Numerical and Experimental Analysis of Optical Frequency Comb Generation in Gain-Switched Semiconductor Lasers

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Abstract—We have performed a systematic experimental and simulation analysis of the optical frequency combs generated by gain-switching single mode laser diodes, either free-running or subjected to optical injection. The simulations have been produced with a dynamical model consisting of three rate equations that include stochastic noise terms. The model parameters have been extracted from experiments. An exhaustive comparison between the simulated and the experimental temporal profiles and optical spectra generated under a wide range of switching and optical injection conditions has been performed. The excellent quantitative agreement found in the entire range of the explored operating conditions supports the validity of the model for the analysis of the comb generation mechanisms. The detailed maps generated provide guidelines for selecting suitable operating conditions for specific comb characteristics.

Index Terms—Semiconductor lasers, rate equations, gain switching, optical injection, optical frequency combs.

I. INTRODUCTION

IN THE last years the generation of optical frequency combs (OFCs) from semiconductor lasers by gain-switching (GS) has attracted a considerable attention. Apart from the acknowledged advantages of the semiconductor sources (high efficiency, low cost and small footprint), this interest is due to the specific advantages of GS as a comb generation technique in comparison with mode locking and electro-optic modula-

tion, in terms of flexibility in the selection of the repetition frequency, low losses and simple implementation.

As an OFC generation method GS consists in driving the diode by a radio-frequency (RF) sinusoidal current superimposed to a direct bias current. The amplitude and frequency of the RF current and the bias current are the key parameters determining for a specific laser the characteristics of the resulting train of pulses in the time domain and therefore the spectral characteristic of the OFC in the spectral domain.

Since the early works demonstrating the OFC generation by GS of edge-emitting single mode lasers [1], an extensive application-driven research has been performed. The main targeted applications have been radio-over-fiber [2]–[4] and multi-carrier optical communications [5]–[7] but also applications such as dual comb spectroscopy have been pursued [8]. The effects of related key issues such as time jitter, phase noise/linewidth and modulation bandwidth, on the device performance in the targeted applications have been analysed [9]–[12]. On the other hand, external optical injection (OI) of the gain-switched lasers has demonstrated to improve the spectral characteristics of the combs when performed at specific injection conditions. The experimental evidences of a dramatic increase of the carrier to noise ratio, as well as, a decrease of the linewidth of each tone have been attributed to the reduction of the pulse-to-pulse phase noise, the time jitter and the relative intensity noise (RIN) [9]–[11], [13]–[15]. Likewise, the maximum attainable frequency spacing of the combs has been demonstrated to increase under strong injection [16].

The OFC generation by GS has also been investigated from a theoretical point of view. A rate equation-based numerical model has been used for qualitatively modelling the generation process at frequencies in the range of the relaxation oscillation frequencies [14], [17], [18]. The model has provided a qualitative physical interpretation of the role of pulse-to-pulse phase noise and time jitter in the quality of the combs and how external OI under specific injection conditions can improve this quality.

We have recently published an experimental analysis of the effects on the OFC spectral characteristics of the switching and optical injection conditions in a limited range of such conditions [15]. The analysis showed some of the trends in the evolution of the OFC characteristics and identified some

conditions for abrupt transitions between high quality coherent combs and broad incoherent spectra.

In summary, at present, some trends in the evolution of the characteristics of the OFCs as a function of the switching and OI conditions are experimentally known in a limited range of conditions. In addition, an initial approach using a rate equation model has provided qualitative agreement with experiments, thus helping the interpretation of the role of peak-to-peak phase noise and time jitter in the quality of the combs. This has been done at GS frequencies around the relaxation oscillation frequencies and at a limited range of OI conditions. However, at this stage, the generation of OFCs by gain switching lacks of a complete comparative, theoretical and experimental, study of the OFC characteristics as a function of the driving conditions and the laser operating point and therefore, it is of high interest a detailed and properly parameterized numerical model capable of producing simulations in not only qualitative but quantitative agreement with experiments in a wide range of driving and injection conditions, thus demonstrating that the main issues affecting the process are properly accounted by the model not only at frequencies in the range of the relaxation oscillation frequencies but also at much lower frequencies. This is actually what this paper aims at. Such a model would help the selection of the suitable operating conditions for a given laser to perform adequately in a targeted application and would provide a deeper insight into the physics underlying the generation of OFCs by gain-switching.

In this paper, we provide a step forward in two directions: from the experimental point of view by completing an exhaustive characterization of the optical spectra of the OFCs generated under a wide range of driving and optical injection conditions; and from the theoretical point of view by building a properly parameterized numerical simulator, capable of producing simulation results quantitatively comparable with the experimental spectra. Special attention has been devoted to minimize the number of assumed parameters to be included in the model. This has been achieved by a detailed fitting of the RIN measurements to an expression including more parameters than the usual analysis. Special care has been taken in the treatment of the stochastic noise terms of the model [19]. The simulation results are compared with experiments in a wide range of GS and OI conditions, including frequencies well below the range of the relaxation oscillation frequencies. The very good agreement found, not only in the main features of the combs, but also in the details of the spectral profiles, points to a proper accounting by the model of the main physical effects involved. Several maps illustrating this agreement are provided. The analysis of these maps provides guidelines for selecting operating conditions for specific performance of a given device at low and high switching frequencies. The simulation results reveal the predominant role of dynamic chirp in the generation of OFCs by gain-switching at high frequencies, while the adiabatic chirp would be mainly responsible for the generation at low frequencies.

The paper is structured as follows. In Section II, we detail the theoretical model and its implementation. The reader is

referred to the Appendix for a detailed description of the numerical algorithm. In Section III, relevant details of the lasers and the experimental setups are provided. Section IV is devoted to the experimental process for the extraction of the device parameters to be used in the model. Finally, the results on the OFC generation by GS, with and without OI, are presented and discussed in Section V and the conclusions are summarized in Section VI.

