Electrical model and optimal design scheme for low work-function tethers in thrust mode

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ABSTRACT

A low work-function tether (LWT), a subclass of bare electrodynamic tether made of a conductor partially coated with a low work function material, can exchange momentum and energy with planetary magnetospheres without any consumable. If fed by an onboard power source to reverse the natural direction of the current given by the motional electric field, a LWT can produce an useful thrust in drag compensation and re-boost scenarios. In the considered scheme, the LWT has an anodic bare segment for passive electron collection, followed by an insulated segment, a power source, and a cathodic segment that emits electrons passively through thermionic and photoelectric effects. Current and voltage profiles along the LWT are obtained and used to compute the system efficiency, i.e. the electrical power to mechanical power conversion rate, and the minimum electrical power to avoid space-charge effects in the cathodic segment. The design conditions to reach a high-efficiency regime are presented. For a given orbit and required thrust, optimal values for the tether geometry, including the fractional lengths of all three tether segments, are found. The results are applied to the preliminary design of a LWT system that would compensate the aerodynamic drag on the International Space Station. The analysis shows that the mission can be performed by a LWT fed with a power source of 4.7 kW, length, width and thickness equal to 5.1 km, 2 cm and 30 μm , work function 1.5 eV, and temperature 600 K.

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1. Introduction

The search of alternatives to conventional propulsion technologies that could lower the cost of space operations is a matter of high interest for the sector. Moreover, thorough analysis [1] indicates that the space debris population in Low Earth Orbit (LEO) is under the Kessler syndrome [2] and an increasing interest exists on developing new technologies for deorbiting and re-boosting spacecraft. Since the propellant represents an important fraction of mass in conventional devices, propellant-less technologies working under physical principles different to the action-reaction law are under investigation. One of them is the sail, a drag augmentation device that uses the aerodynamic drag. However, a sail both produces no re-boost and deorbits inefficiently from the altitudes of interest for space debris due to the very low air density. On the other hand, electrodynamic tethers, i.e. long conductors that exchange momentum with the Earth's magnetosphere, have been discussed broadly as an interesting alternative [3–8]. The peak

on tether interest reached at the end of the last century, when many missions were flown, was followed by a period with important theoretical progresses on key aspects like tether survivability [9–11], mission design [12], dynamics [13] and control [14,15], among others. Two tether missions, DESCENT [16] and TEPCE that are planned for 2019, and very recent works on a great variety of topics, such as tether formation systems [17], tethered space tug-systems [18], tether-assisted payload return with length control [19], show that space tethers are nowadays an active field.

The standard electrodynamic tether concept till 2012 was a bare conductor as passive electron collector equipped with an active electron emitter [20]. Such a system is propellant-free, but it involves an active element that, in the case of a Hollow Cathode (HC), needs expellant. An important progress towards simplicity was given when the thermionic tether was introduced in 2012 [21]. Instead of carrying an active electron emitter, the tether is coated with a low work-function (W) material that emits electron passively through the thermionic effect, thus yielding a system totally free of consumable. Few years later, it was pointed out that the photoelectric effect can be also an important emission mechanism for tether coated with low-W materials and the term Low Work-Function Tether (LWT), which includes thermionic and pho-

Nomenclature

A_t	tether cross-sectional area,.....	m^2	N_0	ambient plasma density,.....	$1/m^3$
$\mathbf{B}$	ambient magnetic field,.....	T	p_t	tether perimeter,.....	m
c_{ch}	jet speed of chemical thruster,.....	m/s	S	Solar energy spectrum,.....	ph/sm ² eV
c_e	jet speed of electrical thruster,.....	m/s	T_e	electron temperature,.....	K
E_m	motional electric field component along $\mathbf{u}_t$,.....	V/m	T_i	ion temperature,.....	K
E	photon energy,.....	eV	T_t	tether temperature,.....	K
e	elementary charge,.....	C	$\mathbf{u}_t$	unit vector along the straight tether	
$\mathbf{F}_D$	aerodynamic drag,.....	N	$\mathbf{v}$	spacecraft velocity,.....	m/s
$\mathbf{F}_M$	Lorentz force,.....	N	W	work function,.....	eV
h_t	tether thickness (or radius),.....	m	W_E	electrical power,.....	W
I	tether current,.....	A	W_M	mechanical power,.....	W
I_{av}	average current,.....	A	w_t	tether width,.....	m
j	current density,.....	A/m ²	x	distance along the tether,.....	m
k_B	Boltzmann constant,.....	m ² kg/s ² K	Y_{ph}	tether photoelectron yield,.....	el/ph
L	tether length,.....	m	α_t	tether related hardware factor	
L_a	anodic segment length,.....	m	α	inverse specific power,.....	kg/kW
L_c	cathodic segment length,.....	m	η_e	thruster conversion efficiency	
L_i	insulated segment length,.....	m	η_{eff}	tether efficiency	
L^*	length characterizing ohmic effects,.....	m	Φ	local bias,.....	V
M_d	system mass,.....	kg	ρ_t	tether density,.....	kg/m ³
$\dot{m}$	mass flow rate,.....	kg/s	σ_t	tether conductivity,.....	1/ Ω m
m_e	electron mass,.....	kg	τ	thruster allotted time,.....	s
m_t	tether mass,.....	kg			

toelectric tethers, was proposed [22]. Besides deorbiting [21], system arrangements based on LWTs for power generation [23] and thrust production [24] have been discussed.

In parallel with development activities on LWT manufacturing, theoretical analyses are necessary to construct an envelope with tether performance and requirements, such as work function and temperature. However, except a work on deorbiting from Geostationary Transfer Orbit [25], the mission analysis of LWTs in space applications is an unspoiled field of research. Among the rich range of potential applications, this work studies for the first time thrust missions by LWTs. In general, tethers are certainly a suitable technology for very long thrust operations [26,27]. These missions include protracted drag compensation or repeated use to move payloads to higher orbits, as space 'tugs', freely re-boosting to meet, dock and grapple a new payload. A drag compensation scenario of particular interest is the re-boost of the *International Space Station* [28–30]. A 10 km long bare tether using less than 10 kW of power was proposed for ISS drag makeup, in the 1990's, soon after the bare-tether concept was introduced in 1993. Estimations suggested that 66 (out of 78) tons of propellant could be saved over a 10-year lifetime, at maximum duty cycle. Tether dynamics and thrusting raised concerns, however, about impact on microgravity payloads and safety issues, at Shuttle rendezvous and flight path [28].

The purpose of this work is to present an electrical model for LWTs in thrust mode and find optimal tether design conditions. Some relevant applications of the study include the re-boost of the ISS and also other extremely large, long-time operative satellites in LEO. New Space Stations might take the place of doomed ones, like the Russian MIR or the Chinese Tiangong-1, re-entered on 23 March 2001 and 2 April 2018, respectively. After revisiting in Sec. 2 some general considerations and key figures of merit about in-space propulsion systems, this work presents the following results. Section 3 introduces an electrical model for a LWT in thrust mode. The model is used to write the efficiency of the LWT, measured in terms of the effectiveness to transform onboard electric power into useful mechanical power, as a function of the design parameters like the lengths of the different tether segments. The design

conditions to optimize the efficiency of the LWT are investigated analytically for the long tether limit in Sec. 4.1, and numerically for a general case in Sec. 4.2. The results are discussed and applied to the re-boost of the ISS in Sec. 5. Section 6 summarizes the conclusions.

