks has been presented in this chapter, focusing on the practical aspects of the application of the FEA transient analysis on the space instruments. The investigation has been carried out for the project of the design of the STEP and EPT-HET instruments of the Solar Orbiter mission, where the shock verification is a key task to finish the development phase of both units.

In the first part of the chapter, an overview of the different numerical analyses with the FEA approach that can be employed to evaluate a structure subjected to a shock environment has been presented. A brief description of each method indicating the main advantages and disadvantages has been included in this work. The conclusions about the characteristics of studied methods are summarized in Table 2-23. The modal transient analysis presents the best precision and is the only method capable of providing all the required results for structural verification in various formats, such as time functions, peak values and SRS curves. Its main drawback is the high computational cost in terms of analysis time and amount of output data. From the transient analyses performed with the STEP FEM, it is concluded that the differences of the acceleration results with respect to the shock test measures are within the tolerable limits, except for TriA7y location, where the difference is mainly due to the high frequency content of the test signals that cannot be reproduced with accuracy in the transient analysis. The rest of numerical methods present high dispersion of results and are quite dependent on the selected analysis option.

Table 2-23: Summary of studied shock analysis methods.

Analysis method	Precision	Time of analysis and amount of output data	Type of results		
			Time functions	Peak values	Acceleration SRS curves
Modal transient	Good	Long	X	X	X
RSA	Low	Short		X	
Sine transmissibility	Low	Long			X
EQS	Low	Short		X*	

*Only for IF forces

If it is not feasible to perform transient analysis due to its high computational cost or if a quick evaluation is needed, the rest of studied methods can be useful to evaluate the shock environment, but their lack of precision in some aspects and their limitations should be taken into account. For a conservative calculation of the response peak values, RSA with ABS option is the recommended analysis. This analysis can be performed together with SRSS option to determine the range of possible peak values for shock analyses. Intermediate options, such as CQC or NRL or by using an adequate value for CLOSE parameter with SRSS or NRL options, provide results more accurate. An alternative to calculate in an easy way the peak values of the interface forces is the EQS method, where the results with decoupled normal modes formulation are as conservative as with RSA ABS option, while with coupled normal modes, the results are similar to those calculated with RSA SRSS. If the calculation of response SRS curves is required to evaluate any critical component or to derive the SRS specification to an internal part, transient analysis is the recommended method because it provides accurate results, while with the sine transmissibility method, the resulting SRS curves must be corrected in the high frequency range.

In the case where, to qualify a space structure, the shock environment is only specified by a SRS format, different possible time functions of acceleration can be calculated from this SRS to be used for transient analysis. As shown in section 2.3.3, the differences between the results (peak values and response SRS curves) are within the tolerable limits considering a varied set of

oscillatory functions determined by damped sinusoids decomposition and wavelet synthesis algorithms, and a rectangular pulse that best matches the specified SRS.

In section 2.5, different techniques to define the input excitation in the modal transient analysis have been compared to evaluate their precision in the out-of-plane shock test simulations, where the effects such as the non-uniformity of the input acceleration field and the non-negligible cross input accelerations have been taken into account in different ways. The simplest and usual way to analyse the shock environment is by assuming a uniform acceleration field for all the interface nodes, where the excitation is only in the intended direction of the test. The input acceleration function can be any of the signals obtained by the reference accelerometers or an average function of these signals. This technique has been applied in the first part of the chapter to simulate the in-plane shock test environment of the STEP instrument, but it cannot reproduce with sufficient precision the more complex input field generated during the out-of-plane shock test due to the aforementioned effects. As a consequence, the results presented in this work with this technique have remarkable differences with respect to the out-of-plane shock test results.

An option to improve the shock simulation is by the addition of the cross input accelerations measured by the reference accelerometers. The comparison performed in this work between the results with this technique and with the application of the input only in the main axis reveals that the influence of the cross input accelerations on the response accelerations is small compared to other effects. Only the response accelerations in the main axis of excitation are presented in this work, but the influence on other results such as the response accelerations in the other axes may be greater.

To find a better simulation of the shock environment measured in the out-of-plane impact test, it is necessary to define a non-uniform input acceleration field. One of the proposed techniques is based on using as many base nodes as reference accelerometers are available in the shock test, where each base node is connected by a rigid element to the set of interface nodes closest to the position of the corresponding reference accelerometer. The acceleration signals measured by each input accelerometer are directly applied to the corresponding base node, generating a non-uniform acceleration field on the interface nodes. The analyses performed with this technique with the FEM of the STEP instrument do not provide significant improvements in the precision of the results with respect to the previous techniques.

Another proposed technique assumes that the support plate, which is a mechanical interface element between the drop table and the tested item, acts as a rigid solid with a motion that combines translational and rotational accelerations. The rotational accelerations of the support plate can be induced by the excitation of the flexural modes of the drop table in the impact shock test. The resulting vertical and rotational accelerations calculated by the procedure described in this work are applied to the single base node connected to all the interface points in the shock analysis. The application of this technique in the STEP instrument case allows us to achieve a better agreement between the FEA results and the test responses. In the last presented technique, the estimation of all or some of the input acceleration functions is considered with the objective to match the FEM results at some of the locations (instrumented with accelerometers) with the test signals measured by these accelerometers. This method is based on the superposition principle of linear models and linear analysis. With the procedure explained in this document and applied to the STEP instrument, the results are very similar to the test data in various locations.

The main conclusion of this part of the investigation is that the simulations of shock tests by the transient analysis method with the FEA approach can achieve a high degree of precision for the acceleration results when the phenomena of non-uniformity of the input acceleration field and the

presence of the cross accelerations are taken into account. This degree of precision has been demonstrated in the practical case of the STEP instrument by comparing the numerical results with the shock test signals. Moreover, different techniques have been proposed to define these aspects in the shock analyses, providing alternatives that can be applied to any space structure subjected to a shock test. The use of the most appropriate technique depends on the available data and on the required degree of precision.

The relevance of the improvement of the shock simulations lies in the fact that an accurate analysis can provide more reliable results, which can be decisive in the structural verification of space structures. This is especially important for the critical components, which in many cases cannot be monitored during the shock test and the only available information about their response against shock loads must be calculated by analysis. This is the case explained in the last part of this chapter, where the planned and executed procedure for the shock verification of the STEP and EPT-HET instruments is detailed, showing the importance of the performed analysis on the success of the final verification. Different options were proposed to achieve the demonstration of the capability of the structural designs of both units to resist the specified shock loads and to protect their sensitive elements, which are essentially electronic components. The most used option for shock verification in the space industry is by testing. This option could not be completed for the EPD units due to the risk of damage for sensitive components in case of over-testing, the unavailability of the EPT-HET QM and the schedule restrictions of the tests facility. Only the qualification test on Y-axis and the low-level tests on Z-axis could be performed on the QM of STEP instrument. Alternative approaches were planned to complete the verification of these instruments against the mechanical shock environments. The justification by comparison with random vibration levels is one of the planned options, but the shock levels are more severe than random vibration levels in the frequency range that contains the main modes of the PCBs. Therefore, the selected verification approach was by FEM analysis, taking into account the adequate level of precision demonstrated in the first part of this chapter with the STEP FEM.

The FEA approach has the advantage of providing a complete set of results (stresses, forces, accelerations, displacements) for all modelled parts and without the risk of damage for sensitive parts that the tests have. In addition, if the precision of the numerical results is within the tolerable limits, the verification by analysis can be accepted to demonstrate the capability of a space system to withstand the specified shock environment. This investigation presents the results extracted from the FEA transient analyses of EPD units, evaluating by the calculation of the corresponding margins of safety whether the structural designs of the instruments are adequate against the specified shock loads. The maximum generated stresses on the structural parts have been calculated and compared with the allowable limits of the materials to determine that both instruments can resist the intense shock loads to which they will be exposed during the launch phase. On the other hand, the maximum levels of accelerations on the PCBs in terms of peak values and SRS curves and the relative displacements calculated from FEA transient analyses have been used to evaluate the risk of damage for the most critical electronic components of EPT-HET unit. High accelerations can produce cracks on fragile elements or malfunctions in relays, while relative displacements on the PCBs are related to the generated stresses on the solder joints and pins. All these aspects have been evaluated for EPD units concluding that all margins of safety are positive and the units are finally verified against the specified shock loads. As a conclusion, the procedure explained in this chapter can be implemented for future projects where the shock verification by analysis is to be applied for space instruments instead of the testing approach.

3 DYNAMIC COUPLING OF SPACE STRUCTURES

3.1 State of the art

The phenomenon of the dynamic or modal coupling is of great interest in the design of space structures. The usual procedure in the space industry is to avoid the dynamic coupling between the launcher with its carried spacecraft, or between a spacecraft and its equipment. The main purpose is to prevent the adverse effects that appear when the frequencies of the dynamic loads generated during the launch phase and other events like landing, testing or ground operations, coincide with the main natural frequencies of the structures, leading to structural resonance, which produces high stresses and forces and the generation of undesirable structural damages [1,3].

To avoid the structural resonance of a satellite at the low frequency vibrations generated by the launch vehicle, the launcher authority specifies minimum limit values for both lateral and longitudinal natural frequencies of the satellite [1]. In a similar way, the satellite designers specify minimum values for the natural frequencies for their payloads and equipment to assure that they are high enough above the fundamental natural frequencies of the satellite. The main reason is to protect the equipment by avoiding the structural resonance with the dynamic loads transmitted through the structure of the satellite, where the frequencies of the highest acceleration amplitudes at the equipment interface correspond to the natural frequencies of the satellite. Another reason is to avoid the undesirable effects caused by the modal coupling with the main structure of the satellite, which can put at risk the structural integrity of the whole system. Usually for early design phases, a fixed minimum frequency requirement on the secondary structure is defined by the primary structure designer. This approach provides in many occasions a very restrictive requirement that leads to oversized designs for the secondary part. In some space projects, particular components like large deployable antennas or massive instruments with low stiff attachment cannot be designed to have the first natural frequency higher enough than the main natural frequencies of the satellite due to the difficulty of achieving the necessary increase in stiffness. The motivation of this second part of the doctoral dissertation is to acquire a better understanding of the effects of modal coupling, which is needed to improve the design parameters of this type of space structure.

Different investigations have dealt with the problem of modal coupling in space structures. The relevance of the modal coupling between parts on large space structures has been described by A. Bertram in [57], indicating its influence on the attitude dynamics and on the structural response during the launch phase. In [58], the shock specification by a shock response spectrum (SRS) envelope is generated by considering a coupled system representing the secondary and the primary structure. The resulting specification is more realistic than that calculated from the usual procedure of considering both parts decoupled. To provide more precise input loads generated during the launch phase, the coupled load analysis (CLA) is the most recommended way to derive the dynamic loads specification for the spacecraft [1]. In CLA simulations, a model composed of the finite element model (FEM) of the launcher and a reduced model of the satellite is considered, and in this way, the influence of modal coupling between launcher and satellite is taken into account for more realistic environments. Different approaches to correlate reduced models of satellites have been presented in [59–61] to have more accurate results in CLA. In [62,63], the

low stiffness attachment of a spacecraft with a vibration isolation system is studied. This type of mechanical interface configuration has the advantage of reducing the levels of vibrations that are transmitted to the spacecraft, but the drawback of decreasing the main natural frequencies of the attached spacecraft to the same frequency range as the modes of the launcher. Therefore, the dynamic coupling between spacecraft and launcher cannot be neglected with this type of interface design.

The modal coupling is also of relevance to the attitude dynamics of spacecraft. The dynamic coupling between the large deployable antenna typically used in communications satellites and the rest of spacecraft is taken into account in [64] to study the attitude dynamics of the entire satellite. This type of payload constitutes a non-negligible mass with a low stiff joint with the satellite after deployment, which leads to a local natural frequency of the same order of magnitude as those of the global modes of the satellite. The modal coupling should also be taken into account to derive a reliable simulation during the mechanical tests [65].

To have a better understanding of the influence of the main parameters on the results of a coupled system, a 2-degree-of-freedom (2-DOF) system has been widely used in numerous research papers. H. Frahm [4] introduced the concept of the tuned mass damper (TMD) and J.P. Den Hartog [66] studied its effects, where the secondary mass is suited to have a natural frequency similar to the main mode of the system. TMDs have been applied in civil engineering to reduce the amplitude of response vibrations of buildings subjected to aerodynamic or earthquake loads [67–71]. In recent years, the concept of TMD has been improved leading to the creation of more sophisticated devices with better performance and effectiveness [72]. In [73], analytical solutions are calculated to find the best position and the optimal damping constants to minimize the vibrations on flexible structures. Semi-active vibration absorbers (SVA) used in conjunction with TMD to control the performance in order to be adapted to the disturbing frequency in real-time are studied in [74,75]. In [76], a semi-active TMD with resettable variable stiffness (RVS-TMD) is proposed to improve the performance of the existing TMD systems. In [77], a TMD with variable inertance is numerically studied to include the effects of different parameters in the performance of a wide range of forcing frequencies. Shape memory alloys (SMA) can be employed as TMD for seismic mitigation [78]. Other designs with the same approach are the tuned liquid mass damper (LTMD) [79], the tuned liquid column gas damper [80], the magnetically tuned mass damper [81], the magnetorheological damper with semi-active TMD [82,83] and the piezoelectric vibration energy harvesting beam as a TMD [84].

TMD concept has also been applied in the space industry. The use of TMDs has been preferred in space industry due to their simplicity and the possibilities that they offer for reducing the response due to particular modes. In [85], TMDs are employed to reduce the oscillations of printed circuit boards (PCB) under vibration loads, being more efficient than increasing the thickness. In [86], the dynamic analysis of space structures with multiple tuned mass dampers (MTMD) is enhanced with formulations of the reverberation matrix method (RMM). In [87], the location and design of TMD devices to improve the performance of space mirrors is evaluated, showing that by means of an eddy current damper a great reduction in the response can be achieved (up to 80%) depending on the TMD location, damping and targeted modes. In [88], a semi-active control system consisting of variable-friction dampers linked to the structure with cables is proposed and studied. In [89], a new mode decomposition method for structures with non-classical damping and very closely distributed modes is proposed and verified with a 2-DOF system with a tuned mass damper (TMD).

Most of these research works about TMD [67–89] have focused on the reduction of the amplitude of oscillations of the main structure, and in some of them [69] also on the decrease of generated

forces and stresses. But some aspects such as the decrease of the natural frequencies of the entire system and the modification of the mode shapes and the modal effective masses have not been studied in depth yet. These aspects are fundamental in the structural design of space structures, especially when modal coupling cannot be avoided. Therefore, the purpose of this doctoral dissertation in this field is to present a parametric study of modal coupling using an undamped 2-DOF system in section 3.2.1. The main results such as the natural frequencies, eigenvectors and modal effective mass fractions are expressed with analytical equations to evaluate the influence of the design parameters. These analytical expressions are applied in the optimization of the structural design in section 3.2.2. The investigation is focused on the case of a satellite as the main structure and an equipment or payload as the secondary part. From a satellite design point of view, the objective is to guarantee the structural integrity of the entire system, which can be at serious risk depending on the design of a particular component, even if the mass of this component is seemingly negligible compared to the mass of the rest of the satellite. The analytical expressions are compared to the FEA results for the case study of the structural design of the university-class 50 kg microsatellite UPMSat-2, which is presented in section 3.3. Finally, conclusions of this investigation about the dynamic coupling are drawn in section 3.4.

3.2 Analytical study of dynamic coupling with the undamped 2-DOF system

Nomenclature

Symbol	Description	Units
$[K]$	Stiffness matrix	
$[M]$	Mass matrix	
$\{J\}$	Response to a static displacement at the base	
f_{10}	Maximum first natural frequency of the system	Hz
f_{20}	Natural frequency of the isolated secondary mass	Hz
f_i	Natural frequency i of the system	Hz
f_{min}	Required minimum natural frequency of the system	Hz
f_r	Reference natural frequency of the system	Hz
K	Stiffness constant of the main structure	N/m
k	Stiffness constant of the secondary part	N/m
M	Large or primary mass	kg
m	Small or secondary mass	kg
$M_{eff,i}$	Modal effective mass of the i mode	kg
$m_{eff,i}$	Modal effective mass fraction of the i mode	
q_{ij}	Component j of the eigenvector i	
u_i	Displacement coordinate	m
x_i	Dimensionless eigenvalue i of the system	

Greek symbol	Description	Units
$\{\phi_i\}$	Eigenvector of mode i	
α	Stiffness ratio	
β	Mass ratio	
Γ_i	Ancillary parameter	
μ	Frequency ratio or tuning parameter	
φ	Ratio between f_{min} and f_{10}	
ω_{10}	Maximum first natural circular frequency of the system	rad/s
ω_{20}	Natural circular frequency of the isolated secondary mass	rad/s
ω_i	Natural circular frequency i of the system	rad/s
ω_r	Reference natural circular frequency of the system	rad/s

Subscript	Description
i	Mode of the system
j	Component of the eigenvector

3.2.1 Mathematical model

The 2-DOF model considered for the entire spacecraft is shown in Figure 3-1. The spacecraft main body (mass M) is fixed to the launcher through the separation system whose stiffness can be modelled as a spring of constant K .

