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**ESCUELA TÉCNICA SUPERIOR
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**EXPERIMENTAL AND THEORETICAL STUDY OF THE
OPTICAL FREQUENCY COMBS GENERATED BY GAIN-
SWITCHING OF SEMICONDUCTOR LASERS**

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Resumen

El trabajo de investigación realizado en el marco de esta tesis doctoral se ha centrado en la generación de Peines Ópticos de Frecuencia (OFC) en láseres de semiconductor mediante la técnica de conmutación de ganancia (GS). El trabajo ha sido fundamentalmente experimental, si bien se han empleado simulaciones para contribuir a una mejor comprensión de la física interna del proceso de generación de OFCs. Se han empleado diversos tipos de láser, fundamentalmente láseres de realimentación distribuida (DFB) y láseres de modo discreto (DML).

Se ha realizado una caracterización experimental exhaustiva de las propiedades de los OFCs generados mediante GS en un rango completo de condiciones de operación a distintas frecuencias de repetición, tanto en el rango de los GHz, con aplicaciones en comunicaciones ópticas, como en el rango de las centenas de MHz, de aplicación en espectroscopía. En el estudio experimental se ha investigado el efecto de la inyección óptica sobre las propiedades de los OFCs, obteniéndose interesantes resultados, como la mejora o empeoramiento de la calidad de los espectros en función de las condiciones de conmutación y de inyección. Se han encontrado las condiciones óptimas de operación para generar OFC de alta calidad a las distintas frecuencias consideradas.

Con el fin de mejorar la comprensión de los diferentes procesos físicos involucrados en la generación de los OFCs por conmutación de ganancia, se ha desarrollado una herramienta de simulación basada en el modelo de ecuaciones de balance para láseres de semiconductor, con especial consideración en el tratamiento de las fuentes de ruido. Para una correcta simulación de los OFCs se requiere el conocimiento de un conjunto de parámetros internos de los láseres; que se obtuvieron mediante un procedimiento original basado en la medida de los espectros del Ruido Relativo de Intensidad (RIN). Mediante este procedimiento se ha reducido la incertidumbre en los resultados simulados, al ser posible extraer mayor número de parámetros que mediante otros procedimientos gracias a un adecuado tratamiento analítico de las ecuaciones del RIN. Las simulaciones han reproducido por completo la respuesta temporal y espectral de diferentes láseres en un rango amplio de condiciones eléctricas y de inyección óptica. A partir de las simulaciones se han comprendido los procesos físicos predominantes que contribuyen a la generación de OFCs, “chirp” dinámico a altas frecuencias de repetición, y “chirp” adiabático a bajas frecuencias.

Se ha demostrado que es posible mejorar las características de los OFCs a baja frecuencia mediante el empleo de excitación con pulsos eléctricos, en lugar de las habituales señales sinusoidales. Se han generado OFCs de alta calidad a 100 MHz (hasta 866 tonos en 10 dB) con buena relación portadora a ruido. También se ha demostrado que es posible emplear “Step-recovery diodes” (SRDs) como fuente de pulsos eléctricos para generar OFCs, reduciendo el costo y tamaño de los equipos involucrados, y manteniendo buena calidad en las características de los OFCs. El papel del “jitter” de la fuente eléctrica en la calidad de los OFCs ha sido estudiado teórica y experimentalmente.

Finalmente, los OFCs generados mediante conmutación de ganancia e inyección óptica han sido empleados para medir los espectros de absorción de diferentes elementos, utilizando tres arquitecturas diferentes. Se ha empleado detección heterodina, espectroscopía de peine dual, y se ha implementado por primera vez la técnica de peine dual con un único láser por conmutación de la frecuencia de GS. Los resultados han sido prometedores de cara a la aplicación de estos OFCs en espectroscopía de gases.

Abstract

The research work carried out within the framework of this PhD has focused on the generation of Optical Frequency Combs (OFC) in semiconductor lasers using the gain-switching (GS) technique. The work has been mainly experimental, although simulations have been used to contribute to a better understanding of the internal physics of the OFCs generation process. Various types of lasers have been used, specially distributed feedback (DFB) lasers and discrete mode lasers (DML).

An exhaustive experimental characterisation of the properties of the OFCs generated by GS in a full range of operating conditions has been performed. Two ranges of repetition frequencies have been explored: the GHz range with applications in optical communications, and the range of hundreds of MHz for application in spectroscopy. The effect of the optical injection (OI) on the properties of the OFCs has been investigated, and interesting results have been obtained, such as the improvement or deterioration of the quality of the spectra depending on the GS and OI conditions. The optimal operating conditions to generate high quality OFCs at the different frequencies considered have been found.

In order to improve the understanding of the different physical processes involved in the generation of the OFCs generated by GS, a simulation tool based on the rate equation model for semiconductor lasers has been developed, with detailed treatment of the noise sources. For a correct simulation of the OFCs the values of a set of internal laser parameters are required. They were obtained through an original procedure based on the measurement of the relative intensity noise (RIN) spectra. Using this procedure, the uncertainty in the simulated results has been reduced, since it allows to extract more parameters than other procedures due to an adequate analytical treatment of the RIN equations. The simulations have completely reproduced the temporal and spectral response of different lasers in a wide range of electrical and optical injection conditions. From the simulations, the predominant physical processes that contribute to the generation of OFCs, dynamic "chirp" at high repetition frequencies, and adiabatic "chirp" at low frequencies, have been identified.

It has been shown that it is possible to improve the characteristics of low frequency OFCs by driving the laser with electric pulses, instead of using sinusoidal driving. High quality 100 MHz OFCs (up to 866 tones in 10 dB) with good carrier-to-noise (CNR) ratio have been generated. It has also been shown that step-recovery diodes (SRDs) can be used as a source of electrical pulses to generate OFCs, thus reducing the cost and size of the equipment involved, while maintaining the quality of the OFCs. The role of the "jitter" of the electrical source in the quality of the OFCs has been studied theoretically and experimentally.

Finally, the OFCs generated by gain-switching and optical injection have been used to measure the absorption spectra of different elements, using three different architectures: heterodyne detection, and dual-comb spectroscopy and a new implementation of

this technique. In this implementation the dual-comb is generated from a single GS laser by alternating the repetition frequency. Promising results for the application of these OFCs in gas spectroscopy have been obtained.

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Chapter 1

Introduction

1.1 Motivation

An optical frequency comb is a group of coherent equally-spaced optical tones. The research in the field of OFC generation began in the 70s. Since then, they are considered as one of the most important advances in photonics during the last century [6]. During the early times of the OFC generation, the OFCs were mostly employed in high-precision spectroscopic measurements.

In 2005, John L. Hall and Theodor W. Hänsch were awarded the Nobel Prize in physics for their pioneering and jointly contributions to the development of laser-based precision spectroscopy, including the optical frequency comb technique. More recently, the research community has paid special attention to the OFCs as they have opened new possibilities in different fields, such as metrology [7], radio-photonics [8], optical communications [9, 10], molecular spectroscopy [5] and arbitrary waveform generation [11].

Semiconductor lasers have become one of the most important coherent light sources during the last century. The first reports of laser emission from a semiconductor laser were published in 1962, when three different research groups reported coherent emission from a diode laser [12–14]. Since then, these laser sources have become the object of an increasing research effort, coming into the scene for numerous applications. Thanks to their inherent characteristics, they have become one of the most important coherent light sources, representing a significant part of the light source market. Some of the advantages of the semiconductor laser are:

- The price of these devices can be very low, reaching the value of less than 5 \$ per laser.
- The power conversion efficiency of these lasers can reach up to 70 % [15].
- They cover a wide range of wavelengths, going from the ultra-violet to the far-infrared.
- The size of these lasers is really small in comparison with other light sources, reaching chip sizes of 1 mm².
- These lasers can be modulated in a wide range of frequencies up to tens of GHz.
- They can be integrated in the well-known Photonic Integrated Circuits (PICs)

Diode lasers are employed nowadays in many applications, such as CD/DVD players, laser printing, smartphones, medicine, spectroscopy, optical communications, etc... Depending on the application, they can be found with quite different output power levels (from $\leq 100 \mu\text{W}$ to several kW), emission spectra and type of packaging.

Different types of lasers have been employed to generate OFCs depending on the targeted application, such as solid-state lasers [16] and fiber lasers [17]. Semiconductor lasers are also relevant in the generation of OFCs due to their usual credentials, such as high efficiency, low cost and small footprint. There are several strategies for generating OFCs from semiconductor lasers, mode locking, gain-switching, electro-optic modulation, and more recently, micro-ring resonators [18].

GS has attracted specific attention due to its easy implementation, high flexibility in the selection of the repetition frequency and low losses. Although the generation of optical pulses by GS has been extensively studied in the 80s [19–21], only in the past decade, the generation of OFCs by gain-switched semiconductor lasers has become the subject of a relatively high number of application-driven works. The simplicity provided by the GS technique makes much easier the implementation of OFCs generated by semiconductor lasers for specific applications, such as optical communications and spectroscopy.

The use of OFCs for spectroscopy is recognized as a simple and useful method to characterise the internal structure of different materials. Recently, much of the attention of the scientific community has been paid to the spectroscopic architecture known as dual-comb spectroscopy. In this technique, two different OFCs with slightly different repetition frequencies are combined in a photo-detector, down-converting the information from optical frequencies into radio frequency (RF), allowing the acquisition with conventional electrical equipment. The employment of this architecture improves the accuracy of the spectroscopic analysis as the measurements can be performed with high-resolution (lower than kHz). Although this scheme is mainly used in molecular spectroscopy, some applications such as Light Detection and Ranging (LIDAR) can also benefit from the implementation of this architecture.

1.2 Objectives of this thesis

This thesis has been developed in the "Grupo de Fotónica Aplicada", which is part of the "Centro de Materiales y Dispositivos Avanzados para las Tecnologías de la Información y la Comunicación (CEMDATIC)" of the Universidad Politécnica de Madrid (UPM). During the last decades, the members of this research group have focused their research activity into the characterisation and theoretical modelling of several semiconductor lasers. Experimental and theoretical investigation on gain-switching in different types of lasers has been carried out by Consoli et al. [22,23], during the development of his Ph.D. thesis. In 2013, the group started to work with a special type of two-section laser called MOPA or Master Oscillator Power Amplifier laser, characterising the response of these lasers under direct modulation and gain-switching [24,25]. The complex dynamics reported in these publications motivated the realisation of an extensive experimental and theoretical investigation of these devices [26–28]. This research activity was performed within the activities of the RANGER and BRITESPACE projects, whose objective was the implementation of two different LIDAR systems for high-precision distance measurements, and atmospheric re-

mote carbon dioxide sensing in future Earth observation missions, respectively. The work of this thesis is performed within the activities of the project COMBINA, devoted to study experimental and theoretically the generation of OFCs from gain-switched semiconductor lasers, subject (or not) to optical injection.

The fundamental objective of this thesis is to study the generation of OFCs from semiconductor laser by GS, with emphasis in understanding the underlying physics to optimise their performance for different applications. This fundamental goal has been divided into several partial goals:

- The determination of the optimal operating conditions to generate OFC with different characteristics, depending on the targeted application.
- To acquire a better understanding of the physical phenomena involved in the different comb generation processes, through the development of simulation tools based on a rate equation model, and the comparison of the simulations with the experimental results.
- To validate the suitability of the OFCs generated by GS in semiconductor lasers for spectroscopic applications.

1.3 Structure of this thesis

After this introductory chapter, the thesis is organized as follows:

- Chapter 2 "**Fundamentals**" presents a summary of the fundamental concepts relevant to this thesis. A brief description of different semiconductor lasers is included. The fundamentals of the OFCs, the central concept of this work, are explained as well as the main techniques used to generate them. Additionally, the heterodyne detection technique and the dual-comb concept is explained in this chapter.
- Chapter 3 "**Dynamic response of a semiconductor laser**" describes the theoretical model that has been used to simulate the response of the laser. A rate equation model was employed to explain the characteristics of the OFCs in terms of the main physical processes involved in the generation. Special attention has been devoted to the correct mathematical treatment of the stochastic differential equations (SDEs).
- Chapter 4 "**Experimental techniques**" is devoted to the description of all the components, equipment and measurement techniques used in the experiments, providing the schematics of the employed setups for the characterisation of the lasers and for the OFC generation. Also, schematics of the spectroscopic setups are presented.
- The results on the experimental characterisation of the different lasers and OFCs is presented in Chapter 5 "**Optical pulse and frequency comb generation in gain-switched semiconductor lasers**". Power-current-voltage characteristics and optical spectra at different conditions of current and temperature of three different types of lasers are provided. The temporal and spectral characterisation of the OFCs generated by edge-emitting laser diodes under gain-switching and optical injection is detailed. The analysis of the characteristics of the optical pulses generated by GS a 1300 nm vertical-cavity surface-emitting laser is also presented in this chapter.

- Chapter 6 "**Numerical and experimental analysis of optical frequency comb generation**" shows an analysis of the OFC generation in gain-switched semiconductor lasers with and without optical injection through the comparison between experimental and simulated results. The internal parameters of the model were extracted by comparing experimental and simulated relative intensity noise spectra.
- A new method to generate low repetition rate high-quality OFCs is presented in Chapter 7 "**OFC generation by pulsed gain-switching and optical injection**". An experimental study on the characteristics of the OFC generated by pulse excitation and OI is presented, employing two different pulsing sources: a pulse pattern generator (PPG) and a step-recovery diode. In the latter, special attention is devoted to the effect of the source timing jitter on the characteristics of the generated OFCs.
- Chapter 8 "**Spectroscopic applications of optical frequency combs**" presents some spectroscopic applications of the OFC generated in externally-injected gain-switched semiconductor lasers. Different techniques, such as the heterodyne detection and the dual-comb spectroscopy, are employed to determine with high resolution the transmittance of a tunable Fabry-Perot (FP) filter and the absorption spectra of two different gas species.
- Chapter 9 "**Conclusions and future work**" summarises the conclusions and future research lines of the thesis.

Finally, the contributions of the author to journal articles and conferences resulting from this work can be found in the Appendix A.

Chapter 2

Fundamentals

Introduction

The fundamentals of semiconductor lasers are introduced in this chapter. Section 2.1 presents general and intuitive ideas about the lasing condition in a resonant cavity, considering a FP cavity. A summary of the main characteristics of the FP lasers is shown in this section. Section 2.2 presents the fundamentals of the main technique for the generation of optical pulses in semiconductor lasers employed in this thesis, the gain-switching. The basis of the optical injection are explained and discussed in Section 2.3. Section 2.4 is devoted to the concept of the optical frequency comb within this section. A historical revision of the development of the OFCs is presented in Section 2.4.1. The principle of operation of the OFCs is explained in Section 2.4.2. Then, some techniques for OFC generation from semiconductor lasers, such as mode-locking or electro-optic modulation are described and their basic characteristics are discussed in Section 2.4.3. Finally, in Section 2.5 the fundamentals of homodyne and heterodyne detection, as well as the principle of operation of the dual-comb spectroscopy are described.

2.1 Semiconductor lasers

2.1.1 Basic structure: Fabry-Perot laser

Fig. 2.1(a) shows the schematics of a typical FP laser diode, which is based on a p-n junction and a FP cavity. The active medium is sandwiched between a p and an n-doped cladding layers. The stimulated emission is produced by the recombination between the electrons and the holes of the conduction and valence band, respectively (Fig. 2.1(b)). When the laser is unbiased, there are no carriers in the active layer. As long as the current applied to the laser is increased, the number of carriers in the active material increases as well and the material reaches transparency. At higher injection currents, the material is in positive optical gain conditions. However, optical gain is not the only condition to generate laser emission. The other key point of the process is the optical feedback, which is provided by the resonant cavity, i.e, the FP cavity. The FP cavity consists of the own facets of the device that are perfectly cleaved along the crystallographic planes. The reader can find more information about semiconductor laser physics in [29, 30].

The optical field inside the FP cavity can be decomposed in two overlapped and op-

Figure 2.1: (a) Schematics of a FP laser diode. (b) Band diagram of the conduction and valence bands in a single quantum-well laser

posite fields (E^+ and E^-) propagating along the longitudinal direction (Fig. 2.2):

$$E(z, \omega) = E^+ e^{-j\bar{\beta}z} + E^- e^{j\bar{\beta}z} \quad (2.1)$$

where $\bar{\beta}$ is the complex propagation constant given by:

$$\bar{\beta} = \beta + \frac{j}{2}(g_m - \alpha_{in}) \quad (2.2)$$

In Eq. 2.2, $\beta = 2\pi n_{eff}/\lambda$ where n_{eff} is the effective refractive index, λ is the wavelength in vacuum, g_m is the modal gain and α_{in} is the internal loss coefficient, arising from scattering and free carrier absorption. The boundary conditions at each facet of the cavity,

Figure 2.2: Schematics of a FP cavity

($z = 0$ and $z = L$) impose the round-trip condition needed for the laser oscillation, i.e, the optical field does not change after a round-trip along the cavity. The first relationship provided by this condition determines the relation between the modal gain at threshold and the losses:

$$g_m|_{th} = \alpha_{in} + \alpha_m \quad (2.3)$$

According to Eq. (2.3), the threshold condition for the laser operation is fulfilled when the modal gain is equal to the cavity losses, given by the addition of the internal losses (α_{in}) and the mirror losses ($\alpha_m = \frac{1}{L} \ln(\frac{1}{\sqrt{R_1 R_2}})$ where R_1, R_2 are the reflectivities of the two mirrors). Although spontaneous emission of photons appears when the semiconductor laser is biased, Eq. (2.3) is only satisfied at the threshold condition, which is reached when

the laser is driven at the threshold current (I_{th}). Below this current, the laser emits only incoherent light with low output power. However, above I_{th} , Eq. (2.3) is satisfied and the light emitted from the laser is predominantly coherent, following a linear evolution with the bias current. Fig. 2.3(a) presents an experimental power-current (P-I) characteristic of a commercial semiconductor laser, illustrating this effect.

Figure 2.3: (a) Measured power-current curve of a commercial discrete mode laser ($I_{th} = 14.5$ mA). More details about the experimental device are provided in Chapter 4. (b) Gain spectra and cavity losses together with the wavelengths of the longitudinal modes of the cavity (extracted from [1])

The second relationship arising from the round-trip condition is related to the real part of the propagation constant:

$$2\beta L = 2m\pi \quad (2.4)$$

where $m = 1, 2, 3, \dots$ is an integer corresponding to the mode number. Then, the frequency of each mode is given by:

$$f_m = \frac{mc}{2Ln_{eff}}, \quad (2.5)$$

where c is the light speed. Therefore, the wavelength of each mode is:

$$\lambda_m = \frac{2Ln_{eff}}{m} \quad (2.6)$$

Fig. 2.3(b) shows the gain spectra, the cavity losses and the cavity modes in a FP laser. When the carrier density increases, the amplitude and width of the gain spectrum increases as well, until the gain of one of the longitudinal modes reaches the threshold condition. Therefore, when the laser is biased above threshold, the FP mode closest to the gain peak starts lasing. As the difference in the gain for the longitudinal modes is very small, there is mode hopping, and several modes appear in the measured spectra. An example of the emission of a FP laser is depicted in Fig. 2.4 (notice the logarithmic scale).

2.1.2 Single frequency semiconductor lasers

Semiconductor lasers can be classified as single longitudinal mode (SLM) or multi-longitudinal mode (MLM). The side-mode suppression ratio (SMSR) establishes the criteria to determine this characteristic. The SMSR is defined as the difference (in dB) between the amplitude of the main emitting mode and the amplitude of the highest side mode. A

Figure 2.4: Measured optical spectrum of a commercial FP laser (Roithner LFO-18-ip) at $I = 45$ mA ($I_{th} = 8$ mA). This figure was extracted from [1]

laser with a SMSR higher than 30 dB is considered as SLM. Different approaches have been followed to achieve single-frequency emission. A wavelength discriminator structure is required to select a single longitudinal mode, either by enhancing the gain of that longitudinal mode or by increasing the losses of the other modes of the cavity. Some approaches for single-frequency operation are described in the following section.

Distributed Bragg reflector and distributed feedback lasers

Figure 2.5: (a) Schematic of a Bragg grating structure: a periodic array of alternating regions with different indices n_1 and n_2 with a period Λ ; (b) Reflected power as a function of β

A Bragg grating is a structure with a periodically varying refractive index (Fig. 2.5(a)). These structures can achieve high reflectivities (Fig. 2.5(b)) at the Bragg wavelength λ_{Bragg} , given by:

$$\lambda_{Bragg} = 2n_{eff}\Lambda \quad (2.7)$$

An example of the reflectivity of a Bragg grating has been plotted in Fig. 2.5(b) as a function of the detuning between the propagation constant of the incident light (β) and the propagation constant corresponding to the Bragg wavelength. As it can be seen in this figure, the reflectivity of the Bragg grating is maximum for the Bragg wavelength.

The Bragg grating technology has been employed to develop high-reflective mirrors to confine efficiently the light inside optical cavities. The distributed Bragg reflector (DBR) lasers are based on this technology. The reflectivity of the longitudinal modes

whose frequencies are close to the maximum is high, whereas the reflectivity is low for those frequencies outside of this region. The typical structure of a DBR laser is shown in Fig. 2.6. The DBR are the mirrors of the cavity. The Bragg gratings act as high-

Figure 2.6: Schematic of a DBR laser

reflectivity narrowband mirrors, and therefore, they provide a high reflectivity for one of the cavity modes. Thus, the mirror losses for that particular mode are lower than for the others, resulting in SLM operation.

If the Bragg grating is extended over the whole cavity and it is acting as the feedback mechanism of the laser, the device is called distributed feedback laser. The typical structure of a DFB laser is shown in Fig. 2.7(a). As it can be seen in the figure, the

Figure 2.7: (a) Schematic of a DFB laser; (b) Measured optical spectrum of a DFB at $I_{bias} = 25$ mA. More details of this laser are provided in Chapter 4.

Bragg grating is fabricated in such a way that the index changes periodically along the longitudinal direction of the device. These lasers select, in principle, two different longitudinal modes. However, different approaches, such as the use of an anti-reflection coating in the front facet and a high-reflection coating in the back facet, allow the change from bi-mode to single-mode emission. Fig. 2.7(b) presents the measured optical spectrum of a commercial DFB laser. The laser exhibits a SMSR larger than 45 dB. In this spectrum, the emission linewidth is expected to be lower than 1 MHz. However, as the resolution of the employed optical spectrum analyser (OSA) is much higher (6.25 GHz), the apparent linewidth is given by the OSA filter.

Discrete mode lasers

The discrete mode laser is a type of edge-emitting SLM laser that was recently developed by the Dublin City University (DCU) and Eblana Photonics [2, 31]. The DML technology is based on the classic FP architecture. However, single-mode operation is achieved by introducing several perturbations of the refractive index in the ridge waveguide. A schematic view of a DML is presented in Fig. 2.8. Using this approach, the multiple longitudinal modes characteristic of a FP cavity are almost suppressed, enhancing just a single longitudinal mode of the cavity. High SMSRs can be achieved using this technology.

Figure 2.8: Schematic view of the DML structure. Top: SEM image of the etched feature pattern. Figure extracted from [2]

It is important to mention that the etched waveguide is not working as in an usual DFB. In this case, the etched features are used as the wavelength selector and not as the primary source of optical feedback. The spacing, the depth and the slot number are key parameters to operate these devices as SLM. An example of the optical spectrum of a commercial DML is provided in Fig. 2.9. As it can be clearly seen in this figure, although

Figure 2.9: Measured optical spectrum of a DML at $I_{bias} = 25$ mA. More details of this laser are provided in Chapter 4.

the modes of the FP cavity are present in the spectrum, all of them are highly suppressed ($\text{SMSR} > 30 \text{ dB}$) in comparison with the main emitting mode.

Vertical-cavity surface-emitting lasers

The vertical-cavity surface-emitting lasers (VCSELs) are laser diodes in which the resonant cavity is perpendicular to the active layer. The light in these devices is emitted from the top or from bottom in contrast to the edge-emitting lasers (such as DFB or DML). A schematics of a VCSEL is shown in Fig 2.10. The active region is sandwiched between

Figure 2.10: Schematic representation of a VCSEL

two DBRs. The combined effect of the high frequency separation between longitudinal modes (due to the small size of the cavity) and the optical bandwidth of the gain selects a single longitudinal mode. Due to the small size of the cavity, the modal gain is low. For this reason, the mirrors of the cavity are high-reflectivity DBRs. This reduces the mirror losses, confining the light within the cavity. The output power of a VCSEL is usually low but the low energy consumption and the threshold current are also low.

Figure 2.11: (a) Measured optical spectrum of a 1300 nm VCSEL. Panel (b) is a zoom of (a). More details of this device can be found in Chapter 4.

An example of the optical spectrum of a commercial VCSEL is presented in Fig 2.11. As it can be clearly seen in Fig 2.11(a), the amplitude of the main emitting mode is several orders of magnitude higher than the side modes ($\text{SMSR} = 57 \text{ dB}$). Although it cannot be seen in Fig 2.11(b), due to the limited resolution of the OSA, an additional orthogonally polarised mode with lower amplitude is expected in the optical spectrum. An in-depth analysis of these devices can be found in [32, 33].

2.2 Gain-switching

Optical pulse generation in semiconductor lasers can be achieved by multiple techniques. One of the simplest techniques is gain-switching. This technique has been extensively used to generate narrow optical pulses from semiconductor lasers [20, 22, 34, 35]. The antecedents of this technique will be reviewed in Chapter 5.

Figure 2.12: (a) Result of simulations of the current (black), carrier density (red) and photon density (blue) during the turn-on process; (b) Enlarged view during the first spike of the relaxation oscillations

In order to understand the underlying physics behind GS, it is necessary to describe the switch-on process. Fig. 2.12 shows the results of a numerical simulation of the behaviour of the current, carrier and photon density during the turn-on of the laser. The equations will be described in Chapter 3. The applied current is current step, i.e, the total current is $I(t) = I_{off} + I_{on}\Delta rect(t)$, where I_{off} is the low level of the current, I_{on} is the amplitude of the step signal and $rect(t)$ is an instantaneous step increment.

If a step current from below to above threshold is applied, the carrier density increases less abruptly, and at given time, it equals its threshold value (dotted line in Fig. 2.12). The output power starts increasing at this point. Later, when the stimulated recombination rate is high enough, the carrier density starts decreasing as the carriers are depleted by stimulated recombination, and the optical pulse is almost extinguished. However, as the applied current is above threshold, the carrier density increases again, yielding an oscillatory process until the steady state is reached. During this transient process, both the carrier density and the output power oscillate. These oscillations are called relaxation oscillations.

The standard gain-switching approach in semiconductor lasers consists in driving the laser with a superposition of two different types of signals: a bias current and a microwave/RF signal, generally with a sinusoidal profile [3, 34]. A representation of this process is shown in Fig. 2.13. When the laser is biased close to I_{th} and it is modulated by a RF signal with a large amplitude, it is possible to generate narrow optical pulses, as the carrier density is being drastically modulated below and above threshold. The idea underlying this technique is exciting the laser only during the time of the initial peak, suppressing the electrical excitation during the relaxation oscillations [36]. This can be easily obtained when the period of the driving current is short, i.e, for high frequency.

The reader can find more information about pulse generation by GS in semiconductor lasers in [36, 37].

Figure 2.13: Results of simulations at 5 GHz of the current (black), carrier density (red) and photon density (blue) in GS operation

2.3 Optical injection

External optical injection in semiconductor lasers has attracted a lot of attention in the last decades, as it is a method to improve the quality of the emission using a simple and flexible scheme [38–41]. OI consists of injecting external light from a laser, which is called master laser (ML), in the cavity of another laser (commonly known as slave laser or SL). The effect of the injection causes a change in the internal field of the slave laser, giving rise to different emission regimes. This configuration can be implemented via free-space optics with lenses, or by using a fiber-based circuit where the two involved lasers are connected by an optical circulator. A schematic diagram of the fiber based approach is presented in Fig. 2.14.

Figure 2.14: Schematics of the configuration for the OI of a laser diode: ML: master laser, SL: slave laser

The OI system can be described by two primary parameters: the injected power (P_{inj}) into the SL and the detuning ($\delta\nu = \nu_{ML} - \nu_{SL}$) between the injected tone and the nominal frequency of the laser in free-running regime. In the ideal OI conditions, i.e., the emission wavelength of the SL is pulled towards the ML wavelength and the SL emission inherits the same characteristics of the ML, such as the phase noise. This transference process is called injection locking. More information about this technique is provided in [42]. Some features of the laser emission, such as the modulation response [43–45], the phase noise [46, 47] and the emission linewidth [3] can be improved using OI.

Fig. 2.15(a) shows an example of the linewidth reduction due to the OI. In the example

Figure 2.15: (a) Measured RF spectra of the delayed self-heterodyne signal of a DFB laser for $I_{bias} = 30$ mA without (blue) and with OI (red). More details are provided in Chapter 5. (b) Simulated RIN spectra for $I_{bias} = 20$ mA without (blue) and with OI (red); external light is injected with $P_{inj} = 12$ dBm and $\delta\nu = 5.4$ GHz

the measured linewidth of the DFB decreases from 600 to 70 kHz after being injected. More details can be found in Chapter 5. The linewidth was measured using the technique called delayed self-heterodyne (DSH), described in Section 2.5. The simulated RIN spectra of a laser is illustrated in Fig. 2.15(b) without and with injection. These simulations have been performed using the model described in Chapter 3. The reader can find more information about this topic in Chapters 3 and 6. As it is illustrated in Fig. 2.15(b), the use of this technique leads to an increase of the relaxation oscillation frequency, and hence the modulation bandwidth [40].

The effect of OI in semiconductor lasers operated in CW regime has been extensively analysed, paying special attention to the non-linear dynamic regimes that can be induced [40, 48]. Examples of these dynamics are self-pulsations, four-wave mixing and chaotic behaviour.

2.4 Optical frequency combs

2.4.1 Introduction

An optical frequency comb consists of an equally spaced group of optical tones (also known as lines, peaks or carriers). These optical tones are coherent, i.e., they are highly phase correlated. In general terms, an ideal OFC arises from a train of narrow coherent pulses whose correspondence in the frequency domain by means of the Fourier transform is a group of mutually coherent optical carriers.

The origin of the OFCs can be attributed to the investigation in this field of the research group of Theodor W. Hänsch by the end of the 70s. OFC generation was initially associated to mode-locked lasers, as the first results in OFC generation were obtained by his group in 1977 [49] in mode-locked femtosecond dye lasers. This research group has become the spearhead in OFC generation demonstrating that stable optical pulses, equally spaced in time and generated by a mode-locked laser represent a group of tones equally spaced in the spectral domain. Since this major breakthrough, OFCs generated by mode-locked lasers were mostly employed in high-precision spectroscopic measurements. In the meantime, the photonic community efforts led to develop ultrafast lasers, improving the

stability and decreasing the pulse width of the optical pulses [50], until the first Ti:Sapphire lasers appeared at the 80s decade. These lasers were capable of generating femtosecond optical pulses.