2. Figures of merit of space thrusters

In comparing thrusting alternatives, the figure of merit, which must be low for high efficiency, is the thrusting-system inverse specific impulse, which is the ratio $M_d/F\tau$ between dedicated system mass M_d and mission impulse provided by an average thrust F during an allotted time τ . For electric propulsion, M_d includes propellant consumed at a mass-flow-rate $\dot{m}$ and power plant αW_E , where α is the inverse specific power of supply and W_E the electrical power [26],

$$\frac{M_d}{F\tau} = \frac{\dot{m}\tau + \alpha W_E}{\dot{m}c_e\tau} = \frac{1}{c_e} + \frac{\alpha c_e}{2\eta_e\tau} \quad (1)$$

where $\eta_e \equiv Fc_e/2W_E$ and $F = \dot{m}c_e$. With reference values of jet speeds $c_e \sim 30$ km/s and 15 km/s, overall conversion efficiencies $\eta_e \sim 0.65$ and 0.50 are found for Ion and Hall thrusters, respectively. Also, α is typically a few tens of kg/kW if a dedicated solar-plant is required, as for a space tug, and just several kg/kW otherwise, as in the case of the *International Space Station*.

In a so-called thruster mode, a tether also requires a power supply lying at the top of a tether collecting electrons, at the satellite, to invert anodic and cathodic ends by overcoming the electromotive force of the magnetically induced motional field $\mathbf{v}_{rel} \times \mathbf{B}$, thus inverting current direction (electrons flowing upwards for eastward LEO orbits). Here $\mathbf{v}_{rel}$ is the tether-to-plasma relative velocity and $\mathbf{B}$ the ambient magnetic field. In the case of a standard tether with active electron emitter, electrons collected in a lower bare segment of length L would be re-emitted in top cathodic-exchange above the power supply, through a HC or a thermionic emitter. In the actual LWT case of interest, thermionic and photoelectric emissions along an upper tether segment coated with a low-W material replace the HC [24] (see Fig. 1).

Fig. 1. Tether configuration in thrust mode.

A LWT involves no consumable mass; on the other hand tether mass m_t , times a factor $\alpha_t \sim 2 - 2.5$, accounting for directly related hardware, does contribute substantially to system mass M_d , thus written as $\alpha W_E + \alpha_t m_t$. In the HC case, its hardware, but not the expellant for typically not too long missions, makes an accountable contribution to mass M_d . Considering the tether as part of the power plant, we may write

$$\frac{M_d}{F\tau} \simeq \frac{\alpha W_E + \alpha_t m_t}{F_M \tau} \approx \frac{\alpha v}{\tau} \frac{W_E}{W_M} \left(1 + \frac{\alpha_t m_t}{\alpha W_E} \right) \equiv \frac{\alpha v}{\tau \eta_{eff}} \quad (2)$$

where $W_M = \mathbf{F}_M \cdot \mathbf{v}$ and $\mathbf{F}_M$ are the mechanical power and Lorentz force, $\mathbf{v}$ the orbital velocity, and η_{eff} is an overall conversion efficiency, which might typically be about 1/3 if using a HC [26].

Electric propulsion may be considered somewhat midway, in the route to make less and less use of consumables. On one extreme one finds rocket propulsion, which involves just consumable mass and has inverse specific impulse

$$\frac{M_d}{F\tau} = \frac{1}{c_{ch}}, \quad (3)$$

with c_{ch} the chemical jet speed. The other extreme is tether propulsion (see Eq. (2)), which has no consumable mass (LWT) or small amount of expellant (bare tether with HC) but involves tether and associated hardware mass. Comparing the right-hand-sides of Eqs. (1)-(3), shows how this route leads to higher and higher specific impulse. With c_{ch} definitely below c_e , taking τ long enough makes electric propulsion, as regards specific impulse, better than rockets. Similarly, for long times, tethers are better than electric propulsion, whatever the relative values of ratios $c_e/2\eta_e$ and v/η_{eff} in (1) and (2). In particular, a tether system with $\eta_{eff} \sim 1/3$ would be twice as light as the corresponding *ion-thruster* for mission duration $\tau(\text{weeks}) \sim \alpha$ (kg/kW), which is about half a year, say, for a space tug, $\alpha \sim 25$ kg/kW, and one month for ISS re-boost, $\alpha \sim 5$ kg/kW[26].

3. LWT model in thruster mode

3.1. General considerations

The left scheme in Fig. 1 shows a tether configuration in thrust mode for a mission in an eastward, low inclination orbit [24]. The tether has anodic and cathodic segments for electron collection and emission, with lengths L_a and L_c , which are deployed downwards and upwards, respectively. An insulated segment of

length L_i is located between a power supply at the satellite and the bare anodic segment in order to increase efficiency; in a fully bare tether, $L_i = 0$, electrons collected near the top, where bias is largest, would do little push work but would fully consume energy from the power source. The satellite, which is schematically represented by a box with a battery inside, should provide an electromotive force $\mathcal{E}$ (volts). The middle and right panels, not drawn in scale, show a scheme of the current and voltage profiles. The bare tether, which is positively polarized with respect to the plasma, works as a giant Langmuir probe and captures electrons, mostly at high bias. Electrons are emitted back to the plasma by the negatively polarized segment through thermionic and photoelectric effects due to the low-W coating.

Thanks to the good and steady electrical contact with the ionospheric plasma provided by the configuration of Fig. 1, a current $\mathbf{I} = I(x)\mathbf{u}_t$ circulates along the tether and generate the Lorentz force

$$\mathbf{F}_M = \int_0^L \mathbf{I} \times \mathbf{B} dx \approx (\mathbf{u}_t \times \mathbf{B}) \int_0^L I(x) dx \equiv LI_{av} (\mathbf{u}_t \times \mathbf{B}), \quad (4)$$

where we ignored magnetic field variations along the tether and took it as a rigid dumbbell perfectly aligned with the local vertical of the orbit. We also used the total tether length

$$\frac{L_i}{L} + \frac{L_a}{L} + \frac{L_c}{L} \equiv \tilde{L}_i + \tilde{L}_a + \tilde{L}_c = 1 \quad (5)$$

and the unit vector $\mathbf{u}_t$ along the current direction that points from C to A in Fig. 1. If we introduce $E_m \equiv \mathbf{u}_t \cdot (\mathbf{v}_{rel} \times \mathbf{B})$ and the orbital velocity $\mathbf{v}$, with $\mathbf{v}_{rel} \approx \mathbf{v}$ for the slow ionospheric plasma corotating with the Earth, we conclude that thrust generation then requires

$$W_M \equiv \mathbf{F}_M \cdot \mathbf{v} = -E_m LI_{av} > 0 \quad (6)$$

indicating that the current should flow in the opposite direction of the motional electric field $\mathbf{v}_{rel} \times \mathbf{B}$, the mission requiring $E_m < 0$; a non-relativistic ($v_{rel} \ll \text{light velocity } c$) Lorentz transformation readily shows that $\mathbf{v}_{rel} \times \mathbf{B}$ is the electric field in the highly-conductive ionospheric background plasma, for an observer moving with the tether. Ultimately, current flows in the opposite direction of the naturally induced electric field as a result of the power $W_E = \mathcal{E} I_B$ supplied at the satellite at the satellite, with I_B the current at points B^+ and B^- .

