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Research Paper

Energy, cost and uniformity in the design of drip irrigation systems

Diego Chamba, Sergio Zubelzu^{*}, Luis Juana

Departamento de Ingeniería Agroforestal, Universidad Politécnica de Madrid, Av. Complutense s/n, 28040, Madrid, Spain

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ABSTRACT

A novel approach for the design of drip irrigation systems is presented. The diameters of the laterals and submain were calculated by minimising an objective cost function that includes investment and operating costs of drip irrigation units. Diameters, head loss and its distribution between laterals and submain, average operating pressure head, upstream end energy head and the distribution uniformity were all deduced from an economic approach. These variables are not matched alone but are the consequence of the economic optimisation. Several examples with self-compensating and non-compensating emitters fed from either a pump station or a water reservoir are presented to illustrate the feasibility of the proposed methodology.

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1. Introduction

The study of drip irrigation systems has occupied the interest of scientists for several decades. Initial work established the foundations of designs (Bralts, Edwards, & Wu, 1987; Keller & Karmeli, 1974; Wu & Gitlin, 1975; Wu, Howell, & Hiler, 1979), while later work developed the initial ideas (Baiamonte, 2016, 2017; Ju, Wu, Weckler, & Zhu, 2017; Juana, Losada, Rodríguez-Sinobas, & Sánchez, 2004; Valiantzas, 1998; Wu & Barragán, 2000; Yurdem, Demir, & Manuchan, 2015; Zayani, Alouini, Lebdi, & Lamaddalena, 2001).

The design of drip irrigation units is in most cases examined from an hydraulic point of view. The lateral and submain

diameters, or their maximum length for a given diameter, are deduced from the equations defining the energy state of the system. Several authors have proposed numerical (Bralts & Segerlind, 1985; Dandy & Hassanli, 1996; Kang & Nishiyama, 1996) or analytical solutions (Baiamonte, 2018; Baiamonte, Provenzano, & Rallo, 2015; Barragán, Cots, Monserrat, & Sanz, 1998; Zhang, Wu, & Zhu, 2013) using with the aforementioned design approach.

Under the traditional design paradigm, design and operating costs are not a design criteria but the result of the hydraulic requirements. Some, including Carrión, Tarjuelo, Hernández, and Moreno (2013), Carrión, Montero, Tarjuelo, and Moreno (2014) and Moreno, del Castillo, Montero, Tarjuelo, and Ballesteros (2016), have studied the

^{*} Corresponding author.

E-mail addresses: diego.chambaz@alumnos.upm.es (D. Chamba), sergio.zubelzu@upm.es (S. Zubelzu), luis.juana@upm.es (L. Juana).
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Nomenclature	
$q, \bar{q}, \bar{q}_u, q_n, q_{min}$	Emitters flow: generic, average, average per unit area, nominal, minimum ($l\ s^{-1}$)
Q, Q_0, Q_r	Flow: generic, unit upstream end, at generic point r ($m^3\ h^{-1}$)
N	Number of units
$h, \bar{h}, h_{min}, h_{inf}, h_{sup}$	Pressure head: generic, average, minimum, compensating range inferior level, compensating range superior level (m)
H_0, H_B	Pressure head: unit, upstream end (m)
$hf, hf_r, hf_t, hf_{rr}, hf_{ty}, hf_T, hf_{Try}, hf_F$	Head losses: generic, lateral, submain, lateral at generic point r , submain at generic point y , aggregated, aggregated to a generic point r, y , accessories of pump station (m)
r_{hf}	Optimum head losses distribution between submain and lateral
$c_i, \acute{c}_i, \acute{c}_l$	Coefficient of the optimum head losses distribution
m, c, n	Coefficients of generic head losses empirical expression: flow exponent, proportionality coefficient, diameter exponent
I	Elevation gradient
$I_0, I_{0t}, I_T, I_{Try}$	Hydraulic gradient: lateral, submain, aggregated including lateral and submain, aggregated to a generic point r, y .
D, D_r, D_t	Diameters: generic, lateral, submain (mm)
le, le_{ar}	Equivalent length: emitters and lateral insertions (m)
x, k	Coefficients of the emitter discharge equation
CV_m	Manufacturing coefficient of variation
u, u_2	Normalised Gaussian random variables
$s, s_r, \acute{s}_r$	Separation: emitters, laterals, laterals in paired units (m)
L, L_t, L_T, L_{Try}	Length: lateral, submain, total, to a generic point r, y (m)
r, y	Generic points at distances r and y in lateral and submain
$UD_{1/4}$	Distribution uniformity
σ_z	Standard deviation of the terrain elevations (m)
Z	Objective cost function
$CI_r, CI_t, CI_B, CA_E, CA_A$	Cost functions: lateral investment, submain investment, pump station investment, updated energy consumption and updated water consumption (€)
Z_u	Objective cost function per unit area
$CI_{ru}, CI_{tu}, CI_{Bu}, CA_{Eu}, CA_{Au}$	Cost functions per unit area: lateral investment, submain investment, pump station investment, updated energy and updated water consumption (€ m^{-2})
CI_{ru}, CI_{tu}	Lateral and submain investment costs per unit area. Paired laterals
$C_{uD}, C_{uDr}, C_{uDt}, C_{uP}, C_{uE}, C_{ua}$	Unitary costs: generic pipe, lateral, submain, power, energy and water (€ m^{-1} , € m^{-1} , € kW^{-1} , € kWh^{-1} , € m^{-3})
$C_{BH}, C_{EHUD}, C_{AUD}, c_r, c_t$	Coefficients of the unitary objective cost function
Γ	Water specific weight ($N\ m^{-3}$)
H	Pump station performance
a_i	Costs updating rate
a, e, a_r, e_r, a_t, e_t	Coefficient and exponent of the pipe investment cost function: generic, lateral and submain
Et_a, Et_{pp}	Evapotranspiration: annual, daily ($mm\ y^{-1}$, $mm\ d^{-1}$)
t_{rd}	Daily irrigation time ($h\ d^{-1}$)

relationship between costs and the design of drip irrigation units, but they do not provide analytical expressions for the optimum economic calculation. There is little research that put the economic variables at the heart of the drip irrigation system design.

The main objective of this work is to propose a method for the drip irrigation units design based on economic criteria. The minimisation of a simplified objective cost function considering the main design and operating costs is used for the design approach. A set of analytical expressions to calculate the diameters and the average operating pressure head are deduced from the minimum condition of the objective function. Under this approach, hydraulic variables are not matched alone but are the consequence of the economic optimisation.

2. Methodology

2.1. Economic approach for drip irrigation units design

The following objective cost function (Z) was proposed for the drip irrigation system (Eq. (1)).

$$Z = CI_r + CI_t + CI_B + CA_E + CA_A$$

$$= NC_{uDr} \frac{L_t}{s_r} L + NC_{uDt} L_t + \frac{\gamma(H_0 + hf_F)Q_0}{\eta} C_{uP} + \frac{Et_a}{a_i UD_{1/4}} \left(C_{ua} + \frac{\gamma(H_0 + hf_F)}{\eta} C_{uE} \right) \quad (1)$$

In which N is the number of units, L_t and L are the lateral and submain lengths, s_r the separation between laterals, γ the

water specific weight, H_0 and Q_0 the unit upstream end pressure head and flow, hf_F the constant head losses of the pump station accessories (the required pressure head at the pump station is $H_B = hf_F + H_0$) and η the pump performance. The costs included in Eq. (1) are:

Lateral (CI_L) and submain (CI_t) investment costs: The unitary cost per pipe unit length can be expressed with a power function dependent on the diameter (D), $C_{uD} = a D^e$. For plastic materials $e \approx 2$ while a depends on the pipe nominal pressure. Specific unitary costs for laterals (C_{uDr}) and submain (C_{uDt}) were also included.

