

Abstract

Exploring the moons of Saturn, particularly Enceladus, is an important goal of planetary science, the more so for search of life outside the Earth. The use of Electrodynamic Tethers (ET) in Jupiter suggests an interesting question: would it be possible to use them for the other three Outer Giant Planets, in particular, Saturn? This work explores the design of the manoeuvres associated with the capture of a tethered S/C in a mission to Saturn.

1. Introduction

WHEREAS use of *Electrodynamic Tethers* is readily possible for Jupiter [1], the case for the other three Outer Giant Planets, Saturn, Uranus and Neptune, presents issues, basically because their magnetic self-field B is grossly weaker. The efficiency of spacecraft capture (S/C-to-tether mass ratio) goes down as B^2 for low enough field. But such low value of B and the ensuing weak Lorentz drag provide a possible gain by a factor of 2 in the efficiency, because no tether spin is required, as opposite the Jovian case, to keep it from bowing.

From the orbital point of view, capture is an essential issue for a tether mission to Saturn, because there will be only one opportunity for success. Once the tethered spacecraft is captured by the gravity of Saturn, a space tether could carry a spacecraft through the neighborhood resorting to neither propellant nor power supply [3]. The basic scenario for the capture is described in [2] for the case of Jupiter; such analysis must be adapted to Saturn, but the corresponding adjustment is not trivial.

In the present work we show how an appropriate design of the trajectory leads to an increase of the feasibility of the capture process. In a first approximation, we face the capture operation with the tether switched off. Thus, the resulting analysis is on the side of safety. Indeed, the effects of the electrodynamic drag associated with the ET provides a plus in the authority control of the capture manoeuvre.

In the near future, however, we will try to increase the efficiency of the ET. First, moving from standard tether use of aluminium to beryllium, in case it could be made ductile enough. This way the ratio between electrical conductivity and density increases by a factor of 1.8, an important parameter for efficiency. Additionally, ensuring that the length-averaged current in the tether is close to its short-circuit, upper-bound value, by choosing it long and thin enough; this raises no issues as in the Jovian case [1], again because of the very low Saturn field B .

This analysis mainly intends to show the feasibility of the mission. Therefore we will introduce some simplifying assumptions. For example, it is assumed that the problem is planar and planets follow circular, coplanar and concentric orbits around the Sun.

2. Hohmann transfer

A Hohmann transfer is, at first sight, the simplest way to reach Saturn. Considering the usual values of the gravitational parameter for the Sun and the semi-major axis of Earth and Saturn, the main results from such a transfer are:

- the time of flight $T_f = 6.05$ years
- the velocity relative to Saturn at the encounter $v_\infty = 5.44$ km/s
- the impulse at launch time $\Delta v_0 = 10.29$ km/s

Capture requires to reduce the relative velocity of the S/C. Tether drag can be used to perform such a reduction. This is a real challenge due to the smallness of Saturn's magnetic field B . Thus the Hohmann transfer is not the best option. The heliocentric velocity of the S/C at the encounter with Saturn can be increased with a previous flyby at Jupiter as the next section shows.

3. Flyby around Jupiter

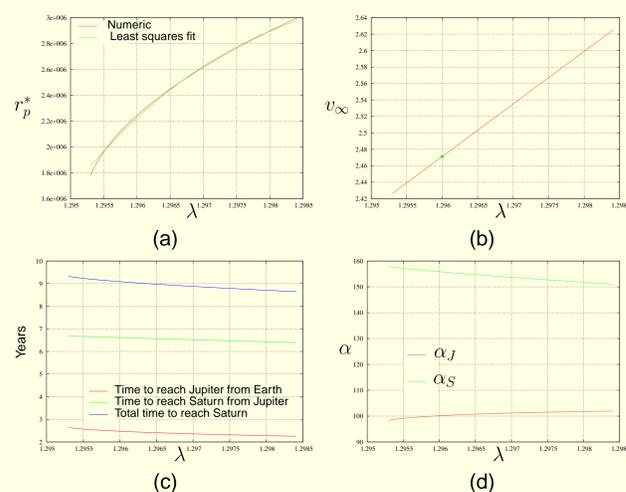


Figure 1: Tail flyby around Jupiter: (a) Perijove distance r_p^* vs. λ ; (b) Relative velocity at the encounter with Saturn; (c) Times spent on each leg of the Saturn approach; (d) Values, in degrees, of the angles α_J and α_S of Jupiter and Saturn at the launch time.