Figure 3-1: 2-DOF system. $M, m; K, k; u_1, u_2$: masses, stiffness and absolute displacements of the whole spacecraft and a small loosely fixed part, respectively [90].

In the spacecraft, a small part (a panel, radiator, instrument, etc.) with a small mass, m , is mounted with a weaker mechanical interface (spring constant $k \ll K$). This configuration is denoted as a “Frahm absorber” and dates back to the beginning of the twentieth century [4]. The model is assumed undamped in order to obtain analytical solutions that can be used for a fast estimation of the coupled response as will be shown in the following sections. In space structures, the usual damping values are low (below 5% of the critical damping factor), and it is considered that the damped natural frequencies of a system can be assumed to be the undamped natural frequencies [1,3]. The analytical results obtained will also apply for systems with proportional damping in terms of the natural frequencies and eigenvectors, and also for the coupled resonances for low damping values.

The aim of this analysis is to study the influence of the design parameters such as the stiffness ratio $\alpha = k / K$ and mass ratio $\beta = m / M$ on the 2-DOF system dynamics (resonant frequencies, eigenmodes, modal effective mass, etc.). Let us define the reference natural frequency $\omega_r^2 = K / M$ (the resonant frequency of the case $m = 0$), the natural frequency of the secondary mass in case of being isolated $\omega_{20}^2 = k / m$, and the dimensionless eigenvalue i of the system $x_i = (\omega_i / \omega_r)^2$.

The equations that describe the dynamics are

$$\begin{aligned} M\ddot{u}_1 + Ku_1 + k(u_1 - u_2) &= 0 \\ m\ddot{u}_2 + k(u_2 - u_1) &= 0 \end{aligned} \quad (3.1)$$

Using complex variable notation, $\{q\} = \{q_0\}e^{i\omega t}$, the system (3.1) can be written as

$$\left(-\omega^2 \begin{bmatrix} 1 & 0 \\ 0 & \beta \end{bmatrix} + \omega_r^2 \begin{bmatrix} 1 + \alpha & -\alpha \\ -\alpha & \alpha \end{bmatrix} \right) \{q_0\} = \{0\} \quad (3.2)$$

The characteristic equation is

$$\begin{vmatrix} -x + 1 + \alpha & -\alpha \\ -\alpha & -\beta x + \alpha \end{vmatrix} = 0 \quad (3.3)$$

and defining $\mu = \frac{\alpha}{\beta} = \frac{k}{m} \frac{M}{K} = \frac{\omega_{20}^2}{\omega_r^2} = x_{20}$ as the tuning parameter, the quadratic equation to calculate the eigenvalues is given by

$$x^2 - x(1 + \mu\beta + \mu) + \mu = 0 \quad (3.4)$$

The exact solutions of Equation (3.4) are

$$x_{1,2} = \frac{1}{2} \left[1 + \mu(1 + \beta) \mp \sqrt{(1 - \mu)^2 + \mu\beta(2 + 2\mu + \mu\beta)} \right] \quad (3.5)$$

where the subscripts “1” and “2” indicate the lowest and the highest eigenvalues respectively.

The variations of the dimensionless eigenvalues x_1 and x_2 are shown in Figure 3-2 as a function of the tuning parameter μ for different values of mass ratio β below unity, and in Figure 3-3 as a function of the mass ratio β , for different constant values of μ . The curves for constant values of the mass ratio β shown in Figure 3-2 converge asymptotically to the straight lines $x_i = 1$ and $x_i = \mu$ for $\beta \ll 1$.

Figure 3-2: Variation of the dimensionless eigenvalues x_1 and x_2 ($x_1 < 1$; $x_2 > 1$) as a function of the tuning parameter, μ . Legend: values of the mass ratio, β [90].

Figure 3-3: Variation of the dimensionless eigenvalues x_1 and x_2 ($x_1 < 1$; $x_2 > 1$) as a function of the mass ratio, β . Legend: values of the tuning parameter, μ [90].

If the tuning parameter is small, $\mu \ll 1$, then the natural frequency of the small part alone is much lower than the reference natural frequency and the first mode x_1 corresponds to the motion of this small part, which in first approximation is $x_1 \approx \mu = x_{20} \ll 1$. The second mode corresponds to the motion of the rest of the structure and its value is close to the reference value ($x_2 \approx 1$). When the

tuning parameter μ takes values much larger than unity, $\mu \gg 1$, the first mode corresponds to the motion of the entire structure and its value tends asymptotically to $x_{10} = 1 / (1 + \beta)$, which is the maximum value that can take the first dimensionless eigenvalue. This value is obtained when the rigidity between both masses k is very high and the structure can be considered as a single-degree-of-freedom (SDOF) system with stiffness K and a total mass $M + m$. The second mode x_2 is, in this case, the motion of the small part and its value tends asymptotically to $\mu (1 + \beta)$.

The proposed approach can be compared with other methods used for the estimations of the natural frequencies of an undamped 2-DOF system, such as Dunkerley's approach and Rayleigh's quotient [3].

The Dunkerley's method can estimate the first natural frequency of a 2-DOF system by considering the following assumption

$$\omega_{1,Dunk}^2 = \left(\sum_{k=1}^2 g_{kk} m_{kk} \right)^{-1} \quad (3.6)$$

where, for the considered 2-DOF system,

$$\begin{aligned} m_{11} &= m & m_{22} &= M \\ g_{11} &= \frac{1}{k} + \frac{1}{K} = \frac{k + K}{kK} & g_{22} &= \frac{1}{K} \end{aligned} \quad (3.7)$$

By using the dimensionless parameters defined in this section, the Dunkerley's estimation for the first dimensionless eigenvalue is given by

$$x_{1,Dunk} = \frac{\omega_{1,Dunk}^2}{K/M} = \frac{\mu}{1 + \mu + \mu\beta} \quad (3.8)$$

For the estimation of the first eigenvalue of a 2-DOF system by the Rayleigh's quotient [3], the static response of the system is assumed to be a close estimation of the first eigenvector leading to the following expression of the dimensionless eigenvalue in terms of the design parameters defined in this chapter

$$x_{1,Ray} = \frac{\omega_{1,Ray}^2}{K/M} = \frac{1}{K/M} \frac{\{\phi\}_1^T [K] \{\phi\}_1}{\{\phi\}_1^T [M] \{\phi\}_1} = \frac{\mu\beta + (\mu + \mu\beta)^2}{\beta(1 + \mu + \mu\beta)^2 + (\mu + \mu\beta)^2} \quad (3.9)$$

The variation of the first dimensionless eigenvalue x_1 as a function of the tuning parameter μ calculated using the exact analytical method proposed in this section and compared with the solutions estimated by Dunkerley's and Rayleigh's approaches is shown in Figure 3-4. The variation of x_1 as a function of the mass ratio β calculated by using these three methods is shown in Figure 3-5.

Figure 3-4: Variation of the dimensionless eigenvalue x_1 as a function of tuning parameter, μ , obtained by using the exact analytical solution, Dunkerley's equation, and Rayleigh's quotient, for different values of the mass ratio, β [90].

Figure 3-5: Variation of the dimensionless eigenvalue x_1 as a function of mass ratio, β , obtained by using the exact analytical solution, Dunkerley's equation, and Rayleigh's quotient, for different values of the tuning parameter, μ [90].

As can be seen from these plots, the differences of the solutions with Dunkerley's method and Rayleigh's quotient with respect to the exact solutions are noticeable for some ranges of values of the design parameters β and μ . The Dunkerley's method gives values for the first eigenvalue below the exact solution for all range of values of mass ratio β and tuning parameter μ , where the highest differences appear for $\mu = 1$. These differences decrease when β increases. Whereas the results provided by the Rayleigh's quotient for the first eigenvalue are larger than the exact solution, showing a good agreement for $\mu > 1$, but with larger differences for $\mu < 1$. As for the Dunkerley's method, the difference with the exact solution decreases when the value of the mass ratio β increases.

The advantage of the 2-DOF model developed in this section is that the solutions are exact for all range of values of β and μ , and the expression (3.5) used to calculate these solutions is not more complicated than the expressions obtained from Dunkerley's method (Equation (3.8)) and from Rayleigh's quotient (Equation (3.9)).

In this study, the range of interest for the tuning parameter μ corresponds to values close to unity, where the phenomenon of dynamic coupling between both masses becomes more relevant. The first case considered is for $\mu = 1$. Using Equation (3.5), the eigenvalues are given by

$$x_{1,2} = 1 + \frac{\beta}{2} \mp \sqrt{\beta} \sqrt{1 + \frac{\beta}{4}} \quad (3.10)$$

For small values of the mass ratio $\beta \ll 1$, and neglecting $O(\beta)$ terms, these dimensionless eigenvalues and the frequencies ratio can be approximated as

$$x_{1,2} \approx 1 \mp \sqrt{\beta} \qquad \frac{f_{1,2}}{f_r} \approx 1 \mp \frac{1}{2}\sqrt{\beta} \quad (3.11)$$

Both solutions are slightly smaller and larger, respectively, than the zero order solution $x = 1$. The separation between these frequencies depends on the mass ratio β , and is given by

$$\sqrt{x_2} - \sqrt{x_1} \approx \sqrt{1 + \sqrt{\beta}} - \sqrt{1 - \sqrt{\beta}} \approx 1 + \frac{1}{2}\sqrt{\beta} - \left(1 - \frac{1}{2}\sqrt{\beta}\right) = \sqrt{\beta} \quad (3.12)$$

Therefore, the influence of the secondary mass on the natural frequencies of the system is higher than expected a priori. For instance, for a mass ratio $\beta = 0.01$, the separation between the frequencies is $\sqrt{\beta} = 0.1$.

In this range both dimensionless eigenvalues x_1 and x_2 are close to 1, and to find out which one is more important in the global response of the entire structure when is excited at its base, it is necessary to calculate the modal effective mass for each mode. As a previous step, the eigenvectors are obtained in the next subsection to determine the motion of the masses in both modes.

3.2.1.1 Eigenvectors

The eigenvectors describe the motion of both masses for each mode and are calculated from Equation (3.2)

$$\{\phi\}_i = (q_{i1} \quad q_{i2})^T \quad (3.13)$$

Whichever of the two equations obtained can be used to determine the eigenvectors. The election is a matter of convenience for analytical simplicity

$$\frac{q_{i2}}{q_{i1}} = \frac{-x_i + 1}{\mu\beta} + 1 \quad (3.14a)$$

$$\frac{q_{i2}}{q_{i1}} = \frac{\mu}{\mu - x_i} \quad (3.14b)$$

To have an expression that relates the ratio of eigenvectors components with both the mass ratio β and the dimensionless eigenvalues x_i , the following expression, obtained directly from Equation (3.4), is necessary as a previous step

$$\frac{x_i}{\mu} = 1 - \frac{x_i}{1 - x_i}\beta \quad (3.15)$$

and combining this expression with Equation (3.14b), the following equation to determine the eigenvectors is obtained

$$\frac{q_{i2}}{q_{i1}} = \frac{\mu}{\mu - x_i} = \frac{1}{1 - x_i/\mu} = \frac{1 - x_i}{x_i\beta} \quad (3.16)$$

The explicit expressions of the eigenvectors components as a function of the mass ratio β and the tuning parameter μ are obtained by substituting Equation (3.5) in Equation (3.14a)

$$\frac{q_{12}}{q_{11}} = \frac{-x_1 + 1}{\mu\beta} + 1 = \frac{1}{2\mu\beta} \left[1 - \mu(1 - \beta) + \sqrt{(1 - \mu)^2 + \mu\beta(2 + 2\mu + \mu\beta)} \right] \quad (3.17a)$$

$$\frac{q_{22}}{q_{21}} = \frac{-x_2 + 1}{\mu\beta} + 1 = \frac{1}{2\mu\beta} \left[1 - \mu(1 - \beta) - \sqrt{(1 - \mu)^2 + \mu\beta(2 + 2\mu + \mu\beta)} \right] \quad (3.17b)$$

Simplified expressions can be obtained considering the first order terms of the power series expansion for the detuned ($|\mu - 1| = O(1)$) and for the tuned ($\mu = 1$) cases, assuming small values of mass ratio ($\beta \ll 1$). For the detuned case, the approximated solutions are

For $\mu \gg 1$

$$x_1 \approx 1 - \frac{\mu}{\mu - 1}\beta \approx 1 - \beta \quad x_2 \approx \mu + \frac{\mu^2}{\mu - 1}\beta \approx \mu(1 + \beta) \quad (3.18a)$$

$$\frac{q_{12}}{q_{11}} \approx \frac{\mu}{\mu - 1} \approx 1 + \frac{1}{\mu} \quad \frac{q_{22}}{q_{21}} \approx -\frac{\mu - 1}{\mu\beta} \approx -\frac{1}{\beta} \quad (3.18b)$$

For $\mu \ll 1$

$$x_1 \approx \mu - \frac{\mu^2}{1 - \mu}\beta \quad x_2 \approx 1 + \frac{\mu}{1 - \mu}\beta \quad (3.19a)$$

$$\frac{q_{12}}{q_{11}} \approx \frac{1 - \mu}{\mu\beta} \quad \frac{q_{22}}{q_{21}} \approx -\frac{\mu}{1 - \mu} \quad (3.19b)$$

The eigenvectors are shown in Figure 3-6. In the case $\mu \gg 1$, the amplitudes of oscillation of the two masses are similar in the first mode. The small mass accompanies the motion of the large mass, in phase for the lower frequency mode, while for the higher mode, large oscillation amplitude motion appears as $\beta \ll 1$, but 180° out of phase. In the case $\mu \ll 1$, in the lower frequency mode the oscillation amplitude of the small mass is large, while in the higher frequency mode, the amplitude of the motion of both masses is quite similar, but 180° out of phase.

Figure 3-6: Eigenvectors associated to eigenvalues x_1 and x_2 respectively for detuned case ($\mu \neq 1$) [90].

For the tuned case, $\mu = 1$, the eigenvectors components have the following relationship obtained by substituting x_i from Equation (3.10) in Equation (3.14b):

$$\frac{q_{i2}}{q_{i1}} = \frac{1}{1 - x_i} = \frac{1}{-\frac{\beta}{2} \pm \sqrt{\beta} \sqrt{1 + \frac{\beta}{4}}} \quad (3.20)$$

As a first approximation, considering low mass ratios $\beta \ll 1$, the relationship between the eigenvectors components becomes

$$\begin{aligned} \frac{q_{12}}{q_{11}} &\approx \frac{1}{\sqrt{\beta}} \\ \frac{q_{22}}{q_{21}} &\approx -\frac{1}{\sqrt{\beta}} \end{aligned} \quad (3.21)$$

As for the detuned case, both motions in the lowest frequency mode are in phase and at the highest frequency mode both motions are 180° out of phase, as are shown in Figure 3-7. The amplitude of the small mass motion is much larger ($1/\sqrt{\beta}$) in both modes than that of the spacecraft. This configuration has the same effects as the TMD studied in several works [67–89], where the amplitudes of the main structure are reduced at the expense of the high amplitudes on the secondary mass. But in the problem studied in this doctoral dissertation, these high amplitudes can be dangerous for the equipment represented by the small mass because they can generate excessive stresses and forces above the allowable limits. Therefore, this condition must be carefully studied taking into account this risk for the secondary part.

Figure 3-7: Eigenvectors associated to eigenvalues x_1 and x_2 for the tuned condition case ($\mu = 1$) [90].

