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**SEISMIC RESPONSE OF STRUCTURES EQUIPPED
WITH PASSIVE ENERGY DISSIPATION SYSTEMS
USING SIMPLIFIED METHODS BASED ON
EQUIVALENT EFFECTIVE PROPERTIES**

*Respuesta sísmica de estructuras con sistemas pasivos de disipación de energía mediante
métodos simplificados basados en el uso de propiedades efectivas equivalentes*

Tesis Doctoral

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ABSTRACT

Analysis of structures with added viscous dampers can be accomplished through methods that range from simplified approaches based on modal analysis to non-linear time-domain step-by-step analysis. The results of the latter typically exhibit a strong dependency on selection, number and scaling of records, require a detailed definition of the system, involve a significant amount of time and computer resources, and offer small insight on the sensitivity of the system behavior to the different variables involved. Therefore, the use of supplementary simpler analysis procedures, at least in the preliminary stages of design, is desirable.

A widely used simplified method is the Non Linear Response Spectrum Analysis procedure (hereinafter referred to as RSA), developed at the end of the XXth century, which relies on two assumptions: i) The shear-displacement relationship obtained by non-linear static analysis under a suitable pattern of lateral loads (pushover curve) is representative of the same relationship under earthquake action; ii) non-linear behavior due to viscous damping and hysteresis can be properly approximated by linear models as long as basic dynamic properties (period and damping) are substituted by equivalent effective values. The method is implemented in North American codes (FEMA P-1051-2015, ASCE/SEI 7-16), for displacement-dependent (hysteretic and friction) and velocity-dependent (viscous and viscoelastic) devices, and it is being considered for inclusion in the new version of Eurocode 8, for structures equipped with velocity-dependent devices. This thesis focuses on the use of RSA to predict the response of systems equipped with viscous dampers.

The method relies on classical modal analysis, where application of modal principles allows the motion equations to become uncoupled through diagonalization of the mass, damping and stiffness matrices by modal vectors. When viscous damping systems are included in an elastic frame, vibration shapes become complex and off-diagonal terms appear in the modal damping matrix. Different non-proportionality indices have been proposed to measure this phenomenon. Through a numerical study it is shown that four existing indices are strongly correlated with the maximum error expected in the procedure for kinematic variables. Limit values of these indices are suggested as a practical validation criterion for applicability of the method.

Through a study based on European accelerograms, spectral correction factors are derived, intended for their use with the Eurocode 8 spectrum. The proposed factors are based on the damping correction factor included in the current version of Eurocode 8, with modifications that take into account the influence of period, ground type and degree of seismicity associated with spectral shape.

It is usual in analysis of structures equipped with viscous dampers to neglect the stiffness of auxiliary elements that form the dissipation system (brace extenders, auxiliary mounting elements and damper itself). In this work it is shown that this stiffness is relevant for the

seismic response of the building, particularly in its top half. The influence is shown to be larger for high damping levels, non-linear damping, and moderate intensities. The influence for a set of accelerograms is softer than the influence for a single accelerogram. The influence is small for displacements and damper forces, larger for drift and drift velocity and considerable for forces in structural elements.

To account for this influence, a numerical approximation to the Maxwell model subjected to forced harmonic oscillation is developed, including the flexibility and viscosity components of the damping system, whose finite stiffness results in decay of energy dissipation efficiency (compared to the pure viscous case). This non-linear viscoelastic model is then included in the RSA method. The added stiffness of the damping system entails a modification of the vibrational properties of the system. Two iterative procedures are proposed to include this effect in a consistent (RSAB) or a simple (RSABS) way.

The predictions obtained with the three methods (RSA, RSAB, RSABS) are compared with results from direct dynamic analyses (TH) for a set of steel frames with different distributions of viscous dampers. The proposed modifications are shown to improve the response accuracy and reduce its variability. In addition, two new expressions are developed for the calculations of static variables, which improve substantially the total shear and damper shear prediction. It is also shown that imposition of a minimum elastic displacement does not improve the accuracy of the response, whereas the CQC combination rule improves the accuracy when compared to the SRSS rule.

Results of the numerical study prove that, when used to predict the response to seismic action defined by a specific motion or set of motions, the accuracy of the methods is poor for verification purposes, but acceptable for preliminary design. When the methods are used to predict the response to seismic action defined by a soft spectrum (such as the Eurocode 8 spectrum), use of appropriate spectral correction factors is shown to be of extreme importance. If suitable spectral correction factors are used (such as the set derived in this study), the accuracy of the methods improves largely and is enough both for verification purposes and preliminary design.

RESUMEN

El análisis de estructuras equipadas con disipadores viscosos puede realizarse con diversos métodos, que abarcan desde planteamientos simplificados basados en análisis modal hasta análisis dinámico no lineal con varios niveles de sofisticación. En este último caso los resultados presentan una fuerte dependencia respecto al número, tipo y escalado de acelerogramas seleccionados, se requiere una definición detallada del sistema analizado, los cálculos consumen una cantidad de tiempo y recursos relativamente elevada y los resultados son poco transparentes respecto a la sensibilidad del sistema ante distintos parámetros. Por ello, el empleo de métodos simplificados, al menos en las fases previas de proyecto, es deseable.

Un método ampliamente empleado es el análisis modal espectral no lineal (RSA), desarrollado a finales del siglo XX, que se basa en dos hipótesis: i) la relación cortante-desplazamiento obtenida de un análisis estático no lineal bajo un patrón de carga lateral (curva pushover) es representativa de la misma relación bajo la acción sísmica; ii) el comportamiento no lineal (debido al amortiguamiento viscoso y a la histéresis estructural) puede aproximarse mediante un modelo lineal con propiedades dinámicas efectivas. Éste es el método implementado en códigos norteamericanos (FEMA P-1051-2015, ASCE/SEI 7-16) para disipadores dependientes del desplazamiento (histeréticos y de fricción), y la velocidad (viscosos y viscoelásticos), y actualmente está siendo considerado para su inclusión en la nueva versión del Eurocódigo 8 para estructuras equipadas con disipadores dependientes de la velocidad. Esta tesis aborda el empleo de RSA para predecir la respuesta sísmica de estructuras equipadas con disipadores viscosos.

El método emplea análisis modal clásico, en el que las ecuaciones del movimiento se desacoplan a través de la diagonalización de las matrices de masa, amortiguamiento y rigidez por los vectores modales. Al añadir amortiguadores viscosos, las componentes de los vectores modales se transforman en números complejos y la matriz de amortiguamiento modal deja de ser diagonal. Este fenómeno puede medirse a través de diferentes índices de no-proporcionalidad. Mediante un estudio paramétrico se demuestra que los valores de cuatro índices existentes en la literatura están correlacionados con el máximo error de las variables cinemáticas obtenido con el método modal. Se sugieren valores límite de dichos índices como criterio práctico de aplicabilidad del método.

A través de un estudio basado en acelerogramas europeos se derivan factores de corrección espectral específicos para su uso con el espectro del Eurocódigo 8. Los factores propuestos están basados en el factor de corrección por amortiguamiento de la versión actual del Eurocódigo 8, con modificaciones que tienen en cuenta la influencia del período, terreno y grado de sismicidad asociado al tipo de espectro.

En el análisis de estructuras equipadas con disipadores viscosos es habitual ignorar la influencia de la rigidez de los elementos que conforman el sistema disipador primario (diagonales, elementos auxiliares de montaje y el propio disipador). En este trabajo se

demuestra que la rigidez del sistema de disipación es relevante para la respuesta sísmica del edificio, particularmente en la parte superior del mismo; la influencia es mayor para niveles elevados de amortiguamiento, disipadores no-lineales e intensidades moderadas. La influencia para un conjunto de acelerogramas es más tenue que para un solo acelerograma. La influencia es pequeña para desplazamientos y esfuerzos en disipadores, mayor para desplomes y velocidades, y muy relevante para esfuerzos estructurales.

Para tener en cuenta esta influencia, se propone una aproximación numérica al modelo de Maxwell bajo oscilación armónica forzada, que incluye la flexibilidad y viscosidad del sistema de disipación cuya limitada rigidez conlleva una pérdida de su eficiencia en términos de disipación de energía. Este modelo viscoelástico no lineal se introduce en el método RSA. La inclusión de la rigidez del sistema de disipación conlleva una modificación en las propiedades vibracionales del sistema. Se proponen dos procedimientos iterativos que tienen en cuenta este efecto, de manera consistente (RSAB) o simplificada (RSABS).

Las predicciones obtenidas con los tres métodos (RSA, RSAB, RSABS) se comparan con los resultados de análisis dinámicos directos (TH) para un juego de pórticos de acero con diferentes distribuciones de disipadores viscosos. Se muestra que las modificaciones propuestas mejoran la precisión de la respuesta y reducen su variabilidad. Se proponen además dos expresiones alternativas que mejoran sustancialmente la predicción de fuerzas totales y en disipadores. Se demuestra también que la imposición del desplazamiento elástico mínimo no mejora la predicción de la respuesta, y que el uso de la regla de combinación CQC reduce los errores respecto a la regla SRSS.

Los resultados del estudio demuestran que al emplear estos métodos para predecir la respuesta ante una excitación sísmica concreta la precisión obtenida es insuficiente como método de verificación, pero basta a efectos de predimensionado. Si (como es habitual) los métodos se emplean para predecir la respuesta a una excitación sísmica definida por un espectro suavizado, tal como el espectro del Eurocódigo 8, el empleo de factores de corrección espectral es crucial. Mediante el empleo de un juego adecuado de dichos factores (como el desarrollado en este estudio), la precisión de los métodos mejora considerablemente, siendo suficiente a efectos de predimensionado y verificación.

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NOTATION

Italics denote variables.

Bold denote matrix variables.

Symbols are generally the same for equivalent concepts in Single or Multiple Degrees of Freedom. For instance, k denotes stiffness for a SDOF, while $\mathbf{k}$ denotes the stiffness matrix in a MDOF.

Conventional units used in the text are indicated in parenthesis.

Some symbols used only in a specific Chapter are not included.

ACRONYMS AND ABBREVIATIONS

AD	Spectral Acceleration vs. spectral Displacement format
CQC	Complete Quadratic Combination
CSM	Capacity Spectrum Method
FVD	Fluid Viscous Dampers
MMPA	Modified Modal Pushover Analysis
MPA	Modal Pushover Analysis
N2	N2 Method for static analysis
NLRSA	Non-Linear Response Spectrum Analysis (abbreviated as RSA)
NLTHA	Non-Linear Time-History Analysis (abbreviated as TH)
NSP	Non-Linear Static Procedure
PED	Passive Energy Dissipation
SRSS	Square-Root of the Sum of Squares
RSA	Response Spectrum Analysis
TH	Time-History Analysis
Vu	Shear vs. displacement format

SYMBOLS

a	Acceleration (g, mm/s/s)
α	Damper exponent

a_g	Ground acceleration (g, mm/s/s)
$a_{g,R}$	Mapped rock acceleration (g, mm/s/s)
A	Spectral acceleration (mm/s/s)
A_b	Brace section (mm ²)
c	Viscous damping coefficient (kN·(s/mm) ^a)
c'	Equivalent Kelvin damping (kN·(s/mm) ^a)
c_a	Added damping coefficient
c_b	Viscous damping coefficient referred to damper axis (kN·(s/mm) ^a)
c_d	Viscous damping coefficient referred to storey drift direction (kN·(s/mm) ^a)
c_I	Inherent damping coefficient
$\mathbf{c}$	Viscous damping matrix
$\mathbf{C}$	Modal damping matrix (kN·s/mm)
$\hat{\mathbf{C}}$	Mass normalized modal damping matrix
c_{cr}	Critical viscous damping coefficient (kN·s/mm)
C_s	Seismic response coefficient
CF	Force coefficient (-)
CFV	Correction factor for velocity (-)
$\mathbf{B}$	Compatibility matrix
D	Spectral displacement (mm)
E	Energy (J)
E	Elastic modulus (kN/mm ²)
EE	Energy error (-)
E_d	Energy absorbed by added damping system (J)
E_I	Energy input (J)
E_k	Kinetic Energy (J)
E_s	Energy absorbed by structure (J)
E_v	Energy dissipated by inherent viscous damping (J)

f	Force (kN)
$\mathbf{f}$	Force vector (kN)
f_b	Damper force parallel to its axis (kN)
f_d	Damping force (kN)
f_{d0}	Force at maximum velocity (kN)
f_s	Elastic restoring force (kN)
f_{s0}	Force at maximum displacement (kN)
f_0	Peak force (kN)
f_{mxa}	Force at stage of maximum acceleration (kN)
f_{mxd}	Force at stage of maximum displacement (kN)
f_{mxv}	Force at stage of maximum velocity (kN)
f_y	Yield force (kN)
g	Gravity acceleration (mm/s/s)
h	Height (mm)
H	Total height (mm)
I	Non-proportionality index
j	Index for modes
J	Total number of modes
k	Stiffness (kN/mm)
K'	Equivalent Kelvin stiffness (kN/mm)
$\mathbf{k}$	Stiffness matrix (kN/mm)
$\mathbf{K}$	Modal stiffness matrix (kN/mm)
k_a	Added stiffness (kN/mm)
k_b	Damping system stiffness parallel to its axis (kN/mm)
k_{bb}	Brace stiffness (kN/mm)
k_{bd}	Damper stiffness (kN/mm)
k_{bo}	Stiffness of auxiliary components other than damper and brace (kN/mm)

k_d	Brace stiffness referred to storey drift direction (kN/mm).
k_e	Total stiffness (structure + added) (kN/mm)
k_s	Storey stiffness (kN/mm)
k_{sa}	Added storey stiffness
k_{se}	Total storey stiffness (structure + added) (kN/mm)
L_b	Brace length (mm)
H	Equilibrium matrix
L_j	Participating mass (kN·s ² /mm)
m	Mass (kN·s ² /mm)
m	Mass matrix (kN·s ² /mm)
M	Modal mass matrix (kN·s ² /mm)
M_j	j^{th} component of modal mass matrix (kN·s ² /mm)
n	Index for storey
N	Number of storeys
K_j	j^{th} component of modal stiffness matrix (kN/mm)
q	Behaviour factor
q	Maxwell parameter (-)
q_D	Behaviour factor component due to ductility and energy dissipation
q_H	Hysteresis loop factor
q_R	Behaviour factor component due to redistribution
q_S	Behaviour factor component due to overstrength
r	Correlation factor for CQC combination rule
R_μ	Strength reduction factor
S	Code-defined soil factor (-)
PHA	Peak horizontal ground acceleration (mm/s/s)
PHD	Peak horizontal ground displacement (mm)
PHV	Peak horizontal ground velocity (mm/s)

P_{NCR}	Probability of exceedance
PS_a	Spectral pseudo-acceleration (mm/s/s)
PS_v	Spectral pseudo-velocity (mm/s)
S_a	Spectral absolute acceleration (mm/s/s)
S_d	Spectral relative displacement (mm)
S_v	Spectral relative velocity (mm/s)
t	Time (s)
T	Period (s)
T_e	Period of structure + added damping (s)
T_B, T_C, T_D, T_E, T_F	Code-defined transition periods (s)
T_{eff}	Effective period (s)
$\mathbf{T}_L$	Lower triangular matrix of ones
T_R	Return period (yr)
$\mathbf{T}_U$	Upper triangular matrix of ones
u	Displacement (mm)
u_b	Damper stroke parallel to its mounting axis (mm)
u_{f0}	Displacement at maximum force (mm)
u_0	Maximum displacement (mm)
u_m	Maximum displacement (mm)
u_p	Displacement at performance point (mm)
V	Shear force (kN)
V_1	Base shear (kN)
V_d	Damper shear (kN)
V_{mxa}	Shear at maximum acceleration (kN)
V_{mxd}	Shear at maximum displacement (kN)
V_{mxv}	Shear at maximum velocity (kN)
V_p	Shear at performance point (kN)

V_s	Storey shear (kN)
V_t	Total shear (kN)
w_n	Weight of storey n (kN)
W	Weight (kN)
W	Work (J)
W_e	External work (J)
W_i	Internal work (J)
$W_{eff,j}$	Effective weight for mode j
W_d	Work dissipated by damping (J)
W_s	Strain energy stored at maximum displacement (J).
W_{tot}	Total weight (kN)
$\mathbf{y}$	Modal coordinate vector
z	Height relative to frame base (mm)

GREEK LETTERS

α	Post-yield stiffness ratio
β_d	Relative peak force at maximum velocity
β_f	Relative peak maximum force
β_s	Relative peak force at maximum displacement
β_u	Relative displacement at maximum force
β_w	Energy dissipation efficiency
γ	Safety factor
Γ	Euler gamma function
Γ_1	Modal participation factor
Γ_e	Modal participation factor of structure + added damping system
δ	Phase angle
δ_n	Interstorey drift ratio for storey n

Δ	Storey drift (mm)
η	Damping correction factor
η_a	Damping correction factor for acceleration
η_d	Damping correction factor for displacement
η_v	Damping correction factor for velocity
θ	Mounting angle of damping system (rad)
$\mathbf{I}$	Influence vector
κ	Modal stiffness (kN/mm)
κ_a	Modal added stiffness (kN/mm)
κ_e	Total modal stiffness (structure + added damping) (kN/mm)
κ_f	Frame modal stiffness at frequency of interest (kN/mm)
λ	Relaxation time
$\boldsymbol{\lambda}$	Force pattern vector
μ	Ductility
μ_T	Period-based ductility
ξ	Viscous damping ratio
ξ_a	Added viscous damping ratio
ξ_e	Viscous damping ratio of equivalent system
ξ_{eff}	Effective viscous damping ratio
ξ_h	Hysteretic viscous damping ratio
ξ_I	Inherent viscous damping ratio
$\xi_{a,eff}$	Effective added viscous damping ratio
$\phi_{n,j}$	n -th component of modal vector for mode j -th
$\boldsymbol{\Phi}_j$	Modal vector for mode j -th
$\boldsymbol{\Phi}_{e,j}$	Modal vector for mode j -th of structure + added damping system
$\boldsymbol{\Phi}$	Modal matrix

$\hat{\Phi}$	Mass normal modal matrix
$\psi_{n,j}$	n -th component of modal drift vector for mode j -th
$\Psi_{e,j}$	Modal drift vector for mode j -th of structure + added damping system
Ψ_j	Modal drift vector for mode j -th
Ψ	Modal drift matrix
ω	Circular frequency (s^{-1})
ω_j	Circular frequency of mode j (s^{-1})
Ω^2	Spectral matrix

Chapter 1

Introduction

1.1 Motivation

Earthquake engineering is barely a century old, and has passed through different stages. At the beginning of the XXth century, the 1908 Messina-Reggio Earthquake motivated the Italian engineer Arturo Danusso to propose the force-based equivalent static approach, implemented in the Japanese Building Ordinance after the 1923 Kanto/Tokyo earthquake [Housner, 1984]. During this period, earthquake resistance was based on elastic analysis under unreduced lateral forces based on fixed fractions of structural weight. The 1960 World Conference held in Japan produced a shift of emphasis from strength to ductility, motivated by the increasing interest in engineering plasticity at the time. It was realized that structures with reduced strength could withstand large earthquake events, as long as they had the ability to sustain large deformations. The lateral forces used in earthquake analysis were reduced accordingly, and emphasis was placed on design criteria to warrant the required ductility levels.

However, cyclic deformation beyond the elastic limit implies low-cycle fatigue and large drift demands may result in severe damage to structural and non-structural elements. The 1994 Northridge and 1995 Kobe earthquakes, with their enormous economic losses, suggested that enhanced seismic performance, beyond mere safety criteria, was needed, particularly in critical and expensive facilities. This realization produced a new shift in earthquake engineering: in the search of improved seismic resilience with a compromise between economy and safety, the natural solution was a probabilistic approach, in which different levels of performance were required for different events associated to different probabilities of occurrence [Reitherman, 2015]. Relative inter-storey drift appeared then as a logical performance indicator, because of its strong correlation with observed damage. Drift control became an essential part of earthquake engineering. This in turn produced a major conundrum: drift reduction required increase of stiffness, which in turn implied increase of structural strength, leading to increased forces and uneconomical structures.

Added viscous damping offers a reasonable solution to this problem, with its beneficial effect on the dynamic performance of conventional structures: an increase in damping results in a reduction of drift, resulting in smaller internal forces associated with the dynamic excitation [Chopra, 2007; Clough & Penzien, 1995]. From the point of view of energy balance, added damping reduces the demand on the structure by assuming a relevant part of the energy dissipation. Hysteretic mechanisms are greatly reduced and

therefore structural damage is diminished or even completely avoided [Christopoulos & Filiatrault, 2006].

Viscous dampers have been widely used in building structures, mostly in U.S.A. and Japan. Their use in buildings throughout Europe has been sparse. This may be partly motivated by the more moderate earthquake requirements, but also by the lack of a simple codified procedure for their verification, such as the RSA method proposed in ASCE-SEI 7/16 [ASCE, 2016]. The new Eurocode 8, currently being drafted, will contain specific provisions for structures equipped with supplemental energy devices, including RSA as a simplified method to verify structures with velocity-dependent devices [Labbé, 2018]. In addition, the new version of the standard EN 15129 [CEN, 2018] includes detailed specifications that ensure adequate reliability levels for anti-seismic devices.

It is expected that these two normative documents may help expand the use of this beneficial technology across seismic areas of Europe and regions of influence. Thus, research on the application of these simplified methods is of interest enough to motivate the present work.

1.2 Objectives and scope

Simplified methods, such as RSA, avoid the complexities and uncertainties involved in time-history analysis (selection and scaling of accelerograms, accurate modelization of the structural system, numerical analysis, and result post-processing). For the purpose of preliminary design of structures with dampers, such methods are of great value. Their application as codified verification tools is more controversial, as their level of accuracy is not well-established, and large errors have been reported by several authors. One first objective of this work is to determine how conservative (or un-conservative) these methods are, in the context of earthquake design.

In the current established RSA procedure, added viscous damping systems are assumed to increase the viscous damping ratio of the structure without major changes in its stiffness and vibrational properties. Recent experimental [Dong et al., 2016; Kasai et al., 2010] and analytical [Akcelyan et al., 2016] research has shown that, in fact, added viscous dampers can contribute significantly to the system stiffness, thus modifying its natural period and mode shapes. A second objective of this work is to identify the extent to which the structural response is modified by the damping system stiffness, and include the necessary modifications in RSA to account for this effect.

The RSA method is based on modal analysis, in which the structure is represented by simple oscillators, one for each mode, defined by their period, viscous damping ratio and modal mass. This representation is possible because the natural mode shapes diagonalize the mass, damping and stiffness matrices. However, the addition of dampers results in a non-diagonalizable damping matrix. If modal analysis is applied, off-diagonal terms in the corresponding modal damping matrix are neglected. A third objective of this work is to

determine the amount of error incurred by this approximation, and try to establish how this error is associated with properties of the structure and its damping system.

The response of the simple oscillators that constitute the modal representation of the structure must be predicted using a response spectrum that should include the important effect of damping. In a codified verification, the 5% damped elastic spectrum is used in combination with spectral correction factors to account for the added viscous damping. Several studies have proved the dependency of these factors from the tectonic setting. A fourth objective of this work is to derive spectral correction factors for specific use with the Eurocode 8 spectrum.

1.3 Organization of the Thesis

The organization of the Thesis is related to the previous objectives, ordered not according to their importance but to the logic of its practical application:

- Chapter 2 contains background information about viscous dampers and simplified methods of analysis, summarizes existing relevant research and establishes the basis and notation used throughout the document.
- Chapter 3 summarizes a study performed to measure the error inherent to the modal solution of non-proportional systems, the connection of this error with mechanical features of the structure and damping system, and its relationship with existing non-proportionality indices.
- Chapter 4 deals with the derivation of spectral correction factors for application with the Eurocode 8 spectrum.
- Chapter 5 presents a numerical model that describes the behavior of the Maxwell system. The Maxwell model includes the effect of the limited flexibility of the brace extender elements. The numerical Maxwell model is used to predict the response of a structure with added viscous damping including limited stiffness, represented by a linear or non-linear standard solid model.
- Chapter 6 uses the model derived in Chapter 5 to modify the RSA method in order to include the limited flexibility of auxiliary mounting elements of the damping system. Two modifications are proposed, a consistent one (RSAB) and a simplified one (RSA). The convergence of the resulting iterative procedure is studied, and the predictive trends of the model are presented.
- Chapter 7 describes a large numerical study carried out in order to investigate the accuracy of the original RSA method and compare it with the proposed modifications. Results of the study are also used to assess the influence of the damping system stiffness in the structural response of structures with added viscous dampers.
- Finally, Chapter 8 contains a summary of observations, results and conclusions, and proposes further research to be addressed.

In every Chapter, a final Section has been added to identify the original contributions of the author. Additional material which is not essential to the main text is presented in Annexes grouped in an independent document.

Chapter 2

Background to Simplified Analysis of Structures with Viscous Dampers

2.1 Introduction

In this Chapter, relevant background information is presented in a succinct way. First, the basic principles of modern Earthquake Design are stated, and it is shown how implementation of Energy Dissipation Devices (EDD) is primarily aligned with those principles. Among the different EDDs, Fluid Viscous Dampers (FVDs) are presented as an efficient alternative. Their main features are examined, along with relevant research on their behavior and use.

Analysis methods available for structures equipped with FVDs are discussed, with an emphasis on Non-Linear Response Spectrum Analysis (RSA), a modification of Response Spectrum Analysis to include non-linear effects in the first mode through the Capacity Spectrum Method (CSM). First, the basis of the RSA method is examined for single-degree of freedom (SDOF) elastic systems. Inelastic systems are then treated and linearization of these systems, (solution of equivalent systems with effective properties) is presented as a simple and efficient analysis technique; different linearization options are reviewed. The extension to multi-degree of freedom (MDOF) systems is performed by conversion of the original system to a SDOF representation using modal analysis. The MDOF non-linear static capacity curve (pushover) is presented, and its use on the CSM and other similar procedures is discussed, followed by extension of these procedures to the case of structures with energy dissipation systems.

2.2 Design principles

Structures should be designed to meet two requirements in the event of earthquakes:

1. Protection of human lives.
2. Limitation of damage.

Additionally, important facilities, such as those needed for civil protection, should remain fully operational after earthquake shocks. This suggests the need for different levels of requirements according to type of use and occupation (Consequence Class).

Due to the fundamentally random nature of earthquake action, the only rational way to achieve these objectives is through a probabilistic framework. However, different levels of accuracy and complexity are possible. Two approaches are currently accepted: Limit State Design (LSD) and Performance Based Design (PBD).

Limit States are defined according to the amount of acceptable damage. An acceptable probability of exceedance for each Limit State is defined. Under certain assumptions, this probability is related to a return period. Performance is measured in terms of exceedance of the Limit State.

In PBD the performance is measured globally, including a holistic view encompassing global and local collapse risk, fatalities, and cost, including repercussion of building functionality loss [FEMA, 2018]. Performance is expressed using a series of measures aligned with the basic objectives of life preservation and damage limitation: casualties, repair cost and time, environmental impact and unsafe occupation. Because a deterministic response to earthquakes is not possible, a probability approach is introduced, considering inherent uncertainties. A key element to the methodology is the development of collapse fragility curves defining the probability of structural collapse as a function of ground motion intensity.

2.2.1 Eurocode 8 requirements

The current version of Eurocode 8 is based on Limit State Design. Limit States are related to a 10% probability of exceedance P_{NCR} of a specific level of the seismic action during a time span. In the current version [CEN, 2004], two fundamental requirements are defined:

- No-Collapse (NC), associated to a 10% P_{NCR} in 50 years (or a return period T_R of 475 years).
- Damage Limitation (DL), associated to a 10% P_{NCR} in 10 years (or a return period T_R of 95 years).

Seismic action for NC is defined through a response spectrum anchored at Peak Horizontal Acceleration (PHA), with spectral shape dependent on seismicity and ground type. The value of PHA is dependent on mapped rock acceleration a_{gR} , building's importance factor η and ground type. Seismic action for DL is defined as 40% or 50% (depending on importance class) of the NC seismic action.

In the new draft of Eurocode 8 [Labbé, 2018], four limit states are being considered: Near Collapse (NC), Significant Damage (SD), Damage Limitation (DL) and Fully Operational (OP). SD is the counterpart of NC with a return period of 475 years. NC and SD are treated as Ultimate Limit States (ULS), whereas DL and OP are treated as Serviceability Limit

States (SLS). Verifications are to be carried out only for SD, and it is assumed that this implies non-exceedance of the NC ULS.

2.2.2 Strength and flexibility

Independently of how requirements are formulated, safety against earthquakes can be conceptually obtained in two different ways: strength or flexibility.

The strength approach implies increasing structural strength to withstand earthquake forces. This increase in strength results in larger stiffness and shorter structural period, which in turn results in increase of acceleration and strength demand. The displacement demand is comparatively small. The extreme example is a rigid solid, infinitely stiff, with no displacement relative to its base, but with the same absolute acceleration as the ground, and large internal forces. With very small drift and velocity, added damping in this type of systems is inefficient.

The flexibility approach is based on reducing stiffness, which in turn results in reduced strength and larger periods, with smaller accelerations but larger displacement demands. Flexible structures are less sensitive to conventional earthquakes, but are prone to suffer damage derived from large interstorey drift demands. The relevant problem is not strength, but drift control.

Added viscous dampers offer an ideal solution to this problem. The added viscosity reduces drift without relevant modification of the fundamental vibration parameters. Therefore, strength requirements are very moderate and can be assumed by the structure with limited amount of inelastic action. For larger earthquake intensities (near collapse), the structure may enter into the inelastic range.

Seismic design can be seen as a balance between drift requirements and strength requirements. Both aspects are discussed hereby.

2.2.2.1 Drift requirements

Drift and displacement have taken a predominant role in earthquake engineering. As indicated, Performance-Based Design (PBD) is concerned with the evaluation of damage. A common damage indicator for frames is the interstorey drift ratio δ_n , defined for storey n as:

$$\delta_n = \frac{\Delta_n}{h_n}, \quad (2.2-1)$$

where Δ_n is the n -th storey interstorey drift (difference between top and bottom storey displacement) and h_n is the storey height. In the 1990's, codes [FEMA-273, FEMA-274] linked the performance objectives with limit values of drift ratios.

The maximum inter-storey drift is limited by a condition of the type (cf. Eurocode 8, Section 4.4.3.2; ASCE/SEI-7, Section 12.12; FEMA-356):

$$\delta_n \leq \delta_{\text{lim}}, \quad (2.2-2)$$

where δ_{lim} is a transient drift ratio limit. A similar condition (with different limits) is applied to residual drift $\delta_{r,n}$. The drift ratio limit is used in two ways: i) as a mean of controlling damage to non-structural elements; ii) as a mean of controlling structural damage. In each case, the limit is imposed to the drift with a certain probability of exceedance during the building lifetime.

Specifications in FEMA-356 are given for both transient and residual drift, whereas Eurocode 8 and ASCE/SEI-7 deal only with transient drift, although ASCE/SEI-7 defines two scenarios for two different return periods.

Listed in Table 2-1 are the requirements in ASCE/SEI-7 and Eurocode 8 for steel MRFs; for conventional buildings (importance class / risk category I or II) and non-structural elements detached from the structure, both codes indicate $0.02 = 1/50$ as a limit. In both cases, this limit applies to drift obtained from a Design Earthquake having a probability $P_R = 10\%$ of being exceeded in a $T_L = 50$ year period, which corresponds to a return period of about $T_R = 475$ years, associated to No Collapse (NC) Limit State (actually, Eurocode 8 defines the limits as half the values listed in the table, but applying to an earthquake with $P_R = 10\%$ in $T_L = 10$ years, equivalent to $T_R = 95$ years, a Damage Limitation Limit State; however, for practical calculations this value is to be approximated by a fraction (0.5 or 0.4, depending on importance class) of the NC drift).

Table 2-1. Inter-storey relative drift limits imposed by Eurocode 8 and ASCE/SEI-7.

Steel and concrete structures		δ for: importance class (EC8) / Risk category (ASCE/SEI 7-16)		
	Non-structural elements	I or II	III	IV
Eurocode 8	Brittle	1.00%	1.25%	1.25%
	Ductile	1.50%	1.88%	1.88%
	Detached from main structure	2.00%	2.50%	2.50%
ASCE/SEI-7	Detached from main structure*	2.50%	2.00%	1.50%
	All other cases	2.00%	1.50%	1.00%

* up to 4 storeys

Although these limits are implemented in current practice, Seo et al [2014] proved that, for a special steel frame equipped with viscous added dampers, with design base shear equal to 75% of the minimum design base shear, a tighter relative drift limit of 1.5% would be needed to guarantee a similar collapse resistance than that of the unreduced steel special moment frame. On the other hand, the addition of dampers generally aims at increasing performance levels, that is, diminishing damage to the main structure and therefore reducing drift.

For structures with added dampers, ASCE/SEI-7 imposes an additional limit to the transient drift ratio calculated with the Maximum Considered Earthquake MCE_R (with a longer return period of 2475 years):

$$\delta_t \cdot \min \left\{ \frac{1.5R}{C_d}, 1.9 \right\} \leq 0.03, \quad (2.2-3)$$

where R is the Response Modification Coefficient, and C_d the Deflection Amplification Factor, both dependent on the type of seismic force-resisting system (as given by Table 12.2-1 in [ASCE, 2016]). For special moment frames, $R = 8$, $C_d = 5.5$, and therefore the relative drift limit given by the previous expression is 3%.

