Acoustic response of a lean premixed flame propagating upward in a tube

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ABSTRACT

An analysis is carried out of the response of a curved flame front rising in a vertical tube to acoustic perturbations of frequency small compared to the ratio of the speed of sound to the radius of the tube. The gas around the flame front, whose characteristic size is the radius of the tube, is then subjected to a oscillatory, nearly uniform acceleration that adds to the acceleration of gravity. As in the better studied case of a downward propagating flame, the response of the flame front to this oscillatory acceleration leads to fluctuations of the front area and the global thermal expansion of the gas, which may inject energy into the acoustic field and thus induce a thermoacoustic instability of the flame plus tube system. Acoustic temperature perturbations associated to adiabatic compression and expansion of the gas are assumed to be much smaller than the Frank-Kamenetskii temperature of the flame and are left out. The transfer function relating thermal expansion fluctuations to infinitesimal acoustic velocity perturbations is computed as a function of the frequency for conditions typical of a very lean flame in the standard flammability tube. The response of the flame front to finite amplitude acoustic perturbations has been also analyzed. At frequencies small compared to the inverse of the residence time of the gas around the flame front, the response is quasi-stationary until the oscillatory acceleration overcomes the acceleration of gravity during a part of the cycle sufficiently long for the front to develop wrinkles and eventually break down under a net upward acceleration. At higher frequencies, the curvature of the front decreases when the amplitude of the acoustic perturbation increases. A secondary instability of the oscillating flame that develops over many oscillation cycles and leads to breakdown appears at a certain value of the amplitude of the perturbation. The onset of this instability seems to be related to the oscillatory motion of the tube wall relative to the flame tip, which induces a flow that carries upwards a part of the front and causes negative velocities of the tip relative to the far fresh gas. This instability features period doubling in a certain range of frequencies.

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1. Introduction

Coupling of the dynamics of a flame front in a combustion chamber with the acoustic modes of the chamber may lead to thermoacoustic instability when this coupling results in transfer of part of the energy released by combustion to the acoustic waves. Various coupling mechanisms exist that may achieve this transfer. These compete with damping mechanisms such as acoustic radiation through chamber openings, viscous dissipation and heat losses, and transfer of energy to non-acoustic modes of the flow in the chamber; see Refs. [1–5] for reviews of the physics and control of combustion oscillations. Shear layers, vortex shedding and turbulence play important roles in the thermoacoustic instability

of realistic chamber configurations with attached flames, and are at the origin of much of the complexity involved in the analysis of these systems. These elements, and the possibility of exciting the flame with an oscillatory velocity around the flame holder, are absent for premixed flames propagating freely in tubes filled with quiescent reactant mixtures, and these have been used to investigate some fundamental features of the response of a flame to acoustic excitations under simplified conditions.

In a very clean experiment, Searby [6] studied the acoustic response and the thermoacoustic instabilities of cellular propane-air flames propagating downwards in a tube open at the upper end and closed at the lower end. A primary acoustic instability of the flame plus tube system was observed when the strength of the flame, measured by the burning velocity of a planar flame propagating in the mixture, rises above a certain threshold. The development of this acoustic instability tends to flatten the flame front,

inhibiting its intrinsic Darrieus–Landau instability, until the acoustic instability saturates. A stronger, secondary instability of the oscillating flame front, involving period doubling, appears when the amplitude of the acoustic forcing acting on the flame (which is determined by the amplitude of the acoustic mode of the tube excited by the primary instability, and thus by the strength of the flame) becomes higher than a second threshold. This secondary instability generates cellular structures on the flame front which are much smaller than the cells of the unforced front, grow rapidly in amplitude, and may disrupt the front. Similar results have been found later for methane–air flames [7] and hydrogen–air flames [8]. The experiment of Searby [6] has prompted much analytical work which is briefly summarized below; see Clavin and Searby [9] for a recent discussion.

Acoustic perturbations cause fluctuations in the heat released by the flame, and thus in the global thermal expansion of the gas. According to Rayleigh’s criterion [10], such fluctuations inject energy into the acoustic field when they make work on it; i.e., when the product of the thermal expansion fluctuation and the pressure perturbation acting on the flame, averaged over a cycle of the oscillation, is positive; see Wu et al. [11] for a rigorous derivation of this result for a flame in a tube.

Different coupling mechanisms between the flame and the acoustic field are at work for a flame propagating in a tube. Pressure coupling, whereby the acoustic temperature variations accompanying adiabatic compression and expansion of the gas modulate the rate of the temperature-sensitive chemical reactions, was originally proposed by Dunlap [12] and has been analyzed by Clavin et al. [13]. These authors conclude that pressure coupling can lead to instability only in some marginal cases.

A stronger destabilizing mechanism is the velocity coupling of Rauschenbakh [14] and Markstein [15]. In a reference frame moving with the fresh gas, the whole flame front region is subjected to a uniform oscillating acceleration that adds to the acceleration of gravity. Owing to the nonuniform density of the gas around the flame, the front is sensitive to this acceleration, which changes its area and the global rate of heat release. This coupling mechanism has been analyzed by Pelcé and Rochwerger [16], Searby and Rochwerger [17], Denet and Toma [18], Clanet et al. [19], and Wu et al. [11] for a cellular flame propagating downwards. In agreement with the experimental results of Searby [6], these authors find that a moderate acoustic forcing opposes Landau’s instability and tends to flatten the flame, while a stronger forcing causes a secondary, parametric instability. Both results are finite amplitude effects requiring non-small values of the acoustic-to-flame velocity ratio when the amplitude of the unforced flame cells is not small.

From their analysis of acoustically forced Bunsen flames, Durox et al. [20] conclude that flame flattening in a tube is an instance of the general tendency for a forced flame front to approach an isobar of the oscillating pressure field for moderately large forcing frequency and amplitude.