Energy cost (CA_E): The annual energy cost depends on the consumed water, which is assumed to be equal to sum of the annual evapotranspiration, Et_a , the pump performance, the required energy head at the pump station and the unitary energy cost (C_{uE}). This term is updated with a rate a_i to be compared with investment costs.

Pump cost (CI_B): The upstream energy head and flow, and the unit power cost (C_{uP}) determine the pump station investment cost.

Cost of consumed volume of water (CA_A): This depends on the unitary water cost (C_{ua}), the amount of water consumed, the cost updating rate and distribution uniformity ($UD_{1/4}$).

In order to simplify the mathematical development, the objective cost function was divided by LL_t to refer it to unit area (Eq. (2)).

$$\begin{aligned} Z_u &= CI_{ru} + CI_{tu} + CI_{Bu} + CA_{Eu} + CA_{Au} \\ &= \frac{C_{uDr}}{s_r} + \frac{C_{uDt}}{L} + \frac{\gamma(H_0 + hf_F)}{\eta} \bar{q}_u C_{uP} + \frac{Et_a}{a_i UD_{1/4}} \\ &\quad \left(C_{ua} + \frac{\gamma(H_0 + hf_F) C_{uE}}{\eta} \right) \\ &= \frac{a_r}{s_r} D_r^{e_r} + \frac{a_t}{L} D_t^{e_t} + c_{BH}(H_0 + hf_F) + \frac{1}{UD_{1/4}} (c_{AUD} + c_{EHUD}(H_0 + hf_F)) \end{aligned} \quad (2)$$

where Z_u is the unitary objective cost function, CI_{ru} , CI_{tu} , CI_{Bu} , CA_{Eu} and CA_{Au} are the respective unitary costs, $\bar{q}_u$ is the average flow per unit area and a_r , a_t , e_r and e_t are the parameters of the laterals and submain cost functions, respectively. The parameters of Eq. (1) that are not dependent on the diameters were grouped in Eq. (2) as follows (Eqs. (3)–(5)).

$$c_{BH} = \frac{\gamma \bar{q}_u C_{uP}}{\eta} \quad (3)$$

$$c_{EHUD} = \frac{1}{a_i} \frac{\gamma Et_a}{\eta} C_{uE} \quad (4)$$

$$c_{AUD} = \frac{1}{a_i} Et_a C_{ua} \quad (5)$$

The proposed method required the diameters that minimise the cost function to be calculated. However, as an intermediate step, the cost function was expressed in terms of the lateral (hf_r) and submain (hf_t) head losses which simplified the deduction of the analytical expressions. As a result, the diameters included in Eq. (2) were substituted for the expressions presented in Eqs. (7) and (9), yielding Eq. (10).

$$hf_r = \frac{c}{m+1} \left(Q_0 \frac{s_r}{L_t} \right)^m D_r^{-n} L \left(1 + \frac{le}{s} \right) \quad (6)$$

$$D_r = \left(\frac{c}{m+1} \left(\frac{s_r Q_0}{L_t} \right)^m \frac{L}{hf_r} \left(1 + \frac{le}{s} \right) \right)^{\frac{1}{n}} \quad (7)$$

$$hf_t = \frac{c}{m+1} Q_0^m D_t^{-n} L_t \left(1 + \frac{le_{ar}}{s_r} \right) \quad (8)$$

$$D_t = \left(\frac{c}{m+1} Q_0^m \frac{L_t}{hf_t} \left(1 + \frac{le_{ar}}{s_r} \right) \right)^{\frac{1}{n}} \quad (9)$$

$$\begin{aligned} Z_u &= CI_{ru} + CI_{tu} + CI_{Bu} + CA_{Eu} + CA_{Au} \\ &= \frac{c_r}{hf_r^{e_r/n}} + \frac{c_t}{hf_t^{e_t/n}} \left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) (H_0 + hf_F) + \frac{c_{AUD}}{UD_{1/4}} \end{aligned} \quad (10)$$

where c , m and n are the parameters of a generic empirical head losses equation $hf = c Q^m D^{-n}$, D_r and D_t are the lateral and submain diameters, le and le_{ar} the emitters and lateral insertion equivalent lengths and s the distance between lateral insertions in the submain.

The parameters independent of the head losses were grouped in Eq. (10) into two coefficients (Eqs. (11) and (12)).

$$c_r = \frac{a_r}{s_r} \left(\frac{c}{m+1} \left(\frac{s_r Q_0}{L_t} \right)^m L \left(1 + \frac{le}{s} \right) \right)^{e_r/n} \quad (11)$$

$$c_t = \frac{a_t}{L} \left(\frac{c}{m+1} Q_0^m L_t \left(1 + \frac{le_{ar}}{s_r} \right) \right)^{e_t/n} \quad (12)$$

In Eq. (10), c_r and c_t , L , L_t , c_{BH} , c_{EHUD} , c_{AUD} , hf_F , CV_m and σ_z were known so the unknowns were hf_r , hf_t and the average operating pressure head ($\bar{h}$). Although these variables were correlated and $\bar{h}$ could be eliminated using $UD_{1/4}$ or H_0 , this would complicate the mathematical deductions and the numerical simulations. Hence, hf_r , hf_t and $\bar{h}$ were the unknowns of the design problem that were deduced from the minimum condition of the objective cost function.

Once the head losses and the average operating pressure head were known, the minimum lateral and submain diameters were deduced and the commercial diameters were selected from the above deduced minimum values.

2.2. Uniformity and hydraulic questions

The emitter discharge (q) depends (Eq. (13)) on the pressure head (h), the coefficients k and x and the Gaussian manufacturing variability ($u CV_m$).

$$q = kh^x (1 + u CV_m) \quad (13)$$

The discharge variability due to the topographic variability within the unit was incorporated through its standard deviation (σ_z) as Eq. (14) shows.

$$q = (1 + u CV_{qm}) k (h + u_2 \sigma_z)^x \quad (14)$$

A constant flow Q_0 is required to be available for each drip irrigation unit. It implies that the average flow per unit

area ($\bar{q}_u$) is constant and that the emitter average flow ($\bar{q}$) is only determined by Q_0 , the lateral and submain lengths and the distances between laterals and between emitters (Eq. (15)).

$$\bar{q} = Q_0 s_r s_t / (LL_t) \quad (15)$$

This approach allows for the homogenisation of each unit design and comparisons between them. Once the average emitters pressure head and flow were defined, the unit pressure head was calculated as follows (Eq. (16)).

$$H_0 \cong \bar{h} + \frac{m+1}{m+2} (hf_r + hf_t) - \frac{1}{2} (I_0 L + I_{0t} L_t) = \bar{h} + \frac{m+1}{m+2} hf_T - \frac{1}{2} I_T L_T \quad (16)$$

where I_0 and I_{0t} are the lateral and submain hydraulic slopes, hf_T the aggregated head losses ($hf_T = hf_r + hf_t$) and I_T and L_T , respectively, the aggregated slope and length of the unit.

