A sample-based approach to estimate the dynamic loads of components with nonlinear uncertain interfaces

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A B S T R A C T

Predicting aircraft dynamics and vibration loads at components' interfaces is a key task for ensuring a robust design and development of the product. Usually, whereas in the case of isolate components the dynamic behavior can be predicted quite accurately, when several components are assembled through discontinuous junctions the predictiveness of a model decreases. The junctions, whose mechanical properties are seldom well characterized experimentally, often introduce nonlinearities in the loads' path. Additionally, their behavior is intrinsically uncertain and as a consequence, the dynamic response of the connected structures becomes stochastic.

We propose a sample-based approach which aims to cope with both aspects, nonlinearities and uncertainties, and can be split in two main tasks. First, the computational cost of each deterministic simulation is minimized considering that the global nonlinear behavior depends on localized sources of nonlinearities at the interfaces. Second, the uncertainties are propagated through the model by a non-intrusive method based on Sobol's low discrepancy design. Attention is paid to the global sensitivity indices, which are estimated by creating a meta-model based on Polynomial Chaos Expansion.

An industrial application considering an aircraft component whose dynamic behavior is affected by uncertain free-plays at its interfaces is presented.

Keywords:

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Free-plays

1. Introduction

Assembled structures in the aerospace industry are often connected through discontinuous and nonlinear junctions whose mechanical properties are seldom well characterized experimentally. From a modeling point of view, they are often reduced to a set of linear springs with some stiffness and damping values. It happens that, during the project, these values are reviewed, argued and fine-tuned to get a reasonable or expected dynamic response [1,2]. Local parameters are changed one at a time to see their effect on the outputs: the higher the number of input parameters, the more tedious the task is and the more complex the understanding of the results as well. In particular, interfaces play a key role in defining the dynamics of components and systems but considering properly their behavior can be a very complex task. This is

especially true for dynamic studies [3,4]. Their behavior, often nonlinear, is intrinsically uncertain and as consequence, making the dynamic response of the connected structures stochastic.

This work looks at the Uncertainty Quantification (UQ) process as a systematic approach to establish, quantitatively and qualitatively, how the uncertain behavior of interfaces among components affects the overall dynamics of a structure. In the UQ process we include also the Global Sensitivity Analysis (GSA), which allows to rank the input parameters according to their influence on the variance of the stochastic Quantities of Interest (QoIs). These QoIs are precisely those outputs which allow to understand and make decisions about the structure.

Some researchers have already investigated the uncertainty within the joints of assembled structures [5,6], as well as in the assembly and manufacture of certain structural applications such as micro-electromechanical systems [7]. Most pragmatic and industrial approaches to UQ are based on sampling the deterministic model according some specific computer design, i.e., the Monte Carlo method, in order to produce a cloud of output points, which can then be used to obtain sample-based statistics. In any case,

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but especially for computationally expensive simulations, the direct use of Finite Element (FE) models can be extremely time-consuming. Hence, usually, a smaller set of simulations is used to generate first a meta-model, which is used to complete the UQ analysis [8,9]. However, when the dynamic response of a structure is affected by strong nonlinearities, for example when the system has free-plays or includes rubber-like materials, the number of simulations required to generate a reliable meta-model can be still considerably high. The main reason is that these types of nonlinearities affect the smoothness of the high-dimensional response surface, which often cases will present discontinuities.

1.1. Outline of the work

In light of the previous observations, and considering that often junctions are represented by lumped elements, the Structural Dynamic Modification Method (SDMM) [10,11] seems a sensible approach for reducing the computational cost of the simulations. The principle behind modification methods is to run the FE Model just once, extracting the modal information at the required degrees of freedom (dofs) and then to update the equations, expressed in modal coordinates, through a set of modifications. While a large body of literature exists in the field of linear modifications, works dealing with nonlinear modifications are significant fewer, i.e. [12, 13]. The formulation of the Nonlinear Structural Dynamic Modification Method (NL SDMM) followed in this work has been introduced by Menga and Hernández [14–16] and is presented in Section 2. It has been proven to be a reliable and efficient approach to reduce drastically the computation burden associated with one deterministic nonlinear simulation when the global dynamic behavior depends on localized sources of nonlinearity, which can be modeled via lumped elements, i.e., elements connecting pairs of dofs.

In the current work, the NL SDMM is used to produce efficiently the initial set of simulations which will be used for generating the meta-model. Then the surrogate, based on a multivariate Polynomial Chaos Expansion (PCE), can be used to generate a broader sample of output points and obtaining the sensitivity indices.