In the configuration shown in Fig. 1, we assumed that the electromotive force $\mathcal{E}$ is large enough to have $\Phi(x) > 0$ and $\Phi(x) < 0$ in the anodic and insulated (AA^* and A^*B^+), and coated (B^-C) segments, respectively. Here $\Phi(x) = \Phi_t - \Phi_{pl}$ is the local bias difference between tether and faraway ambient plasma, increasing from A to B^+ , and being less negative from B^- to C. At both tether ends we impose the boundary conditions $I_A = I_C = 0$ (due to $w_t h_t \ll w_t L$, with h_t the tether thickness or radius, current collection and emission at both tether tips is fully negligible). As regards bias at the ends, note an important difference: bias should keep negative from B^- to C, i.e., Φ_C should not be positive; an electron-attracting segment at end C of the tether would increase current, not allowing to satisfy condition $I_C = 0$. An ion-attracting segment at end A, on the other hand, would have a negative, though insignificant, effect on the circuit.

The tether can be considered as a system that converts electrical power W_E , from solar panels or batteries in Fig. 1, into mechanical power W_M . System efficiency η_{eff} , introduced in Eq. (2) to conceptually present the tether as part of the power plant, can be also conveniently rewritten as

$$\frac{1}{\eta_{eff}} = \frac{W_E + \alpha_t m_t / \alpha}{W_M} \quad (7)$$

explicitly involving, as opposite to Eq. (2), twice the desired mechanical power rather than the supply power, visualizing effective conversion efficiency. This will be later shown to help in system design considerations. System efficiency η_{eff} directly decreases with increased supply power $\mathcal{E}I_B$, which might be required to have Φ_C negative, and increases with increasing magnetic power per unit mass $\propto W_M/\alpha_t m_t$. Calculations leading to system efficiency require evaluation of current profiles to determine both the Lorentz-drag power

$$\frac{W_M}{|E_m|} = I_{av}L = \int_0^{x_{A^*}} I(x)dx + \int_{x_{A^*}}^{x_B} I(x)dx + \int_{x_B}^L I(x)dx \quad (8)$$

and the supply power $W_E = \mathcal{E}I_B$.

3.2. Current-voltage profiles

Determination of system efficiency requires solving current and voltage profiles all along the tether, depending on design parameters and orbital conditions. In a reference frame tied to the tether, the ionospheric plasma is subject to the motional electric field. Far-away plasma potential Φ_{pl} is then given by its gradient $d\Phi_{pl}/dx = E_m < 0$. Current and potential inside the tether satisfy Ohm's law $I(x) = \sigma_t A_t d\Phi_t/dx$. The equation for the local bias $\Phi = \Phi_t - \Phi_{pl}$, valid throughout, is

$$\frac{d\Phi}{dx} = \frac{I(x)}{\sigma_t A_t} + |E_m| \quad (9)$$

Regarding current profile, if tether width (or radius) is not too large, the anodic segment ($\Phi > 0$) will collect electrons following the orbital-motion-limited (OML), high bias law [31]

$$j_{OML}(\Phi) = \frac{eN_0}{\pi} \sqrt{\frac{2e\Phi}{m_e}} \quad (10)$$

with e the elementary charge, m_e the electron mass, and N_0 the ambient plasma density. On the other hand, if Φ is negative enough to avoid space charge effects (a requirement discussed in Sec. 3.4), the emission in the cathodic segment ($\Phi < 0$) is bias-independent and given by [22]

$$j_c = j_{th} + j_{ph} = AT_t^2 \exp\left(-\frac{W_t}{k_B T_t}\right) + f(1-r_t)e \int_0^\infty S(E)Y_{ph}(E)dE. \quad (11)$$

For the thermionic current density j_{th} , we have the Richardson-Dushman law [32] with $A \approx 1.2 \times 10^6 A/m^2 K^2$, and W_t and T_t the tether work function and temperature, respectively. Regarding the photoelectric current density j_{ph} , it involves the energy spectrum of the solar photons (S) with energy E , the tether reflectivity (r_t) and photoelectric yield (Y_{ph}), and a factor f that takes into account that only a fraction of the tether perimeter is illuminated. We then have

$$\frac{dI}{dx} = \begin{cases} p_t j_{OML} & 0 < x < x_{A^*} \\ 0 & x_{A^*} < x < x_B \\ -p_t j_c & x_B < x < x_C \end{cases} \quad (12)$$

where p_t is the tether perimeter and we wrote $B^- = B^+ = B$. The integration of Eqs. (9) and (12) with the boundary conditions $I_A = I_C = 0$ gives the current and voltage profiles that are necessary to compute W_E , W_M , I_{av} and η_{eff} .

3.2.1. Cathodic and insulated segments

Analysis of thermionic and insulated segments is immediate because they do not involve bias Φ profiles when no space charge effect occurs throughout the cathodic segment. Middle and lower equations in (12) are immediately integrated with the boundary condition $I_C = 0$ to give

$$I(x) = I_B \equiv j_c p_t L_c \quad x_{A^*} < x < x_B \quad (13)$$

$$I(x) = I_B - j_c p_t (x - x_B) \quad x_B < x < x_C \quad (14)$$

Therefore, if the electromotive force provided by the power supply is high enough (see Sec. 3.4), the maximum current $I_B = j_c p_t L_c$ just depends on the characteristics of the cathodic segment independently of the electromotive force itself. These equations can now be used to allow direct computation of bias profile from A^* to C , required for determining the battery-voltage range ensuring that bias Φ_C is negative enough. Using Eqs. (13) and (14) in Eq. (9) gives

$$\Phi_C - \Phi_{B^-} = |E_m|L_c \left(1 + \frac{j_c p_t L_c}{2\sigma_t |E_m| A_t}\right) \quad (15)$$

$$\Phi_{B^+} - \Phi_{A^*} = |E_m|L_i \left(1 + \frac{j_c p_t L_c}{\sigma_t |E_m| A_t}\right) \quad (16)$$