Since the commercialisation of these devices, the generation of ultra-short optical pulses became more affordable and feasible, allowing the use of ultrafast lasers for OFC generation. Udem et al. [51] proved the viability of using a femtosecond-laser as a frequency comb synthesizer, reporting OFC generation in mode-locked lasers with a bandwidth of 20 THz. The first absolute frequency measurement was performed in 2000 [52], using a mode-locked based OFC. Comparing with the Cesium clock of the Bureau National de Metrologie, Niering et. al reported for the first time the precise and accurate measurement of the frequency of the two-photon 1S-2S resonance with an uncertainty of 1.9×10^{-14} , improving the accuracy previously achieved by any other optical measurement. This event is considered as the first experiment in high-precision metrology using an OFC. John L. Hall and Theodor W. Hänsch were awarded in 2005 with the Nobel Prize in physics for their pioneering and jointly contributions on OFCs for spectroscopic applications. In recent years, the use of OFCs has spread in other photonic fields. Although the first results in OFC generation were obtained with mode-locked lasers, now several methods can be used to obtain high-quality OFCs at different frequencies.

Although high-resolution metrology [7] was initially the main application of the OFCs, during the recent decades OFCs have found application in other photonics fields. RF photonics [8] and optical communications [9,10] have become areas of interest for the generation of OFCs at high frequencies. More recently, the OFG generation at low frequencies has attracted special attention for spectroscopic applications, specially the dual-comb architecture [53–55].

2.4.2 Comb operation

An ideal OFC can be equivalently described in the time domain as a train of periodic and coherent optical pulses and in the spectral domain as a series of equally spaced spectral components which intensity profile is related with the pulse shape. The relationship between the two pictures is given by the Fourier transform. Fig. 2.16 shows schematically the correspondence between the two descriptions.

In the frequency domain, the frequency of each tone f_n can be expressed as:

$$f_n = f_0 + n f_r, \quad (2.8)$$

where f_r is the frequency separation, i.e., the repetition rate of the pulse train, and f_0 is the carrier envelope frequency offset. Consistently with the duality of this ideal description, the spectral width of the OFC is just the reciprocal of the pulse width.

The carrier envelope frequency offset, f_0 is due to the difference between the phase and group velocity. Though not relevant for the objectives of this thesis, its determination has been very important for applications requiring the accurate knowledge of the absolute value of the frequency. Several methods [16, 56–58] have been used for finding it. The best-known is the f-2f self-referencing method [56]. When the optical spectrum is broader than an octave in frequency, i.e, the highest frequency of the OFC is twice the value of the lowest one, the self-referencing method is greatly simplified. It consists of filtering one line

Figure 2.16: Train of optical pulses and the corresponding OFC

of the OFCs at a frequency f_n (Eq. (2.8)) and then, with the help of a non-linear crystal, doubling the frequency of this tone. If the OFC covers a full optical octave, another tone with a frequency $f_{2n} = f_0 + 2nf_r$ will interfere with the doubled-frequency tone in a photodetector, resulting in a beat tone centred at f_0 .

Apart from the repetition rate and the spectral width, relevant characteristics of a real OFC for applications are also the spectral flatness and the optical carrier-to-noise ratio. Some applications require relatively low repetition frequency (50 to 500 MHz), whereas others require higher frequencies (typically 5 to 20 GHz). The spectral width is usually defined at 3, 10 and even 20 dB, depending on the application. In simple terms the CNR is the ratio between the amplitude of the tones and the noise level and it is a very important parameter for the characterisation of the quality of an OFC.

2.4.3 OFC generation from semiconductor lasers

Different approaches can be applied in order to generate OFCs from semiconductor lasers. The most commonly used are mode-locking, electro-optic modulation, gain-switching and micro-ring resonators [59]. In the following, the more standard methods are summarised.

Mode-locking

As it was previously mentioned, the first OFCs were generated in mode-locked lasers. Mode-locking is a technique that has been extensively used to produce optical pulses in different types of semiconductor lasers [58, 60–64].

In a conventional FP cavity, the longitudinal modes oscillate at its frequency without any phase correlation. Mode-locking consists of inducing a periodical phase correlation between tones so that all modes interfere constructively at the same time, producing optical pulses. The tones of the generated OFCs are spaced by the inverse of time between optical pulses (t_p), which is given by the round-trip time of the cavity:

$$t_p = \frac{2d}{c}, \quad (2.9)$$

Figure 2.17: Schematic diagram of mode-locking principle: (a) each longitudinal mode oscillates without phase correlation; (b) The phase of each mode is locked, i.e, mode-locked operation.

where d is the length of the cavity. Fig. 2.17 illustrates the principle of mode-locking: In panel (a) each longitudinal mode oscillates without a phase correlation, and therefore their superposition does not result in a train of pulses. However, in panel (b) the phase of each mode is correlated and narrow optical pulses are obtained when these cavity modes interfere.

Mode-locking can be achieved by several methods, including active, passive or hybrid techniques [65]. The OFCs generated by mode-locking show really good characteristics in terms of spectral width, as they can be spanned over tens of nanometres with high flatness. However, this technique presents some inconveniences. The main limitation of the method is the lack of flexibility in the repetition rate tunability. As it is determined by the cavity, the separation of the tones cannot be changed. More information about this technique is provided in [36, 37, 58].

Electro-optic modulation

In this technique, the output light from a laser operated in CW regime is modulated by one or several cascaded electro-optic phase or/and intensity modulators, driven by RF generators. This technique has been extensively used to generate OFCs from semiconductor lasers [66–68]. An example of a simple schematics of electro-optic modulation is shown in Fig. 2.18.

The resulting signal is an OFC centred in the wavelength of the semiconductor laser. The frequency spacing is determined by the repetition rate of the electrical input signal, provided by the RF modulators. This confers the method a high flexibility in the selection of the repetition rate. This modulation scheme has been applied in lasers since the 60s, and it has been extensively used during the following decades for the generation of short optical pulses [69]. However, this technique shows some characteristics that can limit the quality of the OFCs. The combs generated by this technique do not show, in principle, a large value of spectral width, exhibiting an asymmetrical shape with suppressed tones and a poor flatness. Different schemes were proposed in order to solve the issues related to the shape and width of the combs [8]. Flat-top OFCs with high bandwidth [70, 71]

Figure 2.18: Schematic diagram of electro-optic modulation technique for OFC generation, LD: laser diode, EOM: electro-optic modulator

can be generated using more complex architectures. Flexibility, low noise and potential for integration [72] are some of the main advantages of this technique. Although flat-top OFCs can be generated using more complex schemes, they require the use of cascaded modulators, implying an increase in the power consumption and higher insertion losses.

Gain-switching

Gain-switching has proven to be a convenient method for generating OFCs [10] with different applications due to its inherent characteristics, such as simplicity and the flexibility in the repetition rate selection. If the optical pulses are highly phase-correlated in the temporal domain, the corresponding optical spectra are OFCs.

Figure 2.19: (a) Experimental train of pulses from a DML driven at $f_R = 5$ GHz, $I_{bias} = 30$ mA and $V_{RF} = 1$; (b) The corresponding optical spectrum in the same driving conditions. More information about this spectrum is provided in Chapter 5.

As an illustrative example Fig. 2.19(a) shows a train of optical pulses, generated by GS with a repetition period of 0.2 ns ($f_R = 5$ GHz), which is equivalent in the spectral domain to a high-quality OFC (Fig. 2.19(b)). More details of this figure can be found in Chapter 5. As it was previously explained, this technique has been extensively used for optical pulse generation in semiconductor lasers but the inherent frequency comb character of the optical pulses was not investigated until recently, due to the lack of high-resolution optical spectrum analysers.

It was not until 2009 when OFC generation was reported for the first time in a gain-switched semiconductor laser by Anandarajah et. al [31]. The initial applications of OFCs

generated by GS in semiconductor lasers were optical communications [73–75] and Radio over Fiber [76, 77]. More recently, OFCs were acknowledged to be a suitable resource in optical spectroscopy, specially in dual-comb architectures [5, 53, 78]. A review of the state of the art in OFC generation by gain-switched semiconductor lasers can be found in Chapter 5.

The use of external OI allows to improve the quality of the optical pulses generated by GS and also the quality of the resulting OFCs. Regarding short optical pulse generation, it has been demonstrated that OI reduces detrimental features of the pulses such as phase noise, timing jitter and frequency chirp, among others [3, 79, 80]. The physical origin of the high timing jittered pulses is attributed to random fluctuations of the starting point of the optical pulses, as these pulses are built-up from spontaneous emission. However, when OI is applied [80], the injected laser couples with the external light, acting as a seed for the optical pulse and reducing the pulse timing jitter.

Figure 2.20: Experimental train of pulses from a GS-DFB: (a) No injection; (b) With OI. This figure is extracted from [3].

A measured train of pulses from a gain-switched DFB is shown in Fig. 2.20. When there is no OI, the temporal fluctuations of the pulse are high due to the high timing jitter (see Fig. 2.20(a)). However, when OI is applied, an apparent reduction of the timing and amplitude jitter is observed. This effect has been previously reported in several publications [3, 80, 81]. The effect of the external injection in the quality of a comb is illustrated in Fig. 2.21. The figure shows the spectrum of a non-injected GS-DFB laser (panel (a)) and the spectrum of the same laser under OI (panel (b)). As it can be seen, when there is no OI, comb lines do not appear. However, when OI is switched on, the injected laser couples with the external light and the spectral coherence improves, generating a high-quality OFC.

Figure 2.21: Optical spectra of a gain-switched DFB laser (a) without external injection and (b) with external injection. This figure was extracted from [3].

In terms of applications, most of the works related to OI GS-OFCs are focused on optical communications. In [82], it is demonstrated that broad OFCs can be generated by a tunable externally-injected FP laser, emitting along all the C-band and beyond it, which is highly convenient for optical networks. An OFC generator, based on GS and OI and integrated into a photonic integrated circuit (PIC), is presented in [83,84]. This device is capable of generating OFCs with a large separation between tones and with a really good performance in terms of spectral width and low noise.

2.5 Homodyne and heterodyne detection

The homodyne/heterodyne detection techniques have been employed to analyse the characteristics of an optical signal by beating it with a local oscillator. In these techniques, two optical fields from different sources are combined and the resultant signal is analysed in the RF spectra. The beating of two optical signals can be described as the interference of two optical fields, $E_1(t)$ and $E_2(t)$:

$$E_1(t) = \sqrt{P_1(t)}e^{j(\omega_1 t + \phi_1(t))}; E_2(t) = \sqrt{P_2(t)}e^{j(\omega_2 t + \phi_2(t))} \quad (2.10)$$

The squared magnitudes of both fields are their correspondent optical powers, as both fields are suitable scaled ($P_i(t) = |E_i(t)|^2$ for $i = 1,2$). The optical angular frequency is labelled as ω_i and the phase of each field is $\phi_i(t)$. Assuming that the polarisation state of both fields is the same, the two optical fields are mixed in a photodetector and the detected photocurrent is proportional to the temporal mean square value of the total optical field:

$$i_{ph}(t) = R [E_1(t) + E_2(t)]^{ms}, \quad (2.11)$$

where R is the detector responsivity and "ms" indicates the mean square value with respect to the optical frequencies. Therefore, the total current can be expressed as:

$$i_{ph}(t) = R \left[P_1(t) + P_2(t) + 2\sqrt{P_1(t)P_2(t)}\cos((\omega_1 - \omega_2)t + (\phi_1(t) - \phi_2(t))) \right] \quad (2.12)$$

An electrical spectrum analyser is capable of obtaining the power spectral density (PSD) of the detected photocurrent. In the spectral domain, the power spectrum of i_{ph} can be separated into two different contributions: the direct detection term of both optical fields $S_d(f)$, i.e, the addition of the Fourier transforms of $P_1(t)$ and $P_2(t)$, namely S_1 and S_2 , respectively; and the mixing product [85]:

$$S(\omega)_{RF} = R^2 S_d(f) + 2R^2 (S_1(\omega_1) \otimes S_2(-\omega_2)) \quad (2.13)$$

The product marked as $\otimes$ is the convolution of these magnitudes. The main difference between the homodyne and the heterodyne detection is the frequency of the two optical fields, which is the same for the homodyne detection and different for heterodyne detection.

Homo/heterodyne detection can be applied to the characterisation of the laser linewidth by using a single laser source (self-homo/ self-heterodyne). Although the self-homodyne approach can be used for this purpose, this technique is limited to symmetrical lineshapes. The delayed self-heterodyne is a heterodyne method that has been extensively used to ob-

tain the linewidth of an unmodulated laser [4, 86, 87]. A schematic diagram of the DSH is shown in the left part of Fig 2.22. The light of the laser is split into two branches. The frequency of the light in one arm is shifted f_{shift} by means of a frequency shifter. In the other arm, the light passes through a fiber spool with a length longer than the coherence length of the laser. Then, both paths are coupled together again and beat in a photodetector. In this way, the beating between both signals is incoherent. The resultant signal is analysed by an electrical spectrum analyser. As it is illustrated in the right part of Fig 2.22, $S_1(f_0)$ and $S_2(f_0 - f_{shift})$ are the PSDs in each optical path. Both PSDs are convolved and the result appears in the ESA centred at f_{shift} .

Figure 2.22: Left: Schematic diagram of the DSH technique: LD: laser diode, PD: photodiode, ESA: electrical spectrum analyser. Right: Fundamental of the DSH technique, reproduced from [4].

Dual-comb architecture

The dual-comb architecture is a multi-heterodyne arrangement, in which two OFCs with slightly different repetition rates are mixed in a photodetector, thus mapping the optical spectrum into the electrical domain [5, 88–90]. The separation of the tones for one OFC is given by f_R while for the other is $f_R + \Delta f_R$, where Δf_R is very small in comparison with f_R . The result of the beating is a series of electrical combs composed of the beat tones between the teeth of each comb.

Figure 2.23: Left: Schematic diagram of the dual-comb fundamentals, based on [5].

A schematic of the dual-comb fundamentals is shown in Fig 2.23. The tone numbers

are labelled as n for the OFC-1 and m for the OFC-2. In the dual-comb scheme, the information of the optical spectra can be retrieved from the electrical spectra. The beating between optical tones with the same tone number provide information corresponding to the optical spectra but located in the RF domain at frequencies given by multiples of the difference between the repetition rates Δf_R .

Figure 2.24: Three different dual-comb architectures: (a) Free-running; (b) Mutually-coherent; (c) Fully referenced. This figure has been extracted from [5].

One of the major issues of the dual-comb architecture is phase referencing. Three approaches are illustrated in Fig. 2.24 according to [5]. When two independent OFCs (Fig. 2.24(a)) are used, both of them have their own central wavelength as well as its own phase fluctuations. The beat signal of these independent sources is completely incoherent, and the result in the analyser is a continuous spectrum without discernible tones. In order to sort out this issue, some techniques can be employed to fix the phase of both OFCs. If both combs are made mutually coherent (Fig. 2.24(b)), clear electrical combs appear in the RF spectra. In this approach, high spectral resolution can be achieved and long-term averaging is possible. If both combs are fully referenced (Fig. 2.24(c)), the accuracy in the measurements, as well as the resolution, can be improved.

This technique has been extensively used in molecular spectroscopy [5, 53, 91]. The fundamentals of the dual-comb spectroscopy are illustrated in (Fig. 2.25(a)). In spectroscopy, this technique is used to down-convert the information of the absorption spectra of a sample from the optical to the electrical domain, thus allowing the analysis of the result with high spectral resolution. This technique can be implemented following two approaches. In the asymmetric configuration (Fig. 2.25(b), upper row), one of the combs goes through the sample, and then, both combs are mixed in the photodetector. This approach yields information on the magnitude and phase of the gas sample. If the two OFCs are mixed before the sample, the configuration is known as the "symmetrical" and it allows to retrieve only the information of the absorption spectra [5]. This approach shares some similarities with the well-known Fourier transform spectroscopy (FTS). In contrast to this technique, the dual-comb technique can retrieve the spectral information of gas faster and with higher accuracy than FTS, as FTS relies on mechanical components that

Figure 2.25: (a) Schematic diagram of the dual-comb spectroscopy; (b) Different configurations in dual-comb spectroscopy: (upper row) asymmetric configuration and (lower row) symmetric configuration. This figure has been extracted from [5].

limit the speed of the measurement [5].

Chapter 3

Dynamic response of a semiconductor laser

Introduction

The dynamic behaviour of the carrier and the photon densities in semiconductor lasers can be described with different theoretical approaches. In this work, the so-called rate equation model (REM) [29,30,92] was chosen because of its simplicity and its easy implementation. The model was formulated for the first time in 1960 and since then, it has been extensively used for semiconductor lasers in hundreds of publications. Some examples can be found in [74,93–100].

The purpose of this chapter is to review the fundamental equations that describe the behaviour of single-mode semiconductor lasers in the framework of the REM, and to explain the modifications to these equations that have been included in this work, in order to consider properly the contribution of different noise sources.

Section 3.1 describes the main equations that have been used in this work to simulate the behaviour of the lasers under gain-switching operation. Section 3.2 is devoted to show the main noise contributions of the model as well as the fluctuations that are present due to these noise contributions. As different stochastic noise sources have been included in the model, Section 3.3 describes the numerical method used to solve these stochastic equations. The changes that have been performed in the equations to replace the usual variables of this model by dimensionless parameters are presented in Section 3.4.

3.1 Rate equation model

The rate equation model provides a clear description of the most relevant features of semiconductor lasers in static and dynamic regimes, assuming uniformity of the carrier and photon densities in the laser cavity. Two alternative approaches have been followed for expressing the set of rate equations. One approach uses two-rate equations describing the dynamics of the carrier density and the complex optical field, taking into account its real and imaginary part [95,99]. The other approach considers three rate equations, one for the carrier density, another one for the photon density (related with the magnitude of the optical field) and an additional one for the phase of the optical field [92,98,100].

In this chapter, the first approach is used to introduce the REM and the second one is employed for the calculations. Assuming that the current, photon and carrier densities along the laser cavity are uniform and that the laser emission is single mode, the dynamics of carrier density ($N(t)$) and optical field ($E(t)$) can be described as [92]:

$$\boxed{\frac{dN(t)}{dt} = \frac{I(t)}{qV_{act}} - R(N) - \frac{\mathcal{G}(N)}{1 + \epsilon|E(t)|^2}|E(t)|^2 + F_N(t)} \quad (3.1)$$

and

$$\boxed{\frac{dE(t)}{dt} = \frac{1}{2}(1 + i\alpha) \left[\Gamma \frac{\mathcal{G}(N)}{1 + \epsilon|E(t)|^2} - \frac{1}{\tau_p} \right] E(t) + E_{sp}(t)} \quad (3.2)$$

In these equations α is the linewidth enhancement factor or alpha parameter, τ_p is the photon life-time, N_{tr} is the carrier density at transparency, Γ is the optical confinement factor, V_{act} is the active volume and ϵ the gain saturation coefficient. Unlike [74, 99], the gain saturation effect has been included in this thesis since it is a relevant effect affecting the dynamics of the laser, as it will be commented later in Chapter 6.

In this work, the following linear dependence of the material gain $\mathcal{G}(N)$ on the carrier density has been assumed:

$$\mathcal{G}(N) = v_g \frac{d\mathcal{G}}{dN}(N(t) - N_{tr}), \quad (3.3)$$

where v_g is the group velocity and $d\mathcal{G}/dN$ is the differential gain. The carrier recombination rate ($R(N)$) has been expressed in the usual way:

$$R(N) = AN(t) + BN(t)^2 + CN(t)^3, \quad (3.4)$$

where A , B and C are the non-radiative, spontaneous, and Auger recombination coefficients, respectively.

When a superposition of a bias current with a sinusoidal signal is employed, the injected current $I(t)$ is expressed as:

$$I(t) = I_{bias} + C_{loss}(f_R) \frac{2\sqrt{2}V_{RF}}{Z_0 + Z_l} \sin(2\pi f_R t), \quad (3.5)$$

where I_{bias} is the bias current, f_R the repetition frequency, $C_{loss}(f_R)$ a loss coefficient accounting for the frequency dependent electrical cable attenuation, V_{RF} the root-mean-square (RMS) voltage value of the generator signal applied to an ideal 50Ω load, $Z_0 = 50 \Omega$ the generator output impedance, and Z_l the impedance of the laser module. The high frequency capacitive effects have been neglected.

Spontaneous emission is included via the term $E_{sp}(t)$, which is formulated later. The term $F_N(t)$ takes into account the shot noise character of the injection current and carrier recombination. The optical field $E(t)$ and the output power coupled to the fiber are related by [29]:

$$P(t) = \eta_f \frac{hf_0 V_c}{\tau_p} |E(t)|^2 \quad (3.6)$$

where η_f is the in-fiber external quantum efficiency given by the product of the differential

quantum efficiency and the coupling efficiency (accounting for the power balance between both facets), h is the Planck's constant, f_0 is the emission frequency and $V_c = V_{act}/\Gamma$ is the volume of the optical mode. It is important to mention that the squared magnitude of the optical field $|E(t)|^2$ is written in units of cm^{-3} , instead of its usual representation in V^2m^{-2} , as that it is normalized to the photon density $|E(t)|^2 \equiv S(t)$.

In the case of external optical injection, Eq. (3.2) has been modified as [97]:

$$\boxed{\frac{dE(t)}{dt} = \frac{1}{2}(1 + i\alpha) \left[\Gamma \frac{\mathcal{G}(N)}{1 + \epsilon|E(t)|^2} - \frac{1}{\tau_p} \right] E(t) + k_c E_{inj}(t) e^{-i2\pi\delta\nu't} + E_{sp}(t)} \quad (3.7)$$

where E_{inj} is the electric field of the OI, $\delta\nu'$ is the detuning of the injected laser field with respect to the frequency of the considered laser at threshold and k_c is the coupling coefficient.

The mathematical description of the noise sources is highly complex as the origin of these noises relies on quantum phenomena [101, 102]. To simplify the mathematical description of these terms, Harder et. al [103] developed a model in which the laser noise is assumed to originate from shot noise, and employed Langevin noise terms proportional to Dirac's Deltas to describe them. In this way they are presented in Eqs. (3.1) and (3.2). The usual approach to present these terms is to write Eqs. (3.1) and (3.2) by means of the photon density $S(t)$ and the phase of the field $\phi(t)$ [30, 92], which is related with the frequency chirp ($\delta f(t)$) through the following relationship:

$$\delta f(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt}, \quad (3.8)$$

where $\delta f(t)$ expresses the difference between the instantaneous frequency and the emission frequency at threshold. The spontaneous emission noise is the dominant noise source in laser diodes [92]. The term $E_{sp}(t)$ in Eq. (3.2) accounts for it. The electric field $E(t)$ in Eq. (3.7) has been substituted by [92]:

$$E(t) = \sqrt{S(t)} e^{j\phi(t)}, \quad (3.9)$$

where $S(t)$ and $\phi(t)$ verify:

$$\frac{dS(t)}{dt} = 2\Re \left\{ \frac{dE(t)}{dt} E(t)^* \right\} \quad \text{and} \quad \frac{d\phi(t)}{dt} = \frac{1}{S(t)} \Im \left\{ \frac{dE(t)}{dt} E(t)^* \right\}, \quad (3.10)$$

then, the complex Eq. (3.7) can be written in terms of the real functions of $S(t)$ and $\phi(t)$ as:

$$\frac{dS(t)}{dt} = \left[\Gamma \frac{\mathcal{G}(N)}{1 + \epsilon S(t)} - \frac{1}{\tau_p} \right] S(t) + 2k_c \Re \left\{ E_{inj}(t) E(t)^* e^{-i2\pi\delta\nu't} \right\} + 2\Re \{ E_{sp}(t) E(t)^* \} \quad (3.11)$$

and

$$\frac{d\phi(t)}{dt} = \frac{\alpha}{2} \left[\Gamma \mathcal{G}(N) - \frac{1}{\tau_p} \right] + \frac{k_c}{S(t)} \Im \left\{ E_{inj}(t) E(t)^* e^{-i2\pi\delta\nu't} \right\} + \frac{1}{S(t)} \Im \{ E_{sp}(t) E(t)^* \} \quad (3.12)$$

where $*$ indicates the complex conjugate, $\Re\{\dots\}$ the real part and $\Im\{\dots\}$ the imagi-

nary part. The spontaneous emission noise can be considered as a white noise source (non-frequency dependent), as it extends over a large frequency range, with the following correlation properties:

$$\langle E_{sp}(t)E_{sp}^*(t') \rangle = \Gamma\beta\mathcal{R}\delta(t-t') \quad (3.13)$$

where β is the fraction of spontaneous emission that is coupled to the lasing mode ($0 < \beta \leq 1$) and $\mathcal{R}$ is the spontaneous recombination rate, $\mathcal{R} = BN(t)^2$. $\langle \dots \rangle$ indicates the average with respect to time because stationary noise processes are considered. With Eq. (3.13) it can be demonstrated [92] that:

$$\langle E_{sp}(t)E^*(t) \rangle = \Gamma\beta\frac{\mathcal{R}}{2}. \quad (3.14)$$

On the other hand, $E_{sp}(t)$ is expressed as a complex white Gaussian random process [97] as:

$$E_{sp}(t) = \sqrt{\frac{\Gamma\beta\mathcal{R}}{2}} (\xi_1(t) + i\xi_2(t)), \quad (3.15)$$

where $\langle \xi_i(t)\xi_j(t') \rangle = \delta_{i,j}\delta(t-t')$. Then it can be found that

$$E^*(t)E_{sp}(t) = \sqrt{\frac{\Gamma\beta S(t)\mathcal{R}}{2}} (\xi_S(t) + i\xi_\phi(t)) + \Gamma\beta\frac{\mathcal{R}}{2}, \quad (3.16)$$

where $\xi_S(t) = \xi_1(t)\cos\phi + \xi_2(t)\sin\phi$ and $\xi_\phi(t) = \xi_2(t)\cos\phi - \xi_1(t)\sin\phi$. Substituting this expression in Eqs. (3.11) and (3.12) and using $E_{inj}(t) = |E_{inj}(t)|e^{i\phi_{inj}(t)}$ the following expressions are obtained:

$$\boxed{\begin{aligned} \frac{dS(t)}{dt} &= \left[\Gamma \frac{\mathcal{G}(N)}{1+\epsilon S(t)} - \frac{1}{\tau_p} \right] S + 2k_c\sqrt{S(t)}|E_{inj}(t)|\cos(\phi(t) - \phi_{inj}(t) + 2\pi\delta\nu't) \\ &\quad + \Gamma\beta\mathcal{R} + F_S(t) \end{aligned}} \quad (3.17)$$

$$\boxed{\begin{aligned} 2\pi\delta f(t) &= \frac{d\phi(t)}{dt} = \frac{\alpha}{2} \left[\Gamma\mathcal{G}(N) - \frac{1}{\tau_p} \right] - \frac{k_c|E_{inj}(t)|}{\sqrt{S(t)}}\sin(\phi(t) - \phi_{inj}(t) + 2\pi\delta\nu't) \\ &\quad + F_\phi(t) \end{aligned}} \quad (3.18)$$

where

$$F_S(t) = \sqrt{2\beta\Gamma\mathcal{R}S(t)}\xi_S(t) \quad \text{and} \quad F_\phi(t) = \sqrt{\frac{\beta\Gamma\mathcal{R}}{2S(t)}}\xi_\phi(t). \quad (3.19)$$

The system formed by Eqs. (3.17) and (3.18) is completed with Eq. (3.1)

$$\boxed{\frac{dN(t)}{dt} = \frac{I(t)}{qV_{act}} - R(N) - \frac{\mathcal{G}(N)}{1+\epsilon S(t)}S(t) + F_N(t).} \quad (3.20)$$

The correlation properties of the different fluctuation terms can be written as [30, 100]:

$$\langle F_i(t) \rangle = 0 \quad \text{and} \quad \langle F_i(t)F_j(t') \rangle = 2D_{ij}\delta(t-t') \quad (3.21)$$

where i and j refer to the variables $S(t)$, $\phi(t)$ and $N(t)$. The non-vanishing diffusion

coefficients D_{ij} are the following

$$D_{SS}(t) = \beta \Gamma B N(t)^2 S(t), \quad (3.22)$$

$$D_{\phi\phi}(t) = \frac{\beta \Gamma B N(t)^2}{4S(t)}, \quad (3.23)$$

$$D_{NN}(t) = \frac{\beta B N(t)^2 S(t)}{\Gamma} + \frac{1}{V_{act}} \left(R(N) + \frac{I(t)}{qV_{act}} \right), \quad (3.24)$$

$$D_{SN}(t) = -\beta B N(t)^2 S(t), \quad (3.25)$$

where it is used $\mathcal{R} = BN(t)^2$. It should be noticed that the fluctuations of the photon and carrier density are correlated. This is the standard model [30, 100], where the shot noise coming from the injection current [104] is taken into account. Some authors consider more correlations but usually the noise in the carriers is neglected.

Additionally, as it is well known [105], when lasers are modulated with low repetition rate, the optical frequency changes due to thermal effects. Basically, as the laser is modulated using a total current $I(t)$, the temperature of the laser changes by simple Joule effect. These variations in the temperature of the laser, induced by the modulating signal, produce a change in the refractive index of the material, which directly affects the phase of the optical field, and therefore, the instantaneous frequency of the laser. Although different approaches were used in the past to model the thermal behaviour of semiconductor lasers in CW and small signal regimes [106–110], none of them was used for gain-switching with a large amplitude signals. In order to account for the induced thermal effects of the large amplitude modulation, the model described below of the dynamic thermal response of the laser has been included for the first time in this thesis.

As the laser is modulated with a large amplitude signal, Eq. (3.18) was changed in order to include the variation of the optical field phase when the temperature of the laser fluctuates because of the large signal modulation. Eq. (3.18) has been modified, yielding:

$$\boxed{\begin{aligned} 2\pi\delta f(t) &= \frac{d\phi(t)}{dt} = \frac{\alpha}{2} \left[\Gamma \mathcal{G}(N) - \frac{1}{\tau_p} \right] + 2\pi \frac{df}{dT} \Delta T(t) \\ &\quad - \frac{k_c |E_{inj}(t)|}{\sqrt{S(t)}} \sin(\phi(t) - \phi_{inj}(t) + 2\pi\delta\nu' t) + F_\phi(t) \end{aligned}} \quad (3.26)$$

where $\frac{df}{dT}$ is the temperature coefficient of the emission frequency and $\Delta T(t)$ is the difference between the temperature of the active region at I_{bias} and at the threshold current (I_{th}).