A brief review of the PCE meta-modeling technique and of variance-based Global Sensitivity Analysis (GSA) is given in Section 3. Also, the mathematical link between the coefficients of the polynomial expansion and the sensitivity indices is discussed, although in a non-exhaustive fashion. The interest reader can refer for example to the work of Saltelli and Ratto [17] in the context of GSA and to the publications of Sudret [18] in the field of PCE.

Section 4 describes an industrial application of the previous ideas, combined in a systematic approach. The example considers an aircraft component whose dynamic behavior is affected by free-play interfaces, which are intrinsically uncertain.

Finally, Section 5 summarizes the main conclusions and results of this research and proposes some possible future work.

2. Nonlinear structural dynamic modifications

In this section the theory behind the NL SDMM is reviewed. The modal matrix and the eigenvalues are assumed to be available from the FE modal analysis of the model. Also, we consider that the only source of nonlinearity in the model is due to the interfaces between components. In this section we formulate the equation in the case of a nonlinear stiffness update, but the same approach can be followed for mass and damping modifications.

The initial stiffness of the spring elements used in the FE model for obtaining the modal properties is called *underlying linear stiffness*, whose value, with reference to the Fig. 1, is denoted k_1 . Starting from the modal information of the model, we want to update the dynamic equations to take into account the nonlinear stiffness

Fig. 1. Multi-linear force-displacement curve.

modifications. Then, the modal base of the linear model is fixed, i.e., the FE model is run just once, and the nonlinear response is the result of an updating process, irrespective of the type of nonlinearities that are incorporated.

The dynamic equilibrium in time-domain can be expressed as:

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = F(t), \quad (1)$$

where M , C , K are, respectively, the mass, damping, and stiffness matrix, and F is the vector of external forces. The eigenvalues and eigenvectors of the system are found by solving the problem:

$$(K - \lambda M)\Phi = 0, \quad (2)$$

where λ is a diagonal matrix with entries equal to the eigenvalues of the system, and Φ is the modal matrix. The i -th column of the modal matrix Φ is the eigenvector corresponding to the eigenvalue λ_i .

The displacements of the system can be expressed as a linear combination of the eigenvectors weighted by some factors $q(t)$, which are called modal coordinates, that is:

$$u(t) = \Phi q(t). \quad (3)$$

When the modal matrix is normalized with the mass, Eq. (1) can be expressed in the well-known modal form:

$$I\ddot{q}(t) + \tilde{C}\dot{q}(t) + \Lambda q(t) = \tilde{F}(t), \quad (4)$$

where I , $\tilde{C}$, and Λ are now the normalized mass, damping and stiffness matrix, respectively, and $\tilde{F}$ is the vector of forces. In Eq. (4) the normalized mass matrix is the identity, and the normalized stiffness matrix is a diagonal matrix whose terms are the eigenvalues.

With reference to Fig. 1, when a relative displacement u_s is applied between the two dofs of the spring, according to the force-displacement curve, the internal force is:

$$F_s = k_1(u_2 - u_1) + k_2(u_3 - u_2) + \dots + k_{n-1}(u_n - u_{n-1}) + k_n(u_s - u_n). \quad (5)$$

The same force can be obtained with a secant stiffness given by:

$$k_{equ} = \frac{\sum_{i=1}^{n-1} k_i(u_{i+1} - u_i) + k_n(u_s - u_n)}{u_s}. \quad (6)$$

Considering an underlying linear stiffness value k_1 , the additional contribution to the current stiffness is:

$$\Delta k = k_{equ} - k_1, \quad (7)$$

and the modification matrix is of the type:

$$\Delta k_{mod} = \begin{bmatrix} \Delta k & -\Delta k \\ -\Delta k & \Delta k \end{bmatrix}. \quad (8)$$

When N_k lumped elements are involved in the modification, the final matrix can be obtained as:

$$\Delta K_{mod} = \sum_{m=1}^{N_k} \Delta k_{mod}^m, \quad (9)$$

which can be normalized by the modal matrix of the underlying linear system:

$$\Delta \tilde{K}_{mod} = \Phi^T \Delta K_{mod} \Phi. \quad (10)$$

Hence, the update stiffness matrix of the structure is:

$$\Lambda_{mod} = \Lambda + \Delta \tilde{K}_{mod}, \quad (11)$$

and Eq. (4) can be re-written as:

$$I\ddot{q}(t) + \tilde{C}\dot{q}(t) + \Lambda_{mod}q(t) = \tilde{F}(t). \quad (12)$$