Across the power source we would just have

$$\Phi_{B^+} - \Phi_{B^-} = \mathcal{E} \quad (17)$$

3.2.2. Anodic segment

Results for current and bias profiles at the anodic segment are readily obtained following well established analysis of tether OML collection [20]. After introducing characteristic magnitudes $I_* \equiv \sigma_t |E_m| A_t$ (short circuit current), $L_* = \ell^{1/3} h_t^{2/3}$, and $\ell \equiv 9\pi^2 m_e \sigma_t^2 |E_m| / 128 e^3 N_0^2$, and variables $i \equiv I/I_*$, $\phi \equiv \Phi/|E_m|L_*$ and $\xi \equiv x/L_*$, Eq. (9) and upper (12) become

$$\frac{d\phi}{d\xi} = i + 1 \quad (18)$$

$$\frac{di}{d\xi} = \frac{3}{4} \sqrt{\phi} \quad (19)$$

with $h_t \approx 2A_t/p_t$ the tether thickness for a tape (or the radius for a round tether). The above system admits the first integral $2i + i^2 - \phi^{3/2} = \text{const}$ and should satisfy the boundary conditions $i(0) \equiv i_A = 0$ and

$$i(x_B) = i_B = \frac{j_c p_t L_c}{\sigma_t |E_m| A_t} \equiv \tilde{j}_c \tilde{L}_c, \quad (20)$$

with $\tilde{j}_c \equiv j_c p_t L_c / \sigma_t |E_m| A_t$ and $\tilde{L}_c = L_c/L$. Tether thickness h_t , appearing in both $\tilde{j}_c \sim p_t/A_t \approx 2/h_t$ and $L_* = \ell^{1/3} h_t^{2/3}$, affect both the cathodic and anodic contacts. With the boundary conditions, the above first integral gives

$$\phi_{A^*}(\phi_A, i_B) = \left(\phi_A^{3/2} + 2i_B + i_B^2\right)^{2/3}, \quad (21)$$

and used in Eq. (18) leads to

$$\frac{\tilde{L}_a}{\tilde{L}_*} = \int_{\phi_A}^{\phi_{A^*}} \frac{d\phi}{\sqrt{1 + \phi^{3/2} - \phi_A^{3/2}}} \quad (22)$$

Equations (21) and (22) determine dimensionless values ϕ_A and ϕ_{A^*} in terms of ambient and tether-design parameters. Fig. 2 shows $\tilde{L}_a/\tilde{L}_*$ versus ϕ_A for several values of i_B . It explicitly shows that the value of the maximum current in the tether i_B can exceed

Fig. 2. $\tilde{L}_a/\tilde{L}_*$ versus ϕ_A for several i_B values.

unity for sufficiently high power supply and design of the cathodic segment. This is a key difference with respect to a tether in passive mode (deorbiting scenarios without power supply), where the maximum current should be below the short circuit current. An alternative form to Eq. (22) that is more appropriate for fast numerical calculations, for instance during orbit propagation, is given in Appendix A.

3.3. Overall LWT efficiency

From Eq. (8), and current profiles obtained in Sec. 3.2, one finds the normalized mechanical power

$$\begin{aligned} \tilde{W}_M &\equiv \frac{W_M}{\sigma_t E_m^2 A_t L} = \frac{I_{av}}{I_*} \equiv i_{av} \\ &= L_* (\phi_{A^*} - \phi_A) - (1 + i_B) \tilde{L}_a + i_B \left(1 - \frac{\tilde{L}_c}{2}\right) \end{aligned} \quad (23)$$

in terms of 4 dimensionless ratios, $\tilde{j}_c$, $\tilde{L}_*$, $\tilde{L}_c$ and $\tilde{L}_a$. Remember that, from Eq. (20), $i_B = \tilde{j}_c \tilde{L}_c$. We note that the first 2 terms in (23) can be readily rewritten, using (21) and (22), as the product of $\tilde{L}_*$ times a function of just ϕ_A and ϕ_{A^*} , that is always negative, whereas W_M must be positive. This directly points to the convenience of system design involving a small L_*/L ratio, as in thrusting with HC tether-plasma cathodic contact [26].

Similarly, adding Eqs. (15)-(17) and using $W_E = I_B \mathcal{E}$ yield the normalized electrical power

$$\begin{aligned} \tilde{W}_E &= \frac{I_B}{\sigma_t |E_m| A_t} \frac{\mathcal{E}}{|E_m| L} = i_B \tilde{\mathcal{E}} \\ &= i_B \left[(\phi_{A^*} - \phi_C) \tilde{L}_* - (1 + i_B) \tilde{L}_a + i_B \left(1 - \frac{1}{2} \tilde{L}_c\right) + 1 \right] \end{aligned} \quad (24)$$

in terms of the same 4 dimensionless ratios above, except for the ϕ_C term. Of course, $\mathcal{E}$ is a design parameter free to select as long as ϕ_C comes out negative, efficiency being larger the smaller is $\mathcal{E}$. Setting $\phi_C = 0$ in (24) would give the minimum $\mathcal{E}$ and minimum power $W_E = \mathcal{E} \times \sigma_t |E_m| A_t \times i_B$ allowed for any particular parameter-arrangement considered, making possible system design in looking for maximum performance. Use of $m_t = \rho_t A_t L$ and Eqs. (23) and (24) immediately determines the overall efficiency of a tether thruster defined in Eq. (2), rewritten in Eq. (7) as

$$\frac{1}{\eta_{eff}} = \frac{W_E}{W_M} + \frac{\alpha_t \rho_t A_t L}{\alpha W_M} = \frac{\tilde{W}_E}{\tilde{W}_M} + \frac{1/\tilde{E}_m^2}{\tilde{W}_M} \quad (25)$$

with $\tilde{E}_m \equiv E_m \sqrt{\alpha \sigma_t / \alpha_t \rho_t}$.

3.4. Minimum electromotive force for avoiding space-charge effects

The dimensionless electromotive force $\tilde{\mathcal{E}}$ is a design parameter free to select. However, there are certain constraints that should be considered in order to make the assumptions of our model valid. Equation (24) shows us that $\tilde{\mathcal{E}} = \tilde{\mathcal{E}}_1^{min} - \tilde{L}_* \phi_C$ with

$$\tilde{\mathcal{E}}_1^{min} \equiv \tilde{L}_* \phi_{A^*} - (1 + i_B) \tilde{L}_a + i_B \left(1 - \frac{\tilde{L}_c}{2}\right) + 1. \quad (26)$$

Therefore, a first condition is $\tilde{\mathcal{E}} > \tilde{\mathcal{E}}_1^{min}$ in order to have $\phi_C < 0$, as required to satisfy condition $I_C = 0$. Moreover, since j_c does not depend on the bias, a larger value of $\tilde{\mathcal{E}}$ will just support a corresponding uniformly-larger negative bias throughout the segment BC, with no change in bias through the anodic and insulated segments, and changes in current nowhere in the tether. Therefore, setting $\tilde{\mathcal{E}} = \tilde{\mathcal{E}}_1^{min}$ would give the minimum electrical power allowed for any particular parameter arrangement considered, making possible system design in looking for maximum performance.