3.2.1.2 Modal effective mass

To analyse the modal response of a system, the effective mass is a parameter often used. The effective mass fraction is the percentage of total mass participating in the motion of the respective eigenmode. This parameter is a measure to identify the relevance of the contribution of each mode to the global response of a system when it is excited at its base. High values of modal effective masses give high values of interface forces. The expression to calculate the modal effective mass from the mass matrix, the eigenvector and the response to a static displacement at the base is

$$M_{eff,i} = \frac{(\{\phi\}_i^T [M] \{J\})^2}{\{\phi\}_i^T [M] \{\phi\}_i} \quad (3.22)$$

where $\{J\} = (1, 1)^T$ is the response to a static unity displacement at the base. The modal effective mass fraction ($m_{eff,i}$) of the 2-DOF system in this work is given by

$$m_{eff,i} = \frac{M_{eff,i}}{M + m} = \frac{M}{M + m} \frac{\left(1 + \beta \frac{q_{i2}}{q_{i1}}\right)^2}{1 + \beta \left(\frac{q_{i2}}{q_{i1}}\right)^2} \quad (3.23)$$

This parameter can be expressed in different ways as a function of the 3 parameters (x_i , β and μ) by combining Equation (3.23) with Equations (3.14a) and (3.14b)

$$m_{eff,i} = \frac{1}{1 + \beta} \frac{(\mu + \mu\beta - x_i)^2}{[(\mu - x_i)^2 + \beta\mu^2]} = \frac{1}{1 + \beta} \left[1 + \beta\mu \frac{\beta\mu + \mu - 2x_i}{(\mu - x_i)^2 + \beta\mu^2} \right] \quad (3.24)$$

$$m_{eff,i} = 1 - \frac{\beta}{1 + \beta} \frac{x_i^2}{(\mu - x_i)^2 + \beta\mu^2}$$

Different expressions are given below that relate the modal effective mass fraction as a function of only two independent parameters.

Using Equation (3.15) and the first term of Equation (3.24), the expressions of $m_{eff,i}$ as a function of the two independent parameters β and x_i is obtained

$$m_{eff,i} = \frac{\beta}{1+\beta} \frac{\left[1 + \left(\frac{1-x_i}{x_i}\right)\right]^2}{\left[\beta + \left(\frac{1-x_i}{x_i}\right)^2\right]} = \frac{\beta}{1+\beta} \frac{1}{[\beta x_i^2 + (1-x_i)^2]} \quad (3.25)$$

From Equation (3.25), the explicit expression of the dimensionless eigenvalue x_i as a function of $m_{eff,i}$ and β is

$$x_i = \frac{1}{1+\beta} \left[1 \mp \sqrt{\beta \frac{1-m_{eff,i}}{m_{eff,i}}} \right] \quad (3.26)$$

The equation that relates the modal effective mass fraction $m_{eff,i}$ with the tuning parameter μ and the dimensionless eigenvalue x_i is calculated by substituting the mass ratio β parameter of Equation (3.25) with Equation (3.15)

$$m_{eff,i} = \frac{1}{1 + \frac{x_i^3 (1-x_i)}{\mu (\mu - x_i)}} \quad (3.27)$$

And finally, to provide the explicit expression of $m_{eff,i}$ as a function of β and μ , the ancillary parameter Γ_i is defined as

$$\Gamma_i \equiv \sqrt{\beta \frac{1-m_{eff,i}}{m_{eff,i}}} \quad (3.28)$$

From Equation (3.15), the tuning parameter μ can be expressed as a function of the modal effective mass fraction $m_{eff,i}$ and β

$$\mu = \frac{1}{\frac{1}{x_i} - \frac{\beta}{1-x_i}} = \frac{1}{\frac{1+\beta}{1-\Gamma_i} - \frac{\beta(1+\beta)}{\beta+\Gamma_i}} = \frac{(1-\Gamma_i)(\beta+\Gamma_i)}{\Gamma_i(1+\beta)^2} \quad (3.29)$$

$$\mu = \frac{\left(1 - \sqrt{\beta \frac{1-m_{eff,i}}{m_{eff,i}}}\right) \left(\beta + \sqrt{\beta \frac{1-m_{eff,i}}{m_{eff,i}}}\right)}{\sqrt{\beta \frac{1-m_{eff,i}}{m_{eff,i}}} (1+\beta)^2} \quad (3.30)$$

The parameter Γ_i can be expressed as an explicit function of the mass ratio β and the tuning parameter μ by solving the following quadratic formula defined from Equation (3.29)

$$\Gamma_i^2 + \Gamma_i[\mu(1+\beta)^2 - 1 + \beta] - \beta = 0 \quad (3.31)$$

The results are

$$\Gamma_i = \frac{1}{2} \left[1 - \beta - \mu(1 + \beta)^2 \pm (1 + \beta) \sqrt{(1 - \mu)^2 + \mu\beta(2 + 2\mu + \mu\beta)} \right] \quad (3.32)$$

Finally, the modal effective mass fractions as a function of β and μ are calculated by considering Equations (3.32) and (3.28)

$$m_{eff,1} = \frac{\beta}{\beta + \Gamma_1^2} \quad (3.33a)$$

$$m_{eff,1} = \frac{\beta}{\beta + \frac{1}{4} \left[1 - \beta - \mu(1 + \beta)^2 + (1 + \beta) \sqrt{(1 - \mu)^2 + \mu\beta(2 + 2\mu + \mu\beta)} \right]^2}$$

$$m_{eff,2} = \frac{\beta}{\beta + \Gamma_2^2} \quad (3.33b)$$

$$m_{eff,2} = \frac{\beta}{\beta + \frac{1}{4} \left[1 - \beta - \mu(1 + \beta)^2 - (1 + \beta) \sqrt{(1 - \mu)^2 + \mu\beta(2 + 2\mu + \mu\beta)} \right]^2}$$

The variation of the modal effective mass fraction, $m_{eff,i}$ as a function of the dimensionless eigenvalue x_i is shown in Figure 3-8. Solid lines are obtained from Equation (3.25) for different constant values of the mass ratio β , while dashed lines represent the functions of Equation (3.27) for different constant values of μ . A change of behaviour is observed in Figure 3-8(a) for the curves of constant μ of the first mode, where for $\mu < 1$, each curve decreases monotonically to zero, while for $\mu > 1$, each curve presents a minimum value of $m_{eff,1}$, which is larger than 0.5. This indicates that, for the range of the tuning parameter μ above unity, the first mode always has more modal effective mass, and therefore, more influence on the generated interface forces than the second mode, when a forced acceleration is applied at the system base. This behaviour is also observed in Figure 3-8(c) for the second mode, with the difference in this case that corresponds to maximum values of the modal effective mass fraction.

For the second mode, the solutions only cover the non-shaded area in the Figure 3-8(b) and Figure 3-8(c). The curve that delimits this area is determined by the calculation of the value of the tuning parameter $\mu_{meff,max}$ that maximizes the modal effective mass fraction of the second $m_{eff,2}$ for a given x_2 . This value is obtained from Equation (3.27)

$$\frac{\partial m_{eff,2}}{\partial \mu} = 0 \quad \mu_{meff,max} = \frac{x_2}{2} \quad (3.34)$$

And substituting $\mu_{meff,max}$ from Equation (3.34) into Equation (3.27), an expression that defines the border line of the shaded area in Figure 3-8(b) and Figure 3-8(c) is obtained

$$m_{eff,2,max} = \frac{1}{1 - 4x_2(1 - x_2)} \quad (3.35)$$

The expression of Equation (3.33a) is plotted in Figure 3-9 in curves for different constant values of mass ratio β , while the same solutions are shown in Figure 3-10 in a graph $m_{eff,i} - \beta$ with curves of constant values of μ .

(a) First mode

(b) Second mode

(c) Second mode

Figure 3-8: Variation of modal effective mass fractions $m_{eff,1}$ and $m_{eff,2}$ as a function of the dimensionless eigenvalues x_1 and x_2 . Constant values of the mass ratio β , solid lines. Constant values of the tuning parameter μ , dashed lines. (a) First mode. (b) and (c) Second mode; the solutions only cover the non-shaded area [90].

Figure 3-9: Variation of the modal effective mass fraction, $m_{eff,1}$ as a function of tuning parameter μ . The values of the mass ratio β are indicated [90].

Figure 3-10: Variation of the modal effective mass fraction, $m_{eff,1}$ as a function of mass ratio, β . The values of the tuning parameter μ are indicated [90].

As shown in Figure 3-9, the curves of $m_{eff,1}$ as a function of the tuning parameter μ increase monotonically from $\beta / (1 + \beta)$, which is the ratio between the small mass and the total mass obtained as a solution of Equation (3.33a) for $\mu = 0$ (low stiffness joint, $k \ll 1$), towards unity for $\mu \gg 1$, which corresponds to the motion of the entire system.

With the aim of determining the design regions where each mode has more modal effective mass than the other one, it is necessary to calculate the values of μ and β where both modal effective mass fractions are equal to 0.5 and therefore where both modes have the same influence on the global response of the system. This condition leads to the following expression obtained from Equation (3.28)

$$\Gamma_1^2 = \Gamma_2^2 = \beta \quad (3.36)$$

And introducing this value into Equation (3.31), the following relationship between μ and β is obtained

$$1 - \beta - \mu(1 + \beta)^2 = 0 \quad \mu_{0.5} = \frac{1 - \beta}{(1 + \beta)^2} \quad (3.37)$$

Interestingly, this value is always $\mu_{0.5} < 1$. The dimensionless eigenvalues are obtained in this case by combining Equations (3.37) and (3.5)

$$x_i = \frac{1 \mp \sqrt{\beta}}{1 + \beta} \quad (3.38)$$

The design region where the first mode has more modal effective mass than the second mode ($m_{eff,1} > m_{eff,2}$) corresponds to the zone B indicated in Figure 3-11. The boundary that separates both regions is given by Equation (3.37).

Figure 3-11: Variation of the mass ratio, β , as a function of the tuning parameter, μ , for different constant values of the dimensionless eigenvalues indicated in each curve. First eigenvalue x_1 , solid lines; second eigenvalue x_2 , dashed lines. Zone A corresponds for $m_{eff,1} < m_{eff,2}$; zone B corresponds for $m_{eff,1} > m_{eff,2}$. Dot-dashed line: $\mu_{0.5}$ according to Equation (3.37) [90].

For $\beta > 1$, the eigenvalues are placed in zone B, which implies that the first mode is the dominant ($m_{eff,1} > m_{eff,2}$), independently of the value of μ . The first mode is also dominant for $\mu > 1$, independently of the value of β . For $\beta \ll 1$, the boundary between zone A and zone B is close to $\mu = 1$.

3.2.2 Design cases

The design cases considered are focused on the determination of the values of the parameters that optimize the design of the structure according to the established requirements.

The requirement usually applied in the space industry that mostly influences the modal behaviour of a satellite consists in establishing a minimum limit value for the main natural frequencies of the satellite f_{min} [1]. The minimum frequency specified in the requirement must be sufficiently larger than the maximum frequency of the excitation to reduce the risk of structural damage. This requirement affects the main modes of the system, which contribute most to the maximum forces and stresses generated by vibrations transmitted through the interface of the system. If the

frequency of the excitation load coincides with one of the main natural frequencies, the effect of resonance increases the generated stresses, which can lead to structural damage. The main modes of the system (e.g. the entire satellite) are defined as those modes whose modal effective mass fractions are above an established threshold $m_{eff,lim}$. For example, a typical value is 10% or 0.1 [1].

In this study, the minimum frequency requirement is only applied for modes whose modal effective mass fraction is above the established threshold. The modes with a modal effective mass fraction below the threshold are considered as local modes, whose contribution to the global response of the system is lower. These modes are related to the motion of small parts of the system and its excitation is less critical for the whole system than the excitation of modes with high modal effective mass, but it may be too severe for the affected small part. The objective of the design cases proposed in this study is to know the limits of the values of the design parameters to assure that the whole system fulfils the imposed requirement, to finally obtain the optimum values of the design parameters within these limits.

Two different design cases are studied. The first design case refers to a design phase with the primary structure already determined (M and K known), while the secondary element is yet to be designed. The second design case is applied when the total mass is fixed but its distribution between the primary and secondary elements is yet to be decided.

To define the requirement of minimum natural frequency of the system in the first design case, the ratio between f_{min} and the reference frequency f_r is a fixed parameter. The values of the parameters of the main structure M and K are pre-established to achieve a reference natural frequency higher than f_{min} .

In the second case, the fixed parameter is the ratio between f_{min} and first natural frequency of the system when k is infinite, f_{10} . This case considers that the total mass, $M + m$, together with the main stiffness K , which directly define the value of f_{10} , are the pre-established parameters.

From the analytical point of view, the main difference between both design cases is that in the first case, the mass ratio β affects only the mass of the secondary part m , while in the second design case, the mass ratio β affects both the primary and the secondary masses. In this second design case, β indicates how the fixed total mass is divided between the primary and the secondary masses.

For both cases, the calculation of the range of values for the mass ratio β and the tuning parameter μ that allow the system to fulfil the specified requirement is determined according to the definition of the requirements, which differ in each design case. Analytical expressions to find the optimum values of the design parameters are obtained from the mathematical model explained in section 3.2.1.

3.2.2.1 Design case 1: f_{min}/f_r as fixed parameter

In the first design case, the parameters related to the main structure are known and have been determined to get a reference natural frequency f_r sufficiently higher than the minimum natural frequency specified to the system,

$$f_r > f_{min}; \quad x_{min} = \left(\frac{f_{min}}{f_r} \right)^2 < 1 \quad (3.39)$$

Figure 3-12: Example of design region (non-shaded area) for the first case according to the specified minimum natural frequency ($x_{min} = 0.85$, ADB limit) and modal effective mass fraction threshold ($m_{eff,lim} = 0.1$, AEC limit). (a) Design region in $m_{eff,1} - x_1$ plane. (b) Design region in $\beta - \mu$ plane [90].

As mentioned in section 3.2.1, the dynamic behaviour of the system is altered by the addition of a small part of mass m with the generation of two modes, where the first mode is always lower than the reference mode.

The design region where the system does not fulfil the specified requirement corresponds to the shaded area in Figure 3-12 and it is defined by the intersection of the regions generated for each of the following inequalities

$$m_{eff,1} > m_{eff,lim} \quad (3.40a)$$

$$x_1 < x_{min} \quad (3.40b)$$

As shown in Figure 3-12(b), there is a value for the mass ratio β_{max} above which the system will never fulfil the requirements. As shown in Figure 3-12(a), the curve associated to a value of $\beta > \beta_{max}$ is entirely in the shaded area. This limit is the maximum between the limits $\beta_{max,1}$ and $\beta_{max,2}$. $\beta_{max,1}$ is related to the established minimum eigenvalue x_{min} (AB), and $\beta_{max,2}$ is related to the modal effective mass fraction threshold $m_{eff,lim}$ (AC).

The first limit $\beta_{max,1}$ is calculated considering the maximum value for the first dimensionless eigenvalue for a 2-DOF system, x_{10} , which is the case when both the stiffness between the two masses, k , and the tuning parameter μ are infinite. This parameter is defined as

$$x_{10} \equiv \left(\frac{\omega_{10}}{\omega_r} \right)^2 = \frac{K/(M+m)}{K/M} = \frac{M}{M+m} = \frac{1}{1+\beta} \quad (3.41)$$

This value of x_{10} constitutes the upper limit for the lowest dimensionless eigenvalue x_1 for a given value of the mass ratio β . The maximum mass ratio $\beta_{max,1}$ that can provide a first eigenvalue of the 2-DOF system above x_{min} is calculated using Equation (3.41) for $x_{10} = x_{min}$

$$\beta_{max,1} = \frac{1 - x_{min}}{x_{min}} \quad (3.42)$$

This value is the asymptotic limit of the design region for $\mu \rightarrow \infty$ indicated in Figure 3-12(b) and that corresponds to point B in Figure 3-12(a).

The value of $\beta_{max,2}$ is the maximum value of mass ratio that can provide a modal effective mass fraction below or equal to the threshold of $m_{eff,lim}$. For a given mass ratio β , the minimum value of modal effective mass fraction of the first mode for a 2-DOF system is obtained from Equation (3.33a) for $\mu = 0$

$$m_{eff,1} = \frac{\beta}{1 + \beta} \quad (3.43)$$

For $m_{eff,1} = m_{eff,lim}$, the value of $\beta_{max,2}$ is

$$\beta_{max,2} = \frac{m_{eff,lim}}{1 - m_{eff,lim}} \quad (3.44)$$

This value is the limit of the design region for $\mu \rightarrow 0$, which corresponds to the point C in Figure 3-12(a).

