Applications of the direct domain decomposition based on cylindrical ports in the hybrid Finite Element/Modal Analysis method

Alfonso Gómez García

Miguel Á. González

Rafael Gómez Alcalá

Jesús García

Yolanda Campos-Roca

José M. Gil

Jesús Rubio

Abstract—In this work, it will be shown some interesting capabilities of a direct domain decomposition methodology, based on the use of the hybrid finite element/modal analysis (FE/MA), which takes advantage of the addition theorems for cylindrical waves. These theorems allow to perform analytical rotations and translations of domains in the whole structure, avoiding recalculations with the FE/MA method. Specifically, it will be used to place via holes between patch antennas on a continuous substrate, in order to prevent mutual coupling through the substrate. It will also be shown how this methodology can be applied to the fast analysis of radial line slot antennas, where each slot is considered as a subdomain defined by a cylindrical port and a hemispherical port; and to the analysis of groove gap waveguides, where each quadrangular post is characterized by a cylindrical port.

Index Terms—direct domain decomposition, cylindrical modes, modal analysis, substrate integrated waveguide (SIW).

I. INTRODUCTION

At the end of the nineties, we introduced a direct domain decomposition methodology for passive microwave circuits [1]. This methodology is based on the use of the hybrid finite element/modal analysis (FE/MA) method to provide the generalized scattering matrix (GSM) from the generalized admittance matrix (GAM) of each domain. In this case, the interfaces between domains correspond to plane waveguide ports, and the whole response of the device is obtained by connecting the GSMs. Because of the used admittance formulation, the methodology is suitable for fast frequency sweep if homogeneous plane ports are used [2]. In [3], the methodology was extended to the efficient analysis of antenna arrays on a ground plane by using hemispherical ports and addition theorems for spherical waves.

Recently, we have included cylindrical ports in the FE/MA method with fast frequency sweep. The use of the addition

theorems for cylindrical waves further enhances the possibilities of the proposed methodology, since it allows to perform analytical rotations and translations of the domains in the whole structure without any FE/MA recalculation.

In the last years, the use of substrate integrated waveguide (SIW) technology has experienced an increase in popularity, due to its high-power performance and its low cost of production. This technology is based on the use of metallized via holes to connect the top and bottom layer of an also metallized substrate, in order to replicate the behaviour of the traditional rectangular waveguide at a frequency band [4]. The cylindrical via holes make the methodology ideal for the analysis and design of SIW circuits, when it is used together with addition theorems for cylindrical waves [5]. Additionally, the simultaneous use of addition theorems for spherical and cylindrical waves is suitable for the efficient analysis of antenna arrays fed by SIW [6].

In this work, it will be shown some another interesting capabilities of the proposed FE/MA methodology that take advantage of the addition theorems for cylindrical waves. Specifically, it will be used to place via holes between patch antennas on a continuous substrate, in order to prevent mutual coupling through the substrate, and thus, replacing a traditional metallic cavity. It will also be shown how this methodology can be applied to the fast analysis of radial line slot antennas, where each slot is considered as a subdomain defined by a cylindrical port and a hemispherical port; and to the analysis of groove gap waveguides (GGW), where each quadrangular post is characterized by a cylindrical port.

II. ANALYSIS OF THE PATCH ANTENNA ARRAY

A. Configuration of the array

The antenna array under study is based on the classic design proposed by F. Zavosh and J. T. Aberle in [7], where an array consistent of circular patch antennas backed by cylindrical metallic cavities is analyzed. Fig. 1 shows an example of a 2×2 array with the same geometry and dimensions as the one being analyzed in this work. The circular patch antennas are fed by using a coaxial probe from the ground plane. The antennas are separated by via holes located in the quadrangular reticle formed between the antennas. The separation between vias is determined so all the vias in the same line are equidistant from each other. These two criteria establish the location of all the via holes in the array. The number of via holes used is the same in each side of each antenna, and is constant through the whole array. Therefore, the configuration shown in Fig. 1 has 3 via holes in each side of each antenna. Table I details the physical dimensions of the array.

Figure 1. Transversal and longitudinal section of a 2×2 array of circular patch antennas backed by substrate integrated cavities using metallized via holes.