The idea is to launch the S/C tangent to the Earth orbit with a velocity v_0 whose magnitude is:

$$v_0 = \lambda \sqrt{\frac{\mu_\odot}{a_E}} \quad a_E = 1 \text{ AU}$$

where $\mu_\odot \approx 3.964 \cdot 10^{14} \text{ AU}^3/\text{s}^2$ is the gravitational parameter of the Sun and λ a dimensionless free parameter. At Jupiter's orbit a tail

flyby takes place governed by the perijove distance r_p . In an usual analysis —neglecting the radius of Jupiter's sphere of influence— the distance of the aphelion of the post-flyby orbit turns out to be a function of (λ, r_p) :

$$r_{ap} = r_{ap}(\lambda, r_p)$$

Let a_S be the orbital radius of Saturn. The condition $r_{ap} = a_S$, provides a relation between λ and r_p that assures that the S/C meets the orbit of Saturn with the minimum relative velocity (tangent to the orbit). Such a relation is plotted in figure fig. 1a for values of λ in the interval: $[1.29530, 1.29841]$

The results of this parametric analysis are summarized in fig. 1. The following consequences should be underlined:

- the time of flight increases; is of the order of $T_f = 9$ years
- the relative velocity at the encounter decreases; $v_\infty \approx 2.5$ km/s
- the impulse at launch time decreases; $\Delta v_0 \approx 8.8$ km/s

The important reduction in the velocity relative to Saturn—from 5.44 to 2.5 km/s—is worth the increase of the total flight time. Fig. 2 shows a typical trajectory designed in this way.

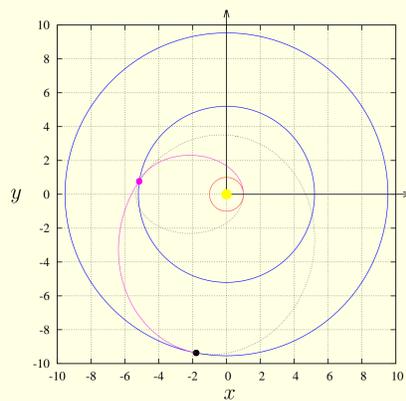


Figure 2: Global trajectory

4. Launch windows

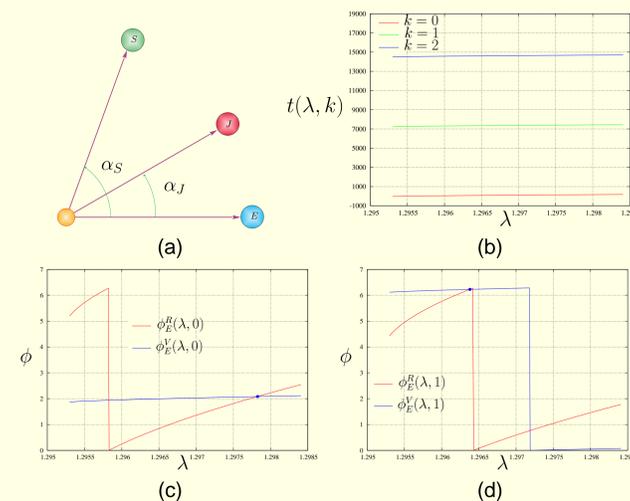


Figure 3: Launch windows: (a) angles relative to Earth; (b) Critical time (days) vs λ ; (c) Punctual launch windows for $k = 0$; (d) Punctual launch windows for $k = 1$.