Therefore, the maximum value of the mass ratio β_{max} for which the system can meet the requirement is

$$\beta_{max} = \max\left(\frac{1 - x_{min}}{x_{min}}, \frac{m_{eff,lim}}{1 - m_{eff,lim}}\right) \quad (3.45)$$

For values of mass ratio β above this maximum value β_{max} , the 2-DOF system will never exhibit a first natural frequency higher than f_{min} . Also, the modal effective mass fraction for the first mode is higher than $m_{eff,lim}$.

There is a value for the mass ratio β_{lim} below which the system always meets the requirement, independently of the value of μ . This configuration is identified as the point A in Figure 3-12. The value of β_{lim} is calculated using Equation (3.25) for $x_1 = x_{min}$ and $m_{eff,1} = m_{eff,lim}$

$$\beta_{lim}^2 + \beta_{lim} \left[1 + \left(\frac{1 - x_{min}}{x_{min}} \right)^2 - \frac{1}{m_{eff,lim}} \left(1 + \frac{1 - x_{min}}{x_{min}} \right)^2 \right] + \left(\frac{1 - x_{min}}{x_{min}} \right)^2 = 0 \quad (3.46)$$

Defining $x'_{min} \equiv \frac{1 - x_{min}}{x_{min}}$, the result is

$$\beta_{lim} = \frac{\frac{(1 + x'_{min})^2}{m_{eff,lim}} - 1 - x'_{min}^2 - \sqrt{\left[\frac{(1 + x'_{min})^2}{m_{eff,lim}} - 1 - x'_{min}^2 \right]^2 - 4x'_{min}^2}}{2} \quad (3.47)$$

The value of the tuning parameter in this point is defined as μ_A , and can be calculated using Equation (3.30) for $\beta = \beta_{lim}$ and $m_{eff,i} = m_{eff,lim}$.

For the case where the mass ratio β are between β_{max} and β_{lim} , there is a range of values for the tuning parameter μ within which the system does not fulfil the requirements. This range corresponds to the segment between points E and D in Figure 3-12. For values higher than the upper limit of this range μ_D , the first natural frequency of the system is above the minimum frequency established in the requirement. For values of the tuning parameter below the lower limit of this range, μ_E , the modal effective mass fraction of the first mode is below the threshold established, and therefore is allowed to have a natural frequency below the required minimum value.

The expression of μ_D for a given mass ratio β between β_{lim} and $\beta_{max,1}$ is calculated from Equation (3.15) for $x_1 = x_{min}$

$$\mu_D = \frac{1 - x_{min}}{\frac{1 - x_{min}}{x_{min}} - \beta} \quad (3.48)$$

This value corresponds to the point D in Figure 3-12.

The lower limit μ_E of the range is identified with the point E in Figure 3-12. This value is calculated from Equation (3.30) for β between β_{lim} and $\beta_{max,2}$ and for $m_{eff,1} = m_{eff,lim}$

$$\mu_E = \frac{\left(1 - \sqrt{\beta \frac{1 - m_{eff,lim}}{m_{eff,lim}}}\right) \left(\beta + \sqrt{\beta \frac{1 - m_{eff,lim}}{m_{eff,lim}}}\right)}{\sqrt{\beta \frac{1 - m_{eff,lim}}{m_{eff,lim}} (1 + \beta)^2}} \quad (3.49)$$

For designs where $\mu > \mu_D$, the lowest natural frequency of the system is $f_1 > f_{min}$ and therefore, the system meets the requirement. In this situation, this first mode is considered as main mode of the system because its modal effective mass fraction is higher than the threshold $m_{eff,lim}$.

For the case where $\mu < \mu_E$, the lowest mode of the system can be considered as a local mode because it has a modal effective mass fraction below the established threshold. From the whole system point of view, this design region is allowed, even though the first natural frequency is below f_{min} , because the excitation of this mode has less influence on the global response of the system. But there is a considerable risk of damage for the small part, which must be taken into account.

After establishing the limits of the design region, the optimum design is defined as the point of this region that minimizes the structural mass, which is directly influenced by the stiffness. In this design case, the variable parameters are related to the secondary mass. In many situations, the main contribution of the total mass of a system like a satellite or a payload is due to the non-structural elements. For this reason, in the optimization example exposed below, the small mass m and the mass ratio β are fixed values independent of the structural parameters as the stiffness k and the tuning parameter μ . The objective is the calculation of the minimum values of the stiffness k and the natural frequency f_{20} of the small part for given values of M , K , m and β .

For the design region where $\mu < \mu_E$, the theoretical optimum design is obtained for $\mu \rightarrow 0$. In this design region, the drawback is the risk of damage to the small part due to the amplification at the resonance of the local mode with the low frequency excitations. For this reason, the design region considered in this optimization is for $\mu > \mu_D$, where the natural frequencies of the system are higher than f_{min} . For a value of the mass ratio between β_{lim} and $\beta_{max,1}$, the optimum design point is when the structural mass and thus the stiffness k of the secondary part is minimum. The minimum stiffness within the design region limits is for $\mu = \mu_D$. From Equation (3.48), the following expression is obtained

$$\frac{\mu_D}{x_{min}} = \frac{1}{1 - \frac{x_{min}}{1 - x_{min}} \beta} \quad (3.50)$$

Expressed in terms of natural frequencies is

$$f_{20,opt} = f_{min} \sqrt{\frac{1}{1 - \frac{f_{min}^2}{f_r^2 - f_{min}^2} \beta}} \quad (3.51)$$

The optimum stiffness of the small part is

$$k_{opt} = 4\pi^2 m f_{min}^2 \frac{1}{1 - \frac{f_{min}^2}{f_r^2} \beta} \quad (3.52)$$

And the optimum stiffness ratio α_{opt} is

$$\alpha_{opt} = \mu_D \beta = \frac{\beta x_{min}}{1 - \frac{x_{min}}{1 - x_{min}} \beta} = \frac{x_{min}}{\frac{1}{\beta} - \frac{x_{min}}{1 - x_{min}}} \quad (3.53)$$

Equation (3.51) is a useful expression that can be employed to derive a specification for the minimum natural frequency to a secondary part, $f_{20,opt}$, with the aim to avoid that the first frequency of the whole system is below a specified f_{min} .

3.2.2.2 Design case 2: f_{10}/f_r as fixed parameter

With the same purpose as in the previous case, a minimum limit is established for the main natural frequencies of the system. Like in the first design case, this requirement only affects the main modes of the system, i.e. those with a modal effective mass fraction above an established threshold. The difference in this case is that the known or fixed parameters are the total mass of the system, $M + m$, and the primary stiffness K . With both parameters, the maximum first natural frequency of the 2-DOF system can be calculated considering that the stiffness k between the two masses is infinite

$$f_{10} = \frac{1}{2\pi} \sqrt{\frac{K}{M + m}} \quad (3.54)$$

$$x_{10} = \left(\frac{f_{10}}{f_r}\right)^2 = \frac{K/(M + m)}{K/M} = \frac{1}{1 + \beta} \quad (3.55)$$

In this second case, the parameter φ is defined as the ratio between f_{min} and f_{10} . The requirement is expressed as follows

$$\varphi \equiv \frac{f_{min}}{f_{10}} < 1 \quad (3.56)$$

The risk region is defined by the intersection of the regions generated for each of these inequalities

$$m_{eff,1} > m_{eff,lim} \quad (3.57a)$$

$$f_1 < f_{min} \quad \left(\frac{f_1}{f_{10}}\right)^2 = \frac{x_1}{x_{10}} < \left(\frac{f_{min}}{f_{10}}\right)^2 = \varphi^2 \quad x_1 < x_{10} \varphi^2 \quad (3.57b)$$

Figure 3-13: Example of design region (non-shaded area) for the second case according to the specified minimum natural frequency ($\varphi = 0.92$, AF limit) and modal effective mass fraction threshold ($m_{eff,lim} = 0.1$, AEC limit). (a) Design region in $m_{eff,1} - x_1$ plane. (b) Design region in $\beta - \mu$ plane [90].

Using Equation (3.55), the expression for the minimum limit for the dimensionless eigenvalue is given by

$$x_{min} = \frac{\varphi^2}{1 + \beta} \quad (3.58)$$

In this second design case, x_{min} depends on the mass ratio β unlike in the first design case, where x_{min} is a constant value. The design region corresponds to the non-shaded area in Figure 3-13.

Expressions that relate the modal effective mass fraction $m_{eff,1}$ with the frequency ratio φ and the minimum dimensionless eigenvalue x_{min} can be obtained from Equations (3.25), (3.26) and (3.58)

$$m_{eff,1} = \frac{1}{1 + \frac{x_{min}}{\varphi^2 - x_{min}}(1 - \varphi^2)^2} \quad (3.59)$$

$$x_{min} = \frac{\varphi^2}{1 + \frac{m_{eff,1}}{1 - m_{eff,1}}(1 - \varphi^2)^2} \quad (3.60)$$

The curve AF that sets the upper limit of the shaded area in Figure 3-13(a) is given by Equation (3.60).

The value of $\beta_{max,2}$ has the same meaning and expression as in the first design case. The difference in this second case is that the maximum value of the mass ratio β_{max} above which the 2-DOF system cannot fulfil the requirement does not exist, i.e. it is possible to fulfil the requirement for any value of β , depending on the value of the tuning parameter μ .

As in the first design case, there is a value for the mass ratio β_{lim} below which the 2-DOF system always complies with the requirement, independently of the value of μ . This value is represented in Figure 3-13 as the point A, and is calculated using Equations (3.25) and (3.56) for x_1 for x_{min} , $m_{eff,1}$ for $m_{eff,lim}$ and $\beta = \beta_{lim}$

$$m_{eff,lim} = \frac{\beta_{lim}}{1 + \beta_{lim}} \frac{\left[1 + \left(\frac{1 - x_{min}}{x_{min}}\right)\right]^2}{\left[\beta_{lim} + \left(\frac{1 - x_{min}}{x_{min}}\right)^2\right]} = \frac{\beta_{lim}}{\beta_{lim} + (1 - \varphi^2)^2} \quad (3.61a)$$

$$\beta_{lim} = \frac{m_{eff,lim}}{1 - m_{eff,lim}} (1 - \varphi^2)^2 \quad (3.61b)$$

The value of μ_A in this point is obtained using Equation (3.15), for $\beta = \beta_{lim}$ and $x_i = x_{min}$

$$\mu_A = \frac{1 - x_{min}}{\frac{1 - x_{min}}{x_{min}} - \beta_{lim}} = \frac{\varphi^2(1 + \beta_{lim} - \varphi^2)}{(1 + \beta_{lim})^2(1 - \varphi^2)} \quad (3.62)$$

$$\mu_A = \frac{\varphi^2(1 - m_{eff,lim})[1 - \varphi^2(1 + m_{eff,lim}) + \varphi^4 m_{eff,lim}]}{(1 - \varphi^2)(1 - 2\varphi^2 m_{eff,lim} + \varphi^4 m_{eff,lim})^2} \quad (3.63)$$

For values of $\beta > \beta_{lim}$, there is a minimum value for the tuning parameter, μ_F , above which the system has a first natural frequency above f_{min} . This design point is indicated as point F in Figure

3-13 for a given mass ratio β . The value of μ_F is obtained from Equation (3.15) and with the same procedure shown in Equations (3.62) and (3.63)

$$\mu_F = \frac{\varphi^2(1 + \beta - \varphi^2)}{(1 + \beta)^2(1 - \varphi^2)} \quad (3.64)$$

As can be seen in Figure 3-13(b), there is a value of the mass ratio β that maximizes the value for μ_F (curve AF). This means that for $\mu > \mu_{F,max}$, the first natural frequency is higher than the specified minimum frequency f_{min} for the whole range of values of β . This maximum value, $\mu_{F,max}$, is obtained by deriving the expression in Equation (3.64)

$$\frac{\partial \mu_F}{\partial \beta} = 0 \quad \beta = -1 + 2\varphi^2 \quad (3.65)$$

and substituting in Equation (3.64)

$$\mu_{F,max} = \frac{1}{4(1 - \varphi^2)} \quad (3.66)$$

The expression of the maximum value $\mu_{F,max}$ in Equation (3.66) is only applicable for $\varphi^2 \geq 0.5$. A design with the mass ratio β given by Equation (3.65) can fulfil the requirement if $\mu > \mu_{F,max}$, even being higher than $\beta_{max,2}$, because in this situation the first mode has a natural frequency above the imposed minimum value and with a modal effective mass fraction above the established threshold.

For $\varphi^2 < 0.5$, the maximum value $\mu_{F,max}$ is for $\beta = 0$ and, using Equation (3.64), is given by

$$\mu_{F,max} = \varphi^2 \quad (3.67)$$

Like in the first design case, for the range of values of the mass ratio between β_{lim} (point A) and $\beta_{max,2}$ (point C), there is a limit μ_E that is defined when the modal effective mass fraction of the first mode is equal to the threshold indicated in the specification. The expression obtained for the first design case to calculate this limit (Equation (3.49)) is also valid for this second design case. For $\mu < \mu_E$, the first natural frequency of the system is lower than the specified minimum value f_{min} , but with an associated modal effective mass fraction below the threshold established to identify the main modes of the system.

The optimum design that gets a first natural frequency of the system above the specified minimum value f_{min} is when $\mu = \mu_F$. The optimum stiffness ratio using Equation (3.64) is

$$\alpha_{opt} = \mu_F \beta = \frac{\varphi^2 \beta (1 + \beta - \varphi^2)}{(1 + \beta)^2 (1 - \varphi^2)} \quad (3.68)$$

The optimum stiffness k_{opt} of the small part can be calculated by using Equations (3.68) and (3.55)

$$k_{opt} = \alpha_{opt} K = \frac{\varphi^2 \beta (1 + \beta - \varphi^2)}{(1 + \beta)^2 (1 - \varphi^2)} K \quad (3.69)$$

$$k_{opt} = \frac{\varphi^2 \beta (1 + \beta - \varphi^2)}{(1 + \beta)^2 (1 - \varphi^2)} (2\pi f_{10})^2 (M + m) = 4\pi^2 m f_{min}^2 \frac{(1 + \beta - \varphi^2)}{(1 + \beta)(1 - \varphi^2)}$$

The minimum natural frequency of the small part when its interface is constrained is given by

$$f_{20,opt} = \frac{1}{2\pi} \sqrt{\frac{k_{opt}}{m}} = f_{min} \sqrt{\frac{(1 + \beta - \varphi^2)}{(1 + \beta)(1 - \varphi^2)}} \quad (3.70)$$

and defining $\beta^* \equiv \frac{m}{M+m} = \frac{\beta}{1+\beta}$, finally

$$f_{20,opt} = f_{min} \sqrt{1 + \beta^* \frac{\varphi^2}{1 - \varphi^2}} = f_{min} \sqrt{1 + \beta^* \frac{f_{min}^2}{f_{10}^2 - f_{min}^2}} \quad (3.71)$$

This expression can be used by the satellite designers to derive a specification of minimum natural frequency to be imposed as a requirement for its subsystems to assure that the first natural frequency of the entire system is above the minimum value f_{min} defined by the launch authority. The input parameters for this equation are f_{min} , the mass ratio of the subsystem or instrument with respect to the total mass β^* and the first natural frequency f_{10} of the satellite considering the subsystem as a rigid body attached to the main structure. Using Equation (3.71), the value of $f_{20,opt}$ increases when the mass ratio β^* increases, indicating that the requirement is more severe for massive subsystems.

3.3 Case Study: numerical results of dynamic coupling with the UPMSat-2 FEM

In this section, the phenomenon of modal coupling is numerically investigated with the design of the microsatellite UPMSat-2 [91]. The main lateral modes in the launch configuration of the entire system can be altered by the mechanical design of the communication antennas, which are four thin beams located vertically on top of the satellite and with very small mass compared to the rest of the satellite. Despite the small mass fraction of these secondary elements, the changes on the values of the natural frequencies of the entire system and on their modal effective masses can be significant. The structural responses obtained under a sine vibration environment such as forces and stresses are also modified due to the modal coupling between antennas and main structure. In section 3.3.1, a general description of the UPMSat-2 satellite is presented. In sections 3.3.2 and 3.3.3, the numerical FEM is described and the obtained numerical results are detailed to evaluate the impact of the modal coupling between antennas and satellite on the dynamic behaviour of the entire satellite and on the changes of the structural responses. The design parameters of the antennas are modified to have different degrees of modal coupling, what allows the assessment of their influence on the results such as natural frequencies, modal effective masses, interface forces and local stresses.