Table I
GEOMETRIC PARAMETERS OF THE ARRAY SHOWN IN FIG. 1

$R1$	$= 2.535cm$	$R2$	$= 3.3312cm$
Dx	$= 8.125cm$	Dy	$= 8.125cm$
a	$= 4.00cm$	h	$= 0.8222cm$
X_c	$= 1.2675cm$	d	$= 0.35cm$
r_o	$= 0.205cm$	r_i	$= 0.065cm$

The coaxial probe is a standard 50Ω SMA connector, whose inner conductor is connected to the circular metallic patch antenna of the top layer, while the outer conductor is connected to the bottom metallic layer of the substrate. The dielectric constant of the material used in the coaxial probe is $\epsilon_r = 1.951$. The height of the substrate, denoted as “h” in Table I, is $0.8222cm$, and its dielectric constant is $\epsilon_r = 2.5$.

The results obtained for the proposed array of circular patch antennas backed by substrate integrated cavities will be compared with its counterpart backed by cylindrical metallic cavities, as proposed in [7]. In Fig. 1 the size and geometry of such metallic cavities is shown. The metallic cavities replace the metallized via holes, but the rest of the geometric and electrical parameters is left untouched.

B. Analysis methodology

The proposed FE/MA methodology is applied to the analysis of this array by dividing it into different subdomains. For the proposed array, there are only three different subdomains: a via hole, a patch antenna on continuous substrate, and a patch antenna backed by metallic cavity. The different subdomains are then analyzed using different types of ports in their geometric frontiers with the hybrid FE/MA method proposed in [1] [2]. The modal expansion of the fields is realized from these ports, characterizing each subdomain by its general scattering matrix (GSM), as explained earlier. We use cylindrical ports (with cylindrical modes) for the ports of the antennas and via holes located in the continuous substrate, and spherical ports (with spherical modes) are used in the radiation region of each antenna. For the proposed array, each antenna requires 24 spherical modes and 13 cylindrical modes, while the via holes are described with just 5 cylindrical modes.

The advantage of the FE/MA methodology is the ability to perform an efficient parametric analysis, since the modification of the geometry of one of the subdomains only requires a new analysis of the affected subdomain, as also the recalculation of the analytic coupling between all the affected elements. If instead the position, rotation or number of the elements is modified, only the analytic coupling needs to be re-evaluated, making the methodology ideal for cases where an optimization process is needed. This same reason makes it ideal for a study on the number of elements or their position in a design, such as the analysis performed in this work.

C. Obtained results

For the study of the proposed substrate-integrated cavity-backed antenna array, a 11×11 array of circular patch antennas is analyzed considering different configurations: microstrip antennas backed by cylindrical metallic cavities, microstrip antennas surrounded by 5 and 6 via holes in each side of the antennas, and microstrip antennas on a continuous dielectric substrate without via holes. Fig. 2 shows the obtained module of the reflection coefficient (S_{11} parameter) of the coaxial feeding for the central element of the 11×11 array. Fig. 3 shows for the same central element, at the resonant frequency of 2.03 GHz, the module of the broadside-matched reflection

coefficient (denoted as $|\Gamma|$) versus the scan angle in the $\phi = 0^\circ$ and $\phi = 90^\circ$ planes ('x' and 'y' direction, respectively). This coefficient defines the scan performance, and is defined in (1),

$$\Gamma(\theta, \phi) = \frac{Z_{in}(\theta, \phi) - Z_{in}(0, 0)}{Z_{in}(\theta, \phi) + Z_{in}^*(0, 0)} \quad (1)$$

where $Z_{in}(\theta, \phi)$ is the input impedance of the coaxial feeding probe. This impedance is calculated from the reflection coefficient when the array is uniformly fed with a relative phase shift to obtain a scan angle pointed towards (θ, ϕ) (active reflection coefficient). Finally, Fig. 4 shows, for the same central element at the resonant frequency of 2.03 GHz, the active element gain diagram in the $\phi = 0^\circ$ and $\phi = 90^\circ$ planes, taking into account the coupling with the rest of the antennas. The simulated gain diagram for the $\phi = 90^\circ$ plane is symmetrical with respect to $\theta = 0^\circ$, and therefore, Fig. 4b) only shows the variation between $\theta = 0^\circ$ and $\theta = 90^\circ$.

Figure 2. $|S_{11}|$ of the center element of the 11x11 array.