However, the choice $\tilde{\mathcal{E}} = \tilde{\mathcal{E}}_1^{min}$ would not be consistent with lower Eq. (12) in our model. When we write $dI/dx = -p_t j_c$, with j_c bias-independent, we are assuming that the bias is negative enough for the potential to increase monotonically away from the emitting material in the plane normal to the tether, otherwise space charge from the emitted electrons results in a local potential minimum that reduces emission itself, pushing emitted electrons not energetic enough back to the tether. If we ignore for simplicity the photoelectrons, the approximate (find a discussion in Ref. [33]) condition to reach a monotonic potential is

$$\frac{e|\Phi_C|}{k_B T_e} > 0.39 \left(\frac{N_{em}}{N_0} \right)^{1.4} \quad (27)$$

with T_e the electron temperature of the unperturbed plasma and N_{em} the density of the emitted electrons. The latter is directly related with j_{th} if we assume a half-Maxwellian emitted distribution function, thus yielding $j_{th} = e N_{em} \sqrt{2k_B T_e / \pi m_e}$. Using this result in the relation $\tilde{\mathcal{E}} = \tilde{\mathcal{E}}_1^{min} - \tilde{L}_* \phi_C$ gives us

$$\tilde{\mathcal{E}}_2^{min} = \tilde{\mathcal{E}}_1^{min} + 0.39 \frac{k_B T_e}{e |E_m| L} \left(\frac{j_{th}}{e N_0 \sqrt{2k_B T_e / \pi m_e}} \right)^{1.4} \quad (28)$$

If $\tilde{\mathcal{E}} > \tilde{\mathcal{E}}_2^{min}$, then the LWT operates free of space charge effects and without developing an anodic segment at B^-C . A design value $\tilde{\mathcal{E}} < \tilde{\mathcal{E}}_2^{min}$ would require splitting analysis of the cathodic segment into 2 sub-segments, with RD emission in one, space charge limited emission applying in the other [34]. Actually, in characteristic tether applications, say $|E_m| \sim 150$ V/km, $T_e \sim 0.1$ eV, and $L \sim 5$ km, one finds $0.39 k_B T_e / e |E_m| L \sim 5.2 \times 10^{-5}$. Therefore, ϕ_C will be comparatively very small for typical values of both N_{em}/N_0 and L/L_* . For that reason, in the analysis of Sec. 4.1 we will reasonably take $\tilde{\mathcal{E}} = \tilde{\mathcal{E}}_1^{min}$, whereas the full Eq. (28) might be required at numerical analysis, as in Sec. 4.2, for extreme particular conditions. Note that our writing $h_t \approx 2A_t/p_t$ involves neglecting thickness-to-width ratios of order 10^{-3} in typical applications of bare tape-tethers.

4. High-efficiency design of LWT in thrust mode

This section discusses the optimal design of a LWT in thrust mode. The proposed scheme is based on the maximization of the

efficiency introduced in Eq. (7) together with rational mission constraints like for instance the required thrust level. Two scenarios are considered: the application of a LWT to re-boost a large space station with a power plant available and independent of the thrust and a tug-thruster mode. For convenience, we first get insight in the optimization problem by performing an analytical analysis in the *long-tether limit* approximation. The general case is studied numerically in Sec. 4.2.

4.1. The long-tether limit

Setting $\phi_C = 0$, i.e. $\tilde{\mathcal{E}} = \tilde{\mathcal{E}}_1^{\min}$, Eq. (24) takes the extremely simple form $\tilde{W}_E = i_B (\tilde{W}_M + \phi_A \tilde{L}_* + 1)$. Efficiency in Eq. (25) then becomes

$$\frac{1}{\eta_{eff}} = i_B + \frac{i_B (1 + \phi_A \tilde{L}_*) + \tilde{E}_m^{-2}}{\tilde{W}_M} \quad (29)$$

with the mechanical power rewritten as $\tilde{W}_M = (1 + \tilde{L}_i) i_B / 2 + \tilde{L}_* G(\phi_A, i_B)$ by using (5) to replace $\tilde{L}_c$ in Eq. (23) and function G defined and discussed in Appendix B.

We will now look for optimal performance conditions, considering small values of the L_*/L -ratio (*long tether limit*), as suggested below Eq. (23), and ignore $\tilde{L}_*$ terms in the above form of $\tilde{W}_M$ and in Eq. (29), which now reads

$$\frac{1}{\eta_{eff}} \approx i_B + \frac{2(i_B + \tilde{E}_m^{-2})}{i_B(1 + \tilde{L}_i)} \quad (30)$$

as function of both ambient $\tilde{E}_m$ -parameter and design $\tilde{L}_i$ -parameter, and related value i_B . While efficiency in Eq. (30) just increases with $\tilde{L}_i$ within the valid range $0 < \tilde{L}_i < 1$, condition $\partial \eta_{eff} / \partial i_B = 0$ shows that it does reach a maximum as function of i_B ,

$$\eta_{eff}^* = \frac{1 + \tilde{L}_i}{2 \left(1 + \sqrt{2\tilde{E}_m^{-2} (1 + \tilde{L}_i)} \right)} \quad (31)$$

at the value $i_B^* = \sqrt{2\tilde{E}_m^{-2} / (1 + \tilde{L}_i)}$. We first note that the efficiency in Eq. (31) tops a maximum, $\tilde{E}_m / (\tilde{E}_m + 2)$ at $\tilde{L}_i = 1$, as in the case of using a HC for cathodic contact [26]. Actually, having $\tilde{L}_i$ near unity, i.e. having negligibly short anodic and cathodic segments (AA^* and BC in Fig. 1, respectively) will most often be far from convenient or possible. Although having a high efficiency, the mechanical power obtained from the system would be small. Consider then how efficiency decreases as $\tilde{L}_i$ is cut in half, from 1 to 0.5, a half-range covering insulated-segment lengths that lead to highest efficiencies. At $\tilde{L}_i = 0.5$, Eq. (31) gives $\eta_{eff}^* = 0.75\tilde{E}_m / (\tilde{E}_m + \sqrt{3})$. The resulting efficiency drop is less than 20% for representative $\tilde{E}_m$ values, from 0.429 to 0.348 for $\tilde{E}_m = 1.5$, and from 0.273 to 0.227 for $\tilde{E}_m = 0.75$. Actually, the full efficiency drop to $\tilde{L}_i = 0$ is less than 40%.

We further note that i_B^* can also be used to write $\tilde{L}_i$ as

$$\tilde{L}_i = \frac{2}{(\tilde{E}_m i_B^*)^2} - 1 \quad (32)$$

for convenient determination of the i_B^* range of values for different ambient and design parameters. The range $0.5 < \tilde{L}_i < 1$ above implies condition

$$2/\sqrt{3} > \tilde{E}_m i_B^* > 1 \quad (33)$$

For $E_m = 150$ V/km (a representative value in LEO orbits at low inclination) and $\alpha = 20$ kg/kW (at the tug-thruster mode), we would have $\tilde{E}_m = 1.5$ for aluminum tethers, with factor $\alpha_t = 2.5$ for tether-related hardware. Equation (33) then reads

$$0.77 \approx 4/3\sqrt{3} > i_B^* > 2/3 \approx 0.67 \quad (34)$$

Moving to $\alpha = 5$ kg/kW, when a power plant is available independently of thrusting (case of the ISS), $\tilde{E}_m$ decreases to 0.75, the i_B^* bounds in Eq. (34) increasing by a factor of 2. A similar result applies for α back to the tug-mode but considering LEO orbits at moderately high inclination, with E_m down to 75 V/km, say. For combined moderately-high inclination and $\alpha = 5$ kg/kW, i_B^* bounds in Eq. (34) would increase by a factor 4.