Eq. (12) needs to be solved by an iterative method which has to take into account that k_{equ} , and consequently Λ_{mod} , are nonlinear functions of the relative displacements of the springs, which, through the modal matrix Φ , depend on the modal displacements. In this work the state space form of Eq. (12) has been implemented in MATLAB[®] and solved with the function ODE45, which uses an explicit Runge-Kutta integration procedure. Once the non-linear modal displacements are obtained, the physical ones, for all the dofs considered in the modal matrix, can be evaluated by Eq. (3). We stress the fact that this approach is computationally very efficient: the linear FE Model is run just once in order to obtain the modal matrix, and usually the number of significant modes is much smaller than the dofs of the model. Additionally, the dofs involved in the updating process are usually a few dozen: the dofs where the loads are applied, those ones where the outputs are required, and finally those ones involved in the modifications.

3. Uncertainty quantification

UQ can be thought as a process with three fundamental phases, namely, pre-processing, uncertainty propagation, and post-processing. The first phase aims to select which input parameters are the source of uncertainties, and to prescribe on them some appropriate probability distribution. The propagation of the uncertainties aims to characterize the distributions of the Quantities of Interest (QoIs) for a given distribution of inputs. That is obtained, in non-intrusive methods, by sampling the deterministic model according to some specific sampling technique. Then, the cloud of results can be post-processed in order to obtain sample-based statistics. The most common design is based on Montecarlo (MC) sampling, which relies on pure randomness. Due its randomness, some points can be clustered closely, while other regions within the space may not contain any samples. Hence, especially when working in high dimensions, most researchers seem to agree that generic, near-uniform, distribution of the data sites is a better strategy. Several sampling strategies try to cover this need, such as the Latin Hypercube (LHD) and the Low-Discrepancy Design (LDD). LDD, in particular, generates a deterministic sequence of quasi-random numbers which fill uniformly the unit hypercube. In the context of this work, Sobol's Low Discrepancy Design (LDD) is considered [19]. It was the first digital sequence, operates in base-2, and has been found by several researchers, i.e. Pianosi [20], that this choice out-performs the classical MC and LHD methods being, in average, more efficient and having a faster convergence rate. For complex systems where, possibly, a very large set of inputs is considered uncertain, it is often necessary to generate first a meta-model and then, using it, complete the analysis.

Table 1

Correspondence between orthogonal polynomials and PDF.

Orthogonal polynomial	Weight function	PDF	Density function
Hermite $H_n(x)$	$e^{-\frac{x^2}{2}}$	Normal	$\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$
Legendre $P_n(x)$	1	Uniform	$\frac{1}{2}$
Laguerre $L_n(x)$	e^{-x}	Exponential	e^{-x}
Jacobi $P_n^{\alpha,\beta}(x)$	$x^\alpha e^{-x}$	Beta	$\frac{x^\alpha e^{-x}}{\Gamma(\alpha+1)}$

3.1. Meta-model: polynomial chaos expansion

The original Hermite Polynomial Chaos Expansion (PCE), also known as homogeneous chaos, was first derived by Wiener [21] for the spectral representation of any stochastic response in terms of Gaussian random variables. Later, Askey [22] studied the orthogonal properties of the Hermite polynomials with respect to the Gaussian probability density function, providing the well-known Wiener-Askey scheme. But only in 2003 Xiu [23] extended the method to others random distributions as summarised in Table 1. To ensure optimal convergence properties of the PCE, the polynomials are required to be orthogonal with respect to a weight function, which is representative of the probability distribution of the Quantities of Interest (QoIs).

To introduce the notation that will be employed later, let us briefly describe next the standard least square approximation problem: if a model is sampled at N_s points of the high-dimensional space $\mathcal{S} \equiv \mathbb{R}^{N_p}$, the generic output $Y_n = Y(X_n)$ evaluated at the input point $X_n = [X_{n1}, X_{n2}, \dots, X_{nN_p}]$, with $n \in [1, 2, \dots, N_s]$ and N_p the total number of input parameters considered, can be approximated by a linear combination of N_k basis or kernel functions $\Psi_k : \mathcal{S} \rightarrow \mathbb{R}$ as

$$Y(X_n) \simeq \sum_{k=1}^{N_k} \beta_k \Psi_k(u_n). \quad (13)$$

In our case, the basis functions are orthogonal polynomials with respect to a defined weight, and usually in as specific domain. For example, Legendre polynomials are orthogonal with respect a uniform probability density function in the domain $[-1, 1]$. Hence, each u_n is a normalised input random variable, and X_n has to be mapped to the domain where the orthogonality is ensured.