The response of a flame propagating upwards in a tube has been less studied. In this configuration the flame front is always far from planar in the absence of acoustic forcing. For lean mixtures of nearly equidiffusional species, this is due to the development of the Rayleigh–Taylor instability, which leads to steadily rising bubble-shaped flame fronts whose curvature radius is of the order of the radius of the tube. Therefore simplified analyses for weakly wrinkled flames, which have allowed gathering essential information for downward propagating flames, are not applicable to upward propagating flames. However, the response of these flames to acoustic forcing is also of interest, both because they appear in practical systems, most notably in the standard flammability tube, and because they show some differences with downward propagating flames. The analysis of this response is the subject of the present paper.

2. Formulation

2.1. Orders of magnitude

The characteristic size of a curved flame front propagating upward in a standard flammability tube filled with a lean methane–air mixture is the radius of the tube, R . This radius is large compared to the thickness of the flame but small compared to the length of the tube, L . The characteristic velocity of the gas relative to a steadily rising flame (U_{st}) is of order $(gR)^{1/2}$ for very lean flames, where g is the acceleration of gravity; see Refs. [21–24]. For the standard flammability tube and other typical tube radii used in laboratory studies, this velocity is somewhat larger than the velocity of a planar flame propagating in the fresh gas. The velocity U_{st} is small compared to the characteristic speed of sound, which for definiteness is taken as that of the fresh gas, a_u . The flame front is thus localized in a region small compared to the length of the tube which drifts upwards with a velocity small compared to the speed of sound. The residence time of the gas in the flame front region is $t_r = R/U_{st}$.

Consider now this flame subjected to an acoustic perturbation with the frequency ω of one of the first longitudinal acoustic modes of the tube. This frequency is of order a_u/L , while the characteristic length of the acoustic field is L . Let $A \ll a_u$ denote the amplitude of the acoustic velocity perturbation immediately ahead of the flame front region. The acoustic pressure and temperature perturbations are of orders $\rho_u a_u A$ and $T_u A/a_u$, respectively, where ρ_u and T_u are the density and temperature of the fresh gas in the tube. Spatial pressure and temperature variations in the flame front region associated with the adiabatic compression and expansion of the gas are smaller than these estimates by a factor of order R/L , owing to the small size of the flame front region. If the effect of temperature perturbations of $O(T_u A/a_u)$ on the reaction rate is small (see next section), then temperature variations associated with pressure variations can be neglected altogether in the flame front region using the quasi-isobaric approximation to the equation of state, which reduces to $\rho T = \rho_u T_u$ when the mean molecular mass of the gas is taken to be constant.

The response of the flame to acoustic perturbations is the subject of this paper, which therefore focuses on the flame front region rather than on the acoustic field in the tube; see Assier and Wu [25] for an analysis of the two-way coupling. The character of this response depends on the value of ωt_r . The perturbed flame front is quasi-stationary when $\omega t_r \ll 1$, while the local acceleration of the gas is large compared to its convective acceleration when $\omega t_r \gg 1$. The distinguished limit $\omega t_r = O(1)$ is considered in what follows.

The condition $A \ll a_u$ introduced above implies that the linearized acoustic approximation is valid in the tube away from the flame front region. In this region, however, a linear analysis can be used only when $A \ll U_{st}$, which amounts to $\omega A \ll g$ if $\omega t_r = O(1)$, but nonlinear effects come into play when this condition is not satisfied.

2.2. Model

An axisymmetric flame front propagates upwards in a vertical tube of radius R filled with a lean fuel–air mixture of density ρ_u , temperature T_u and fuel mass fraction Y_u . The tube is closed at its upper end and open at its lower end, and its wall is kept at temperature T_u . The following simple model is used to simplify the analysis and the numerical treatment of the problem while retaining essential elements of the process. The mixture is a perfect gas with constant specific heats and mean molecular mass. The diffusion of the fuel obeys Fick’s law. The viscosity and thermal conductivity of the gas, μ and k , and the diffusion

coefficient of the fuel, D , are of the form $\mu/\mu_u = k/k_u = \rho D/(\rho_u D_u) = (T/T_u)^\kappa$, where $\kappa = 0.75$ and a subscript u denotes conditions of the fresh mixture. The Prandtl and Lewis numbers, $\text{Pr} = \mu c_p/k$ and $\text{Le} = k/\rho D$, are therefore constant. Combustion is modelled with a single irreversible reaction. The mass of fuel consumed per unit volume and time is $w = \rho B Y \exp(-T_a/T)$, where ρ , T and Y are the local density and temperature of the gas and the mass fraction of the fuel. The frequency factor $B = 2.13 \times 10^9 \text{ s}^{-1}$ and the activation temperature $T_a = 18750 \text{ K}$ are chosen for the single-step chemistry to give planar flame velocities in agreement with experimental results for methane-air mixtures at ambient temperature and pressure, in the range of equivalence ratio $[\phi = 17.39 Y_u/(1 - Y_u)]$ from the lower flammability limit to about 0.7; see [26] for details of the fitting procedure. The adiabatic flame temperature is $T_b = T_u + q Y_u/c_p$, where q and c_p are the heat released per unit mass of fuel consumed and the specific heat of the gas.