Typical values for i_B^* will be used in Appendix B to discuss validity of the approximation of Eq. (30). Here we discuss how such values could be used as help in selecting the fractional lengths of all three tether segments, in the long-tether approximation, compatible with Eq. (5). In order to gain insight into the problem, the study is carried out for the particular case $\phi_A = 1$ because the ratio $\tilde{L}_a/\tilde{L}_*$ takes a very simple analytical form (see Eq. (43) and consider Fig. 2 to extend the results to other ϕ_A values). For $\phi_A = 1$, Eq. (5) can be written as

$$\left(\frac{2}{(\tilde{E}_m i_B^*)^2} - 1 \right) + 4 \left[(1 + i_B^*)^{1/3} - 1 \right] \tilde{L}_* + \frac{i_B^*}{j_c} = 1 \quad (35)$$

where i_B^* is itself related to $\tilde{E}_m$ as discussed above. In Eq. (35) we used Eq. (20) and simple results from Eqs. (21) and (22) for $\phi_A = 1$,

$$\frac{\tilde{L}_a}{\tilde{L}_*} = 4 \left(\phi_A^{1/4} - 1 \right) = 4 \left[(1 + i_B)^{1/3} - 1 \right] \quad (36)$$

Note the simple relation between anodic and cathodic lengths, independent of both Eq. (30) and current i_B though varying with the actual ϕ_A value,

$$\left(1 + \frac{\tilde{L}_a}{4\tilde{L}_*} \right)^3 = 1 + j_c \tilde{L}_c \quad (37)$$

Consider low orbital inclination ($E_m = 150$ V/km and no dedicated power plant ($\alpha = 5$ kg/kW), leading to $\tilde{E}_m = 0.75$ above and i_B^* bounds in Eq. (34) increasing by a factor of 2, $i_B^* = 1.54$ for $\tilde{L}_i = 0.5$. To determine $\tilde{L}_c$ we use values $W_t = 1.5$ eV and $T_t = 580$ K = 0.05 eV in Richardson-Dushman law, with ratio $W_t/k_B T_t = 30$; also, we use aluminum conductivity, and values $L = 10$ km and $h_t = 30$ μ m, finally yielding $j_c \approx 4.7$, $\tilde{L}_c = i_B^* / j_{RD} \approx 0.33$. Using $i_B^* = 1.54$ in $\tilde{L}_a$, at Eq. (36), we first get $\tilde{L}_a/\tilde{L}_* \approx 1.46$, then $\tilde{L}_a = 1 - 0.5 - 0.33 \approx 0.17$, and $\tilde{L}_* \approx 0.17/1.46 \approx 0.116$. Moving to $E_m = 120$ V/km (a proper value to analyze ISS-re-boost at mid-inclinations) could keep the above results if L is reduced to 8 km, which is actually more convenient.

4.2. General regime

Previous section provides useful physical insight on the optimal design of LWTs as thrusters. The assumptions used there for simplicity, such as small $\tilde{L}_*$, no photoelectric effects, and $\phi_A = 1$, are now removed and performance computed numerically. In a general case, and besides tether photoelectric model, one readily finds that tether performance depends on the following mission-related parameters

$$\tilde{E}_m \equiv E_m \sqrt{\frac{\alpha \sigma_t}{\alpha_t \rho_t}}, \quad \kappa \equiv \frac{2j_c}{\sigma_t |E_m|}, \quad \gamma \equiv \frac{k_B T_e}{e |E_m| \ell},$$

$$\nu \equiv \frac{N_{em}}{N_0} = \frac{4\kappa}{3} \sqrt{\frac{e|E_m|\ell}{\pi k_B T_t}} \quad (38)$$

which basically involve ambient conditions and the state-of-the-art of LWT manufacturing. The design parameters are

$$\begin{aligned} \tilde{h}_t &\equiv \frac{h_t}{L}, \quad \tilde{L}_* \equiv \frac{L_*}{L} = \left(\frac{\ell}{L}\right)^{1/3} \tilde{h}_t^{2/3}, \quad \tilde{L}_c \equiv \frac{L_c}{L}, \\ \tilde{L}_a &\equiv \frac{L_a}{L}, \quad \tilde{\mathcal{E}} \equiv \frac{\mathcal{E}}{|E_m|L} \end{aligned} \quad (39)$$

The optimal value of $\tilde{\mathcal{E}}$ avoiding space charge effects is $\tilde{\mathcal{E}} = \tilde{\mathcal{E}}_2^{min}$, which will be taken hereafter. When evaluating $\tilde{W}_E = i_B \tilde{\mathcal{E}}$ and $\tilde{W}_M$ from Eq. (23), and the efficiency from (25), one should keep in mind that: (i) $\tilde{L}_a + \tilde{L}_i + \tilde{L}_c = 1$, (ii) $\tilde{j}_c = \kappa/\tilde{h}_t$, (iii) $i_B = \tilde{j}_c \tilde{L}_c$, (iv) ϕ_A is given by Eq. (21), and (iv) ϕ_A , i_B , and $\tilde{L}_a/\tilde{L}_*$ are linked by Eq. (22). Moreover, in order to meet the condition $\phi_A > 0$, the length of the anodic segment should be below the threshold (see Appendix A)

$$\tilde{L}_a^{max} \equiv \frac{4}{3} \tilde{L}_* F_s \left[\cosh^{-1}(1 + i_B) \right] \quad (40)$$

Otherwise, a cathodic segment ignored in our model would be developed. Therefore the design parameter $\tilde{L}_a$ should be within the range $0 < \tilde{L}_a < \min(1 - \tilde{L}_c, \tilde{L}_a^{max})$, at nominal environment; bias at A will often develop negative values through re-boost, with weak effects.

Hereafter we will consider typical values for a ISS re-boost mission, $E_m = 120$ V/km, at 51.64° mid-inclination, $N_0 = 10^{11} \text{ m}^{-3}$, $T_e = 0.1$ eV, and $\alpha = 5$ kg/kW, and the following tether parameters $\rho_t = 2700 \text{ kg/m}^3$, $\sigma_t = 3.546 \times 10^7 \Omega^{-1}\text{m}^{-1}$, $\alpha_t = 2.5$ and $W_t = 1.5$ eV. We will also adopt the tether photoelectric model presented in Ref. [22], which considers a Fowler-DuBridge law for the photoelectric yield. Since uncertainties exist now on the cathodic capabilities of LWTs, we will consider a *hot* case with $T_t = 600$ K and a *cold* case with $T_t = 550$ K, which are compatible with the aluminum alloy 1100-H19 (melting point at 923 K). If a higher temperature would be needed due to difficulties on manufacturing a LWT with $W_t = 1.5$ eV, a BeCu alloy with melting point at 1300 K could be used. Regarding tether cross-section, we will consider a tape because it is well-known that it presents better performance than round tethers (find studies on current collection [35,36], cut probability [10,11] and dynamics [37] elsewhere). Table 1 summarizes the values of the dimensionless parameters.

Table 1
Parameters used in Sec. 4.2.