The obtained results indicate that the performance derived from using 5 via holes is similar to that achieved with the proposed cylindrical metallic cavities. Furthermore, it can be considered that the cavity formed from more than 5 via holes on each side is completely equivalent to the proposed cylindrical metallic cavity. And it can also be seen how the utilization of no via holes to suppress the surface wave worsens the sweep performance of the array, as it was expected. The SIW design rules established in [4] determine that, for the selected via hole diameter, we should have at least 12 via holes for each size of each antenna to correctly form a SIW cavity. Therefore, it can be considered that there is no need to apply the restricting design criteria used in the SIW technology to implement substrate integrated cavities in a phased array.

III. GROOVE GAP WAVEGUIDES

Groove gap waveguides consist on two parallel metal plates, where one of them is textured with a periodic metamaterial based structure. The GGW are proposed as an alternative to the traditional hollow waveguides for high frequencies, where the wave propagation is controlled by realizing a high surface impedance (ideally a perfect magnetic conductor) in the textured surface [8]. In this section, we will analyze with the proposed FE/MA methodology a small GGW, whose

Figure 3. Broadside-matched reflection coefficient versus scan angle for the central element of the 11x11 array. a) Plane $\phi = 0^\circ$, b) Plane $\phi = 90^\circ$.

Figure 4. Active element gain diagram of the central element of the 11x11 array. a) Plane $\phi = 0^\circ$, b) Plane $\phi = 90^\circ$.

geometrical characteristics are shown in Fig. 5. The design parameters, obtained from [9], are summarized in Table II. The used metallic posts are quadrangular, and the distance between them is constant in all directions.

Figure 5. Transversal section of analyzed GGW with the geometric parameters.

Table II
GEOMETRIC PARAMETERS OF THE GGW SHOWN IN FIG. 5

w	$= 2.54mm$	h	$= 0.81mm$
a	$= 0.19mm$	d	$= 0.30mm$
p	$= 0.90mm$		

The quadrangular posts are analyzed with a cylindrical port using 63 radial modes. The same GGW is also analyzed with the commercial full-wave software CST [10]. The obtained results, shown in Fig. 6, exhibit a good resemblance to the results calculated by CST with a displacement in frequency of about 100 MHz in the minimum.

Figure 6. Comparison of the S-Parameters of the considered GGW, analyzed with the proposed FE/MA methodology and CST.

IV. RADIAL SLOT LINE ANTENNAS

Radial slot line antennas consist of several slots placed on the upper plate of a radial waveguide, arrayed on the aperture so that they can couple with the flowing radial currents and produce circular polarization. Their interest resides in their high-gain, high-efficiency and mass-producible planar phased array antennas for microwave and millimeter-wave applications. Although the simplest desing involves using a spiral distribution of slot pairs, as proposed in [11], the use of concentric circumferences of slot pairs has also been proven successful in the literature [12].

Since the proposed methodology employs cylindrical modes to characterize the fields in a radial port of the subdomains,

we suggest its use as a natural way to analyze the fields in a substrate enclosed in a cylindrical cavity. Therefore, large radial slot line antennas with numerous radiating slot pairs can be analyzed by combining radial ports with the hemispherical ports used for the radiation region, taking advantage of the capabilities of our methodology. The subdomain division would reduce the simulation time, e.g. if all the radiating slot pairs in a radial slot line antenna are equal, we can just analyze one with the FE/MA method, and obtain the GSM for all the slot pairs, while the position and number of the different elements can be easily optimized using the proposed methodology.

There is only one restriction: the minimum circumference that contains a subdomain must not cross any other minimum circumference. The minimum circumference is defined as the circumference with the smallest radius that defines a cylinder (if the region is analyzed with a cylindrical port) or a sphere (if a hemispherical port is used) that contains all the discontinuities in the substrate in the analyzed subdomain. For radiating slots placed on the upper layer of a substrate, these two minimum circumferences are usually equal. In Fig. 7, a top view of a simplified spiral radial slot line antenna is shown, with cylindrical and hemispherical ports (dotted circumferences) located on some slot pairs. The minimum circumference is clearly small enough to not cross any other, making the hypothetical design analyzable with the proposed methodology.

Figure 7. Top view of a simplified spiral radial slot line antenna with cylindrical and hemispherical ports (dotted circumferences) on some slot pairs.

It must be noted that a large enough separation between the radiating slots can be achieved in order to prevent any minimum circumferences to cross each other. Therefore, the proposed methodology could be able to significantly speed-up

the design process of the radial slot line antennas, making it an attractive tool to be used.