Attention is confined to a region of characteristic size R around the flame front. Due to the acoustic waves in the tube, the fresh gas above this region vibrates with a vertical velocity $A \cos \omega t$, except in a Stokes layer by the tube wall where the correction $A \delta u_{\text{el}}$, with

$$\delta u_{\text{el}} = \exp\left(-\sqrt{\rho_u \omega/2\mu_u} y\right) \cos\left(\omega t - \sqrt{\rho_u \omega/2\mu_u} y\right)$$

and y denoting the distance to the wall, must be subtracted to reduce the vibration velocity to zero at the wall. The temperature increment of order $T_u A/a_u$ accompany the acoustic vibration of the gas is assumed to be small compared to the Frank-Kamenetskii temperature T_b^2/T_a and its effect is omitted hereafter by using the quasi-isobaric approximation $\rho T = \rho_u T_u$ mentioned above.

Dimensionless variables are introduced by scaling distances with the radius of the tube, R , velocities with $\sqrt{gR}$, times with $\sqrt{R/g}$, density and fuel mass fraction with ρ_u and Y_u , respectively, (μ, k, D) with their fresh gas values (μ_u, k_u, D_u) , and using the reduced temperature $\theta = (T - T_u)/(T_b - T_u)$. In terms of these variables, the governing equations and boundary conditions are, in a reference frame moving with the tip of the flame at the axis of the tube,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \frac{1}{\text{Re}} \nabla \cdot \boldsymbol{\tau}' + \rho \left(1 + \frac{dU}{dt} + A \omega \sin \omega t \right) \mathbf{i}, \quad (2)$$

$$\rho \left(\frac{\partial \theta}{\partial t} + \mathbf{v} \cdot \nabla \theta \right) = \frac{1}{\text{Re Pr}} \nabla \cdot (k \nabla \theta) + W, \quad (3)$$

$$\rho \left(\frac{\partial Y}{\partial t} + \mathbf{v} \cdot \nabla Y \right) = \frac{1}{\text{Re Pr Le}} \nabla \cdot (\rho D \nabla Y) - W, \quad (4)$$

$$\rho(1 + \gamma\theta) = 1, \quad \mu = k = \rho D = (1 + \gamma\theta)^\kappa, \quad (5)$$

$$W = \text{Da} \rho Y \exp \left[\beta(1 + \gamma) \frac{\theta - 1}{1 + \gamma\theta} \right],$$

$$x \rightarrow -\infty : \mathbf{v} = (U - A \delta u_{\text{el}}) \mathbf{i}, \quad p = \theta = 0, \quad Y = 1, \quad (6)$$

$$x \rightarrow \infty : \frac{\partial \mathbf{v}}{\partial x} = 0, \quad \frac{\partial \theta}{\partial x} = \frac{\partial Y}{\partial x} = 0, \quad (7)$$

$$r = 1 : \mathbf{v} = (U - A \cos \omega t) \mathbf{i}, \quad \theta = 0, \quad \frac{\partial Y}{\partial r} = 0, \quad (8)$$

where dimensionless variables are denoted with the same symbols used before for their dimensional counterparts. Here $U(t)$ is the instantaneous dimensionless upward velocity of the tip of the flame front relative to the fresh gas far ahead of the front, which is to be found as part of the solution; r is the distance to the axis of the tube and x is the distance along the axis measured downward from an origin that is defined by the condition that $\theta_0 = \theta(0, 0, t)$ have a chosen constant value equal to 0.7; and $\mathbf{i}$ is a unit vector pointing downward. Eqs. (1) and (2) are the continuity and momentum equations, where $\boldsymbol{\tau}' = \mu [\nabla \mathbf{v} + (\nabla \mathbf{v})^T]$ is the non-spherical part of the viscous stress tensor, and p is the pressure of the gas augmented with the spherical part of the viscous stress tensor, both scaled with $\rho_u g R$. Eqs. (3) and (4) are the energy and fuel conservation equations. Equations (5) are the equation of state in the quasi-isobaric approximation, and the equations giving the dimensionless transport coefficients and the reaction rate. The fresh gas and the wall of the tube are seen to move downward in the reference frame tied to the flame tip with the velocities that appear in (6) and (8). The slow streamwise variation of the burnt gas conditions due to the heat loss to the wall of the tube is left out in the downstream boundary conditions (7).

The dimensionless parameters that appear in (1)–(8) in addition to the dimensionless frequency and amplitude of the acoustic forcing, ω and A , are

$$\left. \begin{aligned} \gamma &= \frac{T_b - T_u}{T_u}, \quad \beta = \frac{T_a(T_b - T_u)}{T_b^2} \\ \text{Da} &= \left(\frac{R}{g} \right)^{1/2} B \exp \left(-\frac{T_a}{T_b} \right) \\ \text{Re} &= \frac{\rho_u g^{1/2} R^{3/2}}{\mu_u}, \quad \text{Pr}, \quad \text{Le}, \quad \kappa. \end{aligned} \right\} \quad (9)$$

In what follows, a tube with the radius of the standard flammability tube ($R = 25 \text{ mm}$; see Coward and Jones [21]) is considered, for which, using the density and viscosity of the air at ambient conditions for ρ_u and μ_u , the Reynolds number is $\text{Re} = 825$. In addition, the values $\text{Pr} = 0.7$, $\text{Le} = 1$ and $\kappa = 0.75$ are assumed.

2.3. Numerical methods

For the numerical treatment, the mass flux vector $\rho \mathbf{v}$ is first decomposed into solenoidal and irrotational parts. For an axisymmetric flow in cylindrical coordinates (x, r) , this amounts to writing

$$(\rho v_x, \rho v_r) = \left(\frac{1}{r} \frac{\partial \psi}{\partial r}, -\frac{1}{r} \frac{\partial \psi}{\partial x} \right) + \left(\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial r} \right).$$

The only nontrivial component of the curl of the momentum equation, which is free of the pressure term, gives an equation for the only nonzero component of the vorticity, ω , while the function ψ satisfies

$$\frac{1}{r} \frac{\partial}{\partial x} \left(T \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial r} \left(\frac{T}{r} \frac{\partial \psi}{\partial r} \right) = -\omega + \frac{\partial T}{\partial x} \frac{\partial \varphi}{\partial r} - \frac{\partial T}{\partial r} \frac{\partial \varphi}{\partial x}$$

where $T = 1 + \gamma\theta$ with θ satisfying the energy (equation (3)). On the other hand, the continuity (equation (1)) reduces to

$$\nabla^2 \varphi = \frac{1}{T^2} \frac{\partial T}{\partial r},$$

where, again, T comes from the energy equation.