3.3.1 UPMSat-2 project description

UPMSat-2 is a 50 kg microsatellite developed by the research institute “Ignacio Da Riva” (IDR), which belongs to the Spanish university Universidad Politécnica de Madrid (UPM). The technicians, investigators and professors of the research institute, as well as some students of the engineering schools of aerospace and informatics of UPM, have participated in the design,

assembly and testing of this satellite and its equipment, whose main purposes are the space qualification of new technologies, the acquisition of knowledge and experience in the development of a satellite by all the participants and the demonstration of the capability of the UPM in the field of space technology [91,92]. Many works have been published related to different aspects of the design and development of the UPMSat-2, such as the design of its magnetic attitude control subsystem [93–95], the design and tests of the power system [96,97] and of the communications subsystem [98]. The UPMSat-2 satellite will be launched to the space, together with other small satellites, in a multiple payload launch of Vega at the end of 2019.

The UPMSat-2 project was conceived with the aim of developing a multipurpose satellite capable of transporting different payloads and performing missions in a variety of scenarios, keeping the structure and the basic subsystems simple and with standard designs. With this philosophy, the mechanical design selected for UPMSat-2, which is based on its predecessor UPMSat-1, is a quadrilateral prism structure composed of four horizontal trays, which support the main payloads, attached laterally at the corners to four L-shape vertical beams and on the side edges to four double sheet panels (see Figure 3-14). In addition, there are a total of eight vertical shear panels located inside between the lower trays, which stiffer the entire structure. All these structural parts are made of the aluminium alloy EN AW 7075-T6. This concept was chosen by its simplicity and robustness, with the aim of having a standard design that can be suitable for a wide variety of missions and payloads. Furthermore, the main structure has been designed to withstand the most severe load cases extracted from the mechanical environments defined by a large number of launchers in order to have the possibility of being launched by any of these launch vehicles. Finally, the launcher selected has been Vega.

Figure 3-14: UPMSat-2 mechanical design [99].

At the top of the satellite, four communication antennas are located near the corners, where each one is a thin vertical beam made of a copper – beryllium alloy and clamped by its lower end to the satellite structure. Therefore, each antenna is considered as a vertical cantilevered beam from a mechanical point of view, where the first modes are mainly bending and torsional motions. The dimensions of each antenna in the final design are 167 mm long, 8 mm wide and 0.2 mm of thickness, giving a first bending mode of approximately 4 Hz, much lower than the first global mode of 92 Hz of the entire satellite. In this work, the value of the thickness of two opposite antennas is to be modified in the numerical model of the satellite to increase the natural frequency of the local bending modes and, therefore, to achieve different degrees of modal coupling, which reaches its maximum level when the difference in frequency terms between local and global

modes is the minimum. In this way, the calculated results with the different design configurations will allow the numerical evaluation of the dynamic coupling between the two antennas and the main structure of the satellite.

3.3.2 UPMSat-2 finite element model

The FEM of the UPMSat-2 is mainly composed of quadrilateral planar elements (QUAD4), which represent the thin-walled structural pieces (see Figure 3-15). Each payload is represented by a point mass attached to the structure by a rigid element, while the mass of the solar cells is taken into account in the model by the non-structural mass (NSM) parameter defined in the elements that represent the external lateral panels. To join the different structural pieces, CBUSH elements have been created together with RBE2 rigid elements to represent the screws that connect these parts. These CBUSH elements allow the calculation of the interface forces that are generated as a result of the application of the different load cases. The antennas are modelled with 1D beam elements located vertically on top of the UPMSat-2 FEM, whose lower ends are attached to the main structure by rigid elements.

The FEM used in this study includes an adaptor under the lower tray of the satellite, which is the mechanical interface between the satellite and the separation system. The boundary conditions of the satellite in the launch configuration are modelled by the constraints of the six degrees of freedom (DOF) applied to the lower nodes of the CBUSH elements that represent the bolts between adaptor and separation system. The upper nodes of these CBUSH elements correspond to the centre nodes of the RBE2 rigid elements defined in each hole of the adaptor (see Figure 3-16). The results described in the next section have been calculated with this boundary condition in the different proposed analyses.

Figure 3-15: Finite element model of UPMSat-2 [99].

Figure 3-16: (a) FEM of the adaptor and the lower stiffened tray at the bottom face of the UPMSat-2. (b) Detail of the modelling of one interface bolt of the adaptor [99].

3.3.3 Numerical analyses and results

In this section, the results obtained from normal modes analyses and sine vibration simulations with the FEM of the UPMSat-2 are presented, where the thickness of two opposite antennas is modified to have different degrees of dynamic coupling. The selected antennas whose thicknesses will be changed in this study are those in which the thickness direction is perpendicular to the global X-axis. These antennas are located in the corners X+ Y+ and X- Y- (see Figure 3-15). Therefore, the global mode of interest in this investigation is the main lateral mode of the satellite in X-axis, whose motion is in the same direction as the first bending mode of the selected antennas.

3.3.3.1 Normal modes analyses

Some of the numerical results of this section are compared with the analytical solutions of the equivalent 2-DOF model described in section 3.2. In this analytical development for the 2-DOF system, a reference configuration is defined, consisting of the primary structure without the analysed secondary part. Therefore, the first numerical simulation presented here is a normal modes analysis of the FEM of the UPMSat-2 without the two studied antennas, which correspond to the secondary part, to obtain the reference natural frequency considered in this numerical study. This analysis provides a fundamental natural frequency of the satellite of $f_r = 94.8$ Hz, where the corresponding mode shape can be seen in Figure 3-17. The mass of the satellite in this reference configuration is 45.950 kg.

Figure 3-17: Mode shape of the reference main global mode of UPMSat-2 satellite without two opposite antennas [99].

Figure 3-18: (a) Variation of the mass of two antennas with their thickness. (b) Variation of the mass ratio β with the thickness [99].

The following results shown in this work have been obtained with the completed FEM of the satellite that includes the four antennas, where the thicknesses of the two selected antennas vary from 3 mm to 6 mm with steps of 0.1 mm. The linear variation of the mass of these two antennas with their thicknesses can be seen in Figure 3-18(a). The mass ratio β , which is defined in section 3.2.1 as the fraction between the masses of the secondary part and the primary part, corresponds in this numerical example to the ratio between the sum of the masses of the two studied antennas and the mass of the rest of the satellite. The variation of β with the thickness is linear, as can be seen in Figure 3-18(b). It can be observed that the mass fraction of both antennas with respect to the rest of the satellite is very small, with values ranging from 0.15% to 0.32%.

The two parameters used in this numerical investigation to evaluate the results obtained from the normal modes analyses are the dimensionless eigenfrequency x_i , which is described in section 3.2.1 as the square of the ratio between a natural frequency f_i of the model and the reference natural frequency f_r , and the modal effective mass (MEM) fraction, which indicates the quantity of mass involved in the motion of the corresponding mode. The evolutions with the thickness of the two dimensionless eigenfrequencies of interest of the complete model and their associated MEM fractions are shown in Figure 3-19, where the variation step of the thickness is 0.1 mm. These two modes of interest are the global mode of the satellite in X direction and the local mode of the two antennas, which is a simultaneous bending motion of both antennas in the same direction. These results have been extracted from the normal modes analyses, where more modes are obtained from this complex numerical model that are not taken into account in this work, but which may have some influence on the presented results.

Figure 3-19: Variation of the numerical results with the thickness of the antennas: (a) Dimensionless eigenfrequencies. (b) Modal Effective Mass (MEM) Fractions [99].

Both natural frequencies increase with the thickness of the antennas, where the frequency of the Mode 1 is always lower than that of the Mode 2. Particularly, the first dimensionless eigenfrequency is below the unit for all the range of thickness values, while the second dimensionless eigenfrequency is above the unit, which is the minimum asymptotic limit for low thicknesses.

The dependence of the MEM fractions for both modes shown in Figure 3-19(b) indicates a monotonic increase of this value for the first mode and a monotonic decrease for the second mode. It is appreciated that there are three ranges of thickness values depending on the values and variations of the MEM fractions. It can be deduced that, for the first range of thickness values between 3 and 4.5 mm, the first mode corresponds to the local bending mode of the antennas with a MEM fraction below 10%, where the increment of the dimensionless eigenfrequency with the thickness presents a quasi-linear trend. In the same range of thickness values, the second mode corresponds to the main global mode of the satellite with a MEM fraction that decreases from 44 to 36%, but that is clearly higher than that of the first mode. The rest of the MEM fraction is due to the rest of modes of the model that are not taken into account in this study. There is a slight increment of the dimensionless eigenfrequency of the main mode in this first range as a result of the small effect of the modal coupling between the antennas and the primary structure. This effect will increase with the thickness due to the increment of the degree of modal coupling.

The second range of thickness values, which comprises from 4.6 to 5.2 mm, corresponds to the maximum degree of modal coupling between secondary and primary structure. One of the effects of the modal coupling is the higher variation of the MEM fractions for both modes compared to the other ranges. Precisely, with 4.9 mm, the values of the MEM fractions are practically the same

between both modes, making it impossible to distinguish which is the local and which is the global one. Therefore, it can be affirmed that the configuration that appeared for the first design range, with a global mode and a local mode, has been transformed into a configuration with two similar global modes, each one with approximately the half of the MEM fraction of the original global mode. One mode with a natural frequency below the reference one, and the second mode with a natural frequency above. This is one of the most interesting effects of the modal coupling, which in this example has been caused by a secondary part with a practically negligible mass fraction compared to the entire satellite. There are also other effects on the structural results that will be discussed in the next section.

When the thickness of the antennas is higher than 5 mm, the MEM fraction of the first mode overpasses the MEM fraction of the second mode. From this design configuration, the modal behaviour is reversed and the first mode becomes the main global mode, while the second mode, despite being a global mode, has a MEM fraction that is lower than that of the first mode and whose value decreases for higher thicknesses. For the third range of thickness values (5.3 – 6 mm), the MEM fraction of the second mode is reduced below 10%, and therefore, it can be considered as the local bending mode of the antennas. In this case, the situation is similar to that of the first range, where there are one local mode and one global mode. The difference is that the natural frequency of the local mode is higher than that of the global mode in this third range.

One of most interesting aspects observed in the numerical results is the relationship that exists between the dimensionless eigenvalue obtained x_i and the relative MEM fraction $m_{eff,rel,i}$ for the two modes considered. When comparing these numerical results with the analytical equations of the 2-DOF model (see Figure 3-20), it has been found that the numerical results follow with small deviation the analytical expression (3.72), which has been extracted from Equation (3.25) with the addition of a correction factor K_β that multiplies to the mass ratio β to match the numerical results.

$$m_{eff,rel,i} = \frac{K_\beta \beta}{1 + K_\beta \beta} \frac{1}{K_\beta \beta x_i^2 + (1 - x_i)^2} \quad (3.72)$$

Figure 3-20: Comparison between the FEM numerical results and analytical equation of the relationship between the relative MEM fraction and the dimensionless eigenfrequency for the two considered modes [99].

The relative mass fraction is defined as the ratio between the numerical MEM fraction of any of the modes and the sum of the MEM fractions of the two considered modes. The value for the correction factor K_β found for the first mode of this numerical model that best correlates the numerical results is 2.68, while for the second mode is 3. As can be seen in Figure 3-20, the curves calculated with Equation (3.72) interpolate accurately the numerical results for both modes.

The mode shapes for the three different cases are shown in Figure 3-21. For the first case, with 3 mm of thickness, the first mode is clearly the local bending mode of the two considered antennas (see Figure 3-21(a)), while the second mode is the global mode of the satellite (see Figure 3-21(b)). These results agree with the solutions of the analytical 2-DOF model described in section 3.2.1 (see Figure 3-6), where for the first mode, the motion of the secondary part is much higher than the primary mass, while for the second mode, the motions of both masses are similar in intensity but out of phase. However, for the third case, the first mode is the global mode of the satellite (see Figure 3-21(e)), where the motions of the secondary and primary parts are in phase and with a similar intensity, while the second mode is the local mode of the secondary part (see Figure 3-21(f)), with much more intensity in the motion of the antennas. These numerical results also agree with the analytical solutions of the 2-DOF system.

The mode shapes for the situation of the maximum degree of modal coupling correspond to the design case where the thickness of the antennas is 4.9 mm. The first mode shape (see Figure 3-21(c)) is similar to that of the second mode (see Figure 3-21(d)), where the motions of the antennas are amplified to the maximum level as an effect of the dynamic coupling. This result is consistent with the conclusion extracted from the 2-DOF system in section 3.2.1, where the amplitude of the motion of the secondary mass is much greater than that of the primary mass for both modes in the case of modal coupling, and where this difference in intensity is higher for lower values of the mass ratio β . Due to this fact, the motion of the primary structure cannot be well appreciated in the FEM figures for this design case, but theoretically is in phase with the motion of the antennas for the first mode and out of phase for the second mode.

(a) Mode 1 shape for uncoupled design case
(3 mm)

(b) Mode 2 shape for uncoupled design case
(3 mm)

(c) Mode 1 shape for coupled design case
(4.9 mm)

(d) Mode 2 shape for coupled design case
(4.9 mm)

(e) Mode 1 shape for uncoupled design case
(6 mm)

(f) Mode 2 shape for uncoupled design case
(6 mm)

Figure 3-21: Mode shapes of the UPMSat-2 FEM for different design configurations [99].

3.3.3.2 Sine vibration analyses

In this section, the numerical results obtained from the sine vibration analyses applied to the different proposed configurations of the UPMSat-2 FEM are presented. The vibration consists of a forced acceleration defined to a base node that is attached via a rigid element to all the interfaces nodes of the UPMSat-2 adaptor, transmitting this input acceleration to the structure of the satellite. The input acceleration is defined with a constant 0 – peak amplitude of 2.5 g between 5 to 130 Hz, which is very similar to the sine specification defined by the Vega launcher to its payloads. The modal viscous damping for these analyses is a constant value of 0.02 for all the frequency range, which has been chosen because it is a usual value for the space structures [1].

The results considered for this study are the maximum bending stresses calculated at the lower section of the antennas and the lateral forces in the global X direction obtained in the CBUSH elements that represent the interface bolts of the satellite. The bending stresses represent the most severe structural effect on the antennas, while the interface forces are appropriate results related to the primary structure that evaluate the impact of the variation of the modal effective masses on the global behaviour.

The variation of the lateral forces on the 20 interface elements are shown in Figure 3-22. These results are the 0 – peak amplitudes of the dynamic lateral forces calculated when the frequency of the excitation coincides with the natural frequency of the global mode in each design case, that is to say, of the mode with the highest MEM fraction value. The differences in the forces among these elements depend on their location with respect to the direction of the input acceleration. For all elements, the maximum force is reached for the minimum considered thickness of 3 mm. The forces decrease as the degree of modal coupling becomes higher and reach their minimum values just at the design point when the degree of modal coupling is maximum, with a thickness value of 4.9 mm. From this point, there is a change in the trend and the forces increase for higher values of thickness due to the reduction of the degree of the modal coupling. The comparison between the maximum and minimum values of these interface forces is shown in Table 3-1. The modal coupling of the antennas achieves a reduction between 57.1 and 63.7% of these values, which is a remarkable conclusion, taking into account the small mass of the antennas compared to the entire satellite, with a mass ratio of 0.25%. From this point of view, the dynamic coupling seems to be a positive effect for the main structure because it achieves a reduction of the maximum loads supported by the interface bolts, which in turn decreases the risk of bolt failures. But it also has its negative consequences such as the increment of the bending stresses generated in the base sections of the affected antennas, as can be seen in the curve depicted in Figure 3-23. In this graph, the amplitude of the bending stress increases initially with the thickness due to the greater influence of the effect of the increment of the degree of modal coupling compared to the fact of having a more rigid beam. The maximum stress is reached for the design case of 4.9 mm of thickness, which corresponds to the maximum degree of modal coupling. Then, the stress decreases with the thickness due to combined effect of reducing the degree of modal coupling and increasing the rigidity of the antennas. Taking into account that the tensile strength limit of the antennas material (copper – beryllium alloy) is about 330 MPa, a mechanical failure of these elements under the considered sine vibration excitation is probable for the configurations from 3.8 to 5.9 mm of thickness. Therefore, from the secondary part point of view, the modal coupling is an undesirable effect that must be avoided because it can provoke the breakage of these secondary elements.

Figure 3-22: Variation of the lateral forces on the interface elements with the thickness of the antennas [99].

Figure 3-23: Variation of the maximum bending stress on the lower section of the antennas with their thickness [99].

Table 3-1: Comparison of the maximum and minimum interface forces obtained from sine vibration analyses [99].

