Spherical thermal waves in laser plasmas

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The thermal wave produced in a uniform plasma, when energy is absorbed on a spherical surface such that convection is negligible, is analyzed using an integral method, which is very accurate. The curvature speeds up (slightly slows down) the inner (outer) wave front and does not affect the temperature maximum.

Local deposition of energy in fluids with nonlinear heat conduction may give rise to thermal waves and negligible convection. In particular, when energy per unit area and time $\dot{\phi} = \phi_0 / \tau$ is deposited in a given plane within a plasma, where the conductivity is $\kappa = \kappa T / 2 \phi_0$, a thermal wave develops as $\alpha = \left(9k_e / 4m_e\right) / \left(\kappa^2 m_e^2 / \phi_0 K \right)^{2/3}$ is small ($T, \kappa, m_e, \phi_0$), and $k$ is electron temperature and density, ion mass, and Boltzmann's constant, respectively. If $\alpha$ is too small, however, the conductivity will be less than classical.

In laser-plasma applications, radiation absorption may be limited to a thin, spherical layer. For a mono- pulse shapes and moderately small $a$. A spherical wave should develop for a broad class of second order (quasi-planar approximation) solutions.

The initial and boundary conditions are $\hat{T} = 0$ at $\hat{r} = 0$, $\hat{T} = \tau^{-n / 2} \hat{a} = 0$ at $\hat{x} = \hat{x}_f$, where $\hat{x}_f$ being either the inner ($\hat{x}_i$) or outer ($\hat{x}_o$) wave front; it may be shown that $\hat{T} - (1 - \hat{a} / \hat{x}_f)^{2n / 3}$ near $\hat{x}_f$ (Ref. 1).

In addition, the last term in (2) may be dropped if use is made of the condition

$$\frac{\hat{T}^{2n} \hat{a} \hat{T}}{\partial \hat{x}} \bigg|_{\text{out}} = -\hat{g}(\hat{t})^2.$$  

For short times, the curvature effects are small. Expanding in powers of $\hat{t}$ and setting $\hat{g} = \hat{t}^{-\hat{a}}$, we find to second order (quasi-planar approximation)

$$\hat{x}_f = \hat{x}_i^{(n+o) / \hat{a}}_i + \hat{t}^{(n+o) / \hat{a}}_{i}.$$  

The upper (lower) sign should be used for $\hat{x} > 0$ and $\hat{x}_i$ (and $\hat{x}_o$), respectively. We notice that the planar (lowest-order) solution is symmetric, while the second-order correction is antisymmetric. An excellent approximation to $\hat{x}_i$ and $\hat{x}_o$ (well known for $\hat{p} = 1$) is

$$\hat{x}_i = \left(4 / 5\right)\left[1 / ight]^{s / h}(1 - s) / h; \hat{a}_o = \left[1 / ight]^{s / h}(1 - s) / h;$$

both $\hat{a}_i$ (which behaves roughly as $s(1 - s) / h$) and $\hat{a}_o$ are negative. Thus, to lowest order, curvature effects speed up $\hat{x}_i$ and slow down $\hat{x}_o$, do not affect $\hat{T}(\hat{x} = 0)$, and make the inner wave more steplike (the term $2K \hat{T}^{2n} / \hat{T} \hat{g} / \hat{a}$ in (1) is positive for $\hat{x} < 0$).

We now determine $\hat{x}_i, \hat{x}_o, \hat{T}(\hat{x} = 0)$ for arbitrary times. Multiplying Eq. (2) by $(1 + \epsilon \hat{a})^n$ and integrating between $\hat{x}_i$ and $\hat{x}_o$, we get

$$\frac{d}{d\hat{t}} \int_{\hat{x}_i}^{\hat{x}_o} (1 + \epsilon \hat{a})^n \hat{T} d\hat{x} = 0.$$
A discussion of the results, and their accuracy, is simpler for power law pulses, $g = t^p$. For $\tilde{T}_a$ we get

$$\tilde{T}_a = \left[7/8(1+p)\right]^{2/9} t^{2/9} \left(1 - \frac{1}{2} p^{2/9}\right);$$

which differs from the planar (and quasi-planar) result

$$G(\sigma) = 0.878(\epsilon/\epsilon^*)^{9/7}.$$  

For $\epsilon < \epsilon^*$, the solution (9) and (10) breaks down when the pulse ends, at a value $\sigma_1 > 0.557$:

$$G(\sigma_1) = 0.878(\epsilon/\epsilon^*)^{9/7}.$$  

We notice that the integral method does not work in cylindrical geometry, for which only the $q=1$ moment of the energy equation takes a simple form.

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