This relation can be expressed more compactly using matrix notation as in

$$Y(X) \simeq \Psi(u)\beta, \quad (14)$$

where Ψ is the interpolation or kernel matrix, Y is the matrix of the N_{out} QoIs evaluated at N_s sampled points and X is the sample matrix.

Usually, Ψ_k are *multivariate polynomials* that involve products of the one-dimensional polynomials $\psi_i(u_j)$, where i is the order of the polynomial and u_j is the j -th component of the vector u_n . The multivariate expansion is obtained as a tensor product of the one-dimensional basis and consequently presents higher-order terms. These higher order terms are usually truncated considering for the multivariate basis the same maximum order o of the one-dimensional polynomials they come from.

Under these conditions, the total number of coefficients N_k of the polynomial expansion in Eq. (13) is given by:

$$N_k = \frac{(N_p + o)!}{N_p! o!}. \quad (15)$$

As an example, we consider the first six one-dimensional Hermite polynomials:

Table 2
Bivariate second order Hermite polynomials.

$\Psi_0(u)$	$\psi_0(u_1) \cdot \psi_0(u_2)$	$1 \cdot 1$	1
$\Psi_1(u)$	$\psi_1(u_1) \cdot \psi_0(u_2)$	$u_1 \cdot 1$	u_1
$\Psi_2(u)$	$\psi_0(u_1) \cdot \psi_1(u_2)$	$1 \cdot u_2$	u_2
$\Psi_3(u)$	$\psi_2(u_1) \cdot \psi_0(u_2)$	$(u_1^2 - 1) \cdot 1$	$u_1^2 - 1$
$\Psi_4(u)$	$\psi_0(u_1) \cdot \psi_2(u_2)$	$1 \cdot (u_2^2 - 1)$	$u_2^2 - 1$
$\Psi_5(u)$	$\psi_1(u_1) \cdot \psi_1(u_2)$	$u_1 \cdot u_2$	$u_1 u_2$

$$\{\psi(u)\} = \left\{ 1, u, u^2 - 1, u^3 - 3u, \right. \\ \left. u^4 - 6u^2 + 3, u^5 - 10u^3 + 15u, \dots \right\}, \quad (16)$$

and the multivariate polynomials for a second-order expansion over two random variables, presented in Table 2. The higher order terms, such as $\Psi_6(u) = \psi_2(u_1) \cdot \psi_1(u_2) = u_1^2 u_2$, are neglected.

From Eq. (15) it can be noted that increasing the number of random variables or the order of the polynomial will cause a substantial increase in the number of terms N_k of the PCE and the sample size N_s required to find the coefficients of the expansion. Consequently, in the case of complex simulations, the computational cost could be unaffordable.

In this work the least square method is employed to estimate the unknown coefficients β_k of the PCE. Considering Eq. (13) and minimizing the sum of the squares of the residuals:

$$\beta = \arg \min_{\beta^* \in \mathbb{R}^{N_k}} \|\Psi \beta^* - Y\|^2, \quad (17)$$

the least square fitting gives a closed-form solution:

$$\beta = (\Psi^T \Psi)^{-1} \Psi^T Y, \quad (18)$$

where Ψ is assumed to have full column rank.

The number of input points N_s must be higher than the number of unknown coefficients N_k and, depending on the complexity of the response, the recommended number of samples may fluctuate. Following Hosder [24], $N_s \simeq 2 \cdot N_k$ has been employed.

3.2. Global sensitivity analysis

GSA defines a qualitative and quantitative mapping between the input variables and the Quantities of Interest (QoIs). Sensitivity analyses are strictly related to the Uncertainty Propagation (UP) and they can be seen as complementary to each other: UP quantifying the uncertainty in the output of the model, while GSA focuses on apportioning the output's uncertainty to the different sources of uncertain parameters [17].