IF	0 – Peak Lateral Force (N)		Max – Min Ratio
	Maximum	Minimum	
IF1	774	444	57.3%
IF2	494	282	57.1%
IF3	730	417	57.1%
IF4	1031	589	57.1%
IF5	222	132	59.4%
IF6	317	185	58.2%
IF7	937	534	57.0%
IF8	569	326	57.3%
IF9	507	291	57.3%
IF10	818	468	57.2%
IF11	793	455	57.3%
IF12	494	282	57.2%
IF13	702	402	57.3%
IF14	958	550	57.4%
IF15	137	87	63.7%
IF16	216	130	60.1%
IF17	889	509	57.2%
IF18	541	311	57.6%
IF19	496	285	57.5%
IF20	786	450	57.2%

3.4 Conclusions

In this chapter, using both an analytical approach and numerical simulations, the study of the effects of the dynamic coupling between a secondary and a primary structure (in terms of their mass ratio and tuning parameter) on the modification of natural frequencies, mode shapes and modal effective masses of the whole structure is presented. This phenomenon, despite affecting several designs of space structures, such as instruments or components that have natural frequencies close to the global natural frequencies of the spacecraft, is not taken into account in the space industry in the definition of the requirements that limit the natural frequencies of the secondary systems, where generally the modal coupling among the different levels of mechanical assembly is avoided. The standard approach where the requirement is defined as a minimum value for the natural frequencies of the secondary subsystems to be well above the main natural frequencies of the primary system, may be too restrictive for secondary parts whose mechanical design cannot meet this requirement due to other mission requirements. The aim of this second part of the doctoral dissertation is to provide an insight of the phenomenon of modal coupling with the aim of determining which are the effects caused by the design of the secondary part on the dynamic behaviour of the main system in order to establish more precise limits for the values of the natural frequencies of the secondary part, taking into account the modal effective masses of the involved modes.

The dynamic coupling has been evaluated by using an undamped 2-DOF mathematical model, presented and solved in section 3.2. This undamped model has allowed to obtain useful analytical expressions with the advantage of providing a rapid and direct estimation of the influence of design parameters, such as the mass ratio β and the tuning parameter μ , on the main results. These results allow to carry out studies on the preliminary design phase of spacecraft by means of these values and by the existence of feasible and unfeasible design regions. This method has been

compared with other alternatives such as Dunkerley's and Rayleigh's methods to calculate the first natural frequency of the system, showing that the proposed method has the advantage of providing exact solutions for all range of values of the design parameters and with ease-of-use expressions. It has also been demonstrated how the main mode of the system can be appreciably altered when the local mode of the small part has a similar frequency, even for negligible values of the mass of the small part. The effect is the division of the global mode into two modes, where both modes can have similar values of modal effective mass fraction. Another important consequence is the decrease of the first natural frequency of the satellite, which can lead to the non-compliance of the requirement of minimum natural frequency imposed by the launcher to prevent the structural resonance with the loads generated during the launch phase. Therefore, the mathematical equations presented in this work provide an easy calculation method to find out the optimum values for a preliminary design of a secondary part for the case where the modal coupling of this part with the entire satellite cannot be avoided. The objective is to find the design values so that the satellite can fulfil the minimum natural frequency requirement. These equations can be useful to the spacecraft designers to derive a specification of minimum natural frequency to be imposed on the payloads.

In section 3.3, the effects of the modal coupling between a secondary element and the primary structure have been studied using the numerical model of the UPMSat-2 satellite. The results calculated from the normal modes analyses and sine vibration simulations have allowed the investigation of this phenomenon in a case in which small components can remarkably change the dynamic behaviour of the entire system. In particular, the secondary elements are two communication antennas located vertically like cantilevered beams at the top of the satellite, whose masses do not overpass the 1% of the mass of the satellite. In spite of this very small mass fraction of the antennas, they can substantially modify the structural parameters of the entire system, such as the natural frequencies and their associated modal effective masses, to the point of converting a configuration with one global mode and one local mode into a configuration with two global modes for the case of maximum degree of modal coupling. The numerical results obtained have been compared with the analytical results of the undamped 2-DOF system, showing similarities between both approaches. This agreement is interesting to validate the analytical approach in the evaluation of the modal coupling of space systems in the early design phases, when a detailed model is not available yet.

Other structural results are also affected due to the modal coupling, such as the forces generated on the interface elements under a sine vibration load, which in the UPMSat-2 model are reduced to about 57% with the maximum degree of modal coupling. These numerical results show the benefits of the modal coupling on the main structure. However, it also has its negative consequences for the secondary parts. In the example presented in section 3.3.3, these negative effects are materialized in the increment of the maximum bending stresses generated in the antennas, which exceed the maximum allowable limit, what can provoke the structural collapse of these elements.

Therefore, it is important to take into account all these effects of the modal coupling, both positive and negative, for the mechanical design of space systems. These structures will be subjected to intense dynamic loads, especially during the launch phase, which will generate high forces and stresses in all the involved components. Depending on the dynamic design of these systems, these forces and stresses can be intensified or mitigated. Modal coupling is an important aspect that must also be taken into account due to its determining influence on these structural responses.

4 CONCLUSIONS

The investigation carried out in this doctoral dissertation has the purpose of improving the existing knowledge of the structural design and analysis of space systems, focusing on two aspects that nowadays are not fully developed in the space industry: the numerical shock analysis and the dynamic coupling.

The motivation to start the investigation about the numerical methods for shock analysis comes from the experience of participating in various space projects performing the structural evaluation of instruments and spacecraft, where the demand of including the numerical evaluation of the shock environment in the structural assessment procedure of space systems is increasing nowadays. The objective of the numerical analyses is to provide a wide set of results that allow a complete evaluation of the real phenomena, with the advantages of requiring less costs and inducing less risks compared to the tests. One of the required characteristics for the numerical analyses is their capacity of providing reliable results with sufficient accuracy. The finite element approach, which is at present the most employed numerical method for the structural evaluation with the load cases of static inertial acceleration, sine and random vibrations due to its demonstrated accuracy, has been selected in this investigation to be used for the shock simulations. Different methods of analysis exist based on the finite element approach to evaluate the effects of shock loads on the structures, considering different assumptions and strategies. Therefore, with the aim of finding which is the most adequate method for shock analysis, an overview of different numerical methods is presented in this doctoral dissertation, describing them, evaluating their versatility in providing the different structural results (accelerations, stresses, forces...) in the diverse formats (peak values, SRS and time signals) and determining the precision of the numerical results by their application in the real case of the STEP instrument. The shock environment measured during the impact test of the STEP instrument has been simulated with the different methods, where the precision of the numerical results has been analysed by comparison with the test data. From this evaluation, the modal transient analysis has been demonstrated as the best of the studied methods due to its higher precision (with differences of the peak values of acceleration within ± 3 dB with respect to the test data), versatility and capacity of providing all the required results. As a drawback, this method employs more time of analysis and generates a greater amount of output data compared to the other options, because for each node or element, the transient analysis calculates a time function with as many values as time steps, unlike RSA and EQS methods, which only calculate a peak value instead. In particular, the size of the output file generated by transient analysis for the STEP FEM is about 2000 times the size of the output file of the RSA methods, which is directly related to the 2000 time steps used in the functions calculated in the transient analysis. The computational time and amount of output data for the modal transient analysis method also depend on the number of nodes and the number of normal modes of interest of the analysed structure, which are not excessive for small structures like STEP instrument, what minimizes the negative impact of both aspects making the modal transient analysis the most appropriate option.

Once the analysis method has been selected, the next step has been the evaluation of the influence of the input parameters such as the damping and the addition of the cross input accelerations on the obtained results. The influence of the damping is appreciated in the SRS of the response accelerations in the frequency bands that contain the main natural frequencies of the analysed structure, where the SRS curves decrease for higher values of damping. However, the influence of the damping is practically negligible out of these frequencies ranges. In the case of the addition

of the cross input accelerations to simulate the more realistic three-dimensional excitation measured in the impact test, the comparison with the results obtained from the one-directional excitation reveals that the influence of the cross input acceleration on the response accelerations calculated in the direction of the main excitation is negligible. This conclusion has been obtained for the simulations of both the in-plane and out-of-plane tests.

The standard technique to simulate dynamic base excitations in the numerical analyses is based on considering one base node connected by a rigid element to all the interface nodes of the analysed structure, assuming that the input acceleration field transmitted to all these interface points is uniform. This assumption can be considered adequate for the simulations of the in-plane shock tests, as has been demonstrated by the results shown in this document, where the precision of the numerical results compared to the experimental data is good enough. However, the numerical simulations of the out-of-plane impact tests are more complicated due to the presence of the bending modes of the test bench excited by the impact, which generate a non-uniform acceleration field that is transmitted to the mechanical interface of the tested item. With the purpose of improving the accuracy of the numerical results of the simulations of the out-of-plane shock tests, new techniques have been proposed to define the non-uniform input acceleration in the numerical analyses. Among the proposed techniques, two are the most promising. The first one consists of assuming that the adaptor plate, which is the mechanical interface between the test bench and the tested item, is a rigid body that presents a combination of vertical and rotational motions during the shock excitation. The input motions included in the numerical simulations are calculated with the equations presented in this document from the signals measured during the impact test by at least three non-collinear accelerometers located on or near the adaptor. The other technique consists of a numerical simulation with a complex excitation with multiple signals, each one applied to the corresponding base node. Some of the signals are directly measured by the reference accelerometers, while the other input signals are calculated with the algorithm indicated in this document from some of the response accelerometers. The proposed techniques have been applied to simulate the out-of-plane shock test performed on the STEP instrument, where the two aforementioned options present a considerable enhancement of the accuracy of the results compared to the other techniques.

The final part of the investigation of the numerical shock analysis consists of the description of the shock verification procedure performed on the STEP and EPT-HET instruments, indicating the key role of the numerical analysis to achieve the final acceptance of this verification. The verification by analysis includes the calculation of the generated stresses on the structural parts, the accelerations transmitted to the sensitive electronic components and the maximum relative displacements of the PCBs, which are related to the fatigue damage of the pines and solder joints of the electronic components. The capability of the analysed instruments and their critical components to withstand the specified shock loads has been demonstrated numerically by the calculated positive margins of safety related to all these aspects. The verification procedure described in this doctoral dissertation can be useful as an example for the shock verification of future space systems that cannot be tested due to the high risk of damage.

In the second part of the doctoral dissertation, a study about the effects of the dynamic coupling between a secondary subsystem and a primary system is presented. This aspect is not taken into account in the standard designs of the space structures, where it is generally preferred to avoid the dynamic coupling by keeping well separated the fundamental natural frequencies between the satellites and the launchers, and between equipment and satellite. The drawback of this standard approach is that there are situations in which it is impossible to achieve this objective due to the limitations in the mechanical design of a particular subsystem caused by other mission requirements, which leads to the unavoidable modal coupling between the affected subsystem and

the main system. Therefore, it is important to evaluate the structural effects generated by this phenomenon, which can affect both to the secondary and primary structures. The goal of this investigation has been to quantify these effects both analytically and numerically, by using a 2-DOF model and the example of the FEM of the UPMSat-2 satellite respectively. The analytical and numerical results show how the values of the natural frequencies and their corresponding modal effective masses are modified according to the values of the design parameters of the secondary part. One of the most interesting conclusions is the transformation of the main global mode of the system and the local mode of the secondary part, where in the case of the maximum degree of modal coupling, both modes become two similar global modes, each one with approximately the half of modal effective mass fraction than the original global mode. In this design case, the analytical results of the 2-DOF model show that the frequency separation between both modes is approximately the square root of the mass ratio, being the first mode lower and the second mode higher than the reference natural frequency corresponding to the primary system without the secondary part. For example, for a small mass ratio of 0.01, the separation between the natural frequencies is about 10% of the value of the reference natural frequency. The modal effective mass (MEM) fractions for both modes change radically depending of the value of the tuning parameter, even for very small values of the mass ratio. For small values of the tuning parameter, most of the MEM fraction corresponds to the second mode, while for large values of the tuning parameter, the situation is reversed and the first mode has most of the MEM fraction. Precisely, there is an intermediate design region for values of the tuning parameter close to the unity, where the degree of modal coupling is maximum, achieving that the MEM fraction of each mode is equal to 50% of the total for both modes. The consequence of this effect is the reduction of the values of the structural results of the main structure, such as the forces on the interface elements, when the entire system is subjected to a sinusoidal base excitation. In the example of the UPMSat-2 FEM, the maximum forces calculated on the interface bolts, which connect the main structure to the separation system, are reduced by approximately 57%. This is a remarkable consequence taking into account that has been produced by the dynamic coupling of two communication antennas, whose mass is about 0.25% of the mass of the rest of the satellite. However, the stresses and forces generated on the secondary part increase appreciably to the point that can cause its structural collapse.

The analytical equations presented in this document provide mathematical rules that are useful in the preliminary designs of space systems that have a certain degree of dynamic coupling, defining the design ranges of the secondary systems to avoid the negative effects of the dynamic coupling. Two design cases have been developed in this work considering different options for the fixed and variable design parameters, showing by equations and graphs the allowable design regions according to the imposed requirements.

4.1 Future work

One of the aspects related to the evaluation of the mechanical shock on space structures mentioned in section 2.1 that should be improved in the future is the definition of the specification of the shock loads. The SRS approach has its important limitations, which are the origin of many of the negative aspects found in the shock tests and numerical analyses related to the precision and representativeness of the real environment. An interesting future work is the investigation of the application of the proposal developed in [13] to change the SRS approach by the definition of the wavelet coefficients to represent the shock specifications. The proposed investigation should include the practical application of this approach to both the shock tests and numerical analyses, to demonstrate the advantages in terms of representativeness, precision, speed and simplicity compared to the SRS approach.

Another of the future lines of the investigation is the application of the methodology presented in this dissertation for shock simulations to other structures, both simple and complex, to confirm the conclusions drawn in this doctoral dissertation. The difficulty lies in performing the shock tests to have the experimental data needed to evaluate the precision of the numerical results calculated from the analyses that simulate the same shock environment. One new investigation has started with the example of a cantilevered beam, where the first results can be seen in the Appendix B of this document. In this preliminary work, the high level of precision achieved by the modal transient analysis is demonstrated by comparing the numerical results with the experimental results extracted from shock tests defined by pulses applied by an electrodynamic shaker. This investigation will continue with other tested structures to confirm the capability of the modal transient analysis to represent the shock environment. Furthermore, other analysis methods not included in this work for shock simulations can be studied to evaluate their applicability in the structural verification of space systems, taking into account the existing requirements defined in the standards and normative that regularize the use of the numerical simulations in the space industry. Among these methods, the direct transient analysis can be suitable for the shock simulations.

On the other hand, new numerical methods can be proposed to obtain accurate results, at least as precise as for the transient analysis, but with less computational time. In this sense, an idea is to develop a method that can calculate directly the peak values of the structural responses, assuming that the input shock acceleration is a time function composed of the sum of damped sinusoids, which corresponds to the shape of the transmitted acceleration signal from the shock source to the analysed structure due to the excitation of the modes of the structural elements that constitute the transmission path. This assumption may facilitate the development of the proposed method, because the input shock acceleration is defined in a more concise way, unlike the existing approach of considering any type of signal compatible with the given SRS.

Another interesting future research is the development of a shock test facility that can be employed to qualify space systems, achieving the required input high levels and improving the precision of the existing test methods. The precision can be enhanced by considering more complex test methods. One example is a test bench with multiple impacts that act simultaneously to create an input acceleration signal composed of the sum of the signals generated by each impact. In this case, there are more input parameters than for the existing impact test facilities with one hitting piece, which can help to reproduce with sufficient precision the required input shock.

The continuation of the work developed in the field of the dynamic coupling of space systems will consist of the execution of modal and vibration tests on structures with different degrees of dynamic coupling to obtain the experimental results to be compared with the analytical and numerical results exposed in this doctoral dissertation. The most suitable experimental setup is based on simple structures such as beams, where it would be easy to have a representative set of secondary beams with different designs to cover a wide range of degrees of dynamic coupling. Furthermore, more complex analytical methods can be developed in future works to consider the case of modal coupling with more than two involved modes. The final objective for the future work is the creation of a design methodology for space structures with dynamic coupling to be implemented in future projects.

Finally, it would be interesting to develop a study that takes into account the effects of shock loads on structures with dynamic coupling between a local and a global mode. This study can include analytical, numerical and experimental results.