At the early stages of the development of the model, it was decided to take into account the thermal dynamics inside the laser, by including in Eq. (3.26) an additional term $2\pi \frac{df}{dT} \Delta T(t)$ proportional to the difference between the active region temperature at the operating conditions and at threshold. The dynamics of $\Delta T(t)$ term can be described as:

$$C_{th} \frac{d\Delta T(t)}{dt} = (P_{dis}(t) - P_{dis}(I_{th})) - \frac{\Delta T(t)}{\tau_{th}} \quad (3.27)$$

where $\tau_{th} = C_{th} R_{th}$ is the thermal decay time, R_{th} is the thermal resistance, C_{th} is the

thermal capacitance and P_{dis} is the dissipated power in the active area, that is given by:

$$P_{dis}(t) = V(I)I(t) - P(t), \quad (3.28)$$

where $V(I)$ is the junction voltage of the laser diode, that can be approximated at the usual bias conditions by $V(I) = V_0 + R_s I(t)$, then, Eq. (3.27) can be expressed as:

$$\tau_{th} \frac{d\Delta T}{dt}(t) = R_{th} R_s (I(t)^2 - I_{th}^2) + R_{th} V_0 (I(t) - I_{th}) - R_{th} P(t) - \Delta T(t) \quad (3.29)$$

Eq. (3.29) is an additional equation that has to be added to the three previous rate equations (3.17), (3.18) and (3.20):

3.2 Noise treatment

As the Langevin terms $F_S(t)$ and $F_N(t)$ are cross-correlated, it is impossible to model the noise in the carrier and photon densities with two independent randomly distributed Gaussian generators. In order to obtain the independent fluctuations $F_i(t)$ ($i = N, S, \phi$), that describe their correlation properties, a new fluctuation has been introduced following references [98, 104]:

$$F_Z(t) = \frac{F_S(t)}{\Gamma} + F_N(t), \quad (3.30)$$

with correlation diffusion coefficients:

$$D_{ZZ}(t) = \frac{1}{V_{act}} \left(R(N) + \frac{I(t)}{eV_{act}} \right), \quad (3.31)$$

$$D_{ZS} = 0, \quad (3.32)$$

$$D_{Z\phi} = 0, \quad (3.33)$$

$$D_{ZN}(t) = \frac{1}{V_{act}} \left(R(N) + \frac{I(t)}{eV_{act}} \right). \quad (3.34)$$

As shown in [104], $F_S(t)$ and $F_Z(t)$ are orthogonal, as both variable are not cross-correlated ($D_{ZS} = 0$), so that, it is possible to use three different Gaussian noise generators (ξ_i , $i = S, Z, \phi$). Finally, the system of SDEs formed by Eqs. (3.17), (3.18) and (3.20) can be written as

$$\boxed{\begin{aligned} \frac{dS(t)}{dt} &= \left[\Gamma \frac{\mathcal{G}(N)}{1+\epsilon S(t)} - \frac{1}{\tau_p} \right] S(t) + 2k_c \sqrt{S(t)} |E_{inj}(t)| \cos(\phi(t) - \phi_{inj}(t) + 2\pi\delta\nu't) \\ &\quad + \beta B N(t)^2 + F_S(t), \end{aligned}} \quad (3.35)$$

$$\boxed{\frac{d\phi(t)}{dt} = \frac{\alpha}{2} \left[\Gamma \mathcal{G}(N) - \frac{1}{\tau_p} \right] + 2\pi \frac{df}{dT} \Delta T(t) - \frac{k_c |E_{inj}(t)|}{\sqrt{S(t)}} \sin(\phi(t) - \phi_{inj}(t) + 2\pi\delta\nu't) + F_\phi(t),} \quad (3.36)$$

and

$$\boxed{\frac{dN(t)}{dt} = \frac{I(t)}{qV_{act}} - R(N) - \mathcal{G}(S)(N(t) - N_{tr})S(t) + F_N(t),} \quad (3.37)$$

with

$$F_S(t) = \sqrt{2\beta B\Gamma N(t)^2 S(t)} \xi_S(t), \quad (3.38)$$

$$F_\Phi(t) = \sqrt{\frac{\beta B\Gamma N(t)^2}{2S(t)}} \xi_\Phi(t), \quad (3.39)$$

$$F_Z(t) = \sqrt{\frac{2}{V_{act}} \left(R(N) + \frac{I(t)}{eV_{act}} \right)} \xi_Z(t), \quad (3.40)$$

$$F_N(t) = F_Z(t) - \frac{F_S(t)}{\Gamma}, \quad (3.41)$$

where $\xi_i(t)$ is a white Gaussian noise, verifying $\langle \xi_i(t) \xi_j(t') \rangle = \delta_{i,j} \delta(t - t')$ with $i, j = S, \phi, Z$. The usual practise for solving Eqs. (3.35)-(3.37) [98] is to divide the fluctuation terms by $\sqrt{h}$ with h being the time step and solve them by using a 4th order Runge-Kutta algorithm. Other authors have followed similar approaches [74, 97, 104] although the numerical algorithm that they have used is not well clarified. In any case, the numerical algorithms for deterministic systems of ordinary differential equations (ODEs) cannot be used to solve systems of SDEs as they are not taking into account the random nature of the Gaussian noise [111]. The reason for including in this thesis the correct treatment of the Langevin noise sources is that the detailed description of the experimental behaviour of the lasers under GS operation relies on a proper description of the noise.

3.3 Numerical algorithm

A generic system of SDEs for a vector of dynamical variables $x_i(t)$ (where $i = 1, 2, 3$) can be written as:

$$\frac{dx_i(t)}{dt} = f_i(x, t) + \sum_j g_{ij}(x, t) \xi_j(t), \quad (3.42)$$

where $f_i(x, t)$ are called the *drift* terms, $g_{ij}(x, t)$ are called the *diffusion* terms and $\xi_j(t)$ is a white Gaussian noise, which correlation properties are $\langle \xi_i(t) \xi_j(t') \rangle = \delta_{i,j} \delta(t - t')$.

Among all the approaches that can be used to express SDEs [112, 113], the most commonly used are the Itô interpretation (usually used by mathematicians) and the Stratonovich interpretation (normally used by physicists) [113]. Eq. (3.42) shows an equation written in the Ito interpretation. The set of Eqs. (3.35), (3.36) and (3.37) are expressed as Eq. (3.42), and therefore, they are written in the Ito interpretation. One of the main differences between both approaches is in the way of solving the integral of the product between the diffusion terms and the white Gaussian noise. In the case of the Itô interpretation, the integral is expressed as:

$$\sum_j \int_t^{t+h} g_{ij}(x(s)) \xi_j(s) ds = \sum_j g_{ij}(x(t)) \int_t^{t+h} \xi_j(s) ds, \quad (3.43)$$

whereas in the Stratonovich interpretation, the integral is expressed as:

$$\sum_j \int_t^{t+h} g_{ij}(x(s)) \xi_j(s) ds = \sum_j \frac{g_{ij}(x(t)) + g_{ij}(x(t+h))}{2} \int_t^{t+h} \xi_j(s) ds. \quad (3.44)$$

The so-called Heun's method for SDEs was previously used [113–115] for solving these kind of equations. Basically, this method consist in a 2^{nd} order Runge-Kutta algorithm modified to include stochastic noise sources. When the integral is solved as in Eq. (3.44), it can be seen that the coefficient that is placed on the left hand side of the equation corresponds to the coefficient of a 2^{nd} order Runge-Kutta algorithm. Hence, it can be inferred from Eq. (3.44) that the Heun's method is written in the Stratonovich interpretation. However, the system of SDEs formed by Eqs. (3.35)-(3.37) is expressed in the Itô interpretation (Eq. (3.42)). The *Itô-Stratonovich drift correction* formula has been used to change from one interpretation to the other. In this interpretation, the new *drift* terms $\tilde{f}_i(x, t)$ can be written as:

$$\tilde{f}_i(x, t) = f_i(x, t) - \frac{1}{2} \sum_{k,j} g_{kj}(x, t) \frac{\partial g_{ij}(x, t)}{\partial x_k}, \quad (3.45)$$

then the SDE in the Itô interpretation written in Eq. (3.42) is equivalent to the equation

$$\frac{dx_i(t)}{dt} = \tilde{f}_i(x, t) + \sum_j g_{ij}(x, t) \xi_j(t), \quad (3.46)$$

written in the Stratonovich interpretation.

In this particular case, new drift terms $\tilde{f}_i(x, t)$ for the Stratonovich interpretation of the system of SDEs formed by Eqs. (3.35)-(3.37), were calculated using the Eq. (3.45):

$$\begin{aligned} \tilde{f}_1(x, t) &= f_1(x, t) - \frac{1}{2} \left(g_{11}(x, t) \frac{\partial g_{11}}{\partial S} + g_{31}(x, t) \frac{\partial g_{11}}{\partial N} \right) \\ &= \left[\frac{\Gamma g_0(N(t) - N_{tr})}{1 + \epsilon S(t)} - \frac{1}{\tau_p} \right] S(t) + \beta B N(t) \left(\frac{\Gamma N(t)}{2} + S(t) \right), \end{aligned} \quad (3.47)$$

$$\begin{aligned} \tilde{f}_2(x, t) &= f_2(x, t) \\ &= \frac{\alpha}{2} \left[\Gamma g_0(N(t) - N_{tr}) - \frac{1}{\tau_p} \right] - 2\pi \frac{df}{dT} \Delta T(t) \end{aligned} \quad (3.48)$$

and

$$\begin{aligned} \tilde{f}_3(x, t) &= f_3(x, t) - \frac{1}{2} \left(g_{11}(x, t) \frac{\partial g_{31}}{\partial S} + g_{31}(x, t) \frac{\partial g_{31}}{\partial N} + g_{33}(x, t) \frac{\partial g_{33}}{\partial N} \right) \\ &= \frac{I(t)}{eV_{act}} - R(N) - \frac{g_0 S(t)(N(t) - N_{tr})}{1 + \epsilon S(t)} + \beta B N(t) \left(\frac{N(t)}{2} - \frac{S(t)}{\Gamma} \right) \\ &\quad - \frac{1}{2V_{act}} \frac{\partial R(N)}{\partial N} \end{aligned} \quad (3.49)$$

A generic system of SDEs written in the Stratonovich interpretation with multiplicative noise is considered for a vector of dynamical variables $x_i(t)$, as it is be written in Eq.(3.46). The algorithm is a predictor-corrector method, the first step is to calculate the Euler predictor as

$$\tilde{x}_i(t + h) = x_i(t) + h \tilde{f}_i(x(t), t) + \sqrt{h} \sum_j g_{ij}(x(t), t) u_j(t) \quad (3.50)$$

where h is the time step and $u_j(t)$ an independent set of random Gaussian numbers with zero mean value and variance equal to one. Finally, the solution is calculated using

$$\begin{aligned} x_i(t+h) &= x_i(t) + \frac{h}{2} \left[\tilde{f}_i(x(t), t) + \tilde{f}_i(\tilde{x}(t+h), t+h) \right] \\ &\quad + \frac{\sqrt{h}}{2} \sum_j [g_{ij}(x(t), t) + g_{ij}(\tilde{x}(t+h), t+h)] u_j(t) . \end{aligned} \quad (3.51)$$

3.4 Dimensionless model

In order to solve numerically the laser equations, it is convenient to perform the change of variables $N = N_{tr}D$, $S = N_{tr}\psi$ and $t = \tau_p\tau$, to prevent from instabilities due to the different order of magnitude of the implied parameters. Also, the use of a dimensionless model allows to improve the speed of the numerical calculations. However, one should notice that SDEs do not follow the standard calculus rules. The change of variables is performed by using the following two properties:

(i) A SDE written in the Stratonovich interpretation follows the standard chain rule (this is not the case if it is written in the Itô interpretation). The Stratonovich SDE, Eq. (3.46), can be written as

$$dx_i = \tilde{f}_i(x, t) dt + \sum_j g_{ij}(x, t) \circ dW_j(t) , \quad (3.52)$$

where $W_j(t)$ is the vector of Wiener processes and the symbol $\circ$ indicates that the integral must be performed in the Stratonovich interpretation. When the change of variables $y_i = F_i(x)$ is performed and the standard chain rule is applied, it results:

$$dy_i = \left(\tilde{f}_i(y, t) dt + \sum_j g_{ij}(y, t) \circ dW_j(t) \right) \frac{dF_i}{dx_i} . \quad (3.53)$$

(ii) To perform a time scale change the Wiener process scaling factor property is used, i.e.

$$V(t) = \frac{1}{\sqrt{c}} W(\tau) , \quad (3.54)$$

where $\tau = ct$, then the SDE in the Stratonovich interpretation reads as

$$dy_i = \left(\tilde{f}_i(y, \tau) \frac{1}{c} d\tau + \sum_j g_{ij}(y, \tau) \circ \frac{1}{\sqrt{c}} dW_j(\tau) \right) \frac{dF_i}{dx_i} . \quad (3.55)$$

Applying the two properties presented above, the system of SDEs and the change of variables reads as

$$\begin{aligned} \frac{d\psi}{d\tau} &= \left[\Gamma \tilde{\mathcal{G}}(\psi)(D-1) - 1 \right] \psi + 2K_c \sqrt{\psi} |E_{inj}| \cos(\phi - \phi_{inj} + 2\pi\delta\nu'\tau_p\tau) \\ &\quad + \beta\tau_p B N_{tr} D \left(\frac{\Gamma D}{2} + \psi \right) + \tilde{F}_\psi(\tau), \end{aligned} \quad (3.56)$$

$$\frac{d\phi}{d\tau} = \frac{\alpha}{2} \left[\Gamma \tilde{\mathcal{G}}(\psi)(D-1) - 1 \right] + 2\pi\tau_p \frac{df}{dT} \Delta T - \frac{K_c |E_{inj}|}{\sqrt{\psi}} \sin(\phi - \phi_{inj} + 2\pi\delta\nu' \tau_p \tau) + \tilde{F}_\phi(\tau), \quad (3.57)$$

and

$$\frac{dD}{d\tau} = J(\tau) - \tilde{R}(D) - \tilde{\mathcal{G}}(\psi)(D-1)\psi + \beta B \tau_p N_{tr} D \left(\frac{D}{2} - \frac{\psi}{\Gamma} \right) - \frac{1}{2N_{tr} V_{act}} \frac{\partial \tilde{R}(D)}{\partial D} + \tilde{F}_D(t), \quad (3.58)$$

with

$$\tilde{F}_\psi(\tau) = \sqrt{2\Gamma\beta B N_{tr} \tau_p D^2 \psi} \xi_\psi(\tau), \quad (3.59)$$

$$\tilde{F}_\phi(\tau) = \sqrt{\frac{\beta\Gamma B N_{tr} \tau_p D^2}{2\psi}} \xi_\phi(\tau), \quad (3.60)$$

$$\tilde{F}_Z(\tau) = \sqrt{\frac{2}{N_{tr}} \left(\tilde{R}(D) + J(\tau) \right)} \xi_Z(\tau), \quad (3.61)$$

$$\tilde{F}_D(\tau) = \tilde{F}_Z(\tau) - \frac{\tilde{F}_\psi(\tau)}{\Gamma}, \quad (3.62)$$

where

$$\tilde{\mathcal{G}}(\psi) = \tau_p N_{tr} \mathcal{G}(N_{tr} \psi) = \frac{\tau_p N_{tr} v_g \frac{dG}{dN}}{1 + \epsilon N_{tr} \psi}, \quad (3.63)$$

$$\tilde{R}(D) = \tau_p (AD + B N_{tr} D^2 + C N_{tr}^2 D^3), \quad (3.64)$$

$$K_c = \frac{k_c \tau_p}{\sqrt{N_{tr}}} \quad (3.65)$$

and

$$J(\tau) = \frac{\tau_p}{q N_{tr} V_{act}} \left[I_{DC} + \left(2\sqrt{2} \frac{V_{RF} C_{loss}(f_R)}{Z_0 + R_{LD}} \right) \cos(2\pi f_{MOD} \tau_p \tau) \right]. \quad (3.66)$$

The theoretical model has been implemented using a program based on MATLABTM.

Chapter 4

Experimental techniques

Introduction

This chapter is devoted to describe the technical characteristics of the laser devices and the experimental setups used for their characterisation and for the generation of OFCs. Three types of commercial lasers are studied in this thesis: discrete mode lasers, distributed feedback lasers and vertical-cavity surface-emitting lasers. Setups were developed for the following characterisation techniques and operation configurations: power-current and voltage-current (V-I) characteristics, optical and radio frequency spectra under CW and pulsed operation, linewidth measurements, impedance measurements, optical injection and gain-switching configurations, spectral and temporal response under different gain-switching conditions, and setup for dual-comb spectroscopy. The properties of interest derived from each characterisation are summarised in Table 4.1. Routines based on LabVIEWTM and MATLABTM were programmed to acquire and process the experimental data coming from each type of measurement.

This chapter is structured as follows: the main characteristics of the employed lasers are summarised in Section 4.1. The experimental setups developed for the measurements in CW operation are presented in Section 4.2. A description of the experimental setup used to measure the impedance of the lasers is provided in Section 4.3. Section 4.4 is devoted to a detailed description of the setup for gain-switching the lasers and to the implementation of the optical injection. The different RF sources used for this purpose are described in this section. The temporal and spectral characterisation of the laser emission from the GS laser diodes was performed employing the experimental setups described in Section 4.5. The parameters used along this thesis for measuring the quality of the generated OFCs are defined in this section. Finally, Section 4.6 shows a detailed description of the experimental setup employed for the application of the OFCs studied in this thesis in dual-comb spectroscopy.

4.1 Laser devices

Three different types of lasers were used in the experiments: i) a 1310 nm 2.5 Gb/s single-mode VCSEL (Raycan RC240341-FFAam) ; ii) a single frequency 1550 nm DML (Eblana Photonics, EP1550-0-DM-H19-FM) and iii) a single frequency 1550 nm DFB (Gooch and Housego, AA0702-193414-010-SM900-FCA-50). The nominal bandwidths of these two

Type of measurement	Parameter / Effect
Power - current characteristics	Optical power Threshold current Slope efficiency
Voltage - current characteristics	Series resistance
Optical spectrum	Emission spectra Carrier-to-noise ratio Spectral width Laser wavelength
Electrical spectrum	Relative intensity noise
Temporal profile (pulses)	Optical peak-to-peak amplitude Full-width half maximum (FWHM)
Delayed self-heterodyne	Emission linewidth
Scattering parameters	Modulation response Electrical impedance

Table 4.1: Summary of the experimental techniques used in this thesis and the physical properties and parameters measured through them

lasers are 10 and 9 GHz, respectively. The main characteristics of the lasers are shown in Table 4.2.

The VCSEL is a commercial fiber pigtailed device, without an optical isolator and packaged in TO-56. This device is biased through a home-made SMA adaptor which is plugged into the laser anode (central pin) and cathode (case) and is driven by a precision current source (Keithley 236). The laser is mounted in a home-made copper platform which temperature is controlled by a Peltier module and a PT100 (Fig. 4.1(a)). The thermistor is placed into a small hole drilled in the copper platform close to the laser. Both components, the Peltier cell and the thermistor are connected to a temperature controller (ILX Lightwave, LDT-5412) that uses a variable current (from 0 to 2 A) to regulate the temperature of the whole system. The temperature of the VCSEL was set at 20°C in the experiments.

Figure 4.1: Photograph of the employed lasers: (a) VCSEL. (b) DFB laser and (c) DML.

The DFB and the DML are mounted in 7-pin butterfly packages with high-frequency

input connectors and they have no built-in optical isolator to allow external OI. The packaged lasers were mounted on commercial platforms from Arroyo Instruments (ARO 203), for driving and temperature control. Both devices have integrated 10 k Ω thermistors and thermoelectric coolers (TEC) which are connected through their pinout to the platform. The current and the temperature of the lasers were controlled by a laser current source (Arroyo 4205) and a temperature controller (Thorlabs TED 200), respectively, and both connected to the input ports of the platform. It is important to mention that these two devices are anode-grounded. The temperature of both lasers was set at 25°C in the experiments.

	VCSEL	DML	DFB
Operating Power	500 μ W (3.2 mA)	1.3 mW (25 mA)	2.7 mW (20 mA)
Operation Wavelength	1310.8 nm (3.2 mA)	1550.2 nm (25 mA)	1549.2 nm (20 mA)
Threshold Current	0.86 mA	14.5 mA	11.5 mA
Slope Efficiency	0.105 W/A	0.11 W/A	0.267 W/A

Table 4.2: Main measured characteristics of the employed lasers

4.2 CW measurements

The experimental setup for the measurement of the P-I and V-I characteristics is shown in Fig. 4.2(a). The laser is driven with a bias current (I_{bias}), provided by a laser driver, which depends on the employed laser. The output power of the laser goes through an optical isolator, to prevent optical feedback, and then, is measured by a Ge photodetector (Graseby Optronics 261) connected to an electrometer, employed as current-meter (Keithley 136). The value of the responsivity changes with the wavelength of each laser diode. For 1310 nm, the responsivity of the photodetector is 0.78 A/W whereas, for 1550 nm, that value is 0.95 A/W. The diode voltage was measured by the laser driver. The laser driver and the electrometer are connected to a computer via USB/GPIB for data acquisition. Both devices were remotely controlled using LabVIEWTM.

Figure 4.2: (a) Experimental setup for the measurement of the P-I-V characteristics; LD: laser diode, PD: photodetector, OI: optical isolator, EM: electrometer; (b) Example of an experimental P-I and V-I characteristics: in blue P-I, in red V-I

An example of the P-I (in blue) and V-I (in red) characteristics of the DML at 25°C is shown in Fig. 4.2(b).

Fig. 4.3(a) shows the experimental setup for the measurement of the optical spectra. The output signal of the lasers was connected to an optical spectrum analyser (ANDO AQ6315A, wavelength resolution = 0.05 nm). The acquisition and processing of the OSA data were carried out using different programs based on LabVIEWTM and MATLABTM. The optical spectrum of the DFB at $I_{bias} = 50$ mA and 25°C recorded with the OSA is

Figure 4.3: (a) Experimental setup for the acquisition of the optical spectra; LD: laser diode; OSA: optical spectrum analyser, OI: optical isolator; (b) Optical spectrum of the DFB at $I_{bias} = 50$ mA recorded with OSA

shown in Fig. 4.3(b) as example.

For the measurement of the electrical spectra, the output light of the laser was detected with a 45 GHz photodiode (New Focus 1014) and the electrical signal was analysed with a 44 GHz electrical spectrum analyser (Agilent E4446A). A schematic diagram of this setup is shown in Fig. 4.4, as well as an example corresponding to the measured RF spectrum of the VCSEL for $I_{bias} = 4$ mA and 25°C.

Figure 4.4: (a) Experimental setup for the measurement of the RF spectra; LD: laser diode; OSA: optical spectrum analyser, OI: optical isolator, PD: photodetector, ESA: electrical spectrum analyser; (b) RF spectrum of the VCSEL at $I_{bias} = 4$ mA

The laser linewidth measurements require a resolution much higher than the resolution of any OSA and therefore, an alternative technique should be used. The delayed self-heterodyne technique has been employed in this thesis to determine the linewidth of a laser. This technique has been extensively used to measure the linewidth of unmodulated lasers [4, 86, 116].

Fig. 4.5 shows a schematic of the experimental setup used to characterise the linewidth of a laser. The optical path of the light coming from the laser is split into two branches. The frequency of the light travelling through one of the branches is shifted by an amount f_{shift} by means of an acousto-optic modulator (AOM, Gooch & Housego T-M080-0.4C2J-3-F2S). The path of the light travelling through the other branch is enlarged by a 25 km

Figure 4.5: (a) Experimental setup for the measurement of the linewidth: OS: optical source, AOM: acousto-optic modulator, PD: photodetector, ESA: electrical spectrum analyser (b) Measured spectrum of the DFB at $I_{bias} = 30$ mA and fitting with a Voigt profile

fiber spool. Both signals beat in the photodetector (New Focus 1014) and the beating is analysed by the ESA. The lineshape of the resulting signal can be fitted to a Voigt profile, i.e, the convolution of a Gaussian and Lorentzian profile [117]. Fig. 4.5(b) shows an example of the linewidth of the DFB at $I_{bias} = 30$ mA. In this figure, the red points correspond to the measured spectrum while the blue line is the fitting a Voigt profile to the experimental data.

Although a value of the linewidth for the DML and DFB was retrieved, the characterisation of the linewidth in these devices could not be properly performed as several issues related to optical feedback were observed, even with an external optical isolator. The correct value of linewidth of these lasers under OI could be properly obtained.

4.3 Impedance measurements

The internal impedance constitutes a relevant parameter for the modelling of the lasers. It has been determined by measuring the microwave scattering parameters of the devices. The used experimental setup for this purpose is shown in Fig. 4.6(a). The scattering parameters (S_{ij}) of the lasers were measured with a network analyser (Agilent PNA E8363B) in different conditions of I_{bias} . The measurements were performed in the department of "Ciencia de Materiales, Óptica y Tecnología Electrónica" of the Universidad Miguel Hernandez de Elche. The input impedance of the laser is related with the scattering parameter S_{11} as:

$$Z_l(\omega) = \left(\frac{1 + S_{11}}{1 - S_{11}} Z_0 \right) \quad (4.1)$$

where Z_0 is the input impedance of the network analyser ($Z_0 = 50 \Omega$). A proper calibration of the electrical plane of the analyser was necessary in order to obtain the correct value of the laser impedance. The measured input impedance of the DML at $I_{bias} = 35$ mA is shown in Fig. 4.6(b).

Fig. 4.6(b) shows that the value of the laser impedance remains constant at 53Ω up to 0.4 GHz, and then, it drops to less than 50Ω . Besides, the impedance of the laser presents resonant peaks at higher frequencies. This behaviour is attributed to the presence of different electrical parasitics. The values of the input impedance for each device at low

Figure 4.6: (a) Experimental setup for the measurement of the laser scattering parameters, LD: laser diode, PD: photodetector, OI: optical isolator. (b) Measured impedance of the DML at $I_{bias} = 35$ mA.

frequency were calculated from these measurements. A summary is presented in Table 4.3.

	VCSEL	DML	DFB
Z_l	115 Ω	53 Ω	52 Ω

Table 4.3: The input impedance at low frequency Z_L of each laser

The modulation response of the lasers, related with the scattering parameter S_{21} was also measured at different I_{bias} , but the results were not processed due to the predominant role of electrical parasitics

4.4 Gain-switching and optical injection

As it was mentioned in Chapter 2, the standard implementation of gain-switching requires to drive the laser using the combination of a DC bias current superimposed to an alternating RF signal.

Figure 4.7: Schematics of a bias-tee

The superposition of both signals was carried out using a bias-tee. This element is a three-port component used in electrical networks that consists of an inductance and a capacitor. Schematics of a bias-tee is shown in Fig. 4.7.

An external bias-tee (Picosecond labs, PSPL5530B, nominal bandwidth = 20 kHz -12.5 GHz) has been used to drive the VCSEL. In the case of the DML and DFB, as both lasers have integrated inductances in their internal structure, an external DC block (Minicircuits, BLK-18-S+) was included at their RF inputs. Different RF sources were employed in the experiments to provide RF signals with different waveforms and characteristics. A

RF signal generator (Agilent 8648A)	Waveform	Sinusoidal	-
	Power level (P_{RF})	-136 - + 14	dBm
	Range (f_R)	9K - 2G	Hz
	Output impedance	50	Ω
RF signal generator (Rodhe & Schwarz SMB 100A)	Waveform	Sinusoidal	-
	Power level (P_{RF})	-145 - + 30	dBm
	Range (f_R)	11K - 12.75G	Hz
	Output impedance	50	Ω
RF signal generator (Rigol DSG830)	Waveform	Sinusoidal	-
	Power level (P_{RF})	-110 - + 20	dBm
	Range (f_R)	9K - 3.6G	Hz
	Output impedance	50	Ω
Pulse pattern generator (Anritsu MU181020A)	Waveform	Square wave	-
	Signal level (V_{RF})	0.125 - 1.1	V
	Range	0.1 - 12.5	Gb/s
	Output impedance	50	Ω
Step-recovery diode	Waveform	Electrical pulses	-
	Power level (P_{RF})	30 (can be attenuated)	dBm
	Range	90-110	MHz
	Output impedance	50	Ω

Table 4.4: Main parameters of the RF drivers used in all the experiments

summary can be found in Table 4.4. The voltage and the power level refer to the RF signal applied on a nominal 50 Ω load. Along this thesis the label V_{RF} indicates the RMS value of the voltage when the modulating signal is sinusoidal. If the modulating signal is a square waveform, V_{RF} refers to the voltage peak amplitude.

In addition to the implementation of GS, the so-called master-slave configuration is employed for the experiments with optical injection [42, 83]. A master-slave configuration is an operation scheme (Fig. 4.8) where a SL is injected with light from a ML. In the fiber-based circuits, this technique can be implemented with an optical circulator. In this work, a narrow linewidth tunable laser (linewidth = 70 KHz, Pure Photonics PPCL300) is employed as ML. A polarisation controller was used to maximise the coupling between the polarisation of both lasers. A variable optical attenuator was employed to control the input power of the ML external light.

4.5 Temporal and spectral characterisation

An oscilloscope and a high-resolution optical spectrum analyser were used for the characterisation of the temporal and spectral response when the lasers are modulated. Schematics of the experimental setups are shown in Fig. 4.9, without and with OI.

The laser is gain-switched by a bias current (I_{bias}), provided by a laser driver, super-

Figure 4.8: Experimental setup for OI using the master-slave configuration, ML: master laser, SL: slave laser, VOA: variable optical attenuator, OSA: optical spectra analyser; PC: polarisation controller

Figure 4.9: (a) Schematics of the experimental setup for the temporal and spectral characterisation; LD: laser driver, DUT: device under test, PD: photodetector, OSC: oscilloscope, HR-OSA: high-resolution optical spectrum analyser, OI: optical isolator (b) Schematics of the experimental setup for the temporal and spectral characterisation including OI.

imposed to an RF signal. In the case of GS without OI, an optical isolator was connected at the laser output to avoid detrimental effects on the laser emission due to optical feedback (Fig. 4.9(a)). The output signal of the laser was spectrally characterised either with a Brillouin optical spectrum analyser (BOSA) (Aragon Photonics BOSA 210), with 10 MHz resolution, or with a high-resolution (20 MHz) optical spectrum analyser (APEX technologies, AP2050A). The temporal traces were acquired with a digital signal analyser (DSA, Tektronix DSA8200), using an optical input module with 20 GHz bandwidth. The acquisition of the BOSA/APEX and the DSA data were carried out using specific LabVIEWTM routines programmed for these measurements. The master-slave scheme has been implemented for the experiments with OI, substituting the optical isolator at the output of the laser by an optical circulator connected to the ML (Fig. 4.9(b)). Fig. 4.10 shows an example of a temporal profile and an optical spectrum of the DML. Discussion and comments about these measurements are given in Chapter 5.

Two different parameters were chosen to characterise the temporal features of the

Figure 4.10: Example of a temporal profile (a) and optical spectrum (b) for the DML in GS conditions

generated optical pulses: i) the optical peak-to-peak power amplitude (P_{max}) that is defined as the difference, in terms of power, between the minimum and the maximum amplitude of an optical pulse in the temporal domain; and ii) the temporal full-width half-maximum. Both parameters are shown in Fig. 4.11(a). In order to characterise the spectral quality of an OFC, two parameters were defined: i) the 10 dB spectral width (Δf_{10}), defined as the difference between the two most separated tones (at both sides of the optical carrier) which intensity is not more than ten times lower than the intensity of the strongest tone (see Fig. 4.11 (b)); ii) the carrier-to-noise ratio, defined as the average value of the ratio between the intensities of those tones within Δf_{10} and the noise level at frequencies between adjacent tones. Fig. 4.11 (c) shows a magnified view of the spectrum measured with the BOSA in Fig. 4.11 (b), illustrating the value of the CNR. The advantage of high-resolution spectral measurements for the characterisation of OFCs is illustrated in Fig. 4.11(b), where measurements performed with the BOSA and the OSA of the spectrum of an OFC generated at repetition frequency $f_r = 5$ GHz are compared. The tones of the comb are clearly resolved by the BOSA, while the low resolution of the OSA does not allow resolving the individual tones and therefore the quality of the OFC cannot be characterised.