Parameter	Cold case	Hot case
$\tilde{\mathcal{E}}_m$	0.615	0.615
$\kappa \times 10^8$	0.46	5.26
$\gamma \times 10^{19}$	3.6	3.6
ν	8.3	91.9

We start by analyzing the optimum values of the anodic ($\tilde{L}_a^{opt}$), cathodic ($\tilde{L}_c^{opt}$), and insulated $\tilde{L}_i^{opt} = 1 - \tilde{L}_a^{opt} - \tilde{L}_c^{opt}$ segments for a given tether thickness and length. The optimum is found by imposing the conditions $\partial \eta_{eff}/\partial \tilde{L}_a = \partial \eta_{eff}/\partial \tilde{L}_c = 0$, i.e. looking for the maximum in a plot of η_{eff} in the $\tilde{L}_a - \tilde{L}_c$ plane. An example is given in Fig. 3, which corresponds to $h_t = 30 \mu\text{m}$ and $L = L_*$. The red lines correspond to $\tilde{L}_a + \tilde{L}_c = 1$ ($\tilde{L}_i = 0$) and $\tilde{L}_a = \tilde{L}_a^{max}$ in Eq. (40) ($\phi_A = 0$), respectively. In this particular case (see red point in Fig. 3), one finds $\eta_{eff}^{max} \approx 0.026$ for $\tilde{L}_a^{opt} \approx 0.16$ and $\tilde{L}_c^{opt} \approx 0.77$ (and insulated segment $\tilde{L}_i \approx 0.07$). Moving to the hot case (not shown) yields $\eta_{eff}^{max} \approx 0.077$ for $\tilde{L}_a^{opt} \approx 0.54$ and $\tilde{L}_c^{opt} \approx 0.24$ (and insulated segment $\tilde{L}_i \approx 0.22$). Obviously, $\tilde{L}_c^{opt}$ decreases as the parameter κ

Fig. 3. Example of LWT efficiency in the $\tilde{L}_a - \tilde{L}_c$ plane. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

Fig. 4. Optimal tether lengths $\tilde{L}_a^{opt}$ (top) and $\tilde{L}_c^{opt}$ (bottom) versus L/L_* for the parameters of Table 1.

increases, i.e. the cathodic contact of the tether with the plasma is improved by lowering the work function-to-temperature ratio.

Following the previous procedure, i.e. imposing the conditions $\partial \eta_{eff}/\partial \tilde{L}_a = \partial \eta_{eff}/\partial \tilde{L}_c = 0$, we computed $\tilde{L}_a^{opt}(h_t/\ell, L/L_*)$ and $\tilde{L}_c^{opt}(h_t/\ell, L/L_*)$ for a range $0 < L/L_* < 6$ and $h_t/\ell \times 10^{23}$ equal to 1.3, 2.6 and 5.2, which correspond to $h_t \approx 30, 60$, and $120 \mu\text{m}$ (note that $\ell = 2.32 \times 10^{18} \text{ m}$ for the values of Table 1). As shown in Fig. 4, the relative lengths of the optimal anodic and cathodic segments decreases and increases with h_t/ℓ for a given L/L_* ratio. The length of the optimal cathodic segment decreases with the temperature, and both the anodic and cathodic lengths decrease as L/L_* increases. The latter is consistent with the analytical calculations of Sec. 4.1, where it was shown that the maximum efficiency in the long tether limit is reached for $\tilde{L}_i \rightarrow 1$.

If we now use the parameters of Table 1 and set $\tilde{L}_a = \tilde{L}_a^{opt}(h_t/\ell, L/L_*)$ and $\tilde{L}_c = \tilde{L}_c^{opt}(h_t/\ell, L/L_*)$, the optimal efficiency of a LWT in thruster mode can be found. Fig. 5 shows the efficiency versus L/L_* for exactly the same h_t/ℓ ratios of Fig. 4. For convenience, the optimal efficiency given by Eq. (31) with $\tilde{L}_i = 1$, which coincides with the maximum efficiency of a bare tether with

Fig. 5. LWT efficiency versus L/L_* for the parameters of Table 1.

Fig. 6. $\tilde{W}_m$ and $\tilde{W}_e$ efficiency versus L/L_* for the parameters of Table 1.

an ideal HC, is also shown (black line). Clearly, LWT performance are very sensitive to the ratio W_t/T_t : a temperature variation of 50 K changes the efficiency about a factor 3. We also highlight that designing LWT missions with low thicknesses is doubly beneficial. First, since $j_c \sim h_t^{-1}$, decreasing the thickness and keeping L/L_* fixed improves the efficiency because one moves from point A to B in Fig. 5 by improving the performance of the cathodic segment. This effect is less important in the hot case than in the cold case and vanishes as $\kappa \rightarrow \infty$. The behavior can be understood by remembering that the efficiency of a bare tether with HC does not involve h_t when presented versus L/L_* [26]. Second, for given length, decreasing the thickness also improves the performance of the anodic segment because $L/L_* = L/\ell^{1/3}h_t^{2/3}$. This is illustrated in Fig. 5 as a change from point A to C. For completeness, Fig. 6 displays the normalized mechanical and electrical power for optimal lengths of the anodic and cathodic segments.

5. Discussion and ISS application

Previous results can be directly applied to re-boosting of the ISS. The optimal design of a LWT would require a deeper analysis involving not only the efficiency, as made in this work, but also

other considerations as the air drag on the tether itself, tether cut probability considerations, and mechanical constraints, among others. However, we can give here some tentative values. The station, with mass about 420 tons, has an orbit with inclination 51.64° and altitude around 400 km. The aerodynamic drag F_D , which is in the order of 0.2 N, produces the decay of the station at a rate of 2 km per month (mechanical power $W_M^D = F_D \cdot \mathbf{v} \approx 1.5$ kW). As already discussed, low tether thicknesses are beneficial but a lower bound may exist due to mechanical considerations. Let us take $h_t = 30 \mu\text{m}$ and the *hot* case in Table 1. Since the LWT should compensate the drag, tether width and length are linked by

$$w_t(L/L_*) = \frac{W_M^D}{\sigma_t E_m^2 h_t L_*} \times \frac{1}{\tilde{W}_M(L/L_*)} \frac{L_*}{L} \\ = 7.66 \text{ cm} \times \frac{1}{\tilde{W}_M(L/L_*)} \frac{L_*}{L} \quad (41)$$

where $\tilde{W}_M(L/L_*)$ is given in Fig. 6. Increasing the tether length, which produces higher efficiency according to Fig. 5, implies decreasing the width. However, the latter should be above some value to make the tether robust against small debris impacts [11]. Taking $w_t = 2$ cm, that would make the anodic segment operate under the OML regime, one finds that Eq. (41) gives $L \approx 4L_* \approx 5.11$ km with $\tilde{W}_M = 1.55 \text{ kW} > W_M^D$. For that choice, the optimum lengths of the anodic, cathodic, and insulated segments are $L_a^{\text{opt}} \approx 1.74$ km, $L_c^{\text{opt}} \approx 0.77$ km, and $L_i^{\text{opt}} \approx 2.6$ km. These numbers and Eq. (28) show that $\mathcal{E}_1^{\text{min}} \approx \mathcal{E}_2^{\text{min}}$ and, for the ISS case, one could safely take from the beginning $\mathcal{E} = \mathcal{E}_1^{\text{min}}$ and simplify the analysis. The efficiency of the system is $\eta_{\text{eff}} \approx 0.18$, the electrical power $W_E \approx 4.7$ kW, the mass of the tether $m_t \approx 8.3$ kg and the mass of the tether system $m_s \approx 20.7$ kg.