The three equations for ω , ψ and φ are equivalent to the continuity and momentum equations. They are solved together with the

energy and fuel conservation (equations (3) and (4)) with conditions (6) at the tube wall, where the nonslip condition is rewritten in terms of ψ and φ , and conditions (7) and (8) at the upstream and downstream boundaries of the computational domain.

The problem is then rewritten in the partially adaptive coordinates (ξ, r) with $\xi = [h(0, t) + x]/h(r, t)$. Here $h(0, t) = v$ with v a numerical constant of order unity, which sets the position of the flame tip relative to the upstream boundary of the computational domain, $\xi = 0$, and $h(r, t)$ for $0 < r \leq 1$ is chosen at each time step to satisfy the condition $\theta(\xi = 1, r, t) = 0.7$ in a central region of the tube cross-section. The function $h(r, t)$ thus approximately tracks the flame front, and the condition $\theta(\xi = 1, r = 0, t) = 0.7$ determines $U(t)$ in the course of the marching algorithm. The problem is discretized using second order finite differences in a rectangular nonuniform grid in the (ξ, r) plane, which is finer around $\xi = 1$. It is marched in time using a second order scheme that is implicit for the transport terms and explicit for the convection and reaction terms, until the solution becomes time periodic or breaks down (at high values of A). The numerical domain extends from $\xi = 0$ to a certain $\xi = \xi_\infty$ which is typically $\xi_\infty = 3-4$. Numerical tests show that the solution is independent of grid and boundary effects when a 360×120 grid is used.

3. Results and discussion

3.1. Stationary state

Stationary solutions of (1)–(8) without acoustic forcing are computed by setting $A = 0$ and letting the system evolve until the solution becomes time independent. As it has been often reported in the literature (see, e.g. Refs. [22–24]), the flame front for very lean mixtures consists of a spherical cap followed by a long skirt. The shape and velocity of the flame front are nearly independent of the equivalence ratio and similar to the shape and velocity of a bubble rising in a tube of the same radius filled with a liquid. Contours of the reaction rate W and some streamlines of the flow relative to the flame are shown in Fig. 1(a) for $Y_u = 3.23 \times 10^{-2}$ ($\phi = 0.581$), for which $\gamma = 4.31$, $\beta = 9.55$ and $Da = 8.317 \times 10^2$. The dimensionless velocity of the flame front rising in the tube is $U_{st} = 0.458$ (to be compared with the velocity 0.467 predicted by Davies and Taylor's formula for a bubble [27]), while the dimensionless velocity of a planar flame propagating in this mixture is $U_L = 0.132$. The relatively small values of the ratio U_L/U_{st} and the burnt-to-fresh density ratio, $\rho_b/\rho_u = 1/(1 + \gamma) = 0.19$, account for the similarity between the flow of the fresh gas upstream of the flame front and that of the liquid upstream of a bubble. Owing to the small velocity ratio, the flame tip cannot withstand the incoming flow and settles close to the stagnation point of the equivalent bubble flow, while the rest of the flame front in near the surface of the equivalent bubble. The pressure variations in the burnt gas are relatively small compared to the pressure variations in the fresh gas owing to the small density ratio ρ_b/ρ_u .

The burning velocity U_L increases with the equivalence ratio of the mixture. The flame becomes then less dependent of the existence of a low velocity region, until it undergoes a transition and becomes tulip-shaped when the equivalence ratio increases further. A sample tulip flame front computed for $Y_u = 3.7 \times 10^{-2}$ ($\phi = 0.669$), for which $\gamma = 4.94$, $\beta = 8.75$ and $Da = 2.898 \times 10^3$, is shown in Fig. 1(b). The dimensionless velocity of this front is $U_{st} = 0.386$, and the dimensionless velocity of a planar flame in this mixture is $U_L = 0.264$. For convenience, the condition $\theta(0, 1/2, t) = 0.7$ has been used to define the origin of x and $U(t)$ for the tulip flame, instead of the equivalent condition at the axis of the tube used for bubble-shaped flames.

Fig. 1. Flame fronts stationarily rising in the standard flammability tube for two values of the equivalence ratio, $\phi = 0.581$ (a) and $\phi = 0.669$ (b). Red curves are contours of the reaction rate W . Blue curves are streamlines of the flow of the gas relative to the flame front. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

3.2. Small forcing

The conservative form of the energy equation

$$\nabla \cdot \mathbf{v} = \frac{\gamma}{\text{Re Pr}} \nabla \cdot (k \nabla T) + \gamma W \quad (10)$$

can be derived multiplying (3) by γ , adding (1) multiplied by $1 + \gamma\theta$, and using the equation of state in (5). Here T is the temperature scaled with T_u . Eq. (10) can be integrated over the region of the tube containing the flame front, bounded by two cross-sections ($x = x_\infty$ and $x = x_\infty$) far upstream and far downstream of the front, to find the rate of volume generation

$$\pi(u_b - u_u) = \gamma \left(-q_w + 2\pi \int W r dr dx \right), \quad (11)$$

where $\pi u_{u,b} = \int_0^1 2\pi u dr$, with the integrals extended to the upstream and downstream sections x_∞ and x_∞ , respectively, and

$$q_w = -\frac{2\pi}{\text{Re Pr}} \int_{x_\infty}^{x_\infty} \frac{\partial T}{\partial r} \Big|_{r=1} dx$$

is the dimensionless heat lost by conduction to the wall of the tube between these sections. All these magnitudes fluctuate when the flame is subjected to an acoustic perturbation. Denoting the fluctuations with primes, the mean value of $\pi(u'_b - u'_u)p'$ over an oscillation cycle is the rate at which the flame front region injects energy into the acoustic field (see, e.g. Wu et al. [11]). Here p' is the acoustic pressure perturbation at the flame front region. The fluctuation of q_w should be added to the fluctuation of the heat lost in the rest of the tube and is expected to have a stabilizing

effect. Only the fluctuation of the heat released by the chemical reaction is discussed in what follows.