Figure 4.11: (a) Temporal trace of the optical pulse of the VCSEL ($I_{bias} = 4.5$ mA, $f_R = 2$ GHz and $P_{RF} = 16$ dBm). (b) Spectrum of an OFC measured with the OSA (blue) and with the BOSA (red) ($I_{bias} = 30$ mA, $f_R = 5$ GHz and $V_{RF} = 0.4$ V). The arrows indicate the 10 dB spectral width ($\Delta f_{10} = 20$ GHz). (c) Closer look at the 1550.4 nm region of the spectrum. In (b), the carrier-to-noise ratio (CNR = 53 dB) is indicated.

4.6 Dual-comb spectroscopy

Dual-comb spectroscopy is the main targeted spectroscopic application of the OFCs generated in this work. As it is mentioned in Chapter 2, the implementation of this technique requires two OFCs with slightly different repetition rates. The experimental setup for

Figure 4.12: Experimental setup with the symmetrical configuration for the dual-comb architecture: (a) With the AOM and (b) With the OBPF. ML: master laser, SL: slave laser (1&2), PC: polarisation controller, AOM: acousto-optic modulator, DUT: device under test, PD: photodetector, ESA: electrical spectrum analyser, OSC: oscilloscope, OBPF: optical bandpass filter

the dual-comb spectroscopy (see Fig. 4.12) is a dual master-slave configuration [78] using three laser diodes: a narrow linewidth tunable laser used as the master laser, and two different semiconductor lasers, with similar internal characteristics, acting as slave lasers. The external light from the master laser is injected into both slave lasers by employing two optical circulators. Different device under tests (DUTs) have been employed in these experiments: a tunable FP filter (MicronOptics, TF2FMJ, Free Spectral Range = 272 GHz, Bandwidth = 0.375 GHz), a HCN gas cell (HCN-13-H(16.5)-100-FCAPC) and a H₂S gas chamber. In the first setup (Fig. 4.12(a)), the light from one of the SL is shifted f_{shift} by an acousto-optic modulator. The light from both optical paths is coupled and is partially transmitted by the DUT (in the symmetrical configuration [5]). Then, both signals are mixed in a photodetector and the beat signal is analysed by, either, an ESA (Agilent E4466A) or an oscilloscope (MSOS804A, 8 GHz Bandwidth). The polarisation controllers adjust the polarisation of the optical fields, thus maximising the amplitude of the beat signal.

The experimental setup shown in Fig. 4.12(b) was employed for the experiments with H₂S. Here, two different FP lasers (Eblana Photonics) were employed as slave lasers while a tunable external cavity laser (ECL, HP 8168F) was used as the master laser. During these experiments, the acousto-optic modulator was not included in the setup. A tunable optical bandpass filter (OBPF, Yenista XTM-50) was placed at the output port of the second optical coupler, followed by the DUT (in the symmetrical configuration [5]) and then, by the photodetector.

For the experiments with gated dual optical frequency combs, the double master-slave arrangement is substituted by a single slave laser. The main difference between these two techniques is that the slave laser is modulated with an RF signal which frequency switches

Figure 4.13: Experimental setup for the gated dual-comb architecture: ML: master laser, SL: slave laser, PC: polarisation controller, AOM: acousto-optic modulator, VOA: variable optical attenuator, PD: photodetector, ESA: electrical spectrum analyser, OSC: oscilloscope, EDFA: erbium doped fiber amplifier

between two slightly different values, generating an optical signal with two alternating frequencies [118]. The employed experimental setup is presented in Fig. 4.13. The optical signal is split into two branches which optical path are adjusted by a fiber spool to generate a delay corresponding to the semiperiod of the gating signal. Then, the beating takes place between signals with different repetition rates (see Chapter 8). The resulting signal is amplified by an erbium-doped fiber amplifier (EDFA) and both signals beat in the PD. The beat signal is then analysed with the ESA or the oscilloscope (Keysight MSOS804A, with 8 GHz Bandwidth).

Chapter 5

Optical pulse and frequency comb generation in gain-switched semiconductor lasers

Introduction

This chapter reports on the detailed experimental analysis of the generation of OFCs by gain-switching edge-emitting single mode semiconductor lasers that was carried out in this thesis. Therefore, the contents of the chapter are in the core of the original experimental contributions of the thesis to the state of the art.

Section 5.1 provides a review of the state of the art in OFC generation by gain-switching prior to this thesis. Section 5.2 is the central section of the chapter. It provides first, the CW characterisation of two laser sources representative of two edge-emitting single mode laser architectures, namely, the DML and the DFB laser (Section 5.2.1). Then, Section 5.2.2 is devoted to the comparative analysis of the characteristics of the generated combs from both sources as a function of the GS conditions. The combined effects of GS and OI in the improvement of the characteristics of the OFCs are dealt with in Section 5.2.3, while the relevant finding of the advantages of using low frequency pulsed GS deserves a specific chapter devoted to it (Chapter 7). Finally the chapter is complemented with an original study of the pulses generated in 1300 nm VCSELs by GS (Section 5.3). Only the temporal profiles of these pulses are studied in detail while the spectral aspects are not addressed in this thesis.

Part of the work presented in this chapter is based on [119] and [120].

5.1 State of the Art

As mentioned in Section 2.2, since its early times in the 80's, GS is being used as a technique for generating short optical pulses in semiconductor lasers. The technique has been applied to a great variety of lasers diodes including FP [34, 121–124], DFB [35, 125–127], DBR [128–131] and VCSELs [22, 132–134]. Pulses have been generated in a wide range of wavelengths ranging from 400 nm [134, 135] to 2 μm [136]. As switching signals not only sinusoidal RF excitation but also pulsed excitation with and without a

DC component have been employed [19, 35, 123, 136, 137]. Variable pulse repetition rate, possibility of arbitrary pulse sequence, ease-of-use, ruggedness, compactness and low cost are commonly claimed as main advantages of the technique. Although the shortest pulses attained by GS are longer than those of mode-locked semiconductor lasers (at the expenses of a fixed repetition rate given by the cavity length), pulses as short as several tens of picoseconds are easily obtained by GS at a wide range of wavelengths. Femtosecond pulses have also been generated from gain-switched lasers by additional pulse compression [138] and pulse shaping stages [139, 140] and recently even direct generation of femtosecond pulses by gain-switching with strong pulsed optical pumping has been reported [124].

Attention had also been paid to some spectral aspects of the generated pulses. In particular, the spectral purity had been acknowledged as an important issue for optical communications [141] and other applications [127], and several strategies such as spectral windowing [126] and the use of dispersion compensating fibers [142] and fiber Bragg gratings [143, 144] had been applied to achieve transform-limited pulses. However, until relatively recently, more attention had been paid to the temporal aspects than to the inherent frequency comb character of the pulses, partially due to the lack of optical spectrum analysers with enough resolution. It was not until 2009 when Anandarajah et al. [73] demonstrated the OFC generation from gain-switched discrete mode lasers. The performance of the OFCs generated from these lasers in a Differential Phase Shift Keyed transmission system was compared with the performance of gain-switched conventional DFB lasers. It was claimed a superior performance of the first that was attributed to a lower phase noise. Since then, the generation of OFCs by gain-switching edge-emitting single-mode lasers has been the subject of a good deal of application-driven research. The main targeted applications have been Radio-over-Fiber [145–147] and multi-carrier optical communications [148–150]. Some recent achievements in this respect are 100 Gb/s wireless transmission at 200 GHz [147] and 1.6 b/s/Hz spectral efficiency in an all-analogue real-time 100 Gb/s wavelength division multiplexing link using orthogonally overlapping sub-channels [150]. The state of the art in optical communication applications has been recently reviewed by Imran et al. in an excellent review ([10] and references therein). In some interesting contributions to this field the OFCs have been referred to as multi-wavelength sources and the generation method as direct modulation [151, 152]. In what concerns other applications, the group of Prof. P. Acedo at the University Carlos III de Madrid had demonstrated that dual-comb spectroscopy was attainable by the OFCs generated by GS edge-emitting laser diodes [90].

Regarding VCSELs, the generation of OFCs by GS these lasers had also been demonstrated [153] and a significant enlargement of the frequency span by additional non-linear techniques had been shown [154, 155]. It had been demonstrated that the two orthogonal polarisations of the VCSELs generate correlated OFCs, and the polarisation dynamics had been theoretical and experimentally analysed [156, 157]. Relevant for this thesis, it is worth mentioning however, that in the 1300 nm spectral region not even GS pulses had been reported at the beginning of this thesis.

Coming back to edge-emitting lasers, the phase noise/linewidth, the jitter, the modulation bandwidth and the frequency chirp had been analysed as related key issues affecting the performance in the targeted applications of the OFCs generated by GS these lasers [76, 81, 158, 159]. However, the generation of OFCs by GS lacked a systematic study

of the effect of the switching conditions on the characteristics of the generated OFCs, in line with the analysis performed in [156] for the OFCs generated by GS VCSELs. Only the role of the modulation frequency had been studied in a short range of frequencies around the relaxation oscillation frequency [74].

On the other hand, the effect of external OI in semiconductor lasers in single-mode CW operation has been extensively studied and the appearance of a wide variety of induced dynamical regimes, including injection locking, four-wave mixing, self-pulsations and chaotic behaviour through a period-doubling cascade has been described [42]. Regarding OFCs generated by GS, external OI had been demonstrated to reduce related detrimental characteristics such as phase noise, time jitter, frequency chirp and relative intensity noise, thus contributing to improve the spectral quality of the OFCs [3, 76, 80, 81, 158, 160]. In particular, Anandarajah et al. [3] had demonstrated that the broad noisy spectra of a gain-switched DFB laser can be transformed into a high quality OFC by external OI. Furthermore, an increase of the maximum attainable frequency spacing of the comb tones by strong OI had been reported [161] and a large tunability range, spanning the entire C-band, had been demonstrated by tunable injection of the combs generated by GS in Fabry-Perot laser diodes [82]. In summary, OI had been acknowledged as a usual complement for OFCs generated by GS when improved performance was required. And yet, again, a systematic analysis of the effect of the OI conditions on the properties of the injected combs had not been reported when this thesis started. For example, though expected, the fact that OI may not only improve the properties of the injected combs but also worsen them depending on the injection conditions had not been experimentally reported and only a simulation based exploration of the injection power and detuning conditions for efficient locking had been published [99].

In conclusion, when this thesis started the analysis of the OFC generation by gain-switching and OI lacked a systematic experimental study of the effect of the GS and OI conditions on the temporal and spectral characteristics of the generated OFCs, as well as a consistent fundamental explanation of these effects based on the GS induced chirp.

5.2 OFC generation in 1550 nm DML and DFB lasers

As mentioned earlier, the first studies of OFC generation by gain-switching edge-emitting laser had claimed a superior performance of the DML versus the DFB lasers. For this reason it was decided to carry out in this thesis a systematic comparative analysis of the performance of both architectures. The fundamentals of the each type of laser are described in Chapter 2.

5.2.1 CW characterisation

The DML and DFB laser were characterised by means of their P-I and V-I characteristics and their optical spectra at different bias currents. The results of this characterisation are presented in Fig. 5.1. The threshold currents, I_{th} , for the DML and the DFB laser are 14.5 mA and 11.5 mA, respectively. The value of the slope efficiency for the DML and the DFB are 0.133 and 0.283 W/A, respectively. The optical spectra of the two devices at 20 mA are shown in Fig. 5.1(b) and (d). The emission wavelength is close to 1550 nm, and

the SMSR is around 42 dB and 53 dB for the DML and the DFB laser, respectively. The SMSR is almost independent of the bias in the case of the DFB laser, while it increases with the bias in the DML.

Figure 5.1: Left: P-I (in blue) and V-I characteristics at 25°C for the DML (a) and DFB laser (c); Right: Optical spectra at $I_{bias} = 20$ mA for the DML (b) and DFB laser (d)

The evolution of the emission spectra as a function of the heat-sink temperature and the bias current was analysed in both devices in order to determine the thermal and current coefficients of the spectral shifting. Fig. 5.6 illustrates the dependence of the spectra of both lasers on the heat-sink temperature at constant bias (Fig. 5.2(a) and (c)) and on the bias current at constant temperature (Fig. 5.2(b) and (d)). The obtained values were 0.095 and 0.108 nm/K for the thermal coefficients, and 0.005 and 0.017 nm/mA for the current coefficients of the DFB laser and the DML respectively.

5.2.2 OFC generation by gain-switching

Gain-switching experiments were performed in both devices in a parallel study. The setup for these experiments was shown in Fig. 4.9(a). The temporal and spectral responses of the lasers in GS operation were characterised for different values of f_R ranging between 100 MHz and 10 GHz. In the following, only the results at two relevant frequencies will be shown: $f_R = 500$ MHz, within the range used in spectroscopic applications, and $f_R = 5$ GHz, close to the frequencies used in optical communications. The quality of the OFCs was characterised by means of the 10 dB spectral width (Δf_{10}) and the CNR, as they are defined in Chapter 4. The results at other frequencies are qualitatively similar to those presented here.

In Fig. 5.3 shows the temporal traces and the optical spectra of the DML emission at $f_R = 500$ MHz and $I_{bias} = 50$ mA ($\leq 3.2 I_{th}$) and different values of V_{RF} , increasing from left to right. As it can be seen in the temporal traces (upper row), clearly sinusoidal profiles are apparent for the lowest values of V_{RF} (Figs. 5.3(a) and (b)), corresponding to a direct modulation of the laser, i.e, the injected current is higher than I_{th} for the entire period. When increasing V_{RF} , a clear distortion of the sinusoid appears, even though, the

Figure 5.2: DFB Optical spectra for (a) different values of heat-sink temperatures: $T = 15^\circ\text{C}$ (in blue), $T = 24^\circ\text{C}$ (in red), $T = 33^\circ\text{C}$ (in black); and (b) different values of I_{bias} : 40 mA (in blue), 50 mA (in red), 60 mA (in black). DML Optical spectra for (c) different values of heat-sink temperatures: $T = 15^\circ\text{C}$ (in blue), $T = 24^\circ\text{C}$ (in red), $T = 33^\circ\text{C}$ (in black); and (d) different values of I_{bias} : 40 mA (in blue), 50 mA (in red), 60 mA (in black).

laser remains in the ON state during the complete modulation period with a high P_{max} ($P_{\text{max}} = 2.25$ mW). At the highest modulation amplitude (Fig. 5.3(d)) the total injection current becomes lower than I_{th} during the semi-period of negative modulation current, and therefore, the laser is actually switched-off during this period. During the switch-on semi-period a small GS pulse appears followed by a semi-sinusoidal trace corresponding to the positive cycle of the total current.

The optical spectra in the lower row of Fig. 5.3 show very different characteristics regarding CNR and Δf_{10} . The results can be understood by considering the combined effect of amplitude modulation (AM) and frequency modulation (FM) caused by the direct intensity modulation [162]. The first one (Fig. 5.3(e)), corresponding to a small modulation amplitude, shows an almost symmetric spectrum with only three tones within the 10-dB spectral width ($\Delta f_{10} = 1$ GHz) and a high CNR = 43.5 dB, which is due to AM. The second one (Fig. 5.3(f)) corresponds to a higher modulation amplitude although the trace is still rather sinusoidal. It shows an asymmetric spectrum with seven tones in the 10-dB spectral width, a clear suppression of the central tone and also high CNR = 40.5 dB. The carrier frequency suppression is a typical feature of FM, which can be caused either by carrier or by thermal effects [162]. The carrier modulation induces frequency chirp due to either the dynamic response of the laser during the switching (dynamic chirp), or to the dependence of the carrier density, and hence of the emission wavelength, on the photon density through the non-linear gain saturation or through thermal effects (adiabatic chirp) [163]. In this particular case, at a relatively low modulation frequency (500 MHz), the output power can follow the oscillations of the total current, and hence, no dynamic processes are expected. In contrast, these variations of the output power are expected to

Figure 5.3: Temporal traces (upper row) and optical spectra (lower row) of the DML emission at $I_{bias} = 50$ mA and $f_R = 500$ MHz and different values of V_{RF} . Each column corresponds to the value of V_{RF} labelled on the top.

affect to the carrier density through the non-linear gain coefficient, and therefore, to the instantaneous frequency of the light. So that, at this repetition rate, the FM is attributed to adiabatic chirp. In the third case (Fig. 5.3(g)), corresponding to a higher V_{RF} , the optical spectrum shows a much higher number of tones ($\Delta f_{10} = 6.5$ GHz) keeping a high CNR. Regarding CNR and Δf_{10} , this is the best OFC obtained at this repetition frequency. However, some tones are partially suppressed and the shape of the spectral envelope shows a depression in the red region. Finally, in the fourth case (Fig. 5.3(h)), the optical spectrum has a low value of CNR (2 dB) and cannot be considered as a proper OFC with discernible lines. This happens when the modulation amplitude is high enough to switch off the laser and therefore the coherence of the pulses is lost. The subsequent pulses are built up from spontaneously emitted photons with a random initial phase and, in consequence, the CNR is very low.

Figure 5.4: Evolution of the temporal traces (upper row) and optical spectra (lower row) of the DML with the modulation amplitude V_{RF} for $I_{bias} = 30$ mA and $f_R = 5$ GHz. Each column corresponds to the value of the V_{RF} labelled on the top.

The temporal traces and the optical spectra at 5 GHz for the DML at constant bias with different modulation amplitudes are shown in Fig. 5.4. The observed behavior of the temporal traces and the corresponding spectra is similar, but not identical, to the results at 500 MHz. At low V_{RF} a sinusoidal trace is obtained (Fig. 5.4(a)) with a low value of P_{max} ($P_{max} = 0.12$ mW). The corresponding optical spectrum (Fig. 5.4(e)) has a high $CNR = 60$ dB but a low number of tones. At higher V_{RF} the temporal profile is no longer sinusoidal, showing wide optical pulses (FWHM = 69 ps) with P_{max} of 0.75 mW. The FM broadens the spectrum that includes a higher number of coherent tones (Figs. 5.4(b) and (f)). A broader and flatter comb is obtained (Fig. 5.4(g)) when the intensity of the modulation does not switch-off completely the laser between pulses (Fig. 5.4(c)). In this case, the spectral width Δf_{10} is 60 GHz and the asymmetric envelope spectrum shows a maximum at the red side. This is the classical shape of gain-switched laser spectra when measured with low spectral resolution [23, 164]. This type of OFC are attributed to the dynamic frequency chirp caused by the photon-carrier interaction at frequencies close to the relaxation frequency. Finally, the last column in Fig. 5.4 corresponds to the highest V_{RF} yielding the best pulses ($P_{max} = 2.3$ mW and FWHM = 36 ps), with complete switch-off between pulses, which are expected to show temporal high-jitter [3], and giving rise to an optical spectrum with no discernible tones ($CNR < 5$ dB). This type of spectrum in gain-switched lasers has been previously experimentally reported [3, 31, 74, 120] and simulated [74, 80, 100, 157]. As discussed in [74, 100, 157], in this case the pulses are built-up from spontaneous emission and not from the previous pulse; then the coherence between pulses is lost, and in consequence the spectrum does not show resolved tones.

The transition from high quality coherent OFCs such as that shown in Fig. 5.4(g) to broad noisy spectra without discernible tones (Fig. 5.4(h)) has been previously reported by changing the type of device or the operating conditions. In [3] and [31], a good OFC is obtained in a DML while a broad noisy spectrum is obtained in a DFB laser; in [3] the broad spectrum of the DFB evolves to high CNR tones by OI; in [74] the transition to high quality OFCs is observed when increasing the modulation frequency. It was systematically observed the transition from high quality OFCs to broadened spectra at different frequencies in both the DML laser and the DFB when increasing the modulation amplitude at constant bias and when decreasing the bias at constant modulation amplitude. This behaviour is illustrated in Fig. 5.5, where the evolution of the two main parameters of the spectra (CNR and Δf_{10}), for both lasers at $f_R = 500$ MHz and $f_R = 5$ GHz is plotted as a function of the GS conditions, V_{RF} and $(I_{bias} - I_{th})$. As it can be seen in Fig. 5.5, panels (a), (b), (e) and (f), two regimes characterised by very different CNR are clearly distinguished in separate regions of driving conditions: a regime of high CNR (40 to 60 dB), corresponding to coherent OFCs, and a regime of $CNR \sim 0$ dB, corresponding to incoherent broadened spectra. The transition between these regimes by increasing V_{RF} takes place when V_{RF} is high enough to switch off the laser, and this value depends on $I_{bias} - I_{th}$: the higher I_{bias} , the higher the value of V_{RF} needed to deplete the carrier density during a time long enough for the lasing photons in the cavity to be absorbed. The transition is more dramatic at $f_R = 500$ MHz than at $f_R = 5$ GHz, and for the highest bias the maximum experimental V_{RF} was not high enough to switch-off the lasers. The values and the evolution of the CNR for the DML and the DFB are quite similar when plotted as a function of $I_{bias} - I_{th}$, even though the values of the resonance frequencies are

not equal in the two devices. This fact indicates that the transition between regimes is more related to the injection conditions (modulation amplitude and bias current) than to the relative values of the modulation and the relaxation oscillation frequencies.

Figure 5.5: Evolution of CNR and Δf_{10} as a function of the modulation amplitude V_{RF} for the DFB laser (upper row) and the DML (lower row) at $f_R = 500$ MHz (first and third columns) and $f_R = 5$ GHz (second and fourth columns) for different bias currents. The stars indicate the best comb at each bias current.

The results in Fig. 5.5 are a useful guide to select the driving conditions to optimise the OFCs generated by GS. The best OFCs are those having high CNR and Δf_{10} . The driving conditions for generating such combs are the use of a high bias together with a high V_{RF} , but not as high as to switch-off the laser between pulses. The optimum conditions for each laser at 500 MHz and 5 GHz are marked with stars in each plot in Fig. 5.5. CNR around 40 (50) dB with Δf_{10} around 7 (60) GHz can be achieved at a modulation frequency of 0.5 (5) GHz. An interesting result in Fig. 5.5 is that both lasers exhibit the same behaviour, with minimum differences, even though they have different type of cavity and phase noise properties and internal parameters. It is apparent that the driving conditions are more relevant than the internal laser parameters to generate gain-switched OFCs. A more complete analysis of the properties of the OFCs generated from the DML is presented in Chapter 6.

5.2.3 OFC generation by gain-switching with optical injection

In this section the results regarding the generation of OFCs by gain-switching and OI are presented as a function of the injection conditions, namely, the injected power, P_{inj} , and the detuning between the ML and the SL frequency, $\delta\nu$. A systematic study of all dynamic regimes as a function of the five operation parameters (f_m , I_{bias} , V_{RF} , P_{inj} , $\delta\nu$) is not presented. Instead, some significant results are shown obtained by changing the OI parameters in the case of the DFB laser. Similar trends were observed for other GS operation conditions and for the DML.

The first and second row of Fig. 5.6 show respectively the temporal traces and the emission spectra, when increasing the injected power into the DFB laser for small detuning,

Figure 5.6: Temporal traces (upper row) and optical spectra (lower row) of the gain-switched DFB laser ($I_{bias} = 25$ mA, $f_R = 5$ GHz and $V_{RF} = 1.8$ V) under several power levels of OI at $\delta\nu$ of 6 GHz. The injection frequency is indicated with an arrow. The frequencies of the spectra are relative to the emission frequency of the free-running SL in CW operation.

$\delta\nu = 6$ GHz. The frequency detuning, $\delta\nu$, is defined as the frequency difference between the ML and the SL emission in CW operation. Operation conditions ($f_R = 5$ GHz, $I_{bias} = 25$ mA, $V_{RF} = 1.8$ V) producing a broad noisy spectrum without injection have been selected in order to analyse the improvement in the OFC quality with the OI. The pulses in Fig. 5.6 (a) were obtained with very low injected power $P_{inj} = -54$ dBm. They show a small ringing at the pulse end, and the corresponding spectrum in Fig. 5.6 (e) does not show clear comb lines, although very small peaks at frequencies separated from the ML frequency by multiples of f_R are visible. This corresponds to the absence of locking and the generation of incoherent pulses together with the modulation of the injected signal by the carrier dynamics. When increasing the injected power as in Figs. 5.6 (b) and (f), the temporal shape is similar to the previous one, but the spectrum shows clear tones with $CNR = 22$ dB, indicating clear locking. However, the noise level is still high due to the competition of the spontaneous emission and the injected field to build up the emitted pulses. At higher injected power (Figs. 5.6 (c) and (g)), the temporal shape of the pulses remains similar, but now the spectrum is very flat and broad with a high CNR of 51 dB and 10 lines in 10 dB. These are the best injection conditions for generating a high quality OFC from the non-injected broad spectrum. At a higher injection level ($P_{inj} = 6$ dBm) the optical pulses present clear ringing (Fig. 5.6 (d)) and the spectrum shows a high CNR but a narrower width, $\Delta f_{10} = 5$ GHz, as well as poor flatness.

Fig. 5.7 shows the influence of the detuning $\delta\nu$, on the comb generated under switching conditions ($f_R = 5$ GHz, $I_{bias} = 15$ mA, $V_{RF} = 1.6$ V) slightly different than those of Fig 5.6. At very high positive or negative detuning, as shown in Fig. 5.7 (a) for $\delta\nu = 55$ GHz, the emission spectra do not show comb lines but a broad noisy emission together

Figure 5.7: Optical spectra of the gain-switched DFB laser ($I_{bias} = 15$ mA, $f_R = 5$ GHz and $V_{RF} = 1.6$ V) under OI ($P_{inj} = -24$ dBm) at several $\delta\nu$. The arrow indicate the frequency of the injection relative to the emission of the free-running SL.

with the ML line and the side bands due to the gain modulation. For a relatively broad range of detuning (-30 GHz $< \delta\nu < 30$ GHz) the SL is locked to the ML and the spectra correspond to OFCs with high CNR and wide Δf_{10} (Figs. 5.7(b) and (c)). At some particular injection conditions, for instance at $\delta\nu = -37$ GHz (Fig. 5.7(d)), the optical spectrum shows double period dynamics.

Figure 5.8: Optical spectra of the gain-switched DFB laser ($I_{bias} = 25$ mA, $f_R = 5$ GHz and $V_{RF} = 1$ V), under several power levels of OI at $\delta\nu = 5$ GHz. The arrows indicates the injection frequency relative to the SL emission in CW operation.

The improvement by OI of the quality of the OFC (defined in terms of the number of harmonics with high CNR) have been previously reported theoretical [80, 99] and experimentally [3]. However, this is not a general rule. A broad incoherent spectrum can be transformed into a high quality OFC by OI, but the properties of a GS OFC can also deteriorate by OI. Fig. 5.8 is an illustrative example of this behaviour. It shows the evolution of a high quality OFC generated by the DFB laser at 5 GHz when increasing the OI power. The non-injected spectrum (Fig. 5.8 (a)) exhibits a high CNR = 43 dB, with broad $\Delta f_{10} = 60$ GHz. When a relatively low power (-39 dBm) is injected at a detuning of $\delta\nu = 5$ GHz, the SL is not locked to the ML and two overlapped combs, with a slight shift between them, appear (Fig. 5.8 (b)). One of the combs corresponds to the SL while the second is due to the injected power which is amplified and emitted according to the carrier dynamics, thus producing the second comb. When the injected power is increased, as shown in Fig. 5.8 (c), the SL is locked to the ML and a comb is built around the ML wavelength. In addition, at these particular driving and injection conditions, the weak lines between the comb tones may be indicative of a period doubling dynamics. At a high

injection level of - 3 dBm, the spectrum becomes very complex (Fig. 5.8 (d)). It consists of tones at 1 GHz (five times the period), with an irregular intensity distribution over a high noise floor. This kind of spectrum is reported for the first time in this thesis. The distribution of the tones around the comb may suggest that the effect of the OI induces fifth period dynamics or resonant four wave mixing (due to the exact separation of 1 GHz between tones). A further study of these dynamical effects is not presented here. Similar results have been observed in the experimental characterisation of the DML. Apart from this illustrative example of the worsening of the comb quality by OI, many other types of anomalous emission spectra have been observed which descriptions are not included here. A more detailed analysis of these anomalous spectra is presented and discussed in Chapter 6.

A clear advantage of the OI for the generation of OFCs is the reduction of the phase noise. This effect is evidenced by the transferring of the narrow linewidth of the ML to each comb line of the SL. This improvement has been experimentally demonstrated by different authors [3, 81, 82], as well as theoretically studied [99]. The same trend was observed in both, the DML and the DFB laser in locking conditions. Fig. 5.9 compares the CW lineshape of the DFB laser in free-running ($I_{bias} = 30$ mA) conditions and under OI ($P_{inj} = 6$ dBm; $\delta\nu = -1$ GHz), measured using the DSH technique (see Section 4.2). The linewidth is reduced from around 600 kHz to around 75 kHz, which is the measured linewidth of the ML. Same behaviour has been observed in the linewidth of each peak of the gain-switched OFCs, as it is shown in Fig. 5.9 (b), where the lineshapes of the central lines of the non-injected and the optically injected combs are compared ($I_{bias} = 30$ mA, $V_{RF} = 1$ V, $f_m = 5$ GHz). The linewidth reduction caused by the OI is noticeable. Finally, Fig. 5.9 (c) compares the measured linewidths of the different tones of the OFCs in non-injection and injection conditions, showing a reduction around a factor ten.

Figure 5.9: RF spectra of the DSH signal of the DFB laser with and without OI, (a) CW lineshape for $I_{bias} = 30$ mA. (b) Lineshape of the 1st tone of the OFC ($I_{bias} = 30$ mA, $V_{RF} = 1$ V, $f_r = 5$ GHz). (c) Linewidths of the first 5 tones of the OFC. For all panels, without OI is indicated in blue and with OI in red ($P_{inj} = 6$ dBm, $\delta\nu = -1$ GHz).

5.3 Optical pulse generation in gain-switched 1300 nm VCSELs

5.3.1 CW characterisation

For the analysis of the optical pulse generation in 1300 VCSELs, a fiber pigtailed 2.5 Gb/s device was used. The CW-characterisation of the VCSEL consisted in the analysis of its P-I and V-I characteristics as well as the optical spectra under different conditions of current and temperature.

Figure 5.10: (a) P-I (in blue) and V-I (in red) characteristics of the VCSEL at 25°C; (b) VCSEL Optical spectrum at CW operation at $I_{bias} = 3$ mA.

The employed experimental setups for these measurements are shown in Fig. 4.2, Fig. 4.3 and Fig. 4.4. The laser was driven with a bias current I_{bias} provided by a current source. The laser heat-sink temperature was controlled by a TEC based on a Peltier cell. The power-current and the voltage-current characteristics of the VCSEL are shown in Fig. 5.10(a). According to the P-I curve, the measured threshold current is 1.57 mA with a slope efficiency $\eta = 0.162$ W/A. The P-I curve shows also a saturation of the output power at 1 mW for a current of about 8 mA. This is attributed to thermal effects. The emission spectrum (Fig. 5.10(b)) is centred around 1312 nm with a SMSR larger than 45 dB.

Figure 5.11: Schematic diagram of the experimental setup for the polarisation splitting; VCSEL: vertical-cavity surface-emitting laser, PC: polarisation controller, RP: rotatable polariser, OSA: optical spectrum analyser

Although the VCSEL emission can be considered as single-mode, the optical spectrum shows some duplicated structures in the side modes at the low wavelength side. It is well known that these type of devices may show optical spectra consisting of two orthogonally polarised modes at slightly different wavelengths [33]. In order to ascertain this possibility, the experimental setup shown in Fig. 5.11 was used. The output of the VCSEL connected to into a polarisation controller (Newport, F-POL-PC) followed by a polarisation rotator

(OZ Optics, FPR-01-03-1550-S-S-1) in order to acquire polarisation resolved spectra with the OSA. The results of these experiments are shown in Fig. 5.12.