II. RATE-EQUATION MODEL

The interaction between photons and carriers in single mode semiconductor lasers, including the phase of the optical field, can be modelled using the well-known rate equation approach [20], [21]. We have used the following set of equations to describe the dynamics of the carrier density ($N(t)$), the photon density ($S(t)$) and optical phase ($\Phi(t)$):

$$\frac{dN}{dt} = \frac{I(t)}{eV_{act}} - R(N) - \frac{v_g g(N)S(t)}{1 + \epsilon S(t)} + F_N(t) \quad (1)$$

$$\frac{dS}{dt} = \left[\frac{\Gamma v_g g(N)}{1 + \epsilon S(t)} - \frac{1}{\tau_p} \right] S(t) + \beta \Gamma B N^2(t) + Y_S(t) + F_S(t) \quad (2)$$

$$\frac{d\Phi}{dt} = \frac{\alpha}{2} \left[\Gamma v_g g(N) - \frac{1}{\tau_p} \right] + 2\pi \frac{df}{dT} \Delta T + Y_\Phi(t) + F_\Phi(t) \quad (3)$$

where $I(t)$ is the injected current, V_{act} the active volume, e the electron charge, $R(N)$ the carrier recombination rate, v_g the group velocity, $g(N)$ the material gain, ϵ the non-linear gain coefficient, Γ the optical confinement factor, τ_p the photon lifetime, β the fraction of spontaneous emission coupled into the lasing mode and α the linewidth enhancement factor. $Y_S(t)$ and $Y_\Phi(t)$ are terms describing the external optical injection when it is considered, and they will be detailed later.

In Eq. (3), we have included an additional term accounting for the change of the emission frequency due to temperature variations, in which $\frac{df}{dT}$ is the temperature coefficient of the emission frequency and ΔT is the difference between the active region temperature under the operating conditions and under the threshold current.

In Eq. (1)-(3), the Langevin terms $F_N(t)$, $F_S(t)$ and $F_\Phi(t)$ represent additional stochastic noises from different sources, with the following correlation properties [22]:

$$\langle F_i(t') F_j(t) \rangle = 2D_{ij} \delta(t' - t) \quad (4a)$$

$$\langle F_i(t) \rangle = 0, \quad (4b)$$

where $\delta(t)$ is the Dirac delta function and D_{ij} are the diffusion coefficients associated with the corresponding noise sources, with the subindexes i and j referring to the variables S , Φ and N . The non-vanishing diffusion coefficients D_{ij} have the following values [23]:

$$D_{SS} = \beta \Gamma B N^2 S, \quad (5)$$

$$D_{SN} = -\beta B N^2 S \quad (6)$$

$$D_{NN} = \frac{\beta B N^2 S}{\Gamma} + \frac{1}{V_{act}} \left(R(N) + \frac{I}{eV_{act}} \right), \quad (7)$$

$$D_{\Phi\Phi} = \frac{\beta \Gamma B N^2}{4S}. \quad (8)$$

The material gain $g(N)$ is described by:

$$g(N) = \frac{dg}{dN}(N - N_{tr}), \quad (9)$$

where $\frac{dg}{dN}$ is the differential gain and N_{tr} the transparency carrier density. The carrier recombination $R(N)$ is expressed by means of:

$$R(N) = AN + BN^2 + CN^3, \quad (10)$$

where A , B and C are the non-radiative, the spontaneous, and the Auger recombination coefficients, respectively. Neglecting high-frequency electrical effects, the injected current $I(t)$ can be expressed as:

$$I(t) = I_{bias} + C_{loss}(f_R) \frac{2\sqrt{2}V_{RF}}{Z_0 + Z_l} \sin(2\pi f_R t), \quad (11)$$

where I_{bias} is the bias current, f_R the repetition frequency, $C_{loss}(f_R)$ a loss coefficient accounting for the frequency dependent electrical cable attenuation, V_{RF} the root-mean-square (RMS) voltage value of the signal generator applied to an ideal 50Ω load, $Z_0 = 50 \Omega$ the generator output impedance, and Z_l the impedance of the laser module. The output power coupled to the fiber can be expressed as [22]:

$$P(t) = \eta_f \frac{hf_0 V_{act}}{\Gamma \tau_p} S(t), \quad (12)$$

where η_f is the in-fiber external quantum efficiency given by the product of the differential quantum efficiency and the coupling efficiency, h is the Planck's constant and f_0 the emission frequency. In the case of external optical injection, in Eqs. (2) and (3), $Y_S(t)$ and $Y_\Phi(t)$ are expressed as [20]:

$$Y_S(t) = 2k_c \sqrt{S(t)S_{inj}(t)} \cos(\Phi(t) - \Phi_{inj}(t) - 2\pi \delta v' t), \quad (13)$$

$$Y_\Phi(t) = -k_c \sqrt{\frac{S_{inj}(t)}{S(t)}} \sin(\Phi(t) - \Phi_{inj}(t) - 2\pi \delta v' t), \quad (14)$$

where $S_{inj}(t)$ and $\Phi_{inj}(t)$ are the photon density and the phase of the injected laser field, respectively, k_c the master-slave coupling coefficient and $\delta v'$ the detuning of the injected laser field with respect to the frequency of the slave laser (SL) at threshold. The photon density and the phase of the injected signal were obtained using three supplementary rate equations for the master-laser (ML) dynamics.

Eqs. (1)-(3) were solved using a numerical Heun's algorithm for stochastic differential equation (SDE) systems. The Appendix describes in detail the treatment of noise and the numerical algorithm. In order to properly compare with experiments, the reflection of ML signal at the front facet of the SL was included in the spectral calculations, by means of:

$$E_T(t) = \sqrt{P(t)}e^{i\Phi(t)} + \sqrt{R_{SL}P_{inj}(t)}e^{i(\Phi_{inj}(t) + 2\pi \delta v' t + \pi)}, \quad (15)$$

where $E_T(t)$ is the total optical field, R_{SL} is the front facet reflectivity of the SL and P_{inj} is the power injected by the ML, which is calculated from S_{inj} using an expression similar to Eq. (12).