The main advantage of the electrodynamic tether as compared with matured thrust technologies, including chemical and electrical thrusters, is its propellant-less character and, as shown in Sec. 2, its benefits increase with the duration of the mission. Regarding the comparison between LWTs and conventional bare tethers equipped with active electron emitters, a definitive conclusion cannot be found yet. The absence of an active electron emitter in the LWT system decreases its complexity considerably, reduces the power needs, and eliminates the needs for expellant. However, as shown in our analysis, LWT performance are very sensitive to the work function-to-temperature ratio and, the lower the ratio, the better the performance. A comparison among the two types of tethers requires knowing the results of on-going research activities on manufacturing and testing of LWT samples. Taking into account the melting point of alloys with high conductivity-to-density ratio, a value of the work function below around 2 eV would be needed.

6. Conclusions

New advances in material science can open new opportunities for space tether technology. One of them is the LWT, a type of electrodynamic tether that does not need an active electron emitter. This work presented an electrical model for the analysis of LWT operation in thrust mode, where the electromotive force provided by an onboard power source is used to re-boost the spacecraft or compensate the air drag. The proposed scheme involves a bare anodic segment for electron collection, followed by an insulated segment, a power source, and a tether segment coated with a low work function material to enhance the passive electron emission through thermionic and photoelectric effects. The minimum electromotive force that is necessary to avoid the development of a tether segment operating under space charge conditions has been found, as well as key figures of merit of the system as a function of tether design parameters and ambient variables.

Design conditions for high-efficiency operations have been studied analytically under sensible assumptions to simplify the equations. Tether efficiency, measuring the rate of conversion from electrical to mechanical power, increases with the ratio between the tether length and the characteristic length gauging ohmic effects. The optimal efficiency increases, though weakly, with the length of the insulated segment and a dimensionless parameter involving the motional electric field and the tether conductivity-to-density ratio. Useful formulae relating the lengths of the anodic, cathodic and insulated segments with the maximum current value have been obtained.

For a general case, i.e. without taking any simplification in the proposed model, LWT performance was found numerically. For a given mission, and tether length and thickness, optimal lengths for the different tether segments were found and used to present optimal efficiencies. The results are very sensitive to the quality of the cathodic contact, which is mainly governed by the work function-to-temperature ratio. The analysis has been applied to the re-boost of the ISS. It indicates that a LWT of length, width and thickness equal to 5.1 km, 2 cm, and 30 μm can compensate the air drag of the station with no consumable if fed with a power source of 4.7 kW.

Declaration of competing interest

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G. Sanchez-Arriaga and C. Bombardelli. Orbit propulsion system by floating conductor cables, 05 2017. Spanish Patent WO2017081351A1.

The patent is cited in the bibliography of the manuscript.

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Appendix A. The anodic segment

Following a similar procedure to the one presented in Ref. [38] for bare tethers in passive modes, Eq. (22) can be written in a form that is more convenient for fast numerical calculations. For instance, for $\phi_A > 1$, we write $1 + i = \sqrt{\phi_A^{3/2} - 1} \sinh v$ and find

$$\frac{\tilde{L}_a}{\tilde{L}_*} = \frac{4}{3} (\phi_A^{3/2} - 1)^{1/6} \int_{v_{s0}}^{v_{sf}} \cosh^{1/3} v dv \quad (42)$$

A similar procedure for $\phi_A < 1$ and $\phi_A = 1$ yields

Fig. 7. Function G versus i_B for several ϕ_A values.

$$\frac{\tilde{L}_a}{\tilde{L}_*}(\phi_A, i_B) = \begin{cases} \frac{4}{3} (1 - \phi_A^{3/2})^{1/6} [F_s(v_{cf}) - F_s(v_{c0})], & \phi_A < 1 \\ 4 [(1 + i_B)^{1/3} - 1], & \phi_A = 1 \\ \frac{4}{3} (\phi_A^{3/2} - 1)^{1/6} [F_c(v_{sf}) - F_c(v_{s0})], & \phi_A > 1 \end{cases} \quad (43)$$

where

$$F_s(x) = \int_0^x \sinh^{1/3} \zeta d\zeta, \quad F_c(x) = \int_0^x \cosh^{1/3} \zeta d\zeta \quad (44)$$

and the variables $\cosh(v_{cf}) = (1 + i_B)/\sqrt{1 - \phi_A^{3/2}}$, $\cosh(v_{c0}) = 1/\sqrt{1 - \phi_A^{3/2}}$, $\sinh(v_{sf}) = (1 + i_B)/\sqrt{\phi_A^{3/2} - 1}$, $\sinh(v_{s0}) = 1/\sqrt{\phi_A^{3/2} - 1}$. This formulation is advantageous from a computational point of view because functions F_s and F_c are parameter-free and they can be also written explicitly in terms of hypergeometric functions.

Appendix B. The function $G(\phi_A, i_B)$

The dimensionless mechanical power could be written as $\tilde{W}_M = i_B (1 + \tilde{L}_i)/2 + \tilde{L}_* G(\phi_A, i_B)$ with

$$\begin{aligned} G(\phi_A, i_B) &= \int_{\phi_A}^{\phi_A^*} \left(1 - \frac{1 + \sqrt{1 + \phi_A^{3/2} - \phi_A^{3/2}}}{2\sqrt{1 + \phi_A^{3/2} - \phi_A^{3/2}}} \right) d\phi \\ &\equiv \int_{\phi_A}^{\phi_A^*} \left[1 - \frac{1 + (i_B/2)}{\sqrt{1 + \phi_A^{3/2} - \phi_A^{3/2}}} \right] d\phi \end{aligned} \quad (45)$$

where we used again Eq. (5) and ϕ_A^* is given by Eq. (21). Dropping the term $\tilde{L}_* G(\phi_A, i_B)$ in the mechanical power, as done in Sec. 4.1, requires a discussion about function G . For particular test of consistency in our neglect of L_*/L terms, consider the simple case of taking $\phi_A = 1$ in Eqs. (21) and (22), G in Eq. (45) then reading

$$G(i_B) = 3 + 2i_B - (3 + i_B)(1 + i_B)^{1/3} \quad (46)$$

with G increasing very slowly with i_B : $G(i_B < 1) = O(i_B^3)$, $G(1) \approx -0.04$, $G(2) \approx -0.21$, $G(3) \approx -0.52$, certainly yielding $\tilde{L}_* |G| \ll$

$\tilde{L}_* < 1$. This applies to the i_B^+ values considered in Sec. 4.1, ranging from 0.67 to $16/3\sqrt{3} \approx 3.08$. Fig. 7 showing G versus i_B at several ϕ_A values, further confirms general consistency (beyond the particular $\phi_A = 1$ case).

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