FEMA-356 gives limits for transient and residual drift ratios related to diverse return period. Summary values of these limits along with those from ASCE/SEI-7 and Eurocode 8 are given in Table 2-2.

Table 2-2. Inter-storey relative drift limits imposed by Eurocode 8 and ASCE/SEI-7.

Reference	Context	Transient δ	Residual δ_r
FEMA356	Collapse Prevention CP, $P_R = 2\%$, $T_L = 50$ years, $T_R = 2475$ years	5%	5%
	Life Safety LS, $P_R = 10\%$, $T_L = 50$ years, $T_R = 475$ years	2.5%	1%
	Immediate Occupancy IO, $P_R = 20\%$, $T_L = 50$ years, $T_R = 225$ years	0.7%	0%
ASCE/SEI 7-16	Maximum Considered Earthquake MCE_R $P_R = 2\%$, $T_L = 50$ years, $T_R = 2475$ years	3%	—
	Design Earthquake, defined as $2/3 MCE_R$	2%**	—
EN-1998-1:2004 (Eurocode 8)	Damage Limitation DL $P_R = 10\%$, $T_L = 10$ years, $T_R = 95$ years or $P_R = 40\%$, $T_L = 50$ years, $T_R = 95$ years	0.5%, 0.75%, 1%*	—
	No Collapse NC $P_R = 10\%$, $T_L = 50$ years, $T_R = 475$ years	1%, 1.5%, 2%*	—

P_R : Probability of exceedance in life-time period T_L , equivalent to a return period T_R .

(*) For brittle, ductile or detached partitions, respectively.

(**) For frames with 4 storeys or less 2.5% is acceptable.

2.2.2.2 Base strength requirements

ASCE/SEI-7/16

When designing for severe earthquake, minimum strength requirements are imposed [ASCE, 2016]. These requirements are relaxed in the case of structures with added dampers in order to provide more flexible structures. For ASCE-SEI 7/16, the base strength of structures with added dampers can be reduced to 75% of the corresponding value for the structure without added dampers:

$$V_{\min, ASCE} = \max \{ 0.75 V_{1, ASCE}; \eta_e V_{1, ASCE} \} \quad (2.2-4)$$

where $V_{1,ASCE}$ is a given seismic base shear, and η_e a damping correction factor, obtained (from tabulated values) for the addition of inherent viscous damping ratio and added damping ratio in the first mode. $V_{1,ASCE}$ is obtained using the equivalent lateral force (ELF) method as:

$$V_{1,ASCE} = C_s W_{tot}, \quad (2.2-5)$$

where C_s is a seismic response coefficient and W_{tot} the total frame weight. C_s is obtained from spectral ordinates (based on a fundamental period limited to a maximum value of about $0.1N$, N being the number of storeys) divided over the response modification factor R , which is dependent on the structural type. For steel MRFs, R is between 8 (steel special moment frames), and 3.5 (steel ordinary moment frames).

EN-1998-1:2004 (Eurocode 8)

Eurocode 8 does not enforce a minimum base shear value; by extension of the ASCE criteria, a reference value of the base shear in the undamped elastic frame can be obtained as:

$$V_{min,EC8} = \max \{0.75 V_{1,EC8}; \eta_{I+a} V_{1,EC8} \}, \quad (2.2-6)$$

$$V_{1,EC8} = S_e(T_1, q) W_{tot} / g, \quad (2.2-7)$$

where $S_e(T_1, q)$ is the design spectral ordinate, W_{tot} the total seismic weight, η_{I+a} is a damping spectral correction factor (depending on inherent and added viscous damping), and q the behavior factor that takes into account overstrength and ductility. For ductile steel MRFs, q can take values up to 6.5.

2.3 Passive Energy Dissipation Devices

Traditional earthquake design relies on energy dissipation through inelastic action. Yielding of structural elements during earthquake increases flexibility and therefore period, which largely reduces forces but not deformations. The structure maintains its integrity through its capacity to absorb energy by inelastic action. This resilience comes to the cost of structural damage. Energy-wise, the earthquake imparts an amount of energy (energy input). A certain part produces elastic vibration. Other part is dissipated through inherent damping. A substantial part, though, is dissipated through hysteresis in specific locations that experience yielding. Regardless of their special strength and ductility properties, these regions accumulate damage in every inelastic cycle.

An alternative approach is the addition of passive energy dissipation systems (hereby referred to as PEDs), in which mechanical devices dissipate a large part of the energy, thus minimizing or completely preventing hysteretic structural behavior and damage. This is made clear by the energy balance equation at the end of motion [Akiyama, 1985; Uang & Bertero, 1988; Filiatrault & Christopoulos, 2006], see Annex E.

$$E_I - E_v = E_a + E_d, \quad (2.3-1)$$

where E_I is the energy input, E_v the energy dissipated by viscous damping, E_a the energy absorbed and E_d the energy dissipated by the energy dissipation system. If the structure is motionless, E_a contains no strain energy and therefore corresponds to hysteretic energy. This concept is illustrated in Figure 2-1, where the energy input of frame 05D without and with added viscous dampers is compared (frames and damping systems are defined in Annex B and C). The total energy input (top line) is very similar in both cases. However, the proportion in which this energy is divided is drastically changed. Particularly, structural damage is substantially reduced when added damping is included. The work dissipated by hysteretic energy in the first case is about 300 kJ, and in the second case 104 kJ, 35% of the previous value. This reduction in damage is basically achieved through reduction of drift and storey shear, i.e., deformation and internal forces.

Figure 2-1. Energy input of moment frame (a) without dampers; (b) with viscous dampers.

Energy dissipation capacity is the unifying concept behind different kind of systems, every one of which is referred to by its main energy dissipation mechanism: Friction, yielding, viscoelastic, and viscous, illustrated in Figure 2-2 by the corresponding force-displacement loops developed under harmonic excitation at constant frequency and amplitude [Tsai *et al.*, 1993; Filiatrault & Cherry, 1990; Shen & Soong, 1995; Constantinou & Symans, 1992]. Another classification of the devices is according to the kinematic variable they depend upon, as displacement-dependent and velocity-dependent [ASCE, 2016]. In this context, the equivalent terms rate-independent and rate-dependent have also been proposed by Symans *et al.* [2008]. Friction and hysteretic devices are considered as displacement-dependent (or rate-independent) devices, whereas viscous and viscoelastic are velocity-dependent (or rate-dependent) devices.

The word ‘passive’ indicates that these devices perform without any additional source of energy. Other means to achieve a similar level of earthquake protection exist. Base isolation [Filiatrault & Christopoulos, 2006] detaches the building motion from that of the soil, by addition of largely flexible elements between the structure and its foundation. As a result, the fundamental period of the complete system is enlarged and it is seldom affected by the most energetic part of the earthquake signal. However, a part of the energy spectrum is not reflected, and for this reason, and to comply with maximum displacement requirements, base isolation is generally complemented with viscous dampers. Base isolation is efficient but expensive. It requires a considerable volume under the building,

and generates compatibility problems in the connection of services. Replacement of damaged elements is also problematic.

Figure 2-2. Different types of energy dissipation devices; from left to right: yielding (reproduced from Tsai *et al.* [1993]), friction (reproduced from Filiatrault & Cherry, [1990]), viscoelastic (reproduced from Shen & Soong [1995]), viscous (reproduced from Constantinou & Symans [1992]). Top row: pictures of the devices. Bottom row: force-displacement loops under harmonic load.

Active and semi-active systems use a combination of devices and sensors to modify the properties of a structure, generally through some sort of actuator (active bracing, active mass) or through modification of properties of damping systems (semi-active systems) [Soong & Dargush, 1997]. These systems can be highly effective because they adapt the response to the earthquake input; moreover, they can be efficient for other lateral loads such as wind. On the negative side, they are extremely expensive and their reliability is compromised because of their dependence on external energy supply sources.

For the aforementioned reasons, passive energy dissipation systems are a reasonable alternative to base isolation or use of active systems. A large number of experimental studies and practical implementation have demonstrated the successful performance of these systems. An excellent summary can be found in Symans *et al.* [2008].

2.4 Fluid viscous dampers (FVD)

2.4.1 Description

Viscous dampers [Constantinou & Symans, 1992] consist essentially of a piston moving inside a fluid (typically a silicone gel) contained inside a sealed cylinder, see Figure 2-3. The piston is tightly adjusted to the cylinder, dividing it in two chambers, connected through small special orifices in the piston head, which is attached to a rod. The piston damper and rods are attached to two points of the structure whose relative motion is to be damped. The rod enters the cylinder through special seals. When rod and piston move relative to the cylinder, the silicone gel in the upstream chamber flows through the orifices to the adjacent downstream chamber, and in the process dissipates energy as heat. The dissipation is

dependent on the rate of volumetric change, i.e., on the relative velocity of piston and cylinder. For high performance, the orifices need to be designed to ensure large velocities in the fluid. This design determines the damper exponent a , which ranges between 0.07 and 2. The damper coefficient c is determined by other parameters such as piston area and fluid density, with a large practical range of values.

Figure 2-3. Components of a fluid viscous damper.

Every component has especial requirements [Lee & Taylor, 2001]:

- The fluid, generally a silicone gel, must be stable (to both temperature changes and aging), non-toxic and present a low compressibility to ensure that no spring action develops. Paradoxically, low viscosity fluids are ideal.
- The steel cylinder must withstand the pressure imposed by the fluid, (with a safety factor of 1.5 or higher), presenting deformations small enough to minimize spring action.
- The rod must be very rigid and perfectly straight to avoid bending effects due to buckling, and highly polished to minimize friction that would compromise the seal integrity. High alloy stainless steel is typically used.
- The seal, manufactured in Teflon, must be durable and present negligible friction, not allowing seepage or leakage. A 35-year service (or longer) must be warranted. Also, because dampers are in service only intermittently, the seal must allow for long periods of inactivity without sticking.
- The piston head can be made of a different material than the cylinder to compensate for temperature changes.

Other auxiliary elements, such as caps, bearings, plates and clevises are not depicted in Figure 2-3, but form an important part of the set and are generally provided by the same manufacturer and subjected to exhaustive control and testing. Sometimes accumulators or

thermal valves are added to improve the behavior under changing temperature conditions. The high quality of the components results in long warranty periods, but also in increased cost.

One important difference of viscous dampers with respect to other dampers is their durability and stability in time and throughout seismic events; viscous dampers do not experience damage during strong earthquakes, as long as the maximum/minimum damper stroke is not reached and the out-of-plane motion of the frame is within tolerance limits of the auxiliary mounting elements (spherical bearings must be provided at either side of the damper). Hysteretic dampers, on the contrary, require replacement after a strong shaking. Friction dampers are much dependent on aging of the contact surfaces and relaxation of the pressure between them. Viscoelastic dampers show a large dependency of their properties on ambient temperature, which makes them unsuitable for certain uses.

An additional advantage of viscous dampers is that the stiffness of the system is, in theory, not modified, meaning that the fundamental elastic period remains the same (except for the shift due to viscous damping). This is a consequence of the dependency of the velocity-dependence of the device, expressed by Eqn. (2.4-1). Therefore no restoring force (in the structural sense) exists and the main structure must be designed to provide lateral strength. Current codes impose the necessity to provide a seismic-force-resisting system [ASCE, 2016]. It is widely acknowledged [Symans *et al.*, 2008] that viscous damping systems (and, in general, other passive systems) with conventional levels of damping can reduce, but not completely avoid, yielding excursions of the structure under strong earthquake events. Therefore, seismic detailing requirements are not relaxed by the addition of viscous dampers. Nevertheless, for small frequent events (smaller return periods) the main structure can be protected to remain fully operational. The possibility to model the damper as purely viscous (dashpot) is further discussed in Chapter 5.

2.4.2 Historical development of FVDs

As many technological advances, FVDs have their origin in military applications. An interesting account is given in Berquist *et al.* [2019]. The first known use of this technology was as a mean to arrest gun recoil, apparently by the British Army in 1862, and later by the French on a 75mm gun (M1897), Figure 2-4(left). The gun was mounted on a slide with a pneumatic system to recoil shooting energy and return the gun to position. A modification of this gun was installed in the nose of the US B-52 Mitchell Bomber to avoid damage to the aircraft, Figure 2-4(right). The most important problem of these early devices was the poor quality of the seals, not solved until synthetic materials were largely available. This technology was soon used for civil purposes, notably automobile suspension.

The real step forward for viscous dampers was their use as shock absorbers in missile bases during the Cold War period (1945-1990). The performance requirements were very high, and it was soon realized that fluid density was critical. Subsequently, pneumatic

dampers were abandoned and replaced by hydraulic dampers, which allowed fluid pressures up to 350 MPa (over 3400 atm) with simultaneous viscous damping and spring action due to the fluid compressibility. The success of these devices led to their use in other military applications such as aircrafts and ships. An often-stated consequence of their development in the military industry is their reliability and durability.

In the 1990's the end of the Cold War allowed the viscous damper technology to be declassified, and research started to find civil applications. One of the obvious fields was earthquake, shock, vibration and wind protection of structures. A considerable amount of research was carried out at the US National Center for Earthquake Engineering Research (NCEER), showing that implementation of viscous dampers in structures reduced considerably both deformation and stresses. A summary of the most relevant research is reported below. By 1933, the New Arrowhead Regional Medical Center (California) included 186 dampers manufactured by Taylor Devices Inc. in the base isolation system [Asher *et al.*, 2001] and since then viscous dampers have been used in a large amount of practical applications in earthquake engineering [Symans *et al.*, 2008]. Other applications of viscous dampers involve control of vibrations, especially in pedestrian bridges. One famous case was the retrofit of the Millennium Bridge to reduce lateral vibrations [Taylor, 2002]. In this case, inspection and test of 3 (out of 37) dampers after an 11 year period revealed that the damper had experienced no physical or functional deterioration (the tested dampers were, in fact, re-installed in the bridge) [Klembczyk, 2017].

Figure 2-4. Early uses of hydraulic dampers: left, M1897 75mm cannon (from http://www.landships.info/landships/artillery_articles/75mm_Mle_1897.html); right, B25 bomber with damped 75mm cannon mounted on nose (reproduced from Saturday Evening Post).

2.4.3 General behavior

Among passive energy dissipation systems, fluid viscous dampers (FVD) constitute an especial case because their performance is solely dependent on velocity:

$$f_d = c |\dot{u}|^a \operatorname{sgn}(\dot{u}), \quad (2.4-1)$$

where f_d is the damper force, c the damper coefficient, a the damper exponent, and $\dot{u}$ the relative velocity between damper piston and damper cylinder, see Figure 2-3. During

oscillating motion, velocity reaches its peak at the instant of no acceleration, which is approximately coincident with that of no displacement and is also the instant of maximum momentum. Therefore, peak damper forces are out-of-phase with maximum restoring forces, which is beneficial for structural elements. This is shown in Figure 2-5, which displays a section of the time-history for 1st storey structure shear (V_s) and 1st storey damper shear (V_d), for a frame with added viscous dampers subjected to a recorded accelerogram. Both curves have been normalized by their respective peaks. Points of zero damper shear are approximately coincident with maximum storey shear, resembling two out-of-phase harmonic waves. This is more evident in the linear case. However, this relationship is far from perfect, proving that the damper behavior is not purely viscous.

Figure 2-5. Detail of time history of storey shear and damper shear at 1st storey of frame 05D subjected to motion 000414XA with intensity PHA = 0.4g; (a) linear dampers; (b) non-linear dampers.

The significance of the damper exponent a is clear in Figure 2-6(a) where the force developed in three dampers with different exponents, as a function of velocity, is compared. The coefficients have been adjusted to be coincident at approximately 168.5 mm/s. The response of non-linear dampers for high velocity peaks (which might appear during earthquake action) are moderate, thus limiting the forces transmitted to the structure. For small values of velocity, non-linear dampers quickly reach high forces. The shape of the force-velocity response determines the shape of the force-displacement loops, illustrated for the first storey of frame 05D (referred above) with linear and non-linear dampers at two intensity levels. For low velocities, Figure 2-6(b)(c), the peak shear values are higher in the non-linear dampers; for high velocities, Figure 2-6(d)(e), the shear forces are higher in the linear dampers. For early applications it was assumed that $a = 1$ was optimal for earthquake performance, however it was soon realized that smaller exponents might offer the same energy dissipation capacity with reduced damper forces [Lin & Chopra, 2002], see Figure 2-6(a). Current engineering practice favors exponents in the range 0.1 to 0.5 for seismic application. For wind damping applications, exponents in the range 0.5 to 1 are more suitable, and 0.5 is an adequate exponent if both wind and earthquake damping are required.

Figure 2-6. (a): force-velocity relationship for different exponents of viscous dampers. (b) to (e): shear force vs. drift loops of 1st storey dampers in frame 05D, subjected to motion 000414XA; (b) linear dampers, moderate intensity; (c) non-linear dampers, moderate intensity; (d) linear dampers, high intensity; (e) non-linear dampers, high intensity.

2.4.4 Damper characterization

The linear viscous damper behavior is most accurately described by a fractional Maxwell derivative model [Makris & Constantinou, 1990]:

$$f + \lambda \frac{d^r f}{dt^r} = c \frac{d^q u}{dt^q}, \quad (2.4-2)$$

where r and q are positive real numbers (not necessarily integers), and λ is a dimensional parameter (relaxation time). In the tests performed by the cited authors, calibration of the model rendered values of $q = 1$. The tests were performed on a GERB damper with three degrees-of-freedom intended for isolation of pipes and industrial application. The authors proposed an equivalent viscous oscillator with essentially the same response as the original system.

Further study of linear dampers under harmonic steady-state motion was performed by Constantinou & Symans [1992] under different service temperatures. The dampers were similar to the type presented in Figure 2-3, except that the rod did not pass through the piston; motion of the piston implied loss or gain of volume, and this was accounted for by an accumulator with valves. The recorded harmonic force –displacement loops, allowed calculation of λ , which happened to be very small (in the order of 0.006s) for cutoff frequencies above 4 Hz (excitation periods below 0.25s), thus allowing the simplified model described by Eqn. (2.4-1) specialized for $a = 1$ (linear viscous damper). For frequencies above 4Hz, the Maxwell model (including storage stiffness) should be used. Temperature was found to affect the damper performance. Decrease of temperature from 24°C to 0°C (-24°C) resulted in increment of 44% in the damping coefficient c , whereas increase in temperature from 24°C to 50°C (+26°C) produced a decrement of -25% in c . This is small when compared with the variations experienced by viscoelastic dampers [Shen & Soong, 1995]. However, it made clear that the damper properties experienced a considerable variability, a situation that must be addressed in design by bounded solution using minimum and maximum expected values. This problem has been later addressed by manufacturers and current available dampers are less sensitive to temperature changes.

Additional study of linear dampers [Reinhorn et al., 1995] confirmed that the storage stiffness of the devices is virtually non-existent below a certain cut-off frequency. However, this frequency was found to be around 1 to 2 Hz (excitation period between 1 and 0.5s). For larger frequency, significant stiffness contribution happens. Correspondingly, other models might be more accurate, such as a properly calibrated Kelvin or Maxwell model. These models are described in Section 2.4.10. For structural response, the viscous part was found to be accurate enough. For this reason, for practical applications, Eqn. (2.4-1) is, according to these authors, sufficient for linear dampers.

Non-linear dampers were characterized by Seleemah & Constantinou [1997] by fitting model parameters at harmonic steady-state oscillations. Viscous behavior was evident from similar peak force results of tests with identical peak velocity (double frequency, half displacement). Eqn. (2.4-1) was found to be a good description of behavior for a wide range of frequencies, with the exception of very low velocities (less than 15mm/s in the tests performed), where linear behavior was found to be more exact. The cutoff frequency for the appearance of relevant storage stiffness is not given, but from force-displacement loops seems to be around 4Hz. The devices presented also a small amount of friction effect. The temperature dependency of the devices was not explored in this work. The study also showed an apparent larger variability in empirically obtained damping coefficients than for linear devices. Values of parameters c and a were calibrated from test results, and the resulting viscous model was used satisfactorily in numerical estimates of structural response (reported below).

Full scale tests of different dampers were carried out by Kasai *et al.* [2008] at Tokyo University within the framework of the E-Defense full-scale test described below. The tests were performed for harmonic steady-state motion at different peak displacements

(0.5, 12, 24 and 36mm) and four frequencies (0.2, 0.5, 1 and 2Hz), (combination of 36 mm and 2 Hz was not run as it was out of the performance range of most dampers). These deformations were taken for expected drift angles δ of 1/300 (0.0033), 1/150 (0.0067) and 1/75 (0.0133) of a 3600 mm high storey (storey height of the 5-storey specimen where these damper were inserted). 15 additional tests were performed for random deformations, obtained from the 1st storey drift history of 5, 12 and 24 storey building models under Kobe 1995 (JMA NS record) and Taft 1951 (EW Record) earthquakes normalized to peak drifts of 12 and 24mm. For the Kobe earthquake, 36mm amplitude was also used. Simulations and tests showed that the behavior at velocity less than 20 mm/s differed from the analytical prediction (as indicated, this result was also observed by Seleemah & Constantinou). One important difference of these tests with the ones carried in early stages of the technology (reported above) is that the tests included also auxiliary elements (end plates, clevises, brace extenders, etc.). Force-displacement curves obtained presented a more marked effect for stiffness. It was found that, for accuracy, the storage stiffness component of the damper set must be measured and included in the Maxwell model. This is, however, not common practice, as accurate values for this data can only be obtained from tests generally not provided by manufacturers. Hereby it will be shown that the influence of this component is not negligible, therefore it is expected that data relative to storage stiffness becomes available from manufacturers in the future. The limited amount of data for the viscous dampers tested by Kasai *et al.* [2008] is listed in Table 2-3.

Table 2-3. Damper coefficient and stiffness measured in Kasai *et al.* [2008].

c_b kN·(s/mm) ^{0.38}	k_{bd} kN/mm	c_b/k_{bd} s ^{0.38} /mm ^{0.62}
98	193	0.508
196	438	0.447

Note: Damper exponent $a = 0.38$

2.4.5 Test models and performance during earthquake events

2.4.5.1 Tests at NCEER

Some relevant tests performed on structures equipped with viscous dampers are reported hereby. The first test models of structures with linear viscous dampers were conducted by Constantinou & Symans [1992] in a 3-storey model with three configurations: dampers in all storeys; dampers in 1st storey only; dampers in 1st storey with the 2nd and 3rd inter-storey drifts locked (to resemble a single degree of freedom structure). The model was subjected to 5 strong motion records from California and Japan with different levels of scaling. In some tests the structure was stiffened. The structure behaved elastically or with small levels of ductility (up to 1.2). Structural properties were found by modal techniques. It was found that the primary effect of dampers was the increase in viscous damping ratio (for the 3-storey model, increase of 8%, 16% and 18% for 2, 4 and 6 added dampers, respectively), with small effect on period (stiffness), except in the higher modes. For instance, in the configuration with 6 dampers the higher modes were completely suppressed, allowing

analysis only in the first mode. Also, addition of dampers in only the 1st storey altered significantly the modal shapes. Dampers had an effect on reduction of storey drifts (30% to 70%) and storey shear (40% to 70%). In this work the spectral response method was first used to successfully predict the response of the frame.

Reinhorn *et al.* [1995] studied the performance of linear viscous dampers for retrofit of reinforced concrete structures, through analytical and experimental tests on a 1:3 scale reinforced concrete frame. The study included inelastic behavior. The response of the retrofitted structures was good, with enhanced performance through reduction of maximum deformation and small modification of structural forces. The main effect was reduction of hysteretic energy dissipation demand in the main structure. The dampers were found to stiffen the structure at the frequency range of interest, but this effect was considered minor. Larger stiffening effects were observed at higher frequencies, which resulted in increase of accelerations and shear increase for those frequencies. In general, it was found that the internal shear forces in the retrofitted structure were reduced, meaning that damper behavior was almost pure viscous. A simple Kelvin model produced an accurate enough response prediction, although the Maxwell model produced a better one. A push-over analysis was used to investigate the transfer load path of the system, including the static stiffness of the dampers at large deformations. The composite spectrum (AD format spectrum) was suggested as a way to find an estimate of response.

Seleemah & Constantinou [1997] performed model tests using linear and non-linear viscous dampers on one-storey and three-storey MRF structures, and compared the experimental results with the analytical predictions, under ten different earthquake records (including some near-fault records), sinusoidal motions and white noise excitation. The damper configurations were complete and incomplete. The addition of dampers resulted in significant drift reductions, more so for non-linear dampers. In the elastic range, reductions in total shear force response were obtained, which resulted in reduction of structural shear force. The addition of non-linear dampers resulted in high-frequency components in storey response spectra (indicative of stiffening effects).

2.4.5.2 Tests at Lehigh University

Dong *et al.* [2016] studied a 6:10 scale model of a 3-storey high frame with non-linear viscous dampers, subjected to design and maximum earthquake intensities (DBE and MCE) represented by representative suites of scaled motions. The tested structure included only the damping system (dampers and their brace extenders plus the interacting frame elements), whereas the interaction with the rest of the structure was modeled through real-time hybrid simulation (RTHS); this technique consists essentially on the division of the analyzed system in two parts: a physical-experimental model and a numerical-analytical model; both elements are coupled through compatibility and equilibrium conditions imposed on their common interface. Displacements of the latter model are imposed (on real time) to the former model through actuators; the displacements are obtained by

numerical integration of the motion equation, including measured restoring forces from both models. The test results were compared with analytical predictions.

Three different structures were modeled with the same experimental frame. This was achieved increasing the applied weight by 1/3 and 2/3 of the original weight, which produced three structures with the same base shear strength, but decreasing base shear design coefficients of 75% and 60% of the original one (and periods 15% and 29% larger).

The most important result of these tests is the observation of the important effects of the elastic flexibility of elements in the damping systems, resulting in damper forces which are not out-of-phase with structural restoring forces. This is shown to be of particular importance for columns, where large axial forces are simultaneous with large bending moments. The dampers were observed to contribute to the structural shear at maximum drift, thus contributing to the structural stiffness, resulting in larger periods than predicted.

The beneficial effect of dampers was remarkable. The test structures remained basically elastic at design levels (DBE), with negligible residual drift. The residual drifts at MCE were on the order of 0.5%. The reduction of base shear coefficient –in combination with high levels of damping- resulted in reduction of interstorey drift and absolute floor acceleration demands. Therefore, an important conclusion is that MRF equipped with non-linear viscous damping systems can be safely designed for reduced MRF base shear strength and still achieve high seismic performance.

2.4.5.3 E-Defense tests

Passive systems have been installed in buildings since approximately the 1990's. At the time of the first applications these devices had not been subjected to major catastrophic events; the situation was especially critical in Japan, where most relevant buildings constructed after the 1995 Great Hanshin (Kobe) earthquake included either base isolation or passive systems. In order to validate the behavior and prediction of the latter, Kasai *et al.* [2010] conducted very detailed full-scale shake table tests on a 5-storey steel building equipped with different types of passive systems (viscous, viscoelastic, hysteretic and friction). The tests were performed in the E-Defense shaking table, a large facility constructed in Japan after the Kobe earthquake of 1995 with the intention of performing full-scale experiments. The prototype chosen, a 5 storey steel building, is representative of a large number of office buildings in Japan.

The traditional solution for moderate height steel structures in Japan consists on bi-directional frames with square hollow columns (SHS) and I beams. In this solution, popularly known as 'column tree' construction [Bruneau *et al.*, 2011], the column splices are solved at storey level with frontal plates, that also serve to materialize a bidirectional connection with the beams. This joint, represented in Figure 2-7 (with girder in only one side and direction) is shop welded to a girder stub, and the central part of the girder is field-bolted with cover plates.

Building period in Japan is estimated on the basis of total height, as $0.03H$, with H in m (for values of storey height around 3m this expression is similar to the proposal of ASCE-SEI 7/16). This formula was found to be quite accurate for tall buildings, but it gives values too small for medium height buildings. Specifically, for 5 storey buildings, the fundamental period for actual buildings has been found to take values up to $0.07H$. This flexibility results in large interstorey drift ratios that do not comply with code requirements. For this type of structures, designed to remain elastic under design earthquake, the inclusion of added damping is intended primarily to reduce drift demand, thus limiting structural damage.

Figure 2-7. Column-girder joint, Japanese solution.

The 5 storey steel building tested is shown in Figure 2-8 (adapted from Kasai *et al.* [2010]). Modal properties were measured during tests [Ji *et al.*, 2013]; for the bare frame, a natural period of 0.650s (X direction) and 0.705s (Y direction), and inherent damping ratio about 1% was determined for small amplitude. The frame was equipped sequentially with different damping systems and subjected to both components of the Takatori record of the 1995 Kobe earthquake, with 15%, 25%, 40%, 50%, 70%, 83% and 100% scaling. As shown, dampers were not included in the last storey. According to Japanese requirements, the undamped building was designed to remain elastic under the maximum earthquake considered in the analysis. This was verified analytically (numerical models) and empirically by measure of modal properties before and after test, which were found to be virtually identical, proving that the frame had sustained virtually no damage. At the end of the damper tests, the bare building was subjected to test up to 70% the maximum intensity.

Results of the test show the improvement in performance obtained by addition of dampers. Non-linear viscous dampers presented a damper exponent of 0.38. At 100% scaling of intensity, added damping ratio was only 12%, but the maximum interstorey drift ratio was limited below 0.75%, approximately half of the bare frame (estimated). Storey shear was

also greatly reduced. However, the trends are not so clear for storey acceleration, where reduction is observed in the top half of the building, but is very small in the bottom half. Similar improvements were obtained with other type of dampers. Damping ratios for the building with viscous damping increased with increasing earthquake intensity from about 5% to 12% and, interestingly, the natural period after addition of viscous dampers descended to 0.536s (X) and 0.575s (Y); this indicates a substantial contribution of the damping system to structural stiffness; this topic is further discussed in Chapter 5; the period variation in both X and Y direction is about -18%, which corresponds to a very significant stiffness variation of almost +40%.

Figure 2-8. E-Defense 5-storey steel building with dampers (adapted from Kasai et al. [2010]).

The analytical models for the building with different dampers (whose parameters derived from full-scale tests reported above) predicted accurately the response. The analytical model used for the viscous damper included the storage stiffness. State-of-the-art models of the building with viscous and buckling restrained braces (BRBs) for this building were developed by Akcelyan *et al.* [2016]. The models proved very accurate, and the study proved some important points, namely:

- i) no significant differences in prediction are found by use of two- or three-dimensional analysis for regular buildings with dampers;
- ii) simple numerical methods are suitable for steel frames with dampers where small amount of damage is expected;
- iii) however, accurate modelization of the damper non-linear behavior, including flexibility of the auxiliary system is important, and improves the numerical response;
- iv) the effective damping formulation in ASCE for FVD might not be conservative enough.

2.4.5.4 2011 Tohoku Earthquake and Japan Society of Seismic Isolation.

The 2011 Tohoku Earthquake (M_w 9.0) has been the largest ever recorded in Japan, producing large damage on its east coast (including the Fukushima nuclear accident produced by the tsunami caused by the earthquake). It is considered the worst natural disaster in history in economic terms. During the last years of the XXth century, a large interest had arisen in Japan in passive protection technologies, materialized in the constitution of the Japan Society of Seismic Isolation (JSSI) in 1993. This interest was boosted by the occurrence of the 1995 Kobe earthquake, and by the time of the Tohoku earthquake about 950 buildings in Japan incorporated passive energy dissipation systems, of which about 37% were hysteretic dampers, 19% viscous dampers and 20% oil dampers (a type of viscous damper with oil as fluid, very popular in Japan but seldom used abroad) [JSSI, 2020]. Moreover, some buildings were instrumented with sensors and their true response was recorded and analyzed, including 11 buildings with supplemental damping, 5 among them with viscous or oil dampers. Characteristics of these buildings, all of them located in Tokyo prefecture, are listed in Table 2-4 (adapted from Kasai et al. [2013]). The PHAs were not extremely high, but the actual motion duration was long, about 160s. The buildings' inherent damping was estimated at 2%. The actual period and damping properties of the buildings with added dampers were obtained by modal analysis techniques applied to the recorded response and base excitations.

The fundamental observation is that no failures were observed for buildings with supplemental damping. The largest benefit is the reduction of drift and peak acceleration response. The authors estimated that the same excitation input in the undamped structure would result in response increases of 1.78 (displacement) and 1.75 (acceleration) for the 14-storey building. For the 54-storey building, these figures are 1.32 (displacement) and 1.64 (acceleration). These are very significant values, considering the small added damping. Due to the long excitation history, the reduction in drift results in important reduction of both damage and component fatigue. The acceleration reduction is valuable in reducing fear and discomfort of the occupants during the event. More importantly, acceleration-based hazard such as the fall of non-structural components is greatly reduced.

Table 2-4. Buildings with viscous damping with response recorded during 2011 Tohoku Earthquake (adapted from Kasai et al. [2013]).