Fig. 1. Schematic diagram of the experimental setup in GS operation. ML: master laser, SL: slave laser, PC: polarization controller, VOA: variable optical attenuator, BOSA: Brillouin optical spectrum analyzer, PD: photodiode, OSC: oscilloscope.

III. EXPERIMENTAL TECHNIQUES

A Discrete Mode Laser (DML) (Eblana Photonics EP1550-0-DM-H19-FM) was used in all the experiments. The laser was extensively characterized in continuous wave (CW) and GS operation. The device is packaged in a 7-pin butterfly with high frequency input connectors and has not built-in optical isolator in order to allow external optical injection. Its nominal modulation bandwidth is 10 GHz. The operating temperature was set to 25 °C by means of a temperature controller (Thorlabs, TED200). The CW characterization included standard power-current (P-I) characteristics, optical spectra and RIN measurements at different bias currents. The RIN spectra were measured with a 45 GHz photodiode (Newfocus 1014) followed by a 44 GHz Electrical Spectrum Analyser (ESA) (Agilent E4446).

The experimental setup in GS operation, with (or without) OI, is shown in Fig. 1. An optical circulator was used to inject the output signal of the tunable ML (Pure Photonics PPCL300) into the gain-switched DML, operating as SL. In order to maximize the coupling between both lasers, a polarization controller was used. A variable optical attenuator (VOA) was employed to control the injected power. The SL was driven using a combination of two electrical signals: a bias current (I_{bias}), provided by a current source (Arroyo 4205) and a sinusoidal signal provided by a microwave/RF generator (Rohde & Schwarz SMB100A). The output spectra and the temporal profiles were respectively measured using a Brillouin optical spectrum analyser (BOSA) (Aragon Photonics BOSA 210) with 10 MHz resolution, and a digital signal analyser (DSA, Tektronix DSA8200) with a 20 GHz bandwidth optical input module. In a separate set-up, the impedance of the laser module, Z_l , was obtained as a function of the frequency by measuring the scattering parameters in a network analyser, yielding a value close to 50 Ω up to several GHz. The loss coefficient of the cables was experimentally determined as a function of the frequency using the RF generator and the DSA.

IV. PARAMETER EXTRACTION

The extraction of the laser parameters needed by the model was accomplished in several stages by an original procedure similar to that in [21], [24], [25]. The power-current (P-I) characteristics (Fig. 2 (a)), provided a threshold current $I_{th} = 14.8$ mA and a slope efficiency equal to 0.133 W/A, and in consequence, two relationships between model parameters.

Fig. 2. (a) Power-Current Characteristics. (b) Evolution of the temperature increase of the active region with respect to threshold as a function of the bias current.

Fig. 3. Measured RIN spectra for different values of I_{bias} . From top to bottom, $I_{bias} = 18$ mA, 22 mA, and 26 mA. Red: experimental measurements, black line: simulations.

Measurements of the emission wavelength at a constant I_{bias} close to threshold for different heat-sink temperatures were used to determine $df/dT = -13.5$ GHz/K. This value was used, in conjunction with optical spectrum measurements as a function of I_{bias} at constant heat sink temperature, to determine ΔT vs the injected current (Fig. 2 (b)). We have used the value of ΔT as input variable in Eq. (3).

The RIN spectra were measured for different values of I_{bias} . Fig. 3 shows, as an example, the measured RIN spectra (red points) for different I_{bias} in comparison with the simulated RIN, as it will be later commented. The experimental curves were calibrated according to the RIN definition [22], and the contribution of the background noise was subtracted. RIN spectra were used to obtain relationships between the model parameters by a procedure similar to those in [24] and [26]. An analytical expression for the RIN was obtained from a first-order perturbation analysis of Eqs. (1)-(2) around steady-state conditions above threshold, yielding:

$$RIN(\omega) = \frac{A' [\omega^2 + \omega_s^2]}{\omega^4 + (\gamma^2 - 2\omega_r^2)\omega^2 + \omega_r^4} \quad (16)$$

where the parameters ω_r^2 , A' , and γ depend on the bias conditions and can be expressed as:

$$\omega_r^2 = \frac{\Gamma v_g \frac{dg}{dN} (I_{bias} - I_{th})}{q V_{act}}, \quad (17)$$

$$A' = \frac{2B\beta N_{th}^2 q V_{act}}{\tau_p} \frac{1}{I_{bias} - I_{th}}, \quad (18)$$

Fig. 4. (a) ω_r^2 (blue squares) and $1/A'$ (red squares) vs $I_{bias} - I_{th}$; (b) γ vs ω_r^2 . The lines correspond to linear fits.

TABLE I
SUMMARY OF MAIN SIMULATION PARAMETERS

Symbol	DML Values	Units	Obtained
V_{act}	1.53×10^{-17}	m^3	Assumed [27]
Γ	0.06	-	Assumed [27]
N_{tr}	1.3×10^{24}	m^{-3}	Assumed [27]
B	1.5×10^{-16}	$m^3 s^{-1}$	Assumed [27]
$\frac{dg}{dN}$	4.38×10^{-20}	m^2	RIN + P-I
τ_p	2.17	ps	RIN + P-I
A	2.8×10^8	s^{-1}	RIN + P-I
C	9.0×10^{-41}	$m^6 s^{-1}$	RIN + P-I
β	5.3×10^{-6}	-	RIN + P-I
η_f	0.17	-	P-I
ϵ	1.97×10^{-23}	m^3	Fit to optical spectra
α	3	-	Fit to optical spectra
R_{SL}	0.1	-	Fit to optical spectra
k_c	4.23×10^{10}	s^{-1}	Fit to optical spectra