Eqs. (1)–(8) can be linearized about the stationary state when the acoustic forcing is small ($A \ll 1$). In these conditions, the fluctuation of the rate of volume generation, $\pi(u'_b - u'_u)$, oscillates sinusoidally with time at the frequency ω of the velocity forcing, u'_u , though with a different phase. The pressure fluctuation p' is spatially uniform in first approximation in the region around the flame, and proportional to the velocity fluctuation u'_u , though in phase quadrature with it (90° out of phase). The linear response of the flame can be summarized by writing $u'_u = A \text{Re}(e^{i\omega t})$ and $2\gamma \int W' r dr dx = B \text{Re}(e^{i(\omega t - \varphi)})$, and introducing the complex transfer function $\mathcal{T} = (B/A)e^{-i\varphi}$, which depends on ω and the dimensionless parameters (9). This function, i.e. B/A and φ , can be found from the solution of the linearized form of (1)–(8). Its modulus is the ratio of the amplitude of the heat release fluctuation to the amplitude of the velocity forcing far above the flame. Its argument is the phase lag of the heat release fluctuation relative to the velocity forcing.

The forcing frequency ω is a free parameter as far as the region around the flame is concerned. In practise, however, the flame propagates in a flammability tube and ω must be the frequency of one of the longitudinal acoustic modes of the tube. These frequencies have been computed by Clanet et al. [19] assuming that the temperature of the fresh and burnt gases are $\theta = 0$ and $\theta = 1$, and imposing the conditions that p' and u' be continuous across the flame front. The result is

$$\tan\left(\frac{1-s}{\sqrt{1+\gamma}}\Omega\right) \tan(s\Omega) = \sqrt{1+\gamma},$$

where s is the distance from the flame front region to the upper end of the tube scaled with the total length of the tube L , and $\Omega = \omega M_u L/R$, with M_u denoting the ratio of the velocity of the unperturbed steadily rising flame front to the speed of sound in the fresh gas.

The frequency ω becomes complex, of the form $\omega - i\sigma$, when the fluctuation of the heat release and the acoustic losses, leading to $u'_b \neq u'_u$, are taken into account. Assuming that the growth rate σ is small compared with the frequency ω of the destabilized mode, a perturbation analysis gives the contribution of the fluctuation of the flame heat release to the growth rate as (see Pelcé and Rochwerger [16] and Clanet et al. [19])

$$\sigma = F \text{Im}(\mathcal{T}) \quad \text{with} \quad F = \frac{R}{M_u L} \frac{1+\gamma}{(1+s\gamma) \tan(s\Omega) + \sqrt{1+\gamma} \tan\left(\frac{1-s}{\sqrt{1+\gamma}}\Omega\right)}.$$

The factor F is positive for the fundamental mode (Clanet et al. [19]), so that the contribution of the flame is destabilizing when $\text{Im}(\mathcal{T}) > 0$. The imaginary part of the transfer function is shown in Fig. 2 as a function of the forcing frequency for the two stationary flames discussed in Section 3.1. Here $\tau\omega$ is the forcing frequency scaled with the inverse of the transit time of the gas across a planar flame propagating in the given mixture, computed in the asymptotic limit $\beta \rightarrow \infty$; see e.g. Williams [28]. This transit time is $\delta_L^\infty/U_L^\infty$ with $\delta_L^\infty = k_u/\rho_u c_p U_L^\infty$ and $U_L^\infty = (2Da/\beta^2(1+\gamma)^{1-\kappa})^{1/2} (gD_u/R)^{1/4}$ in terms of the parameters (9). Thus, in the dimensionless variables introduced above, $\tau = (\delta_L^\infty/U_L^\infty)/\sqrt{R/g} = \beta^2(1+\gamma)^{1-\kappa} \text{Le}/2Da$. For the two mixtures of Section 3.1, $\tau = 8.11 \times 10^{-2}$ when $\phi = 0.581$ and $\tau = 2.03 \times 10^{-2}$ when $\phi = 0.669$.

The result in Fig. 2 is qualitatively similar to the analytical transfer function of Pelcé and Rochwerger [16] for a weakly corrugated downward propagating flame. This, in turn, is not unlike the transfer function of Birbaud et al. [29] for a burner-attached flame in a tube (see also Refs. [30–33]), though in this case the

Fig. 2. Imaginary part (solid) and argument (φ , dashed, right-hand side scale) of the transfer function relating fluctuations of the global heat release and fresh gas acoustic velocity perturbation. Curves labeled 1 are for the flame front of Fig. 1(a) and curves labeled 2 for that of Fig. 1(b).

flame is excited by an acoustic velocity rather than an acoustic acceleration, and its response is mediated by vortex shedding from the rim of the burner. The main difference with these previous results is in the range of very small frequencies, for which the effect of the flame is stabilizing in the present case.