Figure 5.12: (a) VCSEL Experimental optical spectra at 6 mA for the three different polarisations. (b) Closer look at the 1312.6 nm region of the spectra. Each polarisation is labelled in the legend of both figures.

The labels "X" and "Y" correspond to the spectra of the light transmitted by the rotator at two perpendicular orientations of its transmittance axis, while the label "X+Y" correspond to the spectrum recorded without the polarisation rotator. For obtaining the trace "Y" the PC was adjusted so as to minimise the intensity of the peak at 1312.6 nm, up to point that the relative intensity of the peaks at 1312.8 and 1312.6 nm resulted to be maximum. The trace "X" was obtained after rotating 90° the PR.

Figure 5.13: VCSEL Optical spectra for (a) different values of I_{bias} (blue: 3 mA, red: 6 mA, black: 9 mA) and (b) different values of heat-sink temperature (blue: 20°C, red: 25°C, black: 30°C)

It is apparent in Fig. 5.12(a) that the duplicated structure of the side modes at the lower wavelength side disappears in the spectra X and Y (the high wavelength structure in X and the low wavelength structure in Y). It is also apparent in Fig. 5.12(b) that the ratio between the intensity of the peaks at 1312.8 and 1312.6 nm is much higher in the Y spectrum. Both facts are consistent with the coexistence in the spectrum X+Y of pairs of polarisation modes, giving rise to the spectral structure of the main peak (not well resolved by the OSA) and the duplicated structure of the side modes. It is important to point out that since no polarisation maintaining fiber was used in the setup, the linear polarisation of the emitted light may be partially lost when it arrives to the OSA. The absence of total suppression of the peak at 1312.6 nm in the spectrum Y may be also partially due to the

use of a PC and a PR optimised for 1550 nm.

In order to find the current coefficient of the wavelength emission, the optical spectra of the VCSEL were measured for different bias currents, at a constant heat-sink temperature ($T = 20^\circ\text{C}$). The results in Fig. 5.13(a) indicate that the current dependence of the wavelength emission is not linear in the range of 3-9 mA. For this reason, the current coefficient of the wavelength emission was calculated in the region between 4 and 6 mA, obtaining a value of 0.42 nm/mA.

Optical spectrum measurements as a function of heat sink temperature at constant I_{bias} were also performed in order to determine the coefficient of the thermal dependence of the emission wavelength (see Fig. 5.13(b)). A value of 0.091 nm/K was found.

5.3.2 Optical pulse generation by gain-switching

Figure 5.14: Temporal traces (upper row) and optical spectra (lower row) of the VCSEL emission at $P_{RF} = 18\text{ dBm}$ and $f_R = 2.5\text{ GHz}$ and different values of I_{bias} . Each column corresponds to the value of I_{bias} labelled on the top

In this section the effects of the GS conditions on the characteristics of the optical pulses generated in the VCSEL are analysed. The setup for pulse generation is shown in Fig. 4.9. The range of the explored GS conditions includes variations of I_{bias} in the range 2 – 8 mA and variations of the RF amplitude corresponding to powers delivered by the RF generator ranging between 2 and 18 dBm.

Fig. 5.14 shows the temporal and spectral profiles of the pulses generated at a fixed RF power ($P_{RF} = 18\text{ dBm}$) for three values of I_{bias} (2, 5 and 8 mA). The temporal traces (upper row) show clear pulse profiles. For $I_{bias} = 2\text{ mA}$ (see Fig. 5.14(a)), although the RF amplitude is high enough to produce pulses, the value of I_{bias} is so low, that the amplitude of the pulses is very low ($P_{max} = 77\text{ }\mu\text{W}$) with a FWHM = 59 ps. As I_{bias} increases, P_{max} also increases and the width decreases. For $I_{bias} = 5\text{ mA}$ (Fig. 5.14(b)) the peak power and the pulse width reach optimum values ($P_{max} = 471\text{ }\mu\text{W}$ and FWHM = 50 ps), the pulses are symmetric and the light is almost totally extinguished in the time between

pulses, revealing that the total injection current becomes lower than I_{th} for some time along each cycle. However, a high jitter is expected from such pulses since each pulse is built up from the incoherent spontaneous emission that remains when the laser light is extinguished [80, 165]. When I_{bias} increases up to 8 mA (Fig. 5.14(c)) the pulses become asymmetric and their width increases (FWHM = 57 ps). The pulse profile suggests that not only the first but higher order relaxation oscillations are involved in this case. The corresponding optical spectra for each injection conditions (lower row in Fig. 5.14) were recorded with the OSA, whose resolution is only around 6 GHz. Although clear differences between them and in comparison to the CW spectrum (Fig. 5.12(b)) are apparent, the resolution of the OSA does not allow to ascertain to which extend the features appearing in the spectra could be related with the spectral envelope of two OFC each one corresponding to a polarisation mode as described in [166].

Figure 5.15: Temporal traces (upper row) and optical spectra (lower row) of the VCSEL emission at $I_{bias} = 5$ mA and $f_R = 2.5$ GHz and different values of P_{RF} . Each column corresponds to the value of P_{RF} labelled on the top

The evolution of the temporal and spectral profiles as a function of the RF amplitude for fixed $I_{bias} = 5$ mA is shown in Fig. 5.15. For $P_{RF} = 2$ dBm (Fig. 5.15(a) and (d)), since the RF amplitude is low, the laser is always biased above the threshold level, and therefore, the temporal profile of the output signal is rather a modulation-like profile with low power and high FWHM ($P_{max} = 102 \mu\text{W}$ and 163 ps). The corresponding optical spectrum is narrow and reminds the CW spectrum. As P_{RF} increases, the peak power, the extinction ratio and the symmetry of the pulses also increase and the width decreases. For $P_{RF} = 10$ dBm (Fig. 5.15(b)), $P_{max} = 300 \mu\text{W}$ and FWHM = 80 ps, while at $P_{RF} = 18$ dBm (Fig. 5.15(c)) the peak power and the pulse width reach their optimum values as mentioned above. Apart from some spectral features (not well resolved by the OSA), it is clear that the envelope spectral profiles become broader as P_{RF} increases. This is in line with what was commented in the previous section for the gain-switched DML and DFB lasers.

Fig. 5.16 summarises the evolution of P_{max} , FWHM and δf_{10} as a function of the switching conditions. The dependence on I_{bias} at $P_{RF} = 18$ dBm is shown in Figs. 5.16(a) and (b), and the dependence on P_{RF} at $I_{bias} = 5$ mA in Figs. 5.16(c) and (d). As expected, the pulse amplitude increases as I_{bias} is higher. The evolution of FWHM as a function of I_{bias} shows a minimum. This is due to the fact that by increasing I_{bias} the ratio of the sinusoid above I_{th} and the frequency of the relaxation oscillations increase. Therefore, for low I_{bias} the laser is switched off before reaching the maximum of the first spike. As I_{bias} increases, the pulse is generated from the whole first spike, so that the FWHM reaches a minimum.

Figure 5.16: Evolution of P_{max} (left) and the FWHM of the optical pulses of the optical pulses (right, in blue); and the corresponding value of 10-dB spectral width of the optical spectra (right, in red) generated in the VCSEL at $f_R = 2.5$ GHz: (a) and (b) $P_{RF} = 18$ dBm and I_{bias} variable; (c) and (d) $I_{bias} = 5$ mA and P_{RF} variable

If I_{bias} is increased beyond this point the laser is biased well beyond the threshold, its response is faster and consequently secondary oscillations appear, hence increasing the FWHM. The evolution of the spectral width shows a complementary behaviour with respect to the FWHM. Regarding the evolution as a function of P_{RF} it is clear that for low P_{RF} the laser is directly modulated showing a low amplitude and the high width corresponding to the modulating signal. When P_{RF} increases the peak power also increases, the laser is actually switched off and the switching is faster as the reverse bias increases giving rise to decreasing values of the FWHM. Again the spectral width evolves in a complementary way.

A qualitatively similar behaviour was observed for switching frequencies in the range of 2 - 6 GHz. The results for the 1300 nm VCSEL presented here are similar as those reported by Consoli et al. [22] in a 1550 nm VCSEL. Finally, although it has not been pursued in this thesis, it is clear that a further insight into the GS process and the generation of OFCs from these VCSELs would be gained by investigating the optical spectra with a high resolution OSA, dealing with the polarisation issues and applying appropriated simulation tools in

line with the works done in 1550 nm VCSELs [166].

Chapter 6

Numerical and experimental analysis of optical frequency comb generation

Introduction

An in-deep understanding of the physics underlying the performance of semiconductor lasers requires not only an experimental investigation but also a proper modelling of the devices with a simulation model relying on as few unknown parameters as possible. This is particularly true in the case of laser diodes performing as OFC generators under gain-switching. For this reason, after having experimentally analysed the generation of OFCs in Chapter 5, this chapter is devoted to the implementation of a rate equation simulation model entailing a set of experimentally determined parameters and the comparison of the simulations with the experimental characterisation of the OFCs generated under a wide range of GS and OI conditions.

The chapter starts with a short review of the state of the art regarding the simulation of GS with rate equation models, and more specifically the simulation of the generation of OFCs by this method (Section 6.1). Since RIN measurements are a main source for the experimental extraction of simulation parameters, Section 6.2 is devoted to an analytical modelling of RIN in terms of these parameters. Subsequently, in section 6.3 the parameter extraction process followed in this thesis is described. Finally, Section 6.4 is the central section of the chapter, where the simulated OFCs are compared with the experimental results and the details of the generation with and without OI are analysed on the basis of physical grounds.

The main contents of this chapter were the subject of the journal paper [100].

6.1 State of the Art

Although several approaches have been employed to model the dynamics of laser diodes [28, 167, 168], the simplicity of the rate equation model [20, 21, 37, 74, 96, 99, 169] has made this model the most preferred choice for simulating these devices.

The traditional approach using the REM [21, 123, 169] has been to model the behaviour

of the photon and carrier density when the laser is operated either in CW regime or under modulation with different electrical waveforms. In some of these works the non-linear response of the laser gain has been modelled through the non-linear gain saturation coefficient [29].

All these approaches rely on a set of parameters, some of which can be extracted from experimental measurements. Several methods have been applied to make this extraction [98, 170–172]. All of these methods are based on the analysis of the P-I characteristics and the experimental determination of the modulation response of the laser. The usual approach has been to analyse the RIN spectra of the lasers, expressing them in terms of the relaxation oscillation angular frequency (ω_r) and the damping factor (γ). The characterisation of the FM response has also been used to obtain the value of the linewidth enhancement factor (α) in some of these publications [170]. The rest of the parameters has to be assumed or used as fitting parameters.

The use of the REM to model gain-switched semiconductor lasers has been traditionally restricted to the analysis of the generated optical pulses. However, when the emphasis was put on the OFC character of the pulses, some theoretical works dealing with the application of the REM to the analysis of the OFC generation by GS were published [74, 99]. In this [74], the role of the repetition frequency was theoretically analysed for frequencies in the range of the relaxation oscillation frequency. However, a systematic analysis of the evolution of the OFC characteristics at different frequencies in a wider range of GS conditions was missing. The use of OI to generate OFCs increases the complexity of the involved equations, requiring additional terms to simulate the response of the GS laser under external injection. Some theoretical analysis of the generation of OFCs by GS and OI has been carried out [99], but yet again a systematic theoretical analysis considering the operating conditions is missing.

6.2 Analytical expression for the RIN spectrum

An analytical expression for the RIN spectrum of laser diodes can be derived from the rate equation model described in Chapter 3. This expression includes the same parameters involved in the REM and therefore, several relevant parameters can be experimentally extracted by fitting the experimental RIN spectra to the corresponding analytical expressions. In the following the derivation of this analytical expression is presented. In summary the procedure consists in first solving the rate equations in the steady-state to obtain the steady-state carrier and photon densities, then accounting for the noise through a perturbation analysis around the steady-state, and finally, performing a Fourier analysis to obtain the spectral density of intensity noise which is essentially the RIN spectrum. Eqs. (6.1) - (6.3) for the dynamics of the carriers ($N(t)$), the photons ($S(t)$) and the phase ($\Phi(t)$) are repeated here for easy reading:

$$\frac{dN(t)}{dt} = \frac{I(t)}{eV_{act}} - R(N) - \frac{g_0 S(t)(N(t) - N_{tr})}{1 + \epsilon S(t)} + F_N(t) \quad (6.1)$$

$$\frac{dS(t)}{dt} = \left[\frac{\Gamma g_0 (N(t) - N_{tr})}{1 + \epsilon S(t)} - \frac{1}{\tau_p} \right] S(t) + \beta \Gamma B N(t)^2 + F_S(t) \quad (6.2)$$

$$\frac{d\Phi(t)}{dt} = \frac{\alpha}{2} \left[\Gamma g_0 (N(t) - N_{tr}) - \frac{1}{\tau_p} \right] + F_\Phi(t) \quad (6.3)$$

In the CW operation, Eqs. (6.1) and (6.2) read as:

$$\frac{I_0}{eV_{act}} - R(N_0) - \frac{g_0 S_0 (N_0 - N_{tr})}{1 + \epsilon S_0} = 0 \quad (6.4)$$

$$\left[\frac{\Gamma g_0 (N_0 - N_{tr})}{1 + \epsilon S_0} - \frac{1}{\tau_p} \right] S_0 + \beta \Gamma B N_0^2 = 0, \quad (6.5)$$

where N_0 , S_0 and I_0 are the steady-state carrier density, photon density and injected current, respectively. When the laser is working well above the threshold current, the spontaneous emission term $\beta \Gamma B N_0^2$ can be neglected and Eq. (6.5) gives N_0 as:

$$N_0 = N_{th} + \frac{\epsilon S_0}{\Gamma g_0 \tau_p}, \quad (6.6)$$

where the threshold carrier density N_{th} is given by:

$$N_{th} = N_{tr} + \frac{1}{\Gamma g_0 \tau_p} \quad (6.7)$$

Eq. (6.4) can be solved taking into account Eq. (6.5) and it provides S_0 as:

$$S_0 \approx \frac{\tau_n \Gamma g_0 \tau_p}{q V_{act} (\epsilon + g_0 \tau_n)} (I_0 - I_{th}) \quad (6.8)$$

where the recombination rate has been linearised around its threshold value $R(N_{th})$ and the differential carrier lifetime τ_n is defined as:

$$\tau_n^{-1} = \left[\frac{dR}{dN} \right]_{N_{th}} = A + 2BN_{th} + 3CN_{th}^2 \quad (6.9)$$

A perturbation analysis can be performed around the steady-state in order to account for the intensity noise. The instantaneous densities of carriers and photons and the instantaneous phase can be expressed as:

$$N(t) = N_0 + n(t); \quad (6.10a)$$

$$S(t) = S_0 + s(t) \quad (6.10b)$$

$$\Phi(t) = \Phi_0 + \phi(t) \quad (6.10c)$$

where $n(t)$, $s(t)$ and $\phi(t)$ are small perturbations. By substituting them in Eqs. (6.1) - (6.3) and linearising the terms $R(N)$, $\frac{g_0}{1+\epsilon S}$ and $\beta \Gamma B N^2$ around N_0 and S_0 , these equations transform into a set of equations in $n(t)$, $s(t)$ and $\phi(t)$. Now, by taking the Fourier transform and using the derivative property ($\mathcal{F}(f') = i\omega \mathcal{F}(f)$), the equations read as:

$$i\omega n(w) - F_N(w) = -\frac{n(w)}{\tau_n} - \frac{g_0 S_0 n(w)}{1 + \epsilon S_0} + \frac{g_0 S_0 \epsilon s(w) (N_0 - N_{tr})}{(1 + \epsilon S_0)^2} - \frac{g_0 s(w) (N_0 - N_{tr})}{1 + \epsilon S_0} \quad (6.11)$$

$$i\omega s(w) - F_S(w) = \frac{\Gamma g_0 s_0 n(w)}{1 + \epsilon S_0} + \frac{\Gamma g_0 (N_0 - N_{tr}) s(w)}{1 + \epsilon S_0} - \frac{s(w)}{\tau_p} - \frac{\Gamma g_0 \epsilon (N_0 - N_{tr}) S_0 s(w)}{(1 + \epsilon S_0)^2} + 2N_0 B \beta \Gamma n(w) \quad (6.12)$$

$$i\omega \Phi(w) - F_\Phi(w) = \frac{\alpha}{2} \Gamma g_0 n(w) \quad (6.13)$$

where $n(w)$, $s(w)$ and $\Phi(w)$ are the Fourier transforms of $n(t)$, $s(t)$ and $\phi(t)$, respectively. These equations can be solved for $n(w)$, $s(w)$ and $\Phi(w)$ by using the Cramer's rule. The resulting $s(w)$ is given by:

$$s(w) = \frac{(\frac{1}{\tau_n} + \frac{g_0 S_0}{1 + \epsilon S_0} + i\omega) F_S(w) + (\frac{\Gamma g_0 S_0}{1 + \epsilon S_0} + 2\beta \Gamma B N_0) F_N(w)}{-w^2 + iw\gamma + w_r^2} \quad (6.14)$$

where:

$$w_r^2 \equiv \frac{1}{\tau_n \tau_p} + \frac{g_0 S_0}{\tau_p (1 + \epsilon S_0)} + \frac{2N_0 B \Gamma \beta g_0 (N_0 - N_{tr})}{(1 + \epsilon S_0)^2} - \frac{\Gamma g_0 (N_0 - N_{tr})}{\tau_n (1 + \epsilon S_0)^2} \quad (6.15)$$

$$\gamma \equiv \frac{1}{\tau_n} + \frac{1}{\tau_p} + \frac{g_0 S_0}{1 + \epsilon S_0} - \frac{\Gamma g_0 (N_0 - N_{tr})}{(1 + \epsilon S_0)^2} \quad (6.16)$$

The use of some approximations [29] (Chapter 2) allows to simplify the expressions for w_r^2 and γ as:

$$w_r^2 \approx \frac{g_0 S_0}{\tau_p (1 + \epsilon S_0)} \approx \frac{\Gamma g_0 (I_0 - I_{th})}{q V_{act}} \quad (6.17)$$

$$\gamma \approx \frac{1}{\tau_n} + [\tau_p + \frac{\epsilon}{g_0}] \omega_r^2. \quad (6.18)$$

The RIN spectra is defined as [92]:

$$RIN = \frac{S_P(w)}{S_0^2} \quad (6.19)$$

where $S_P(w)$ is the spectral density of noise given by the Fourier transform of the auto-correlation of $s(t)$. Then, by using the Wiener-Khinchin theorem:

$$S_P(w) = \mathcal{F} \langle s(t) s(t') \rangle = \mathcal{F} (s(t))^* \mathcal{F} (s(t)) = s(w)^* s(w) = |s(w)|^2. \quad (6.20)$$

$|s(w)|^2$ can be calculated from Eq. (6.14) by applying to the products $F_i^*(w) F_j(w)$ the correlation theorem

$$F_i^*(w) F_j(w) = \mathcal{F} \langle F_i(t) F_j(t') \rangle = \mathcal{F} (2D_{ij} \delta(t - t')) = 2D_{ij} \quad (6.21)$$

And finally the RIN spectra can be expressed as:

$$RIN = \frac{2D_{ss}}{\Phi S_0^2} \left[\omega^2 + (\Gamma_N - \Gamma_{NN})^2 + \Gamma_{NN}^2 \left(\frac{D_{NN}}{D_{SS}} - 1 \right) - \frac{2\Gamma_N \Gamma_{NN}}{\Gamma} \right] \quad (6.22)$$

where:

$$\Gamma_N \equiv \frac{1}{\tau_n} + \frac{g_0 S_0}{1 + \epsilon S_0} \quad (6.23a)$$

$$D_{SS} \equiv \beta \Gamma B N_0^2 S_0 \quad (6.23b)$$

$$D_{SN} \equiv -\beta B N_0^2 S_0 \quad (6.23c)$$

$$D_{NN} \equiv \frac{\beta B N_0^2 S_0}{\Gamma} + \frac{1}{V_{act}} \left(R(N) + \frac{I_0}{q V_{act}} \right) \quad (6.23d)$$

$$\Phi \equiv \omega^4 + (\gamma^2 - 2\omega_r^2)\omega^2 + \omega_r^4 \quad (6.23e)$$

$$\Gamma_{NN} \equiv \frac{\Gamma g_0 S_0}{1 + \epsilon S_0} + 2\beta \Gamma B N_0 \quad (6.23f)$$

After some algebra, Eq. (6.22) can be expressed in terms of the parameters A' , ω_s , γ and ω_r as:

$$RIN = \frac{A' [\omega^2 + \omega_s^2]}{\omega^4 + (\gamma^2 - 2\omega_r^2)\omega^2 + \omega_r^4} \quad (6.24)$$

where:

$$A' \equiv \frac{2D_{ss}}{S_0^2} \quad (6.25a)$$

$$\omega_s^2 \equiv (\Gamma_N - \Gamma_{NN})^2 + \Gamma_{NN}^2 \left(\frac{D_{NN}}{D_{SS}} - 1 \right) - \frac{2\Gamma_N \Gamma_{NN}}{\Gamma}. \quad (6.25b)$$

These expressions can be further simplified and A' and ω_s can be written as:

$$A' \approx \frac{2B\beta N_{th}^2 q V_{act}}{\tau_p} \frac{1}{I - I_{th}} \quad (6.26)$$

$$\omega_s^2 = \left(\frac{1}{\tau_n} + \tau_p \omega_r^2 \right)^2 + \frac{\Gamma^2 D_{NN}}{D_{SS}} \tau_p^2 \omega_r^4 - 2 \left(\frac{\tau_p \omega_r^2}{\tau_n} + \tau_p^2 \omega_r^4 \right). \quad (6.27)$$

By fitting Eq. (6.24) to the experimental data for different values of I_{bias} , it is possible to establish relationships between the parameters of the model through A' , ω_s^2 , γ and ω_r^2 . However, the usual parameter extraction procedure [170, 171] is based on modelling the RIN spectra in terms of only γ and ω_r^2 and therefore uses the relationships only for I_{th} , γ and ω_r^2 . In this thesis the modelling of RIN in terms of the additional parameters A' and ω_s provides additional relationships between the parameters of the model, thus minimizing the number of assumed parameters and improving the reliability of the model.

Using the same mathematical treatment now for $\phi(w)$ allows to obtain its analytical expression as:

$$\phi(w) = \frac{\frac{\alpha}{2} \Gamma g_0 F_N(w) [i\omega + \varphi] - \frac{\alpha}{2} \Gamma g_0 F_S(w) - F_\phi(w) (\omega^2 - i\omega\gamma - \omega_r^2)}{-i\omega(\omega^2 - i\omega\gamma - \omega_r^2)} \quad (6.28)$$

where

$$\varphi \equiv \frac{1}{\tau_p} - \Gamma G \quad (6.29)$$

and

$$G \equiv \frac{g_0(N_0 - N_{tr})}{(1 + \epsilon S_0)^2} \quad (6.30)$$

The spectral density of frequency noise is defined as:

$$S_{FM} = \phi_P(w) \omega^2 \quad (6.31)$$

where $\phi_P(w)$ is the Fourier transform of the autocorrelation of $\phi(t)$. Then, by using again the Wiener-Khinchin theorem:

$$\phi_P(w) = \mathcal{F}\langle\phi(t)\phi(t')\rangle = \mathcal{F}(\phi(t))^* \mathcal{F}(\phi(t)) = \phi(w)^* \phi(w) = |\phi(w)|^2, \quad (6.32)$$

and using (6.21) and (6.31), the spectral density of the frequency noise can be written as:

$$S_{FM} = F_\phi(w)F_\phi(w)^* + \left(\frac{\alpha\Gamma g_0}{2}\right)^2 \left[\frac{2D_{nn}(\omega^2 + \varphi^2) - 4D_{sn}G\varphi + 2D_{ss}G^2}{\Phi} \right] \quad (6.33)$$

6.3 Parameter extraction

As mentioned in the previous section, the usual approach for the parameter extraction is based on the modelling the RIN spectra in terms of only γ and ω_r [170,171,173]. However, the procedure followed in this thesis goes a step forward, establishing additional relations between parameters, allowed by the modelling of the RIN spectra in terms of the additional parameters A' and ω_s . Most of the parameters were extracted by a combination of the information provided by the CW characterisation and the RIN spectra analysis.

The RIN spectra for different values of I_{bias} were acquired by using the setup presented in Fig. (4.11). It is important to mention, that the raw measurements acquired with the ESA were processed using the following formula [29]:

$$RIN = 10 \times \log_{10} \left(\frac{p_{opt}^2(\omega)}{P_{opt}^2} \right) [\text{in dB}] - 10 \times \log_{10}(RBW [\text{in Hz}]) \quad (6.34)$$

where $p_{opt}(\omega)$ and P_{opt} are the fluctuations of the output optical power in the frequency domain and the averaged optical output power, respectively; and RBW is the resolution bandwidth of the ESA, 3 MHz in all measurements. The total output power was measured with a Ge photodetector using a similar procedure as in the P-I-V measurements. The following conversion was required in order to obtain the real RIN spectra from the measured spectra acquired by the ESA:

$$p_{opt}^2(\omega) = \frac{i_{ph}^2(\omega)}{R_\lambda^2} = \frac{p_{elec}(\omega)}{R_{input}R_\lambda^2} \quad (6.35)$$

where $R_{input} = 50 \, \Omega$ is the impedance of the ESA, $R_\lambda = 0.45 \, \text{A/W}$ the responsivity of the photodetector and $p_{elec}(\omega)$ the PSD measured by the ESA for a given RBW. To properly obtain the correct profile of the RIN spectra, it is necessary to subtract the contribution of the floor noise, as several parameters directly depend on the *plateau* of the RF spectrum at low frequencies. Different noise sources contribute to increase the low frequency noise (thermal, shot noise, etc...) [4]. To reduce these contributions, the RF spectra were measured with and without light, and the spectra were subtracted.

Fig. 6.1 shows a comparison between an "as recorded" RF spectrum (with the contribution of the low frequency noise) and its corresponding processed RIN spectrum for $I_{bias} = 20 \, \text{mA}$. Both spectra show peaks at 3.4 GHz. As it is predicted by Eq. (6.24), the RF spectra should show a resonance peak at the frequency of the relaxations oscillations, and therefore, the peak is attributed to the relaxation oscillations. It is apparent that

the shape of both spectra at frequencies lower than 1 GHz is completely different. In Fig. 6.1(a), the amplitude of the signal increases below 1 GHz while in Fig. 6.1(b), that part of the spectrum is completely flat, since the low frequency contributions have been removed by the subtraction.

Figure 6.1: Semi-logarithmic plots of the RF spectra of the DML at $I_{bias} = 20$ mA (a) "as recorded" and (b) after processing according to Eq. (6.34)

Fig. 6.2 shows the processed RIN spectra of the DML for different values of I_{bias} together with the fitting of the spectra to Eq. (6.24). As it can be seen in Fig. 6.2, two effects appear when the value of I_{bias} increases: i) the main peak shifts towards higher frequencies, which is consistent with the fact that the relaxation oscillation frequency shifts to higher frequencies as the bias current is increased; and ii) the value of the amplitude decreases as it is expected from Eq. (6.26). The measured RIN spectra were fitted to Eq. (6.24) by using a Gauss-Newton algorithm [174]. This algorithm minimises the sum of the squared differences between the values of Eq. (6.24) and the measured RIN, thus determining the parameters w_r^2 , A' , γ and ω_s that provide the best fitting for each value of I_{bias} . The fittings (red line in Fig. 6.2) are good in the entire frequency range, thus giving support to the validity of the analytical expression.

Figure 6.2: Semi-logarithmic plots of the RIN spectra of the DML at different values of I_{bias} ; (a) $I_{bias} = 20$ mA, (b) $I_{bias} = 24$ mA, (c) $I_{bias} = 28$ mA. Blue: experimental measurements. Red Lines: fits of measured RIN spectra to Eq. (6.24).

Fig. 6.3 shows the plots of the extracted values of w_r^2 , A'^{-1} , γ and ω_s^2 and the linear fits to the following expressions

$$w_r^2 = a_1(I_{bias} - I_{th}) \quad (6.36a)$$

$$A'^{-1} = a_2(I_{bias} - I_{th}) \quad (6.36b)$$

Figure 6.3: Parameters extracted from the fit of the RIN expression to the measured spectra for the DML: (a) w_r^2 (blue squares) and $1/A'$ (red squares) vs $I_{bias} - I_{th}$; (b) γ vs w_r^2 ; (c) ω_s^2 vs w_r^2 . The lines correspond to numerical fits.

$$\gamma = a_3 + a_4 w_r^2 \quad (6.36c)$$

$$\omega_s^2 = a_5 + a_6 w_r^4 \quad (6.36d)$$

where a_i for $i = 1, 2, \dots, 6$ are fitting coefficients. These fitting coefficients are directly related with the model parameters as it can be seen from the comparison between Eqs. (6.36) and Eqs. (6.17), (6.18), (6.26) and (6.27), thus providing a set of 6 relationships between the model parameters. However, not all these relationships turned out to be equally reliable. In fact, as it will be commented later in section 6.4.2, when using the set of model parameters verifying the relationships given by the fitting of Eq. (6.36d) (Fig. 6.3) it was impossible to reproduce both the low and high repetition rate OFC spectra for any sensible choice of the assumed parameters. Therefore, it was concluded that, in the range of frequencies of interest, ω_s^2 is almost negligible in comparison with ω^2 , so that the value extracted from the fitting was not reliable and the relationships coming from this fitting were discarded.

The total number of unknown relevant physical parameters of the model for the simulation of GS without OI is twelve (see Table I). Obviously not all of them could be experimentally extracted. After a literature survey, four of them (V_{act} , N_{tr} , Γ and B) were assumed based on reported values for high speed $1.55 \mu\text{m}$ lasers [175], showing similar characteristics. The parameters α and ϵ were obtained by comparing the simulated and measured optical spectra in GS operation at different frequencies, while the other six parameters were calculated from the relationships given by the P-I and RIN measurements as commented before.

For the simulation of GS with OI, the same parameters for the SL and the ML were employed. In order to compare with experiments, an additional term accounting for the reflection of ML signal at the front facet of the SL was included in the spectral calculations, by means of:

$$E_T(t) = \sqrt{P(t)} e^{i\Phi(t)} + \sqrt{R_{SL} P_{inj}(t)} e^{i(\Phi_{inj}(t) + 2\pi\delta\nu' t + \pi)}, \quad (6.37)$$

where $E_T(t)$ is the total optical field, R_{SL} is the front facet reflectivity of the SL and P_{inj} is the injected power by the ML.

In this way, two more parameters are required: the coupling coefficient k_c (see Eq. (3.7)) and the reflectivity of the front facet of the SL, R_{SL} (see Eq. (6.37)). These two additional

parameters were obtained by comparing the simulated and experimental optical spectra under the same OI and driving conditions.

A summary of the parameters employed in the simulations is shown in Table 6.1. The response of the laser was simulated by using the Heun's method to solve numerically the equations of the REM, described in Chapter 3. The internal consistency of the procedure was verified by simulating the RIN spectra at different conditions of I_{bias} using the parameters shown in Table 6.1. The temporal fluctuations of the output power were calculated from the simulated photon density. The average value of the output power was obtained in CW regime and it was subtracted from the temporal trace of the output power. The PSD of the power fluctuations were calculated by means of the Fourier transformation of the temporal trace, properly normalised. The obtained results are plotted in Fig. 6.4 for different values of I_{bias} (in black), and compared with the measured RIN spectra (in blue). The results provide a very good quantitative agreement indicating the validity of the model and of the procedure for parameter extraction. The same set of parameters of Table 6.1 were employed for the simulations of the ML.