The minimum pressure head appears at the downstream end in the horizontal units (or units with unfavourable gradient). In units with favourable gradient, the point with minimum pressure head is located at distances r and y along the lateral and submain. Particularising Eq. (16) for a general case in which the minimum pressure head (h_{min}) places at r and y points yields (Eq. (17)).

$$H_0 \cong \bar{h} + \frac{m+1}{m+2} hf_T - \frac{1}{2} I_T L_T = h_{min} + hf_{Try} - I_{Try} L_{Try} \quad (17)$$

pressure head losses at r (hf_{rr}) and y (hf_{ty}) points were deduced as follows (Eqs. (20) and (22)).

$$r = L \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{1}{m}} \right) \quad (19)$$

$$hf_{rr} = hf_r \left(1 - \left(1 - \frac{r}{L} \right)^{m+1} \right) = hf_r \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} \right) \quad (20)$$

$$y = L_t \left(1 - \left(\frac{L_t I_{0t}}{(m+1)hf_t} \right)^{\frac{1}{m}} \right) \quad (21)$$

$$hf_{ty} = hf_t \left(1 - \left(1 - \frac{y}{L_t} \right)^{m+1} \right) = hf_t \left(1 - \left(\frac{L_t I_{0t}}{(m+1)hf_t} \right)^{\frac{m+1}{m}} \right) \quad (22)$$

The above equations are valid for $r, y \geq 0$. Using logical variables, the expressions were generalised as follows.

$$r = L \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) \quad (23)$$

$$hf_{rr} = hf_r \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) \quad (24)$$

Substituting Eq. (23) and Eq. (24) into Eq. (16) yields.

$$H_0 \cong h_{min} + hf_{Try} - I_{Try} L_{Try} = h_{min} + hf_r \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) + hf_t \left(1 - \left(\frac{L_t I_{0t}}{(m+1)hf_t} \right)^{\frac{m+1}{m}} (I_{0t} > 0) \right) \left(\frac{L_t I_{0t}}{(m+1)hf_t} < 1 \right) - I_0 L \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) - I_{0t} L_t \left(1 - \left(\frac{L_t I_{0t}}{(m+1)hf_t} \right)^{\frac{1}{m}} (I_{0t} > 0) \right) \left(\frac{L_t I_{0t}}{(m+1)hf_t} < 1 \right) = \bar{h} + \frac{m+1}{m+2} hf_T - \frac{1}{2} I_T L_T \quad (25)$$

In which I_{Try} and L_{Try} are the aggregated hydraulic slope and length to the point r, y , h_{min} the minimum operating pressure head and hf_{Try} the aggregated head losses to the

Finally, the expression proposed in Burt et al. (1997) for the distribution uniformity was used (Eq. (26)).

$$UD_{1/4} \cong 1 - \sqrt{(1.27 CV_m)^2 + \left(1 - UD_{h1/4}^x \right)^2 + \left(1.27 x \frac{\sigma_z}{h} \right)^2} \cong 1 - \sqrt{(1.27 CV_m)^2 + \left(1 - \left(\frac{h_{min}}{h} \right)^x \right)^2 + \left(1.27 x \frac{\sigma_z}{h} \right)^2} \quad (26)$$

generic point r, y ($hf_{Try} = hf_{rr} + hf_{ty}$). The point r places where the terrain gradient (I) equals the hydraulic gradient (I_0), $I = I_0$, so the flow at this point Q_r can be deduced as follows (Eq. (18)).

$$I_0 = I = \text{const} Q_r^m \rightarrow LI_0 = \text{const} Q_r^m L = (m+1) hf_r \left(\frac{Q_r}{Q_0} \right)^m \quad (18)$$

$$\cong (m+1) hf_r \left(1 - \frac{r}{L} \right)^m$$

Rearranging for r in Eq. (18), and similarly for y , the

3. Results

The objective cost function presented in Eq. (10) was used to deduce the analytical expressions to be used for the drip irrigation system design. For this purpose, the derivatives of that function with respect to the lateral and submain head losses,

Fig. 1 – Schematic diagram of the analysed system.

and, occasionally, with respect to the average operating pressure head, were calculated.

In the following sections analytical expressions will be presented and applied to different drip irrigation units (with self-compensating or non-compensating emitters, horizontal or favourable gradient, fed from pump stations or from water reservoirs) in order to illustrate the feasibility of the proposed

method. A hypothetical drip irrigation unit was utilised (Fig. 1, Table 1).

3.1. Self-compensating emitters

3.1.1. Horizontal units

For self-compensating emitters, provided that the operating pressure head is maintained within the emitters self-compensating range, distribution uniformity does not depend on head losses. Since the distribution uniformity is constant ($UD_{1/4} = 0.96$), water cost has no influence on the minimum of the objective function and for this reason, CA_{Au} was not included in Z_u .

For horizontal units, once the emitter operating pressure head was set to coincide with the minimum value of the self-compensating range, the average operating pressure head only depended on head losses (see Eq. (17)). Thus, hf_r and hf_t became the unique unknowns of the design problem. Hence, the minimum of the cost function yields the

Table 1 – Characteristics of the analysed unit.

Variable	Value
A (ha)	2.50
Q_0 ($m^3 h^{-1}$)	15
N	10
L (m)	50
L_t (m)	50
$\bar{q} = q_n$ ($L h^{-1}$)	3
h_{inf} (m)	4
h_{sup} (m)	40
CV_m	0.03
s (m)	0.5
s_r (m)	1
I_0	0/0.03
I_{0t}	0/0.01
σ_z (m)	0.25
le (m)	0.3
le_{ar} (m)	0.1
hf_F (m)	5
m	1.75
n	4.75
c	0.0246
a_r	0.001
e_r	2
a_t	0.0009
e_t	2
C_{uP} ($€ kW^{-1}$)	1000
η	0.75
C_{uE} ($€ kWh^{-1}$)	0.1
Et_a (mm season $^{-1}$)	600
Et_{pp} (mm d $^{-1}$)	7
a_t (year $^{-1}$)	0.065
C_{ua} ($€ m^{-3}$)	0.025

In which ET_{pp} is the daily peak evapotranspiration. Constant values of Eqs. (3)–(5) and (11) and (12) are displayed in Table 2.

Table 2 – Constant values of the cost function.

Variable	Value
c_{BH} ($€ m^{-3} ha^{-1}$)	21.78
c_{EHUD} ($€ m^{-3} ha^{-1}$)	33.50
c_{AUD} ($€ ha^{-1}$)	2308
c_r ($€ m^{e/n} ha^{-1}$)	2002
c_t ($€ m^{e/n} ha^{-1}$)	550

Table 3 – Characteristics of the optimal solution for the analysed unit (self-compensating emitters, horizontal units).

hf_r (m)	hf_t (m)	D_r (mm)	D_t (mm)	hf_T (m)	H_0 (m)	$\bar{h}$ (m)	H_B (m)
6.69	2.69	9.48	44.86	9.38	13.38	6.50	18.38
CI_{ru} ($€ ha^{-1}$)	CI_{tu} ($€ ha^{-1}$)	CI_{Bu} ($€ ha^{-1}$)	CA_{Eu} ($€ ha^{-1}$)	CA_{Au} ($€ ha^{-1}$)	Z_u ($€ ha^{-1}$)		
899	362	400	640	–	2302		

optimum relationship between both head losses as follows (Eq. (29)).

$$dZ_u = 0 = -\frac{e_r}{n} \frac{c_r}{hf_r^{1+\frac{e_r}{n}}} dhf_r - \frac{e_t}{n} \frac{c_t}{hf_t^{1+\frac{e_t}{n}}} dhf_t + \left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) (dhf_r + dhf_t)$$

$$\begin{cases} -\frac{e_r}{n} \frac{c_r}{hf_r^{1+\frac{e_r}{n}}} dhf_r + \left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) dhf_r = 0 \\ -\frac{e_t}{n} \frac{c_t}{hf_t^{1+\frac{e_t}{n}}} dhf_t + \left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) dhf_t = 0 \end{cases}$$

$$hf_r = \left(\frac{e_r}{n} \frac{c_r}{c_{BH} + \frac{c_{EHUD}}{UD_{1/4}}} \right)^{\frac{n}{e_r+n}} \quad (27)$$

$$hf_t = \left(\frac{e_t}{n} \frac{c_t}{c_{BH} + \frac{c_{EHUD}}{UD_{1/4}}} \right)^{\frac{n}{e_t+n}} \quad (28)$$

$$\frac{e_t = e_r = e}{n} \rightarrow r_{hf} = \frac{hf_t}{hf_r} = \left(\frac{c_t}{c_r} \right)^{\frac{n}{e+n}} \quad (29)$$

where r_{hf} represents the optimum head losses distribution between lateral and submain. The lateral and submain head losses were easily calculated using Eqs. (27) and (28), while the other design variables were deduced using these values and the equations presented in section 2.