Let α_J and α_S be the angles of the Jupiter and Saturn relative to Earth (see fig. 3a). Let ϕ be the celestial longitude of a planet. In this simple model their time evolution is linear:

$$\phi_E \approx 5.7060 + \omega_E t, \quad \omega_E \approx 1.99 \cdot 10^{-7} \text{ rad/s} \quad (1)$$

$$\phi_J \approx 3.6328 + \omega_J t, \quad \omega_J \approx 1.68 \cdot 10^{-8} \text{ rad/s} \quad (2)$$

$$\phi_S \approx 4.6466 + \omega_S t, \quad \omega_S \approx 6.76 \cdot 10^{-9} \text{ rad/s} \quad (3)$$

where ω is the angular frequency of the planet. Here, t is the time (seconds) with origin in 20/08/2017 at 00:00:00 UTC.

For a fixed values of λ , there is a virtual manoeuvre reaching Saturn. At launch time the angles of Jupiter and Saturn are functions of λ : $\alpha_J(\lambda)$ and $\alpha_S(\lambda)$. An opportunity to launch requires that the following relation is met:

$$\alpha_J(\lambda) - \alpha_S(\lambda) + 2k\pi = \phi_J(t) - \phi_S(t) = -1.0138 + (\omega_J - \omega_S)t$$

where the difference $\phi_J(t) - \phi_S(t)$ is taken from relations (2,3). This equation provides the critical time in which the relative positions of Jupiter and Saturn are appropriate for the launch:

$$t(\lambda, k) = \frac{\alpha_J(\lambda) - \alpha_S(\lambda) + 1.0138}{\omega_J - \omega_S} + k T_{JS} \quad (4)$$

The difference $\omega_J - \omega_S = 1.0016 \cdot 10^{-8} \text{ rad/s}$ provides the synodic period of the relative rotation between Jupiter and Saturn:

$$T_{JS} = \frac{2\pi}{\omega_J - \omega_S} \approx 6.273 \cdot 10^8 \text{ s}$$

therefore the relative positions Jupiter/Saturn are repeated each $T_{JS} \approx 7260.52$ days (19.88 years).

Fig. 3b shows the critical time $t(\lambda, k)$ given by (4) for three cases: $k = 0, 1, 2$. By fixing λ the critical time becomes fixed and Jupiter and Saturn have the relative positions appropriate for launch. This is not the case of Earth; in general, its virtual position:

$$\phi_E^V(\lambda, k) = \phi_J(t(\lambda, k)) - \alpha_J(\lambda)$$

is different from its actual position: $\phi_E^R(\lambda, k) = 5.7060 + \omega_E t(\lambda, k)$. A launch opportunity appears when

$$\phi_E^V(\lambda, k) = \phi_E^R(\lambda, k)$$

Fig. 3c and d show the virtual and actual position of the Earth for $k = 0$ and $k = 1$ respectively. Two launch opportunities appear in this analysis, corresponding to the following dates: **21/01/2018 and 19/09/2037**. Much more opportunities can be found in their neighborhood by introducing a deep space manoeuvre which requires a small Δv .

5. Flyby around Titan and capture

The worst-case scenario in which the capture at Saturn has to be accomplished without the aid of the tether, is considered here. An appropriate flyby at Titan guarantees the capture. Moreover under suitable circumstances the periape of the post-flyby orbit is lower than the synchronous orbit (389 190 km radius), which is a requirement for the ET to deorbit the S/C. Let $r_T \approx 1\,221\,865$ km be the orbital radius of Titan and $\mu_S = 3.7922 \cdot 10^7 \text{ km}^3/\text{s}^2$ the gravitational parameter of Saturn. The flyby at Titan is searched through a loop over three parameters:

- the relative velocity $v_\infty \in [2.42, 2.64]$ at the encounter with Saturn (see fig. 1b)
- the eccentricity $e \in [1.01, e_{max}]$ of the hyperbola relative to Saturn. Here $e_{max} = r_T v_\infty^2 / \mu_S + 1$ is the maximum allowed value for an encounter with Titan to occur
- the pericenter altitude $h_T \in [1000, 15000]$ km of the flyby at Titan.