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A. APPENDIX A: FATIGUE ANALYSIS OF ELECTRONIC COMPONENTS BY STEINBERG METHOD

Electronic components in spacecraft are subjected to vibration loads during the ascent phase of the launcher. It is important to verify by tests and analysis that all parts can survive in the most severe load cases. The purpose of this appendix is to present the methodology and results of the application of the Steinberg's fatigue model to estimate the life of electronic components of the EPT-HET instrument for the Solar Orbiter space mission. A NASTRAN finite element model (FEM) of the EPT-HET instrument was created and used for the structural analysis. The methodology is based on the use of the FEM of the entire instrument to calculate the relative displacement spectral density (RDSD) and the root mean square (RMS) values of the PCBs from random vibration analyses. These values are used to estimate the fatigue life of the most susceptible electronic components with the Steinberg's fatigue damage equation and the Miner's cumulative fatigue index. The estimations are calculated for two different configurations of the instrument and three different inputs in order to support the redesign process. Finally, these analytical results are contrasted with the inspections and the functional tests made after the vibration tests, concluding that this methodology can adequately predict the fatigue damage or survival of the electronic components.

A.1 Theoretical background

The principal formulation used for this analysis can be found in the book of Dave S. Steinberg 'Vibration Analysis for Electronic Equipment' [56]. In addition, the software created by Tom Irvine (Vibration Data) has a module of fatigue of electronic components that has been used in this appendix for more precise calculations. It employs an extension of the Steinberg approach to take into account the RDSD curve of each PCB in the fatigue analysis [100].

Steinberg's fatigue limit equation

The maximum relative displacement (3σ -RMS) at the centre of a PCB that will give a fatigue life of 20 million of cycles in a random vibration environment for a particular component mounted on the PCB is given by

$$Z_{3\sigma\text{limit}} = \frac{0.02816B}{Chr\sqrt{L}} \quad (\text{A.1})$$

The input data for this equation (see Table A-1) are the dimensions of the PCB and the length, position (see Table A-2) and type (see Table A-3) of the electronic component.

Table A-1: Parameters of Steinberg's fatigue limit equation [56].

Symbol	Parameter name	Units
$Z_{3\sigma\text{limit}}$	Maximum relative displacement (3σ -RMS) of the PCB.	mm
B	Length of the circuit board edge parallel to the component.	mm
L	Length of the electronic component.	mm
h	Thickness of the circuit board.	mm
r	Relative position factor for the component mounted on the board (Table A-2).	-
C	Constant for different types of electronic components (Table A-3) $0.75 \leq C \leq 2.25$.	-

Table A-2: Relative position factor values for the electronic components on the PCB [56].

r	Component location (rectangular board simply supported on all sides)
1	Component located at the centre of the PCB
0.707	Component located near one side of the PCB
0.5	Component located near one corner of the PCB

Table A-3: Constants for different types of electronic components [100].

C	Type of electronic component
0.75	Axial leaded through hole or surface mounted components, resistors, capacitors, diodes.
	Fine-pitch surface mounted axial leads around perimeter of component with four corners bonded to the circuit board to prevent bouncing.
	Standard dual inline package (DIP).
1.0	Through-hole Pin grid array (PGA) with many wires extending from the bottom surface of the PGA.
	DIP with side-brazed lead wires.
	Surface-mounted leaded ceramic chip carriers with thermal compression bonded J wires or gull wing wires.
1.26	Any component with two parallel rows of wires extending from the bottom surface, hybrid, PGA, very large scale integrated (VLSI), application specific integrated circuit (ASIC), very high scale integrated circuit (VHSIC), and multichip module (MCM).
1.75	Surface-mounted ball grid array (BGA).
2.25	Surface-mounted leadless ceramic chip carrier (LCCC).

Derivation of the fatigue curve

The number of cycles (N) of fatigue life as a function of the maximum relative displacement ($Z_{3\sigma}$) obtained from FEA and the limit value from the Steinberg's equation ($Z_{3\sigma\text{limit}}$) is given by

$$N = 20 \times 10^6 \left(\frac{Z_{3\sigma\text{limit}}}{Z_{3\sigma}} \right)^b \quad (\text{A.2})$$

The parameters involved are summarized in Table A-4.

Table A-4: Parameters of fatigue curve [56].

Symbol	Parameter name
N	Number of cycles of fatigue life of the electronic component related to the results of the random vibration analysis.
$Z_{3\sigma\text{limit}}$	Maximum relative displacement (3σ -RMS) of the PCB related to a fatigue life of 20 million of cycles.
$Z_{3\sigma}$	Maximum relative displacement (3σ -RMS) of the PCB obtained from FEA.
b	Fatigue exponent. Takes a value of 6.4 for the Steinberg's model.

Number of cycles accumulated during random vibration test

In order to calculate the number of cycles (n) that are accumulated during the random vibration test, Steinberg assumes only the main natural frequency of the PCB (f_1), and multiplies this value by the duration of the random vibration test (T) as shown in

$$n = f_1 T \quad (\text{A.3})$$

In Tom Irvine's software [100], more than one mode of the PCB are taken into account by the RDSD curve, which is introduced as input to calculate the number of cycles (n). In the next sections, the results obtained from this software are compared with the results obtained with Equation (A.3).

Miner's cumulative fatigue

The accumulated fatigue of the electronic components subjected to m different mechanical vibration environments is given by

$$CDI = 4 \sum_{i=1}^m \frac{n_i}{N_i} \quad (\text{A.4})$$

It compares the number of cycles (n_i) that a component mounted on a PCB is subjected by a particular mechanical environment with the number of cycles (N_i) that the same component can withstand in the same environment. Later, each fraction is added to get the total accumulated fatigue damage. The application of the safety factor of 4 is recommended in the standards for European space projects [1] to take into consideration the uncertainties of the results. It is assumed that the component fails when $CDI = 1.0$.

A.2 Structural verification and redesign process for EPT-HET instrument

The use of a Qualification Model (QM) to be tested in the qualification test campaign and two Flight Models (FM), which must be tested in the acceptance test campaign before being integrated into the Solar Orbiter spacecraft, were included in the initial verification plan of the EPT-HET instrument. After qualification vibration tests, some anomalies were detected during the functional test. The later inspections revealed that some pins and glued spots of the main electronic components were broken by fatigue during the random vibration tests (see Figure A-1). These damages could not be prevented because fatigue analysis was not considered necessary before the qualification tests. Because of this, the qualification tests had to be repeated, but in this occasion on one of the FMs, which becomes the Proto-Flight Model (PFM). With the aim of avoiding the same failures for the next test campaign, the following actions were implemented:

- Modification of the PCBs assembly design by adding stiffeners attached to centre points of the PCBs. These structural elements can achieve the decrease of the maximum deflections of the PCBs during vibration environment, and therefore, they can improve the fatigue life of the electronic components. The original design without these stiffeners corresponds to the QM, and the second design, with the stiffeners, is the PFM.
- Review and change of the levels of the random vibration specifications. The first random vibration input, estimated conservatively by the European Space Agency (ESA) with a 1σ -RMS value of 26.2 g, is referred as ‘Input 1’ in this appendix. This environment, which was applied for the qualification tests, provoked cracks in the pins of some electronic components. After the qualification tests, ESA proposed a second environment (‘Input 2’) from a more realistic estimation, with a RMS value of 11.6 g. Finally, a third input (‘Input 3’) was obtained by the reduction of the levels of the second specification in a frequency band that includes the main natural frequencies of the instrument and of its PCBs in a notching procedure to prevent the over-testing. This ‘Input 3’, with a RMS value of 10.9 g, was finally applied for the proto-flight vibration tests. All these specifications are shown in Figure A-2.
- Implementation of the fatigue analysis explained in this study to evaluate the fatigue damage considering all these changes of the design and input levels.

Figure A-1: Broken pins found during the visual inspections after qualification vibration tests (Courtesy of the Institute of Experimental and Applied Physics of the University of Kiel) [49].

Figure A-2: Random vibration input specifications. The 1σ -RMS values are 26.2 g for ‘Input 1’, 11.6 g for ‘Input 2’ and 10.9 g for ‘Input 3’ [49].

The methodology explained here was applied in order to get estimations for the fatigue life of the electronic components in a simple and rapid way for all the combinations considering the two design configurations and the three different random vibration loads. Only the random excitation along the axis perpendicular to the PCBs (Z-axis) is considered in this study, because it is the most severe load condition. However, all random excitations in each of the three principal axes can also be taken into account. The advantage of this procedure is that the same FEM of the entire instrument used for the typical structural analyses (to calculate stresses and IF forces) is also

employed to obtain the maximum relative displacements (RDSD curves and RMS values). This option avoids the creation of new finite element models where every electronic component has to be modelled in detail, as proposed in various studies [101,102], what requires more time of analysis and more amount of data.

A.3 Fatigue analysis of the electronic components

The first step is to run the random vibration analyses on the FEM of the instrument (see Figure A-3) to obtain the relative displacement results of the PCBs. Two versions of the FEM were created and analysed with NASTRAN: the first one (see Figure A-4(a) and Figure A-5(a)) without PCB stiffeners (QM) and the second one (see Figure A-4(b) and Figure A-5(b)) with the PCB stiffeners (PFM). The approach of modelling the PCBs was the globally smeared technique, where the electronic components are not explicitly represented, but their total mass is taken into account with the non-structural mass (NSM) parameter for each PCB. The FEM is correlated with the results of the qualification vibration tests in order to get the same natural frequencies and modal damping factor as those obtained experimentally.

Figure A-3: (a) FEM of the EPT-HET instrument. (b) FEM of the PCBs [49].

Figure A-4: FEM of the Digital PCB assembled into the Instrument Housing for (a) QM design and (b) PFM design [49].

Figure A-5: FEM of the Analogic PCB assembled into the Instrument Housing for (a) QM design and (b) PFM design [49].

The maximum relative displacement results (RMS values) obtained from FEA are shown in Table A-5 and Table A-6. The distributions of the absolute displacement on the three PCBs of the QM when ‘Input 1’ random load is applied are shown in Figure A-6. These distributions indicate that the maximum displacements are located at the centre of each PCB. The Relative Displacement curves between the centre node and the boundary of the Digital PCB and Analogic PCB of the QM are shown in Figure A-7. Each one of these PCB has one Field Programmable Gate Array (FPGA) electronic component attached on the centre, whose pins were broken during the qualification tests.

Table A-5: Maximum relative displacement (3σ -RMS) for QM design [49].

PCB	Maximum Relative Displacement $Z_{3\sigma}$ (mm)		
	Input 1	Input 2	Input 3
LVPS PCB	0.5623	0.2410	0.1448
Digital PCB	0.5336	0.2287	0.1377
Analogic PCB	1.1167	0.4811	0.3042

Table A-6: Maximum relative displacement (3σ -RMS) for PFM design [49].

PCB	Maximum Relative Displacement $Z_{3\sigma}$ (mm)		
	Input 1	Input 2	Input 3
LVPS PCB	0.4458	0.1871	0.1007
Digital PCB	0.4637	0.1945	0.1042
Analogic PCB	0.1314	0.0550	0.0396

Figure A-6: Absolute displacement distributions (in meters) for the PCBs of the EPT-HET QM design [49].

Figure A-7: Relative displacement spectrum density (RDSD) curves for the centre node of the Digital and Analogic PCBs in the QM configuration [49].

Table A-7: Values of the parameters for the Steinberg's fatigue limit equation [49].

Component	PCB	B (mm)	h (mm)	L (mm)	r	C	$Z_{3\sigma\text{limit}}$ (mm)
FPGA	Digital	100	1.5326	40	1.0	1.26	0.23
FPGA	Analogic	120	1.5702	40	1.0	1.26	0.27

Table A-8: Number of cycles of fatigue life (N) for FPGA component on Digital PCB [49].

Random input	Number of cycles of fatigue life (N)	
	QM	PFM
Input 1	9.3×10^4	2.3×10^5
Input 2	2.1×10^7	5.9×10^7
Input 3	5.4×10^8	3.2×10^9

Table A-9: Number of cycles of fatigue life (N) for FPGA component on Analogic PCB [49].

Random input	Number of cycles of fatigue life (N)	
	QM	PFM
Input 1	2.3×10^3	2.0×10^9
Input 2	5.0×10^5	5.3×10^{11}
Input 3	9.3×10^6	4.3×10^{12}

The second step is to calculate with Equation (A.1) the maximum relative displacement ($Z_{3\sigma\text{limit}}$) related to 20 million of cycles of fatigue life for the more susceptible electronic components (in this case, two FPGA components). The main parameters and results of the Steinberg's fatigue limit equation are summarized in Table A-7.

The next step is to determine the number of cycles of fatigue life of each electronic component with Equation (A.2) for the three different random inputs and for the two design configurations, taking into account the values of $Z_{3\sigma}$ provided from the FEA random analysis and the values of $Z_{3\sigma\text{limit}}$ calculated in the previous step. The fatigue exponent is 6.4 for electronic structures [56]. The results are indicated in Table A-8 and in Table A-9. In order to calculate the number of cycles accumulated during random vibration tests (n), two methods have been taken into account. The first one consists in assuming only the first main natural frequency of each PCB (f_1) with Equation (A.3). The second method consists in using the fatigue module of the Vibrationdata software [100], introducing as input the RDSD curves (see Figure A-7 as an example) and the maximum relative displacement ($Z_{3\sigma\text{limit}}$) from Steinberg's fatigue limit equation (see Table A-7). This second estimation takes into account the whole relative displacement spectrum. The specified duration of random vibration test for each axis is 120 seconds for qualification and 60 seconds for acceptance. In order to better compare the different cases (two designs and three random inputs) among them and with the qualification tests, the value of 120 seconds has been considered

in the following calculations. In Table A-10 are indicated the main natural frequencies obtained by FEA for each PCB for both design configurations.

The results obtained with both methods are the number of cycles accumulated during random vibration environment (see Table A-11 and Table A-12) and the corresponding CDI values (see Table A-13 and Table A-14). The results from these two different methods are very similar each other, showing the same order of magnitude for the CDI values.

Table A-10: Fundamental natural frequencies of the EPT-HET PCBs [49].

PCB	Main natural frequencies (Hz)	
	QM	PFM
Digital PCB	660.8	745.2
Analogic PCB	628.6	1048.3

Table A-11: Number of accumulated cycles for random vibration Z-axis for FPGA component on Digital PCB [49].

Random Input	Number of accumulated cycles (<i>n</i>) with Equation (A.3)		Number of accumulated cycles (<i>n</i>) with Vibrationdata software	
	QM (120 s)	PFM (120 s)	QM (120 s)	PFM (120 s)
Input 1	7.9×10^4	8.9×10^4	8.1×10^4	9.1×10^4
Input 2	7.9×10^4	8.9×10^4	8.0×10^4	9.1×10^4
Input 3	7.9×10^4	8.9×10^4	7.9×10^4	9.1×10^4

Table A-12: Number of accumulated cycles for random vibration Z-axis for FPGA component on Analogic PCB [49].

Random Input	Number of accumulated cycles (<i>n</i>) with Equation (A.3)		Number of accumulated cycles (<i>n</i>) with Vibrationdata software	
	QM (120 s)	PFM (120 s)	QM (120 s)	PFM (120 s)
Input 1	7.5×10^4	1.3×10^5	7.7×10^4	1.1×10^5
Input 2	7.5×10^4	1.3×10^5	7.6×10^4	1.1×10^5
Input 3	7.5×10^4	1.3×10^5	7.6×10^4	1.2×10^5

Table A-13: Miner's cumulative fatigue (CDI) values for random vibration Z-axis for FPGA component on Digital PCB [49].

Random Input	CDI Considering only the fundamental mode of the PCB		CDI obtained with Vibrationdata software	
	QM (120 s)	PFM (120 s)	QM (120 s)	PFM (120 s)
Input 1	3.4	1.6	3.5	1.7
Input 2	1.5×10^{-2}	6.0×10^{-3}	1.5×10^{-2}	6.3×10^{-3}
Input 3	5.8×10^{-4}	1.1×10^{-4}	5.5×10^{-4}	1.1×10^{-4}

Table A-14: Miner's cumulative fatigue (CDI) values for random vibration Z-axis for FPGA component on Analogic PCB [49].