Table 3 shows the results obtained for a horizontal unit with self-compensating emitters whose characteristics were presented in Table 1.

Fig. 2 – Costs variation with the unit pressure head (H_0) for the optimal distribution of head losses between laterals and submain ($hf_t/hf_r \approx 0.4$) (a). Costs variation with the optimal distribution of head losses between laterals and submain (hf_t/hf_r) for the optimum unit pressure head ($H_0 = 13.38$ m) (b). Self-compensating emitters, horizontal units (CI_{Eu}, continuous thin black; CI_{ru}, continuous thick black; CI_{Bu}, dashed thin blue; CA_{Eu}, dashed thick blue; Z_u, dashed-dotted red; Optimum, green point).

Figure 2 shows the evolution of the cost functions as H_0 (Fig. 2a) or hf_r/hf_r (Fig. 2b) vary. The curves were calculated substituting each H_0 , or hf_r/hf_r , value into the general hydraulic and costs equations for drip irrigation units while the optimum points were calculated from the expressions presented in this paper.

The unit layout can also be defined at the design stage so it is worth analysing the sensitivity of the optimum solution to changes in the lateral and submain lengths. Therefore, Fig. 3 shows the variation of the cost functions as the relationship L/L_t varies.

As Fig. 3 shows, the optimal solution was found for laterals smaller than submain ($L/L_t \approx 0.7$).

The method also allowed for analysing paired laterals. Only the separation between laterals ($s_r' = s_r/2$) and the unitary pipes investment costs ($CI_{tu} = CI_{tu}/2$; $CI_{ru} = CI_{ru}/2$) were modified to incorporate this change. Table 4 shows the results of the unit design while Figs. 4 and 5 present the cost functions variation as H_0 (Fig. 4a), hf_r/hf_r (Fig. 4b) and L/L_t (Fig. 5) vary. It can be observed by comparing Tables 3 and 4 that using paired laterals significantly reduced costs.

3.1.2. Units with favourable gradient

The optimum condition for units with favourable gradient is expressed as follows (Eq. (30)).

$$dZ_u = 0$$

$$= -\frac{e_r}{n} \frac{c_r}{hf_r^{1+\frac{e_r}{n}}} dhf_r - \frac{e_t}{n} \frac{c_t}{hf_t^{1+\frac{e_t}{n}}} dhf_t + \left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) \times \left(\left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) dhf_r + \left(1 - \left(\frac{L_t I_{0t}}{(m+1)hf_t} \right)^{\frac{m+1}{m}} (I_{0t} > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) dhf_t \right) \quad (30)$$

The respective optimum conditions for hf_r and hf_t are (Eqs. (31) and (32)).

$$\frac{e_r}{n} \frac{c_r}{hf_r^{1+\frac{e_r}{n}}} = \left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \times \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) \quad (31)$$

$$\frac{e_t}{n} \frac{c_r}{hf_t^{1+\frac{e_r}{n}}} = \left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) \left(1 - \left(\frac{L_t I_{0t}}{(m+1)hf_t} \right)^{\frac{m+1}{m}} (I_{0t} > 0) \right) \times \left(\frac{LI_0}{(m+1)hf_t} < 1 \right) \quad (32)$$

Rearranging Eqs. (31) and (32) as Eqs. (31a) and (32a) allowed the iterative calculation of hf_r and hf_t .

Fig. 3 – Influence of the unit layout (L/L_t) on the costs items for the optimum solution. Optimum unit shape that minimises the cost function ($L = 35$ m; $L_t = 71$ m). Self-compensating emitters, horizontal units (CI_{tu} , continuous thin black; CI_{ru} , continuous thick black; CI_{Bu} , dashed thin blue; CA_{Eu} , dashed thick blue; Z_u , dashed-dotted red; Optimum, green point).

Table 4 – Characteristics of the optimal solution for the analysed unit (self-compensating emitters, horizontal units, paired laterals).

hf_r (m)	hf_t (m)	D_r (mm)	D_t (mm)	hf_r (m)	H_0 (m)	$\bar{h}$ (m)	H_B (m)
3.8	2.76	7.15	45.44	6.57	10.57	5.75	15.57
CI_{ru} (€ ha ⁻¹)	CI_{tu} (€ ha ⁻¹)	CI_{Bu} (€ ha ⁻¹)	CA_{Eu} (€ ha ⁻¹)	CA_{Au} (€ ha ⁻¹)	Z_u (€ ha ⁻¹)		
511	372	339	542	–	1764

Fig. 4 – Costs variation with the unit pressure head (H_0) for the optimal distribution of head losses between laterals and submain ($hf_t/hf_r \approx 0.72$) (a). Costs variation with the optimal distribution of head losses between laterals and submain (hf_t/hf_r) for the optimum unit pressure head ($H_0 = 10.57$ m) (b). Self-compensating emitters, horizontal units, paired laterals (CI_{tu} , continuous thin black; CI_{ru} , continuous thick black; CI_{Bu} , dashed thin blue; CA_{Eu} , dashed thick blue; Z_u , dashed-dotted red; Optimum, green point).

$$hf_r = \left(\frac{e_r}{n} \frac{c_r}{\left(c_{BH} + \frac{c_{FHUD}}{UD_{1/4}} \right) \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right)} \right)^{\frac{n}{e_r+n}} \quad (31a)$$

$$hf_t = \left(\frac{e_t}{n} \frac{c_t}{\left(c_{BH} + \frac{c_{FHUD}}{UD_{1/4}} \right) \left(1 - \left(\frac{LI_{0t}}{(m+1)hf_t} \right)^{\frac{m+1}{m}} (I_{0t} > 0) \right) \left(\frac{LI_{0t}}{(m+1)hf_t} < 1 \right)} \right)^{\frac{n}{e_t+n}} \quad (32a)$$

As a result of the above expressions, the optimal head losses distribution is Eqs. (33) and (34).

$$r_{hf} = \frac{hf_t}{hf_r} = \left(\frac{c_t}{c_r} \right)^{\frac{n}{e_r+n}} \quad (33)$$

where

$$c_l = \frac{\left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right)}{\left(1 - \left(\frac{LI_{0t}}{(m+1)hf_t} \right)^{\frac{m+1}{m}} (I_{0t} > 0) \right) \left(\frac{LI_{0t}}{(m+1)hf_t} < 1 \right)} \quad (34)$$

In horizontal units $I_0 = I_{0t} = 0$, so $c_l = 1$ and Eq. (33) is a generic form of Eq. (27). In this case, r_{hf} depends on hf_t and hf_r . This implies that the optimal distribution cannot be defined a priori and that the solution has to be found with iterative routines. Figure 6 show the evolution of hf_t/hf_r as the optimum H_0 varies.

Fig. 5 – Influence of the unit layout (L/L_t) on the costs items for the optimum solution. Optimum unit shape that minimises the cost function ($L = 32.5$ m; $L_t = 38$ m). Self-compensating emitters, horizontal units, paired laterals (CI_{tu} , continuous thin black; CI_{ru} , continuous thick black; CI_{Bu} , dashed thin blue; CA_{Eu} , dashed thick blue; Z_u , dashed-dotted red; Optimum, green point).

Fig. 6 – Evolution of the optimal head losses distribution between submain and lateral (hf_t/hf_r) as the optimum unit pressure head (H_0) varies. (hf_t/hf_r , continuous thick grey; Optimum hf_t/hf_r , dashed grey).