GSA allows to identify the most relevant parameters of the model, and score them according to their relevance in the QoIs. In this context, a sensitivity index is a measure of the influence of an uncertain quantity on an output variable. The most popular GSA techniques are based on the decomposition of the variance's output probability distribution and allow the calculation of Sobol's sensitivity indices. They refer to Sobol', because the decomposition of the variance can be formulated analytically from the High Dimensional Model Representation (HDMR) of Sobol' [25]. From a practical point of view, particular attention is paid to the *first order* and *total order sensitivity indices*, denoted respectively as S_i^I and S_i^T , also called the *main effects* and the *total effects*. If a model Y is a function of N_p stochastic variables X_i with $i = 1, \dots, N_p$, the main effects represent the contribution of each parameter X_i to the output variance, or equivalently, the expected percentage reduction of the output variance $V(Y)$ obtained when the uncertainty of input parameter X_i is eliminated. The total effects represent the total contribution of each input parameter considering its individual effect and its interaction with the remaining factors.

Table 3
Multi-index: bivariate PCE of 2nd order.

Multivariate PCE	Multi-Index			Coefficients	Function
$\Psi_0(u)$	α_0	0	0	β_0	1
$\Psi_1(u)$	α_1	1	0	β_1	u_1
$\Psi_2(u)$	α_2	0	1	β_2	u_2
$\Psi_3(u)$	α_3	2	0	β_3	u_1^2
$\Psi_4(u)$	α_4	0	2	β_4	u_2^2
$\Psi_5(u)$	α_5	1	1	β_5	$u_1 u_2$

Table 4
Equivalence between Sobol's HDMR and PCE.

Sobol's HDMR	PCE functions	Variance	Variance PCE
f_0	β_0	$V(f_0)$	β_0^2
f_1	$\beta_1 u_1 + \beta_3 u_1^2$	$V(f_1)$	$\beta_1^2 + \beta_3^2$
f_2	$\beta_2 u_2 + \beta_4 u_2^2$	$V(f_2)$	$\beta_2^2 + \beta_4^2$
f_3	$\beta_5 u_1 u_2$	$V(f_3)$	β_5^2

3.3. Polynomial chaos and Sobol's indices

According to the PCE theory, the mean and the variance of the QoIs can be estimated directly by means of the PCE coefficients. In fact, assuming that the polynomials are normalized with the variance, the following relationships hold:

$$E(Y(X)) \approx \beta_0, \quad (19)$$

$$V(Y(X)) \approx \sum_{k=1}^{N_k} \beta_k^2. \quad (20)$$

In the context of multivariate polynomials, the concept of a multi-index α_k is often presented. This notation allows to describe the order of the multivariate terms from the tensor product of the univariate polynomials. Table 3 provides the multi-indices in the case of a second-order, bivariate PCE.

It is straightforward to see a close similarity between the PCE and the High Dimensional Model Reduction (HDMR) proposed by Sobol. In fact, the output functions can be seen as the sum of higher dimensional terms defined according the multi-index α_k . In addition, Sobol's condition for the uniqueness of the decomposition requires expressing the output function as the sum of a constant term plus a higher dimensional, zero-mean function, a condition that is satisfied thanks to the orthogonality of the polynomials.

Continuing the previous example on a bivariate random variable, Sobol's HDMR considers the terms of increasing dimensionality:

$$Y(X) = f(X_1, X_2) = f_0 + f_1(X_1) + f_2(X_2) + f_{12}(X_1, X_2), \quad (21)$$

while looking at the PCE, according to Table 3, the following expansion is obtained:

$$Y(X) \simeq \beta_0 + \beta_1 u_1 + \beta_2 u_2 + \beta_3 u_1^2 + \beta_4 u_2^2 + \beta_5 u_1 u_2. \quad (22)$$

Comparing Eq. (21) and Eq. (22), it is possible to identify the equivalence between the two formulations, which allows an easier calculation of the partial variances and later of the sensitivity indices (Table 4).

In fact, the total variance, defined by Eq. (20) can be decomposed as:

$$V(Y) = V(f_0) + V(f_1) + V(f_2) + V(f_{12}) \\ = \beta_0^2 + (\beta_1^2 + \beta_3^2) + (\beta_2^2 + \beta_4^2) + \beta_5^2, \quad (23)$$

Fig. 2. A380 - RAI System and Air Intake detail (source|AIRBUS OPERATIONS).