The inner structure of the flame ceases to be quasi-stationary when $\tau\omega$ becomes large. As noted by Clavin and Searby [34] in their analysis of the pressure coupling mechanism, a kinetic scheme more realistic than the single overall reaction assumed here would have an effect on the response of the flame at these high frequencies. However, the simple scheme may be appropriate in the range of frequencies where the transfer function has significant values.

The dashed curves in Fig. 2 show the phase delay of the rate of volume generation relative to the acoustic velocity. The delay increases with frequency from about π to about $3\pi/2$, monotonically for the tulip flame of Fig. 1(b) and with a single oscillation associated to the secondary maximum of $\text{Im}(\mathcal{T})$ for the bubble flame of Fig. 1(a).

3.3. Finite amplitude forcing

The response of the flame front region ceases to be linear when A ceases to be small, though the acoustic approximation is still valid in the rest of the tube.

Consider first small values of ω . The flow in the flame front region should be quasi-stationary in the limit of very low frequencies. For a bubble-like flame, the flow around the flame cap and the velocity of the front U should adjust to the instantaneous acceleration $1 + A\omega \sin \omega t$, varying as its square root with little change of the front shape. The length of a stationary skirt should be proportional to the velocity of the front in order to burn the varying flow rate of fresh gas reaching the flame.

Figures 3 and 4 show some numerical results for the bubble-like flame of Fig. 1(a) and $\omega = 0.274$. As can be seen, the limiting conditions are approached around the flame cap. However, non-stationary effects and the presence of the wall, which is seen to move with a velocity different from that of the front in the reference frame used (see Eq. (8)), affect the long skirt and cause the heat release to lag behind the acoustic acceleration (Fig. 3). The increase of the heat release past the maximum acceleration outweighs the decrease past its minimum, so the value of the heat release averaged over a cycle of the oscillation increases with the amplitude of forcing. This is opposite to the trend for higher frequencies discussed below.

Fig. 3. Upward velocity of the flame tip relative to the far fresh gas (a) and global heat release (b) as functions of time during one cycle of the acoustic perturbation for $\omega = 0.274$ and $A = 1.826, 2.739, 3.651$ and 4.108 , increasing as indicated by the arrows. Dashed curves (right-hand side scale) show the phase of the acoustic velocity ($A \cos \omega t$) and acceleration ($A \omega \sin \omega t$). The five black circles in (a) key the five snapshots in Fig. 4.

The minimum acceleration becomes negative (upward) during part of the cycle when $A > 1/\omega = 3.651$. The negative acceleration tends to destabilize the flame [6,16,19]. Wrinkles appear on the flame front when the value and the duration of the negative acceleration increase, and last until positive accelerations restabilize the flame. The flame front breaks down, and the numerical computation relying on the assumption of a continuous front fails, when A becomes larger than about 4.108. Figure 4 shows some snapshots of the flame front during an oscillation cycle for the highest value of A for which a solution has been obtained. In Figs. 3 and 4 the acceleration $1 + dU/dt + A\omega \sin \omega t$ is negative for $12.93 < t < 20.99$. A single wrinkle develops on the front that is marginally visible at $t = 16.06$ (not displayed) and apparent at $t = 18.34$ (Fig. 4(d)). The amplitude of the wrinkle increases without significant displacement while the acceleration remains negative. Once the acceleration becomes again positive (downward), the wrinkle undergoes a single oscillation (Fig. 4(e)) and disappears. It must be noted that the condition of axisymmetry used in these computations may be restricting the development of cellular flames. However, the fully three-dimensional computations that would be needed in the periods of negative acceleration are beyond the scope of this work. The blue contours in Fig. 4 mark the boundary of the region of upwards gas velocity, which extends between this contour and the wall of the tube. This region appears when $U - A \cos \omega t < 0$ and is essentially the thin Stokes layer where δu_{BL} is not zero, though modified by the variation of the gas velocity around the flame. The green contours show the unforced stationary flame front.

Moving now to somewhat higher frequencies, Fig. 5 shows the mean and rms values of the heat release, $\bar{Q} = (\omega/2\pi) \int_0^{2\pi/\omega} Q dt$ and $Q_{rms} = (\omega/2\pi) \int_0^{2\pi/\omega} (Q - \bar{Q})^2 dt$ respectively, with $Q = \int W r dr dx$, scaled with the heat release of the stationary front (Q_{st}), as functions of the forcing amplitude for various forcing frequencies. The mean heat release decreases when the amplitude or the frequency increase. The rms heat release begins

Fig. 4. Five snapshots of the flame front (red) for $\omega = 0.274$ and $A = 4.108$ at the instants keyed in Fig. 3(a); i.e. $t = 4.59$ (a), 9.17 (b), 13.76 (c), 18.35 (d) and 22.34 (e). For reference, the green contours mark the flame front in the absence of acoustic perturbation. The axial velocity of the gas relative to the flame tip is negative (upwards) between the blue curve and the tube wall ($r = 1$) in (a), (d) and (e). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 5. Mean (a) and root mean square (b) values of the global heat release, $Q(t) = \int W r dr dx$, scaled with the global heat release of the stationary flame front (subscript st; $Q_{st} = \int W_{st} r dr dx$), as functions of the forcing amplitude A for $\omega = 1.095$ (1), 1.643 (2), 2.739 (3), 3.834 (4), 5.477 (5), 21.909 (6) and 43.818 (7), increasing as indicated by the arrow in (a) and by the numbers in (b). Results for the two highest frequencies are not shown in (b) because they are too small to be seen at the scale of the figure. The jumps of curves (3), (4) and (5), for $\omega = 2.739$, 3.834 and 5.477, mark the onset of period doubling.