Table 5 – Characteristics of the optimal solution for the analysed unit (self-compensating emitters, units with favourable gradient).

hf_t/hf_r	hf_r (m)	hf_t (m)	D_r (mm)	D_t (mm)	hf_T (m)	H_0 (m)	$\bar{h}$ (m)	H_B (m)
0.401	6.78	2.72	9.46	44.76	9.50	11.80	5.83	16.80
CI_{ru} (€ ha ⁻¹)	CI_{tu} (€ ha ⁻¹)	CI_{Bu} (€ ha ⁻¹)	CA_{Eu} (€ ha ⁻¹)		CA_{Au} (€ ha ⁻¹)	Z_u (€ ha ⁻¹)		
894	361	366	585		–	2206		

Figure 6 shows that, as H_0 increases, the optimal solution gets closer to the optimum point for horizontal units ($hf_t/hf_r \approx 0.4$). It proves that, as H_0 increases, the terrain gradient becomes less important in comparison with the head losses. The results of the unit design are presented in Table 5 and the trend of the cost functions with H_0 and hf_t/hf_r are shown in Fig. 7.

3.1.3. Units fed from a reservoir

Unlike the previous cases, H_B is not a design variable for systems fed from a water reservoir, but a constant determined by the reservoir elevation head. The design has to ensure that the required H_0 is provided to each

unit. It implies that hf_T cannot overcome a given value such that the minimum operating pressure head does not drop below the minimum value of the self-compensating range.

Given that $UD_{1/4}$, H_0 and H_B are constant, the optimum point only depends on the pipes investment costs. The derivative of the cost function with respect to H_B is presented in Eq. (35).

$$\frac{dZ_u}{dH_B} = \frac{\partial Z_u}{\partial hf_r} \frac{dhf_r}{dH_B} + \frac{\partial Z_u}{\partial hf_t} \frac{dhf_t}{dH_B} \quad (35)$$

$r_{hf} = hf_t/hf_r$ is constant if the terrain gradient is null, so Eq. (35) was developed as follows.

Fig. 7 – Costs variation with the unit pressure head (H_0) for the optimal distribution of head losses between laterals and submain ($hf_t/hf_r \approx 0.4$) (a). Costs variation with the optimal distribution of head losses between laterals and submain (hf_t/hf_r) for $hf_t + hf_r = 9.5$ m (b). Self-compensating emitters, units with favourable gradient ($I_0 = 0.03$, $I_{0t} = 0.01$). (CI_{tw} , continuous thin black; CI_{ru} , continuous thick black; CI_{Bu} , dashed thin blue; CA_{Eu} , dashed thick blue; Z_u , dashed-dotted red; Optimum, green point).

$$\begin{aligned}
\frac{dZ_u}{dH_B} &= -\frac{e_r}{n} \frac{c_r}{hf_r^{1+\frac{e_r}{n}}} \frac{1}{1+hf_t/hf_r} - \frac{e_t}{n} \frac{c_t}{hf_t^{1+\frac{e_t}{n}}} \frac{1}{1+hf_r/hf_t} \\
&= \xrightarrow{e_r=e_t=e} -\frac{e}{n} \frac{c_r}{hf_r^{\frac{e}{n}}} \frac{1}{hf_r + hf_t} \left(1 + \frac{c_t/c_r}{\left(\frac{hf_t}{hf_r}\right)^{\frac{e}{n}}} \right) \\
&= -\frac{e}{n} \frac{c_r}{hf_r^{\frac{e}{n}}} \frac{1}{hf_r} = -\frac{e}{n} \frac{CI_{ru}}{hf_r} = -\frac{e}{n} \frac{CI_{tu}}{hf_t} = \frac{dCI_{ru}}{dhf_r} = \frac{dCI_{tu}}{dhf_t} \quad (36)
\end{aligned}$$

If the gradient is positive, the development differs slightly from that shown in Eq. (36) as can be seen in Eq. (37).

$$\begin{aligned}
\frac{dZ_u}{dH_B} &= -\frac{e}{n} \frac{c_r}{hf_r^{\frac{e}{n}}} \frac{1}{hf_r + hf_t} \left(1 + \frac{c_t/c_r}{\left(\frac{hf_t}{hf_r}\right)^{\frac{e}{n}}} \right) \\
&= -\frac{e}{n} \frac{c_r}{hf_r^{\frac{e}{n}}} \frac{1}{hf_r + hf_t} \left(1 + \frac{hf_t}{hf_r \cdot c_i} \right) = -\frac{e}{n} \frac{c_r}{hf_r^{\frac{e}{n}}} \frac{1}{hf_r} \frac{hf_r c_i + hf_t}{hf_r + hf_t} \quad (37)
\end{aligned}$$

This result is similar to that found by Labye, Olson, Galand, and Tsiortis (1988) where a continuous method for branched distribution networks was proposed. Submain and laterals behave as sections in series of branched irrigation networks. As a result, $dCI_{ru}/dhf_r = dCI_{tu}/dhf_t = d(CI_{Bu} + CA_{Eu})/dH_B$.

Therefore, if the optimum point for pumped horizontal units was $H_B = 18.38$ m, $hf_r = 6.69$ m, $hf_t = 2.69$ m, and the pipe investment costs were $CI_{ru} + CI_{tu} = 899 + 362 = 1262$ € ha⁻¹ (see Table 3), these would be the results if the available energy head at the reservoir was $H_B = 18.38$ m. If H_B varies by a metre ($\Delta H_B = 1$ m), i.e. $H_B + \Delta H_B = 19.38$ m or $H_B - \Delta H_B = 17.38$ m, the objective function $Z_u = CI_{ru} + CI_{tu}$ will vary, approximately by amounts equal to dCI_{ru}/dhf_r or dCI_{tu}/dhf_t , which give the result 56.6 (€ ha⁻¹ m⁻¹).

$$\frac{dCI_{ru}}{dhf_r} = -\frac{e_r}{n} \frac{CI_{ru}}{hf_r} = -\frac{2}{4.75} \frac{899}{6.69} = -56.6 (\text{€ ha}^{-1} \text{ m}^{-1})$$

$$\frac{dCI_{tu}}{dhf_t} = -\frac{e_t}{n} \frac{CI_{tu}}{hf_t} = -\frac{2}{4.75} \frac{362}{2.69} = -56.6 (\text{€ ha}^{-1} \text{ m}^{-1})$$

The previous results approximately coincide with the values calculated with the cost function: $(Z_u)_{H_B=18.38} - (Z_u)_{H_B=19.38} = 1262 - 1209 = 53$ (€ ha⁻¹), $(Z_u)_{H_B=17.38} - (Z_u)_{H_B=18.38} = 1323 - 1262 = 61$ (€ ha⁻¹). The error between both results (with the derivative previously referred to and with the cost function) will reduce as ΔH_B decreases.

The objective function $Z_u = CI_{ru} + CI_{tu}$, and its derivative dZ_u/dH_B , both considering the optimal head losses distribution, are shown in Fig. 8.

Given that $Q_0 = 15$ m³ h⁻¹, $A = 2.5$ ha, $Et_{pp} = 7$ mm d⁻¹ and $UD_{1/4} = 0.96$, the daily irrigation duration was $t_{rd} = 12.1$ h d⁻¹. Increasing Q_0 will reduce the irrigation duration, but it will change the pipes investment costs as the graphics in Fig. 9 show (given $H_B = 18.38$ m). Otherwise, increasing Q_0 , and maintaining A constant, will only be achieved incrementing the emitters nominal flow rate and/or reducing the separation between emitters and/or laterals.