	<i>N</i>	X-Direction					Y-Direction				
		<i>T</i> (s)	ξ_1	<i>PHA/g</i>	<i>PRA/g</i>	δ (%)	<i>T</i> (s)	ξ_1	<i>PHA/g</i>	<i>PRA/g</i>	δ (%)
1 BRB + Viscous wall	14	1.21	5.8%	0.11	0.22	0.12%	1.66	4.0%	0.13	0.16	0.11%
2 Steel + viscous wall	21	1.87	-	0.08	0.12	0.10%	2.03	-	0.07	0.13	0.12%
3 BRB + Viscous wall	38	4.05	-	0.07	0.17	0.22%	3.97	-	0.07	0.23	0.28%
4 Oil damper	41	3.97	-	0.08	0.12	0.27%	4.1	-	0.07	0.13	0.21%
5 Oil damper	54	5.37	2.9%	0.10	0.24	0.23%	6.25	3.9%	0.14	0.16	0.24%

Notes: *PRA* is the Peak Response Acceleration

2.4.6 Advantages of added viscous damping

Figure 2-9 presents the trends in peak values of relevant structural variables when subjected to a single earthquake (000414XA) progressively scaled for different levels of intensity, measured in $S_a(T_1)$ (spectral acceleration at fundamental period). The result is referred to an incremental dynamic analysis (IDA) curve [Vamvatsikos & Cornell, 2002]. IDA curves are generally performed with multiple accelerograms in order to obtain collapse probability information, however here a single curve serves to illustrate the performance of a 5 storey frame (05D, see Annex B) without dampers and with linear and non-linear dampers. The linear dampers are designed to provide 11% additional viscous damping ratio. Due to the inherent variability in earthquake action, the results obtained here (although representative) cannot be extrapolated without caution.

The most relevant variable for earthquake performance is the maximum inter-storey drift ratio δ , a good indicative of damage, subjected to strict limits in all seismic codes. For the design earthquake level, a typical limitation is 0.025. Figure 2-9(a) shows that addition of dampers results in very significant reductions for the same value of $S_a(T_1)$. For instance, at 0.2g the value of drift ratio with dampers falls to 60% of the corresponding value in the bare frame. Conversely, the same drift limit is reached for substantially higher intensity (approx. 25% more) when viscous dampers are added.

Residual drift ratio is critical to post-earthquake repair. For IO (immediate occupancy) this value should be close to 0, and for design earthquake, values of 0.01 are required. This limit is reached abruptly for certain intensity, as shown in Figure 2-9(b). Inclusion of viscous dampers increases the critical intensity (in this case by 42%).

Inclusion of dampers reduces the value of storey shear, Figure 2-9(d), but it increases base shear and therefore foundation cost.

As already shown, the relative earthquake input energy is almost unaffected by the addition of damping, Figure 2-9(e). The fraction of this energy to produce structural damage, however, is largely reduced as shown by Figure 2-9(f). At 0.2g it falls to 33%, and at 0.4g to 55%.

As a summary, addition of viscous dampers generally reduces structural damage by correspondingly reducing drift, beam and column rotations and shear and axial forces in columns. Residual drift is also reduced, although this is a less general result. The energy dissipated by the structure falls dramatically. The most important benefits of viscous dampers are i) control of drift and ii) reduction of damage and low-cycle fatigue.

If strength is critical, for the same earthquake intensity level a structure with viscous dampers can be designed with smaller elements. In structures designed for wind, inclusion of dampers reduces drift and avoids over-sizing the structural elements.

Figure 2-9. Evolution of peak values for different variables with intensity $S_a(T_1)$; frame 05D without dampers, and with added damping systems Q.1.1 and Q.1.5.

On the other side, maximum internal forces are not reduced unless the structure remains elastic (which is not to be expected with reasonable amounts of damping). The structure still undergoes inelastic cycles, and detailing must provide the necessary ductility. Base shear increases, which might increase the cost of foundation.

Figure 4-6 (in Chapter 4) shows the reduction in spectral displacement obtained for increasing levels of damping with respect to the 5% spectrum. The first increase ($\xi_a = +5\%$) results in a reduction of 20%. To achieve a subsequent reduction of 20%, additional increase of ξ_a by +10% is needed. To achieve a 40% reduction, an increase of +15% is needed in ξ_a , etc. Therefore, added damping is most effective at small values. Several authors point at 30%-40% as the economical limit for added viscous damping ratio, however no cost calculations to backup this remark are found in the literature. Some codes limit the amount of damping acceptable for application of simplified methods to 35%, including inherent damping [ASCE, 2016].

2.4.7 Mounting arrangements

Transformation equations for mounting arrangements

Different configuration arrangements must be related to the frame DOFs (interstorey drift and storey shear) in order to perform analysis. Derivation of equations for a usual case, dampers mounted diagonally, Figure 2-10(a)(b), is given hereby. The properties of the damping system $k_{b,n}$, $c_{b,n}$, are transformed to $k_{d,n}$, $c_{d,n}$, Figure 2-10(c), allowing the storey to be mathematically modeled as shown in Figure 2-10(d). Damping system force $f_{b,n}$ and displacement $u_{b,n}$ measured along the bracing axis (parallel to damper) are related to storey shear V_n and drift Δ_n as shown by Figure 2-10 (e)(f). For storey n :

$$V_n = f_{b,n} \cos \theta_n, \quad (2.4-3)$$

$$\Delta_n = \frac{u_{b,n}}{\cos \theta_n}, \quad (2.4-4)$$

$$k_{d,n} = \frac{V_n}{\Delta_n} = \frac{f_{b,n}}{u_{b,n}} \cos^2 \theta_n = k_{b,n} \cos^2 \theta_n, \quad (2.4-5)$$

where $k_{b,n}$ is the brace extender stiffness measured along its axis:

$$k_{b,n} = \chi_{b,n} \frac{A_{b,n} E_{b,n}}{L_{b,n}}. \quad (2.4-6)$$

In this expression $A_{b,n}$ is the n -th storey brace cross-section area, E_b the n -th storey brace Young Modulus, and L_b the n -th storey brace length. The correction factor $\chi_{b,n}$ takes into account that the stiffness is participated by auxiliary mounting elements such as plates, bolts and rods, and that the damper length and other elements must be subtracted from the total brace length L_b . An accurate value can be obtained by the flexibility method applied to the set of components that form the bracing system or by testing (see Section 5.1.2). This is, however, not relevant for the following discussion. For the brace-mounted damper, Eqn. (2.4-3) is valid; in addition:

$$\dot{\Delta}_n = \frac{\dot{u}_{b,n}}{\cos \theta_n}, \quad (2.4-7)$$

$$c_{d,n} = \frac{V_n}{|\dot{\Delta}_n|^{a_n}} = \frac{f_{b,n}}{|\dot{u}_{b,n}|^{a_n}} |\cos \theta_n|^{1+a_n} = c_{b,n} |\cos \theta_n|^{1+a_n}. \quad (2.4-8)$$

In the previous expressions, the damping exponent a_n can take different values at each level, although current engineering practice favors the use of constant damper exponents throughout the whole structural system. Eqns. (2.4-3) and (2.4-8) allow conversion of the system in Figure 2-10(a) to the equivalent system in Figure 2-10(c). The diagonal angle should be measured coherently with drift. If positive drift is defined as depicted in Figure 2-10(b), the diagonal shown will experience positive lengthening and $\cos(\theta_n)$ must be positive; if the diagonal is placed reversely (from lower right corner to upper left corner), positive drift results in negative lengthening of the diagonal and $\cos(\theta_n)$ must be negative.

In either case, Eqns. (2.4-5) and (2.4-8) produce the same results, but Eqns. (2.4-3) and (2.4-7) are sign-sensitive. Similar equations can be derived for other mounting configurations [Constantinou et al., 2001].

Figure 2-10: Dampers mounted diagonally and conversion to storey coordinates.

2.4.8 Brace and auxiliary elements

Element type

Brace elements are subjected primarily to axial forces, with negligible bending moments; ideally, they should be elements with bi-axial symmetry, low slenderness and compact sections. Hot-finished hollow sections, square (SHSH) or circular (CHSH) are a good solution, as their radius of gyration (and therefore their buckling strength) is constant for any orientation. Hot-finishing is the most reasonable option, due to absence of residual stress, which allows these sections to be ascribed to buckling curve ‘a’ according to EN-1993-1:2005 [CEN, 2005]. Compact sections, preferably class 1 (according to the cited code), should be used; for SHSH this means that the ratio between width and thickness should be (roughly) smaller than $33 \cdot (235 \text{ MPa} / f_y)^{0.5}$, where f_y is the yield stress in MPa.

Their non-dimensional slenderness $\bar{\lambda}$, as defined in the mentioned code, should be smaller than 2. If axial forces are very high, catalogue sections might not be enough; in that case, welded box sections with thick plates can be a suitable alternative; the corresponding buckling curve is ‘c’ but very high strength values can be achieved.

Brace strength

As for the element strength, according to ASCE/SEI-7, the bracing elements (including their connections) must be sized to resist the forces corresponding to the average Maximum Considered Earthquake (MCE_R) response ($P_R = 2\%$, $T_L = 50$ years, $T_R = 2475$ years) with a safety factor $\gamma = 1.2$. The MCE_R spectrum is taken as 1.5 times the Design Based Earthquake (DBE) spectrum.

According to the new EN-1998-1 draft, the bracing elements (including their connections) must be sized to remain elastic under the forces corresponding to the design earthquake ($P_R = 5\%$, $T_L = 50$ years, $T_R = 475$ years) with a safety factor $\gamma = 1.5$.

There is no direct relation between both values (ASCE and CEN); assuming linearity (which is not realistic) the ASCE code would be equivalent to a safety factor $1.2 \cdot 1.5 = 1.8$.

Brace stiffness

Regarding stiffness, no condition is given in the mentioned codes, except for the indication that all elements contributing to the damping system flexibility must be accounted for in the structural model. The importance of auxiliary element stiffness is addressed in Chapter 5.

Other elements

Some critical elements of the system are generally provided by the manufacturer of the device. These include two clevises at either end of the device and brace extender (for connection with the main structure) and the corresponding spherical bearings. Frontal plates and bolts, gusset plates, etc. are not provided by the manufacturer.

2.4.9 Marketplace devices

Several manufacturers of viscous devices are currently available in the market. Three of them have been selected here as representative. All manufacturers offer a similar range of products. Particularly, all of them guarantee that the devices are maintenance-free and that the seals do not leak; fluid in viscous dampers is (generally) not pre-stressed, which largely prevents this problem. The issue of temperature dependency detected in the first devices tested [Constantinou & Symans, 1992] seems to have been solved, as all manufacturers claim that the device properties are unchanged for a very wide range of temperatures, typically -40°C to $+40^\circ\text{C}$.

Taylor Devices Inc.,

Taylor Devices offers viscous dampers with exponents a between 0.2 and 2.0. For seismic application, the manufacturer recommends $a = 0.3$, and for wind and earthquake $a = 0.5$. No limits are given for damper coefficients. The damper set includes edge clevis for attachment to the structure, one of them with a spherical bearing bore for adjustment tolerance. Forces range between 250 kN to 8,000 kN. Clevis-to-clevis length from 867 mm to 2,293 mm, with typical strokes from 150 mm (± 75 mm) to 250 mm (± 125 mm); strokes can be extended with a reduction in force. Cylinder diameters range between 114 mm and 565 mm. Service temperature ranges between -40°C to $+70^\circ\text{C}$. The devices are maintenance free.

Jarret Structures

Jarret structures provides pure viscous dampers with values of damper exponent a between 0.07 to 0.8, with recommended values between 0.1 and 0.2. No limits are given for damper coefficients or forces. The damper set includes clevis and end plates for attachment in the structure. Conventional sizes (from end-plate to end-plate) extend from 961 mm to 2,990 mm, with damper strokes ranging from 100mm (± 50 mm) to 600mm (± 300 mm) and cylinder diameters between 140mm and 360mm. The manufacturer claims that temperature ranges between -55°C to $+80^{\circ}\text{C}$ have no effect on damper performance, and that viscous silicon fluid presents no aging.

Maurer

Maurer provides dampers for use in civil engineering, with a very small exponent $a = 0.04$ (although other exponents up to 1.00 are possible), and forces between 500 kN and 8,000 kN. Although no information is given, from the force-displacement loops presented in the brochures it seems that the damper presents certain storage stiffness. The strokes range from 200 mm (± 100 mm) to 1200 mm (± 600 mm), and the length (excluding anchor devices) varies accordingly between 1,140 mm to 5,670 mm. The devices comply with the European Standard EN-15129. The service temperature ranges from -40°C to $+40^{\circ}\text{C}$. The devices are maintenance-free.

Other

Sumitomo Construction Company manufactures a viscous damping wall consisting on a thin steel box filled with viscous liquid, attached to a storey. This device presents pure viscous damping and has been used successfully in a number of applications in Japan.

2.4.10 Mathematical models for viscous dampers

Different mathematical models are available for viscous dampers. The models considered are summarized in Figure 2-11. The pure viscous damper (also known as dashpot) is shown in Figure 2-11(a); this model ignores the stiffness components of the damper and is enough for most practical applications. The Kelvin model (also known as Kelvin-Voigt or Voigt model), Figure 2-11(c), is the parallel combination of a pure damper and a spring, Figure 2-11(c); this model is suitable to represent the viscous damper inserted in a elastic structure, but it can also be used to approximate the response of the Maxwell model, Figure 2-11(d), in which the stiffness components of the damping system are considered in series with the viscous damper; the stiffness component k_d might contain different contributions (damper itself, auxiliary components, brace elements, etc., [Guo *et al.*, 2015]). Finally, Figure 2-11 (e) shows the standard solid model (also known as Zener model), that considers a Maxwell model in parallel with a spring and is suitable to represent the more complex behavior of damper, brace elements and storey. In these models the spring components associated with the damping system are assumed always as elastic, but the spring components representing the main structure might present yielding.

Figure 2-11. Different mathematical models for dampers and auxiliary elements: (a) pure viscous damper; (b) pure spring; (c) Kelvin model; (d) Maxwell model; (e) standard solid model.

The main features of these models are discussed by subjecting them to forced harmonic vibration, of amplitude $u_0 > 0$ and circular frequency ω ; this input excitation is defined by:

$$u = u_0 \sin(\omega t) \quad (2.4-9)$$

$$\dot{u} = u_0 \omega \cos(\omega t) = \omega \sqrt{u_0^2 - u^2} \quad (2.4-10)$$

$$\ddot{u} = -u_0 \omega^2 \sin(\omega t) = -\omega^2 u \quad (2.4-11)$$

Although this is a purely harmonic excitation, the resonant case (harmonic excitation at the system's frequency) is often considered representative of seismic transient input, where the structure is excited mostly on its fundamental modes.

Figure 2-12. Force-displacement loops under forced harmonic excitation for (a) pure viscous model; (b) pure spring model; (c) Kelvin model.

2.4.10.1 Pure viscous damper model

In the pure viscous damper model, Figure 2-11(a), the force response is proportional to an exponent of velocity, Eqn. (2.4-1) repeated here for convenience:

$$f = c |\dot{u}|^a \text{sgn}(\dot{u}), \quad (2.4-12)$$

using Eqn. (2.4-10), for the positive range of f_d and u :

$$f = c \omega^a \sqrt{(u_0^2 - u^2)^a}, \quad (2.4-13)$$

that can be expressed in the form:

$$\left[\left(\frac{f}{f_{d0}} \right)^2 \right]^{1/a} + \left(\frac{u}{u_0} \right)^2 = 1. \quad (2.4-14)$$

Eqn. (2.4-14) is the implicit form of a generalized super-elliptical loop displayed in Figure 2-12(a) and Figure 2-13, where f_{d0} is the maximum damping force, found at maximum velocity, i.e. zero displacement, as:

$$f_{d0} = c \omega^a u_0^a. \quad (2.4-15)$$

Integration of the differential work $dW = f du$ throughout the harmonic cycle renders the energy dissipated by damping, equal to the area enclosed by the loop in Figure 2-12 (a), which is dependent on excitation frequency [Constantinou *et al.*, 1998]:

$$W_d = \left(2^{2+a} \frac{\Gamma^2(1+a/2)}{\Gamma(2+a)} \right) c \omega^a u_0^{1+a}, \quad (2.4-16)$$

where Γ is the gamma function. The bracketed term is tabulated in Ramirez *et al.*, [2001]; here it is referred to as γ ,

$$\gamma = 2^{2+a} \frac{\Gamma^2(1+a/2)}{\Gamma(2+a)}. \quad (2.4-17)$$

For the linear case ($a = 1$),

$$W_{d,a=1} = \pi c \omega u_0^2. \quad (2.4-18)$$

A more meaningful non-dimensional form of Eqn. (2.4-16) is obtained normalizing the dissipated energy:

$$b = \frac{W_d}{\pi f_{d0} u_0}, \quad (2.4-19)$$

Using b , Eqn. (2.4-16) can be rearranged to

$$W_d = \pi b c \omega^a u_0^{1+a}, \quad (2.4-20)$$

For exponents ranging from 0.2 to 2.0, the value of b is given in Table 2-5; the closed-form expression for b is:

$$b = \frac{2^{2+a}}{\pi} \frac{\Gamma^2(1+a/2)}{\Gamma(2+a)} = \frac{\gamma}{\pi}, \quad (2.4-21)$$

The meaning of b is clear in Figure 2-13(b), as the quotient between areas of the corresponding non-linear viscous force-displacement loop and linear viscous loop with equivalent maximum force and displacement. Clearly, smaller exponents produce a better performance in terms of energy dissipation for equal values of force and displacement. The left plot shows hysteresis loops for different damping exponents with equal energy dissipation W_d and maximum displacement u_0 .

Table 2-5. Values of coefficients b and γ .

a	0	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.2	1.5	2
b	$4/\pi$	1.201	1.170	1.140	1.113	1.087	1.063	1.041	1.020	1	0.963	0.915	0.849
γ	4	3.774	3.675	3.582	3.496	3.416	3.341	3.270	3.204	π	3.027	2.876	2.667

Figure 2-13: Hysteresis loops for pure damping with different damping exponent a , but equal displacement u_0 and excitation frequency ω . (a) c adjusted for equal dissipated energy W_d (left); (b) c adjusted for equal damping force f_{d0} (right).

2.4.10.2 Kelvin model with linear viscous damping

In this case, Figure 2-11(c), a spring component, Figure 2-11(b), acts in parallel with the pure linear damper, Figure 2-11(a), with $a = 1$; compatibility conditions dictate equal displacement in both elements; equilibrium conditions impose that the applied force equals the addition of both components, $f = f_s + f_d$; finally, the constitutive equations are used to render the well-known expression:

$$f = k_s u + c \dot{u}, \quad (2.4-22)$$

and, after substitution of (2.4-10), it is found that:

$$f = k_s u + c \omega \sqrt{u_0^2 - u^2} = k_s u_0 \sin(\omega t) + c u_0 \omega \cos(\omega t). \quad (2.4-23)$$

Eqn. (2.4-23) represents an ellipse with its principal axis inclined; at maximum displacement u_0 the response is found to consist solely on the spring component f_{s0} , and at maximum velocity, i.e. zero displacement, there is no spring component and only pure damping f_{d0} :

$$f_{s0} = k_s u_0, \quad (2.4-24)$$

$$f_{d0,a=1} = c \omega u_0. \quad (2.4-25)$$

The maximum force is found by equating to 0 the derivative of Eqn. (2.4-23):

$$f_0 = k_s u_0 \cos \delta + c \omega u_0 \sin \delta = f_{s0} \cos \delta + f_{d0,a=1} \sin \delta, \quad (2.4-26)$$

$$\delta = \arctan(2\xi); \quad (2.4-27)$$

$$\xi = \frac{c \omega}{2k_s}. \quad (2.4-28)$$

In these expressions, δ is referred to as the phase lag angle, and ξ is the viscous damping ratio, discussed below. Maximum force happens at a point slightly out of phase with the maximum displacement. The corresponding displacement at maximum force is:

$$u_{f=f_0} = u_0 \frac{1}{\sqrt{1+(2\xi)^2}}. \quad (2.4-29)$$

Finally, the energy dissipated in one cycle is given by Eqn. (2.4-18), showing that the linear spring does not contribute to energy dissipation.

Effective damping ratio by Jacobsen's equation

The effective viscous damping ratio ξ can be expressed using (2.4-28). An alternative way, which is often used experimentally, is defined here. It requires that the system is subjected to resonant excitation ($\omega = \omega_1$ where ω_1 is the natural frequency of the SDOF system). For the linear case, W_d is given by Eqn. (2.4-18), with $\omega = \omega_1$. The elastic energy stored in the system at maximum displacement is:

$$W_s = k u_0^2. \quad (2.4-30)$$

Using Eqns. (2.4-18) and (2.4-30):

$$c = \frac{W_d}{2\pi W_s} \frac{k}{\omega}. \quad (2.4-31)$$

Recalling from SDOF dynamic considerations (see Section 2.5.1.1) that ξ represents the proportion between c and a critical value $c_{cr} = 2k/\omega$, the expression can be transformed to:

$$\xi = \frac{W_d}{4\pi W_s}. \quad (2.4-32)$$

This equation was first proposed by Jacobsen [1930, cited in Gülkan & Sozen, 1974]. It is a general expression most efficient to calculate the effective damping of a system, provided that it is subjected to resonant harmonic motion. This equation is profusely used in this text, and forms the theoretical basis of the concept of effective damping.

2.4.10.3 Kelvin model with non-linear viscous damping

In this case, because the spring component does not participate in the dissipative mechanism, the total energy dissipated in one cycle can be found using (2.4-20). The effective damping ratio can be expressed in different forms:

$$\xi = \frac{W_d}{4\pi W_{s,\max}} = \frac{1}{4\pi} \frac{b\pi f_{d0} u_0}{(1/2)k_s u_0^2} = \frac{1}{2} \frac{bc\omega^a u_0^{a-1}}{k_s} = \frac{1}{2} \frac{b f_{d0}}{f_{s0}}. \quad (2.4-33)$$

Eqn. (2.4-33) shows that the effective damping ratio for non-linear viscous damping is dependent on amplitude. The system constitutive equations are:

$$f = k_s u + c|\dot{u}|^a \operatorname{sgn}(\dot{u}), \quad (2.4-34)$$

$$f = k_s u + c\omega^a \sqrt{(u_0^2 - u^2)}^a = k_s u_0 \sin(\omega t) + c u_0^a \omega^a |\cos(\omega t)|^a \operatorname{sgn}(\cos^a(\omega t)). \quad (2.4-35)$$

The maximum force happens for the value of t_0 found equating the derivative of (2.4-35) to 0; in the range $0 \leq \omega t \leq \pi/2$, the trigonometric functions are positive, and upon derivation and reordering, the following expression is obtained:

$$\frac{\cos^{2-a}(\omega t)}{\sin \omega t} = \frac{2a\xi}{b}. \quad (2.4-36)$$

This equation does not present an explicit solution; an approximate solution is derived in [Ramirez *et al*, 2001], assuming that the solution takes place at a value of $\omega t = \pi - \delta$, where δ is a small phase angle. Using trigonometric equalities:

$$\frac{\sin^{2-a} \delta}{\cos \delta} = \frac{2a\xi}{b}, \quad (2.4-37)$$

and, for small values of δ :

$$\delta \approx \left(\frac{2a\xi}{b} \right)^{\frac{1}{2-a}}. \quad (2.4-38)$$

With this value replaced in Eqn. (2.4-35), and after substitution for the complementary angle:

$$f_0 \approx k_s u_0 \cos(\delta) + c u_0^a \omega^a \sin^a(\delta). \quad (2.4-39)$$

This equation can take different forms. First, introducing (2.4-24) and (2.4-15):

$$f_0 \approx f_{s0} \cos(\delta) + f_{d0} \sin^a(\delta). \quad (2.4-40)$$

This expression is often written as [Ramirez *et al*., 2001; Tsopelas *et al*., 1991]:

$$f_0 \approx f_{s0} CF_1 + f_{d0} CF_2, \quad (2.4-41)$$

with

$$CF_1 = \cos \delta, \quad (2.4-42)$$

$$CF_2 = \sin^a \delta, \quad (2.4-43)$$

where CF_1 and CF_2 are referred to as force coefficients, and this approximate method is referred to as the *force coefficient method*.

Dividing over mass the equation can also be expressed in terms of spectral acceleration, see [Ramirez *et al.*, 2001]. The displacement at maximum force is:

$$u_{f0} \approx u_0 \cos \delta, \quad (2.4-44)$$

Figure 2-14 shows hysteresis loops for systems with different values of a , calibrated to obtain similar damping force (left) or dissipated energy (right).

Figure 2-14: Hysteresis loop for Kelvin model.

2.4.10.4 Maxwell model for linear damping

The Maxwell model, Figure 2-11(d), represents the coupling between stiffness and pure viscous components of the damping system. It consists of a spring component and a pure viscous damper in series. The spring k_d represents the stiffness contribution k_i from each flexible element i in the damping system; for example, the damper itself, the cylinder and rod, compressibility of the fluid, clevises, front plates, and most importantly, the brace element. If there are S elements contributing, the equivalent stiffness is:

$$k_d = \left(\sum_{i=1}^S k_i^{-1} \right)^{-1} \quad (2.4-45)$$

The model is simply introduced here, and a more extensive treatment is given in Chapter 5. Equilibrium conditions impose that the force f in the set is the same as the force in the spring f_s and the force in the damper f_d :

$$f = f_d = f_s. \quad (2.4-46)$$

The system displacement is found by addition of damper and spring displacement components:

$$u = u_s + u_d = \frac{f_s}{k_d} + u_d. \quad (2.4-47)$$

Using Eqns. (2.4-46) and (2.4-47):

$$\dot{u} = \dot{u}_s + \dot{u}_d = \frac{\dot{f}_s}{k_d} + \frac{f_d}{c} = \frac{\dot{f}}{k_d} + \frac{f}{c}, \quad (2.4-48)$$

that can be expressed as:

$$c\dot{u} = f + \lambda \dot{f}, \quad (2.4-49)$$

where

$$\lambda = c / k_d \quad (2.4-50)$$

is known as the relaxation time, expressed in s, representing the timescale of force decay of the model. Eqn. (2.4-49) is the constitutive equation of the Maxwell model for linear damping ($a = 1$).

For forced harmonic excitation, substitution of Eqn. (2.4-10) in Eqn. (2.4-49) renders:

$$c u_0 \omega \cos(\omega t) = f + \frac{c}{k_d} \dot{f}, \quad (2.4-51)$$

with the following analytical solution for zero initial conditions:

$$f = A \exp\left(-\frac{t}{\lambda}\right) + \frac{c^2}{c^2 \omega^2 + k_d^2} \left[\omega^2 k_d u_0 \sin(\omega t) + \frac{k_d^2}{c^2} c u_0 \omega \cos(\omega t) \right]. \quad (2.4-52)$$

The first term of this solution is a transient component that vanishes with time. As can be seen, small values of λ accelerate this effect, thus the name relaxation time for this parameter. The second term is a steady-state component, which presents a cyclic behavior with vibration frequency equal to the excitation frequency. Force-displacement loops are presented in Figure 2-16(a). The steady-state term of the expression can be presented as:

$$f^{ss} = \frac{c^2 \omega^2}{c^2 \omega^2 + k_d^2} k_d u_0 \sin(\omega t) + \frac{k_d^2}{c^2 \omega^2 + k_d^2} c u_0 \omega \cos(\omega t), \quad (2.4-53)$$

$$f^{ss} = \frac{\omega^2 \lambda^2}{1 + \omega^2 \lambda^2} f_{s0} \sin(\omega t) + \frac{1}{1 + \omega^2 \lambda^2} f_{d0} \cos(\omega t). \quad (2.4-54)$$

Comparison of the last expression with (2.4-23) reveals that the Maxwell linear model can be represented by a Kelvin model in which the stiffness and damping coefficients are scaled, respectively, by:

$$\beta_s = \frac{c^2 \omega^2}{c^2 \omega^2 + k_d^2} = \frac{\omega^2 \lambda^2}{1 + \omega^2 \lambda^2}, \quad (2.4-55)$$

$$\beta_d = \frac{k_d^2}{c^2 \omega^2 + k_d^2} = \frac{1}{1 + \lambda^2 \omega^2}, \quad (2.4-56)$$

with

$$\frac{\beta_d}{\beta_s} = \frac{1}{\lambda^2 \omega^2}. \quad (2.4-57)$$

c is scaled by β_d ; therefore the effectiveness of damping is dependent on this scaling factor. Introducing the notation c' and k' for the equivalent Kelvin damping and stiffness:

$$k' = k_d \beta_s, \quad (2.4-58)$$

$$c' = c \beta_d, \quad (2.4-59)$$

all equations presented for the Kelvin model apply upon substitution of k_d and c by their modified counterparts:

$$f'_{s0} = k' u_0 = \beta_s k u_0 = \beta_s f_{s0}, \quad (2.4-60)$$

$$f'_{d0} = c' \omega u_0 = \beta_d c \omega u_0 = \beta_d f_{d0, a=1}, \quad (2.4-61)$$

$$\xi' = \frac{c' \omega}{2k'} = \frac{\beta_d}{\beta_s} \frac{c \omega}{2k_d} = \frac{\beta_d}{\beta_s} \xi \quad (2.4-62)$$

$$f'_0 \approx k' u_0 \cos \delta' + c' \omega u_0 \sin \delta' = \beta_s f_{s0} \cos \delta' + \beta_d f_{d0, a=1} \sin \delta', \quad (2.4-63)$$

$$\delta' = \arctan(2\xi'); \quad (2.4-64)$$

$$u_{f0} \approx u_0 \frac{1}{\sqrt{1 + (2\xi')^2}}. \quad (2.4-65)$$

$$W'_d = \pi c' \omega u_0^2 = \beta_d W_d. \quad (2.4-66)$$

The notation u_{f0} has been introduced to refer to the displacement u for which the peak force f_0 is reached. β_s and β_d in expressions (2.4-55) and (2.4-56) add up to unity; therefore they are indices representing predominance of the spring behavior or damper behavior. The terms $k\beta_s$ and $c\omega\beta_d$ are often referred to as storage stiffness and loss stiffness. If

$\beta_s = 1$, then the response is similar to the one provided by the spring element without damping. If $\beta_d = 1$ the response is dominated by the damper with no influence of spring elements. Plots of β_s and β_d as a function of period for different values of λ are shown in Figure 2-15. It can be seen that:

1. The behavior of the set is frequency-dependent: For low periods (high frequencies), the behavior is always dominated by the spring element; for long periods (low frequencies), the behavior is always dominated by the damper.
2. Increasing values of λ reduce the effectiveness of the damper element in terms of dissipated energy.

Consequently, for structures with viscous dampers, forced harmonic excitation at higher modes frequencies results in very small amounts of added viscous damping. For these modes, however, the damping system will feature high spring stiffness. This behavior is apparent in harmonic tests. Further discussion is presented in Chapter 5.

Figure 2-15. Coefficients indicating prevalence of spring behaviour (β_s) or damper behaviour (β_d) in a Maxwell-type bracing element subjected to forced harmonic vibration, for different values of the relaxation time λ : (a) $\lambda = 0.01s$; (b) $\lambda = 0.02s$; (c) $\lambda = 0.05s$; (d) $\lambda = 0.1s$.

Figure 2-16. Force-displacement loops for Maxwell systems under forced harmonic excitation; (a) linear damping; (b) non-linear damping.

2.4.10.5 Maxwell model for non-linear viscous damping

For the non-linear case ($a \neq 1$), and using Eqn. (2.4-12) the damper velocity can be expressed as:

$$\dot{u}_d = \frac{|f_d|^{1/a}}{c^{1/a}} \text{sgn}(f_d). \quad (2.4-67)$$

Introducing this expression in Eqn. (2.4-47):

$$\dot{u} = \dot{u}_s + \dot{u}_d = \frac{\dot{f}_s}{k_d} + \frac{|f_d|^{1/a}}{c^{1/a}} \text{sgn}(f_d) = \frac{\dot{f}}{k_d} + \frac{|f|^{1/a}}{c^{1/a}} \text{sgn}(f), \quad (2.4-68)$$

$$\dot{f} = k_d \dot{u} - \frac{k_d}{c^{1/a}} |f|^{1/a} \text{sgn}(f), \quad (2.4-69)$$

Eqn. (2.4-68) – or (2.4-69) – is the differential motion equation for the Maxwell damper. For harmonic motion:

$$u_0 \omega \cos(\omega t) = \frac{\dot{f}}{k_d} + \frac{|f|^{1/a}}{c^{1/a}} \text{sgn}(f); \quad (2.4-70)$$

$$\dot{f} = k_d u_0 \omega \cos(\omega t) - \frac{k_d}{c^{1/a}} |f|^{1/a} \text{sgn}(f). \quad (2.4-71)$$

These equations present two limit cases:

- for small values of k_d , the first term on the right side of Eqn. (2.4-70) predominates, and after integration, the equation reduces to $f = k_d u$, that is, a pure spring with small stiffness and no damping;
- for large values of k_d , the first term on the right side of Eqn. (2.4-70) vanishes and the equation reduces to the pure non-linear damper.

Loops of force-displacement for this case are presented in Figure 2-16(b).