The minimum flyby altitude achieved by the Cassini S/C was 1000 km. Below this height, the atmosphere causes braking effects on the trajectory [4] which cannot be neglected in the analysis.

The hypothesis is made that when the S/C crosses the orbit of Titan, an encounter occurs. The parameters of the post-flyby orbit relative to Saturn are computed and the capture is accomplished if the new two-body energy is negative, i.e. the post-flyby trajectory is elliptical, and the orbital period does not exceed a reasonable maximum value (set to 250 days in our numerical experiments) for the resulting orbit to be useful.

Fig. 4a shows an example: the entry speed relative to Saturn is $v_\infty = 2.42$ km/s, its hyperbole eccentricity $e = 1.15$ and the pericenter altitude $h_T = 1000$ km. In this case the pericenter radius of the post-flyby ellipse is of 824 738 km and its orbital period is of 228 days. Note that within the above given limits for the parameters, no post-flyby ellipse is possible with a pericenter below the synchronous orbit.

As an alternative strategy, we considered only the encounters with Titan which result in capture ellipses *resonant with the moon*. In this way the S/C performs a second flyby with Titan. The pericenter radius of the corresponding hyperbola is made to vary between the above defined limits. Only the post-flyby ellipses whose closest approach with Saturn takes place below the synchronous orbit and whose orbital period is shorter than 100 days are retained. Fig. 4b shows one example of this strategy: the arrival hyperbola with respect to Saturn has an asymptotic entry speed of 2.43 km/s, its eccentricity is 1.09 and the pericenter altitude h_T is of 1200 km and 1000 km, respectively at the first and second Titan flyby. The pericenter radius of the first post-flyby ellipse is just below 17 million km and its period is of 813 days, whereas the second post-flyby ellipse reaches a distance of 339 047 km from the center of Saturn and its orbital period is of 79 days. This strategy permits to switch on the tether for the second pass. The capture manoeuvre would benefit from the thrust and power provided by the ET inside the synchronous orbit. The control of the electrodynamic drag would make the navigation easier. More strategies will be analyzed in the future.

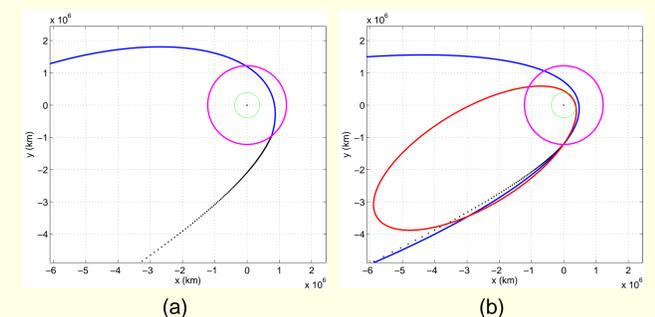


Figure 4: Some examples: arrival hyperbola relative to Saturn in black, post-flyby ellipse(s) in blue (and red), Titan's orbit in purple, synchronous orbit in green

References

- [1] Juan R. Sanmartín, Mario Charro, Henry B. Garrett, Gonzalo Sánchez-Arriaga, and Antonio Sánchez-Torres. Analysis of tether-mission concept for multiple flybys of moon Europa. *Journal of Propulsion and Power*, 33:338–342, 2017.
- [2] Juan R. Sanmartín, Mario Charro, Enrico C. Lorenzini, Henry Garrett, Claudio Bombardelli, and Cristina Bramanti. Electrodynamic tether – I: Capture operation and constraints. *IEEE Transactions on Plasma Science*, 36(5):2450–2458, 2008.
- [3] Juan R. Sanmartín and Enrico C. Lorenzini. Exploration of outer planets using tethers for power and propulsion. *Journal of Propulsion and Power*, 21(3):573–576, 2005.
- [4] EP Turtle, J. Barnes, B.J. Buratti, G. Collins, S. Fussner, R. Lopes, R.D. Lorenz, J.I. Lunine, T.B. Mccord, A.S. McEwen, et al. Exploring the surface of titan with cassini-huygens. In *AGU Fall Meeting Abstracts*, 2005.