Figure 6.4: RIN spectra of the DML for different values of I_{bias} ; (a) $I_{bias} = 20$ mA, (b) $I_{bias} = 24$ mA and (c) $I_{bias} = 28$ mA. Blue: experimental measurements, black line: simulations. (See text for details)

Table 6.1: Summary of main simulation parameters

Symbol	DML Values	Units	Obtained
V_{act}	1.53×10^{-17}	m^3	Assumed [175]
Γ	0.06	-	Assumed [175]
N_{tr}	1.3×10^{24}	m^{-3}	Assumed [175]
B	1.5×10^{-16}	m^3s^{-1}	Assumed [175]
$\frac{dg}{dN}$	4.38×10^{-20}	m^2	RIN + P-I
τ_p	2.17	ps	RIN + P-I
A	2.8×10^8	s^{-1}	RIN + P-I
C	9.0×10^{-41}	m^6s^{-1}	RIN + P-I
β	5.3×10^{-6}	-	RIN + P-I
η_f	0.17	-	P-I
ϵ	1.97×10^{-23}	m^3	Fit to optical spectra
α	3	-	Fit to optical spectra
R_{SL}	0.1	-	Fit to optical spectra
k_c	4.23×10^{10}	s^{-1}	Fit to optical spectra

6.4 Comparison between experiments and simulations

6.4.1 OFC generation by gain-switching without optical injection

The temporal and spectral response of the DML in GS operation was extensively characterised with and without OI for different values of f_R , P_{inj} and $\delta\nu$ and a wide range of values of I_{bias} and V_{RF} . The quality of the OFCs was characterised by means of the 10 dB spectral width (Δf_{10}) and the CNR, as they are defined in Chapter 4. The results of this characterisation were compared with the simulations. In order to properly compare both spectra, the simulated spectra were convolved with the filter response of the BOSA (10 MHz), and they were shifted by adding the measured CW emission frequency at threshold (193.36 THz). Three basic operating conditions were selected: i) high frequency without OI (5 GHz, in the range used in optical communications); ii) relatively low frequency without OI (200 MHz, within the window of the spectroscopic applications); and iii) high frequency (5 GHz) with OI. The measurements were performed using the experimental setup presented in Fig. 4.9.

High Frequency (5 GHz)

Fig. 6.5 shows the experimental and the simulated optical spectra at $I_{bias} = 34$ mA, $f_R = 5$ GHz, and two values of V_{RF} . At this bias current, the relaxation oscillation frequency (f_{RO}) is 6.6 GHz. As it was already shown in Chapter 5, for the lower V_{RF} a high quality OFC with a high CNR = 39 dB and a broad $\Delta f_{10} = 60$ GHz is observed (Fig. 6.5 (a)), while when the applied voltage is increased (Fig. 6.5 (c)), a continuous and noisy spectrum appears, without any appearance of OFC (CNR = 6 dB, $\Delta f_{10} = 70$ GHz). The simulated spectrum at low voltage (Fig. 6.5 (b)) is quite similar to the experimental one in terms of

Figure 6.5: Experimental (left) and simulated (right) optical spectra of the gain-switched DML laser at $I_{bias} = 34$ mA, $f_R = 5$ GHz, and two values of V_{RF} (top row $V_{RF} = 1$ V, bottom row $V_{RF} = 1.8$ V).

shape, flatness, CNR and width ($CNR = 39$ dB, $\Delta f_{10} = 65$ GHz). The simulated spectrum at high voltage (Fig. 6.5 (d)) is also very similar to the experimental one, corresponding to a high and broad noise level with low amplitude tones ($CNR = 6$ dB, $\Delta f_{10} = 80$ GHz). The only clear difference between simulations and experiments is the BOSA background noise, which has not been taken into account in the simulations.

Same similarities between experimental and simulated spectra, including shape, flatness, width and noise level, were found for the complete range of measurements conditions at $f_R = 5$ GHz. This set of driving conditions were chosen to complete the measurements shown in the previous chapter. The comparison between the experimental and simulated spectra is made by means of the colour maps presented in Fig. 6.6, which show the values of CNR, Δf_{10} and the product $CNR \cdot \Delta f_{10}$ for different operating conditions. For constructing these maps the optical spectra of the laser were recorded for 35 values of I_{bias} and 30 values of V_{RF} , accounting to a total of 1050 spectra for each value of the repetition rate. The automatization of the measurement was carried out by an in-house program based on LabVIEWTM. Then, the optical spectra were processed to obtain the value of the CNR and Δf_{10} from each spectrum. The processing was accomplished by means of a home-made MATLABTM program that calculates these parameters. The experimental and simulated maps are qualitatively and quantitatively almost identical, indicating the validity of the model and of the procedure for parameter extraction.

Three different regions can be seen in the maps of the CNR (Figs. 6.6 (a) and (b)). In region A, the modulation amplitude is high enough to switch off the laser, the subsequent pulses build-up from spontaneous emission, and the temporal coherence between pulses is lost. As a result, the generated spectra show high values of Δf_{10} (Figs. 6.6 (c) and (d)) but CNRs lower than 10 dB. Region B is a narrow transition region, and in region C,

Figure 6.6: Colour maps with the dependence on V_{RF} and I_{bias} of the experimental (left) and simulated (right) spectral characteristics for $f_R = 5$ GHz: (a) and (b), CNR; (c) and (d), Δf_{10} ; and (e) and (f), $\text{CNR} \cdot \Delta f_{10}$.

OFCs with high CNR (50-60 dB) are generated. The transition between regions A and C is very abrupt, and it takes place at higher V_{RF} when I_{bias} is increased. At constant V_{RF} , the width of the OFCs in region C is maximum close to the transition region (Figs. 6.6 (c) and (d)); in this case, the optical power between pulses is very low and the carrier swing is maximum, yielding the broadest OFCs. It is clear that this border region produces the best quality OFCs, as it can be clearly observed in Figs. 6.6 (e) and (f), where the maximum product $\text{CNR} \cdot \Delta f_{10}$, considered as the figure of merit, is observed within a narrow region close to the border. It is interesting to observe in Figs. 6.6 (c) and (d), that for a contour of equal Δf_{10} (equal colour) there is a bias current yielding a minimum value of V_{RF} ; for this bias, between 25 and 30 mA, the relaxation oscillation frequency is close to the repetition frequency ($f_{RO} = 4.8$ GHz and $f_{RO} = 5.9$ GHz, respectively) and therefore the modulation efficiency is maximum, yielding broader OFCs than those generated at higher bias.

The simulation results can be used as a guideline to select the optimal driving con-

Figure 6.7: Temporal profiles at the same GS conditions as in Fig. 6.5 ($I_{bias} = 34$ mA, $f_R = 5$ GHz and $V_{RF} = 1$ V (left) and 1.8 V (right)): (a) and (b), experimental power; (c) and (d), simulated power (black) and chirp (orange); (e) and (f), simulated carrier density (black) and calculated threshold carrier density (orange); and (g) and (h), simulated current (black) and threshold current (orange)

ditions to obtain the best OFCs in terms of the product $CNR \cdot \Delta f_{10}$. Within the range of driving conditions previously analysed, Fig. 6.6 (f) indicates a maximum product of around 2600 dB·GHz at the maximum driving voltage (1.8 V) and a bias current of 40 mA. Some simulations were performed at higher values of V_{RF} . The product $CNR \cdot \Delta f_{10}$ continues increasing when the value of the bias is correspondingly increased, without apparent saturation. For instance, for $V_{RF} = 2.6$ V and $I_{bias} = 55$ mA, $CNR \cdot \Delta f_{10}$ reaches 3200 dB·GHz. This result suggests the use of electrical RF amplification stages to improve the quality of the OFCs, with the experimental limit of the internal heating of the device.

The excellent agreement between simulations and experiments indicates that the analysis of the internal variables can be used for a better understanding of the real physical processes involved in the OFC generation. This has been done by plotting the temporal profile of the injected current, the output power, the carrier density and the frequency chirp. An example is provided in Fig. 6.7, where the temporal response of the internal variables, corresponding to the spectra shown in Fig. 6.5, is presented. The measured temporal trace of the output power is also plotted for comparison. For lower V_{RF} , the temporal trace presents narrow optical pulses (FWHM = 42 ps) with a high value of $P_{max} = 11.5$ mW (Fig. 6.7(a)). As it is expected, when V_{RF} increases P_{max} is higher

($P_{max} = 14.8$ mW), and the value of FWHM decreases (29 ps). The simulated temporal pulses present a similar evolution of the pulse characteristics with the driving conditions than the experiments. The optical power increases from 13 mW (Fig. 6.7(c)) to 18.6 mW (Fig. 6.7(d)), with a reduction of the FWHM (from 34 ps to 24 ps). The comparison between simulated and experimental temporal power profiles indicates small differences in the peak power and the FWHM. Besides, the small irregularities observed at the falling edge of the experimental pulses are not reproduced by the simulations. However, the general features are again similar.

The temporal responses of the carrier density, frequency chirp and output power in Fig. 6.7 correspond to the well known GS process [3,74,92,176]: non-sinusoidal variations of the carrier density around N_{th} are induced by the sinusoidal modulation of the total current. As a result, the quick change in the state of the laser (on and off) is capable of producing short optical pulses. The emission frequency follows the variations of the carrier density due to the carrier density dependence of the refractive index. The frequency and power variations are not in phase, with the frequency being a fraction of the period in advance, thus yielding a relatively small difference in the power of the spectral components, and therefore a relatively flat spectrum. It is clear that in this case the so-called dynamic chirp, caused by transient effects, is the main physical effect producing the broadening of the spectra. This is consistent with the fact that, in Figs. 6.5(a) and (b), the red tail of the spectra falls more abruptly than the blue tail, as the maximum chirp is produced during the rising part of the optical pulse.

The difference between the spectra in Fig. 6.5 for different modulation voltages can be understood by analysing the temporal profiles shown in Fig. 6.7. For a higher modulation voltage the current during the negative cycle is lower than the threshold current (see Fig. 6.7 (h)), producing a deep carrier depletion (see Fig. 6.7 (f)). Then, the optical pulse is switched off and the random spontaneous emission is predominant; the calculated chirp shows random fluctuations as it can be observed in Fig. 6.7 (d)). As it was discussed in [80], the new pulse is built-up from spontaneous emission without any phase correlation with the previous pulse, losing the temporal coherence, and thus, showing absence of tones in the optical spectrum.

Low frequency (200 MHz)

In the following paragraphs, the characteristics of the OFCs generated at a relatively low frequency $f_R = 200$ MHz are analysed.

In Fig. 6.8, the experimental and the simulated spectra are compared at driving conditions producing one of the best quality OFCs at this frequency ($I_{bias} = 60$ mA, $V_{RF} = 1.5$ V). The similarity between the experimental spectrum shown Fig. 6.8 (a) (CNR = 30 dB and $\Delta f_{10} = 8.6$ GHz.) and the simulated spectrum shown in Fig. 6.8 (b) (CNR = 29 dB, and $\Delta f_{10} = 8.4$ GHz) is remarkable. Both are asymmetric, with higher intensities at lower wavelengths, and both show the typical suppression of tones characteristic of frequency modulation [105]. The difference between the experimental and the simulated emission wavelengths is attributed to an increase in the device temperature caused by the sinusoidal excitation. The model does not consider this additional heating.

The comparison between experiments and simulations at this repetition frequency was

Figure 6.8: Experimental (left) and simulated (right) optical spectra at $I_{bias} = 60$ mA, $f_R = 200$ MHz and $V_{RF} = 1.5$ V.

performed in a wide range of operating conditions. Again, CNR and Δf_{10} were used as parameters for the characterisation of the spectra. The results are shown in the colour maps of Fig. 6.9. An excellent agreement between simulations and experiments is again observed for the complete range of I_{bias} and V_{RF} .

Figs. 6.9 (a) and (b) show the experimental and simulated value of the CNR. Two different regions (A and B) can be clearly distinguished: in A, the values of CNR are below 10 dB corresponding to noisy and almost continuous spectra, with a total absence of tones. In this regime, the current modulation switches off the laser during a significant part of the modulation period, and in the rest of the period the output power follows the modulation current. This produces incoherent spectra, although with a different shape than the noisy spectra at high frequency previously commented. An example of this can be found in Fig. 5.3(d). On the other hand, in B, the laser is actually modulated above threshold and OFCs are achievable, as it is shown in Fig. 6.8.

Figure 6.9: Colour maps with the dependence on V_{RF} and I_{bias} of the experimental (left) and simulated (right) spectral characteristics for $f_R = 200$ MHz: (a) and (b), CNR; (c) and (d), Δf_{10} ; and (e) and (f), $CNR \cdot \Delta f_{10}$.

As commented in Chapter 5, in this regime the laser is actually working under direct modulation and not in real GS conditions, although the term gain-switching is used as it is usual in this context. In this region, the spectral width increases with the modulation amplitude, but it does not depend on the bias current, as it can be clearly seen in Figs. 6.9 (c) and (d), where the colour contours corresponding to a constant Δf_{10} are parallel to the abscise axis. This fact is a clear difference with respect to the high frequency maps of Figs. 6.6 (c) and (d), and it is a consequence of using a frequency much lower than the relaxation oscillation frequency. This implies that a different mechanism is involved in the OFC generation, adiabatic chirp instead of dynamic chirp, as it will be detailed later. The product of CNR and Δf_{10} , shown in Fig. 6.9 (e) and (f), also indicates that the best OFCs are obtained at the highest V_{RF} , irrespective of the I_{bias} , but fulfilling the condition of not switching-off the laser.

An additional analysis of the temporal profiles was performed to support the results of the spectral characterisation. For these driving conditions ($I_{bias} = 60$ mA and $V_{RF} = 1.5$

Figure 6.10: Temporal profiles for $I_{bias} = 60$ mA, $f_R = 200$ MHz and $V_{RF} = 1.5$ V: (a) simulated current (black) and threshold current (orange); (b) simulated carrier density (black) and calculated threshold carrier density (orange) and (c) simulated output power (black) and chirp (orange).

V), the temporal profile of the output power of the laser corresponds to a sinusoidal profile with $P_{max} = 11.2$ mW. The variations of the internal photon density produce changes in the gain through the non-linear gain coefficient (see Eqs. (6.1)-(6.2)), and consequently variations of the carrier density to maintain the gain equal to loss condition. At this frequency, the carriers are able to follow the gain variations, and an almost sinusoidal carrier profile is obtained (Fig. 6.10(b)). As the frequency chirp (Fig. 6.10(c), orange line) is proportional to the carrier density, its temporal profile follows the same evolution. Here, as the repetition rate of the GS signal is much lower than the frequency of the relaxations oscillations, the frequency chirp only changes due to the adiabatic variation of the carrier density, the so-called adiabatic chirp. In this case, as it can be observed in Fig. 6.10 (c), the optical power profile and the frequency chirp are in-phase. Then, lower frequencies correspond to lower optical power, and the spectra exhibit the previously commented asymmetry, with higher suppression for those tones at the red side of the spectra (see Fig. 6.8).

It is interesting to comment that the initial use of Eqs. (6.36d) together with Eqs. (6.36a)-(6.36c) for the parameter extraction, led to a value of ϵ around half of the value used in the final simulations. Using that value of ϵ , the shape of the simulated low frequency spectra was similar, but the spectral width was around half of the experimental one. Then, in order to reproduce the measured width, the possibility of a temperature induced adiabatic chirp was considered as an additional physical mechanism contributing to the OFC generation. In order to account for this possibility, a dynamical thermal equation was included (see Eq. (3.29)). The thermal solution provided broader spectra, but the asymmetry of the spectra in Fig. 6.8 disappeared. The reason of this result is clear, temperature variations are expected to be in phase, or slightly out of phase with respect to the current, and therefore the temperature induced chirp would be in phase opposition to the carrier induced chirp, thus correcting the asymmetry of the optical spectra. After this analysis, it is concluded that the thermal effects are not relevant to explain the OFC generation of this particular laser at this operating frequency.

6.4.2 OFC generation by gain-switching with optical injection

In this section, the results of the comparison between experimental and simulated OFCs generated by the combination of GS and OI are shown and discussed. A systematic

characterisation of the laser in CW operation under external injection was carried out, showing a wide range of dynamical regimes, similar to [42]. Here only operating conditions producing high frequency broad noisy spectra without injection are analysed (similar to those of Figs. 6.5(c) and (d)), in order to determine the best injection conditions to improve the OFC quality. The experimental results have been obtained by using a scheme similar to that shown in Fig. 4.9(b). The evolution of CNR and Δf_{10} in a wide range of P_{inj} and $\delta\nu$ have been analysed in the same GS conditions of Fig. 6.5 ($I_{bias} = 32$ mA, $f_R = 5$ GHz, $V_{RF} = 1.8$ V). For the realisation of this comparison, it was necessary to measure the experimental OFC in 30 conditions of $\delta\nu$ and P_{inj} , resulting in colour maps of 900 points. 900 simulations at different OI conditions were performed in order to sweep the same area than in the experiments.

Figure 6.11: Experimental (left column) and simulated (right column) optical spectra with OI for fixed GS conditions ($I_{bias} = 32$ mA, $f_R = 5$ GHz and $V_{RF} = 1.8$ V), using different values of P_{inj} and $\delta\nu$, (a) and (b), $P_{inj} = 0$ dBm and $\delta\nu = 2$ GHz, (c) and (d), $P_{inj} = -36$ dBm and $\delta\nu = 2$ GHz, (e) and (f), $P_{inj} = 0$ dBm and $\delta\nu = -66$ GHz. The arrow indicates the injection frequency.

Fig. 6.11 shows the experimental (left) and the simulated (right) optical spectra for fixed driving conditions ($I_{bias} = 32$ mA, $f_R = 5$ GHz, $V_{RF} = 1.8$ V) and different injected conditions. The optical spectra in Figs. 6.11(a) and (b), generated using a low detuning ($\delta\nu = 2$ GHz) and an intermediate injected power ($P_{inj} = 0$ dBm), are very similar, shown high-quality OFCs with high values of CNR and Δf_{10} (CNR = 45 dB and 46 dB, and

$\Delta f_{10} = 60$ GHz and 60 GHz, for the experiments and simulations, respectively). Under these operating conditions, without OI, the pulses from the slave laser are built-up from the spontaneous emission, so that, the corresponding optical spectrum is does not show an OFC. However, when OI is applied, the ML photons are predominant, thus fixing the initial phase of each pulse and resulting in the coherence of the pulse train and in the high quality of the OFC [99]. When the injected power is decreased ($P_{inj} = -36$ dBm, see Figs. 6.11(c) and (d)), the spectrum shows a clear OFC with low CNR (10 dB) above the noisy spectral envelope, indicating that there is a clear competition between the spontaneous emission and the injected field to build up the emitted pulses: some pulses are locked to the external injection while others are not. Here, some difference between the simulated and experimental spectra are clearly visible. In the case of the experimental spectra, the tones are distributed with similar amplitudes along the noisy envelope, i.e, the OFC is rather flat. However, in the simulated OFC the amplitude of each tone decreases asymmetrically.

A similar phenomenon can be clearly observed in Figs. 6.11(e) and (f), corresponding to an intermediate value of injected power ($P_{inj} = 0$ dBm) and a large detuning ($\delta\nu = -66$ GHz). In both cases, the emission spectra do not show OFCs. The line at the ML frequency is due to the combination of the reflection at the SL facet, together with internal amplification giving rise to its sidebands, due to the gain modulation. Broad noisy spectra, due to unlocked emission of the SL, can be observed in experiments and simulations. Under these conditions, the coupling between the ML and the SL dramatically decreases because of the large value of the detuning. The experimental results shown in Fig. 6.11(e) could not be quantitatively reproduced in simulations (Fig. 6.11(f)) without taking into account the effect of reflection term in the spectral calculations.

In order to analyse the conditions of the OI to provide high quality OFCs, the evolution of CNR and Δf_{10} was studied in a wide range of P_{inj} and $\delta\nu$, for the GS conditions of Fig. 6.11 ($I_{bias} = 32$ mA, $f_R = 5$ GHz, $V_{RF} = 1.8$ V). The experimental and simulation results are compared in the colour maps of Fig. 6.12. The evolutions of CNR and Δf_{10} as a function of the OI conditions are shown in Figs. 6.12 (a) and (b); and (c) and (d), respectively. Two regions, A and B, with different detuning can be distinguished. In region A, ($50 > \delta\nu > -50$ GHz in the simulations and $40 > \delta\nu > -60$ GHz in the experiments), the OI influences the SL emission above a certain power level. At low power levels, broad spectra similar to those shown in Fig. 6.5 (c) and (d) appear. As the injected power is increased, the coupling between the master and the slave laser improves and so the coherence of the pulse trains, leading to OFCs with low values of CNR (Figs. 6.11 (c) and (d)) over broad noisy spectral profiles similar to those of the non-injected GS laser (Figs. 6.5 (c) and (d)). At higher injected power, the generated OFCs show high quality in terms of CNR and Δf_{10} (Figs. 6.11 (a) and (b)). The power level needed to achieve a fixed value of the CNR increases with the detuning as indicated by the constant colour contours in Figs. 6.12 (a) and (b). Injection levels around 0 dBm and detuning around 0 GHz are the best injection conditions for generating a high quality OFC from the non-injected broad spectrum. At higher injection levels, the spectrum shows a high CNR but a narrower width, as well as poor flatness. The difference between experiments and simulations regarding the detuning range separating the two regions, (around 10 GHz towards negative detuning in the case of experiments), is attributed

to the temperature increase caused by the sinusoidal excitation in comparison with the reference CW conditions.

Figure 6.12: Colour maps with the dependence on P_{inj} and $\delta\nu$ of the experimental (left) and simulated (right) of: (a) and (b) CNR , (c) and (d) δf_{10} , and (e) and (f) $CNR \cdot \Delta f_{10}$ for $f_R = 5$ GHz with a GS DML at $I_{bias} = 32$ mA, $f_R = 5$ GHz, $V_{RF} = 1.8$ V

In the region of high detuning (region B), the injected light has no effect on the noisy envelope, generating a spectrum very similar to that shown in Figs. 6.11 (e) and (f), with the ML emission and a broad and continuous spectrum. The high values of Δf_{10} at medium power (red regions in Figs. 6.12 (c) and (d)) are an artefact caused by the definition of Δf_{10} , as the considered spectral width is calculated taking into account not only the width of the noisy envelope, but also the difference between the SL emission and the CW tone of the injected light.

Figs. 6.12 (e) and (f) show the product $CNR \cdot \Delta f_{10}$, and they are a useful guide to select the best injection conditions to optimise the OFCs generated by the combination of GS and OI. As it was previously mentioned, the spectral performance of the generated OFCs can be improved in a relatively broad range of detuning (around $|\delta\nu| < 50$ GHz) and a broad range of injection levels (P_{inj} between -20 and 0 dBm, depending on the detuning).

The optimum range, given by the red regions in Figs. 6.12 (e) and (f), is not critical, thus being easy to find suitable injection conditions. The degradation of the product $\text{CNR} \cdot \Delta f_{10}$ by increasing the injected power above 0 dBm, caused by the reduction of the spectral width, is due to the reduction of the chirp by the external OI. However, if the maximum value of $\text{CNR} \cdot \Delta f_{10}$ is compared without and with OI, a higher value (3300 dB·GHz) is observed in the case of OI. The reason for this is that in the case of OI, at constant V_{RF} , it is possible to use lower bias and therefore, the carrier excursion and the corresponding chirp increases.

Figure 6.13: Simulated optical spectra of a GS laser driven at $I_{bias} = 32$ mA, $f_R = 10$ GHz and $V_{RF} = 1$ V under OI at different P_{inj} and $\delta\nu$: (a) $P_{inj} = -2$ dBm and $\delta\nu = 16$ GHz; (b) $P_{inj} = 2$ dBm and $\delta\nu = 16$ GHz, (c) $P_{inj} = 12$ dBm and $\delta\nu = 20$ GHz

The simulations have also reproduced some of the dynamic regimes observed experimentally under GS with OI shown in Section 5.2.3. The simulated optical spectra in different conditions of P_{inj} and $\delta\nu$, with fixed electrical conditions ($I_{bias} = 32$ mA, $f_R = 10$ GHz and $V_{RF} = 1$ V) are plotted in Figs. 6.13, illustrating the different dynamical regimes. In these driving conditions the optical spectrum without injection corresponds with a proper OFC, similar to the spectrum shown in Figs. 6.5 (b), but with a tone separation of 10 GHz. As it is illustrated in Fig. 6.13(a), the OI conditions lead to double period dynamics, where the separation between tones is half the repetition rate of the GS electrical signal, i.e, 5 GHz. In other condition of OI (see Fig. 6.13(b)), changing slightly the injected power, it is possible to generate an OFC whose tones are separated 2.5 GHz. In the last example, the optical spectrum shown in Fig. 6.13(c) corresponds to an incoherent and broad spectrum, as the phase noise is very high with some tones separated 10 GHz. The presence of tones over the incoherent spectra is an indicative of chaotic dynamics with strong periodic components. A systematic study of the different dynamic regimes in gain-switching conditions with OI is beyond the scope of this work.

Chapter 7

OFC generation by pulsed gain-switching and optical injection

Introduction

Having emphasized in chapter 6 the difficulties for achieving high quality OFCs at low repetition rates by sinusoidal GS, in this chapter it is demonstrated that the combination of pulsed GS and OI results in the generation of broad and flat OFCs at low frequencies, making them appropriated candidates for applications such as high resolution absorption spectroscopy. This chapter presents a systematic study of the properties of the OFCs generated in semiconductor lasers by pulsed GS and emphasises the crucial role played by OI in the generation process. The findings reported here are a significant step forward in the direction of extending the performance range of the OFCs generated by GS to lower frequencies, while keeping simplicity as the distinct feature of the GS method.

Section 7.1 provides a short review of the state of the art in OFC generation by gain-switching with electrical pulses prior to this thesis, as well as the rationale for the combination of pulsed GS and OI as a method for the generation of high quality OFCs at low frequencies. Section 7.2 presents the temporal and spectral characterisation of a gain-switched DML when modulated with the low repetition rate electrical pulses, provided by a pulse pattern generator. The combined effects of GS and OI in the improvement of the characteristics of the low frequency OFCs are demonstrated in this section. Finally, Section 7.3 is devoted to describe an optimised operation mode for generating high-quality low repetition rate OFCs, in which the PPG is substituted for a step-recovery diode as the pulsing source.

Some of the results shown in the first section of this chapter have been published in [177].

7.1 State of the Art and rationale

As already commented in previous chapters, the general approach to produce high-quality OFCs in GS semiconductor lasers is based on using a bias current superimposed to a sinu-

soidal waveform [3, 74, 144]. This approach is only fully successful when the frequency of the sinusoidal gain-switching signal is of the order of the relaxation oscillation frequencies as it was shown in chapters 5 and 6. For low frequencies (< 1 GHz) the range of driving conditions for getting broad and well resolved combs is very small. Only at high values of I_{bias} (well above the threshold level) and the highest values of V_{RF} avoiding switching off the laser, OFCs such as those shown in Figs. 5.3(g) and 6.8 can be obtained. These optimised OFCs, apart from being relatively narrow, are plagued with imperfections such as bad planarity, asymmetry and tone suppressions. Out of this small range either the pulses are spectrally narrow when the laser is just modulated, or there is a switch-off period long enough for the coherence to be lost, resulting in a spectrum without discernible lines.

And yet, it is in this low repetition frequency range where OFCs generated by GS have recently found interesting advantages in dual-comb optical spectroscopy [90]. These advantages can only be fully exploited if the number of spectral lines of the OFC and the separation between them can sustain the high resolution in a sufficiently broad spectral range which is characteristic of dual-comb spectroscopy. The authors in [90] illustrate the potential of their approach with the OFC generated by a 1 GHz sinusoidal RF current. Admittedly, they had to make a meticulous optimization of the switching and optical injection conditions in order to get a comb with enough number of lines. However, when higher resolution (lower repetition frequency) is required, the approach is clearly hindered by these critical conditions, thus compromising the usefulness of the OFC generated by sinusoidal GS for this application.

In this context, the idea of making GS at these frequencies with a low duty-cycle pulsed signal instead of using sinusoidal driving looks attractive for two reasons: first, such pulsed signal could switch-off the laser after exciting the first relaxation oscillation, thus making real GS and producing much shorter pulses than those produced when the sinusoidal excitation only modulates the laser. Therefore, the spectrum of these pulses is expected to be broader. And second, in addition, the fast dynamics of the current injection along the excitation pulse is expected to activate the dynamics of the carrier and photons in a similar way as when the laser is driven by a high frequency sinusoidal signal. This is expected to be beneficial for a broader spectrum since the dynamic component of the chirp activated in this case has been demonstrated (Chapter 6) to be much higher than the adiabatic one which is the only component in the case of low frequency sinusoidal driving.

In fact, since the early times of GS, not only sinusoidal RF excitation but also pulsed excitation conditions, have been regularly used for generating short optical pulses at a wide range of wavelengths and repetition frequencies [19, 35, 123, 137, 178–181]. Several types of pulse sources have been used. Among them, step-recovery diodes [19, 123, 178, 179, 182] have been employed to generate narrow optical pulses with different pulse widths (10 - 100 ps) in semiconductor lasers emitting in the spectral regions of 1300 and 1550 nm. However, although the spectral width of the trains of pulses generated by pulsed GS has been analysed and manipulated [35, 127, 136, 138], the coherence between pulses and their potential as OFCs had not been studied. In spite of the expected advantages of pulsed GS commented in the previous paragraph, and taking into account that the coherence relies on the preservation of stimulated emission during the switch-off period as a seed out of which to build up the next pulse, a poor coherence is expected from low repetition

frequency trains of pulses having long switch-off periods between them. Therefore, broad unresolved spectra rather than high quality OFCs are expected from just pulsed GS. It is at this point where OI and its demonstrated capacity for preserving the coherence of trains of optical pulses (sections 5.2.3 and 6.4.2), comes to the rescue of the whole argument making plausible the generation of high quality OFCs by a combination of GS and OI. This possibility has been explored and demonstrated as a part of this thesis. The results are reported in the next two sections.

7.2 OFC generation with a pulse pattern generator

Figure 7.1: Temporal traces (upper row) and optical spectra (lower row) of the DML emission under pulsed GS for fixed $V_{RF} = 0.9$ V and $f_R = 500$ MHz and different values of I_{bias} . Each column corresponds to the value of I_{bias} labelled on the top

An experimental setup similar to that employed in Chapter 5 (section 5.3) has been used to generate OFCs under pulsed GS. The RF signals in these experiments were pulses with different pulse widths (τ_{elec}) and peak amplitudes (V_{RF}), provided by a PPG (Anritsu MU181020A).