2.4.10.6 Standard solid model for linear dampers

The standard linear solid model shown in Figure 2-11(e) applies to a linear Maxwell system (damping system composed of linear viscous damper and stiffness component), inserted within a structure; by equilibrium considerations the total force is the addition of the damping system force (brace with damper) and the storey force:

$$f = f_M + f_s. \quad (2.4-72)$$

Here the term f_M refers to the response force in the damping system (subscript M stands for 'Maxwell'). Replacement of damping system force f_M by Eqn. (2.4-69), and storey force f_s by the spring equation renders:

$$f + \lambda \dot{f}_M = k_s u + c \dot{u}. \quad (2.4-73)$$

Noticing that

$$\dot{f}_M = \dot{f} - \dot{f}_s = \dot{f} - k_s \dot{u}, \quad (2.4-74)$$

the following model equation is found:

$$f + \lambda \dot{f} = k_s u + \lambda(k_d + k_s)\dot{u}. \quad (2.4-75)$$

When subjected to forced harmonic excitation, replacement of Eqns. (2.4-9) and (2.4-10) into Eqn. (2.4-75) results in

$$f + \lambda \dot{f} = k_s u_0 \sin \omega t + \lambda(k_d + k_s)u_0 \omega \cos \omega t. \quad (2.4-76)$$

The explicit solution to this differential equation with zero initial conditions is of the form:

$$f = -\frac{\lambda \omega}{1 + \lambda^2 \omega^2} k_d \exp\left(\frac{-t}{\lambda}\right) + u_0 \left(k_s + \frac{\lambda^2 \omega^2}{1 + \lambda^2 \omega^2} k_d \right) \sin \omega t + u_0 \omega \left(\frac{c}{1 + \lambda^2 \omega^2} \right) \cos \omega t, \quad (2.4-77)$$

where the first term is the transient part that vanishes with time; the steady-state solution adopts the following form:

$$f^{ss} = u_0 k_s \sin \omega t + \beta_s u_0 k_d \sin \omega t + \beta_d u_0 \omega c \cos \omega t, \quad (2.4-78)$$

that is, the same form as the linear Maxwell solution, Eqn. (2.4-54), with the first added term due to the storey stiffness. β_s and β_d are given by Eqns. (2.4-55) and (2.4-56). The first and second terms do not participate in energy dissipation, and therefore the dissipated energy and viscous damping ratio of the system are independent from the storey stiffness, as long as the forced displacement u_0 and excitation frequency ω are the same. It is clear

that the damping effectiveness is only impaired by k_d ; therefore, the expressions derived for the Maxwell model are valid.

Figure 2-17 displays the hysteresis loop corresponding to Eqn. (2.4-78) for four cases with identical excitation frequency, amplitude and damping coefficient, but different values of k_s and k_d . The plots show how, in all cases, the relative energy dissipation is independent of k_s . If k_d presents high values, β_s is very small (bottom plots), and the global stiffness is primarily influenced by k_s ; for moderate values of k_d , the contribution of the brace is considerable and the global stiffness is participated by k_s and k_d .

Figure 2-17. Force-displacement loops for standard linear solid model (SSM) under forced harmonic excitation with different values of k_d .

2.4.10.7 Standard solid model for non-linear dampers

In this case, the viscous damping law is non-linear. Building upon the equilibrium equation, (2.4-70), after derivation with respect to time:

$$\dot{f} = \dot{f}_M + \dot{f}_s. \quad (2.4-79)$$

The first term on the right side corresponds to the Maxwell model, and can be replaced by Eqn. (2.4-69), resulting in:

$$\dot{f} = k_d \dot{u} - \frac{k_d}{c^{1/a}} |f_M|^{1/a} \text{sgn}(f_M) + k_s \dot{u}. \quad (2.4-80)$$

Noticing that

$$f_M = f - f_s = f - k_s u, \quad (2.4-81)$$

Eqn. (2.4-80) can be written as:

$$\dot{f} = (k_d + k_s) \dot{u} - \frac{k_d}{c^{1/a}} |f - k_s u|^{1/a} \text{sgn}(f - k_s u). \quad (2.4-82)$$

This is the defining differential equation for the non-linear standard solid model. The following change of variable can be performed:

$$F = f - k_s u; \quad (2.4-83)$$

whereupon Eqn. (2.4-82) can be rearranged as:

$$\dot{F} = k_d \dot{u} - \frac{k_d}{c^{1/a}} |F|^{1/a} \text{sgn}(F), \quad (2.4-84)$$

which is the equation of a Maxwell model, Eqn. (2.4-69), with parameters a , c , ω , u_0 , k_d . That is, the solution to Eqn. (2.4-84) can be found solving the equivalent Maxwell model with parameters a , c , ω , u_0 , k_d , and adding the term $k_s \cdot u$ due to the storey stiffness. Further discussion is offered in Chapter 5.

2.4.11 Methods of analysis

Different solution methods are available for structures with damping systems. The methods are briefly described here. A thorough treatment of the response spectrum analysis is given in the next Section.

Non-linear time history analysis (TH or NLTHA)

NLTHA is the more general form to solve the motion equations for systems involving any type of non-linearity. Essentially, the differential motion equations (described in next Section) are solved by numerical time-stepping techniques. Starting with known initial conditions (generally a motionless structure), all non-linear effects are assumed as constant for each time step; the linearized motion equations are solved, and the properties of the system are updated at the end of the step. Time increments must be very small in order to achieve convergence. Several methods to check convergence exist. In this study, an energy method proposed by Filiatrault & Christopoulos [2006] is used for THNLA (see Annex E).

NLTHA is only valid to find the response to a specific excitation. For soft spectra, such as code-defined spectra, a set of motions must be assembled that are representative of the chosen spectrum. For structures with added dampers, typically 7 or more accelerograms are selected. The accelerograms must be scaled so that their averaged spectrum matches

the properties of the target spectrum within certain tolerances. THNLA is performed for each accelerogram; peak values for variables of interest are then retrieved and treated statistically to find the overall response. For design earthquake levels the mean can be used, although most authors use the median calculated under the assumption of lognormal distribution of results.

Incremental dynamic analysis (IDA)

In this technique, THNLA is used with increasing levels of scaling for the corresponding accelerogram. Generally the process is continued until structural collapse is reached. The results are expressed as a curve relating *IM* (intensity measure, generally spectral acceleration at T_1 and 5% damping) to *DM* (damage measure, for instance inter-storey drift ratio, etc.). This is referred to as an IDA curve.

The process is repeated for all accelerograms in a representative suite, ideally for a similar set of values of *IM*. For a certain value of *IM*, the *DM* results present a lognormal distribution, and statistical results can be used to obtain the probability of exceedance of different values of *DM*. The most interesting result is the fragility curve for collapse, expressing the probability of reaching collapse for every intensity measure.

Response Spectrum Analysis (RSA or NLRSA)

The original system is decomposed into several single-degree-of-freedom (SDOF) systems using modal analysis. The modal properties of every system are calculated and the peak response for each one is found using response spectra. Finally, individual responses are combined using a combination rule. A detailed description follows.

Strictly speaking, modal analysis can only be applied to linear systems. Therefore, the RSA method is approximate. The properties of each SDOF system are modified in approximate ways according to the degree of inelasticity of the system and the added viscous dampers, which increases the error. This is discussed further in Chapter 3.

Lateral force (ELF)

Equivalent lateral force (ELF) is a further simplification of RSA. Acknowledging that higher modes are of minor importance to the system's response, their effect is combined in a single 'residual mode', with fictitious modal properties.

Energy-based analysis (EBA)

Energy-based analysis uses the energy concept to find the response. The hysteretic energy (energy dissipated by damage) can be obtained as the earthquake energy input at the end of motion minus the energy dissipated by viscous damping. This energy demand is compared to the energy capacity (the capacity to dissipate energy by hysteresis). Drift demands and other variables are obtained by assimilating the structure to non-linear SDOF systems and equating the energy demand to the energy dissipated in a certain number of cycles at maximum displacement. This technique is promising, but not fully developed yet.

2.5 RSA applied to systems with FVD

2.5.1 Elastic Single Degree-of-Freedom (SDOF) systems

2.5.1.1 Elastic SDOF systems without added damping

The basic motion equation for an elastic SDOF system [Chopra, 2007] can be written as:

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = -ma_g(t), \quad (2.5-1)$$

where t indicates time, m is the system vibrating mass (kg), c is the viscous damping coefficient (N·s/m), k is the system stiffness (N/m), u is the relative displacement between base and mass, a_g the ground acceleration (m/s/s). The corresponding undamped free-vibration problem corresponds to the same equation without the damping term on the left side and equating 0 on the right side. The solution to the undamped free-vibration problem returns the undamped natural frequency ω , or undamped natural period T , the most important dynamic system properties:

$$\omega = \sqrt{k/m}, \quad (2.5-2)$$

$$T = 2\pi\sqrt{m/k}. \quad (2.5-3)$$

Using these expressions, Eqn. (2.5-1) can be expressed in normalized form:

$$\ddot{u} + 2\xi\omega\dot{u} + \omega^2 u = -a_g(t), \quad (2.5-4)$$

$$\ddot{u} + 2\xi\left(\frac{2\pi}{T}\right)\dot{u} + \left(\frac{2\pi}{T}\right)^2 u = -a_g(t), \quad (2.5-5)$$

where ξ is the viscous damping ratio, relating the damping coefficient c to a critical damping coefficient c_{cr} :

$$\xi = \frac{c}{c_{cr}} = \frac{c}{2m\omega} = \frac{c\omega}{2k}. \quad (2.5-6)$$

Eqn. (2.5-5) shows that the system behavior is totally defined by its period $T (= 2\pi/\omega)$ and damping ratio ξ . When a_g is a recorded time history acceleration, this equation can be solved numerically [Nigam & Jennings, 1969] to obtain u . The peak relative displacement, relative velocity and absolute acceleration response are defined as

$$S_d = |u|_{\max}, \quad (2.5-7)$$

$$S_v = |\dot{u}|_{\max}, \quad (2.5-8)$$

$$S_a = |\ddot{u} + a_g|_{\max}. \quad (2.5-9)$$

Generally ‘relative displacement’ is referred to simply as ‘displacement’, ‘relative velocity’ as ‘velocity’, and ‘absolute acceleration’ as acceleration. The reason is that, for structural damage and verifications, the interesting variable is the displacement relative to the ground, whereas to evaluate inertial forces the absolute acceleration is needed.

The peak displacement occurs at a time of zero velocity. The acceleration at this time, known as pseudo-acceleration, is derived from Eqn. (2.5-4):

$$PS_a = \omega^2 S_d. \quad (2.5-10)$$

Due to the presence of a viscous component (damping), pseudo-acceleration is not the maximum system acceleration S_a , but is generally accepted as a good approximation. For zero damping, $S_a = PS_a$. Pseudo-velocity is defined as:

$$PS_v = \omega S_d = S_a / \omega, \quad (2.5-11)$$

which is an approximation to the actual maximum velocity S_v , albeit a poor one as will be shown in Chapter 4. The peak restoring force of the system is $k \cdot S_d$; using Eqn. (2.5-2):

$$f_0 = m PS_a \quad (2.5-12)$$

The solution of Eqn. (2.5-1) for the same acceleration history a_g and different systems (characterized by different periods and damping ratio), gives place to the spectral displacement function $S_d(T, \xi, a_g)$, and using Eqns. (2.5-10) and (2.5-11) to spectral pseudo-acceleration $PS_a(T, \xi, a_g)$ and spectral pseudo-velocity $PS_v(T, \xi, a_g)$; these functions return peak displacement, pseudo-acceleration and pseudo-velocity, respectively. In conventional engineering practice, spectral pseudo-acceleration is generally referred simply to as ‘spectral acceleration’ and labeled as $S_a(T, \xi, a_g)$. To simplify the notation, the last term (a_g) may be dropped. In this text a difference is made between acceleration and pseudo-acceleration.

The graphical display of the spectral functions is known as spectrum. Figure A-8 in Annex A shows the displacement and acceleration spectrum for a single motion. If a hazard analysis is carried out taking into account all magnitudes and faults contributing to a given site seismicity, a softened uniform hazard spectrum can be obtained, that presents spectral ordinates with a specific probability of being exceeded over a certain time span. Design spectra are a simplification of uniform hazard spectrum with standard shapes that can be evaluated according to mapped values of spectral acceleration or ground acceleration, generally given for 5% damping ratio. The main features of spectral shapes were first introduced by Newmark & Hall [1982] and acknowledge the existence of three spectral regions (acceleration-constant, velocity-constant and displacement-constant) which affect short, medium and large period systems, respectively. A design spectrum for a given site can be matched with a collection of ground motions, as shown in Figure A-5 in Annex A. These deterministic spectra can be used to represent design seismicity at a specific site, with softened shapes when compared to the jagged shapes of simple motion spectra. Both ASCE SEI-7-16 [ASCE 2016] and Eurocode 8 [CEN 2004] accept this representation of

earthquake action. If the elastic spectrum for a certain damping ratio is available, the peak displacement and accelerations are directly available. Internal forces are derived from displacements (multiplying by the stiffness) and inertial forces from accelerations (multiplying by mass). Use of design elastic spectra is the most extended technique for the solution of building structures subjected to earthquakes.

If the viscous damping ratio is different from the standard 5%, the spectrum ordinates need correction. This is generally done through scaling by a damping correction factor (DCF) for displacement η_d , which can be made to depend on several parameters; primarily, but not only, added damping ratio ξ and period T . By application of η_d to S_d , in conjunction with Eqn. (2.5-10), the corrected ordinate for PS_d is also obtained.

Spectrum in ADRS format

Because spectral displacement and acceleration are related through Eqn. (2.5-10), a parametric plot of spectral acceleration versus spectral displacement (PS_d-S_d) can be produced, termed as acceleration-displacement response spectra (ADRS), hereby referred to as AD format. In this plot, lines radiating from the origin with slope ω^2 are representative of a single period, with a vertical line corresponding to zero period and a horizontal line corresponding to infinite period.

Such a plot is displayed in Figure 2-18 for a single accelerogram (000414XA, see Annex A) and the Eurocode 8 [CEN, 2004] elastic spectrum. Spectra are displayed for three different damping ratios (5%, 10%, 20%), calculated using the current EC8 expression for η (see Chapter 4). The beneficial effect of damping in reduction of displacement and acceleration is clear. Moreover, another interesting trend is that the reduction obtained by a 5% to 10% increase in damping ratio is very similar to the reduction obtained by a 10% to 20% increase, indicating that added damping is relatively more effective for small values of added damping; several authors point at 30% as a reasonable upper limit. Most building structures will be represented by the section comprised between T_B and T_E .

Figure 2-18. Elastic spectra for single accelerogram (000414XA) and Eurocode 8 in AD format; 5%, 10% and 20% damping ratios.

The ADRS format was introduced by Mahaney *et al.* [cited in Freeman, 2004], and has since become the standard format for all non-linear static methods [Freeman, 1998; Chopra & Goel, 2002; Fajfar, 2000; ATC, 1996; FEMA, 2000b; FEMA, 2005]. Currently several terminologies (AD diagram, AD format or ADRS) are used for the same plot. The term ‘diagram’ is deemed more accurate than the term ‘spectrum’ by some authors [Chopra & Goel, 1999], as spectrum is generally reserved for plot of quantities versus period or frequency.

Eurocode 8 elastic spectral functions

The following elastic spectral displacement functions are defined in Eurocode 8 [CEN 2004] and used throughout this document:

$$0 \leq T \leq T_B : \quad S_d(T, \xi) = (T/2\pi)^2 PHA [1 + T(2.5\eta - 1)/T_B] \quad (2.5-13)$$

$$T_B \leq T \leq T_C : \quad S_d(T, \xi) = (T/2\pi)^2 PHA 2.5\eta \quad (2.5-14)$$

$$T_C \leq T \leq T_D : \quad S_d(T, \xi) = (T/2\pi)^2 PHA 2.5\eta T_C / T \quad (2.5-15)$$

$$T_D \leq T \leq T_E : \quad S_d(T, \xi) = (T/2\pi)^2 PHA 2.5\eta T_C T_D / T^2 \quad (2.5-16)$$

$$T_E \leq T \leq T_F : \quad S_d(T, \xi) = \left(\frac{T}{2\pi} \right)^2 PHA \left[2.5\eta + \frac{T - T_E}{T_F - T_E} (1 - 2.5\eta) \right] T_C T_D / T^2 \quad (2.5-17)$$

$$T_F \leq T : \quad S_d(T, \xi) = (T/2\pi)^2 PHA T_C T_D / T^2 = PHD \quad (2.5-18)$$

where PHA is the peak ground horizontal acceleration, η is a damping coefficient dependent on ξ , and T_B , T_C , T_D , T_E , T_F are period limits also called corner periods: T_B and T_C define the constant acceleration region, T_C and T_D the constant velocity region, T_D and T_E the constant displacement region, up to T_F where the spectral displacement is coincident with the peak horizontal displacement (PHD), see Figure 2-18. PS_a and PS_v can be found using Eqns. (2.5-10) and (2.5-11). PS_a is only defined for periods below T_E ; expressions are obtained simply removing the term $(T/2\pi)^2$ from the right hand side of Eqns. (2.5-13) to (2.5-16). Normalized plots of these spectral functions are given in Annex C. Two types of spectra are defined in Eurocode 8, depending on the seismicity of the regions. PHA is usually calculated from a mapped acceleration a_{gR} (design peak base acceleration for a 475 year return period measured on rock) modified to account for site class and building importance.

Treatment of inherent damping

Inherent damping in SDOFs is defined by c in Eqn. (2.5-1). This parameter contains multiple contributions from different phenomena [Kareem & Gurley, 1996]: viscous components, microscopic material friction, connection friction, heat radiation, aerodynamic phenomena, etc, not only from structural elements but also from other constructive units, including soil interaction effects. These components are wrapped up

into a term that, purely for mathematical convenience, is assumed to be velocity proportional. For the aforementioned reasons, inherent damping is not established analytically, but rather defined in terms of other measurable structural properties. Two definitions are common:

1. As a fraction of a structural-dependent critical value, see Eqn. (2.5-6).
2. As a ratio between dissipated energy and stored energy, Jacobsen's expression, see Eqn. (2.4-32).

Inherent damping has a beneficial effect in response reduction; because of its variability, caution must be exerted in its assignment.

2.5.1.2 Elastic SDOF systems with added viscous dampers

The general equation for SDOF systems is obtained from Eqn. (2.5-1) adding to the right the term due to the added damping contribution:

$$m\ddot{u}(t) + c_I \dot{u}(t) + k u(t) + c_a |\dot{u}(t)|^a \text{sgn}(\dot{u}) = -m a_g(t), \quad (2.5-19)$$

where c_I is the inherent damping coefficient, c_a the added damping coefficient and a the added damper exponent.

For linear dampers ($a = 1$) Eqn. (2.5-19) is expressed as:

$$m\ddot{u}(t) + (c_I + c_a) \dot{u}(t) + k u(t) = -m a_g(t). \quad (2.5-20)$$

This is simply a version of Eqn. (2.5-1). Using Eqn. (2.5-6):

$$\xi = \xi_I + \xi_a = \frac{c_I}{2m\omega} + \frac{c_a}{2m\omega}, \quad (2.5-21)$$

and Eqn. (2.5-19) can be expressed in normalized form as:

$$\ddot{u} + 2(\xi_I + \xi_a) \left(\frac{2\pi}{T} \right) \dot{u} + \left(\frac{2\pi}{T} \right)^2 u = -a_g. \quad (2.5-22)$$

This expression can be solved using the response spectrum, modified for the total level of damping ξ . This modification is performed using damping correction factors. The displacement correction factor η_d is defined as the ratio between elastic spectral displacement and 5% damped elastic spectral displacement, both evaluated at the corresponding period:

$$\eta_d(T, \xi) = \frac{S_d(T, \xi)}{S_d(T, 5\%)}. \quad (2.5-23)$$

Thus:

$$S_d(T, \xi) = \eta_d(T, \xi) S_d(T, 5\%). \quad (2.5-24)$$

Spectral pseudo-acceleration and pseudo-velocity can be found by application of Eqns. (2.5-10) and (2.5-11). However, several authors [Pekcan et al, 1999; Lin & Chang, 2003] have shown that, for increasing levels of damping the differences between pseudo-acceleration and acceleration increase. For pseudo-velocity and velocity, the differences are even larger. For this reason, specific correction factors must be derived for velocity and acceleration. This topic is further addressed in Chapter 4.

2.5.2 Inelastic SDOF systems

2.5.2.1 Inelastic SDOF without added damping

The motion equation for inelastic systems in conventional and normalized form can be expressed as [Clough & Penzien, 2003; Chopra, 2007; Guyader & Iwan, 2006]:

$$m\ddot{u}(t) + c\dot{u}(t) + f_s = -m a_g(t), \quad (2.5-25)$$

$$\ddot{u}(t) + 2\xi\omega\dot{u}(t) + f_s / m = -a_g(t), \quad (2.5-26)$$

where f_s is the inelastic restoring force response. f_s is not a linear function of displacement; instead it is dependent on both current displacement and load path. Eqn. (2.5-25) can only be solved rigorously by numerical step-by-step procedures, such as the Newmark-beta method [Bathe, 1996; Chopra, 2007].

Generally the relationship between f_s and u is defined through a hysteretic law; the simplest one is the bilinear elastoplastic law, Figure 2-19(c), defined by elastic behavior up to yield point b, (where yield force is f_y and yield displacement is u_y), with elastic stiffness k ($= f_y/u_y$); after yielding (point b), the stiffness falls to a fraction $\alpha \cdot k$ of the initial one; α is generally referred to as post-yield stiffness ratio. After reaching the peak point c at f_m , u_m , the system unloads following a path parallel to the initial one. In the reverse cycle, the yield force depends on the peak value (point c), and the initial yield strength f_y , so that $f_m - f_{y,neg} = 2f_y$. A special case of the elastoplastic system is the elastic-perfectly-plastic system, with $\alpha = 0$, i.e., no post-yield stiffness.

Figure 2-19. Elastoplastic SDOF system: (a) monotonic force-displacement curve; (b) normalized monotonic curve; (c) hysteretic behaviour.

2.5.2.2 Equivalent linearization by strength and ductility factors

An approach for the direct solution of Eqn. (2.5-25) is the equivalent linearization, that is, replacement of the nonlinear system by a linear system with effective properties that performs as the original system in one or several aspects. Assuming that points c and d in Figure 2-19(a) correspond to the same ground excitation, the following quantities can be defined:

$$R_\mu = f_0 / f_y, \quad (2.5-27)$$

$$\mu = u_m / u_y, \quad (2.5-28)$$

where R_μ is the strength reduction factor (relating maximum strength in the elastic system to yield strength in the inelastic system), and μ is the ductility demand. Using basic considerations, the relationship between the elastic system and the inelastic system forces and displacements can be expressed as:

$$f_m = \frac{1 + \alpha(\mu - 1)}{R_\mu} f_0, \quad (2.5-29)$$

$$u_m = \frac{\mu}{R_\mu} u_0 \quad (2.5-30)$$

Using these expressions, the response of the non-linear SDOF system can be calculated from the linear SDOF, i.e., the elastic spectral response for 5% damping. Using Eqn. (2.5-12) on Eqn. (2.5-29),

$$PS_a = \frac{1 + \alpha(\mu - 1)}{R_\mu} PS_{ae}, \quad (2.5-31)$$

$$S_d = \frac{\mu}{R_\mu} S_{de} = \frac{\mu}{1 + \alpha(\mu - 1)} \left(\frac{T}{2\pi} \right)^2 PS_a, \quad (2.5-32)$$

where PS_{ae} and S_{de} are the elastic spectral pseudo-acceleration and displacement, respectively. For the elastic-perfectly-plastic case ($\alpha = 0$) the equations reduce to:

$$PS_a = \frac{PS_{ae}}{R_\mu}, \quad (2.5-33)$$

$$S_d = \frac{\mu}{R_\mu} S_{de} = \mu \left(\frac{T}{2\pi} \right)^2 PS_a, \quad (2.5-34)$$

The inelastic system is referred to the corresponding elastic system by two parameters, μ and R_μ . If one of them is fixed, the other has to be calculated, albeit with different procedures.

If R_μ is fixed, f_y can be calculated from the maximum response of the elastic system f_0 with no need for iteration, and the corresponding inelastic system (defined by T , ξ , and f_y) can be analyzed, u_m can be calculated and the ductility is found from Eqn. (2.5-28). With this procedure, the spectrum for a fixed value of R_μ can be easily obtained, and is referred to as constant R_μ spectra. If only μ is fixed, however, the spectral response must be calculated by successive iterations. The spectral response of the elastic system f_0 is known. The value of R_μ must be changed iteratively, and in every step f_y is determined with Eqn. (2.5-27), the non-linear response u_m is calculated, and the ductility μ_1 is assessed with Eqn. (2.5-28), until the difference between μ and μ_1 is smaller than a desired tolerance.

Constant ductility spectra for two values of ductility ($\mu = 1, 2$ and 4) are shown in Figure 2-20 for motion 000414XA. For the PS_a spectrum, (a) and (c), it is clear that ductility has a reduction effect, with the direct consequence that structural forces are diminished when comparing equal-period elastic and inelastic systems. The following trends are observed:

- Throughout the velocity and displacement period range, PS_a can be obtained approximately reducing the elastic spectrum by μ . This implies that μ is approximately equal to R_μ . This observation is confirmed by the S_d spectrum, which takes similar values to the elastic spectrum without regard to the ductility value. This observation is the underlying principle to the equal displacement rule. At very long periods (very flexible structures), R_μ is rigorously equal to μ .
- As the period approaches 0 (rigid structures), the value of R_μ falls to 1; the force reduction is virtually negligible; however the displacement demand increases further.

Figure 2-20. Motion 000414XA, constant ductility spectra ($\mu = 1, 2$ and 4).

Eqns. (2.5-33) and (2.5-34) give the exact expressions to calculate S_d and S_a for an inelastic system from the elastic spectral values S_{de} and S_{ae} , using R_μ and μ ; dependency of these two parameters from the fundamental period of the system T has been studied by several researchers. An excellent review is given by Miranda & Bertero [1994]. Because the method is not suitable for analysis of systems with added damping, only the most relevant results are summarized hereby:

Nassar & Krawinkler [1992]

This study is reported because is the only one that incorporates the effect of post-yield stiffness ratio α . The expressions were derived from statistical studies performed on 15 Western U.S. strong ground motions recorded on firm soils, using SDOF with periods T ranging from 0.1s to 4.0s, target ductility ratios μ from 1 to 8, and strain hardening ratios $a = 0, 2\%$ and 10% , through a least square fit regression process.

$$R_\mu = [1 + c(\mu - 1)]^{1/c}, \quad (2.5-35)$$

$$c = \frac{T^a}{1 + T^a} + \frac{b}{T}; \quad (2.5-36)$$

where a and b are parameters dependent on α , listed in Table 2-6.

Table 2-6. Coefficients for Equation (2.5-36), adapted from Nassar & Krawinkler [1992].

α	a	b
0.00	1.00	0.42
0.02	1.00	0.37
0.10	0.80	0.29

Vidic et al., [1994]

In this study the authors derive strength factors using 20 records from California and Montenegro, and additional records from Friuli 1976 and Chile, 1985. The factor R_μ was found to be mildly dependent from T in the velocity and displacement range, and roughly equal to μ . For individual records, peaks appear in R_μ . The expressions were derived for two types of SDOF systems (bilinear and degrading stiffness), with post-yield stiffness ratio $\alpha = 10\%$. Inherent damping ratio was assumed as 5% and two different inherent damping models (mass-proportional, and effective stiffness-proportional) were used. In the second type viscous damping varies as the effective stiffness varies during time history. The expressions proposed are:

$$T \leq T_0, \quad R_\mu = 1 + a(\mu - 1)^b \frac{T}{T_0}; \quad (2.5-37)$$

$$T \leq T_0, \quad R_\mu = 1 + a(\mu - 1)^b; \quad (2.5-38)$$

$$T_0 = c\mu^d T_C \leq T_C, \quad (2.5-39)$$

where T_C is the corner period between acceleration and velocity ranges. The parameters in the expression are listed in Table 2-7.

Table 2-7. Coefficients for Eqns. (2.5-37), (2.5-38), (2.5-39), adapted from Vidic *et al.* [1992].

Hysteresis Type	Damping Type	a	b	c	d
Degrading	Mass-proportional	1.00	1.00	0.65	0.30
	Stiffness-proportional	0.75	1.00	0.65	0.30
Bilinear	Mass	1.35	0.95	0.75	0.20
	Stiffness-proportional	1.10	0.95	0.75	0.20

A simpler version of these expressions was proposed by Fajfar [2000]:

$$T \leq T_C, \quad R_\mu = 1 + (\mu - 1)T / T_C; \quad (2.5-40)$$

$$T_C < T, \quad R_\mu = \mu. \quad (2.5-41)$$

These expressions correspond approximately to the case of degrading systems with mass proportional inherent damping and have been used in the context of the N2 method [Fajfar, 2000], reported below.

Miranda

Miranda [1993] focused on the influence of the variability in local site conditions, magnitude and epicentral distance on strength reduction factors. According to their characteristics, 124 ground motions were divided in three groups corresponding to rock, alluvium and soft soil deposits; the motions were also binned by magnitude and epicentral distance. Strength reduction factors were obtained for SDOF systems with bilinear hysteretic characteristics, target ductility between 2 and 6 and 5% inherent damping ratio. It was found that strength reduction factors were influenced by site class, but not by magnitude and only mildly by epicentral distance and hysteretic behavior. Different analytical expressions were proposed according to site class (not given here for brevity). The difference between rock and alluvium sites was found to be small, but larger differences happened for soft soil sites, especially at large ductility levels. An additional work by Miranda [1994] compared the factors derived from different authors. All factors show similar tendencies in the long period range, but variability increases for the short period range.

Comparison of expressions

The previous expressions for R_μ are plotted in Figure 2-21 for different values of μ and period range 0-4s. Only mass-proportional damping is considered (stiffness-proportional expressions by Vidic *et al.* are dismissed). For small ductility values (up to $\mu = 2$), all expressions are virtually coincident in the velocity and displacement range, except the Vidic-Bilinear, which produces values above all others. For medium ductility ($\mu = 4$) the Nassar-Krawinkler expression with $\alpha = 0$ approximates the Vidic-Bilinear expressions, and all others are below. For high ductility ($\mu = 8$), the Nassar-Krawinkler expression with $\alpha = 0$ gives the maximum value, and the Vidic-bilinear is well approximated by Nassar-Krawinkler with higher values of α . The simple expression by Fajfar (virtually coincident with the Vidic-Degrading expression), is the most conservative and, due to its simplicity, probably the best candidate for design purposes.

Figure 2-21. Comparison of strength reduction factors according to different authors.

2.5.2.3 Equivalent linearization by effective properties

A different approach for the solution of Eqn. (2.5-25) is linearization, i.e., the search of an equivalent linear system with effective properties to represent the inelastic behavior of the original non-linear system. The linear system is often referred to as ‘substitute’ structure following the terminology introduced by Shibata & Sozen [1974]. The motion equation of the effective system is [Guyader & Iwan, 2006]:

$$m\ddot{u}_{el} + c_{eff}\dot{u}_{el} + k_{eff}u_{el} = -ma_g(t), \quad (2.5-42)$$

where u_{el} indicates an elastic response and c_{eff} and k_{eff} are the effective damping ratio and stiffness. The equation can be mass-normalized:

$$\ddot{u}_{el} + 2\xi_{eff}\omega_{eff}\dot{u}_{el} + \omega_{eff}^2u_{el} = -a_g(t), \quad (2.5-43)$$

$$\ddot{u}_{el} + \frac{4\pi\xi_{eff}}{T_{eff}}\dot{u}_{el} + \left(\frac{2\pi}{T_{eff}}\right)^2u_{el} = -a_g(t), \quad (2.5-44)$$

Using basic dynamics, the fundamental parameters T_{eff} and ξ_{eff} can be expressed as:

$$\xi_{eff} = \frac{c_{eff}}{2\sqrt{mk_{eff}}}, \quad (2.5-45)$$

$$T_{eff} = 2\pi \sqrt{\frac{m}{k_{eff}}}. \quad (2.5-46)$$

It is important to notice that in these equations, ξ_{eff} implicitly includes the inherent damping component ξ_I . Different approaches to the linearization process are available; according to Miranda & Ruiz-García [2001], the main differences between methods lie in the way to treat the effective period and the differences in the hysteretic models considered. From the point of view of the variables approximated, two families of methods exist: energy-based and displacement-based.

Figure 2-22. Forced resonant harmonic vibration: (a) viscous energy dissipated in one cycle for Kelvin system; (b) elastic energy stored at maximum displacement; (c) hysteretic energy dissipated in one cycle for elastoplastic system; (d) elastic energy stored at maximum displacement.