increasing linearly with the amplitude, but saturates and decreases slightly when the amplitude is further increased. Similar results have been reported in the literature for other configurations, including flames propagating downwards in tubes [6,11,16–18] and Bunsen flames and bluff-body stabilized flames in enclosures [20,35–38]. The decrease of the heat release has been traced to a decrease of the flame front area. This is also the case here. Leaving aside the very low frequencies discussed above, inspection of the numerical results shows that the curvature of the flame front away from the tube wall decreases with increasing forcing amplitude, the effect being more pronounced the higher the frequency. Figures 6 and 7 illustrate this result for the rather extreme case $\omega = 43.818$. The trend is the same for lower frequencies, though flames as flat as that in Fig. 7(d) are not attained. Near the wall, the front bends downward due to the radially diverging downward flow of the fresh gas that prevails during most of the oscillation cycle. This condition, however, ceases to be satisfied when the forcing amplitude increases too much. Its failure seems to be related to the breakdown of the flame front at a certain maximum forcing, as will be discussed later in this section.

The tendency of the flame to flatten can be qualitatively understood by looking at the response of an infinitely thin front propagating at a constant velocity relative to the gas to high frequency and amplitude forcing, $(\omega, A) \gg 1$; see Ref. [20]. The velocity and pressure of the gas around the flame, as well as other variables, can be decomposed into mean and oscillatory parts; e.g., $\mathbf{v}(x, t) = \mathbf{v}_0(x) + \mathbf{v}_a(x, t)$ and $p(x, t) = p_0(x) + p_a(x, t)$, at either side of the front. At leading order in the high frequency limit, the momentum equation for the oscillatory variables reduces to $\rho \partial \mathbf{v}_a / \partial t =$

Fig. 6. Upward velocity of the flame tip relative to the far fresh gas (a) and global heat release (b) as functions of time during one cycle of the acoustic perturbation for $\omega = 43.818$ and $A = 0.0913$, 0.456, 0.639 and 0.913, increasing as indicated by the arrow in (a) and from top to bottom in (b). Dashed curves (right-hand side scale) show the phase of the acoustic velocity and acceleration.

$-\nabla p_a + \rho(dU_a/dt + A\omega \sin \omega t)\mathbf{i}$ for any $A \ll \omega$, with $\rho = 1$ ahead of the flame front and $\rho = 1/(1 + \gamma)$ behind. The continuity equation is $\nabla \cdot \mathbf{v}_a = 0$ in the region around the flame. Since the oscillatory force is irrotational, the oscillatory velocity is of the form $\mathbf{v}_a = \nabla \varphi_a$ with φ_a depending sinusoidally on time. The oscillatory velocity and pressure are continuous across the front, the second condition reflecting that there is no oscillatory force concentrated at the front. But, since the burnt-to-fresh gas density ratio is fairly small, the oscillatory pressure and the body force (the last term of the momentum equation) are smaller in the burnt gas than in the fresh gas, and the condition of continuity of the oscillatory pressure across the front can be satisfied only if this pressure is nearly uniform at the front. The flame front must approach one of the isobars of the oscillatory pressure in the fresh gas, which are then horizontal planes in the present configuration.

The momentum equation for the mean velocity and pressure, obtained by averaging Eq. (2), is

$$\rho \mathbf{v}_0 \cdot \nabla \mathbf{v}_0 + \rho \nabla \left(\frac{1}{2} \overline{|\nabla \varphi_a|^2} \right) = -\nabla p_0 + \frac{1}{\text{Re}} \nabla \cdot \tau'_0 + \rho \mathbf{i}, \quad (12)$$

where the bar denotes time averaging. Due to the density jump across the flame front, the second term on the left-hand side cannot be accounted for using a modified pressure. This term, together with similar terms in the averaged jump conditions, may drive the mean flow towards the required near-flat flame state, though detailed computations would be required to ascertain the conditions under which this occurs.

Comparison of Figs. 3(a) and 6(a) for large forcing amplitudes shows that, as the frequency increases, the velocity of the flame tip relative to the fresh gas, $U(t)$, changes from oscillating nearly in phase with the acoustic acceleration $A\omega \sin \omega t$ to oscillating nearly in opposition with the acoustic velocity $A \cos \omega t$, with an ampli-

Fig. 7. Snapshots of the flame front (red) for $\omega = 43.813$ and $A = 0.456$ (a), 0.639 (b), 0.913 (c) and 1.278 (d) at representative times during the oscillation cycle; namely $t = 0.0372$ (a), 0.137 (b), 0.0189 (c) and 0.0216 (d). For reference, the green contours mark the flame front in the absence of acoustic perturbation. The axial velocity of the gas relative to the flame tip is negative (upwards) between the blue curve and the tube wall in (b), (c) and (d). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

tude that is small compared to A . The velocity $U(t)$ would be a constant for a strictly planar flame that is merely convected back and forth by a uniform oscillating flow that does not change its internal structure. The time variation of $U(t)$ may be due to the motion of the wall in the reference frame tied to the flame tip. As indicated by the boundary condition (8), the downward velocity of the wall is smaller than $U(t)$ when $A \cos \omega t > 0$. This retards the motion of the gas between the wall and the flame skirt, reducing the outflow of fresh gas from the region ahead of the flame front and thus decreasing $U(t)$ to satisfy the condition of incompressibility. Conversely, the drag of the wall enhances the outflow of fresh gas and by this means increases $U(t)$ when $A \cos \omega t < 0$.