$$\gamma = \frac{1}{\tau_n} + \left[\tau_p + \frac{\epsilon}{v_g \frac{dg}{dN}} \right] \omega_r^2, \quad (19)$$

$$\omega_s^2 = \left(\frac{1}{\tau_n} + \tau_p(1 - \Gamma)\omega_r^2 \right)^2 + \frac{\Gamma^2 D_{nn}}{D_{ss}} \tau_p^2 \omega_r^4 - 2 \left(\frac{\tau_p \omega_r^2}{\tau_n} + \tau_p^2 \omega_r^4 \right) \quad (20)$$

with τ_n being the differential carrier lifetime at threshold, which is given by:

$$\tau_n^{-1} = \frac{dR(N)}{dN}|_{N_{th}} = A + 2BN_{th} + 3CN_{th}^2. \quad (21)$$

where N_{th} is the carrier density at threshold. The measured RIN spectra were fitted to Eq. (16) and the parameters ω_r^2 , A' , γ and ω_s were extracted for each value of I_{bias} . Fig. 4 (a) shows the extracted values of ω_r^2 and A' vs I_{bias} , and Fig. 4 (b) the values of γ vs ω_r^2 . The three extracted parameters follow clearly the theoretical Eqs. (17)-(19), providing four relationships between the model parameters.

The total number of unknown relevant physical parameters of the model in the case of the solitary SL is twelve (see Table I). After a literature survey, we assumed four of them (V_{act} , N_{tr} , Γ and B) based on reported values of high speed $1.55 \mu m$ lasers. Two parameters (α and ϵ) were obtained by comparing the simulated and measured spectra in GS operation at different frequencies, while the other six

parameters were calculated from the relationships given by the P-I and RIN measurements. The results provide a very good quantitative agreement with the experimental optical spectra, as it will be shown in next section, as well as a good reproduction of the RIN characteristics, as it is shown in Fig. 3.

It is interesting to comment here that initially Eq. (20) was used together with Eqs.(17)-(19) for the parameter extraction, thus removing one more degree of freedom. However, as it is commented with more detail in Section V.B, we were not able to reproduce simultaneously the low and high frequency optical spectra for any combination of the four assumed parameters. Therefore, we conclude that, in the range of frequencies of interest, ω_s^2 is almost negligible in comparison with ω^2 , so that the value extracted from the fitting procedure to the RIN spectra is not reliable.

In the simulations with OI the parameters used for the ML were the same as for the SL, as we consider irrelevant the fluctuations in the output of the ML which is operating in CW conditions. A detailed analysis of the frequency noise properties of the OFCs, which is beyond the scope of this work, would require a better characterization of the noise of the ML. Two additional parameters, k_c and R_{SL} , were obtained by comparing the simulated and experimental optical spectra under OI conditions. The main simulation parameters are shown in Table I.

V. RESULTS AND DISCUSSION

The temporal and spectral response of the laser in GS operation was extensively characterized with and without OI for different values of f_R , I_{bias} , V_{RF} , P_{inj} and δv . The quality of the OFCs was characterized by means of the 10 dB spectral width (Δf_{10}) and the carrier-to-noise ratio (CNR), as they are defined in [15]. The results of this characterization were compared with the simulations. In order to properly compare both spectra, the simulated spectra were convolved with the filter response of the BOSA (10 MHz), and they were shifted by adding the measured CW emission frequency at threshold (193.36 THz). We have selected three different basic operating conditions: i) high frequency (5 GHz) without OI, in the range used in optical communications; ii) relatively low frequency (200 MHz) without OI, within the window of the spectroscopic applications; and iii) high frequency (5 GHz) with OI.

A. High Frequency OFCs

Fig. 5 shows the experimental and the simulated optical spectra at $I_{bias} = 34$ mA, $f_R = 5$ GHz, and two values of V_{RF} . At this bias current, the relaxation oscillation frequency is 6.6 GHz. As we already showed in [15], for lower V_{RF} , a high quality OFC with a high CNR = 39 dB and a broad $\Delta f_{10} = 60$ GHz is observed (Fig. 5 (a)), while when the applied voltage is increased (Fig. 5 (c)), a continuous and noisy spectrum appears, without any appearance of OFC (CNR = 6 dB, $\Delta f_{10} = 70$ GHz). The simulated spectrum at low voltage (Fig. 5 (b)) is quite similar to the experimental one in terms of shape, flatness, CNR and width (CNR = 39 dB, $\Delta f_{10} = 65$ GHz). The simulated spectrum at high voltage (Fig. 5 (d)) is also very similar to the experimental one, corresponding to a high and broad noise level with low

Fig. 5. Experimental (left) and simulated (right) optical spectra of the gain-switched DML laser at $I_{bias} = 34$ mA, $f_R = 5$ GHz, and two values of V_{RF} (top row $V_{RF} = 1$ V, bottom row $V_{RF} = 1.8$ V).

amplitude tones (CNR = 6 dB, $\Delta f_{10} = 80$ GHz). The only clear difference between simulations and experiments is the BOSA background noise, which has not been taken into account in the simulations.

Same similarities between experimental and simulated spectra, including shape, flatness, width and noise level, were found for the complete range of measurements conditions. The comparison is made by means of the colour maps in Fig. 6, which show the values of CNR, Δf_{10} and the product CNR $\cdot \Delta f_{10}$ for different operating conditions. The experimental and simulated maps are qualitatively and quantitatively almost identical, indicating the validity of the model and of the procedure for parameter extraction.