Energy-based linearization

In this approach, the effective system is selected to equal the energy dissipated by hysteresis of the original system, both measured in one cycle of steady-state harmonic oscillation with equal amplitude. This is the basis of the simplified method proposed in ASCE-SEI 7-16 [ASCE, 2016]. The concepts of dissipated and stored energy were used by Jacobsen [1930, cited in Güllkan & Sozen, 1974] for the alternative definition of damping, see Eqn. (2.4-32). From an energy point of view, the effect of inelastic action is similar to the effect of damping in the sense that both dissipate energy and both reduce the force response (PS_a). However, damping also reduces the displacement response (S_d). Figure 2-22(a)(b) present a Kelvin system formulated to present a displacement similar to the original hysteretic system, Figure 2-22(c)(d), under harmonic base excitation. The equivalence is obtained using energy concepts, as follows: Figure 2-19(c) displays a full cycle of harmonic motion for the hysteretic system, in which the energy dissipated W_d is 4 times the shadowed area, also marked in Figure 2-22(c). At the point of maximum displacement, defined by f_m , u_m , the effective stiffness of the Kelvin system is:

$$k_{eff} = f_m / u_m = k(1 + \alpha(\mu - 1)) / \mu. \quad (2.5-47)$$

The period of this system is inversely dependent on the square root of the stiffness; under the assumption of equal mass of both systems:

$$T_{eff} = T \sqrt{\frac{\mu}{1 + \alpha(\mu - 1)}}. \quad (2.5-48)$$

The energy dissipated at one loop of the hysteretic system is:

$$W_d = 4f_y u_y (\mu - 1)(1 - \alpha), \quad (2.5-49)$$

and the energy stored in the Kelvin system at maximum displacement is:

$$W_s = 0.5 f_m u_m = 0.5 f_y u_y (1 + \alpha(\mu - 1)) \mu, \quad (2.5-50)$$

and using Jacobsen's expression, Eqn. (2.4-32):

$$\xi_h = \frac{2}{\pi} \frac{(\mu - 1)(1 - \alpha)}{\mu(1 + \alpha(\mu - 1))} = \frac{2}{\pi} \left(\frac{1}{1 + \alpha(\mu - 1)} - \frac{1}{\mu} \right). \quad (2.5-51)$$

The hysteresis loop is generally not perfect. For instance, the actual force-displacement diagram might be not exactly bilinear. This can be taken into account with a hysteresis loop factor q_H , applied to Eqn. (2.5-51).

$$\xi_h = q_H \frac{2}{\pi} \left(\frac{1}{1 + \alpha(\mu - 1)} - \frac{1}{\mu} \right). \quad (2.5-52)$$

This so-called hysteretic damping must be added to the system's inherent damping to provide the total effective damping:

$$\xi_{eff} = \xi_I + \xi_h. \quad (2.5-53)$$

Equations (2.5-48), (2.5-52) and (2.5-53) define the effective properties T_{eff} , ξ_{eff} , of the equivalent viscously damped system. For the case of no post-yield stiffness ($\alpha = 0$), the equations can be further simplified:

$$T_{eff} = T \sqrt{\mu}, \quad (2.5-54)$$

$$\xi_{eff} = \xi_I + q_H \frac{2}{\pi} \left(1 - \frac{1}{\mu} \right). \quad (2.5-55)$$

Effective values obtained for different combinations of μ and α are listed in Table 2-8.

Table 2-8. Effective period and hysteretic viscous damping ratio by Eqns. (2.5-48) and (2.5-52).

μ	α									
	0.00		0.02		0.05		0.10		0.15	
	T_{eff}/T	ξ_h	T_{eff}/T	ξ_h	T_{eff}/T	ξ_h	T_{eff}/T	ξ_h	T_{eff}/T	ξ_h
1.25	1.12	13%	1.13	12%	1.14	12%	1.17	11%	1.20	10%
1.50	1.22	21%	1.23	21%	1.25	20%	1.28	18%	1.31	17%
2.00	1.41	32%	1.42	31%	1.44	29%	1.47	26%	1.50	24%
4.00	2.00	48%	2.01	44%	2.03	39%	2.06	33%	2.11	28%
6.00	2.45	53%	2.46	47%	2.48	40%	2.53	32%	2.59	26%
8.00	2.83	56%	2.84	48%	2.87	39%	2.92	29%	2.99	23%

These effective properties are derived under the assumption of resonant harmonic oscillation in both systems, requiring the excitation frequency to be different for every system (T and T_{eff} , respectively). The validity of this approximation is questionable for other input signals. For earthquake excitation the approximation is, in general, reasonable: The earthquake signals can be seen as a random excitation with wideband frequency content. The linear response of lightly damped SDOF systems to this Gaussian input process is a narrowband random process with the central frequency at the natural system frequency ω (or T) [Clough & Penzien, 2003]. In other words, under conventional earthquake action the structure (with moderate ductility values) oscillates predominantly with its own frequency, and the approximation suggested here is reasonable. It will give poorer results for cases where the frequency content of the excitation is biased (for instance, impulsive earthquakes) or for high equivalent damping ratios (higher than 10%). This is illustrated in Figure 2-23, where the relative frequency content of the displacement response is shown for 4 different systems with natural periods ($T = 0.5s, 1s, 1.5s$ and $2s$), and with different ductility demands (1.5, 4, 8) subjected to the 40 accelerograms comprising motion set ESB (see Annex A). As seen, the scatter in frequency response increases with increasing ductility, suggesting that the method is not reliable for high values of ductility.

Figure 2-23. Relative frequency content of displacement response for 4 yielding SDOFs ($T = 0.5s, 1s, 1.5s, 2s$) with given ductility ($\mu = 1.5, 4, 8$) subjected to ground motion set ESB.

Energy-based linearization by substitute damping method

Gülkan & Sozen [1974] tested three single-storey concrete frames, one subjected to harmonic motion, the second subjected to a recorded ground motion, and the third to pseudo-static cyclic motion following the amplitudes recorded in the second frame. All frames presented ample inelastic excursions. The hysteresis loops of the last frame (of the Takeda type) resembled those described by Kelvin systems, suggesting that the behavior

could be represented by a substitute vibrating linear system with an effective viscous damping, obtained from the tests equating the energy dissipated by viscous damping with the total energy input, both measured in one cycle of harmonic motion:

$$2m\left(\frac{2\pi}{T}\right)\xi_{eff}\int_0^T \dot{u}^2 dt = -m\int_0^T a_g \dot{u} dt, \quad (2.5-56)$$

where m and T are the mass and period of the SDOF system that represents the single-storey frame, $a_g(t)$ is the ground excitation (shaking table excitation), $u(t)$ the recorded response. The following expression was obtained under the assumption of 2% inherent damping:

$$\xi_{eff} = \xi_I + 0.2\left(1 - \frac{1}{\sqrt{\mu}}\right). \quad (2.5-57)$$

The shift in period was referred to the stiffness softening of the system with zero post-yield stiffness, measured by the ductility value, resulting in Eqn. (2.5-54).

Substitute structure method

An extension of the substitute damping method was proposed by Shibata & Sozen [1974] and referred to as substitute-structure method, originally intended for the analysis of concrete structures using earthquake spectra. In this procedure, the original structure is replaced for analysis purpose by an effective structure (substitute structure) with effective properties (stiffness, period and damping). To obtain these properties, the structure is decomposed in its members (beams, columns) and the individual value of viscous damping $\xi_{eff,i}$ and flexural stiffness $k_{eff,i}$ is calculated for each element i according to its individual ductility level μ_i . For viscous damping, Eqn. (2.5-57) is used, with ξ_{eff} replaced by $\xi_{eff,i}$ and μ replaced by μ_i . For stiffness $k_{eff,i}$, the following expression is proposed:

$$k_{eff,i} = k_i / \mu_i. \quad (2.5-58)$$

The substitute structure period is calculated replacing these individual values in the global structural model. Total viscous damping (referred to as smeared damping) is calculated weighing the contribution of each element by its relative flexural strain energy (calculated using modal shapes):

$$\xi_{eff} = \frac{\sum_{i=1}^{NE} E_i \xi_{eff,i}}{\sum_{i=1}^{NE} E_i}, \quad (2.5-59)$$

where E_i is the flexural strain energy of element i , given by:

$$E_i = \frac{L_i}{6(EI)_i} (M_{1,i}^2 + M_{2,i}^2 - M_{1,i}M_{2,i}). \quad (2.5-60)$$

The basic concepts of the substitute structure method were later included in the development of displacement-based seismic design [Priestley *et al.*, 2007].

Statistical linearization

In this method, the effective system properties are determined to render a similar displacement (in a statistical sense) to that of the original system under earthquake excitation described by a set of representative earthquakes. Any type of hysteretic law can be used in the inelastic system. Following these guidelines, Iwan & Gates [1979] found the effective period and damping for six types of systems with similar monotonic response but different hysteretic characteristics, representing different types of strength and stiffness deterioration and including a bilinear hysteretic system, all of them with 5% post-elastic stiffness ratio and inherent damping ratio $\xi_I = 2\%$. The conclusions were obtained for 9 different periods between 0.4s and 4s. The ground motion excitation was defined by 12 strong motion accelerograms pre-scaled to represent a specific design spectrum. Displacement was defined as the median of the peak displacements over accelerograms. For a given ductility value μ , the set of motions was scaled until the desired ductility was achieved, and the inelastic displacement D_{in} was found. Using the same scaling value, the elastic displacement D_{el} was calculated for different combinations of period and damping. A measure of the error for a specific combination of parameters was given by $\varepsilon = D_{el}/D_{in} - 1$. The overall error was found as the SRSS over all periods considered. Maps of error were traced and the minimum error located, corresponding to the optimal combination of period and effective damping. The results were given in form of tabulated data, not analytical expressions. Some relevant results are summarized in Table 2-9, where some trends are observed:

- For the bilinear system, the values of effective period and effective damping obtained from the study are very different (much smaller) than those derived from the energy criterion (listed in Table 2-8, see column $\alpha = 0.05$). However, the effect over response displacement of increasing period and damping are opposite, so this does not mean that the actual response calculated by both systems presents large differences.
- For ductility values up to 1.5, the effective period is smaller than $1.05T$, regardless of the kind of system. For ductility values up to 2, smaller than $1.21 \cdot T$.
- For ductility values up to 1.5, the effective damping is smaller than 8%, regardless of the kind of system. For ductility values up to 2, smaller than 10%.
- Bilinear systems present values of effective period and damping that lie within the minimum and maximum for all systems contemplated.

The error obtained in the approximation is not listed, but is similar for all hysteretic systems considered, below 6%, and generally increases with increasing ductility.

Table 2-9. Effective period and hysteretic viscous damping ratio according to Iwan & Gates [1979].

μ	Bilinear system		Other systems (min)		Other systems (max)	
	T_{eff}/T	$\xi_{eff} - \xi_I$	T_{eff}/T	$\xi_{eff} - \xi_I$	T_{eff}/T	$\xi_{eff} - \xi_I$
1.5	1.000	2.00%	0.946	1.68%	1.043	5.87%
2	1.130	6.06%	0.979	2.37%	1.202	7.68%
4	1.317	8.75%	1.281	7.88%	1.559	11.44%
8	1.573	10.25%	1.574	11.00%	2.050	12.00%

By a least-square fit process of the previous data, Iwan [1980] suggested the following empirical expressions to fit all hysteretic systems:

$$T_{eff} = T \left(1 + 0.121(\mu - 1)^{0.939} \right) \quad (2.5-61)$$

$$\xi_{eff} = \xi_I + 0.0587(\mu - 1)^{0.371}. \quad (2.5-62)$$

In these expressions, α is not a parameter, as it was fixed in the parametric study reported above. Comparison of the results obtained with these expressions with exact values indicates errors below 20% in the range $0.4s \leq T \leq 4.0s$, $2 \leq \mu \leq 8$. Comparison with approximate values obtained by the substitute damping method (see above) indicates that the expressions proposed are more accurate. Effective values obtained with Eqns. (2.5-61) and (2.5-62) are listed in Table 2-10.

The work was further expanded by Guyader & Iwan [2006], who adjusted both T_{eff} and ξ_{eff} using a statistical approach with an optimization criterion to minimize the error, described as the relative difference between maximum displacement response of linear and inelastic systems. The systems were subjected to a set of recorded earthquake acceleration time histories (rather than harmonic excitation), and the optimization criterion was aimed at achieving the maximum number of acceptable results within a certain engineering acceptability range of $[-10\%, +20\%]$, where minus indicates un-conservative result, and plus indicates conservative result. This range was allegedly determined by asking a number of practicing engineers. Expressions were derived for T_{eff} and ξ_{eff} , with 12 parameters depending on the type of hysteretic system. These expressions were later included in FEMA-440 [FEMA, 2005], Eqns. (2.5-63) to (2.5-68), and Table 2-11. In this document, a set of parameters were proposed, optimized to give reasonable results for all hysteretic models and values of post-yield stiffness (row labeled as ‘GEN’ in Table 2-11).

$$1.0 < \mu < 4.0: \quad T_{eff} = T \left[1 + G(\mu - 1)^2 + H(\mu - 1)^3 \right] \quad (2.5-63)$$

$$\xi_{eff} = \xi_I + A(\mu - 1)^2 + B(\mu - 1)^3; \quad (2.5-64)$$

$$4.0 \leq \mu \leq 6.5: \quad T_{eff} = T \left[1 + I + J(\mu - 1) \right]; \quad (2.5-65)$$

$$\xi_{eff} = \xi_I + C + D(\mu - 1); \quad (2.5-66)$$

$$T_{eff} = T \left[1 + K \left(\sqrt{\frac{\mu - 1}{1 + \alpha(\mu - 2)}} - 1 \right) \right]; \quad (2.5-67)$$

$6.5 < \mu$:

$$\xi_{eff} = \xi_I + E \left(\frac{T_{eff}}{T} \right)^2 \frac{F(\mu - 1) - 1}{[F(\mu - 1)]^2}; \quad (2.5-68)$$

Table 2-10. Effective period and hysteretic viscous damping ratio according to Iwan [1980] and FEMA-440 [FEMA, 2005].

		μ						
		1	1.5	2	3	4	6	8
Iwan [1980]	T_{eff}/T	1	1.06	1.12	1.23	1.34	1.55	1.75
	$\xi_{eff} - \xi_I$	0%	4.54%	5.87%	7.59%	8.82%	10.66%	12.08%
FEMA-440	T_{eff}/T	1	1.05	1.16	1.50	1.67	1.93	2.14
	$\xi_{eff} - \xi_I$	0%	1.09%	3.80%	10.80%	14.96%	15.60%	15.04%

Note: FEMA-440 expressions valid for $0.2 \leq T \leq 2s$.

Table 2-11. Coefficients in FEMA-440 [FEMA, 2005] expressions, Eqns. (2.5-63) to (2.5-68).

System	α	A	B	C	D	E	F	G	H	I	J	K	L
EP	0.00	3.2	-0.66	11.0	0.12	19	0.73	0.11	-0.017	0.27	0.090	0.57	0.00
	0.02	3.3	-0.64	9.4	1.10	19	0.42	0.10	-0.014	0.17	0.120	0.67	0.02
	0.05	4.2	-0.83	10.0	1.60	22	0.40	0.11	-0.018	0.09	0.140	0.77	0.05
	0.10	5.1	-1.10	12.0	1.60	24	0.36	0.13	-0.022	0.27	0.100	0.87	0.10
	0.20	4.6	-0.99	12.0	1.10	25	0.37	0.10	-0.015	0.17	0.094	0.98	0.20
SD	0.00	5.1	-1.10	12.0	1.40	20	0.62	0.17	-0.032	0.10	0.190	0.85	0.00
	0.02	5.3	-1.20	11.0	1.60	20	0.51	0.18	-0.034	0.22	0.160	0.88	0.02
	0.05	5.6	-1.30	10.0	1.80	20	0.38	0.18	-0.037	0.15	0.160	0.92	0.05
	0.10	5.3	-1.20	9.2	1.90	21	0.37	0.17	-0.034	0.26	0.120	0.97	0.10
	0.20	4.6	-1.00	9.6	1.30	23	0.34	0.13	-0.027	0.11	0.110	1.00	0.20
SSD	-0.03	5.3	-1.20	14.0	0.69	24	0.90	0.18	-0.033	0.17	0.180	0.76	-0.03
	-0.05	5.6	-1.30	14.0	0.61	22	0.90	0.20	-0.038	0.25	0.170	0.71	-0.05
GEN		4.9	-1.10	14.0	0.32	22	0.90	0.20	-0.038	0.28	0.130	0.89	0.05

EP: Elasto-Plastic. SD: Stiffness-Degrading. SSD: Strength- and Stiffness- Degrading.

Simplification of FEMA-440 expressions

Based on the results of the FEMA expressions, a very simple yet conservative expression is proposed hereby:

$$1.0 < \mu \leq 1.5: \quad T_{eff} = T, \quad (2.5-69)$$

$$\xi_{eff} = \xi_I, \quad (2.5-70)$$

$$1.5 < \mu \leq 4: \quad T_{eff} = T \left(1 + 0.5 \frac{\mu - \mu_1}{4 - \mu_1} \right), \quad (2.5-71)$$

$$\xi_{eff} = \xi_I + \xi_4 \left(\frac{\mu - \mu_1}{4 - \mu_1} \right), \quad (2.5-72)$$

$$4 < \mu \leq 8: \quad T_{eff} = T \left(1.5 + 0.5 \frac{\mu - 4}{4} \right), \quad (2.5-73)$$

$$\xi_{eff} = \xi_I + \xi_4, \quad (2.5-74)$$

in which $\mu_1 = 1.5$, and ξ_4 must be taken as 0.11 (EP systems) or 0.15 (all other cases). It is a tri-linear relationship, shown in Figure 2-25.

Figure 2-24. Comparison of effective period (a) and damping (b) from different expressions.

Figure 2-25. Effective period (a) and damping (b) from FEMA-440.

2.5.2.4 Inelastic SDOF with added viscous dampers

The equation of motion in this case contains non-linear terms from both added damping and restoring force:

$$m\ddot{u}(t) + c_I \dot{u}(t) + f_S + c_a |\dot{u}(t)|^a \text{sgn}(\dot{u}) = -m a_g(t), \quad (2.5-75)$$

This equation must be solved using numerical time-stepping procedures. Simplified solutions are available mixing different techniques reported above:

Linear viscous dampers

For linear viscous dampers, the equation can be further simplified as:

$$m\ddot{u}(t) + (c_I + c_a)\dot{u}(t) + f_s = -m a_g(t), \quad (2.5-76)$$

This equation can be solved by equivalent linearization techniques. However, specific treatment must be given to the damping terms. First, the added damping is expressed as viscous damping ratio using Eqn. (2.4-31):

$$\xi_{a,el} = \frac{c_a \omega}{2k} = \frac{c_a \pi}{T k}. \quad (2.5-77)$$

In this equation, k , ω and T are the elastic stiffness, frequency and period of the system (before yielding). Using the equal energy approach, for the inelastic system these values must be replaced by the effective values k_{eff} , Eqn. (2.5-47), and T_{eff} , Eqn. (2.5-48). The resulting expression is

$$\xi_{a,in} = \frac{c_a \pi}{T k} \sqrt{\frac{\mu}{(1 + \alpha(\mu - 1))}} = \xi_{a,el} \sqrt{\frac{\mu}{(1 + \alpha(\mu - 1))}}. \quad (2.5-78)$$

This expression gives the added viscous damping ratio of the effective system. As discussed, there is no physical ground to assume that ξ_I is dependent on the degree of yielding (the hysteretic damping component is already accounted for). For this reason, it makes no sense to apply a similar correction to inherent damping. The total effective damping is:

$$\xi_{eff} = \xi_I + \xi_{a,in} + \xi_h = \xi_I + \xi_{a,el} \sqrt{\frac{\mu}{(1 + \alpha(\mu - 1))}} + \xi_h, \quad (2.5-79)$$

where ξ_h was defined in Eqn. (2.5-51).

Non-linear viscous dampers

In this case, the expression of the elastic viscous damping ratio is displacement-dependent. That means that a different value is obtained depending on the elastic point at which the viscous damping is calculated. The expression of the effective viscous damping ratio will then be different. The most general expression is, once again, Jacobsen's equation. For practical application, two elastic points can be of significance:

- i) The yielding point (u_y, f_y) .
- ii) The elastic point with equal displacement as the inelastic point $(\mu u_y, \mu f_y)$.

Reference to viscous energy calculated at yielding point:

At yielding point the strain energy $W_{s,el}^Y$, is

$$W_{s,el}^Y = 0.5k u_y^2, \quad (2.5-80)$$

And the strain energy for ductility μ is given by Eqn. (2.5-50), that can be expressed as:

$$W_{s,in} = \mu [1 + \alpha(\mu - 1)] W_{s,el}^Y, \quad (2.5-81)$$

The energy dissipated by viscous damping at frequency ω and amplitude u_y (at yielding point) $W_{d,el}^Y$, is given by:

$$W_{d,el}^Y = \pi b c \omega^a u_y^{1+a}, \quad (2.5-82)$$

and the corresponding energy at the inelastic point is:

$$W_{d,in} = \pi b c \omega^a \left(\frac{1 + \alpha(\mu - 1)}{\mu} \right)^{a/2} \mu^{1+a} u_y^{1+a} = W_{d,el}^Y (1 + \alpha(\mu - 1))^{a/2} \mu^{1+a/2}, \quad (2.5-83)$$

whereupon replacing $W_{s,in}$ and $W_{d,in}$ in Jacobsen's equation:

$$\xi_{a,in} = \xi_{a,el}^Y (1 + \alpha(\mu - 1))^{a/2-1} \mu^{a/2}, \quad (2.5-84)$$

For $a = 1$, this expression reduces to Eqn. (2.5-78).

Reference to viscous energy calculated at equal displacement: In this case the strain energy at the elastic point with equal displacement $W_{s,el}^D$ is:

$$W_{s,el}^D = 0.5k \mu^2 u_y^2, \quad (2.5-85)$$

And the strain energy for ductility μ can be expressed as:

$$W_{s,in} = \left(\frac{1 + \alpha(\mu - 1)}{\mu} \right) W_{s,el}^D, \quad (2.5-86)$$

The energy dissipated by viscous damping at frequency ω and amplitude μu_y is:

$$W_{d,el}^D = \pi b c \omega^a \mu^{1+a} u_y^{1+a}, \quad (2.5-87)$$

and the corresponding energy at the inelastic point is expressed as:

$$W_{d,in} = W_{d,el}^D \left(\frac{1 + \alpha(\mu - 1)}{\mu} \right)^{a/2}, \quad (2.5-88)$$

whereupon:

$$\xi_{a,in} = \xi_{a,el}^D \left(\frac{\mu}{1 + \alpha(\mu - 1)} \right)^{1-a/2}, \quad (2.5-89)$$

For $a = 1$, as in the previous case, this expression reduces to Eqn. (2.5-78). The result for both equations is the same only if the equal displacement rule applies.

Force coefficients for inelastic MDOF systems with added damping

Ductility influences the force coefficients. For viscous dampers with $a \leq 1$, it can be shown that [Ramirez *et al.*, 2001]:

$$\delta \approx \left(\frac{2a}{b} (\xi_{eff} - \xi_h) \right)^{\frac{1}{2-a}}. \quad (2.5-90)$$

$$\mu \leq 1, \quad CF_1 = \cos \delta \quad (2.5-91)$$

$$\mu > 1, \quad CF_1 = \min\{\mu \cos \delta; 1\} \quad (2.5-92)$$

CF_2 can still be calculated with Eqn. (2.4-43).

2.5.3 Multi -Degree-of-Freedom (MDOF) systems

2.5.3.1 Elastic MDOF systems

Modal analysis of MDOF systems, provide the basis for non-linear static procedures. In this analysis procedure, the MDOF system with N degrees of freedom is converted into a set of N SDOF systems, whose peak response can be found using the response spectrum. The N solutions are then combined to give the original system solution. The procedure uses modal orthogonality properties and can be found in any dynamics textbook [Chopra, 2007; Clough & Penzien, 2003]. Strictly speaking, the procedure can be applied only to elastic systems. A concise summary is given here, as the equations to convert MDOF systems to SDOF systems form the basis for the non-linear static procedures.

The MDOF system presents two fundamental modal vibration properties at mode j : vibration period T_j ; and modal shape $\boldsymbol{\phi}_j = [\phi_{1,j}, \dots, \phi_{N,j}]^T$. The equation of motion for the elastic multi-degree of freedom (MDOF) system is:

$$\mathbf{m} \ddot{\mathbf{u}}(t) + \mathbf{c} \dot{\mathbf{u}}(t) + \mathbf{k} \mathbf{u}(t) = -\mathbf{m} \mathbf{1} a_g(t), \quad (2.5-93)$$

where $\mathbf{m}$ is the mass matrix (generally a diagonal matrix where m_n is the n^{th} storey mass), $\mathbf{c}$ is the damping matrix, $\mathbf{k}$ is the stiffness matrix, $\mathbf{1}$ is an influence vector (relating the base displacement to the structural degrees of freedom) and $\mathbf{u}$ is the displacement vector. Eqn. (2.5-93) represents a system of N coupled equations, because $\mathbf{c}$ and $\mathbf{k}$ are generally not diagonal. The homogeneous equation without the damping term represents the free undamped vibration problem:

$$\mathbf{m} \ddot{\mathbf{u}} + \mathbf{k} \mathbf{u} = \mathbf{0}, \quad (2.5-94)$$

which is expressed as an eigenproblem

$$(\mathbf{k} - \omega_j^2 \mathbf{m}) \boldsymbol{\phi}_j = \mathbf{0}, \quad (2.5-95)$$

and solved to render the structural natural modes $\Phi = [\phi_1, \dots, \phi_N]$, with $\phi_j = [\phi_{1,j}, \dots, \phi_{N,j}]^T$, and natural circular vibration frequencies ω_j , the latter generally grouped in the spectral matrix $\Omega = \text{diag}(\omega_j)$, where $\text{diag}(x_j)$ indicates a diagonal matrix with x_j as its j^{th} diagonal term. Natural modes represent relative displacement proportions and can be arbitrarily scaled. Throughout this text, it will be assumed that all modes are normalized to the roof, that is, $\phi_{Nj} = 1$. The natural modes diagonalize the mass and stiffness matrices, that is,

$$\Phi^T \mathbf{m} \Phi = \mathbf{M}, \quad (2.5-96)$$

$$\Phi^T \mathbf{k} \Phi = \mathbf{K}, \quad (2.5-97)$$

where $\mathbf{M} = \text{diag}(M_j)$ (modal mass) and $\mathbf{K} = \text{diag}(K_j)$ (modal stiffness), are diagonal matrices, linked by $K_j = \omega_j^2 M_j$, where $M_j = \phi_j^T \mathbf{m} \phi_j$ and $K_j = \phi_j^T \mathbf{k} \phi_j$. The transformation

$$\mathbf{u}_j(t) = \phi_j \mathbf{u}_N(t), \quad (2.5-98)$$

can be applied to expression (2.5-93); $\mathbf{u}_N$ represents the vector containing the roof displacement at each mode, $\mathbf{u}_N = [u_{N,1}, \dots, u_{N,j}]^T$. After pre-multiplication by ϕ_j^T :

$$\mathbf{M} \ddot{\mathbf{u}}_N(t) + \mathbf{C} \dot{\mathbf{u}}_N(t) + \mathbf{K} \mathbf{u}_N(t) = -\Phi^T \mathbf{m} \mathbf{a}_g(t), \quad (2.5-99)$$

where $\mathbf{C} = \Phi^T \mathbf{c} \Phi$ is the modal damping matrix. If $\mathbf{C}$ is diagonal or approximately diagonal (further discussion about this assumption is given in Chapter 3), Eqn. (2.5-99) can be transformed to a system of N uncoupled equations:

$$M_j \ddot{u}_{N,j}(t) + C_j \dot{u}_{N,j}(t) + K_j u_{N,j}(t) = -\phi_j^T \mathbf{m} \mathbf{a}_g(t) = -L_j a_g(t), \quad (2.5-100)$$

$L_j = \phi_j^T \mathbf{m} \mathbf{l}$. After normalization,

$$\ddot{u}_{N,j}(t) + M_j^{-1} C_j \dot{u}_{N,j}(t) + \omega_j^2 u_{N,j}(t) = -\Gamma_j a_g(t), \quad (2.5-101)$$

where Γ_j is the modal participation factor for mode j , given by:

$$\Gamma_j = \frac{L_j}{M_j} = \frac{\phi_j^T \mathbf{m} \mathbf{l}}{\phi_j^T \mathbf{m} \phi_j}. \quad (2.5-102)$$

Γ_j can be positive or negative. The damping matrix $\mathbf{c}$ is generally not formulated in explicit form, but rather defined using modal damping ratios:

$$C_j = 2\xi_j M_j \omega_j. \quad (2.5-103)$$

Using Eqn. (2.5-103), Eqn. (2.5-101) can be rewritten as:

$$\ddot{u}_{N,j}(t) + 2\xi_j \omega_j \dot{u}_{N,j}(t) + \omega_j^2 u_{N,j}(t) = -\Gamma_j a_g(t), \quad (2.5-104)$$

which is similar to Eqn. (2.5-4), except for the scaling factor in the right term. If the spectral displacement for the unscaled equation is $S_d(T_j, \xi_j)$ (found using the response spectrum), then the peak roof displacement at mode j is:

$$u_{N,j} = \Gamma_j S_d(T_j, \xi_j). \quad (2.5-105)$$

Roof acceleration is obtained with the same equation replacing S_d by PS_a . Modal storey displacements are found from Eqn. (2.5-98); this equation can also be used to find storey pseudo-accelerations. The inertial force at storey n is given by:

$$f_{n,j} = \omega_j^2 m_j \phi_{n,j} \Gamma_j S_d(T_j, \xi_j) = m_j \phi_{n,j} \Gamma_j PS_a(T_j, \xi_j). \quad (2.5-106)$$

The system can be subjected to static analysis under this set of forces to find other relevant parameters at mode j . Base shear can be obtained by addition of all storey inertial forces:

$$V_{1,j} = \boldsymbol{\phi}_j^T \mathbf{m} \mathbf{1} \Gamma_j PS_a(T_j, \xi_j) = M_{eff,j} PS_a(T_j, \xi_j), \quad (2.5-107)$$

where $M_{eff,j}$ is the effective mass for mode j .

$$M_{eff,j} = L_j \Gamma_j = \frac{L_j^2}{M_j}. \quad (2.5-108)$$

Using modal properties, and with the proposed modal normalization, it can be shown [Ramirez et al, 2001] that $\sum_{j=1}^N M_{eff,j} = \sum_{n=1}^N m_n = M_{tot}$ (where M_{tot} is the total mass), and that $\sum_{j=1}^N \Gamma_j = 1$.

The peak solutions found for each mode do not happen at the same time instant. Therefore strictly speaking they cannot be added. By adding them a conservative solution is found. Rather, a so-called combination rule is used to find a synthetic solution. Two options are generally accepted in current codes, the Square Root of the Sum of Squares (SRSS) rule and the Complete Quadratic Combination (CQC) rule:

SRSS rule:

The SRSS rule [Goodman *et al.*, 1952] is widely used due to its simplicity:

$$X_{P,n} = \text{SRSS}(\mathbf{X}_n) = \sqrt{\sum_{j=1}^N X_{n,j}^2}, \quad (2.5-109)$$

Where $X_{P,n}$ is the peak value of the variable to be calculated at storey n , and $X_{n,j}$ the modal value of the variable at storey n and mode j .

CQC rule:

The CQC rule [Der Kiureghian, 1979, 1980; Wilson et al, 1981] should be applied if modal frequencies are not well-spaced. This is generally verified with the following condition:

$$\frac{T_i - T_j}{T_i + T_j} > \xi_i + \xi_j \quad (2.5-110)$$

Where $T_i > T_j$. The combination rule reads:

$$X_{p,n} = \text{CQC}(\mathbf{X}_n) = \sqrt{\sum_{i=1}^N \sum_{j=1}^N X_{n,i} r_{ij} X_{n,j}}, \quad (2.5-111)$$

where the correlation factor r_{ij} is obtained from:

$$r_{is} = \frac{8\sqrt{\xi_i \xi_j} (\xi_i + \rho_{ij} \xi_j) \rho_{ij}^{3/2}}{(1 - \rho_{ij}^2)^2 + 4\xi_i \xi_j \rho_{ij} (1 + \rho_{ij}^2) + 4(\xi_i^2 + \xi_j^2) \rho_{ij}^2}, \quad (2.5-112)$$

in which $\rho_{ij} = T_i / T_j$. In order to achieve an approximate solution, not all modes need to be included in the procedure; a small number of modes comprising the first modes are generally sufficient. The number of modes can be determined using the cumulative effective mass ratio $\bar{m}_{eff,J}$. If J modes are included:

$$\bar{m}_{eff,J} = \frac{\sum_{j=1}^J M_{eff,j}}{M_{tot}} \quad (2.5-113)$$

Generally a target cumulative effective mass ratio of 90% is enough for design purposes.

2.5.3.2 Storey properties of regular MDOF systems

In this Section, the storey properties of regular MDOF systems, used throughout this text, are introduced. Figure 2-26 shows a regular frame. The term ‘regular’ is used to denote structural and inertial regularity, such as defined in current codes (see for instance, [CEN, 2004], section 4.2.3, or [ASCE, 2016], section 12.3).

Figure 2-26. Frame vibrating in the j^{th} mode: a) full frame view; b) shear model representation for mode j ; c) frame displacement according to mode j ; d) consistent static forces.