The trend towards flatter flame fronts with increasing forcing amplitude comes to an end for each forcing frequency at a certain maximum amplitude, at which the front apparently ceases to be continuous and the computation breaks down. The maximum amplitude is shown in Fig. 8 as a function of the frequency. Period doubling, which is prominent for downward propagating flames [6,11,17,18], occurs here as the maximum amplitude for $\phi = 0.581$ (solid curve) is approached in a range of frequencies corresponding roughly to the plateau of Fig. 8, from about $\omega = 2.7$ to about $\omega = 13.7$, but it is not observed outside of this range. Figure 9 il-

Fig. 8. Maximum amplitude of the velocity perturbation as a function of the frequency for $\phi = 0.581$ (solid) and 0.669 (dashed).

lustrates the time dependence of $U(t)$ and the heat release in a case with period doubling.

The process leading to breakdown at very low frequencies has been discussed above; the maximum amplitude increases then

Fig. 9. Upward velocity of the flame tip relative to the far fresh gas (a) and global heat release (b) as functions of time during two cycles of the acoustic perturbation for $\omega = 10.954$ and $A = 0.0913, 0.456, 0.730$ and 0.913 , increasing as indicated by the arrow in (a) and from top to bottom in (b). The curves for $A = 0.913$ show period doubling. Dashed curves (right-hand side scale) show the phase of the acoustic velocity and acceleration.

with decreasing frequency in order for the maximum acoustic acceleration $A\omega$ to overcome the acceleration of gravity. The process leading to breakdown at higher frequencies, in the rest of Fig. 8, is different and extends over many oscillation cycles. The velocity of the flame tip relative to the fresh gas far above the flame takes negative values during part of the cycle when the maximum amplitude is approached; see Fig. 6(a). The flame still advances in the fresh gas in these conditions, but the effect of the motion of the wall discussed above is now very intense. The wall moves upward in the part of the cycle considered and induces negative (upward) gas velocities. Furthermore, these upward velocities are not confined to a thin Stokes layer, contrary to the picture envisaged by Kaskan [39]. In the part of the cycle where $U < 0$ the velocity of the gas far above the flame is upward in the whole cross-section of the tube (see boundary condition (6) for the velocity). The gas moves massively upward, and this upward flow persists around the flame front in the region between the blue contour and the wall in Fig. 7(b–d). The upward velocity of the gas, augmented by the burning velocity of the flame, displaces part of the flame front upward, and this displacement accumulates in successive oscillation cycles until the front breaks down. The process is illustrated in Fig. 10 for $\omega = 21.909$, where the snapshots are spaced 15 cycles.

To qualitatively account for this process, contours of the high frequency approximation to the apparent force leading to steady streaming (the negative of the second term on the left-hand side of (12)) have been included in the first snapshot of Fig. 10 (red curves). The force concentrates around the part of the flame front that drifts upwards, and it has been nearly unchanged for some hundred cycles before this instant (not displayed). This may explain the continuous upward displacement. The coherence of the

Fig. 10. Three snapshots of the flame front for $\omega = 21.909$ and $A = 0.959$, spaced 15 cycles of the acoustic perturbation immediately before breakdown. The red curves in (a) are contours of the apparent force leading to steady streaming. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

force is lost in the other two snapshots of Fig. 10, which represent the very last stage of the process. By then, however, the flame is extinguished in a large region of the tube, and its fate may be already decided. For comparison, at smaller values of A the force is distributed in a smooth layer that covers a large fraction of the flame front (result not displayed). Apparently this distributed force can be largely balanced by a pressure force without causing a continuous displacement of the flame front.

Acoustically forced tulip flames are less relevant than bubble flames near the lower flammability limit and have been given less attention here. The dashed curve in Fig. 8 shows the maximum forcing amplitude as a function of frequency for the flame of Fig. 1(b) ($\phi = 0.669$). This flame is somewhat more robust than the bubble flame of Fig. 1(a) in a range of moderate frequencies, in the sense that A_{max} is larger. However, the maximum forcing amplitude abruptly drops when the frequency increases above $\omega \approx 16.4$. Inspection of the numerical results shows that at these forcing frequencies the tip of the oscillatory flame aperiodically switches between tulip- and bubble-shaped. These changes induce strong perturbations that extend to the rest of the flame front, eventually leading to pinchoff of a region of the front around the axis of the tube and to failure of the numerical computations. The process is similar to the last stages of the collapse of tulip flames described by Gonzalez et al. [40].

4. Conclusions

The response of a flame front propagating upward in a vertical tube to acoustic perturbation has been analyzed in conditions typical of lean flames in the standard flammability tube. A simple model of an axisymmetric flame has been used which includes a single overall irreversible chemical reaction and transport coefficients that increase with a power of the temperature. A velocity coupling between the global expansion rate of the gas at the flame and the acoustic perturbation has been considered for frequencies such that the wavelength of the perturbation is large compared to the characteristic size of the flame front, which is the radius of the tube. The flame front is then subjected to a uniform oscillatory acceleration which modulates its area and the heat release rate, and which may induce a thermoacoustic instability of the flame plus tube system. Pressure coupling is explicitly excluded.

The transfer function linearly relating the fluctuations of the heat release rate to the amplitude of infinitesimal velocity perturbations has been computed and compared to the transfer function for downward propagating flames.

The response of the flame front to finite amplitude perturbations depends on their frequency. At frequencies small compared to the inverse of the residence time of the gas around the flame front, the response of most of the front is quasi-stationary until wrinkles appear on the front when the acoustic acceleration overcomes the acceleration of gravity during part of the oscillation cycle.

At higher frequencies, increasing the amplitude of the perturbation tends to flatten the flame front and to decrease the mean and rms values of the heat release rate, in agreement with known results for downward propagating flames. The result is rationalized in terms of the oscillatory pressure variations at both sides of the front in the high frequency limit. A secondary acoustic instability of the oscillatory front appears at a threshold amplitude of the perturbation that depends on the frequency. In the conditions of the standard flammability tube, this instability seems to originate in the relative motion of the flame to the wall of the tube. It displays period doubling in a certain range of frequencies.

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