Three different regions can be clearly distinguished in the CNR maps of Figs. 6 (a) and (b). In region A, the modulation amplitude is high enough to switch off the laser, the subsequent pulses build-up from spontaneous emission, and the temporal coherence between pulses is lost. As a result, the generated spectra show high values of Δf_{10} (Figs. 6 (c) and (d)) but CNRs lower than 10 dB. Region B is a narrow transition region, and in region C, OFCs with high CNR (50-60 dB) are generated. The transition between regions A and C is very abrupt, and it takes place at higher V_{RF} when I_{bias} is increased, as it was already reported in [15]. At constant V_{RF} , the width of the OFCs in region C is maximum close to the transition region (Figs. 6 (c) and (d)); in this case, the optical power between pulses is very low and the carrier swing is maximum, yielding the broadest OFCs. It is clear that this border region produces the best quality OFCs, as it can be clearly observed in Figs. 6 (e) and (f), where the maximum product CNR $\cdot \Delta f_{10}$, considered as the figure of merit, is observed within a narrow region close to the border. It is interesting to observe in Figs. 6 (c) and (d), that for a contour of equal Δf_{10} (equal colour) there is a bias current yielding a minimum value of V_{RF} ; for this bias, between 25 and 30 mA, the relaxation oscillation frequency is close to the repetition frequency ($f_{ROF} = 4.8$ GHz and

Fig. 6. Colour maps with the dependence on V_{RF} and I_{bias} of the experimental (left) and simulated (right) spectral characteristics for $f_R = 5$ GHz: (a) and (b), CNR; (c) and (d), Δf_{10} ; and (e) and (f), $\text{CNR} \cdot \Delta f_{10}$.

$f_{ROF} = 5.9$ GHz, respectively) and therefore the modulation efficiency is maximum, yielding broader OFCs than those generated at higher bias.

The simulation results can be used as a guideline to select the optimal driving conditions to obtain the best OFCs in terms of the product $\text{CNR} \cdot \Delta f_{10}$. Within the range of driving conditions previously analysed, Fig. 6 (f) indicates a maximum product of around 2600 dB · GHz at the maximum driving voltage (1.8 V) and a bias current of 40 mA. We have simulated at higher values of V_{RF} and we have observed that $\text{CNR} \cdot \Delta f_{10}$ continues increasing when the value of the bias is correspondingly increased, without apparent saturation. For instance, for $V_{RF} = 2.6$ V and $I_{bias} = 55$ mA, $\text{CNR} \cdot \Delta f_{10}$ reaches 3200 dB · GHz. This result suggests the use of electrical RF amplification stages to improve the quality of the OFCs, with the experimental limit of the internal heating of the device.

The excellent agreement between simulations and experiments indicates that the analysis of the internal variables can be used for a better understanding of the real physical processes involved in the OFC generation. This has been done by plotting the temporal profile of the injected current, the output power, the carrier density and the frequency chirp. An example is provided in Fig. 7, where the temporal response

Fig. 7. Temporal profiles at the same GS conditions as in Fig. 5 ($I_{bias} = 34$ mA, $f_R = 5$ GHz and $V_{RF} = 1$ V (left) and 1.8 V (right)): (a) and (b), experimental power; (c) and (d), simulated power (black) and chirp (orange); (e) and (f), simulated carrier density (black) and calculated threshold carrier density (orange); and (g) and (h), simulated current (black) and threshold current (orange).

of the internal variables, corresponding to the spectra shown in Fig. 5, is presented. The measured temporal trace of the output power is also plotted for comparison. The comparison between simulated and experimental temporal power profiles indicates small differences in the peak power. Besides, the small irregularities observed at the falling edge of the experimental pulses are not reproduced by the simulations. However, the general features are again similar.

The temporal responses of the carrier density, frequency chirp and output power in Fig. 7 correspond to the well known GS process [13], [18], [22], [28]: the sinusoidal modulation of the injected current induces non-sinusoidal variations of the carrier density below and above N_{th} , and in consequence, the fast switch-off and switch-on of the laser resulting in the emission of short pulses. The emission frequency follows the variations of the carrier density due to the carrier density dependence of the refractive index. The frequency and power variations are not in phase, with the frequency being a fraction of the period in advance, thus yielding a relatively small difference in the power of the spectral components, and therefore a relatively flat spectrum. It is clear that in this case the so-called dynamic chirp, caused by transient effects, is the main physical effect producing the broadening of the spectra. The asymmetry observed in the tails of the spectra,

Fig. 8. Experimental (left) and simulated (right) optical spectra at $I_{bias} = 60$ mA, $f_R = 200$ MHz and $V_{RF} = 1.5$ V.

with the blue tail falling less abruptly than the red tail (see Figs. 5(a) and (b)) is a consequence of the phase difference, as the maximum chirp is produced during the rising part of the optical pulse.

The difference between the spectra in Fig. 5 for different modulation voltages can be understood by analysing the temporal profiles shown in Fig. 7. For a higher modulation voltage the current during the negative cycle is lower than the threshold current (see Fig. 7 (h)), producing a deep carrier depletion (see Fig. 7 (f)). Then, the optical pulse is switched off and the random spontaneous emission is predominant; the calculated chirp shows random fluctuations as it can be observed in Fig. 7 (d)). As it was discussed in [14], [15], the new pulse is built-up from spontaneous emission without any phase correlation with the previous pulse, losing the temporal coherence, and thus, showing absence of tones in the optical spectrum.

B. Low Frequency OFCs

In the following paragraphs we analyse the OFCs generated at a relatively low frequency $f_R = 200$ MHz.

In Fig. 8, we compare the experimental and the simulated spectra at driving conditions producing one of the best quality OFCs at this frequency ($I_{bias} = 60$ mA, $V_{RF} = 1.5$ V). The similarity between the experimental spectrum shown Fig. 8 (a) (CNR = 30 dB and $\Delta f_{10} = 8.6$ GHz.) and the simulated spectrum shown in Fig. 8 (b) (CNR = 29 dB, and $\Delta f_{10} = 8.4$ GHz) is remarkable. Both are asymmetric, with higher intensities at lower wavelengths, and both show the typical suppression of tones characteristic of frequency modulation [29]. The difference between the experimental and the simulated emission wavelengths is attributed to an increase in the device temperature caused by the sinusoidal excitation. The model does not consider this additional heating.