For the n^{th} storey the seismic mass is m_n . The frame is assumed to present modal inherent damping ratio $\xi_{l,j}$ in mode j . Only horizontal displacements in the frame plane are

considered. Figure 2-26(a) and Figure 2-26(c) show the frame vibrating in mode j , with a given arbitrary amplitude at roof $u_{N,j}$. Nodal displacements at each storey $\mathbf{u}_j = [u_{1,j}, \dots, u_{N,j}]^T$ are proportional to the j^{th} modal shape $\boldsymbol{\phi}_j = [\phi_{1,j}, \dots, \phi_{N,j}]^T$. Consider the frame subjected to a set of forces $\mathbf{f}_j = [f_{1,j}, \dots, f_{N,j}]^T$ producing displacements $\mathbf{u}_j$,

$$\mathbf{u}_j = \boldsymbol{\phi}_j r, \quad (2.5-114)$$

where r is a displacement that depends on the magnitude of $\mathbf{f}_j$, Figure 2-26(d); if modal displacements are normalized to the roof, r is merely the peak roof displacement $u_{N,j}$ for each mode. Displacements are assumed to be small enough so that the frame does not yield, P - Δ effects are negligible, and the behavior is linear. The static force vector producing this deformation is:

$$\mathbf{f}_j = \mathbf{k} \mathbf{u}_j = \mathbf{k} \boldsymbol{\phi}_j r, \quad (2.5-115)$$

where $\mathbf{k}$ is the $N \times N$ stiffness matrix of the frame (obtained from static condensation of the actual stiffness matrix [Clough & Penzien, 2003; Chopra, 2007]). Shear forces $\mathbf{V}_j = [V_{1,j}, \dots, V_{N,j}]^T$ due to the applied forces are:

$$\mathbf{V}_j = \mathbf{T}_U \mathbf{f}_j = \mathbf{T}_U \mathbf{k} \mathbf{u}_j = \mathbf{T}_U \mathbf{k} \boldsymbol{\phi}_j r, \quad (2.5-116)$$

where $\mathbf{T}_U$ is an $N \times N$ lower triangular matrix of ones. The inter-storey drifts (referred hereby simply as drifts) $\boldsymbol{\Delta}_j = [\Delta_{1,j}, \dots, \Delta_{N,j}]^T$ are:

$$\boldsymbol{\Delta}_j = \mathbf{B} \mathbf{u}_j = \mathbf{B} \boldsymbol{\phi}_j r = \boldsymbol{\Psi}_j r, \quad (2.5-117)$$

where $\mathbf{B}$ is the $N \times N$ compatibility matrix relating displacements with drifts, $\mathbf{B} = \mathbf{T}_L^{-1}$; $\mathbf{T}_L$ is an $N \times N$ upper triangular matrix of ones; and $\boldsymbol{\Psi}_j = [\psi_{1,j}, \dots, \psi_{N,j}]^T$ is the j -th mode modal drift vector, where $\psi_{n,j} = \phi_{n,j} - \phi_{n-1,j}$ for $n > 1$ and $\psi_{1,j} = \phi_{1,j}$ for $n = 1$. These vectors can be grouped in the modal drift matrix $\boldsymbol{\Psi}$:

$$\boldsymbol{\Psi} = \mathbf{B} \boldsymbol{\Phi}. \quad (2.5-118)$$

The storey stiffness for mode j and storey n can be defined as the relationship between applied shear and resulting drift:

$$k_{s,n,j} = \frac{V_{n,j}}{\Delta_{n,j}}, \quad (2.5-119)$$

which is independent from r , i.e., it depends solely on the frame structural properties. Thus, the former steps can be performed with any arbitrary value, for instance $r = 1$ mm.

Repeating the process for all natural modes, an $N \times N$ matrix $\mathbf{k}_s$ is formed, whose j^{th} column is $\mathbf{k}_{s,j} = [k_{s,1,j}, \dots, k_{s,N,j}]^T$. $\mathbf{k}_{s,j}$ can be seen as the stiffness of the structure when the storey displacements are proportional to natural mode j . That is the case when the structure is

subjected to forced base harmonic excitation of frequency ω_j . $\mathbf{k}_s$ is referred here as *modal storey stiffness matrix*; it might contain negative terms, and is non-symmetrical.

The former procedure allows construction of the modal storey stiffness matrix from the stiffness matrix $\mathbf{k}$. The opposite process, i.e., construction of the stiffness matrix $\mathbf{k}$ from the modal storey stiffness matrix $\mathbf{k}_s$, can be performed as follows: first an arbitrary drift matrix Δ is produced:

$$\Delta = \mathbf{B}\Phi \mathbf{1} = \mathbf{B}\Phi = \Psi, \quad (2.5-120)$$

where the arbitrary displacement has been taken as 1. The modal shear forces $\mathbf{V}$ consistent with the modal drift matrix Δ are obtained as:

$$V_{n,j} = k_{s,n,j} \Delta_{n,j} = k_{s,n,j} \psi_{n,j}, \quad (2.5-121)$$

where $\Delta_{n,j}$, $V_{n,j}$ and $\psi_{n,j}$ are the n^{th} storey - j^{th} mode component of Δ , $\mathbf{V}$ and Ψ , respectively. Finally, the structural stiffness $\mathbf{k}$ is given by:

$$\mathbf{k} = \mathbf{H}\mathbf{V}\Delta^{-1}\mathbf{B}, \quad (2.5-122)$$

where $\mathbf{H}$ is the $N \times N$ equilibrium matrix relating external forces with shear forces, $\mathbf{H} = \mathbf{B}^T$.

An overall modal stiffness for mode j , κ_j , can also be defined, relating the base shear to the roof displacement:

$$\kappa_j = \frac{V_{1,j}}{\phi_{N,j} r}. \quad (2.5-123)$$

If the modal shapes are assumed normalized to roof displacement, $\phi_{N,j} = 1$, using Eqn. (2.5-121):

$$\kappa_j = \psi_{1,j} k_{s,1,j}. \quad (2.5-124)$$

2.5.3.3 Inherent damping in MDOF systems

Analytical determination of inherent damping for MDOF systems is, at the current state of knowledge, not possible. The uncertainty of how inherent damping affects each mode adds up to the motives already stated in Section 2.5.1.1. For all these reasons, MDOF damping is more often assumed to be modally uncoupled [Chopra, 2007], and damping is treated assigning a viscous damping ratio to each mode. There are several ways of constructing inherent damping matrices. Two of them are used in this document and described hereby:

Modal damping

By defining the viscous damping ratio in each mode ξ_j , and applying Eqn. (2.5-103), the diagonal modal damping matrix $\mathbf{C}$ is obtained; $\mathbf{c}$ is defined as:

$$\mathbf{c} = (\Phi^T)^{-1} \mathbf{C} \Phi^{-1}. \quad (2.5-125)$$

Modal damping allows control over each mode damping ratio. Moreover, some modes can be un-damped or over-damped, if necessary.

Rayleigh damping

Because the mass and stiffness matrices are diagonalized by the undamped modal shapes, any linear combination of them is also diagonalized. Rayleigh damping is defined as:

$$\mathbf{c} = a_0 \mathbf{m} + a_1 \mathbf{k}, \quad (2.5-126)$$

where a_0 and a_1 are coefficients to be determined. In this case, it is only possible to fix the viscous damping ratio in two modes. Using modal properties it can be shown that, if the target damping ratio is ξ_r and ξ_s in modes r and $s > r$, respectively, the coefficients a_0 and a_1 can be found from solution of:

$$\frac{1}{2} \begin{bmatrix} 1/\omega_r & \omega_r \\ 1/\omega_s & \omega_s \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \xi_r \\ \xi_s \end{bmatrix}. \quad (2.5-127)$$

The modal damping ratio will be exact in modes r and s , slightly smaller in between r and s and larger for modes outside this range, according to:

$$\xi_j = \frac{a_0}{2\omega_j} + \frac{a_1\omega_j}{2}. \quad (2.5-128)$$

r should always be taken as 1; otherwise damping in the first mode, which is the more significant, will not be properly controlled. Two especial cases happen when $a_0 = 0$ (stiffness-proportional damping) and $a_1 = 0$ (mass-proportional damping). With mass-proportional damping in the first mode, higher modes will present smaller damping ratios. Stiffness-proportional damping in the first mode produces over-damping in higher modes.

2.5.3.4 Inelastic MDOF systems

The equation of motion for an inelastic MDOF is [Chopra, 2007]:

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{c}\dot{\mathbf{u}} + \mathbf{f}_s = -\mathbf{m}\ddot{\mathbf{u}}_g(t), \quad (2.5-129)$$

where $\mathbf{f}_s$ is the inelastic restoring force vector. This vector is dependent in the system hysteretic properties (system response) and excitation, including damping. Eqn. (2.5-129) can only be solved by numerical time-stepping non-linear methods, such as the Newmark beta method [Bathe, 2006; Chopra, 2007; Clough & Penzien, 2003], or, if non-linearity is restricted to a small number of elements (such as plastic hinges), Fast Non-Linear Analysis (FNA) [Wilson, 2000].

Approximate modal solution

A possible approach to approximate the solution of Eqn. (2.5-129) is to apply modal analysis and neglect off-diagonal terms. This is the basis of some simplified methods, particularly non-linear response spectrum analysis (NLRSA) and modal pushover analysis

(MPA). The second term in the motion Eqn. (2.5-129), can be decomposed into its modal components using basic modal procedures; for the j -th mode the earthquake excitation takes the form:

$$-\Gamma_j \mathbf{m} \boldsymbol{\phi}_j a_g(t). \quad (2.5-130)$$

Eqn. (2.5-129) is then transformed into N MDOF systems, one for each mode. Each system represents the structural response to the j -th modal component of the earthquake excitation. The equations are numerically solved and the response $\mathbf{u}$ obtained is subjected to modal decomposition. It has been observed [Chopra & Goel, 2002] that the contribution of mode j to the response of the j -th modal component of the earthquake excitation is much larger than the contribution of other modes. From this observation it follows that uncoupled terms in the motion equation derived from inelastic restoring forces will be small and can be neglected. That is, if modal concepts are applied to Eqn. (2.5-129), the motion equation is expressed as:

$$\ddot{u}_j + 2\xi_j \omega_j \dot{u}_j + \boldsymbol{\phi}_j^T \mathbf{f}_s / M_j = -\Gamma_j a_g(t), \quad (2.5-131)$$

in which the restoring force term is expected to depend mostly on u_j and $\dot{u}_j$, and so other modal components can be neglected. Eqn. (2.5-131) represents a non-linear SDOF system that can be solved approximately using the equivalent linearization techniques discussed. Particularly, the non-linear SDOF equation can be replaced by an equivalent linear SDOF, with energy-based or statistical-based linearization.

2.5.3.5 MDOF equipped with viscous dampers

Figure 2-27 shows a frame equipped with viscous dampers subjected to earthquake excitation. The dampers are mounted diagonally, Figure 2-27(a), but for generality the damper properties are referred to the storey displacement, Figure 2-27(b). In this way, any mounting arrangement can be dealt with, using the transformation equations introduced in Section 2.4.7. The general expression for the response of the damping system is expressed as the SDOF equation for the Maxwell system, Eqn. (2.4-68), specialized for storey n :

$$\dot{\Delta}_n = \frac{\dot{V}_{d,n}}{k_{d,n}} + \left| \frac{V_{d,n}}{c_{d,n}} \right|^{1/a_n} \text{sgn}[V_{d,n}]. \quad (2.5-132)$$

Forces due to the damping system are expressed as:

$$\mathbf{f}_d = \mathbf{H} \mathbf{V}_d, \quad (2.5-133)$$

and the general MDOF motion equation is:

$$\mathbf{m} \ddot{\mathbf{u}}(t) + \mathbf{c} \dot{\mathbf{u}}(t) + \mathbf{f}_s(t) + \mathbf{f}_d(t) = -\mathbf{m} \mathbf{1} a_g(t). \quad (2.5-134)$$

Eqn. (2.5-134) in combination with the N equations described by Eqn. (2.5-132) define the system. These equations can only be solved numerically with time stepping procedures.

Figure 2-27. Frame with dampers: a) full frame view; b) shear model representation for mode j ; c) frame displacement according to mode j ; d) consistent static forces; e) f) g) h) storey representation.

If the Maxwell system representing brace + diagonal is simplified to a pure viscous system, Figure 2-27, Eqn. (2.5-132) can be rewritten as:

$$V_{d,n} = c_{d,n} |\dot{\Delta}_n|^{a_n} \text{sgn}[\dot{\Delta}_n] = c_{d,n} \dot{\Delta}_n |\dot{\Delta}_n|^{a_n-1}. \quad (2.5-135)$$

Recalling that $\Delta = \mathbf{B} \cdot \mathbf{u}$, Eqn. (2.5-133) can be expressed as:

$$\mathbf{f}_d = \mathbf{c}_a \dot{\mathbf{u}}, \quad (2.5-136)$$

where the matrix c_a , not constant but dependent on drift velocity, is defined as:

$$\mathbf{c}_a = \mathbf{H} \text{diag} \left[c_{d,n} |\dot{\Delta}_n|^{a_n-1} \right] \mathbf{B}. \quad (2.5-137)$$

Using Eqn. (2.5-135):

$$\mathbf{M} \ddot{\mathbf{u}}(t) + (\mathbf{c}_1 + \mathbf{c}_a(\dot{\mathbf{u}})) \dot{\mathbf{u}}(t) + \mathbf{f}_s(t) = -\mathbf{m} \mathbf{1} a_g(t). \quad (2.5-138)$$

This expression is similar to Eqn. (2.5-129); however, $\mathbf{c}_a$ is dependent on velocity. Modal decomposition of this equation assuming undamped modal shapes results in:

$$\mathbf{M} \ddot{\mathbf{u}}_N(t) + \mathbf{C}_1 \dot{\mathbf{u}}_N(t) + \Phi^T \mathbf{c}_a \Phi \dot{\mathbf{u}}_N(t) + \Phi^T \mathbf{f}_s(t) = -\Phi^T \mathbf{m} \mathbf{1} a_g(t), \quad (2.5-139)$$

where matrices $\mathbf{M}$ and $\mathbf{C}_1$ are diagonal, but the last two terms on the left side are not; in addition the third term on the left side is dependent on velocity.

An approximate solution can be found for the case of linear dampers ($a_n = 1$), in which $\mathbf{c}_a$ does not present dependency on velocity:

$$\mathbf{c}_a = \mathbf{H} \text{diag} [c_{d,n}] \mathbf{B}. \quad (2.5-140)$$

Although the velocity dependence is removed, coupled terms still exist in both matrices $\Phi^T \mathbf{c}_a \Phi$ and $\Phi^T \mathbf{f}_s$; if these coupled terms are neglected:

$$\ddot{u}_j + 2(\xi_{I,j} + \xi_{a,j}) \omega_j \dot{u}_j + \phi_j^T \mathbf{f}_s / M_j = -\Gamma_j a_g(t), \quad (2.5-141)$$

This equation can be solved by equivalent linearization. For non-linear damping, the same process can be repeated by previously replacing the non-linear damping by an equivalent linear damper, as shown in Section 2.5.1.2.

It is convenient to summarize the assumptions behind Eqn. (2.5-141):

1. The brace elements (and other auxiliary elements of the damping system) contributing to stiffness are assumed to be infinitely rigid ($k_{d,n} = \infty$). The implication is that the model does not need to include the stiffness of these elements and the damper behaves as a pure dashpot.
2. The viscous component of the damper is linear ($a_n = 1$) or can be modeled by an equivalent linear damper.
3. Modal shapes and frequencies are unchanged by the addition of viscous dampers.
4. The added damping matrix is approximately diagonalized by the undamped modal shapes. Added damping matrix can then be substituted by an equivalent modal viscous damping ratio.
5. Non-linear restoring force components are approximately diagonalized by modal shapes.

Under these assumptions, the MDOF system can be converted using modal analysis to SDOF non-linear systems that can be solved using equivalent linearization by the response spectrum approach.

One critical assumption in the previous steps is the diagonalization of the added damping matrix. This topic is further discussed in Chapter 3.

2.5.3.6 Estimation of added viscous damping ratio

Resonant harmonic steady-state motion at mode j of the frame with ADSs is now considered. Under assumption of unchanged modal properties, the frequency of excitation is ω_j . The peak amplitude of this cyclic motion is measured by the peak roof displacement $u_{N,j} > 0$. The roof velocity is given by $\omega_j \cdot u_{N,j}$. Under the assumption of unchanged modal shapes, each storey of the frame displays harmonic motion of fixed amplitude equal to the inter-storey drift:

$$\Delta_j = \boldsymbol{\psi}_j u_{N,j}, \quad (2.5-142)$$

where $\boldsymbol{\psi}_j$ is the j^{th} modal drift vector, calculated from the roof-normalized modal displacement vector $\boldsymbol{\psi}_j = \mathbf{B} \cdot \boldsymbol{\phi}_j$. The energy dissipated by viscous damping in storey n is given by Eqn. (2.4-20):

$$W_{d,n,j} = \pi b_n c_{d,n} \omega_j^{a_n} |\Delta_{n,j}|^{1+a_n} = \pi b_n c_{d,n} \omega_j^{a_n} |\boldsymbol{\psi}_{n,j}|^{1+a_n} u_{N,j}^{1+a_n}. \quad (2.5-143)$$

The maximum strain energy of the whole system is obtained at maximum displacement as:

$$W_{s,j} = \frac{1}{2} (\mathbf{k} \mathbf{u}_j)^T \mathbf{u}_j, \quad (2.5-144)$$

or, alternatively, can be equated to the maximum kinetic energy:

$$W_{s,j} = \frac{1}{2} \dot{\mathbf{u}}_j^T \mathbf{m} \dot{\mathbf{u}}_j = \frac{1}{2} \boldsymbol{\phi}_j^T \mathbf{m} \boldsymbol{\phi}_j \omega_j^2 u_{N,j}^2, \quad (2.5-145)$$

where the storey velocities have been expressed as $\omega_j \boldsymbol{\phi}_j u_{N,j}$. Finally, the equivalent viscous damping ratio of the system can be obtained using Jacobsen's formula as:

$$\xi_{a,j} = \frac{W_{d,j}}{4\pi W_{s,j}} = \frac{\sum_{n=1}^N b_n c_{d,n} \omega_j^{a_n} |\psi_{n,j}|^{1+a_n} u_{N,j}^{1+a_n}}{2 \omega_j^2 u_{N,j}^2 \boldsymbol{\phi}_j^T \mathbf{m} \boldsymbol{\phi}_j}. \quad (2.5-146)$$

This expression allows for different damper exponents at different storeys. For the more frequent design case in which all damper exponents are similar ($a_n = a$), the expression reduces to:

$$\xi_{a,j} = \frac{b \omega_j^{a-2} u_{N,j}^{a-1} \sum_{n=1}^N c_{d,n} |\psi_{n,j}|^{1+a}}{2 \boldsymbol{\phi}_j^T \mathbf{m} \boldsymbol{\phi}_j}. \quad (2.5-147)$$

In this equation, $\xi_{a,j}$ is dependent on the amplitude of motion $u_{N,j}$. For conventional systems, $a < 1$ and the displacement term exponent ($a-1$) is negative; as the displacement increases, added viscous damping ratio is expected to diminish. This effect is conspicuous in elastic systems, but it is moderate in inelastic systems, as the effective frequency also changes. For linear dampers at all storeys:

$$\xi_{a,j} = \frac{\sum_{n=1}^N c_{d,n} |\psi_{n,j}|^2}{2 \omega_j \boldsymbol{\phi}_j^T \mathbf{m} \boldsymbol{\phi}_j}. \quad (2.5-148)$$

Eqn. (2.5-148) is equivalent to the one presented by Constantinou & Symans [1992]. Eqn. (2.5-147) is similar to the extension for non-linear dampers presented in Ramirez et al [2001].

Modal damping ratios derived with the previous equations are valid for harmonic motion. Their use for earthquake excitation is only approximate. It is also assumed that uncoupling of the modal equations is valid, so the system can be analyzed by modal analysis. In each mode, viscous damping ratio must be calculated for a harmonic motion with the same roof amplitude of the modal transient response. This requires an iterative process, except in the case of linear dampers, where viscous damping ratio is not dependent on vibration amplitude.

Application of any of the previous equations to higher modes is problematic because the damper behavior is affected by the cut-off frequency; therefore the assumption of pure viscous damper behavior is no longer valid. This topic is further addressed in Chapter 5.

2.5.4 Capacity curve (pushover curve)

The capacity of a structure to resist lateral forces can be expressed by the non-linear relationship between applied loads and resulting displacements. This relationship is strictly dependent on the excitation and response of the structure, including damping, as shown by Eqn. (2.5-129) reformulated as:

$$\mathbf{f}_s = -\mathbf{m}\ddot{\mathbf{u}} - \mathbf{m}\mathbf{1}a_g(t) - \mathbf{c}\dot{\mathbf{u}}. \quad (2.5-149)$$

If a static analysis is carried out under forces which are representative of the right-side term, the structural response $\mathbf{f}_s$ can be found. This is the underlying principle in non-linear static procedures (NSP). The capacity curve is an expression of $\mathbf{f}_s$ for different intensities. The incremental static analysis that produces the capacity curve is sometimes referred to as pushover analysis, and the capacity curve obtained is known as pushover curve.

For simplicity and to obtain a single representative curve, the vector $\mathbf{f}_s$ is generally expressed in terms of a single scalar: storey shear V_n , storey shear ratio $\bar{V}_n$, base shear V_1 , or base shear ratio $\bar{V}_1$; whereas the intensity is expressed as storey drift Δ_n , storey displacement u_n , relative drift δ_n or, more often, roof relative displacement ($\delta_{tot} = u_N/h_{tot}$). The shear ratio $\bar{V}_n$ is defined as:

$$\bar{V}_n = \frac{V_n}{\sum_{i=n}^N w_i}, \quad (2.5-150)$$

where w_i is the i^{th} storey weight. The denominator in the right term is the cumulative weight for storey i . For global structural performance, the most adequate variables are base shear V_1 and roof displacement u_N (or their relative counterparts $\bar{V}_1$, δ_{tot}). These are the variables chosen in all non-linear static procedures.

The pushover curve is obtained by applying in successive steps an increasing set of lateral forces to the structure and monitoring its lateral displacements. The lateral force pattern applied can be constant (relative values of lateral forces are kept during pushover) or adaptive (relative values of lateral forces change after each step). Because seismic forces are primarily lateral inertial forces, during earthquake action the distribution of lateral forces changes; so-called adaptive load patterns try to simulate this uneven excitation; unfortunately, they are more difficult to compute and their claimed higher accuracy has not been rigorously proved [Pinho *et al.*, 2013]. A more established option is the use of constant lateral load patterns. These can be described by a non-dimensional vector λ . Typical load patterns are: concentrated load, uniform, triangular, first mode, modal, power, SRSS [FEMA, 2005; Reinhorn *et al.*, 2009]. The most usual pattern in real applications is the first mode pattern, as it is assumed that, during earthquake, first-mode vibration is predominant. Some typical load patterns are defined by:

$$\lambda_{CONCENTRATED,n} = 1 \cdot (n = N) \quad (2.5-151)$$

$$\lambda_{UNIFORM,n} = w_n / \sum_{i=1}^N w_i \quad (2.5-152)$$

$$\lambda_{TRIANGULAR,n} = w_n z_n / \sum_{i=1}^N w_i z_i \quad (2.5-153)$$

$$\lambda_{FIRSTMODE,n} = \phi_{n,1} w_n / \sum_{i=1}^N \phi_{i,1} w_i \quad (2.5-154)$$

$$\lambda_{MODEJ,n} = \phi_{n,j} w_n / \sum_{i=1}^N \phi_{i,j} w_i \quad (2.5-155)$$

$$\lambda_{POWER,n} = z_n^a w_n / \sum_{i=1}^N z_i^a w_i \quad (2.5-156)$$

The SRSS pattern is based on elastic pushover analysis for a number of modes $J \leq N$; the elastic storey shear for storey n is obtained for each mode; the modal shear are combined according to the SRSS rule and thus a relative pattern of forces is defined. The number of modes is chosen to achieve a 90% cumulative effective mass ratio.

Figure 2-28. Frame pushover, setup and lateral load pattern.

As shown in Figure 2-28, prior to pushover analysis the building weight must be applied. P-Δ effects should be included in analysis. This is divided in two parts: the actual tributary weight applied on the frame; and the rest up to total seismic weight resisted by the frame, applied on a lean-on column element. The lean-on column is a vertical element with nodes constrained to the corresponding storey nodes, with large area but negligible bending stiffness. Second order effects should be considered in the model.

The values of base shear V_1 and roof displacement u_N can be represented in a plot. This presentation is termed hereby as Vu format. For comparison with spectral demands, the curve must be converted to AD format, by conversion of the MDOF system to a SDOF system, a process that is based on concepts summarized in previous Sections. Rigorously speaking, the method is valid only if the deflected shape of the MDOF ϕ is constant. That means that the method is only strictly valid for a displacement-controlled pushover

analysis, where a constant displacement pattern rather than a constant force pattern is enforced. Nevertheless, the conversion is used liberally for constant force patterns.

Displacement forced analysis is difficult to perform, and past experience has shown that, for well-designed frames, constant force patterns produce reasonable results [Krawinkler & Seneviratna, 1997; Chopra, 2002; FEMA, 2005]. Collapse mechanisms under first mode loads are expected to be complete mechanisms of the form shown in (a), rather than incomplete mechanisms as shown in (c)(d). In that case, the assumption of a constant shape throughout complete pushover is reasonable. For incomplete mechanisms, the assumption can be misleading, resulting in unreliable spectral values.

Figure 2-29. Frame pushover, complete collapse mechanism (a)(b) vs. partial collapse mechanisms (c)(d).

2.5.4.1 Conversion of MDOF to SDOF for constant displacement shape ϕ

The conversion of MDOF to SDOF is performed under the assumption [Krawinkler & Seneviratna, 1997; Reinhorn, 1997] that the deflected shape of the MDOF system throughout earthquake excitation is represented by a single constant vector ϕ . This vector is assumed here as roof-normalized. The displacement vector is expressed as:

$$\mathbf{u} = \phi u_N. \quad (2.5-157)$$

If ϕ is not roof-normalized, the second term must simply be divided by ϕ_N . The MDOF equation for a non-linear system, Eqn. (2.5-129), can be transformed using Eqn. (2.5-157) and pre-multiplying by ϕ^T :

$$M \ddot{u}_N + C \dot{u}_N + \phi^T \mathbf{f}_S = -L a_g(t), \quad (2.5-158)$$

Where, by analogy with 2.5.3.1:

$$M = \boldsymbol{\Phi}^T \mathbf{m} \boldsymbol{\Phi}, \quad (2.5-159)$$

$$C = \boldsymbol{\Phi}^T \mathbf{c} \boldsymbol{\Phi}, \quad (2.5-160)$$

$$F_S = \boldsymbol{\Phi}^T \mathbf{f}_S, \quad (2.5-161)$$

$$L = \boldsymbol{\Phi}^T \mathbf{m} \mathbf{1}, \quad (2.5-162)$$

$$\Gamma = L / M. \quad (2.5-163)$$

The SDOF displacement u^* is defined as:

$$u^* = u_N / \Gamma. \quad (2.5-164)$$

Replacing $u_N = \Gamma u^*$ in Eqn. (2.5-158) and normalizing by L , the following SDOF equation is obtained:

$$\ddot{u}^* + (C / M) \dot{u}^* + \boldsymbol{\Phi}^T \mathbf{f}_S / L = -a_g(t). \quad (2.5-165)$$

Eqn. (2.5-165) represents a SDOF system with mass $m^* = L$, damping coefficient $c^* = \Gamma C$ and restoring force $f^* = \boldsymbol{\Phi}^T \mathbf{f}_S$. The transformation equations from MDOF to normalized SDOF are:

$$S_d = u_N / \Gamma, \quad (2.5-166)$$

$$S_a = \boldsymbol{\Phi}^T \mathbf{f}_S / L. \quad (2.5-167)$$

As pointed out in the previous section, the main implication of the basic assumption given by Eqn. (2.5-157) is that a ‘strict’ pushover analysis should be displacement-based. In current engineering practice, due to the approximate nature of NSP, Eqns. (2.5-166) and (2.5-167) are liberally applied also in case of force-based pushover.

If a yield point, defined by roof displacement $u_{y,N}$, with applied forces $\mathbf{f}_{S,y}$, is chosen in the pushover curve, its base shear is given by:

$$V_{y,1} = \mathbf{1}^T \mathbf{f}_{S,y}, \quad (2.5-168)$$

whereupon the stiffness of the corresponding SDOF is defined as:

$$k^* = \frac{\boldsymbol{\Phi}^T \mathbf{f}_{S,y}}{u_{N,y}} \Gamma, \quad (2.5-169)$$

and the period can be calculated as:

$$T^* = 2\pi \sqrt{\frac{u_{N,y}}{\boldsymbol{\Phi}^T \mathbf{f}_{S,y}} \frac{L}{\Gamma}} = 2\pi \sqrt{\frac{u_{N,y}}{\boldsymbol{\Phi}^T \mathbf{f}_{S,y}} M}. \quad (2.5-170)$$

2.5.4.2 Conversion of MDOF to SDOF for modal shape φ_1

The especial and common case $\boldsymbol{\varphi} = \boldsymbol{\varphi}_1$ is discussed in this Section (the process can be applied to any other mode $\boldsymbol{\varphi} = \boldsymbol{\varphi}_j$ by exchanging every first mode modal property X_1 by its j -th mode counterpart X_j). When modal shapes are chosen in Eqn. (2.5-157), some simplification is obtained. From basic modal considerations, if the structure remains elastic and the response is dominated by the first mode, forces are given by Eqn. (2.5-106), specialized for mode $j = 1$; the expression is repeated here in matrix format for convenience:

$$\mathbf{f} = \mathbf{m} \boldsymbol{\varphi}_1 \Gamma_1 S_a = \mathbf{m} \boldsymbol{\varphi}_1 \omega_1^2 \Gamma_1 S_d, \quad (2.5-171)$$

This expression indicates that, if the forces applied in the pushover analysis are proportional to the storey mass and $\boldsymbol{\varphi}_1$, the displacements are proportional to the first mode shape (at least during the elastic stage). The pushover forces can be expressed as:

$$\mathbf{f} = p \mathbf{m} \boldsymbol{\varphi}_1 = \mathbf{m} \boldsymbol{\varphi}_1, \quad (2.5-172)$$

The base shear of the MDOF system is given by the addition of forces:

$$V_1 = \mathbf{1}^T \mathbf{f} = \mathbf{1}^T \mathbf{m} \boldsymbol{\varphi}_1 p = L_1 p. \quad (2.5-173)$$

It follows that:

$$\mathbf{f}_s = \frac{V_1}{L_1} \mathbf{m} \boldsymbol{\varphi}_1, \quad (2.5-174)$$

and using this expression on Eqn. (2.5-165):

$$\ddot{u}^* + 2\xi_1 \omega_1 \dot{u}^* + V_1 / M_{eff,1} = -a_g(t), \quad (2.5-175)$$

where $M_{eff,1}$ is the effective mass defined in Eqn. (2.5-108). The properties of the system are:

$$u^* = u_N / \Gamma_1. \quad (2.5-176)$$

$$m^* = L_1, \quad (2.5-177)$$

$$c^* = \Gamma_1 C_1 = 2\Gamma_1 M_1 \omega_1 \xi_1 = 2L_1 \omega_1 \xi_1, \quad (2.5-178)$$

$$f^* = V_1 / \Gamma_1. \quad (2.5-179)$$

And the corresponding spectral ordinates are:

$$S_d = u_N / \Gamma_1, \quad (2.5-180)$$

$$PS_a = V_1 / M_{eff,1}. \quad (2.5-181)$$

If the yield point in the pushover curve is identified by $V_{y,1}$, $u_{y,N}$, the corresponding yield stiffness κ_1 of the MDOF is:

$$\kappa_1 = \frac{V_{y,1}}{u_{y,N}} \quad (2.5-182)$$

In a bilinear simplification of the pushover curve, κ_1 is not strictly located in the pushover curve. The yield stiffness and period of the SDOF are:

$$\omega^* = \sqrt{\frac{V_1 / M_{eff,1}}{u_N / \Gamma_1}} = \sqrt{\frac{V_1}{u_N L_1}}, \quad (2.5-183)$$

$$T^* = 2\pi \sqrt{\frac{u_{y,N}}{V_{y,1}}} L_1, \quad (2.5-184)$$

$$k^* = (\omega^*)^2 L_1 = \frac{V_{y,1}}{u_{y,N}} = \kappa_1 \quad (2.5-185)$$

c^* can also be calculated using any other standard expression ($2\xi_1 k^* / \omega^*$, $2\xi_1 (k^* m^*)^{0.5}$, etc.).

It is important to acknowledge that, due to the change in stiffness as the structure progressively degrades, modal properties change during the different stages of the pushover. Therefore, the former expressions must be evaluated at each point of the pushover curve with the updated modal properties. This is problematic when the structure is at an advanced yielding stage and the stiffness is correspondingly softened. As a simplification, current established procedures consider constant modal shapes throughout the conversion process.