The initial experiments were performed without OI (the setup can be found in Fig. 4.9(a)). Fig. 7.1 shows the temporal profiles and the optical spectra recorded when pulses of 100 ps with a repetition rate of $f_R = 500$ MHz, $V_{RF} = 0.9$ V and different values of I_{bias} are used. For $I_{bias} = 14$ mA, slightly below the threshold current, short optical pulses (FWHM = 50 ps) are generated with $P_{max} = 5.7$ mW (Figs 7.1(a)). The corresponding spectrum is broad and continuous, with a total absence of peaks (Figs 7.1(d)). Under these driving conditions, the laser is driven below the threshold level for the elapsed time between pulses and the emission is incoherent. When I_{bias} is slightly increased ($I_{bias} = 18$ mA), the laser is not any more switched but modulated above the threshold, producing optical pulses with higher values of FWHM (55 ps) and high peak power ($P_{max} = 8.65$ mW)(see

Fig. 7.1(b)). The train of pulses includes a CW component as well as secondary pulses due to the secondary relaxation oscillation. The optical spectrum shown in Fig. 7.1(e) corresponds to an irregular OFC with three tones in the first 10 dB ($\Delta f_{10} = 1$ GHz) with a CNR of 18 dB. Finally for $I_{bias} = 26$ mA (Fig. 7.1(c) and (f)), the generated optical pulses show similar characteristics than those observed in the previous case, but with lower P_{max} ($P_{max} = 6.6$ mW) and higher FWHM (70 ps). As the bias current is almost twice the value of the threshold current, the strong CW component is always present. In addition, tiny pulses due to the secondary relaxation oscillations appear together with the main pulse. As in the previous case, the spectrum is narrow ($\Delta f_{10} = 1.5$ GHz) with a high value of CNR (CNR = 32 dB). The shape of the optical pulses and the spectra are attributed to the combined effect of the predominant CW component of the emission and the dynamical behaviour of the laser during the switch-on process, as it is described in the previous chapter. This effect is illustrated by the simulations shown in Fig. 7.2. The

Figure 7.2: Simulated temporal profile of the output power (blue) and carrier density (red) at the same GS conditions as in Fig. 7.1: $V_{RF} = 0.9$ V and $f_R = 500$ MHz and different values of I_{bias} . Each column corresponds to the value of I_{bias} labelled on the top

figure shows in terms of the simulated output power and carrier density of the laser for $V_{RF} = 0.9$ V, $f_R = 500$ MHz and different values of I_{bias} . The simulations were performed using the REM described in Chapter 6 with the parameters obtained for the DML. In this case, the sinusoidal input signal was substituted by electrical pulses with 100 ps of pulse width, reproducing the driving conditions of the experiments.

When the bias current is below threshold (Fig. 7.2(a)), the quick change in the carrier density produced by the electrical pulse generates a short optical pulse, slightly out of phase with respect to the carrier density, giving rise to the dynamical behaviour of the laser. When I_{bias} is increased above threshold (see Fig. 7.2(b) and (c)), the laser remains in the on-state for the whole modulation cycle, and therefore, the dynamic response of the carrier density is lower. Also, the carrier density shows the typical relaxation oscillations, which are characteristics of the switch-on process (see Chapter 2). The profile of the output power corresponds to a narrow optical pulse followed by a smaller and broader pulse, which is attributed to the secondary relaxation oscillations. As I_{bias} increases, the frequency of the relaxation oscillations increases and the stationary state between pulses is reached sooner. Consequently, the carrier density and therefore, the instantaneous frequency remain constant, thus explaining the narrowing of the optical spectra of Figs. 7.1(b) and (c). The appearance of multiple tones in these spectra is due to dynamic chirp during the switch-on process.

The measured evolution of the CNR and Δf_{10} as a function of I_{bias} and V_{RF} is shown by the colour maps in Fig. 7.3. Two regions can be distinguished in Fig. 7.3(a). The

Figure 7.3: Colour maps showing the dependence on V_{RF} and I_{bias} of the spectral characteristics of the pulses generated by pulsed GS with a PPG at $f_R = 500$ MHz: (a) CNR and (b) Δf_{10}

first region, coloured in blue, is constrained to the left part of the map (low I_{bias}) almost independently of the value of V_{RF} . The optical spectra in this region are similar to that shown in Fig. 7.1(d), namely, broad and showing no discernible tones. In the red region the CNR takes high values but the spectral width of the OFCs is always low. The laser is modulated above the threshold and narrow OFC are generated, as it can be seen in Fig. 7.1(f). The maximum value of the figure of merit $CNR \cdot \delta f_{10}$ is 65 dB GHz, much lower than the best value obtained at this frequency using sinusoidal excitation (~ 300 dB GHz). Although optical pulse generation is achievable, these results show the impossibility of generating high-quality OFC just using electrical pulses.

Figure 7.4: Temporal trace of the output power (a) and optical spectrum (b) of the light emitted by the laser under 500 MHz pulsed electrical excitation ($I_{bias} = 14.4$ mA and $V_{RF} = 1.1$ V) and OI ($P_{inj} = -14$ dBm, $\delta\nu = 16$ GHz). The injection frequency is indicated by an arrow in (b). 3(c) is a zoom of 3(b).

Here is where the role of the OI becomes crucial. Fig. 7.4 illustrates this role for specific OI conditions. Same electrical driving conditions as in Fig. 7.1(a) and (d) were used, with an injected optical power $P_{inj} = -14$ dBm at a detuning $\delta\nu = \nu_{ML} - \nu_{SL} = 16$ GHz. In this case, although the FWHM of the optical pulses ($\Delta t = 40$ ps)(Fig. 7.4(a)) is lower than the width achieved without OI (Fig. 7.1(a)), an extremely broad and flat OFC is obtained, as shown in Fig. 7.4(b), in contrast with the unresolved spectrum of Fig. 7.1(d). The CNR is 37 dB and δf_{10} is 54 GHz (corresponding to 108 lines), a factor 12 higher than that of the best OFC obtained at this frequency with sinusoidal excitation. Furthermore, the flatness reaches 54 tones at 3 dB instead of the 3 tones observed in the case of sinusoidal excitation, without OI. The measured linewidth of each tone is ~ 10 MHz, limited by

the BOSA resolution (Fig. 7.4(c)). However, the real linewidth is expected to be close to 75 kHz, the linewidth of the ML. This is expected based on experimental evidence of a transfer of the linewidth to the SL tones close to the injection frequency, as reported for the OFCs generated by GS at higher repetition rates [120], although some broadening cannot be ruled out for far apart tones. In addition to the dynamic chirp in the fast GS process, the origin of the high-quality of this broad OFC is the OI. Now the externally injected photons are acting as a seed out of which the GS pulses grow coherently.

Figure 7.5: Experimental optical spectra under pulsed GS conditions ($I_{bias} = 14$ mA, $f_R = 500$ MHz, $V_{RF} = 1.1$ V) and different conditions of OI: (a) $P_{inj} = 12$ dBm and $\delta\nu = 9$ GHz and (b) $P_{inj} = -24$ dBm and $\delta\nu = -30$ GHz. The injection frequency is indicated with an arrow.

Not surprisingly, the improvement of the OFC quality under OI in the case of pulsed GS operation, far from being a general rule, depends very much on the injection conditions. As an illustrative example, Fig. 7.5 shows the resulting spectra after subjecting the pulse gain-switched DML to two optical injection conditions. In both cases the GS conditions are the same as the conditions giving rise to the OFC shown in Fig. 7.4. However, the spectra are very different from the spectrum of Fig. 7.4(b). A significant increase of the injected power ($P_{inj} = 12$ dBm vs. $P_{inj} = -14$ dBm) at a not very different detuning results in a much narrower OFC (a single tone in the first 10 dB) as seen in Fig. 7.5(a). On the other hand, when the detuning is made large such that the injection frequency lies out of the incoherent envelope of the non-injected spectrum, and the injected power is low ($P_{inj} = -24$ dBm), the spectrum remains incoherent (Fig. 7.5(b)) as the injected light is so low that cannot lock the SL emission. The line of the ML in the optical spectrum of the SL is due to the reflection at the SL facet

The evolution of the properties of the generated OFCs by pulsed GS and OI were systematically analysed in a complete set of OI conditions for the same values of I_{bias} and V_{RF} , which give rise to the spectra of Fig. 7.4. The results of this characterisation are presented in Fig. 7.6. Different regions can be distinguished in this figure. In the main region (region A), delimited by -25 GHz $< \delta\nu < 40$ GHz and $P_{inj} < -5$ dBm, the obtained spectra are high-quality OFCs whose characteristics depends on the injection conditions. In general terms, the OFCs generated in this region have high CNR and Δf_{10} . This region can be easily appreciated in Fig. 7.6(c), where the product of CNR and Δf_{10} is plotted as a figure of merit. Here is where the product of both quantities is higher than 750 dB GHz.

For high absolute value of the detuning ($\delta f_{10} < -25$ GHz, $\delta f_{10} > -40$ GHz), there

Figure 7.6: Colour maps showing the dependence on P_{inj} and $\delta\nu$ of the experimental spectral characteristics at $f_R = 500$ MHz, $I_{bias} = 14$ mA, $V_{RF} = 1.1$ V: (a) CNR; (b) Δf_{10} ; and (c) $\text{CNR} \cdot \Delta f_{10}$

is a limiting value of the injected power given by approximately $P_{inj} = -30$ dBm below which the CNR drops down to less than 5 dB and the spectra are broad and incoherent showing only the ML tone at the corresponding side of the spectra (see Fig. 7.5(b)). This is region B. The calculated δf_{10} may reach high values in this region but this is only an artefact caused by the definition of δf_{10} (Section 4.5), since the spectral width is calculated considering the total width of the noise envelope and the difference between the spectral envelope and the ML tone. Finally, region C encompasses the rest of the OI conditions. In this region the calculated CNR is higher than 30 dB but δf_{10} is narrower than 15 GHz. The spectra in this region look like the one shown in Fig. 7.5(a). It should be noticed that the border between region B and C is artificial in the sense that the characteristic spectra of region B (Fig. 7.5-like) appear also in region C below approximately $P_{inj} = -12$ dBm. The only difference between these spectra and those in region B is that difference between the intensity of the ML line and the intensity of the noise envelope is higher than 10 dB. This is again a consequence of the limitation of the parameters CNR and δf_{10} for characterising broad spectral profiles without discernible tones.

For fixed conditions of OI giving rise to high quality OFCs ($P_{inj} = -14$ dBm $\delta\nu = 16$ GHz), the influence of the electric pulse amplitude and width on the characteristics of the optical pulses and the corresponding OFCs was also analysed. For these experiments, it is easier to describe the input variables I_{bias} and V_{RF} in terms of I_{ON} and I_{OFF} . The

relationship between these magnitudes is given by:

$$I_{OFF} = I_{bias} - \frac{4V_{RF}}{Z_0 + Z_L} f_R \tau_{elec} \quad \text{and} \quad I_{ON} = I_{OFF} + \frac{4V_{RF}}{Z_0 + Z_L}. \quad (7.1)$$

where $Z_0 = 50 \, \Omega$ is the internal impedance of the RF source, Z_L the impedance of the laser module ($Z_L \approx 53$, see Section 4.3) and $f_R \tau_{elec}$ the duty cycle of the pulsed electrical signal.

The evolution of the FWHM optical pulse width Δt , δf_{10} and the CNR were characterised as a function of the electrical excitation width τ_{elec} for constant $I_{OFF} = 12.2$ mA and different values of I_{ON} . The results are shown in Figs. 7.7(a)-7.7(c). The optical

Figure 7.7: Evolution of the FWHM of the optical pulse Δt , δf_{10} and CNR, as a function of the electric pulse width τ_{elec} at $f_R = 500$ MHz, $I_{OFF} = 12.2$ mA, $P_{inj} = -14$ dBm, $\delta\nu = 16$ GHz and different values of I_{ON} . The lines are drawn as a guide to the eye.

pulses are narrower for higher excitation amplitude. At constant excitation voltage, the evolution of Δt as a function of the excitation pulse width shows a minimum lying at lower excitation width for higher excitation amplitude (Fig. 7.7(a)). These results are expected from the well known properties of the GS pulses generated under sinusoidal excitation [37]. In this case, at constant repetition frequency, the width of the optical pulses decreases as the amplitude of the excitation signal increases [22, 123, 183]. The decreasing is due to the faster growth of the carrier density that provides the gain needed to build-up the optical pulse and reaches a lower limit related to the photon lifetime [21]. This is also observed in our case where the narrowest pulses are obtained for the highest excitation amplitudes. The minimum value of Δt as a function of τ_{elec} is attributed to two counteracting effects: On one hand, when τ_{elec} decreases the amount of charge injected into the laser decreases as well. Therefore, the accumulated carrier density and the corresponding gain giving rise to the onset of the optical pulse also decrease and consequently, the rise up of the internal photon density becomes slower. But on the other hand, when τ_{elec} increases, the time to switch off the laser also increases and therefore the optical pulses are wider. The minimum shifts towards lower τ_{elec} when increasing the excitation amplitude due to the faster growth of the carrier density that reduces the switch-on time.

The evolution of δf_{10} as a function of τ_{elec} (Fig. 7.7(b)) is similar, but opposite, to that of the optical pulse width Δt . An inverse dependence of δf_{10} on Δt is expected in short optical pulses, but δf_{10} also depends on the amplitude of the frequency chirp in the optical pulse. The broadest OFCs are obtained at the highest excitation amplitude at $\tau_{elec} \sim 90$ -100 ps, which also produces the shortest optical pulses and the maximum chirp. δf_{10} vs. τ_{elec} shows a maximum at a value of τ_{elec} corresponding to the minimum value of Δt . The CNR of the OFCs is high (~ 34 dB) for all electrical injection conditions (Fig. 7.7(c)),

and it does not show a clear dependence on τ_{elec} or on the excitation amplitude.

The generation of OFCs at even lower repetition frequencies which would be useful in applications requiring a higher number of tones in the comb was also investigated. It was observed that at fixed electrical and optical injection conditions the quality of the OFCs degrades when decreasing the repetition rate. The peak arising from the injected optical power becomes higher than the amplitude of the comb tones, which is attributed to the longer time that the laser remains switched-off, i.e., to the low duty cycle. However, if the electric and optical injection conditions are optimised, high-quality OFCs can be also generated at lower repetition rates. The optimization procedure consists of selecting switching conditions leading to a non-injected broad and incoherent spectrum as in Fig. 7.1(d) and then, selecting an injection frequency in the central region of the spectrum but slightly shifted to the low-intensity side of the asymmetric spectrum. The injected power should be as low as required for equalizing the spectrum and high enough for ensuring optical locking. Fig. 7.8 shows, as an example, the optical pulse profile (Fig. 7.8(a)), the corresponding spectrum (Fig. 7.8(b)) and a zoom of the spectrum (Fig. 7.8(c)) generated at a repetition frequency of 100 MHz. The duration of the electrical excitation pulse is 100 ps with $I_{OFF} = 6.5$ mA and $I_{ON} = 156.5$ mA (achieved by an additional electrical amplification stage).

Figure 7.8: Temporal trace of the output power (a) and optical spectrum (b) of the light emitted by the laser under 100 MHz pulsed electrical excitation, $I_{OFF} = 6.5$ mA, $I_{ON} = 156.5$ mA and optical injection ($P_{inj} = -30$ dBm, and detuning, $\delta\nu = 16$ GHz). (c) is a zoom of (b).

The injected power has been reduced to -30 dBm and the detuning is 17 GHz. In this case, and due to the lower average output power, the pulses were measured after optical amplification with an Erbium-doped fiber amplifier. The pulses are short ($\Delta t \sim 34$ ps) and exhibit several relaxation oscillations. The OFC is very broad ($\delta f_{10} = 86.4$ GHz, corresponding to 866 tones), and its CNR value is 18 dB. OFCs at frequencies lower than 100 MHz were also achieved with these techniques, but a low CNR was measured, probably due to the limit imposed by the BOSA resolution.

Finally, although the experiments reported here were made for DMLs, the improvement of the comb quality by the combination of pulsed driving conditions and optical injection is expected to be similar in other single-frequency semiconductor lasers, such as DFBs or VCSELs.

7.3 OFC generation using step-recovery diodes

The analysis presented in the previous section demonstrates the high quality of the OFCs achievable by GS with pulsed sources of optically-injected laser diodes. However, the use of a PPG is not the most practical choice in terms of size and cost of the experimental setup. In this section, a study on the generation of low-frequency OFCs in an externally injected DML using electrical pulses provided by a SRD is reported.

A SRD is a P-I-N diode which special doping profile allows it to generate high-power short electrical pulses when it is modulated with a sinusoidal signal. The dynamic response of the SRDs is illustrated in Fig. 7.9.

Figure 7.9: Transient behaviour of a SRD: (a) Basic schematic circuit; (b) Transient current response;

These devices can store a significant amount of charge while they are biased in the forward direction. When the polarity of the voltage changes, a high reverse current flows for a short time, until all the stored charge in the diode is depleted. After this period, the current flow in the device stops until it is forward biased again. The transition between the non-conductive and the conductive state is typically less than 100 ps, thus producing narrow electrical pulses. A detailed discussion about the physics of these diodes can be found in [184].

The experiments presented in this section were performed during a research internship carried out in the Photonics Systems and Sensing Lab of the School of Electronic Engineering of the Dublin City University, supervised by Dr. Prince M. Anandarajah.

A setup similar to the one shown in Fig. 4.9(b) was used. The PPG was substituted by an assembly consisting of an RF source and the SRD. The SRD when driven with a 100-MHz sinusoidal waveform, provided by a microwave/RF source, produced the short electrical pulses that gain-switched the laser. Alternatively, for comparison purposes a PPG followed by an RF amplifier with a 3 GHz bandwidth was used to produce electrical pulses. The spectral response of the laser was measured with a high-resolution (20 MHz) optical spectrum analyser, using a span of 1.5 nm with an acquisition time of 3.5 s. The temporal profile of the output power was measured using the optical module (53 GHz bandwidth) of a digital communication analyser (Agilent, DCA-J 86100C).

The temporal electrical response of the SRD was characterised separately using the digital communication analyser with an electrical module of 63 GHz bandwidth, while its spectral electrical response was characterised using an electrical spectrum analyser (Rodhe & Schwarz, FSL spectrum analyser) with a 6 GHz-bandwidth.

Fig. 7.10 shows a schematic diagram of the experimental setup for the SRD charac-

Figure 7.10: (a) Experimental setup for the electrical characterisation of the SRD. Experimental temporal trace (b) and RF spectrum (c) of the electrical pulses generated in SRDs

terisation, as well as the temporal profile and the RF spectrum of the electrical pulses generated by the SRD. They were obtained by applying to the SRD a 100 MHz sine wave of peak-to-peak amplitude $V_{RF-input} = 0.7$ V, which was amplified in the internal pre-amplifier up to 11.25 V. The spectra were recorded after attenuating the output signal to a peak-to-peak amplitude $V_{RF-output} = 10$ V, because this amplitude was excellent to provide high-quality OFCs. The measured temporal FWHM of the electrical pulses is around 200 ps. The pulses exhibit a temporal profile that has a large and sharp peak at the beginning, followed by a broad shoulder at the end of the pulse (see Fig. 7.10 (b)). As shown in Fig. 7.10 (c), the electrical comb exhibits tones with very narrow linewidths (< 3 Hz, limited by the resolution of the ESA) that are separated by the repetition rate of the pulses. All of them display similar values of amplitude, except for the 1st and 2nd tone. Also, as clearly seen in Fig. 7.10 (c), the noise floor of the spectrum is higher as the frequency increases, with a visible broadening in the base of the peaks, attributed to timing jitter.

The electrical pulses generated by the SRD were employed in combination with a bias current to generate low repetition rate OFCs from the optically-injected laser. For the comparative analysis with the OFCs generated by using the PPG, an additional amplification stage was used to amplify the voltage provided by the PPG to the required voltage. An example of the optical spectrum is presented in Fig. 7.11. The injection conditions in

Figure 7.11: (a) Experimental optical spectrum of the optically-injected GS laser. The driving conditions are: $f_R = 100$ MHz, $I_{bias} = 1.34$ mA, $V_{RF-output} = 10.5$ V, $P_{inj} = -17$ dBm, and injection wavelength $\lambda_{inj} = 1549.8$ nm. The injection frequency is indicated with an arrow; (b) is a zoom of (a)

this case were $P_{inj} = -17$ dBm and a injection wavelength (λ_{inj}) of 1549.8 nm. Although

the quality of the generated OFC is good in terms of CNR and spectral width and flatness (CNR = 14 dB and $\delta f_{10} = 112$ GHz), it should be noted that the comb shows a non-uniform local CNR. In fact, the local CNRs at the low and high wavelength limits defined by δf_{10} , 1549.5 and 1550.4 nm, are around 17 dB and 5 dB, respectively. As commented previously, the effect of the OI is to lock the SL phase and as a consequence to improve the coherence of the train of pulses. However, it can be observed here that the OI enhancement is not uniform but it is higher in the region close to the ML wavelength, giving rise to a non-uniform local CNR.

In order to ascertain the origin of the non-uniformity along the CNR in the optical spectrum, the spectral behaviour of the GS-DML under OI was simulated. The rate equation model presented in Chapter 3 was employed to simulate the behaviour of the carrier and photon density, including the phase of the optical field for an externally-injected GS single mode semiconductor laser, employing jittered and non-jittered electrical pulses at a repetition rate of 100 MHz. The results of these numerical simulations are shown in Fig. 7.12. Both spectra were simulated using the same driving and injection conditions except for the timing jitter of the electrical pulses. It can be clearly seen that the effect of the different timing jitter is noticeable in the simulated spectra. Using the peak structure located at 1550.3 nm as reference, the number of tones within δf_{10} was found to be similar ($\delta f_{10} = 70$ GHz) in both spectra. However, their mean CNR is quite different. The spectrum in Fig. 7.12 (a) shows a much higher CNR (9 dB) in comparison to the lower CNR (3 dB) in the case of Fig. 7.12 (b). Also, the CNR in the last case is found to be highly non-uniform as in the experiments shown in Fig. 7.11. The results from the simulations indicate that the origin of this effect can be attributed to the timing jitter of the modulating source.

Figure 7.12: Simulated optical spectra of the optically-injected GS laser. The driving conditions were: $V_{RF} = 3.5$ V mA, $I_{bias} = 1$ mA, $f_R = 100$ MHz and $P_{inj} = 12$ dBm with $\delta\nu = 50$ GHz; (a) Using non-jittered electrical pulses and (b) Using 10-ps jittered electrical pulses

In order to check the simulation results, an experimental study of the effect of the timing jitter in the OFC quality was carried out. It was observed that the timing jitter of the electrical pulses generated by the SRD depended on the value of the input voltage of the SRD ($V_{RF-input}$), although the peak-to-peak amplitude of the output pulse ($V_{RF-output}$) remained constant at 10 V. Different electrical pulses with varying levels of timing jitter were generated as described above and used to characterise the timing jitter of the optical pulses generated and their corresponding optical spectra. These results are shown in Fig. 7.13.

Figure 7.13: (a) Evolution of the electrical timing jitter of the pulses generated by the SRD; (b) Evolution of the optical timing jitter of the generated optical pulses in the externally-injected GS laser for different electrical timing jitter conditions; (c) Experimental optical spectra for the externally-injected GS laser for different electrical timing jitter conditions.

The evolution of the timing jitter of the electrical and optical pulses is shown in Fig. 7.13(a) and (b), respectively. For fixed conditions of laser operation ($I_{bias} = 4.08$ mA, $V_{RF-output} = 10.5$ V and $P_{inj} = -6$ dBm and $\lambda_{inj} = 1550.06$ nm), the timing jitter of the optical pulses follows the same trend as the timing jitter of the electrical pulses, reaching a minimum value of 24.9 ps for $V_{RF-input} = 0.48$ V. Regarding the spectral behaviour, it is observed that the shape and quality of the resulting OFC depends on the SRD operation point. The results are shown in Fig. 7.13(c), where it can be observed that the quality of the OFCs depends on the timing jitter of the electrical signal. Although the average value of the CNR slightly depends on the jitter (18.1 and 17.3 dB for 24.9 and 35.5 ps, respectively), the asymmetry of the CNR changes dramatically: the local CNR is around 19 dB for the low wavelength limit defined by δf_{10} , while it takes the values of 2.5, 6 and 8 dB for timing jitters of 35.5, 29 and 24.9 ps, respectively. These results, together with the numerical simulations, indicate that the timing jitter produced by the SRD is the responsible for the non-uniformity of the CNR in the obtained OFC. It is expected that by minimising the jitter, high quality OFCs can be generated by using SRDs.

A comparison between the results generated by a PPG and the SRD in terms of the pulse shape and spectral quality of the OFC was carried out and the results are shown in Fig. 7.14. Figs. 7.14(a) and (b) show the input electrical pulses provided by the SRD and the PPG, respectively. Both electrical pulses present similar values of $V_{RF-output}$, but with different profiles. The electrical pulses from the SRD exhibit their usual characteristics. In the case of the electrical pulses generated by the PPG, they show a first peak followed by consecutive pulses with decreasing amplitudes. The shape of these pulses is attributed to an impedance uncoupling between the source and the oscilloscope, due to the limited bandwidth of the amplifier. As it can be observed in Figs. 7.14(c) and (d), both optical spectra have similar characteristics in terms of shape and flatness. Fig. 7.14(c) shows an asymmetric OFC with higher CNR at lower wavelengths. The OFC is very broad ($\delta f_{10} = 112$ GHz) and the CNR value is 13.7 dB. In the other case (Fig. 7.14(d)), in which

Figure 7.14: Temporal traces (upper row) and optical spectra (lower row) of the DML emission using as pulse source: (a) and (c) the PPG; and (b) and (d) the SRD . The driving conditions were in (a) and (c): $f_R = 100$ MHz, $I_{bias} = 1.7$ mA, $V_{RF-output} = 7$ V and $P_{opt} = -27$ dBm and $\lambda_{inj} = 1550.2$ nm; and in (b) and (d): $f_R = 100$ MHz, $I_{bias} = 1.45$ mA, $V_{RF-output} = 7$ V and $P_{opt} = -25$ dBm and $\lambda_{inj} = 1550.195$ nm

the SRD has been used as the pulsing source, the corresponding optical spectrum shows similar values of CNR ($CNR = 16$ dB) with a smaller value of spectral width ($\delta f_{10} = 90$ GHz). Although the spectral width in the second case is lower, the performance of an SRD as a low repetition rate OFG generator is good enough to use this type of device in those applications that require low repetition rate frequency combs, considering that. Although the quality of the generated OFCs is slightly lower, the cost of a PPG is much greater than the cost of a SRD together with a low frequency RF source, making the second option a much cheaper alternative for generating OFCs at low repetition rates.

Chapter 8

Spectroscopic applications of optical frequency combs

Introduction

Some spectroscopic applications of the OFCs generated by GS and OI are presented in this chapter. Section 8.1 provides a short review of the state of the art in spectroscopic applications of the OFCs, centred in the homodyne/heterodyne detection and the dual-comb spectroscopy. Section 8.2 shows an heterodyne scheme which employs an OFC in combination with the emission from a CW laser to characterise the transmittance profile of a tunable Fabry-Perot filter. The results on the experimental determination of the absorption profile of two gases: HCN and H_2S in air, carried out by employing the dual-comb spectroscopy technique, are shown in Section 8.3. A novel experimental approach based on the dual-comb architecture is presented in Section 8.4. The experimental results shown in this thesis demonstrate that this new approach can be employed to generate two mutually coherent OFCs at slightly different frequencies with a single gain-switched optically-injected semiconductor laser for dual-comb spectroscopy.

8.1 State of the Art

Among their multiple applications, OFCs have been recognized as light sources with an enormous potential in absorption spectroscopy. Fourier-Transform spectroscopy or, more specifically, Fourier-Transform infrared spectroscopy (FTIR) are nowadays the standard spectroscopic techniques for optical absorption in the infrared. However, the conventional FTIR spectrometers does not provide enough resolution to characterise the absorption spectra of many interesting species. Besides, the speed of the scanning mirrors limits the velocity of the measurement.

In order to achieve higher spectral resolutions, alternative approaches have been employed. The down conversion of the optical spectra into RF frequencies allows to analyse the information of the optical spectra with electrical equipment. Some techniques, such as the homodyne and heterodyne detection, were originally used to characterise the linewidth of lasers [4, 87]. In these techniques, the light from a local oscillator interferes with the emission of a laser (modulated or not) [86, 185–187], and the result is analysed with an

electrical spectrum analyser. These techniques can be used to characterise the absorption spectra of a specific gas, through the beating of an OFC with a local oscillator.

The dual-comb spectroscopy is the other usual approach that implies the use of OFCs to characterise the absorption profile of a gas. This technique [5, 53, 54, 78, 89–91, 188, 189] has attracted specific attention due to its resolution and measurement speed. The technique has been applied to different types of spectroscopic measurements such as spectrally resolved LIDAR [190] or greenhouse gas monitoring [191]. Ref. [5] constitutes an excellent tutorial on the state of the art of these technique. OFCs generated by GS and OI have been demonstrated in dual-comb spectroscopy. In [90], Jerez et al. used low repetition rate OFCs (~ 1 GHz) generated by GS and OI to characterise the absorption profile of a HCN gas cell. An in-depth analysis on the OI conditions of the OFCs generated by GS for dual-comb spectroscopy is provided in [78], demonstrating that the shape of the OFC changes with the OI conditions.

Based on the dual-comb architecture, Carlson et. al [118] proposed a new approach for dual-comb spectroscopy. In this new approach, the OFCs are generated with an electro-optic modulator at two slightly different frequencies, by using an external gating source that changes periodically the frequency of the OFCs. A similar approach is proposed in this thesis but using the OFCs generated by a gain-switched externally-injected semiconductor laser.

8.2 Heterodyne spectroscopy

An experimental arrangement for heterodyne spectroscopy was set up. In simple terms, the objective of this setup is to measure the OFC spectrum by mapping it into the RF region where the spectrum can be analysed with high resolution by an ESA. The mapping is obtained by beating the OFC generated by a GS optically-injected laser with a local oscillator. The results obtained with this technique were compared with those provided by the BOSA. Also, the transmission of a FP filter was measured with the heterodyne spectroscopy technique.

Figure 8.1: Schematic diagram of the experimental setup used for heterodyne spectroscopy: ML: master laser; VOA: variable optical attenuator, CW-SL(A): continuous wave slave laser A, GS-SL(B): gain-switched slave laser B; AOM: acousto-optic modulator; DUT: device under test; PD: photodetector, ESA: electrical spectrum analyser

	SL(A)	SL(B)
I_{bias}	25 mA	35 mA
V_{RF}	-	1.5 V
f_R	-	200 MHz
P_{inj}	13.5 dBm	-16.5 dBm
λ_{inj}	1549.57 nm	
f_{shift}	80 MHz	

Table 8.1: Summary of the operation conditions for the heterodyne experiments

A schematic diagram of the experimental setup is shown in Fig. 8.1. A dual master-slave configuration is used in the setup. The SL (A) is a DFB laser operated in CW regime, acting as the local oscillator. The SL (B) is a GS-DFB that generates an OFC at low frequency. The OFC beats with the local oscillator (which frequency has been shifted by the AOM) in the photodetector and the result is characterised with the ESA. A summary of the operation conditions is presented in Table 8.1. Under these GS and OI conditions, the SL-A is completely locked to the ML injection. In the case of the SL-B, these driving conditions would lead to the obtention of an incoherent continuous spectrum (similar to those shown in Chapter 5 at low frequency, Fig. 3(h)). However, the OI allows to generate a low repetition rate OFC.