V. CONCLUSIONS

A methodology based on the hybrid FE/MA method, and the application of the addition theorems for and spherical cylindrical waves, was previously introduced to analyze arrays fed by SIW in [6]. Its use has been extended in this work to the analysis of the location of via holes between patch antennas on a continuous substrate to suppress mutual coupling through the substrate, obtaining a response similar to a metallic cavity with less via holes than a SIW cavity. The viability of the methodology for the analysis of the radial slot line antennas has also been theorized, making it a valid candidate due to its use of cylindrical modes, which are able to naturally simulate the fields in a radial waveguide. The application of the methodology to the analysis of GW has also been demonstrated, increasing the possible applications of the methodology

ACKNOWLEDGMENT

This work has been funded by the Ministry of Economy and Business of Spain (MINECO), the State Research Agency of Spain (AEI) and the FEDER program (European Union) under projects TEC2017-83352-C2-2-P and TEC2017-83352-C2-1-P; by the Junta of Extremadura and the FEDER program (EU) under the project GR18055; and by the Ministry of Education, Culture and Sport of Spain (MECD) via a PhD grant conceded to Alfonso Gómez García (FPU17/00825).

REFERENCES

- [1] J. Rubio, J. Arroyo, and J. Zapata, "Analysis of passive microwave circuits by using a hybrid 2-D and 3-D finite-element mode-matching method," *IEEE Trans. Microwave Theory Tech.*, vol. 47, no. 9, pp. 1746–1749, Sep. 1999.
- [2] J. Rubio, J. Arroyo, and J. Zapata, "SFELP-an efficient methodology for microwave circuit analysis," *IEEE Trans. Microwave Theory Tech.*, vol. 49, no. 3, pp. 509–516, Mar. 2001.
- [3] J. Rubio, M. A. González, and J. Zapata, "Generalized-scattering-matrix analysis of a class of finite arrays of coupled antennas by using 3-D FEM and spherical mode expansion," *IEEE Trans. Antennas Propag.*, vol. 53, no. 3, pp. 1133–1144, Mar. 2005.
- [4] Ke Wu, D. Deslandes and Y. Cassivi, "The substrate integrated circuits - a new concept for high-frequency electronics and optoelectronics," 6th International Conference on Telecommunications in Modern Satellite, Cable and Broadcasting Service, 2003. TELSIKS 2003., Nis, Yugoslavia, 2003, pp. P-III..
- [5] J. Rubio, A. Gómez-García, R. Gómez-Alcalá, Y. Campos-Roca, J. Zapata, "Overall formulation for multilayer SIW circuits based on addition theorems and the generalized scattering matrix," *IEEE Microw. Wireless Comp. Lett.*, vol. 28, no. 6, pp. 485–487, June 2018.
- [6] J. Rubio, A. Gómez-García, R. Gómez-Alcalá, J. García, Y. Campos-Roca, J.M. Gil, M.A. González, "Fast analysis of arrays of antennas fed by substrate integrated waveguides", *Eur. Conf. Antennas Propag.* (EuCAP'19), April. 2019.
- [7] F. Zavosh and J. T. Aberle, "Infinite phased arrays of cavity-backed patches," in *IEEE Transactions on Antennas and Propagation*, vol. 42, no. 3, pp. 390–398, March 1994.
- [8] E. Rajo-Iglesias and P. Kildal, "Groove gap waveguide: A rectangular waveguide between contactless metal plates enabled by parallel-plate cut-off," *Proceedings of the Fourth European Conference on Antennas and Propagation*, Barcelona, 2010, pp. 1–4.
- [9] Aljarosha, Alhassan. Millimeter-wave microstrip-to-groove gap waveguide transition for use in gap-waveguide-integrated grid amplifiers and antenna arrays. Diss. Master's thesis, Chalmers University of Technology, Sweden, 2016.
- [10] CST Suite 2019, CST AG, Darmstadt, Germany, 2019.
- [11] M. Ando, K. Sakurai, N. Goto, K. Arimura and Y. Ito, "A radial line slot antenna for 12 GHz satellite TV reception," in *IEEE Transactions on Antennas and Propagation*, vol. 33, no. 12, pp. 1347–1353, December 1985.
- [12] M. Sierra-Castaner, M. Sierra-Perez, M. Vera-Isasa and J. L. Fernandez-Jambrina, "Low-cost monopulse radial line slot antenna," in *IEEE Transactions on Antennas and Propagation*, vol. 51, no. 2, pp. 256–263, Feb. 2003.