2.5.4.3 Energy transformation

The work performed by the applied forces in the original system at any step of the pushover is:

$$\Delta W = \mathbf{f}^T \delta \mathbf{u}, \quad (2.5-186)$$

where $\delta \mathbf{u}$ is the incremental displacement vector between two steps. Using Eqns. (2.5-179) and (2.5-157), the equation can be transformed into:

$$\Delta W = \frac{V_1 \delta u_N}{\Gamma_1}. \quad (2.5-187)$$

The corresponding work in the SDOF system can be obtained by:

$$\Delta W^* = f^* \delta u^*. \quad (2.5-188)$$

Replacing Eqns. (2.5-179) and (2.5-176):

$$\Delta W^* = \frac{V_1 \delta u_N}{\Gamma_1^2} = \frac{\Delta W}{\Gamma_1}. \quad (2.5-189)$$

The former equations were presented for ϕ_1 , but are also valid for an arbitrary ϕ , simply replacing Γ_1 by Γ .

2.5.4.4 Modification of modal properties with yielding

Figure 2-30 and Figure 2-31 offer information about pushover and approximate modal properties for two different frames, 05D and 05C (see definition in Annex B). In Figure 2-30, relevant plots are given for frame 05D and two cases: small (0.01%) post-yield stiffness and relevant P- Δ effects (left column, referred to as 05D $_{\alpha=0\%}$), and moderate (3%) post-yield stiffness and small P- Δ effects (right column, referred to as 05D $_{\alpha=3\%}$). Likewise, in Figure 2-31, relevant plots are given for frame 05C and two cases, small (0.01%) post-yield stiffness (left column, referred to as 05C $_{\alpha=0\%}$) and moderate (3%, referred to as 05C $_{\alpha=3\%}$) post-yield stiffness (right column); frame 05C presents a soft storey (4th storey).

The pushover curve is formed by successive steps. The s^{th} step can be characterized by the roof ductility demand μ (hereby referred to simply as ductility or ductility demand), defined as:

$$\mu = \frac{u_N^{(s)}}{u_{y,N}}, \quad (2.5-190)$$

where $u_N^{(s)}$ is the roof displacement at step s . There are several options for the definition of the yield point. Remarkable variations in yield displacement (and therefore in ductility) are obtained for the different definitions of the yield point. Here $u_{y,N}$ has been calculated with an equal energy criterion at the point of maximum shear, discussed under the heading ‘energy’ in Section 2.5.4.5, and is included as reference in the pushover curves shown in Figure 2-30(a)(b) for frame 05D and Figure 2-31(a)(b) for frame 05C. In all plots, the indication $\mu = 0$ refers to the initial conditions at the origin of the pushover curve, i.e. pure elastic conditions. Thus $T(0)$ refers to the initial first mode period, etc.

The progression of the storey displacements relative to the initial storey displacements is tracked in Figure 2-30 (c)(d) for frame 05D and Figure 2-31(c)(d) for frame 05C. It is clear that, as yielding progresses, the displacement shape departs from the initially assumed shape (ϕ_1), as a consequence of the constant force vector applied in the pushover analysis. The expectation of the first modal shape ϕ_1 to be representative of the actual displacements after yielding is proved to be valid only for frames with complete collapse mechanisms and very moderate values of ductility, around 1.4 for frame 05D $_{\alpha=0\%}$, and 2 for frame 05D $_{\alpha=3\%}$ (20% error in modal shape). For frame 05C, regardless of the level of ductility, the modal shapes are inadequate after soft-storey yielding.

If the actual relative displacement vector at every step is taken as an approximate representative modal shape, approximate modal properties can be calculated. These are presented in Figure 2-30(e)(f) for frame 05D and Figure 2-31(e)(f) for frame 05C. For frame 05D with complete collapse mechanism, the modal properties are quite robust in

spite of the remarkable variations in modal shapes. For this reason, the conversion between the MDOF and SDOF properties can be performed with the constant vector ϕ_1 , with a very small error, Figure 2-30(a)(b). For frame 05C large variations happen in modal shapes after yielding and the conversion using the constant vector fails for inelastic behavior, Figure 2-31(a)(b).

Finally, the variation of period with ductility, calculated with the approximated modal shapes, is presented in Figure 2-30(g)(h) for frame 05D and Figure 2-31(g)(h) for frame 05C, and compared with the effective period estimated from Eqn. (2.5-48), evaluated for $\alpha = 0$. The expression gives a very good estimate of period for 05D $\alpha=3\%$, a frame with well-defined bi-linear hysteretic law with no strength degradation, but it under-predicts the period for all cases with P- Δ or strength degradation effects.

Figure 2-30. Frame 05D: (a)(b) conversion of pushover curve to AD format. (c)(d) Modification of deformed shape with yielding. (e)(f) Approximate modification of modal properties with yielding. (g)(h) Approximate modification of first modal period with yielding.

Figure 2-31. Frame 05C: (a)(b) conversion of pushover curve to AD format. (c)(d) Modification of deformed shape with yielding. (e)(f) Approximate modification of modal properties with yielding. (g)(h) Approximate modification of first modal period with yielding.

2.5.4.5 Bilinear approximation to pushover curve

The pushover curve is used in the NSP to identify the performance point. This is the point closer to or lying at the curve chosen to represent the seismic demand. The performance point is defined by its coordinates $u_{p,N}$, $V_{p,1}$, see Figure 2-32. For the purpose of simplified analysis it is common to search a bilinear approximation to the pushover curve. This is defined by a first elastic branch with stiffness κ , up to a yield point ($u_{y,N}$, $V_{y,1}$) and a second post-yield branch with post-yield stiffness $\alpha\kappa$. The initial stiffness can be defined by the

tangent stiffness at the origin, in which case $\kappa = k_1 = \omega_1^2 L_1$, or it can be defined by a selected point $(u_{e,N}, V_{e,1})$ in the curve. A typical point [ATC, 2005] is defined by $V_{e,1} = 0.6V_{m,1}$, where $V_{m,1}$ is the peak shear in the pushover curve.

Figure 2-32. Bilinear approximation to pushover curve.

There are several possibilities to linearize the pushover curve. Figure 2-34 shows some options for a frame (05D, described in Annex B). In the left column, pushover plots are offered for the frame analyzed considering a 3% post-yield stiffness, where the influence of P- Δ is very moderate. In the right column, the post-yield stiffness is negligible and the P- Δ effects are relevant. Five different bi-linearization techniques have been used. All of them preserve the tangent stiffness at the origin, which is coherent with the modal properties.

- Envelope: as shown in Figure 2-34(a)-(b), the yield point is characterized by the maximum value of V_1 , i.e., $V_{y,1} = V_{m,1}$, and the initial stiffness κ , whereupon $u_{y,N} = V_{m,1}/\kappa$. Post-yield stiffness is assumed as 0. This option has the advantage of simplicity, offering a single approximation for all performance points. However, the value of $V_{p,1}$ is systematically overestimated and so is the energy dissipation.
- Reduced envelope: as shown in Figure 2-34(c)-(d), the yield point is defined by the performance point shear, $V_{y,1} = V_{p,1}$, and the initial stiffness κ , whereupon $u_{y,N} = V_{p,1}/\kappa$. Post-yield stiffness is 0. This option over-estimates energy dissipation before the peak point and under-estimates it after the peak point.
- Tangent: shown in Figure 2-34(e)-(f), the yield point is defined by intersection of two lines, one going through the origin with stiffness κ , and the other tangent to the pushover curve at the performance point. Post-yield stiffness is defined as the slope of the second line. This option over-estimates energy dissipation. For negative post-yield stiffness, it also gives values of yield shear above the peak shear $V_{m,1}$. The yield displacement is defined by:

$$u_{y,N} = \frac{V_{p,1} - \alpha \kappa u_{p,N}}{\kappa - \alpha \kappa}. \quad (2.5-191)$$

- Energy: shown in Figure 2-34(g)-(h), the yield point is defined so that the area below the bilinear curve equals the area A_p below the pushover curve, both areas measured from the origin to the performance point. The energy dissipation is exact. The post-yield stiffness is the slope of the second line and can be positive or negative. The expression for $u_{y,N}$ is:

$$u_{y,N} = \frac{2A_p - V_{p,1}u_{p,N}}{ku_{p,N} - V_{p,1}}. \quad (2.5-192)$$

- Capped energy: shown in Figure 2-34(i)-(j); a maximum value is imposed on yield shear: $V_{y,1} \leq V_{m,1}$; if the performance point is located before the peak point, the limitation has no effect and energy dissipation is exact. If the performance point is located after the peak point, the estimation of dissipated energy is conservative (smaller than in the actual curve), which is not especially relevant considering that this is a region of potential instability.

An important implicit assumption in the bi-linearization process of the capacity curve is that yielding at all storeys happens at approximately the same step in the pushover curve. This is clear by using modal properties on $u_{y,N}$:

$$\Delta_{y,n} = \psi_{n,1} u_{y,N}, \quad (2.5-193)$$

This approximate assumption is reasonable for moment resisting frames (MRFs), as long as they are designed for a complete mechanism in the first mode. Figure 2-33 shows a comparison between storey yield shear $V_{y,n,j}^{(\text{exact})}$ and yield drift $\Delta_{y,n,j}^{(\text{exact})}$ obtained from modal pushover analysis ('exact' values) vs. their 'modal' counterparts calculated using

$$\Delta_{y,n,j}^{(\text{modal})} = \psi_{n,j} u_{y,N,j}, \quad (2.5-194)$$

$$V_{y,n,j}^{(\text{modal})} = k_{s,n,j} \psi_{n,j} u_{y,N,j}, \quad (2.5-195)$$

referred to as 'modal' values. The process to obtain the values was the following:

1. Non-linear pushover analysis for first and second mode lateral load patterns was performed; the values of storey displacement, storey drift and storey shear were retrieved.
2. The overall pushover curve ($V_{1,j} - u_{N,j}$) was plotted and its yield point found by intersection of lines tangent to elastic and post-elastic branches; this point defines $V_{y,1,j}$ and $u_{y,N,j}$.
3. The storey pushover curve for each storey ($V_{n,j} - \Delta_{n,j}$) was plotted and each storey yield point found by intersection of lines tangent to elastic and post-elastic branches; these points define $V_{y,n,j}^{(\text{exact})}$ and $\Delta_{y,n,j}^{(\text{exact})}$ for each storey. These values are referred to as 'exact'.

4. Eqns. (2.5-194) and (2.5-195) were applied to $V_{y,1,j}$ to obtain the ‘modal’ values $\Delta_{y,n,j}^{(\text{modal})}$ and $V_{y,n,j}^{(\text{modal})}$.

Comparison of ‘exact’ and ‘modal’ results are shown for 13 steel MRFs, 2 to 9 storeys high, designed to form complete lateral weak beam – strong column mechanisms. The frames are described in Annex B. These results, obtained for modes 1 and 2, show that storey yield values obtained from Eqns. (2.5-194) and (2.5-195) are reasonably accurate for the types of frames examined, particularly in the first mode.

For frames with soft storeys, the conversion of pushover curve to AD format and the bi-linearization process are compromised because modal shapes are not representative of the post-yield shape. The pushover curve cannot be considered representative of the frame capacity, not even at moderate ductility values.

Figure 2-33. Comparison of yield properties obtained by exact pushover and modal approximation; (a)(b) yield shear, (c)-(d) yield drift.

Figure 2-34. Different approaches to linearization of the pushover curve.

2.5.4.6 Estimation of capacity based on limit analysis

For well-designed frames an estimate of the capacity can be easily obtained. This is possible because the 1st mode yield pattern in moment resisting frames designed for earthquake loads derives from the strong-column weak-beam criterion; hinges are expected to form at beams and column bases only. An idealized version of the collapse mechanism is shown in Figure 2-29(a). Using this configuration, it is possible to predict the yield base shear of the 1st mode load pattern using kinematic limit analysis, i.e., equating the external work produced by the loads to the internal work dissipated by the plastic hinges, assuming that the post-yield stiffness is negligible. The procedure is outlined in Ramirez *et al* [2001], and is summarized here because a modification of this method is used in Chapter 6.

Referring to the complete mechanism in Figure 2-29(a), the external work W_e is calculated as the sum of external force times displacement. At yield, the relationship between force vector $\mathbf{f}_y$ and base shear $V_{y,1}$ can be expressed using Eqn. (2.5-174), rewritten here as:

$$\mathbf{f}_y = \frac{\mathbf{m} \boldsymbol{\phi}_1}{\boldsymbol{\phi}_1^T \mathbf{m} \mathbf{1}} V_{y,1}. \quad (2.5-196)$$

The horizontal displacement of storey n is given by $z_n \cdot \theta$, with z_n and θ defined in Figure 2-29. Notating $\mathbf{z} = [z_1, \dots, z_N]^T$, the virtual external work W_e is given by:

$$W_e = \mathbf{f}_y^T \mathbf{z} \theta = \frac{\boldsymbol{\phi}_1^T \mathbf{m} \mathbf{z}}{\boldsymbol{\phi}_1^T \mathbf{m} \mathbf{1}} V_{y,1} \theta. \quad (2.5-197)$$

z_n must be measured relative to the approximate height of hinge formation in the frame base, generally taken as one column height over the base. The virtual internal work dissipated in beams and columns is:

$$W_i = \sum_{n=1}^N \sum_{r=1}^R 2M_{by,n,r} \theta_{n,r} + \sum_{r=1}^{R+1} M_{cy,r} \theta, \quad (2.5-198)$$

where R is the number of spans; $M_{by,n,r}$ is the yield moment of beam at storey n and span r ; $M_{cy,r}$ is the column base yield moment for the r -th column; and $\theta_{n,r}$ is defined in Figure 2-29(b), and can be expressed as $\theta_{n,r} = \theta L_r / L_{c,n,r}$, with L_r being the length of the r -th span, and $L_{c,n,r}$ the clear length between hinges of beam at storey n and span r . W_i is then a function of geometrical and strength properties of the frame and θ , this can be expressed as:

$$W_i = \left(\sum_{n=1}^N \sum_{r=1}^R 2M_{by,n,r} \frac{L_r}{L_{c,n,r}} + \sum_{r=1}^{R+1} M_{cy,r} \right) \theta = X_1 \theta \quad (2.5-199)$$

X_1 has units of moment, is given by the bracketed term in Eqn. (2.5-199), and is independent on the force pattern applied, as long as the collapse mechanism is unchanged. The subscript '1' indicates first mode. Equating W_i , Eqn. (2.5-199) with W_e , Eqn. (2.5-197), the value of yield base shear can be derived as:

$$V_{y,1} = \frac{\boldsymbol{\phi}_1^T \mathbf{m} \mathbf{1}}{\boldsymbol{\phi}_1^T \mathbf{m} \mathbf{z}} X_1. \quad (2.5-200)$$

The yield displacement can be found using basic modal analysis as:

$$u_{y,N} = \frac{V_{y,1}}{\omega_1^2 L_1}. \quad (2.5-201)$$

2.5.4.7 Simplified P-Δ effect

The simple method outlined in the previous Section renders values of yield shear and yield displacement. No indication is given for the post-yield branch, generally assumed to be horizontal. The influence of P-Δ effects can be introduced in this simplified model through the model presented by Gupta & Krawinkler [1999].

2.5.4.8 An energy procedure to avoid the use of an effective yielding point

Bi-linearization of the pushover curve is needed to define a yield point, used to calculate ductility μ and post-yield stiffness α in order to apply Eqn. (2.5-52); this expression gives the effective energy dissipated for one hysteretic loop at maximum displacement, used to assess the demand. Paradoxically, definition of the yield point is dependent on the performance point chosen; for this reason, the ductility values measured at different performance points are not consistent, unless a unique yield point is defined. For numerical application, if the target is definition of effective damping, an alternate procedure based on energy concepts, is suggested hereby.

First, pushover data is used to obtain the work W_p performed by forces applied, up to every step p (> 1) of the pushover, using the trapezoidal integration rule:

$$W_p = W^{(p)} = W^{(p-1)} + \frac{1}{2} (\mathbf{f}^{(p-1)} + \mathbf{f}^{(p)})^T (\mathbf{u}^{(p-1)} - \mathbf{u}^{(p)}), \quad (2.5-202)$$

with $W^{(1)} = 0$. Likewise, the work performed in the SDOF system representation of the MDOF system must be obtained using Eqn. (2.5-188) as:

$$W_p^* = W^{*(p)} = W^{*(p-1)} + \left(f^{*(p-1)} + f^{*(p)} \right) \left(u^{*(p-1)} - u^{*p} \right). \quad (2.5-203)$$

The work can also be obtained from the V_1 - u_N curve, however, as per Eqn. (2.5-187), this is not the true work of the MDOF system. Instead, the work so obtained must be divided by Γ_1 :

$$W_p = W^{(p)} = W^{(p-1)} + \frac{1}{2\Gamma_1} \left(V_1^{(p-1)} + V_1^{(p)} \right)^T \left(u_N^{(p-1)} - u_N^p \right) = \frac{W_p^{Vu}}{\Gamma_1}, \quad (2.5-204)$$

This is the area enclosed under the pushover curve in Vu format (referred to as W_p^{Vu}) calculated up to point p in Figure 2-35, and divided over Γ_1 . The hysteretic loop energy, though, is four times the (corrected) area of the rhomboid 0ypr, which can be expressed as:

$$W_{h,p} = \frac{4}{\Gamma_1} [2W_p^{Vu} - V_{p,1} u_{p,N}] \quad (2.5-205)$$

Figure 2-35. Energy method.

The (corrected) elastic energy stored under the triangle 0pq is:

$$W_{s,p} = \frac{V_{p,1} u_{p,N}}{2\Gamma_1}, \quad (2.5-206)$$

This equation allows the capacity curve to be expressed in terms of hysteretic energy. The effective viscous damping is:

$$\xi_{h,p} = \frac{W_{h,p}}{4\pi W_{s,p}} = \frac{2}{\pi} \left[\frac{2W_p^{Vu}}{V_{p,1} u_{p,N}} - 1 \right]. \quad (2.5-207)$$

This equation allows the capacity curve to be expressed in terms of viscous damping ratio. Because viscous damping ratio is a fraction, the term Γ_1 disappears from the expression. The expression can be rephrased using the effective frequency ω_{eff} :

$$V_{p,1} = \omega_{eff}^2 L_1 u_{p,N}, \quad (2.5-208)$$

which finally leaves:

$$\xi_{h,p} = \frac{2}{\pi} \left[\frac{W_p^{Vu}}{\omega_{eff}^2 L_1 u_{p,N}^2} - 1 \right]. \quad (2.5-209)$$

If the PS_a - S_d curve is used instead of the V_1 - u_N curve, the expression converts to:

$$\xi_{h,p} = \frac{2}{\pi} \left[\frac{W_p^{AD}}{\omega_{eff}^2 M_{eff,1}^2 S_d^2} - 1 \right], \quad (2.5-210)$$

where the notation W_p^{AD} has been introduced to indicate the area below the AD format curve. Finally, if the MDOF work W_p is used:

$$\xi_{h,p} = \frac{2}{\pi} \left[\frac{W_p \Gamma_1}{\omega_{eff}^2 L_1^2 u_{p,N}^2} - 1 \right]. \quad (2.5-211)$$

With this definition of $\xi_{h,p}$, the consistent way to express the degree of inelasticity of the system is to use the monotonically increasing parameter:

$$\mu_T = \frac{T_{eff}^2}{T^2} = \frac{4\pi^2 S_d / PS_a}{T^2}, \quad (2.5-212)$$

which represents the quotient between secant stiffness at any point and the tangent stiffness at the origin. This parameter is referred to as ‘period ductility’ and is similar in value to the displacement ductility. This concept is similar to the ‘damage ratio’ used as a stiffness softening value in the substitute structure method proposed by Shibata & Sozen [1974]. The previous concepts are summarized in Figure 2-35.

Incidentally, the true value of $\Gamma^{(p)}$ can be obtained using the previous concepts, as:

$$\Gamma_1^{(p)} = \frac{W_p^{Vu}}{W_p}. \quad (2.5-213)$$

As an example, Figure 2-36 displays the capacity curve for the frames analyzed in the previous sections (05D and 05C with 0% and 3% post-yield stiffness ratio), expressed in work - spectral displacement (W- S_d) format (left column). The figure displays also the effective viscous damping ratio. The beginning of yielding can be traced with accuracy as the point where the W_h curve (or hysteretic damping) first appears.

Figure 2-36. Frames 05D and 05C with 0% and 3% post-yield stiffness ratio: Capacity diagram in W-Sd format (left column). Effective viscous damping ratio ξ_{eff} vs. S_d (right column).

Energy-based SDOF

Hernández-Montes *et al.* [2004] presented a method for the conversion from MDOF to SDOF based on energy principles. In this method, the work increment ΔW_p performed in the MDOF system at every step p is first calculated by Eqn. (2.5-186). The SDOF is defined with similar base shear $V_1^{(p)}$ and similar work increment:

$$\Delta u_{ENERGY}^{(p)} = \frac{\Delta W_p}{V_1^{(p)}}, \quad (2.5-214)$$

Comparing this Equation with Eqn. (2.5-187), it is obvious that:

$$\Delta u_{ENERGY}^{(p)} = S_d^{(p)} \quad (2.5-215)$$

That is, the displacement obtained is directly the spectral displacement. Spectral acceleration is obtained with Eqn. (2.5-181).

2.5.5 Non-linear Static Procedures (NSP)

The RSA method summarized in Section 2.5.3.1 can only be rigorously applied to elastic systems. However, approximate procedures have been derived to approximate the solution of inelastic systems using modifications of RSA. A family of methods known as non-linear static procedures (NSPs) relies on non-linear static analysis to approximate the dynamic response of non-linear structures.

NSPs are based on comparison of capacity versus demand; it is often advantageous to express this comparison graphically. Capacity is defined by the non-linear static incremental pushover curve discussed in Section 2.5.4. Demand is expressed by the spectral acceleration - spectral displacement curve. To compare demand and capacity, both curves must be expressed in similar coordinates, and the most suitable option is the AD format presented in Section 2.5.1.1. Therefore, expression of the pushover curve in AD format is convenient, although for engineering purposes the Vu format is more useful. The conversion parameters were given in Section 2.5.4.1 (general shape ϕ) or Section 2.5.4.2 (first mode shape ϕ_1).

Existing NSP based on non-adaptive lateral force vectors are:

- The capacity spectrum method (CSM).
- The N2 method.
- The modal pushover analysis (MPA) method.

Other methods exist, that rely on adaptive lateral force vectors [Antoniou & Pinho, 2004; Kalkan & Kunnath, 2006]. As pointed out in Section 2.5.4, parametric studies [Pinho et al, 2013] have proved that the accuracy of these methods is not superior to that obtained using conventional non-adaptive lateral patterns, and their analysis requires especial tools. For this reason, these methods are not discussed in this study. A brief summary of methods based on constant lateral load patterns follows:

2.5.5.1 Capacity spectrum method (CSM)

The capacity spectrum method (CSM) was first introduced by Freeman [1975, cited in Freeman, 2004], as a rapid verification procedure of seismic vulnerability of a large set of buildings of very different types. A very simple capacity curve was derived using three points (origin, incipient damage and ultimate limit), determined by a mixed procedure of on-site observation, analytical calculation and engineering judgement. The method was

later improved by the introduction of the AD format [Mahaney *et al.*, 1993]; a procedure to identify the relevance of higher modes was suggested by Paret *et al.* [1996], and further developed by Sasaki *et al.* [1998]. An excellent description of the basic method can be found in Freeman [1998]. The CSM is the basis of the ATC-40 verification procedure [ATC, 1996]. The method was tested and some improvements included in the FEMA-400 report [FEMA, 2005]. An outline of the historical development of the method can be found in Freeman [2004]. As indicated in this work, the method can be used for several purposes: as a quick evaluation technique for existing buildings or as a design verification procedure for new buildings; it can also be used to correlate damage in existing buildings to ground motion intensities.

The application starts with the creation of the capacity curve and its transformation to AD format, using the procedures indicated in Section 2.5.4 with the first mode vector ϕ_1 . In this step, it is suggested that relevant benchmarks, such as yielding in important elements, are marked in the curve. An effective yield point must be defined. This is an important landmark, because ductility (and therefore effective damping) will be referred to this point.

Then, demand spectra in AD format are superimposed for different values of viscous damping representing corresponding values of ductility. For a given value of ductility μ , ξ_{eff} is determined using Eqn. (2.5-52); then ξ_{eff} is used to modify the spectrum; if earthquake action is represented by an accelerogram, the elastic spectrum for the corresponding damping is obtained analytically. If the earthquake action is represented by a softened spectrum, this is scaled, either multiplying by the corresponding damping correction factor $\eta_d \leq 1$ (as in Eurocode 8 [CEN, 2004]) or dividing over the corresponding spectral reduction factors $B \geq 1$ (as in North American codes).

The different demand curves (each characterized by its ductility level μ_d) may intersect the capacity curve at different points characterized by their ductility μ_c . If, for a certain demand curve, $\mu_d = \mu_c = \mu$, the performance point is defined by the intersection of both curves. Spectral displacement and acceleration can be read and the roof displacement and base shear determined with Eqns. (2.5-180) and (2.5-181).

As an example, shows the implementation of the method for frame 05D with post-yield stiffness 3% and 2% inherent damping. Figure 2-37(a)(b) display the procedure applied to a recorded motion (000414XA, see Annex A). Figure 2-37(c)(d) show the procedure for the elastic Eurocode 8 spectrum type 1 in site class B. Both spectra have been scaled to be anchored at $PHA = 0.4g$. The spectral acceleration at T_1 is similar at both spectra: $PS_a(T_1) = 0.29g$ for the recorded motion and $0.30g$ for the soft spectrum.

Figure 2-37. CSM for frame 05D with $\alpha = 3\%$, for: (a)(b), motion 000414XA scaled to PHA = 0.4g; (c)(d), Eurocode 8 spectrum for PHA = 0.4g, site class B, spectrum type 1.

Focusing on the solution for the recorded motion, Figure 2-37(a) shows a complete view of the AD format plot, whereas Figure 2-37(b) shows a detail of the region of interest. The capacity curve has been linearized, using the energy criterion with reference to a performance point corresponding to 5% roof displacement ratio. This produces the reference yield point, ($S_{d,y} = 116\text{mm}$; $PS_{a,y} = 0.1155\text{g}$), traced with a circle, and also the post-yield stiffness ratio $\alpha^{AD} = 0.0748$, measured on this plot (the actual α^{Vu} post-yield ratio is found multiplying by L_1). The line radiating from the origin through the yield point defines ω_1^2 , and thus the initial period $T_1 = 2.01\text{s}$. The intersection of this line with the 2% demand spectrum (demand curve for $\mu = 1$) represents the elastic behavior. However, because this intersection happens outside the elastic branch of the capacity curve, it is clear that the frame is not strong enough to withstand elastic forces and will enter into the inelastic realm.

Demand curves for different values of ductility are plotted. To obtain one of these curves, μ is fixed and the hysteretic damping ξ_h evaluated using Eqn. (2.5-52) with q_H taken as 1. Then the elastic spectrum is found for $\xi = \xi_h + \xi_i$. The same value of ductility is represented by its effective period T_{eff} calculated with Eqn. (2.5-48), that intersects the capacity curve at point $(\mu \cdot S_{d,y}; PS_{a,y} + \omega_1^2 \cdot \alpha^{AD} \cdot (\mu - 1))$. In Figure 2-37(b) the same colour is used to trace the spectrum, effective period and capacity point corresponding to the same value of ductility.

The demand curve for $\mu = 1.2$ (marked in red and continuous line) intersects the capacity curve at about $S_d = 220\text{mm}$, whereas the effective period (marked in red and dashed line) does so at about 140mm (red square); so the demand ductility is larger than the capacity ductility $\mu_d > \mu_c$. For the next ductility value, $\mu = 1.5$, it is observed that the situation is reversed, $\mu_d < \mu_c$. This is indicative that the actual solution is within these two values of ductility. By iteration the actual value of $\mu = 1.43$ is reached. The corresponding effective period T_{eff} is 2.37s, represented by the solid black line radiating from the origin. The final values obtained are $S_d = 166\text{mm}$ and $PS_a = 0.119g$, that convert to frame roof displacement using Eqns. (2.5-180) and (2.5-181) with $\Gamma_1 = 1.36$ and $M_{eff,l} = 1.02 \text{ kN} \cdot \text{s}^2/\text{mm}$, giving $u_N = 226\text{mm}$ and $V_1 = 1187 \text{ kN}$. The actual values calculated by NLTHA for this frame and the scaled excitation is $u_{N,\text{exact}} = 217\text{mm}$, which indicates a very good approximation, but $V_{1,\text{exact}} = 1740 \text{ kN}$, which is 47% higher than the value obtained by the CSM. This is indicative of one of the drawbacks of this method, namely, that it approximates well kinematic variables, particularly displacement, but it can be un-conservative for static variables. The same procedure with $q_H < 0$ would produce larger displacements.

With the value of spectral acceleration, Eqn. (2.5-106) can be used to obtain the vector of forces $\mathbf{f}_s$; non-linear static analysis under this force vector renders the internal forces in the members. Alternatively, this data can be retrieved at the step of corresponding roof displacement (u_N) in the pushover curve.

Some relevant remarks:

- The method has been profusely verified to produce reasonable approximations to displacement [FEMA, 2005; Goel & Chopra, 2004]; there is less evidence regarding its accuracy with respect to other structural parameters, but from the existing data it seems that the method produces poor approximations to drift, shear and overturning moments.
- The merit of the method is its ability to render a quick estimate of the roof displacement and of the relative position of this point in the capacity curve (i.e., the ductility demand μ) can be considered indicative of damage. In first generation performance-oriented codes, such as ATC-40 [ATC, 1996] or FEMA-356 [FEMA, 2000] roof displacement ratio serves to verify the performance objectives.
- Due to the inherent approximations included in the method (variability in models, properties, earthquake excitation, etc.), the level of accuracy does not seem to be relevant. The method is more qualitative than quantitative.

- The method does not include the effect of strength and/or stiffness degradation, although simplified approaches to include these effects exist.
- The influence of higher modes is neglected; this is particularly troublesome for tall buildings.

The choice of the yield point is ambiguous; for the usual bi-linearization criterion (equating energy under the bilinear curve and the actual capacity curve) the yield point depends on the performance point and is therefore changed with the level of ductility. This is disturbing from the theoretical point of view. To avoid over-complication, a unique yield point can be chosen from a pre-selected maximum performance point. Maximum acceptable displacement points (for instance, $u_N/h_{tot} = 0.025$) are a reasonable choice. This was the option adopted in Figure 2-37.

The method introduced in Section 2.5.4.8, can be used for enhanced coherency. Because effective damping ratio is directly available as a function of S_d (see Figure 2-36), every point in the curve can be directly assigned a damping ratio that can be compared with that of the intersecting demand curve. The performance point is reached when both values are coincident. The period-based-ductility is defined by Eqn. (2.5-212); for the case shown, $\mu_T = 1.39$, similar but not equal to μ . No bi-linearization is needed and the performance point is located exactly on the capacity curve.

Influence of higher modes

The influence of higher modes is not contemplated in the simple version of the method. Several authors [Paret *et al.*, 1996; Sasaki *et al.*, 1998] showed that this influence is relevant in tall buildings (the definition of tall building in the literature is somewhat fuzzy) and proposed the following procedure to quantify this influence: First, pushover curves for higher modes are produced using the corresponding modal shapes. Then, the capacity curves are plotted in AD format and superimposed to the elastic demand spectrum. If the straight part of the modal capacity curve intersects the spectrum, then the mode is not likely to produce yielding. Otherwise, the mode will participate in yielding. To quantify this procedure, a modal criticality index *MCI* is proposed:

$$MCI_j = \frac{PS_{a,e}}{PS_{a,y}}, \quad (2.5-216)$$

where $PS_{a,e}$ is the spectral ordinate of the intersection of the modal period line with the elastic spectrum, and $PS_{a,y}$ is the ordinate of the spectral yielding point. The procedure is illustrated in Figure 2-38 for the same example discussed in the previous section (frame 05D _{$\alpha=3\%$} with 2% damping ratio subjected to accelerogram 000414XA). The three modal pushover curves are shown in Figure 2-38(a) in Vu format; these curves have been truncated at the point where the maximum interstorey drift ratio reaches 0.025. Figure 2-38 (b) displays the three modal pushover curves in AD format and their corresponding bi-linearization; the yield point for mode j is labeled as Y_j ; the elastic 2% spectrum is

superimposed and the elastic demand is found at the intersection of the mode initial stiffness line with the spectrum; this point is labeled as X_j . Dividing the ordinate of Y_j over the ordinate of X_j , $\mathbf{MCI} = [2.53, 1.63, 0.75]^T$. The maximum value corresponds to the first mode, which indicates that this is the most critical. However, MCI_2 is larger than one, indicating that yielding at this mode is also to be expected. Finally, the third mode is remaining elastic. The definition of the yield points is again ambiguous, and a better criterion would be the starting point of yielding (calculated as indicated in Section 2.5.4.8) because this is the starting point of damage in the structure. Another issue is the number of modes to include, as convergence difficulties in the pushover increase with increasing mode number. A cumulative effective modal mass ratio over 90% is often used as a reasonable criterion. Most studies use two or three modes for buildings up to 9 storeys; Goel & Chopra [2004] used five modes for a 20 storey building.