Figure 8.2: (a) Experimental RF spectrum of the beat signal from an OFC generated at 200 MHz from SL(B) and the single tone from the CW-SL(A). Panel (b) is a zoom of panel (a). This measurement was performed in the operation conditions shown in Table 8.1. ESA parameters: RBW = 75 kHz; Average = 10; Sweep time = 30 s

Fig. 8.2 shows the result of beating the single tone from the local oscillator with the OFC, with both lasers locked to the injection. The resulting spectrum in the ESA consist of two type of lines: i) the lines due to the direct detection corresponding to the intensity modulation of the GS laser, and ii) the beat tones between each peak of the OFC and the local oscillator. The direct detection terms are located at multiples of the modulating frequency:

$$f_d = m f_R; m = 1, 2, 3, \dots \quad (8.1)$$

where f_R is the frequency of the modulating RF signal. The frequencies of the beat tones

are given by:

$$f_b = |nf_R - f_{shift}|; n = 0, \pm 1, \pm 2, \pm 3, \dots \quad (8.2)$$

It is important to mention that, as the ESA only shows positive values of the frequency, the RF spectrum shown in Fig. 8.2(a) corresponds to the folded electrical spectrum of the beating between the OFC and the CW emission, i.e, the beating terms corresponding to negative frequencies are also displayed in the positive part of the RF spectrum.

The interference between the OFC and the local oscillator gives as a result a first tone located at f_{shift} (in this case $f_{shift} = 80$ MHz). Using this peak as the reference, the other tones were isolated and processed to reconstruct the original optical spectrum. The reconstruction of the optical spectra were performed by an in-house MATLABTM program. In summary, the program saves the values of amplitude and frequency of each tone relocating it in its real position. The reconstructing process is performed in several steps:

1. The program saves the position and amplitude of the peak located at $f_{shift} = 80$ MHz and uses it as the reference.
2. The program locates the position of the "m" peaks due to direct detection.
3. For each value of "m", the program saves in two vectors, the amplitude and the frequency of the tones adjacent to the direct detection term. These tones are located at $mf_R + f_{shift}$ and $mf_R - f_{shift}$.
4. The program sweeps for all the values of m, generating two matrix (one for amplitude and the other for frequency) of two rows and m columns constituted by the vectors of the previous step for each m.
5. In the case of the frequency matrix, the elements of the first row, which correspond with the tones located at $f_b = nf_R - f_{shift}$ ($f_b = 120, 320, 520$, etc...) with positive n in Fig 8.2, are shifted to negative multiples of the repetition rate. The elements of the second row, which are at $f_b = |nf_R - f_{shift}|$ ($f_b = 280, 480, 680$, etc...), with negative n, in Fig 8.2, are shifted to positive multiples of the repetition rate, and both rows merge into a vector. The same algorithm is applied to the amplitude matrix.

The background noise was measured and subtracted from measurement. The reconstructed spectrum is centred at zero frequency. In order to obtain the corresponding optical wavelength, the value of frequency of each tone was shifted by the ML frequency and converted into wavelength. It is important to point out that this reconstruction process does not provide information of the sign of the frequencies, i.e, the reconstructed spectra can be correct or inverted.

Fig. 8.3 shows an example of a reconstructed spectrum in comparison with the spectrum in similar conditions measured with the BOSA. The spectrum corresponds to an asymmetric OFC, what makes the reconstruction process easier, as the amplitude of the tones at the left and right side of the reference is different. As it can be seen in Fig. 8.3(a), the blue (higher wavelengths) part of both spectra matches well while the red parts of the spectra are different. The reconstructed spectrum shows higher values of the peak

Figure 8.3: Experimental optical spectrum (black) recorded with a BOSA and the reconstructed optical spectrum (blue) from the RF spectrum of the beating between the local oscillator and the OFC under test

amplitude for wavelengths below 1549.4 nm. The differences between the measured and reconstructed spectra are attributed to different injection conditions, as the experiments were not performed simultaneously. The BOSA is located in the Instituto de Fisica de Cantabria (Santander) while the RF measurements were carried out in Madrid. Taking this fact into account, both spectra shown a reasonable resemblance.

Figure 8.4: Measured RF spectra of the unfiltered (a) and the filtered OFC (b). The OFC for these measurements was generated at the same operation conditions shown in Table 8.1. ESA parameters: RBW = 75 kHz; Average = 10; Sweep time = 6 s

The transmittance of a tunable FP filter in the spectral region covered by the same OFC was measured using this technique. The RF spectra of the beat signals obtained with the unfiltered and the filtered OFC were used to determine the filter transmittance. The results are shown in Fig. 8.4. The presence of the filter can be clearly noticed in Fig. 8.4(b) as the amplitude of the tones decreases far from the center. The optical spectra of the unfiltered and filtered OFC were reconstructed and subtracted (in logarithmic scale) to obtain the transmittance of the filter. Fig. 8.5 shows the measured filter transmittance (in red) compared with the theoretical transmittance of a FP filter. As it can be seen in Fig. 8.5, although some points of the red line are above and below the blue line, the reconstructed profile of the transmission clearly follows the theoretical profile of this FP filter. The discrepancies are attributed to the fact that the insertion losses are not considered for the

Figure 8.5: Measured (red) and theoretical (blue) transmittance of the tunable FP filter

calculations of the theoretical profile of the filter. The linewidth of the reconstructed profile is ~ 400 MHz while the value of the linewidth of the filter provided by the manufacturer is 360 MHz. As the repetition rate of the OFC is 200 MHz, it is very difficult to match a line of the OFC with the maximum amplitude value of the filter transmittance, and therefore, both spectra show clear differences in the first 3 dB. The characterisation of the filter transmittance could be improved by employing OFCs with lower repetition rates.

8.3 Dual-comb spectroscopy

This section is devoted to the spectral characterisation using the dual-comb spectroscopy technique of two absorption profiles: i) the single line located at 1549.73 nm of a commercial gas cell of *HCN* at a pressure of 100 Torr, and ii) multiple absorption lines around 1574.8 nm from *H₂S* confined (at a partial pressure of 45 mBar) within a gas chamber of 6.6 meters filled with 1 atmosphere of air. A schematic diagram of the experimental setup is shown in Fig. 8.6. The reader can find more information about the complete setup in Chapter 4.

Figure 8.6: Schematic diagram of the experimental setup used for the characterisation of the *HCN* gas cell: ML: master laser; VOA: variable optical attenuator, GS-SL(A): gain-switched slave laser A, GS-SL(B): gain-switched slave laser B; AOM: acousto-optic modulator; DUT: device under test; PD: photodetector, ESA: electrical spectrum analyzer

	SL(A)	SL(B)
I_{bias}	4.7 mA	6 mA
V_{RF}	3 V	3 V
f_R	1 GHz	1.001 GHz
P_{inj}	3.5 dBm	3.5 dBm
Temp.	23.7°C	26.4°C
λ_{inj}	1549.49 nm	
f_{shift}	80 MHz	

Table 8.2: Summary of the operation conditions for the dual-comb experiments with the gas cell of HCN

For these experiments it is necessary to use a dual master-slave configuration where two different slave laser are injected by the same master laser. The experimental setup of Fig. 8.6 is based on the symmetric configuration for dual-comb spectroscopy [5]. The two lasers are modulated with two sinusoidal RF signal with slightly different repetition rates. In this setup, one of the OFCs is frequency shifted by the AOM. Both optical paths are coupled again, partially transmitted by the DUT and then, mixed in the PD. The result is analysed by the ESA. A summary of the operation conditions is presented in Table 8.2.

The operating temperatures of both lasers were adjusted, so that their optical spectra measured with a standard OSA completely overlapped. Under these driving and injection conditions, the generated OFCs were similar. The resultant beat signal between both OFCs is an electrical comb centred at f_{shift} and exhibiting the shape of the original OFCs, but with tones separated 1 MHz. The RF spectra were measured with and without the cell, and the gas transmittance was calculated by subtracting the amplitude of each tone of the RF spectra in logarithmic scale.

Figure 8.7: Measured electrical comb a) without gas and b) with gas; (c) Calculated transmittance for the gas cell. The driving and optical conditions of this measurement are summarised in Table 8.2. ESA parameters: RBW = 30 kHz; Average = 10; Sweep time = 0.2 s

Fig. 8.7 shows the resulting electrical comb without and with the HCN gas cell, as well as, the calculated transmittance of the gas. The beat signal of the two OFCs is a flat and wide electrical comb with peaks separated 1 MHz (Fig. 8.7(a)). In the case of the spectrum measured with the gas cell (Fig. 8.7(b)), the effect of the absorption of the gas

can be clearly observed at frequencies close to 50 MHz. The ratio between peaks provides the gas transmittance, shown in Fig. 8.7(c). It is important to mention that the ratio between electrical spectra, in dB, is a factor two greater than the optical transmittance, in dB, due to the symmetric configuration. The measured spectrum corresponds with the absorption line of the HCN "SRM2519a" that is located at 1549.73 nm. According to its nominal characteristics, the absorption depth of this line is 2.5 dB. The measured transmittance in Fig. 8.7(c) drops down to 2.5 dB at those frequencies where the gas cell absorbs, and therefore, the measured absorption depth of the line is consistent with the value provided by manufacturer. According to Fig. 8.7(c), the value of the transmittance at half of the minimum (-2.5 dB is the minimum, -1.25 dB is half this value) is 8 MHz, corresponding to 8 GHz in the optical spectra, which corresponds to the expected value

This spectroscopic approach was also employed to characterise the absorption spectrum around 1574.8 nm of H_2S , confined in a 6.6 m gas chamber. These experiments were performed in the Physics department of the University College Cork (UCC), during the research internship in Ireland. For these experiments, it was also necessary to use the dual master-slave configuration described above. In this case, the frequency of one of the branches was not shifted. A schematic diagram of the experimental setup is shown in Fig. 8.8. The reader can find more information about the complete setup in Chapter 4.

Figure 8.8: Schematic diagram of the experimental setup used for recording the optical spectrum of the H_2S gas cell: ML: master laser; VOA: variable optical attenuator, GS-SL(A): gain-switched slave laser A, GS-SL(B): gain-switched slave laser B; AOM: acousto-optic modulator; OBPF: optical band pass filter, GC: gas cell; PD: photodetector, ESA: electrical spectrum analyser

The lasers used in this setup were two optically-injected GS FP lasers acting as slave lasers and an external cavity laser acting as the master device. These two slave lasers were gain-switched with two sinusoidal RF signals with slightly different frequencies (f_{R1} and f_{R2}). Both OFCs are filtered by an optical band pass filter and beat in the photodetector after passing through the gas cell. The beat signal is analysed by an ESA. A summary of the operation conditions is presented in Table 8.3.

Fig. 8.9 shows the electrical combs resulting from the beating of the two OFCs after passing through the gas cell either filled only with air (Fig. 8.9(a)) or filled also with H_2S at a partial pressure of 45 mBar (Fig. 8.9(b)). The ratio between these two spectra is shown in (Fig. 8.9(c)). The optical filter removes the region of the optical spectra close to

	SL(A)	SL(B)
I_{bias}	8 mA	8.5 mA
V_{RF}	7 V	7 V
f_R	1.25 GHz	1.25025 GHz
Temp.	26.4°C	27.7°C
P_{inj}	Unknown	Unknown
λ_{inj}	1574.36 nm	
f_{shift}	No AOM	

Table 8.3: Summary of the operation conditions for the dual-comb experiments with the gas cell of H_2S

the ML frequency, and avoids that the part of the comb at negatives frequencies overlap with positive frequencies.

Figure 8.9: Electrical combs recorded by the ESA when (a) the gas cell is filled only with air (reference) and when (b) it also contains H_2S at 45 mBar; (c) Ratio (blue) and position of the absorption lines extracted from HITRAN database. These measurements were performed in the operation conditions shown in Table 8.3. ESA parameters: RBW = 1 kHz; Average = 30; Sweep time = 6 s

The reference electrical comb (Fig. 8.9(a)) shows really good characteristics in terms of width, flatness and dynamic range. The profile of the electrical comb in Fig. 8.9(b) shows the features evidencing the gas absorption which is clearly visible in the region close to 3 MHz. More precisely, the absorption of the gas is accounted by the ratio of the tones from Fig. 8.9(a) and (b). In order to obtain the corresponding optical spectrum as a function of the wavelength (Fig. 8.9(c)), the value of the frequency of each tone was shifted by 190.420 THz and converted into wavelength. Fig. 8.9(c) shows also the position and the relative intensity of the H_2S absorption lines provided by the high-resolution transmission molecular absorption database (HITRAN). The agreement is fairly good, thus demonstrating the potential of the method.

8.4 Gated dual-comb generation

8.4.1 Principle of operation

A new spectroscopic technique based on a dual-comb architecture was proposed by Carlson et. al [118]. In this technique, the two OFCs with slightly different repetition rates are generated by a single laser. This new approach was called PHIRE (Parallel Heterodyne Interferometry via Repetition-rate Exchange) in [118], but here it is referred as "Gated dual-comb" as it is similar to the gated delay self-homodyne technique in [185].

A schematic diagram of the technique as implemented in [118] is shown in Fig. 8.10.

Figure 8.10: Schematic diagram of the gated dual-comb architecture according to [118].

The OFCs are generated by electro-optic modulation of a single laser diode. The electro-optic modulator is driven by an RF generator which, in turn, is gated by an external source at a frequency f_{mod} . The objective of this is to drive the EOM with an RF signal which frequency switches periodically between two slightly different values, f_{rep} and f'_{rep} at a frequency $f_{mod} \ll f_{rep}$. Consequently, the optical signal generated by the EOM ((1) in Fig. 8.10) consists of a train of optical pulses which repetition rate switches between f_{rep} and f'_{rep} at a frequency f_{mod} ; in other words, two OFCs at f_{rep} and f'_{rep} delayed by a time $1/f_{mod}$. Subsequently, the optical path is split into two branches. The light travelling through one of the branches ((2) in Fig. 8.10) is delayed by a fiber spool and the light travelling through the other ((3) in Fig. 8.10) is just frequency shifted (f_{shift}) by an AOM. The key point here is to adjust the length of the fiber spool, L_{delay} , and f_{mod} such that, when the light travelling through each branch beat in the photodetector, the beating occurs always only between the OFCs at f_{rep} and f'_{rep} , thus generating the same kind of RF signal produced by the conventional dual-comb generators. For this matching the relationship between L_{delay} and f_{mod} should be:

$$L_{delay} = \frac{v_g}{2f_{mod}} \quad (8.3)$$

where v_g is the group velocity in the fiber. As usual, the only role of the AOM here is to shift the RF spectrum by f_{shift} to avoid centring the beat signal at zero frequency. This technique shows some similarities with the gated delayed self-homodyne (GDSH) [185], in the sense that both techniques require a gated signal to modulate a laser. However, in the case of the PHIRE approach, the frequency of the modulating current, instead of the amplitude, changes between two different values due to the gating signal.

8.4.2 Experimental results

The implementation of the technique carried out in this thesis uses as OFC generator an optically-injected GS laser instead of the electro-optically modulated laser diode. Fig. 8.8 shows a schematic diagram of this experimental setup.

Figure 8.11: Schematic diagram of the experimental setup used for the gated dual-comb experiment: ML: master laser; SL: slave laser; AOM: acousto-optic modulator; SMF: single-mode fiber, EDFA, erbium-doped fiber amplifier, PD: photodetector, ESA: electrical spectrum analyser, OSC: oscilloscope

The gating signal was a square RF signal produced by the PPG. The laser diode was the DFB described in Chapter 4. The optical signal was amplified by means of an EDFA. The reader can find more information about the complete setup in Chapter 4. A summary of the operation conditions is presented in Table 8.4.

	OFC-1	OFC-2
I_{bias}	6 mA	
V_{RF}	3 V	
f_{mod}	100.4 kHz ($L_{fiber} = 1$ km)	
f_{rep} and f'_{rep}	1 GHz	1.001 GHz
P_{inj}	-16.5 dBm	
λ	1549.49 nm	
f_{shift}	100 MHz	
I_{EDFA}	34 mA	

Table 8.4: Summary of the working conditions for the gated dual-comb operation

The electrical spectrum of the dual OFC generated under these conditions is shown in Fig. 8.12. The spectrum, recorded with the ESA, shows an electrical comb with a separation between peaks of 1 MHz and a low dynamic range (10-15 dB) (Fig. 8.12(a)).

Figure 8.12: (a) Dual electrical comb spectrum generated under the working conditions summarised in Table 8.4; panel (b) is a zoom of panel (a). The coloured arrows indicate lines corresponding to two overlapped combs. ESA parameters: RBW = 10 kHz; Average = 20; Sweep time = 1.5 s

However, a closer look at the spectrum reveals that it actually correspond to two overlapped sub-combs. The profiles of the electrical comb spectra as recorded by the ESA are always symmetric no matter the driving and OI conditions even though the OFCs are not. The "symmetrization" of the comb is due to the long integration time of the measurements in the ESA. This effect is illustrated in Fig. 8.13. For the first semi-period

Figure 8.13: Explanation of the symmetric combs obtained with the gated dual OFCs experiments when the measurement of the signals is performed with the ESA. T_1 , first semi-period of the gating cycle; T_2 , second semi-period of the gating cycle; f_{rep} , repetition frequency to generate the OFC1; f'_{rep} , repetition frequency to generate the OFC2. The OFC is assumed to be strongly asymmetric

of the gating signal, the beating of the n line of the OFC 1 (f_{rep}) with the n line of the OFC 2 (f'_{rep}) give rise to the line of the RF comb at a frequency of $f_{shift} + n(f_{rep} - f'_{rep})$. It means that the OFC is directly mapped into the RF domain around f_{shift} . However, for the other semi-period the beating of the same lines gives rise to the line of the RF comb at a frequency $f_{shift} - n(f_{rep} - f'_{rep})$. It means that the mapping of the OFC in the RF domain is now an inverted spectrum respect to f_{shift} . Therefore, the average made by the ESA along hundreds of cycles results in the observed symmetric profile.

In order to avoid the effect of the "symmetrization" of the electrical comb, the beat signal between both OFCs has been acquired by an oscilloscope used as analog-to-digital converter (ADC). In order to perform these measurements, the oscilloscope was triggered

with the external gating source. The fast Fourier transform algorithm, implemented in MATLABTM, was applied to obtain the spectrum of the temporal trace.

Figure 8.14: (a) Temporal trace of the measured interferogram for the gated dual-comb configuration in the driving and injection conditions mentioned in Table 8.4; (b) the corresponding electrical spectrum calculated by applying the Fourier transform of the interferogram. Oscilloscope parameters: Sampling Rate = 1.25 GSamples/S; Points = 6250; Average = 10

Fig. 8.14(a) shows the acquired signal during a single semi-period of the interference signal between both OFCs. The resulting signal consists of multiple structures (interferograms) periodically distributed every 1 μ s, time corresponding to the inverse of the frequency separation between both combs (1 MHz). The temporal trace has been processed in MATLABTM to calculate the Fourier transform of the temporal signal. The result is presented in Fig. 8.14(b). Here, in contrast to Fig. 8.12(a), the corresponding spectrum shows a single asymmetrical comb with tones separated by 1 MHz, thus demonstrating that the effect of the "symmetrization" is due to the ESA. Although the resulting signal is noisy with a relatively low dynamic range (15-20 dB), these results demonstrate for the first time that a dual-comb approach can be implemented with a single semiconductor laser by gated GS and OI, opening new opportunities for the application of this technique to gas spectroscopy.

Chapter 9

Conclusions and future work

Conclusions

The conclusions of this work are summarised as follows:

(A) Optical pulse and OFC generation

- The experimental characterisation of a 1310 nm VCSEL, a 1550 nm DML and a DFB in CW operation has been carried out by means of the P-I-V characteristics and the optical spectra in different conditions of current and temperature. Relevant parameters of these lasers were obtained in this analysis for simulation purposes.
- A systematic experimental study of the effect of the GS conditions on the temporal and spectral characteristics of the OFCs generated from the DML and the DFB laser has been performed. The combined information provided by the optical spectra, the temporal profiles and linewidth measurements is used for ascertaining the origin of the comb characteristics and their evolution as a function of a wide range of switching conditions, revealing the important role of the adiabatic and dynamic chirp in the generation of OFCs at low and high frequencies, respectively.
- It has also been observed that the characteristics of the OFCs depend on the switching conditions rather than on the type of cavity or other internal parameters of the laser; as the DFB laser and the DML behave similarly in GS operation.
- An exploration of the effect of the OI under specific conditions of fixed detuning and increasing power, and fixed power and variable detuning has been performed showing the improvement or worsening of the quality of the generated OFCs, depending on the injection conditions. The results provide guidelines for selecting the suitable switching conditions for specific OFC characteristics.
- The generation of narrow optical pulses in a gain-switched 1300 nm VCSEL is reported for the first time in this thesis, as far as the author's knowledge. Special attention has been paid to the evolution of the characteristics of these pulses (FWHM and P_{max}), with the gain-switching conditions, I_{bias} and V_{RF} . This analysis provides similar results than those previously reported for VCSELs

emitting in other spectral regions. The shortest optical pulses ($\text{FWHM} = 45$ ps) were obtained at 2.5 GHz optimising the driving conditions.

(B) Numerical analysis and comparison with experiments

- A systematic experimental and numerical study on OFC generation has been performed under a wide range of GS and OI conditions.
- A properly parametrised numerical simulator has been developed, capable of producing simulation results quantitatively comparable with the experimental spectra in GS and OI. Special attention has been devoted to minimise the number of assumed parameters to be included in the model. This has been achieved by a detailed fitting of the RIN measurements to the theoretical expression and by comparison with the experimental optical spectra. Special care has been taken in the treatment of the stochastic noise terms of the model.
- The very good agreement between experiments and simulations in the main features of the combs and also in the details of the spectral profiles points out to a proper accounting by the model of the main involved physical effects.
- The model has confirmed the relevant role of the dynamic chirp in the comb generation at high frequencies and the role of the adiabatic chirp at low frequencies.

(C) Generation of OFCs by pulse excitation and optical injection

- An experimental analysis is performed on the generation of OFCs in semiconductor lasers by GS with electrical pulses, provided by two different pulsing sources: a pulse pattern generator and a step-recovery diode. The use of the electrical pulses provided by the PPG in combination with OI allows the direct generation by GS of broad OFCs, containing an unprecedented number of well-resolved spectral lines separated by repetition frequencies lower than 1 GHz, without further expansion stages. This is reported for the first time in this thesis. Moreover, an experimental systematic analysis of the effects of the GS and OI on the quality of the OFCs is presented.
- The use of an SRD as the electrical pulse generator allows to produce high-quality OFCs with over 900 comb tones spaced by 100 MHz. Nevertheless, comparing the OFC quality obtained using a SRD with those obtained with a costly PPG, it has been concluded that despite of the slightly worse OFC quality, the use of SRDs could reduce the cost, size and footprint of the equipment required for low repetition rate OFC generation.
- Through experimental measurements, supported by simulations, it has been demonstrated that the jitter of the electrical input pulses impacts the quality of the generated OFCs, giving rise to a non-uniform CNR which is higher around the injection frequency and smaller far from it.

(D) Applications of OFCs to spectroscopy.

- Three different spectroscopic schemes based on OFCs are proposed in this thesis to characterise the absorption spectra of different devices and gas species. The first technique, known as heterodyne detection, has been successfully employed to characterise the transmittance of a tunable Fabry-Perot Filter with OFCs generated by GS and OI.
- The dual-comb spectroscopy was employed to characterise the molecular absorption spectra of two different gas species: i) an absorption line of hydrogen cyanide (HCN) gas cell located at 1549 nm and ii) a broad absorption spectra of a 6.6 m chamber with 1 atm of air and H₂S, providing results in a very good agreement with the nominal characteristics of the gas.
- The gated dual-comb technique, based on generating and beating two OFCs from a single laser has been developed for the first time, as far as the author's knowledge, using gain-switched semiconductor lasers. The initial results are promising, revealing the need of using a triggered ADC in time domain to recover the RF-OFC.

Future work

Open questions have arisen as a result of this work. Although they have not been addressed due to the limitation of time, the following issues deserve special attention and further investigation:

- A further experimental characterisation of the laser response under OI and GS, analysing the evolution of the features of the generated under a wide range of OI conditions, including those in where additional dynamical regimes can appear. The characterisation of the generated OFC in these conditions can increase the current knowledge of these dynamical regimes and the OFC generated in these conditions can be applied in some applications, such as chaotic LIDARs.
- The SRDs have been demonstrated to be a suitable pulsing source to develop a low-cost OFC generator. These devices can be implemented, in combination with the recently developed PICs to obtain a commercial OFC generator based on pulsed GS. These devices would benefit from the advantages provided by the PICs, such as flexibility and low size, with the low cost of the SRD, obtaining multi-purpose devices for several applications. In order to improve the performance of the OFCs generated with these devices, an in-depth analysis of the generation of OFC can be carried out devoting special attention to the timing jitter.
- Due to the lack of commercial optical spectrum analysers in the 1300 nm, the heterodyne detection technique can be used to study the characteristics of the generated OFCs in a 1300 nm VCSELs, paying special attention to the orthogonally polarised modes of the VCSEL and their effects on the generation of OFCs.
- To continue the research in dual-comb spectroscopy using OFCs generated by semiconductor lasers by pulsed GS and OI, improving the sensitivity to reduce the measurement time.

- To validate the gated dual-comb technique with GS semiconductor lasers for spectroscopic applications.
- To develop LIDAR systems for remote gas sensing based on the dual-comb approach.

Appendix

List of publications

Papers in peer-reviewed international journals

1. **A. Rosado**, A. Pérez-Serrano, J. M. G. Tijero, Á. Valle, L. Pesquera, and I. Esquivias, Experimental study of optical frequency comb generation in gain-switched semiconductor lasers, *Opt. Laser Technol.*, 108, 542-550, 2018.
2. **A. Rosado**, A. Pérez-Serrano, J. M. G. Tijero, Á. Valle, L. Pesquera, and I. Esquivias, "Enhanced optical frequency comb generation by pulsed gain-switching of optically injected semiconductor lasers," *Opt. Express* 27, 9155-9163, 2019.
3. **A. Rosado**, A. Pérez-Serrano, J. M. G. Tijero, Á. Valle, L. Pesquera, and I. Esquivias, "Numerical and Experimental Analysis of Optical Frequency Comb Generation in Gain-Switched Semiconductor Lasers". *IEEE. J. Quantum. Electron.* (2019), vol. 55, pp. 1-12, 2019.

International Conferences

1. **A. Rosado**, M. Vilerá Suárez, A. Pérez-Serrano, J.M. García Tijero e I. Esquivias, "Gated Delayed Self-Heterodyne Measurements of Frequency Combs Generated by Gain Switched Laser Diodes". European Semiconductor Laser Workshop, Copenhagen, Denmark, 15-16 Sept., 2017.
2. **A. Rosado**, A. Pérez-Serrano, J. M. G. Tijero, Á. Valle, L. Pesquera, and I. Esquivias, "Optical frequency comb generation from semiconductor lasers using pulsed excitation and optical injection". European Semiconductor Laser Workshop, Bari, Italy, 21-22 Sept. 2018.
3. **A. Rosado**, A. Pérez-Serrano, J. M. G. Tijero, A. Valle, L. Pesquera and I. Esquivias, "Optical Frequency Comb Generation in Gain Switched DFB and Discrete Mode Lasers: Comparison and Optical Injection Effects," 2018 European Conference on Optical Communication (ECOC), Rome, 2018, pp. 1-3.
4. **A. Rosado**, A. Pérez-Serrano, J. M. G. Tijero, A. Valle, L. Pesquera and I. Esquivias, "Optical Frequency Comb Generation from Semiconductor Lasers using Optical Injection and Pulsed Gain Switching," 2019 Conference on Lasers and Electro-Optics Europe & European Quantum Electronics Conference (CLEO/Europe-EQEC), Munich, Germany, 2019, pp. 1-1.

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5. **A. Rosado**, A. Pérez-Serrano, J. M. G. Tijero, A. Valle, L. Pesquera and I. Esquivias, "Analysis of Optical Frequency Comb Generation in Gain-Switched Semiconductor Lasers," 2019 Conference on Lasers and Electro-Optics Europe & European Quantum Electronics Conference (CLEO/Europe-EQEC), Munich, Germany, 2019, pp. 1-1.
 6. **A. Rosado**, E. P. Martin, A. Pérez-Serrano, J. M. G. Tijero, I. Esquivias and P. M. Anandarajah, "Optical frequency comb generation via pulse excitation using step-recovery diodes in an optically-injected semiconductor laser". European Semiconductor Laser Workshop, Cork, Ireland, 27-28 Sept. 2019.
 7. **A. Rosado**, J. Díez, A. Quirce, A. Pérez-Serrano, J. M. G. Tijero, A. Valle, L. Pesquera, and I. Esquivias, "Experimental and numerical analysis of optical frequency comb generation in gain-switched semiconductor lasers subject to optical injection". SPIE Photonic Europe 2020, 2020, Strasbourg, France

National Conferences

1. **A. Rosado**, M. Vilera Suárez, A. Pérez-Serrano, J.M. García Tijero e I. Esquivias, Characterization of optical pulses generated by Gain Switching a 1310 nm VCSEL. X Reunión Española de Optoelectrónica OPTOEL2017, Santiago de Compostela, 2017
2. **A. Rosado**, A. Pérez-Serrano, J.M. García Tijero, A. Valle, L. Pesquera e I. Esquivias, Characterization of Optical Frequency Combs Generated by Gain-Switching Semiconductor Lasers. XI Reunión Española de Optoelectrónica OPTOEL2019, Zaragoza, 2019

Manuscripts submitted to peer-reviewed international journals

1. **A. Rosado**, E. P. Martin, A. Pérez-Serrano, J. M. G. Tijero, I. Esquivias and P. M. Anandarajah, "Optical Frequency Comb Generation via Pulsed Gain-Switching in Externally-Injected Semiconductor Lasers using Step-Recovery Diodes". Submitted to Opt. Laser Technol.

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List of acronyms

ADC	Analog-to-digital converter
AM	Amplitude modulation
AOM	Acousto-optic modulator
BOSA	Brillouin optical spectrum analyser
CNR	Carrier-to-noise ratio
CW	Continuous wave
DBR	Distributed Bragg reflector
DFB	Distributed feedback laser
DML	Discrete mode laser
DSH	Delayed self-heterodyne
DUT	Device under test
ECL	External cavity laser
EDFA	Erbium-doped fiber amplifier
EOM	Electro-optic modulator
ESA	Electrical spectrum analyser
FM	Frequency modulation
FP	Fabry-Perot
FTS	Fourier transform spectroscopy
FTIR	Infrared FTS
FWHM	Full-width half-maximum
GS	Gain-switching
HITRAN	High-resolution transmission molecular absorption database
LIDAR	Light Detection and Ranging
ML	Master laser
MLM	Multi-longitudinal mode
MOPA	Master Oscillator Power Amplifier
ODE	Ordinary differential equation
OFC	Optical frequency comb
OI	Optical Injection
OSA	Optical spectrum analyser
PIC	Photonic integrated circuit
PPG	Pulse pattern generator
PSD	Power spectral density
REM	Rate equation model
RF	Radio frequency
RIN	Relative intensity noise
RMS	Root-mean-square
SDE	Stochastic differential equation

SL	Slave laser
SLM	Single longitudinal mode
SMSR	Side-mode suppression ratio
SRD	Step-recovery diode
VCSEL	Vertical-cavity surface-emitting laser
TEC	Thermoelectric cooler