The comparison between experiments and simulations at this repetition frequency was performed in a wide range of operating conditions. Again, CNR and Δf_{10} were used as parameters for the characterization of the spectra. The results are shown in the colour maps of Fig. 9. An excellent agreement between simulations and experiments is again observed for the complete range of I_{bias} and V_{RF} . Figs. 9 (a) and (b) show the experimental and simulated value of the CNR. Two different regions (A and B) can be clearly distinguished: in A,

the values of CNR are below 10 dB corresponding to noisy and almost continuous spectra, with a total absence of tones. In this regime, the current modulation switches off the laser during a significant part of the modulation period, and in the rest of the period the output power follows the modulation current. This produces incoherent spectra, although with a different shape than the noisy spectra at high frequency previously commented. An example of these spectra is shown in Fig. 4 (l) of [15]. On the other hand, in B, the laser is actually modulated above threshold and OFC generation is achievable, as it is shown in Fig. 8. As commented in [15], in this regime the laser is actually working under direct modulation and not in real GS conditions, although we continue using the term gain switching as it is usual in this context. In this region, the spectral width increases with the modulation amplitude, but it does not depend on the bias current, as it can be clearly seen in Figs. 9(c) and (d) (the colour contours corresponding to a constant Δf_{10} are parallel to the abscise axis). This fact is a clear difference with respect to the high frequency maps of Figs. 6 (c) and (d), and it is a consequence of using a frequency much lower than the relaxation oscillation frequency. This implies that a different mechanism is involved in the OFC generation, adiabatic chirp instead of dynamic chirp, as it will be detailed later. The product of CNR and Δf_{10} , shown in Fig. 9 (e) and (f), also indicates that the best OFCs are obtained at the highest V_{RF} , irrespective of the I_{bias} , but fulfilling the condition of not switching-off the laser.

The variations of the internal photon density produce changes in the gain through the non-linear gain coefficient (see Eqs. (1)-(2)), and consequently variations of the carrier density to maintain the gain equal to loss condition. At this frequency, the carriers are able to follow the gain variations, and an almost sinusoidal carrier profile is obtained (Fig. 10 (b)). The frequency chirp (Fig. 10 (c), orange line) is then due to the adiabatic change of the carrier density, the so-called adiabatic chirp. In this case, as it can be observed in Fig. 10 (c), the optical power profile and the frequency chirp are in-phase. Then, lower frequencies correspond to lower optical power, and the spectra exhibit the previously commented asymmetry, with higher suppression for those tones at the red side of the spectra (see Fig. 8).

It is interesting to comment that the initial use of Eq. (20) together with Eqs. (17)-(19) for the parameter extraction, led to a value of ϵ around half of the value used in the final simulations. Using that value of ϵ , the shape of the simulated low frequency spectra was similar, but the spectral width was around half of the experimental one. Then, in order to reproduce the measured width, we considered the possibility of a temperature induced adiabatic chirp as an additional physical mechanism contributing to the OFC generation. In order to account for this possibility, we added a dynamical thermal equation similar to that of [29], with the thermal response time as an additional parameter. The thermal solution provided broader spectra, but the asymmetry of the spectra in Fig. 8 disappeared. The reason of this result is clear, temperature variations are expected to be in phase, or slightly

Fig. 9. Colour maps with the dependence on V_{RF} and I_{bias} of the experimental (left) and simulated (right) spectral characteristics for $f_R = 200$ MHz: (a) and (b), CNR; (c) and (d), Δf_{10} ; and (e) and (f), $\text{CNR} \cdot \Delta f_{10}$.

out of phase with respect to the current, and therefore the temperature induced chirp would be in phase opposition to the carrier induced chirp, thus correcting the asymmetry of the optical spectra. We conclude then that thermal effects are not relevant to explain the OFC generation of this particular laser at this operating frequency.

C. OFCs Generation With GS and OI

In this section, the results of the comparison between experimental and simulated OFCs generated by the combination of GS and OI are shown and discussed. It is well known that the properties of a GS OFC can be improved by OI [13], and also that for a range of driving conditions the OI can have detrimental effects [15]. A wide range of dynamical regimes have been observed in the case of OI with the SL in CW conditions [30], and similar behaviours for the generated OFCs have been theoretically predicted [17] and experimentally observed [15]. Here we only analyse operating conditions producing at high frequency broad noisy spectra without injection (similar to those of Figs. 5 (c) and (d)), in order to determine the best injection conditions to improve the OFC quality. A systematic study of all dynamic regimes

Fig. 10. Temporal profiles for $I_{bias} = 60$ mA, $f_R = 200$ MHz and $V_{RF} = 1.5$ V: (a) simulated current (black) and threshold current (orange); (b) simulated carrier density (black) and calculated threshold carrier density (orange); and (c) simulated power (black) and chirp (orange).

as a function of the five operation parameters (f_R , I_{bias} , V_{RF} , the injected power (P_{inj}) and the detuning (δv)) is out of the scope of this work.

For the following analysis, we have defined the detuning for a given I_{bias} as $\delta v = v_{ML} - v_{SL}$, where v_{ML} is the emission frequency of the ML and v_{SL} is the CW emission frequency of the SL at I_{bias} . The difference between δv and $\delta v'$ (used in Eqs. (13)-(15)) is due to the adiabatic chirp.