3.2. Non-compensating emitters

3.2.1. Horizontal units

The average operating pressure head was not previously matched in this case, so the unknowns were hf_r , hf_t and $\bar{h}$. In this case $UD_{1/4}$ is not constant so CA_{Au} was included in Z_u .

The derivative of the cost function with respect to $\bar{h}$ was used to calculate the optimum average operating pressure head.

$$\frac{\partial Z_u}{\partial \bar{h}} = 0 = c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} - \frac{c_{EHUD}(H_0 + hf_r) + c_{AUD}}{UD_{1/4}^2} \frac{\partial UD_{1/4}}{\partial \bar{h}} \quad (38)$$

Fig. 8 – Variation of the cost function and its derivative with respect to the upstream end pressure head (H_B). (Z_u , dashed-dotted red; dZ_u/dH_B , continuous thick green).

Fig. 9 – Relationship among pipes investment costs and its derivative with the irrigation duration in the peak period (t_{rd}), the unit flow (Q_0), and the emitters nominal flow (q_n), with $H_B = 18.38$ m and $Et_{pp} = 7$ mm $\cdot$ d $^{-1}$. (Z_u , dashed-dotted red; dZ_u/dq_n , Continuous thin green; t_{rd} dashed green).

$$\begin{aligned} \frac{\partial UD_{1/4}}{\partial \bar{h}} &\cong -\frac{1}{2} \frac{1}{1 - UD_{1/4}} \left(-2 \left(1 - \left(\frac{h_{min}}{\bar{h}} \right)^x \right) \left(\frac{x}{h_{min}} - \frac{x}{\bar{h}} \right) \left(\frac{h_{min}}{\bar{h}} \right)^x \right. \\ &\quad \left. - \frac{2}{\bar{h}} \left(1.27x \frac{\sigma_z}{\bar{h}} \right)^2 \right) \\ &= \frac{1}{1 - UD_{1/4}} \frac{1}{\bar{h}} \left(x \left(1 - \left(\frac{h_{min}}{\bar{h}} \right)^x \right) \left(1 - \frac{h_{min}}{\bar{h}} \right) \left(\frac{h_{min}}{\bar{h}} \right)^{x-1} \right. \\ &\quad \left. + \left(1.27x \frac{\sigma_z}{\bar{h}} \right)^2 \right) \end{aligned} \quad (39)$$

An implicit equation for $\bar{h}$ (Eq. (40)) was deduced substituting Eq. (39) into Eq. (38).

$$\bar{h} = \frac{1}{UD_{1/4}^2} \frac{c_{EHUD} (H_0 + hf_F) + c_{AUD}}{(1 - UD_{1/4}) (c_{BH} + \frac{c_{EHUD}}{UD_{1/4}})} \left(x \left(\frac{h_{min}}{\bar{h}} \right)^{x-1} \left(1 - \left(\frac{h_{min}}{\bar{h}} \right)^x \right) \left(1 - \frac{h_{min}}{\bar{h}} \right) + \left(1.27x \frac{\sigma_z}{\bar{h}} \right)^2 \right) \quad (40)$$

The iteration routine required matching $\bar{h}$ to calculate the head losses, h_{min} , $UD_{1/4}$ and finally again $\bar{h}$ to check the difference between the resulting and the initial values of $\bar{h}$. Otherwise, the optimum lateral and submain head losses were calculated using the derivatives of Z_u with respect to hf_r and hf_t .

$$\begin{aligned} \frac{\partial Z_u}{\partial hf_r} &= 0 \\ &= -\frac{e_r}{n} \frac{c_r}{hf_r^{1+e_r/n}} + \left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) \frac{m+1}{m+2} \\ &\quad - \frac{c_{EHUD} (H_0 + hf_F) + c_{AUD}}{(UD_{1/4})^2} \frac{\partial UD_{1/4}}{\partial hf_r} \end{aligned} \quad (41)$$

$$\frac{\partial UD_{1/4}}{\partial hf_r} = \frac{x}{1 - UD_{1/4}} \left(1 - \left(\frac{h_{min}}{\bar{h}} \right)^x \right) \frac{1}{h_{min}} \left(\frac{h_{min}}{\bar{h}} \right)^x \frac{\partial h_{min}}{\partial hf_r} \quad (42)$$

$$\begin{aligned} \frac{\partial h_{min}}{\partial hf_r} &= \frac{m+1}{m+2} - \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) \\ &= \xrightarrow{I_0 \leq 0} -\frac{1}{m+2} \end{aligned} \quad (43)$$

From the above expressions, a relationship for the iterative calculation of hf_r was deduced (Eq. (44)). An expression to determine hf_t was easily obtained following an equivalent procedure (Eq. (45)).

$$hf_r = \left(\frac{e_r}{n} \frac{c_r}{\left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) \frac{m+1}{m+2} + \frac{c_{EHUD} (H_0 + hf_F) + c_{AUD}}{(UD_{1/4})^2} \frac{x}{1 - UD_{1/4}} \left(1 - \left(\frac{h_{min}}{\bar{h}} \right)^x \right) \frac{1}{h_{min}} \left(\frac{h_{min}}{\bar{h}} \right)^x \frac{1}{m+2}} \right)^{\frac{n}{e_r+n}} \quad (44a)$$

Table 6 – Characteristics of the optimal solution for the analysed unit (non-compensating emitters, horizontal units).

hf_t/hf_r	hf_r (m)	hf_t (m)	D_r (mm)	D_t (mm)	hf_T (m)	H_0 (m)	$\bar{h}$ (m)	H_B (m)	h_{min} (m)	$UD_{1/4}$
0.403	4.03	1.62	10.55	49.92	5.65	11.24	7.1	16.24	5.6	0.88
CI_{ru} (€ ha ⁻¹)		CI_{tu} (€ ha ⁻¹)		CI_{Bu} (€ ha ⁻¹)		CA_{Eu} (€ ha ⁻¹)		CA_{Au} (€ ha ⁻¹)		Z_u (€ ha ⁻¹)
1114		449		354		619		2625		5159.7

Fig. 10 – Costs variation with the average operating pressure head ($\bar{h}$) for the optimum distribution of head losses between laterals and submain ($hf_t/hf_r \approx 0.4$) (a). Costs variation with the optimal distribution of head losses between laterals and submain (hf_t/hf_r) for $\bar{h} = 7.1$ m and $hf_t + hf_r = 5.65$ m (b). Non-compensating emitters, horizontal units. (CI_{tu} , continuous thin black; CI_{ru} , continuous thick black; CI_{Bu} , dashed thin blue; CA_{Eu} , dashed thick blue; CA_{Au} , dashed purple; Z_u , dashed-dotted red; Optimum, green point; $UD_{1/4}$, dashed double dotted black; $CI_{ru} + CI_{tu}$, dashed double dotted red).

$$hf_t = \left(\frac{e_t}{n} \frac{C_t}{\left(C_{BH} + \frac{C_{EHUD}}{UD_{1/4}} \right)^{\frac{m+1}{m+2}} + \frac{C_{EHUD}(H_0+hf_r)+C_{AUD}}{(UD_{1/4})^2} \frac{x}{1-UD_{1/4}} \left(1 - \left(\frac{h_{min}}{\bar{h}} \right)^x \right) \frac{1}{h_{min}} \left(\frac{h_{min}}{\bar{h}} \right)^x \frac{1}{m+2}} \right)^{\frac{n}{e_t+n}} \quad (45a)$$

Table 7 – Characteristics of the optimal solution for the analysed unit (non-compensating emitters, units with favourable gradient).