Fig. 3. A380 - RAI System Flaps (source|AIRBUS OPERATIONS).

and Sobol's indices can be obtained by the ratio between the partial variances and the total one. For example, the first and total effects with respect the first input variable X_1 are:

$$S_1^I = \frac{V(f_1)}{V(Y)} \approx \frac{\beta_1^2 + \beta_3^2}{\sum_{m=1}^p \beta_m^2}, \quad (24)$$

$$S_1^T = \frac{V(f_1) + V(f_{12})}{V(Y)} \approx \frac{\beta_1^2 + \beta_3^2 + \beta_5^2}{\sum_{m=1}^p \beta_m^2}. \quad (25)$$

Hence the calculation of the PCE coefficients provides a straightforward and computational inexpensive way to estimate the mean, the variance and Sobol's sensitivity indices. It is worth highlighting that Sobol's HDMR is an exact decomposition, while the polynomial expansion would be exact only if the summation were extended to infinite terms. In practice, the PCE is truncated to an order that in most applications does not exceed the third degree.

4. Industrial application: A380 RAM Air Inlet free-play nonlinear vibrations

In this section the UQ is considered in the context of an industrial case of study: A380 RAM Air Inlet nonlinear vibrations.

A RAM refers, in this context, to any moving part of a machine that exerts a force onto another one, by pushing or applying pressure on the latter. The RAM Air Inlet(RAI) and the RAM Air Outlet(RAO) openings provide a forced ventilation system to control the air temperature around the Air Generation Unit (AGU). This system supplies and maintains the air in the pressurised fuselage

compartments at the correct pressure, temperature, and freshness for passenger comfort and equipment cooling. Also, it provides air for ventilation in the unpressurised fuselage bays. Fig. 2 shows, on the left, the Airbus A380 RAI air intake and two RAO outtakes. Air flow enters through the RAI intake, wets the forward and progressively the reward flaps, enables the AGU heat exchange by forced ventilation, and then comes out through the RAO outtakes.

This study focuses on the RAI system, shown in Fig. 3: The RAI forward (FWD) and rearward (RWD) flaps are two moving panels whose relative position defines the height of the RAI section and then the amount of air required for heat exchange. In Fig. 3 two extreme positions are shown: the maximum opening (RAI 100%) and the minimum one (RAI 0%). An actuator, through a kinematic chain, shown in Fig. 4, adjusts the RAI section height and consequently the flaps' relative position. 3D details of the kinematic chain can be seen in Fig. 5: the actuator drives the shaft, in green, and through two flap links, moves two rods (gray color) which regulate the height by acting on the RWD flap.

During a flight, the two flaps are subjected to an unsteady pressure field which induces vibrations in the RAI system. Additionally, due to tolerances and wear of the bearings, the connections between the flap links and the rods present free-plays, which make the dynamic problem and hence the rods' reaction forces, nonlinear. Free-play behavior is shown in Fig. 10 by a nonlinear force-displacement curve: the stiffness is zero when the dynamic relative displacements of the interfaces is within a certain tolerance δ and assumes a value given by Eq. (6) when they are out of this range.

Fig. 4. Kinematic scheme of flaps control system (source|AIRBUS OPERATIONS).

Fig. 5. 3D design of the flaps control system (source|AIRBUS OPERATIONS). (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

Fig. 6. RAI FE Model.

Large flight test campaigns have facilitated the identification of the worst dynamic load conditions, both in terms of relative flaps' position and unsteady pressure distributions. In all cases, a reliable prediction should take into account the following aspects:

- the free-plays interfaces between the flap links and the rods make the response of the system strongly nonlinear;
- the uncertainties regarding both gaps' size δ and stiffness of the free-plays;
- the uncertainties about the magnitude of the resultant forces on the flaps and their dominant frequency.

4.1. Computational approach

Based on the previous considerations, a computational approach is proposed next to estimate the stochastic dynamic response of connected structures with uncertain nonlinear interfaces. The study relies on the NL SDMM presented in Section 2 to reduce the computational cost associated with each simulation and on Sobol's LDD to sample efficiently the deterministic model. Then, GSA is used to post-process the results and identify the most influential variables of the problem.

The modal matrix is obtained in NASTRAN[®] from the FE Model showed in Fig. 6, with the rods disconnected from their arms, or equivalently, by setting the stiffness of the junctions to zero.