Random Input	CDI Considering only the fundamental mode of the PCB		CDI obtained with Vibrationdata software	
	QM (120 s)	PFM (120 s)	QM (120 s)	PFM (120 s)
Input 1	1.3×10^2	2.5×10^{-4}	1.4×10^2	1.9×10^{-4}
Input 2	6.1×10^{-1}	9.5×10^{-7}	6.5×10^{-1}	7.0×10^{-7}
Input 3	3.2×10^{-2}	1.2×10^{-7}	3.3×10^{-2}	8.4×10^{-8}

As can be appreciated in Table A-11 and Table A-12, the results obtained by using Equation (A.3) are independent of the input environment, whereas the results from Vibrationdata software vary slightly for the different input environments. The final results are the accumulated fatigue damage index (CDI) values. There are some CDI values that exceed the limit of 1.0, which are considered as cases with high risk of fatigue damage. This risk is higher for the worst combination of random input load ('Input 1') and design configuration (QM without PCB stiffeners) for both electronic components. There is also risk of fatigue damage for the FPGA on the Digital PCB for the combination of 'Input 1' load and PFM design. Although the method that only takes into account the fundamental natural frequency of each PCB is simpler than using the Vibrationdata software, which considers the whole Relative Displacement Spectrum Density curve, both methods lead to very similar CDI values for each case.

A.4 Summary of the results

The following conclusions of the fatigue analysis for the EPT-HET instrument have been deduced from the results obtained in this study:

- For the FPGA component on the Digital PCB, the most determining action to improve its fatigue life is the reduction of the random input levels. For the most severe random input load ('Input 1'), there is a high risk of fatigue damage for both design configurations. This fact agrees with the failures observed on the pins of the FPGA component after the random vibration test performed with this load on the Qualification Model. On the other hand, the stiffener for this PCB slightly decreases the fatigue damage risk, but not as much as with the reduction of the input levels. This fact can be explained because although it gets the decrease of the maximum relative displacement, this benefit is minimized by the increment of the number of the accumulated cycles during random vibration environment (n) produced by the increase of the first natural frequency of the PCB.
- For the FPGA component on the Analogic PCB, the stiffener contributed the most to reduce the fatigue damage risk. This action reduces the CDI values about 6 orders of magnitude, while the change of the random specification input can only reduce the CDI values by 4 orders of magnitude as much. The high value of CDI for the QM design with 'Input 1' random specification indicates a serious risk of fatigue damage, which agrees with the broken pins found after the vibration tests performed with this load on the QM.
- The final design configuration was the Proto-Flight Model (with stiffeners). The proto-flight campaign was performed by applying the random vibration environment identified in this appendix as 'Input 3'. No anomalies were detected during the functional tests performed after these vibration tests. This fact agrees with the low CDI values calculated applying the method explained in this study, but considering the duration of the random test of 60 seconds for each axis for proto-flight tests. The resulting CDI values for this case are the half of the values indicated in the bottom right cell of the Table A-13 and Table A-14, which are far below of the limit of 1.0.

A.5 Conclusions

The objective of this study is to present the application of the Steinberg's method to predict the fatigue life of electronic components of the EPT-HET instrument for the Solar Orbiter program. The advantage of this methodology lies in the fact that the order of magnitude of the CDI values can indicate in a quick way the risk of fatigue damage of the electronic components, avoiding the need to create a new and more detailed finite element model. Therefore, it is a useful method that

can be implemented in the structural verification procedure for future space projects to get an easy estimation about the fatigue damage of the electronic components before the vibration tests.

In this study, this analytical estimation is made for different combinations of designs and random inputs, where the results agrees with the fatigue damage found on some pins of the electronic components during the inspections and functional tests made on the EPT-HET instrument after the qualification vibration tests. Additionally, it was possible to predict with this method the survival of the electronic components for the tests on the PFM.

B. APPENDIX B: STUDY OF SHOCK ANALYSIS IN A CANTILEVERED BEAM

B.1 Introduction

The aim of this work is the validation of the method of the modal transient analysis with the NASTRAN finite element approach to be applied for mechanical shock simulations in simple structures. The selected simple structure is a vertical cantilevered beam with a constrained lower end and a free upper end. In the first part of this appendix, a comparison is made between the experimental results of the shock tests performed in a configuration that includes the analysed vertical beam and the mechanical fixtures, with the numerical results calculated with the FEM of the tested assembly. The same input accelerations measured in the tests are used in the numerical analyses. The objective is to demonstrate the good agreement of the numerical results with the experimental results. In the second part of the work, the results of various transient analyses are presented, each one with a different input acceleration that has been calculated from the same SRS. The purpose of this second part is to find whether the differences between the numerical results in terms of peak values and response SRS curves of the different analyses are within the tolerable limits.

B.2 Description of the tested configuration

The tested configuration consists of a mechanical assembly with a vertical thin beam that is attached at its lower end by a set of pieces (mechanical fixtures) that join it with the slip table of the shaker. The tested beam and the mechanical fixtures are made of aluminium alloy. The design and dimensions of the vertical beam are shown in Figure B-1, while the test assembly is shown in Figure B-2, where two L-shape pieces hold the beam by a contact force at the lower section of 40 x 50 mm and 7 mm thickness. The free part of the beam of 200 x 50 mm is 1.5 mm thick.

Figure B-1: Tested vertical thin beam design and dimensions in millimetres.

Figure B-2: Test assembly with the vertical thin beam.

B.3 Mechanical shock tests

The shock tests were conducted in the experimental facilities of the Department of Mechanical and Aerospace Engineering of the La Sapienza University, in Rome. The test equipment are composed of an electrodynamic shaker “Dongling Air-Cooled Vibration Test” model ES-6-200 (see Figure B-3(a)), a slip table model GT 700M for lateral vibrations, a test controller LMS SCADAS III Dual-Channel and a set of one-directional accelerometers to measure the input (A1) and the response accelerations (see Figure B-3(b)). Two of the response accelerometers are located at the upper free end (A2 and A3) and the other two response accelerometers (A4 and A5) are located at the middle of the beam.

(a) Electrodynamic shaker

(b) Tested assembly with accelerometers

Figure B-3: Equipment for the shock tests of the vertical thin beam.

The shock tests are applied in the lateral direction with the shaker joined to the slip table (see Figure B-2(a)), where the excitation is in the same direction as the thickness of the tested beam. Different input shock loads have been applied, consisting of acceleration signals with a main half-sine pulse that is accompanied by a pre-pulse and a post-pulse to ensure that the ending input acceleration, velocity and displacement are all zero. The characteristics of the main pulses (maximum amplitude and duration) of the input accelerations for the different shock tests are indicated in Table B-1. The input signal for the test case “HS_15g_6ms” is shown in Figure B-4, where it can be appreciated the main pulse together with the two auxiliary pulses.

Table B-1: Characteristics of the input pulses for the shock tests of the vertical thin beam.

Test Name	Maximum amplitude (g)	Duration (ms)
HS_15g_11ms	15	11
HS_15g_6ms	15	6
HS_10g_11ms	10	11
HS_10g_6ms	10	6
HS_5g_11ms	5	11
HS_5g_6ms	5	6

Figure B-4: Input acceleration of the shock test “HS_15g_6ms” composed of the main half-sine pulse with a pre-pulse and a post-pulse.

B.4 Finite element model

To calculate the numerical responses of the tested pieces with the different shock cases, a finite element model (FEM) has been created in NASTRAN code. The FEM (see Figure B-5) includes the mechanical fixtures modelled with 3D elements, the vertical beam modelled with 2D shell elements and the accelerometers modelled with point masses joined by rigid RBE2 elements to the beam. RBE2, RBE3 and CBUSH elements are used to connect the different parts.

Figure B-5: FEM of the tested assembly with the vertical cantilevered beam for the shock simulations.

Before the shock simulations, FEM suitability has been validated by comparing the numerical normal modes with the experimental modes obtained from a low-level sine sweep test performed with the same shaker. The comparison of the transmissibilities at the sensors locations between numerical and experimental results is shown in Figure B-6, where the peaks represent the main modes. The numerical model is well correlated for the first modes, except for the last experimental mode with approximately 1700 Hz, which has not been calculated from the numerical analysis. The first two modes shapes calculated numerically are shown in Figure B-7, which correspond to bending modes of the vertical beam. The comparison between numerical and experimental natural frequencies indicated in Table B-2 reveals a good agreement of the numerical results with respect to the test data, with differences below 2%.

(a) Transmissibility between A2 sensor location and base

(b) Transmissibility between A4 sensor location and base

Figure B-6: Comparison between numerical and experimental transmissibilities at two locations.

(a) Mode 1: 25 Hz

(b) Mode 2: 154 Hz.

Figure B-7: Mode shapes of the first two modes of the vertical thin beam calculated with FEM.

Table B-2: Comparison of the first natural frequencies of the vertical thin beam between numerical and experimental results.

Modes	Natural frequencies (Hz)		Difference
	Test	FEM	
1	25.0	24.8	-0.84%
2	154.0	153.6	-0.23%
3	433.0	434.8	0.41%
4	857.0	873.6	1.93%
5	1400.0	1421.7	1.55%

B.5 Numerical simulations of the shock tests

In this section, the shock environments measured during the shock tests are simulated with the FEM of the tested assembly by using the transient modal analysis method, where the input acceleration signals measured by the reference accelerometer are applied directly in the corresponding numerical simulations. Each input signal is a forced base acceleration applied to a node connected by a rigid element to all the nodes of the lower face of the base fixture. In this way, it is assumed that the input acceleration field is uniform for the entire lower face of the quadrilateral base fixture, which is in contact with the slip table. All the normal modes of the beam calculated from 0 to 10000 Hz are taken into account in the numerical analyses. The damping used for all analyses is a constant modal damping factor of 0.001 for all the frequency range of interest (0 – 10000 Hz). The results obtained from these analyses are the response accelerations calculated at the accelerometers locations, which are compared with the experimental signals to evaluate the precision of the numerical method (see from Figure B-8 to Figure B-13). As can be seen in these figures, the time signals of response accelerations calculated numerically are in good agreement with the experimental signals. The precision is higher for the analyses with the highest amplitude of 15 g for the input main pulse. The absolute peak values obtained from both the numerical and experimental signals for all the considered load cases are shown in Table B-3, indicating the differences of the numerical results with respect to the experimental results in decibels. All the differences are within ± 3 dB.

Figure B-8: Comparison Test-FEM results for the shock case with a Half-Sine pulse of 15 g and 11 ms.

Figure B-9: Comparison Test-FEM results for the shock case with a Half-Sine pulse of 15 g and 6 ms.

Figure B-10: Comparison Test-FEM results for the shock case with a Half-Sine pulse of 10 g and 11 ms.

Figure B-11: Comparison Test-FEM results for the shock case with a Half-Sine pulse of 10 g and 6 ms.

Figure B-12: Comparison Test-FEM results for the shock case with a Half-Sine pulse of 5 g and 11 ms.

Figure B-13: Comparison Test-FEM results for the shock case with a Half-Sine pulse of 5 g and 6 ms.

Table B-3: Comparison of the absolute peak values of response accelerations between experimental and numerical results of the shock load cases with the vertical thin beam.

Load case	Location	Absolute peak values (g)		Difference (dB)
		Test	FEM	
HS_15g_11ms	A2	31.3	31.5	0.06
	A3	31.8	31.5	-0.09
	A4	16.9	16.7	-0.09
	A5	16.9	16.7	-0.09
HS_15g_6ms	A2	33.0	32.8	-0.05
	A3	34.0	32.8	-0.30
	A4	16.1	15.6	-0.23
	A5	16.0	15.7	-0.17
HS_10g_11ms	A2	23.5	24.1	0.23
	A3	23.7	24.1	0.15
	A4	11.6	12.6	0.70
	A5	11.3	12.6	0.93
HS_10g_6ms	A2	22.6	23.2	0.23
	A3	23.0	23.2	0.08
	A4	11.3	11.3	0.03
	A5	11.4	11.3	-0.05
HS_5g_11ms	A2	12.0	12.4	0.28
	A3	12.0	12.4	0.24
	A4	5.6	6.7	1.60
	A5	5.6	6.7	1.52
HS_5g_6ms	A2	11.3	12.3	0.79
	A3	11.3	12.3	0.73
	A4	5.4	5.9	0.77
	A5	5.4	5.9	0.78

B.6 Numerical analysis with different input signals from a given SRS

The objective of this section is to evaluate the differences of the results calculated numerically by modal transient analyses with the FEM of the tested assembly, using as inputs different time functions of acceleration obtained from a given input SRS. In particular, the same oscillatory functions calculated in section 2.3.3.1 for STEP FEM from the specified SRS curve (see Figure 2-16) by the algorithms of the damped sinusoids decomposition and the wavelet synthesis with durations of 20, 40 and 100 ms are used in this section as input base accelerations. The modal damping value defined for all these analyses is 0.001 for the frequency range between 0 and 10000 Hz. The results to be compared are the response accelerations in terms of both SRS curves and peak values calculated at the locations A2 to A5 of the vertical thin beam. The purpose is to know the influence of the chosen input function on the calculated results when the only available information about the input shock environment is defined by a SRS specification.

The results in terms of SRS of response accelerations are shown in Figure B-14, while the differences of these curves with respect to the corresponding average SRS curve for each location are shown in Figure B-15. It can be appreciated the similarities between the response curves for each location, with peaks that are produced for each natural frequency of the analysed beam. However, the differences in some frequency bands are above the ± 6 dB, which is an important aspect to determine that the results depend on the chosen input function. Additionally, it is observed for this example in particular that the response SRS curves are higher for longer durations of the input functions.

Figure B-14: Comparison of response SRS curves calculated by transient analyses considering different inputs from a given SRS by the algorithms of damped sinusoids decomposition (DS) and wavelet synthesis (WL) with durations of 20, 40 and 100 ms.

Figure B-15: Differences of the SRS curves with respect to the average values.

The comparison of the absolute peak values of response accelerations between the performed analyses is shown in Table B-4, and the differences of these values with respect to the average values are shown in Table B-5. In this case, the differences range from -7.53 to 6.13 dB.

Table B-4: Absolute peak values of response accelerations calculated from the transient analyses of the vertical thin beam.

Location	Absolute peak values (g)					
	DS 20 ms	DS 40 ms	DS 100 ms	WL 20 ms	WL 40 ms	WL 100 ms
A2	1880.42	2862.76	4331.68	2695.59	3228.74	7638.67
A3	1877.06	2857.87	4318.52	2686.86	3221.20	7588.62
A4	1876.67	3248.26	4928.47	3768.93	4023.03	8789.31
A5	1871.53	3253.14	4957.38	3772.19	4018.29	8851.23

Table B-5: Differences of the peak values with respect to the average values for the transient analyses of the vertical thin beam.

Location	Differences (dB)					
	DS 20 ms	DS 40 ms	DS 100 ms	WL 20 ms	WL 40 ms	WL 100 ms
A2	-6.05	-2.40	1.20	-2.92	-1.35	6.13
A3	-6.03	-2.38	1.21	-2.92	-1.34	6.10
A4	-7.48	-2.71	0.91	-1.42	-0.85	5.93
A5	-7.53	-2.73	0.93	-1.44	-0.89	5.97

B.7 Conclusions

The first part of this appendix presents a comparison between the experimental and numerical results of a vertical cantilevered beam subjected to shock loads. The input transient accelerations are different signals composed of a main half-sine pulse with two auxiliary pulses that are used to achieve that the ending acceleration, velocity and displacement are zero. The numerical simulations are performed with the FEM of the tested assembly, applying as base forced acceleration the measured input accelerations and obtaining the time functions of response accelerations in the same positions where the accelerometers were located during the shock tests. The comparison reveals a high degree of accuracy of the numerical results with respect to the test data in both the time functions and the peak values, with differences within ± 3 dB. The level of precision is higher for the simulations with higher values of the input load, and decays when the amplitude of the main pulse decreases. One of the probable reasons is the damping, which numerically is the same value of 0.001 for all the analyses, but experimentally may be different depending on the amplitude of the input pulse.

The second part of this work is a comparison of the results calculated with different input accelerations that represent the same shock environment defined by a given SRS curve. This comparison shows appreciable differences among the obtained response SRS curves, which indicates the non-negligible influence of the chosen input function on the numerical results. The differences are not quite large (within ± 10 dB), but they are large enough to be considered when choosing the time acceleration function that best characterizes the shock environment represented by an SRS. These differences are higher than those of the study performed with the same inputs with the STEP FEM in section 2.3.3.3, because the damping value used for the cantilevered beam is lower (0.001) than the damping used for STEP FEM (0.05). It can be concluded that for lower damping values, the differences between the results are higher. In some projects, additional aspects such as the type of input signal and its duration are specified together with the input SRS to reduce the possible options to represent the shock environment.