hf_i/hf_r	hf_r (m)	hf_i (m)	D_r (mm)	D_i (mm)	hf_T (m)	H_0 (m)	$\bar{h}$ (m)	H_B (m)	h_{min} (m)	$UD_{1/4}$
0.403	4.11	1.62	10.51	49.90	5.73	8.55	5.34	13.55	4.42	0.89
CI_{ru} (€ ha ⁻¹)		CI_{tu} (€ ha ⁻¹)		CI_{Bu} (€ ha ⁻¹)		CA_{Eu} (€ ha ⁻¹)		CA_{Au} (€ ha ⁻¹)		Z_u (€ ha ⁻¹)
1105		448		295		506		2571		4924.27

Fig. 11 – Costs variation with the average operating pressure head ($\bar{h}$) for the optimum distribution of head losses between laterals and submain ($hf_i/hf_r \approx 0.4$) (a). Costs variation with the optimal distribution of head losses between laterals and submain (hf_i/hf_r) for $\bar{h} = 5.34$ m and $hf_i + hf_r = 5.73$ m (b). Non-compensating emitters, units with favourable gradient ($I_0 = 0.03$, $I_{0t} = 0.01$). (CI_{tu} , continuous thin black; CI_{ru} , continuous thick black; CI_{Bu} , dashed thin blue; CA_{Eu} , dashed thick blue; CA_{Au} , dashed purple; Z_u , dashed-dotted red; Optimum, green point; $UD_{1/4}$, dashed double dotted black; $CI_{ru} + CI_{tu}$, dashed double dotted red).

The optimal relationship between hf_r and hf_i was calculated as follows (Eqs. (44) and (45)).

$$r_{hf} = \frac{hf_i}{hf_r} = \left(\frac{c_t}{c_r} \right)^{\frac{n}{c+n}} \quad (44)$$

$$c_l = \frac{\left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) \frac{m+1}{m+2} - \frac{c_{EHUD} (H_0 + hf_r) + c_{AUD}}{(UD_{1/4})^2} \frac{x}{1-UD_{1/4}} \left(1 - \left(\frac{h_{min}}{h} \right)^x \right) \frac{1}{h_{min}} \left(\frac{h_{min}}{h} \right)^x}{\left(c_{BH} + \frac{c_{EHUD}}{UD_{1/4}} \right) \frac{m+1}{m+2} - \frac{c_{EHUD} (H_0 + hf_r) + c_{AUD}}{(UD_{1/4})^2} \frac{x}{1-UD_{1/4}} \left(1 - \left(\frac{h_{min}}{h} \right)^x \right) \frac{1}{h_{min}} \left(\frac{h_{min}}{h} \right)^x} \quad (45)$$

The system presented in Table 1 was designed using the above equations. The results are presented in Table 6.

Figure 10 shows the costs variation as $\bar{h}$ (Fig. 10a) or hf_r/hf_t vary (Fig. 10b). The distribution uniformity is also displayed. It can be perceived that $UD_{1/4}$ increases as $\bar{h}$ increments.

3.2.2. Units with favourable gradient

The Eqs. (40)–(43), presented in the previous section, are valid for units with favourable gradients. However, the formulae for the head losses calculation differ due to the need to incorporate the lateral and submain slopes. The results are presented in Eqs. (46) and (47).

The results obtained for units with favourable gradient and non-compensating emitters are presented in Table 7 and Fig. 11.

3.2.3. Units fed from a reservoir

Given that H_B and H_0 are constant, the optimal solution must trade-off between the higher average operating pressure head, which reduces the water cost, and the higher head losses that reduce pipes investment costs. Hence, once H_0 is matched, the relationship between the average operating pressure head and the head losses was analysed by means of Eqs. (50)–(52).

$$hf_r = \left(\frac{e_r}{n} \frac{C_r}{\left(C_{BH} + \frac{C_{EHUD}}{UD_{1/4}} \right)^{\frac{m+1}{m+2}} - \frac{C_{EHUD}(H_0+hf_r)+C_{AUD}}{(UD_{1/4})^2} \frac{x}{1-UD_{1/4}} \left(1 - \left(\frac{h_{min}}{h} \right)^x \right) \frac{1}{h_{min}} \left(\frac{h_{min}}{h} \right)^x \left(\frac{m+1}{m+2} - \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) \right)} \right)^{\frac{n}{e_r+n}} \quad (46)$$

$$hf_t = \left(\frac{e_t}{n} \frac{C_t}{\left(C_{BH} + \frac{C_{EHUD}}{UD_{1/4}} \right)^{\frac{m+1}{m+2}} - \frac{C_{EHUD}(H_0+hf_t)+C_{AUD}}{(UD_{1/4})^2} \frac{x}{1-UD_{1/4}} \left(1 - \left(\frac{h_{min}}{h} \right)^x \right) \frac{1}{h_{min}} \left(\frac{h_{min}}{h} \right)^x \left(\frac{m+1}{m+2} - \left(1 - \left(\frac{LI_0}{(m+1)hf_t} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_t} < 1 \right) \right)} \right)^{\frac{n}{e_t+n}} \quad (47)$$

Consequently, the optimum head losses distribution included the aforementioned slopes, producing Eqs. (48) and (49).

$$r_{hf} = \frac{hf_t}{hf_r} = \left(\frac{C_t}{C_r} \right)^{\frac{n}{e_r+n}} \quad (48)$$

$$H_0 = \bar{h} + \frac{m+1}{m+2} (hf_r + hf_t) = h_{min} + hf_r + hf_t \quad (50)$$

$$d\bar{h} = -\frac{m+1}{m+2} (dhf_r + dhf_t) \quad (51)$$

$$-dh_{min} = dhf_r + dhf_t \quad (52)$$

$$C_I'' = \frac{\left(C_{BH} + \frac{C_{EHUD}}{UD_{1/4}} \right)^{\frac{m+1}{m+2}} - \frac{C_{EHUD}(H_0+hf_r)+C_{AUD}}{(UD_{1/4})^2} \frac{x}{1-UD_{1/4}} \left(1 - \left(\frac{h_{min}}{h} \right)^x \right) \frac{1}{h_{min}} \left(\frac{h_{min}}{h} \right)^x \left(\frac{m+1}{m+2} - \left(1 - \left(\frac{LI_0}{(m+1)hf_r} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_r} < 1 \right) \right)}{\left(C_{BH} + \frac{C_{EHUD}}{UD_{1/4}} \right)^{\frac{m+1}{m+2}} - \frac{C_{EHUD}(H_0+hf_t)+C_{AUD}}{(UD_{1/4})^2} \frac{x}{1-UD_{1/4}} \left(1 - \left(\frac{h_{min}}{h} \right)^x \right) \frac{1}{h_{min}} \left(\frac{h_{min}}{h} \right)^x \left(\frac{m+1}{m+2} - \left(1 - \left(\frac{LI_0}{(m+1)hf_t} \right)^{\frac{m+1}{m}} (I_0 > 0) \right) \left(\frac{LI_0}{(m+1)hf_t} < 1 \right) \right)} \quad (49)$$

Table 8 – Characteristics of the optimal solution for the analysed unit (non-compensating emitters, units fed from a reservoir).

hf_r/hf_t	hf_r (m)	hf_t (m)	D_r (mm)	D_t (mm)	hf_T (m)	H_0 (m)	$\bar{h}$ (m)	H_B (m)	h_{min} (m)	$UD_{1/4}$
0.403	4.28	1.72	10.42	49.29	6.00	11.24	6.84	16.24	5.24	0.87
CI_{ru} (€ ha ⁻¹)	CI_{tu} (€ ha ⁻¹)		CI_{Bu} (€ ha ⁻¹)		CA_{Eu} (€ ha ⁻¹)		CA_{Au} (€ ha ⁻¹)		Z_u (€ ha ⁻¹)	
1086	437		–		–		2660		4183.13

Fig. 12 – Costs variation with the average operating pressure head ($\bar{h}$) for the optimum distribution of head losses between laterals and submain ($h_{f_t}/h_{f_r} \approx 0.4$) (a). Costs variation with the optimal distribution of head losses between laterals and submain (h_{f_t}/h_{f_r}) for $\bar{h} = 6.84$ m and $h_{f_t} + h_{f_r} = 6.00$ m (b). Non-compensating emitters, horizontal units fed from a reservoir. (CI_{tu} , continuous thin black; CI_{ru} , continuous thick black; CA_{Au} , dashed purple; Z_u , dashed-dotted red; Optimum, green point; $UD_{1/4}$, dashed double dotted black; $CI_{ru} + CI_{tu}$, dashed double dotted red).