First, let us consider applying a set of linear modifications on the model in order to check if the SDMM is able to update the dynamic response in case of a disruptive change of the stiffness at the interfaces, which change from $K_0 = 0$ to $K_1 = 1.5e^4$ [N/mm]. The displacements at the interfaces have been evaluated in NASTRAN[®] considering two models having respectively connected, i.e. K_1 , and disconnected, i.e. K_0 , interfaces. Additionally, by means of the SDMM, the results of the model with connected interfaces have been obtained by updating the disconnected generalized stiffness matrix with an increment of the spring stiffness from K_0 to K_1 (see Eq. (11)). The comparison, shown in Fig. 7,

in both time and frequency domain, validates the capability of the SDMM to account for a disruptive change of local stiffness.

When rod interfaces are affected by free-plays the system response is no longer linear because the junctions' stiffness depends on the relative displacements of the lumped elements. In particular, Fig. 8 shows that until the relative displacements are within the interval $[-0.5; 0.5]$ [mm], which corresponds to a nominal gap of 1 [mm], the interface stiffness and the rod's reaction force are zero. In contrast, when the gap is saturated, and the absolute value of the relative displacement is greater than 0.5 mm, the interface stiffness has an equivalent value given by Eq. (6). Consequently, the nonlinear force is obtained as the product of the equivalent stiffness and the relative displacement. Fig. 9 compares the rod's reaction force at nominal conditions defined in Table 5.

Finally, let us consider interfaces whose behavior is nonlinear and uncertain. The uncertain free-play force-displacement curve is shown in Fig. 10. Independently for each of the two rods, the size of the gap and stiffness are considered as random variable with prescribed normal distributions according to the values in Table 5. Additionally, uniform probability distributions are prescribed on the magnitude and frequency of the force exciting the RWD Flap. When uncertainties are considered, the system's response, and in particular the rods' reactions are uncertain as well. The root mean square (RMS) of the nonlinear rods' reaction forces, over a time period of 10 [s], i.e., $F1_{rms}$ and $F2_{rms}$, are the selected stochastic QoIs. The deterministic model is sampled according a Sobol's LDD of size $N_{run} = 400$, which could be already a very expensive computational cost considering the time required by the model due to its nonlinearities. This initial cost is considerably reduced by the NL SDMM discussed in Section 2.

The least square method is used to find the approximation's coefficients, 84 according to Eq. (15), of a 3rd order PCE over a sample of $N_s = 250$ training points. The remaining $N_{ev} = N_{run} - N_s = 150$ are reserved exclusively for the validation of the PCE surrogate and the error on this unseen set of data is:

$$Err_{ev} = \frac{\|Y_{ev} - \Psi_{ev}\beta\|}{N_{ev}\|\Psi_{ev}\beta\|}. \quad (26)$$

Legendre and Hermite polynomials are respectively employed for the uniform and normal probability distributions according to

Fig. 7. SDMM updating method vs. NASTRAN: in this Figure both time and frequency domain response of the system are compared when the springs at interfaces between the rods and the arms are disconnected (red line) and connected (blue line). Both solutions are obtained in NASTRAN. Additionally, the response with connected springs is obtained through updating the disconnected response by means of the SDMM. The obtained response (orange line) overlap the NASTRAN's solution. Hence, the SDMM is able to consider disruptive change of local stiffnesses in both domains.

Fig. 8. Nonlinear stiffness and reaction force at nominal conditions as defined in Table 5.

Fig. 9. Comparison between linear and nonlinear rod's reaction force at nominal conditions as defined in Table 5.

Fig. 10. Nonlinear interfaces behavior: uncertain free-play stiffness.

Table 5
Statistical distributions of the random variables.

Random Variable x	Distribution	Nominal	Lower bound	Upper bound
Force [N]	Uniform	500	350	650
Frequency [Hz]	Uniform	35	29.5	40.5
Random Variable x	Distribution	Mean value	Standard deviation	
K_1 [N/mm]	Normal	$1.5e^4$	$1.5e^3$	
K_2 [N/mm]	Normal	$1.5e^4$	$1.5e^3$	
Gap ₁ [mm]	Normal	1.0	0.3	
Gap ₂ [mm]	Normal	1.0	0.3	

Table 5. The independence of the random variables preserves the orthogonality of the multivariate PCE when different types of polynomials are used. The approximation error is on the order of 10^{-3} .

Usually, the PCE shows worse performance in case of strong nonlinearities, and increasing the degree of the polynomials does not always guarantee an improvement because of over-fitting: the meta-model trusts the data on the training set too much and performs poorly over unknown sets.