Fig. 11 shows the experimental and the simulated optical spectra for fixed gain-switching conditions ($I_{bias} = 32$ mA, $f_R = 5$ GHz, $V_{RF} = 1.8$ V) and different optical injection conditions. The optical spectra in Figs. 11 (a) and (b), generated using a low detuning ($\delta v = 2$ GHz) and an intermediate injected power ($P_{inj} = 0$ dBm), are very similar, showing high-quality OFCs with high values of CNR and Δf_{10} (CNR = 45 dB and 46 dB, and $\Delta f_{10} = 60$ GHz and 60 GHz, for the experiments and simulations, respectively). Under these operating conditions, without OI, the pulses from the slave laser are built-up from the spontaneous emission, so that, the corresponding optical spectrum is totally incoherent. However, when OI is applied, the slave laser emission couples with the emission of the master laser, thus fixing the initial phase of each pulse and resulting in the coherence of the pulse train and in the high quality of the OFC [14]. When the injected power is decreased, ($P_{inj} = -36$ dBm, Figs. 11 (c) and (d)) the spectrum shows a clear OFC with low CNR (10 dB) above the noisy spectral envelope, indicating that there is a clear competition between the spontaneous emission and the injected field to build up the emitted pulses: some pulses are locked to the external injection while others are not.

A similar phenomenon can be clearly observed in Figs. 11 (e) and (f), where we considered an intermediate value of injected power ($P_{inj} = 0$ dBm), and a large detuning ($\delta v = -66$ GHz). In both cases, the emission spectra do not show an OFC. The line at the ML frequency is due to

Fig. 11. Experimental (left column) and simulated (right column) optical spectra with optical injection at $I_{bias} = 32$ mA, $f_R = 5$ GHz, $V_{RF} = 1.8$ V, with different values of injected power and detuning, (a) and (b), $P_{inj} = 0$ dBm and $\delta\nu = 2$ GHz, (c) and (d), $P_{inj} = -36$ dBm and $\delta\nu = 2$ GHz, (e) and (f), $P_{inj} = 0$ dBm and $\delta\nu = -66$ GHz. The injection frequency is indicated with an arrow.

the combination of the reflection at the SL facet, together with internal amplification giving rise to its sidebands, due to the gain modulation. Broad noisy spectra, due to unlocked emission of the SL, can be observed in experiments and simulations. Under these conditions, the coupling between the ML and the SL dramatically decreases because of the large value of the detuning. The experimental results shown in Fig. 11 (e) could not be quantitatively reproduced in simulations (Fig. 11 (f)) without taking into account the reflection term (Eq. (15)) in the spectral calculations.

In order to analyse the conditions of the OI to provide high quality OFCs, we have studied the evolution of CNR and Δf_{10} in a wide range of P_{inj} and $\delta\nu$, for the GS conditions of Fig. 11 ($I_{bias} = 32$ mA, $f_R = 5$ GHz, $V_{RF} = 1.8$ V). The experimental and simulation results are compared in the colour maps of Fig. 12. The evolutions of CNR and Δf_{10} as a function of the optical injection conditions are shown in Figs. 12 (a) and (b); and (c) and (d), respectively. Two regions, A and B, with different detuning can be distinguished. In region A ($50 > \delta\nu > -50$ GHz in the simulations and $40 > \delta\nu > -60$ GHz in the experiments), the OI influences the SL emission above a certain power level. At low power levels, broad spectra similar to those shown in

Fig. 12. Colour maps showing the dependence on P_{inj} and $\delta\nu$ of the experimental (left) and simulated (right) spectral characteristics at $I_{bias} = 32$ mA, $f_R = 5$ GHz, $V_{RF} = 1.8$ V: (a) and (b), CNR; (c) and (d), Δf_{10} ; and (e) and (f), $\text{CNR} \cdot \Delta f_{10}$.

Fig. 5 (c) and (d) appear. As the injected power is increased, the coupling between the master and the slave laser improves and so the coherence of the pulse trains, leading to OFCs with low values of CNR (Figs. 11 (c) and (d)) over broad noisy spectral profiles similar to those of the non-injected GS laser (Figs. 5 (c) and (d)). At higher injected power, the generated OFCs show high quality in terms of CNR and Δf_{10} (Figs. 11 (a) and (b)). The power level needed to achieve a fixed value of the CNR increases with the detuning as indicated by the constant colour contours in Figs. 12 (a) and (b). Injection levels around 0 dBm and detuning around 0 GHz are the best injection conditions for generating a high quality OFC from the non-injected broad spectrum. At higher injection levels, the spectrum shows a high CNR but a narrower width, as well as poor flatness. The difference between experiments and simulations regarding the detuning range separating the two regions, (around 10 GHz towards negative detuning in the case of experiments), is attributed to the temperature increase caused by the sinusoidal excitation in comparison with the reference CW conditions.

In the region of high detuning (region B), the injected light has no effect on the noisy envelope, generating a spectrum very similar to that shown in Figs. 11 (e) and (f), with

the ML emission and a broad and continuous spectrum. The high values of Δf_{10} at medium power (red regions in Figs. 12 (c) and (d)) are an artifact caused by the definition of Δf_{10} , as the considered spectral width is calculated taking into account not only the width of the noisy envelope, but also the difference between the SL emission and the CW tone of the injected light.

Figs. 12 (e) and (f) show the product $\text{CNR} \cdot \Delta f_{10}$, and they are a useful guide to select the best injection conditions to optimize the OFCs generated by the combination of GS and OI. As it was previously mentioned, the spectral performance of the generated OFCs can be improved in a relatively broad range of detuning (around $|\delta\nu| < 50$ GHz) and a broad range of injection levels (P_{inj} between -20 and 0 dBm, depending on the detuning). The optimum range, given by the red regions in Figs. 12 (e) and (f), is not critical, thus being easy to find suitable injection conditions. The degradation of the product $\text{CNR} \cdot \Delta f_{10}$ by increasing the injected power above 0 dBm, caused by the reduction of the spectral width, is due to the reduction of the chirp by the external optical injection [13], [31]. However, if we compare the maximum value of $\text{CNR} \cdot \Delta f_{10}$ without and with OI (Figs. 6 (f) and 12 (f)), we observe a higher value ($3300 \text{ dB} \cdot \text{GHz}$) in the case of OI. The reason for this is that in the case of OI, at constant V_{RF} , it is possible to use lower bias and therefore, the carrier excursion and the corresponding chirp increases.