The expression to calculate the lateral head losses was deduced from the derivative of the cost function as follows (Eq. (53)).

$$\begin{aligned}
 \frac{\partial Z_u}{\partial h_f} = 0 &= -\frac{e_r}{n} \frac{c_r}{h_f^{1+e_r/n}} - \frac{c_{AUD}}{(UD_{1/4})^2} \frac{\partial UD_{1/4}}{\partial h_f} \\
 \frac{\partial UD_{1/4}}{\partial h_f} &= \frac{\partial UD_{1/4}}{\partial \bar{h}} \frac{\partial \bar{h}}{\partial h_f} \rightarrow (1.27 CV_m)^2 + \left(1 - \left(\frac{h_{min}}{\bar{h}}\right)^x\right)^2 + \left(1.27 x \frac{\sigma_z}{\bar{h}}\right)^2 = (1 - UD_{1/4})^2 \\
 -2x \left(1 - \left(\frac{h_{min}}{\bar{h}}\right)^x\right) \left(\frac{h_{min}}{\bar{h}}\right)^x \left(\frac{1}{\bar{h}} \frac{m+1}{m+2} - \frac{1}{h_{min}}\right) &+ 2 \left(1.27 x \frac{\sigma_z}{\bar{h}}\right)^2 \frac{1}{\bar{h}} \frac{m+1}{m+2} = -2(1 - UD_{1/4}) \frac{\partial UD_{1/4}}{\partial h_f} \\
 \frac{e_r}{n} \frac{c_r}{h_f^{1+e_r/n}} &= \frac{c_{AUD}}{(UD_{1/4})^2 (1 - UD_{1/4})} \frac{1}{\bar{h}} \frac{m+1}{m+2} \left(x \left(1 - \left(\frac{h_{min}}{\bar{h}}\right)^x\right) \left(\frac{h_{min}}{\bar{h}}\right)^x \left(\frac{m+2}{m+1} \frac{h}{h_{min}} - 1\right) + \left(1.27 x \frac{\sigma_z}{\bar{h}}\right)^2\right) \\
 h_f &= \left(\frac{e_r}{n} \frac{c_r}{\left(\frac{c_{AUD}}{(UD_{1/4})^2 (1 - UD_{1/4})} \frac{1}{\bar{h}} \frac{m+1}{m+2} \left(x \left(1 - \left(\frac{h_{min}}{\bar{h}}\right)^x\right) \left(\frac{h_{min}}{\bar{h}}\right)^x \left(\frac{m+2}{m+1} \frac{h}{h_{min}} - 1\right) + \left(1.27 x \frac{\sigma_z}{\bar{h}}\right)^2\right)\right)^{\frac{n}{n+e_r}}} \right)
 \end{aligned} \tag{53}$$

Fig. 13 – Relationship between uniformity, pipes investment cost and unitary cost of water for the optimum point, ($H_B = 16.24$ m; $UD_{1/4}$, dashed double dotted thick black; $CI_{ru} + CI_{tu}$, dashed double dotted thick red), and two additional values ($H_B = 20$ m: $UD_{1/4}$, dashed double dotted thin grey; $CI_{ru} + CI_{tu}$, dashed double dotted thin orange; and $H_B = 12$ m: $UD_{1/4}$, dashed double dotted thick grey; $CI_{ru} + CI_{tu}$, dashed double dotted thick orange).

hf_t is obtained following an equivalent procedure (Eq. (54)).

$$hf_t = \left(\frac{e_t}{n} \frac{c_t}{\frac{C_{AUD}}{(UD_{1/4})^2 (1-UD_{1/4})} \frac{1}{h} \frac{m+1}{m+2} \left(x \left(1 - \left(\frac{h_{min}}{h} \right)^x \right) \left(\frac{h_{min}}{h} \right)^x \left(\frac{m+2}{m+1} \frac{\bar{h}}{h_{min}} - 1 \right) + \left(1.27x \frac{e_z}{h} \right)^2 \right)} \right)^{\frac{n}{n+e_t}} \quad (54)$$

Setting the upstream end energy head to be $H_B = 16.24$ m (which is the upstream end energy head of the horizontal unit, see Table 6) and the unit pressure head to be $H_0 = 11.24$ m, the optimum design is presented in Table 8. The trend of the cost functions, and $UD_{1/4}$ as well, are shown in Fig. 12.

If the cost of water is not considered, the cost function of an unit fed from a reservoir only depends on the pipes investment costs. In this case, reducing the diameters leads to cost savings although it occurs at the expense of reducing the distribution uniformity. Hence, the optimum solution depends on the relationship between cost and distribution uniformity. Additionally, a water cost that yields the same optimum point can be found. The relationship between distribution uniformity, the cost function without water cost and the water cost that gives the same optimum point are shown in Fig. 13.

4. Discussion

The design method presented in this paper, unlike conventional hydraulic optimisations, puts economic criteria at the heart of the design. Previous methods have relied on matching some hydraulic parameters, such as average

pressure head or distribution uniformity, to find the optimum solution. However, in this method these parameters are the consequence of the economic optimum point. The design approach presented in this paper is aligned with the approach presented in Carrión et al. (2013), Carrión et al. (2014) or Moreno et al. (2016), in that they also used a cost function. However, there are relevant differences to the extent that these works do not provide analytical expressions for the optimum economic calculation but numerical algorithms to find the minimum cost. Zhu, Wu, Wang, and Zhang (2013) also analysed the relationship between the economic questions and certain drip irrigation design variables using the average operating pressure head. However, they did not provide analytical solutions for the design either. The method presented here allows the design of any layout including paired laterals, so the deduced method is more general than, for example, the approaches from Ju et al. (2015) or Monserrat, Barragán, and Cots (2018).

The results presented in this paper are based on general cost functions that are included in common irrigation network costs. Further research could be conducted to customise the cost function to different drip irrigation conditions and costs categories. This would lead to specific mathematical formulations which could vary with respect to

the formula presented here. Although different analytical expressions or numerical routines should be required for these cases, the foundations established in the present paper should be suitable for such applications.

In order to simplify the mathematical procedure, continuous diameters have been considered in this paper. For practical applications, they can be conceived as approximated solutions to deduce the discrete diameters of pipes that are commercially available. However, optimal solutions are not very sensitive to small diameter variations in the vicinity of the optimum points (see Z_u curves in Figs. 2, 4, 7, 10 and 11) and therefore this implies that selecting commercial diameters close to the optimum solution will not significantly modify costs.

5. Conclusions

The present paper focuses on the drip irrigation units design using economic criteria. The laterals and submain diameters were deduced by minimising an objective cost function that includes the investment and operating costs of drip irrigation systems. Pipes investment costs and, if applicable, pump station investment costs and the costs of the energy consumed were included. Also, the costs of the amount of water consumed were also considered.

The average operating pressure head of emitters, distribution uniformity and head loss distribution between lateral and submain are the consequences of the optimum economic point.

A mathematical formulation was provided and analytical expressions were deduced to obtain the optimal results for rectangular units. The developed examples, with self-compensating and non-compensating emitters fed from pump station or reservoir, showed the viability of proposed methodology.

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