4.2. Results

Table 6 shows the mean, the variance, the error on the evaluation set, and Sobol's sensitivity indices. The frequency is the stochastic parameter which affects the outputs variance the most. This is in agreement with the fact that both intact and disconnect structures present amplification peaks in the prescribed range of variability (Fig. 7). The nonlinear interfaces behavior is shown in Fig. 10: the uncertainty on the size of the gaps has a much more relevant effect on the variance of the reaction loads than the uncertainty on the stiffness. According to the results in Table 6, there exist differences between the first and total order effects, meaning that the interaction among parameters is significant. This is particularly true for the total indices regarding the gap parameters. It makes sense because some interactions are expected, at least between the applied force and the gap. Then, for example, the $ST_5 = S_5 + S_{15} + S_{25} + S_{35} + S_{45} + S_{65}$ is expected to have, at least, the term S_{15} different from zero.

The sensitivity results of the stochastic QoI $F1_{rms}$ are summarized in the pie charts of Fig. 11. It is worth noting that when the

Fig. 11. Reaction force $F1_{rms}$ - Pie charts of first and total indices. The output's variance cannot be explained completely by the first order effects because interactions among inputs are relevant. Particularly, seeing at the total effects, the stiffness K_1 and K_2 acquire more relevance. In any case the uncertainty of the response is dominated by the frequency of excitation first, and then by the magnitude of the applied force and by the size of the gaps of free-plays.

Table 6
RAI vibrations - statistical moments and Sobol's Indices.

Parameter	Results	$F1_{rms}$	$F2_{rms}$
	Mean(Y)	140.10	207.49
	V(Y)	3286.55	8145.43
	error on N_{eval}	0.0015	0.0024
Force	S_1	0.14	0.17
Frequency	S_2	0.43	0.30
K_1	S_3	0.00	0.01
K_2	S_4	0.00	0.01
Gap ₁	S_5	0.12	0.17
Gap ₂	S_6	0.15	0.06
Force	ST_1	0.21	0.28
Frequency	ST_2	0.48	0.38
K_1	ST_3	0.06	0.10
K_2	ST_4	0.06	0.09
Gap ₁	ST_5	0.20	0.27
Gap ₂	ST_6	0.22	0.21

interactions among parameters are significant, the sum of the first order indices is less than one, and the pie chart does not cover the 100% of the total output variance. In contrast, the sum of the total effects can be greater than one, because for example the factor S_{15} contributes to both ST_5 and ST_1 , hence the percentages in the pie chart of total effects are redistributed according to the value of their sum.

5. Conclusions and future work

This research presents a pragmatic approach for estimating the stochastic dynamic response of assembled structures whose interfaces are nonlinear and uncertain. Non-intrusive, or sample-based, UQ has received more and more attention in the engineering community during the last decade and is a very wide and active field of research. The novelty of this work is in incorporating in the UQ process the NL SDMM, which is presented as a reliable and efficient method to avoid the computational burden due to nonlinearities when they can be considered localized and then modeled through lumped elements.

The main issue regarding SDMM, in both linear and nonlinear analyses, is whether the baseline modal base, obtained by using the FE Model, can be updated correctly when the modified local impedance matrix is introduced, or not. The method is more sensitive to the magnitude of the local increments (stiffness, damping, mass) than to the linearity of the modifications. Hence, we consider that free-play behavior is particularly interesting: disruptive changes of local stiffness are considered. A numerical implemen-

tation of the SDMM takes care of a reliable modal truncation and eventually to enrich the modal base with specific residual vectors. These are the static vectors of the dofs involved into the modification and can be calculated by applying a unit force on them.

Focus is also put on the similitude between the Sobol's HDMR and the PCE, which allows to obtain the sensitivity indices directly from the coefficients of the expansion. Some theoretical aspects of the key items, i.e., NL SDMM, PCE and GSA, are presented.

The feasibility of the proposed approach is tested on an industrial application. An aircraft component whose dynamic behavior is affected by uncertain free-play junctions is considered and its nonlinear time-domain response is evaluated when it is loaded by a stochastic force. In particular, the root mean square values of the reactions forces at interfaces are selected as QoIs. GSA is proposed as a reliable tool to post-process the outcomes, quantify the effect of each uncertain variable on the QoIs, and select the most relevant ones. The results are discussed looking at their physical coherence and some perspective on the use of PCE in the case of nonlinear problems is given.

The efficiency and predictiveness of meta-models for nonlinear models is an open field of research and further investigations will compare the PCE with other surrogates, such as those based on Radial Basis Functions (RBFs).

Conflict of interest statement

The authors have no conflict of interest to declare.

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