For GS conditions already leading to good quality OFCs without OI, the optically injected photons compete with the internally generated photons to switch-on the pulses, and double combs and/or unstable dynamics can be obtained [15]. It is remarkable that this type of effects and dynamic regimes (not shown in this work) are also quantitatively reproduced by the simulations, indicating that the main physical mechanisms are properly considered. Details of these effects and the dynamic regimes induced in optically injected GS lasers will be further reported.

VI. CONCLUSION

We have systematically analyzed the optical spectra of the OFCs generated under a wide range of gain switching and optical injection conditions with the help of a properly parameterized three rate equation numerical model. The remarkable quantitative agreement found has demonstrated that our numerical model considers adequately the most relevant physical phenomena/effects determining the generation process and the characteristics of the OFCs generated by gain-switching free-running or optically injected laser diodes. Consequently, when the laser parameters are properly determined and the noise terms adequately treated, the model can be used for predicting the evolution of the OFC characteristics as a function of the switching and optical injection conditions for a wide range of such conditions, including switching frequencies well below the relaxation oscillation frequencies. It also provides guidelines for selecting suitable operating conditions for a specific performance requirement and can

be used for ascertaining the suitability of a laser for a given application requiring specific comb characteristics. The model has confirmed the relevant role of the dynamic chirp in the comb generation at high frequencies and the role of the adiabatic chirp in the generation at low frequencies and can be used for further analyzing the details of the mechanisms underlying the effects of the optical injection.

APPENDIX NUMERICAL ALGORITHM

In order to solve numerically the system of SDEs formed by Eqs. (1), (2) and (3), first we have to describe their correlation properties correctly, Eq. (4), noticing that the photons and carriers fluctuations are correlated. We introduce a new fluctuation [21], [23] as

$$F_Z(t) = \frac{F_S(t)}{\Gamma} + F_N(t), \quad (22)$$

with the following diffusion coefficients,

$$D_{ZZ} = \frac{1}{V_{act}} \left(R(N) + \frac{I}{eV_{act}} \right), \quad (23)$$

$$D_{ZS} = 0, \quad (24)$$

$$D_{Z\Phi} = 0, \quad (25)$$

$$D_{ZN} = \frac{1}{V_{act}} \left(R(N) + \frac{I}{eV_{act}} \right). \quad (26)$$

In this way, the fluctuation terms can be written as

$$F_S(t) = \sqrt{2\beta B \Gamma N^2 S} \xi_S(t), \quad (27)$$

$$F_\Phi(t) = \sqrt{\frac{\beta B \Gamma N^2}{2S}} \xi_\Phi(t), \quad (28)$$

$$F_Z(t) = \sqrt{\frac{2}{V_{act}} \left(R(N) + \frac{I}{eV_{act}} \right)} \xi_Z(t), \quad (29)$$

$$F_N(t) = F_Z(t) - \frac{F_S(t)}{\Gamma}, \quad (30)$$

where $\xi_i(t)$ is a white Gaussian noise, $\langle \xi_i(t) \xi_j(t') \rangle = \delta_{i,j} \delta(t-t')$ being $i, j = S, \Phi, Z$. From the mathematical point of view, the system of equations formed by Eqs. (1), (2) and (3) is a system of SDEs written in the Itô interpretation with multiplicative noise. However, most of the useful numerical algorithms to solve SDEs are written for the Stratonovich interpretation. The way to move from one interpretation to the other is called the *Itô-Stratonovich drift correction formula* [32] that we describe in the following. We consider a generic system of SDEs written in the Itô interpretation for a vector of dynamical variables $x_i(t)$ written as

$$\frac{dx_i(t)}{dt} = f_i(x, t) + \sum_j g_{ij}(x, t) \xi_j(t), \quad (31)$$

where $f_i(x, t)$ are called the *drift* terms, $g_{ij}(x, t)$ are called the *diffusion* terms and $\xi_j(t)$ is a white Gaussian noise vector, which correlation properties are $\langle \xi_i(t) \xi_j(t') \rangle = \delta_{i,j} \delta(t-t')$.

As its name indicates, the Itô-Stratonovich drift correction formula, consists in modifying the drift terms $f_i(x, t)$ as

$$\tilde{f}_i(x, t) = f_i(x, t) - \frac{1}{2} \sum_{k,j} g_{kj}(x, t) \frac{\partial g_{ij}(x, t)}{\partial x_k}, \quad (32)$$

then the SDE in the Itô interpretation written in Eq. (31) is equivalent to the equation

$$\frac{dx_i(t)}{dt} = \tilde{f}_i(x, t) + \sum_j g_{ij}(x, t) \xi_j(t), \quad (33)$$

written in the Stratonovich interpretation. Notice that both interpretations are equivalent in the case of additive noise, i.e. $g_{i,j} \neq g_{i,j}(x, t)$.

In order to solve the laser equations, we use the so-called Heun's method for SDEs [19], [33]. The algorithm is a predictor-corrector method, the first step is to calculate the Euler predictor as

$$\tilde{x}_i(t+h) = x_i(t) + h \tilde{f}_i(x(t), t) + \sqrt{h} \sum_j g_{ij}(x(t), t) u_j(t) \quad (34)$$

where h is the time step and $u_j(t)$ is an independent set of random Gaussian numbers with zero mean value and variance equal to one. Finally, the solution is obtained using

$$x_i(t+h) = x_i(t) + \frac{h}{2} \left[\tilde{f}_i(x(t), t) + \tilde{f}_i(\tilde{x}(t+h), t+h) \right] + \frac{\sqrt{h}}{2} \sum_j [g_{ij}(x(t), t) + g_{ij}(\tilde{x}(t+h), t+h)] u_j(t). \quad (35)$$

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