Figure 2-38. Influence of higher modes in CSM, frame 05D $_{\alpha=3\%}$: (a) pushover curves for modes 1 to 3; (b) determination of modal criticality index MCI for motion 000414XA scaled to PHA = 0.4g.

The influence of higher modes was studied by Nassar *et al.* [1991] (presented with more detail in Nassar & Krawinkler [1992]), using MDOF frames with three different failure modes (by beam hinging, BH, by column hinging, CH and by soft-storey at first storey, WS). The systems, with heights from 2 to 40 storeys, hardening ratios of $\alpha = 0$ and 10%, and different ductility demands in the range $\mu = 1-8$, were designed to storey shear distributions obtained from equivalent static lateral loads distributions, and then subjected to 25 strong ground motions. The target was to study storey ductility, shear and overturning moment ductility demand and their correlation with the base shear, estimated from an equivalent SDOF of similar period and ductility, subjected to the same ground motions, and to use these data to estimate the base shear required in MDOF systems (compared to that of the equivalent SDOF) in order to keep the inter-storey ductility demand below the level prescribed. It was found that the base shear demand of the MDOF compared to that of the SDOF increases with period, with ductility, and is always much larger for systems with soft-storeys. For instance, for a period of $T = 2s$ and $\mu = 2$, the base shear strength required in a soft-storey system is around 1.5 than for the other two types, whereas for $\mu = 8$ it is more than double.

Use of displacement-based effective properties

The capacity method described presents some problems: i) the bilinear hysteretic model adopted is a simplified approach and does not describe accurately the actual behavior of conventional structures; ii) the model is based on an equal-energy criterion for harmonic motion and is only approximate for actual earthquake motion.

When soft spectra are used, the equations developed by Iwan can be used.

2.5.5.2 N2 Method

The N2 method [Fajfar & Fischinger, 1988; Fajfar & Gašperšič, 1996; Fajfar, 1999; Fajfar, 2000] is a version of the capacity spectrum method in which the reduced spectral demand and target points are calculated using strength reduction factors (discussed in 2.5.2.2) instead of effective properties. This method is the non-linear static procedure included in the current version of the Eurocode 8 [CEN, 2004].

The strength reduction factor R_μ is defined as the ratio of required elastic strength to yield strength; post-yield stiffness ratio is not contemplated in the formulation. For this reason the method requires that the capacity curve presents a zero post-yield stiffness ratio. Damping is not included in the formulation of the demand reduction. For these two reasons the method is not amenable to use with energy dissipation devices, that are formulated using effective damping and, in many cases, also provide stiffness in the post-yield range.

2.5.5.3 Modal Pushover Analysis (MPA)

A logical extension of the CSM is to consider the contribution of higher modes in the response. The method proposed by Paret *et al.* [1996] serves to quantify the influence of higher modes, but does not include the contribution in the response. Modal pushover analysis (MPA) [Chopra & Goel, 2002; Goel & Chopra, 2004; Goel & Chopra, 2005] and its modified version (MMPA) [Chopra *et al.*, 2004] attempt to contemplate the contribution of significant modes through a modified version of modal analysis applied to inelastic systems, which is a logical extension to the CSM.

The original MPA proposed by Chopra & Goel [2002] is based on two assumptions: i) non-linear motion equation can be considered approximately uncoupled by the modal shapes; ii) for a elastic structure, the modal response can be obtained by elastic static pushover if the target roof displacement is coincident with the modal roof displacement $u_{N,j}$. This is apparent from Eqn. (2.5-106), that is rephrased here as:

$$\mathbf{f}_j = \omega_j^2 \mathbf{m} \boldsymbol{\varphi}_j u_{N,j} = \mathbf{k} \boldsymbol{\varphi}_j u_{N,j} = \mathbf{k} \mathbf{u}_j. \quad (2.5-217)$$

Using these two concepts the MPA procedure is simply the modal combination of the results of modal pushover analyses to target modal roof displacement. If $J (< N)$ modes are included, the procedure for every mode j is summarized here (slightly modified under the assumption that the yield point lies on the tangent stiffness line)

1. Determine modal properties of the system, including T_j , Γ_j , L_j , M_{effj} and $\boldsymbol{\varphi}_j$.

2. For every mode, apply vertical loads and retrieve response results r_g , including lateral roof displacement $u_{N,g}$ that is expected to be negligible for symmetrical structures.
3. Find inelastic modal roof displacement $u_{N,j} = \Gamma_j \cdot S_d(T_j, \xi_{I,j})$. For a single motion, this involves direct non-linear solution of the SDOF equation of motion for a system with period T_j and modal inherent damping ratio $\xi_{I,j}$. For a soft spectrum, equivalent linearization, as discussed in Section 2.5.2.3, can be used.
4. With vertical loads applied, carry out static pushover analysis under a lateral force pattern $\mathbf{m} \cdot \boldsymbol{\phi}_j$, up to target roof displacement $u_{N,j} + u_{N,g}$. The results of interest (base shear, storey displacements, internal forces and deformations) are retrieved from the analysis, and constitute the modal response to lateral force plus gravity r_{j+g} .
5. Find the dynamic response as $r_j = r_{j+g} - r_g$.
6. Determine the total response r combining gravity and the peak modal responses obtained by the chosen combination rule.

$$r \approx \max[r_g \pm \text{SRSS}(\mathbf{r})], \quad (2.5-218)$$

where the SRSS rule has been used, but it can be substituted by any other, such as CQC.

The method, although intuitive, has no rational basis; it has not been formally proved that the uncoupled terms in the modal conversion of inelastic systems are small. However, it must be pointed out that many other simplified empirical rules have been successfully used in earthquake engineering (an outstanding example is the equal displacement rule). It is clear that the method converges to exact values for elastic systems, so it is expected that its accuracy will increase for small values of ductility.

Goel & Chopra [2004] checked the method using six SAC buildings 3- and 20-storey-high, defined in Gupta & Krawinkler [1999], for inter-storey drift and beam plastic rotation (derived from storey drifts by an approximate method), with the following findings:

- The accuracy of the method is reasonable for buildings not deforming much into the inelastic range. It is unacceptable for large ductility values, producing loss of lateral stiffness by P-Δ effects.
- The trends in drift and beam plastic rotation values derived from MPA procedure are similar to those of NLTHA, meaning that MPA captures well the influence of higher modes.
- The bias introduced by neglecting modal coupling is small.

Other interesting findings of this work are:

- The RSA method for linearly elastic systems leads to significant underestimation of drift, in the range of 15% to 28% for the buildings analyzed.
- The FEMA-356 lateral force distributions produce large underestimation (up to 75%) of drift and fail to predict the plastic rotation of beams. This is particularly so for the

uniform force distribution, that the authors recommend to be ignored, and happens also for elastic buildings.

- The MPA produces results more accurate than the mentioned FEMA distributions.

Improvement of internal forces solution

One drawback of the method is that the combination rules of internal forces over modes produce results that exceed the member capacity. This is a major drawback, because in the verification procedure involves that the member would be rejected. Goel & Chopra [2005] presented an extension of the MPA method to compute member forces. In this extension, rules are given for beams and columns. First, their internal forces (bending moment M for beams; bending moments M and axial forces P for columns) are determined with the original procedure and the chosen combination rule. After that, the following corrections are performed:

- For beams: if bending moments exceed the yielding bending moments, they must be reevaluated from a Moment-Rotation diagram, from computed beam rotations. Shear forces are derived using capacity concepts, i.e., equilibrium considerations under the assumption of yielding at both ends and applied gravity load.
- For columns: if bending moments exceed the yielding column moments obtained from the P-M interaction diagram under load P , they must be reevaluated adding this yielding moment and a strain hardening moment obtained from a Moment-Rotation diagram from computed bottom and top column rotations. Again, shear forces are obtained by capacity concepts, except that in this case the moment at both sides of the element is different.

The method was verified by the same six SAC buildings and conclusions for force deviations were similar to those previously stated: good approximations except for cases of high ductility demand, and accuracy superior to the FEMA-356 lateral force method, especially if the uniform load pattern is used.

Modified MMPA

Finally, a modified procedure, referred to as MMPA [Chopra *et al.*, 2004], has been proposed, in which the higher modes ($j > 1$) are treated as elastic. This procedure can be seen as a combination of CSM and modal analysis, and has the advantage of smaller computational demand. Verification of the method (for the same mentioned SAC buildings and a collection of generic frames) was performed for drift demands, with the following results:

- The method produces over-estimation of drift demands with respect to MPA, except for the case of lightly damped frames where the over-estimation is unacceptably large.
- As a consequence of over-estimation, for cases where MPA underestimates drifts, MMPA produces generally more accurate estimates.

- Likewise, if MPA over-estimates the response, MMPA is still more conservative.
- The dispersion of error is similar between both methods.

2.5.6 NLRSA for structures equipped with added viscous dampers

This Section presents the modification of NLRSA (also referred to as linear dynamic method) to account for supplemental viscous damping. The method is of application to other types of devices, but for every one a specific formulation of stiffness and viscous damping must be used. Details can be found in Ramirez et al. [2001]. The procedure is based on the implementation in ASCE-SEI 7/16 [ASCE, 2016], and relies on the following general principles [Symans et al., 2008]:

- The building can be represented with a single-degree-of freedom per story in each principal direction.
- For both directions, the structure collapse mechanism is complete, and dampers are uniformly distributed so that modal shapes are approximately valid to estimate relative modal drifts before and after yielding.
- For both directions, nonlinear response of the building can be represented by a bilinear elasto-plastic relationship.
- The yield shear can be approximated by static pushover analysis or using simplified code formulae (based on system's seismic performance factors).

The following simplifications are used:

- Response spectrum method is used to find peak modal values. Combination rules (SRSS, CQC) are used to find peak response.
- Inelastic effects are considered only in the first mode through CSM.
- Higher modes are treated as elastic.
- Harmonic motion at first mode frequency and roof displacement amplitude is used as the basis to evaluate added damping ratio (using Jacobsen's formula).
- Hysteretic energy dissipation for the first mode is based on a bilinear model modified with empirical hysteresis loop factors and no post-yield stiffness ($\alpha = 0$).
- Spectral displacement demands are modified with enhanced damping correction factors derived specifically for SDOFs with added damping.
- Damper response is found using correction factors for velocity (used to estimate velocity from pseudo-velocity), derived for SDOFs with added damping.
- Roof displacement must be equal or larger than the elastic roof displacement.
- The response is found for three different 'stages': maximum displacement, maximum velocity and maximum acceleration.

- The verification of the main structural system is performed for the forces at maximum displacement (reduced by overstrength factors).
- Verification of the damping system is performed for the three stages. Maximum force responses for the damping system at the maximum acceleration stage are found using the force coefficient method.

2.5.6.1 Previous conditions

In order to ensure that the principles and simplifications can be applied, the following conditions must be met by the structural system:

1. Viscous dampers with the same exponent a are placed at all storeys. In each direction at least two dampers configured to resist torsion are placed at each storey.
2. Total effective damping ξ_{eff} in the first mode is not greater than 35% of critical.
3. The devices and connecting elements are elastic for the maximum considered earthquake MCE_R . Additional 20% safety is required to auxiliary elements at this seismic level.
4. The method is not allowed in areas of high seismicity.

Condition 1) and 2) ensure that the analysis can be performed in two independent directions and that modal properties of the damped system are not too different from undamped modal properties. Condition 3 avoids undesired non-linear effects due to auxiliary elements.

2.5.6.2 Spectral values

The structure is subjected to earthquake excitation, defined by its displacement spectral function $S_d(T, \xi)$. If the excitation is a soft spectrum (code spectrum), S_d can be found using the DCF for displacement η_d as:

$$S_d(T, \xi) = S_d(T, 5\%) \eta_d(T, \xi). \quad (2.5-219)$$

The spectral velocity is approximated as:

$$S_v(T, \xi) = \frac{2\pi}{T} S_d(T, \xi) CFV(T, \xi). \quad (2.5-220)$$

If the excitation is a single accelerogram, $S_d(T, \xi)$ must be obtained solving Eqn. (2.5-5). In that case, $S_v(T, \xi)$ can be obtained from the same equation or approximated by the pseudo-velocity calculated from S_d .

2.5.6.3 Inherent damping

Inherent damping ratio is generally defined as modal damping with a constant value for all modes. In this method, ξ_I is limited to 3%. This limit is especially important for concrete

structures, where a 5% damping ratio is usual. For steel structures, inherent damping ratio is taken between 1% and 3%.

The procedure must be repeated for each direction of interest. The steps to be taken are described hereby:

2.5.6.4 Modal properties of the structure

Modal properties of the structure must be calculated (period T_j , modal displacement vector ϕ_j , modal drift vector ψ_j , modal participation factor Γ_j , effective modal weight $W_{eff,j}$) at all modes of interest $j = 1$ to J (the cumulative effective weight of the last mode J must exceed 90% of total weight).

2.5.6.5 Pushover curve

Non-linear static curve (pushover curve) of the structure is performed under a 1st mode lateral load pattern (a bilinear approximation can be obtained with the simple methods discussed in Section 2.5.4.6.). The initial stiffness $\kappa_1 = \omega_1^2 \cdot W_{eff,1} / \Gamma_1$ is obtained.

2.5.6.6 Elastic roof displacement response in the first mode

The elastic roof displacement response $u_{e,N,1}$ at first mode is found.

For linear dampers, the calculation is direct. First, $\xi_{a,1}$ is calculated as:

$$\xi_{a,1} = \frac{\sum_{n=1}^N c_{d,n} |\psi_{n,1}|^2}{2\omega_1 \phi_1^T \mathbf{m} \phi_1}. \quad (2.5-221)$$

Then $u_{e,N,1}$ is found as:

$$u_{e,N,1} = \Gamma_1 S_d(T_1, \xi_{l,1} + \xi_{a,1}). \quad (2.5-222)$$

For non-linear dampers, iteration is needed because the amount of added damping is dependent on the amplitude of displacement. First $u_{e,N,1}^{(0)}$ is calculated using Eqn. (2.5-222) with $\xi_{a,1} = 0$. Then:

1. $\xi_{a,1}$ is found as:

$$\xi_{a,1} = \frac{b\omega_1^{a-2} (u_{e,N,1}^{(0)})^{a-1} \sum_{n=1}^N c_{d,n} |\psi_{n,1}|^{1+a}}{2 \phi_1^T \mathbf{m} \phi_1}. \quad (2.5-223)$$

2. $u_{e,N,1}$ is found using Eqn. (2.5-222).

3. If the difference between $u_{e,N,1}$ and $u_{e,N,1}^{(0)}$ is larger than a certain tolerance, $u_{e,N,1}^{(0)} = u_{e,N,1}$ and the analysis is repeated from step 1.

2.5.6.7 Inelastic roof displacement response in the first mode

An iterative procedure is needed. There are several ways to perform this procedure, but the most common one is based on ductility. An initial tentative displacement $u_{N,1}^{(0)}$ is chosen. A good candidate is the elastic roof displacement $u_{e,N,1}$ previously calculated.

1. Base shear $V_{b,push}$ corresponding to $u_{N,1}^{(0)}$ must be identified on the pushover curve.
2. The corresponding effective yield point $u_{y,N}$, must be calculated by bi-linearization of the pushover curve, using the ‘reduced envelope’ criterion (Section 2.5.4.5), where the post-yield stiffness ratio is assumed as 0, the yield shear $V_{y,1}$ is taken as $V_{b,push}$ and the yield displacement is:

$$u_{y,N} = \frac{V_{y,1}}{\kappa_1} \quad (2.5-224)$$

3. The corresponding tentative ductility μ is calculated as:

$$\mu = \frac{u_{N,1}^{(0)}}{u_{y,N}} \quad (2.5-225)$$

4. $T_{eff,1}$ and $\xi_{eff,1}$ are calculated using:

$$T_{eff,1} = T_1 \sqrt{\mu}, \quad (2.5-226)$$

$$\xi_h = q_H \frac{2}{\pi} \left(1 - \frac{1}{\mu} \right). \quad (2.5-227)$$

$$\xi_{eff,1} = \xi_{I,1} + \xi_{a,1} \mu^{1-a/2} + \xi_h. \quad (2.5-228)$$

$$q_H = \min \left\{ 1; \max \left\{ 0.67 \frac{T_C}{T_1}; 0.5 \right\} \right\} \quad (2.5-229)$$

5. The roof displacement based on these effective properties is calculated:

$$u_{N,1} = \Gamma_1 S_d(T_{eff,1}, \xi_{eff,1}). \quad (2.5-230)$$

If the difference between $u_{N,1}$ and $u_{N,1}^{(0)}$ is larger than a predetermined tolerance, $u_{N,1}^{(0)} = u_{N,1}$ and the iterative calculation is repeated from step 1.

After convergence, base shear from the pushover curve $V_{b,push}$ should be approximately equal to that calculated using spectral values:

$$V_{1,1} = \frac{W_{eff,j}}{g} \left(\frac{2\pi}{T_{eff,1}} \right)^2 S_d(T_{eff,1}, \xi_{eff,1}), \quad (2.5-231)$$

Graphical summary of this procedure is given in Figure 2-39

Figure 2-39. RSA concept.

2.5.6.8 Kinematic response in the first mode

The peak roof displacement in the first mode $u_{N,1}$ is taken as the maximum between $u_{e,N,1}$ and $u_{N,1}$. The peak roof velocity in the first mode is taken as:

$$\dot{u}_{N,1} = \frac{2\pi}{T_{eff,1}} u_{N,1} \text{CFV}(T_{eff,1}, \xi_{eff,1}). \quad (2.5-232)$$

First mode drift vector Δ_1 is obtained as $\psi_1 \cdot u_{N,1}$. First mode drift velocity vector ∇_1 is obtained as $\psi_1 \dot{u}_{N,1}$.

2.5.6.9 Static response in the first mode

The restoring force vector in the first mode is obtained as:

$$\mathbf{f}_{mxd,1} = \mathbf{m}\phi_1 \Gamma_1 S_a(T_{eff,1}, \xi_{eff,1}), \quad (2.5-233)$$

where the subscript 'mxd' indicates maximum displacement. Storey shear in the first mode can be found as $\mathbf{V}_{mxd,1} = \mathbf{T}_U \cdot \mathbf{f}_{mxd,1}$. Design forces are obtained dividing these forces by the corresponding overstrength factor.

Shear force in the damping devices at storey n and mode 1 are calculated at the stage of maximum velocity:

$$V_{mxv,n,1} = c_{d,n} \left| \nabla_{n,1} \right|^a. \quad (2.5-234)$$

The corresponding horizontal forces can be calculated as $\mathbf{f}_{mxv,1} = \mathbf{H} \cdot \mathbf{V}_{mxv,1}$.

Finally, the forces at the stage of maximum acceleration are calculated combining the maximum displacement and velocity stages using force coefficients:

$$\mathbf{f}_{mxa,1} = CF_1 \mathbf{f}_{mxd,1} + CF_2 \mathbf{f}_{mxv,1}, \quad (2.5-235)$$

$$\mathbf{V}_{mxa,1} = CF_1 \mathbf{V}_{mxd,1} + CF_2 \mathbf{V}_{mxv,1}, \quad (2.5-236)$$

where CF_1 and CF_2 are dependent on δ , as given by Eqns. (2.5-90), (2.5-91), (2.5-92) repeated here for convenience:

$$\delta \approx \left(\frac{2a}{b} (\xi_{eff} - \xi_h) \right)^{\frac{1}{2-a}}. \quad (2.5-237)$$

$$\mu \leq 1, \quad CF_1 = \cos \delta \quad (2.5-238)$$

$$\mu > 1, \quad CF_1 = \min\{\mu \cos \delta; 1\} \quad (2.5-239)$$

$$CF_2 = \sin^a \delta.$$

For $a \leq 0.25$, $CF_1 = CF_2 = 1$; and for $a > 1$, $CF_2 = \sin \delta$.

2.5.6.10 Response in higher modes

The elastic roof response in mode $j > 1$ is found with exactly the same procedure as the elastic roof response in the 1st mode (Section 2.5.6.6), but assuming:

- The exponent a for higher modes is taken as 1.
- The damping coefficient $c_{d,n}$ at storey n is calculated as:

$$c_{d,eff,n} = a c_{d,n} \left| \nabla_{n,1} \right|^{a-1} \quad (2.5-240)$$

This expression is based on the higher modes considered as linear with small vibration superimposed to the first mode vibration [Ramirez et al., 2001]. so the procedure does not need iteration, just direct use of Eqns. (2.5-221) and (2.5-222), replacing the corresponding modal values of the 1st mode for the values of the j^{th} mode and using $c_{d,n,eff}$. The response in higher modes is assumed as elastic, so there is no need to calculate the inelastic response. The viscous damping is directly estimated as:

$$\xi_{a,j} = \frac{\sum_{n=1}^N c_{d,n} \left| \psi_{n,j} \right|^2}{2\omega_j \boldsymbol{\phi}_j^T \mathbf{m} \boldsymbol{\phi}_j}. \quad (2.5-241)$$

The kinematic response is calculated using modal properties instead of effective properties:

$$\dot{u}_{N,j} = \frac{2\pi}{T_j} u_{N,j} \text{CFV}(T_j, \xi_{I,j} + \xi_{a,j}). \quad (2.5-242)$$

and correspondingly, $\Delta_j = \psi_j \cdot u_{N,j}$; and $\nabla_j = \psi_j \dot{u}_{N,j}$.

The static response is found as indicated in Section 2.5.6.9, using modal properties, and the effective linear damper defined by Eqn. (2.5-240). Additionally, in the calculation of CF_1 , the ductility is taken as μ .

2.5.6.11 Verification of main structure

The seismic design internal forces must be obtained by modal combination of design internal forces at all modes. These forces must be combined as required with other actions.

2.5.6.12 Verification of the damping system

The maximum internal forces in the damping system must be obtained as the maximum of the following three modal combinations:

1. stage of maximum displacement,
2. stage of maximum velocity,
3. stage of maximum acceleration.

2.5.7 Numerical Example

The proposed procedure is applied to frame 05D (defined in Annex B) with 2% inherent damping and 3% post-yield stiffness ratio. The frame is equipped with non-linear viscous dampers in configuration Q.1.5, as defined in Annex D. The frame is assumed to be located on a site with soil class B, characterized by spectrum Type 1, with a 475-year return acceleration $a_{gR} = 0.33g$ (rock). This corresponds to Peak Horizontal Acceleration $PHA = 0.4g$, (soil parameter $S = 1.2$ according to CEN [2004], Table 3.2).

The pushover curve of the bare frame has been presented in Section 2.5.4. The curve is discretized at roof displacement intervals of 50mm, as shown in Figure 2-40.

Figure 2-40. Frame 05D. Discretization of pushover curve.

2.5.7.1 Frame properties

The storey weight and stiffness matrix are:

$$w^T = [2601.7 \ 2601.7 \ 2601.7 \ 2601.7 \ 2440.3] \text{ kN}$$

$$k = \begin{bmatrix} 280.4 & -180.3 & 56 & -11.6 & 1.9 \\ -180.3 & 247.9 & -158.6 & 46.9 & -7.6 \\ 56 & -158.6 & 209.2 & -122.7 & 27.8 \\ -11.6 & 46.9 & -122.7 & 139.3 & -54.3 \\ 1.9 & -7.6 & 27.8 & -54.3 & 32.6 \end{bmatrix} \frac{\text{kN}}{\text{mm}}$$

The modal properties period, frequency, modal displacement matrix, modal drift matrix are:

$$T^T = [2.0098 \ 0.6916 \ 0.3417 \ 0.2035 \ 0.1387] \text{ s}$$

$$\omega^T = [3.126 \ 9.085 \ 18.388 \ 30.876 \ 45.301] \frac{1}{\text{s}}$$

$$\Phi = \begin{bmatrix} 0.138 & -0.457 & 1.237 & -2.944 & 7.995 \\ 0.35 & -0.898 & 1.2 & 0.509 & -9.26 \\ 0.575 & -0.837 & -0.675 & 2.496 & 7.113 \\ 0.805 & -0.099 & -1.418 & -2.663 & -3.59 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\Psi = \begin{bmatrix} 0.138 & -0.457 & 1.237 & -2.944 & 7.995 \\ 0.212 & -0.441 & -0.036 & 3.453 & -17.255 \\ 0.224 & 0.061 & -1.876 & 1.987 & 16.373 \\ 0.23 & 0.739 & -0.743 & -5.159 & -10.703 \\ 0.195 & 1.099 & 2.418 & 3.663 & 4.59 \end{bmatrix}$$

$$\Gamma^T = [1.3638 \ -0.5078 \ 0.2009 \ -0.0718 \ 0.0149]$$

$$W_{eff}^T = [9954.8 \ 1787.3 \ 669.7 \ 311.1 \ 124.1] \text{ kN}$$

$$W_{eff, ratio}^T = [77.5\% \ 13.9\% \ 5.2\% \ 2.4\% \ 1\%]$$

Initial stiffness is taken as tangent at origin to the pushover curve. It can be calculated as $\kappa_1 = \omega_1^2 \cdot W_{eff,1} / \Gamma_1$.

$$\kappa_1 = 7.27 \frac{\text{kN}}{\text{mm}}$$

2.5.7.2 Damping system properties

Inherent damping using the Rayleigh model, with 2% damping in the 1st and 3rd mode:

$$\xi_I^T = [2\% \ 1.43\% \ 2\% \ 3.04\% \ 4.33\%]$$

Damping exponent a_n , corresponding values of b_n , coefficients $c_{b,n}$, and conversion to storey drift coefficients $c_{d,n}$:

$$a^T = [0.5 \ 0.5 \ 0.5 \ 0.5 \ 0.5]$$

$$b^T = [1.113 \ 1.113 \ 1.113 \ 1.113 \ 1.113]$$

$$c_b^T = [30.3 \ 30.3 \ 30.3 \ 30.3 \ 30.3] \text{ kN} \cdot \left(\frac{s}{mm} \right)^{0.5}$$

$$c_d^T = [25.8 \ 25.8 \ 25.8 \ 25.8 \ 25.8] \text{ kN} \cdot \left(\frac{s}{mm} \right)^{0.5}$$

2.5.7.3 Elastic roof displacement in the first mode

Initial value of damping contemplates only inherent damping:

$$\xi_e^T = [2\% \ 1.4\% \ 2\% \ 3\% \ 4.3\%]$$

Roof response is calculated using Eqn. (2.5-222), as:

$$u_{e_{N,1}} = 404.4 \text{ mm}$$

The corresponding added viscous damping can be estimated from Eqn. (2.5-223) as $\xi_{a,1} = 10.7\%$. With this value of added viscous damping, the roof displacement is recalculated as $u_{e_{N,1}} = 255.4\text{mm}$; this results in 13.5% added damping, etc. Further iteration results in a stable value of roof displacement and added viscous damping of:

$$u_{e_{N,1}} = 234.51 \text{ mm}$$

$$\xi_{a_1} = 14.06\%$$

2.5.7.4 Inelastic roof displacement in the first mode

In the first step, the roof displacement is assumed as the elastic roof displacement, $u_{N,1}^{(0)} = u_{e_{N,1}}$ calculated in the previous step. The corresponding shear value is located on the pushover curve:

$$V_{b, push} = 1194.6 \text{ kN}$$

And the yield point is obtained dividing this value by κ_1 :

$$u_{y_{N,1}} := \frac{V_{b, push}}{\kappa_1} = 164.2 \text{ mm}$$

The ductility demand can now be calculated, as the ratio between the tentative value $u_{N,1}^{(0)}$ and $u_{y,N,1}$:

$$\mu = 1.43$$

And the effective properties are calculated with Eqns. (2.5-226) to (2.5-229):

$$T_{eff,1} = 2.402 \text{ s}$$

$$q_H(T_1) = 0.5$$

$$\zeta_h = 9.54\%$$

$$\zeta_{eff,1} := \zeta_{I_1} + \zeta_{a_1} \cdot \mu^{1-0.5 \cdot a_1} + \zeta_h = 29.91\%$$

With this value of effective damping, the roof is calculated using Eqn. (2.5-222) as:

$$u_{N,1} = 182.9 \text{ mm}$$

Because there is a substantial difference between the assumed roof displacement and the final result, iteration is needed. The new value $u_{N,1}$ is used as starting point. After several iterations, convergence is achieved at:

$$u_{N,1} = 197.4 \text{ mm}$$

$$T_{eff,1} = 2.244 \text{ s}$$

$$u_{y,N,1} = 158.4 \text{ mm}$$

$$\zeta_{eff,1} = 24.87\%$$

The base shear value obtained from the pushover curve is:

$$V_{b,push} = 1152.3 \text{ kN}$$

And the base shear calculated from the response spectrum, Eqn. is

$$V_{b,1} = 1152.5 \text{ kN}$$

The ductility demand is moderate, $\mu = 1.246$.

2.5.7.5 Kinematic response in the first mode

The correction factor for velocity is:

$$cfv_1 = 1.095$$

And correspondingly, the roof velocity is (the symbol v is used here to denote velocity):

$$v_{N,1} = 719.3 \frac{mm}{s}$$

With the estimated values of roof displacement and velocity, modal equations are used to find the first mode displacement, drift, velocity and drift velocity (the symbol Y is used for drift velocity):

$$u_1^T = [32.4 \ 82.1 \ 134.7 \ 188.7 \ 234.5] \text{ mm}$$

$$\Delta_1^T = [32.4 \ 49.7 \ 52.6 \ 53.9 \ 45.8] \text{ mm}$$

$$v_1^T = [99.5 \ 251.9 \ 413.3 \ 578.8 \ 719.3] \frac{mm}{s}$$

$$Y_1^T = [99.5 \ 152.4 \ 161.4 \ 165.4 \ 140.6] \frac{mm}{s}$$

2.5.7.6 Static response in the first mode

Stage of maximum displacement

At the stage of maximum displacement, the forces are calculated using the modal response, Eqn. (2.5-233). This is equivalent to distributing the previously calculated shear using the first mode lateral load pattern:

$$f_{mxd,1}^T = [56.8 \ 143.8 \ 236 \ 330.5 \ 385.3] \text{ kN}$$

$$V_{mxd,1}^T = [1152.5 \ 1095.7 \ 951.8 \ 715.8 \ 385.3] \text{ kN}$$

First mode design forces for the main structure can be obtained dividing these by the overstrength factor (calculated as $q_R \cdot q_S$).

$$f_{design,1}^T = [29.1 \ 73.8 \ 121 \ 169.5 \ 197.6] \text{ kN}$$

$$V_{design,1}^T = [591 \ 561.9 \ 488.1 \ 367.1 \ 197.6] \text{ kN}$$

Stage of maximum velocity

Shear forces due to the damping system are obtained from drift velocity using Eqn. (2.5-234); by projection, the forces in the damping devices f_b at the first mode are obtained:

$$V_{mxd,1}^T = [257.6 \ 318.8 \ 328.2 \ 332.2 \ 306.2] \text{ kN}$$

$$f_{b,1}^T = [286.5 \ 354.5 \ 364.9 \ 369.4 \ 340.5] \text{ kN}$$

Stage of maximum acceleration

The phase lag angle is with Eqn. (2.5-237) to be $\delta = 0.303$. With this value, $CF_1 = 1$, and $CF_2 = 0.546$. Then, the forces and shear forces at maximum displacement and velocity are combined according to Eqn. (2.5-235) and (2.5-236) to obtain:

$$f_{mxal}^T = [90.2 \ 148.9 \ 238.2 \ 344.7 \ 552.5] \text{ kN}$$

$$V_{mxal}^T = [1293.1 \ 1269.7 \ 1131 \ 897.2 \ 552.5] \text{ kN}$$

2.5.7.7 Response in higher modes

First, the behavior of the dampers is linearized, based on drift velocity of the first mode using Eqn. (2.5-240):

$$c_{d,eff}^T = [1.295 \ 1.046 \ 1.016 \ 1.004 \ 1.089] \text{ kN} \cdot \frac{s}{mm}$$

With these effective damper coefficients, taking $a_{eff} = 1$, the elastic viscous damping ratio in the higher modes is found using Eqn. (2.5-221), replacing the first mode by the corresponding mode:

$$\xi_a^T = [14.06\% \ 18.22\% \ 20.06\% \ 18.18\% \ 15.64\%]$$

The response in higher modes is elastic, therefore it needs no iteration.

Kinematic response

$$u = \begin{bmatrix} 32.44 & 12.91 & 4.55 & 1.42 & 0.38 \\ 82.11 & 25.38 & 4.42 & -0.25 & -0.44 \\ 134.75 & 23.65 & -2.49 & -1.21 & 0.34 \\ 188.68 & 2.79 & -5.22 & 1.29 & -0.17 \\ 234.51 & -28.26 & 3.68 & -0.48 & 0.05 \end{bmatrix} \text{ mm}$$

$$\Delta = \begin{bmatrix} 32.44 & 12.91 & 4.55 & 1.42 & 0.38 \\ 49.68 & 12.46 & -0.13 & -1.67 & -0.81 \\ 52.63 & -1.72 & -6.9 & -0.96 & 0.77 \\ 53.94 & -20.87 & -2.74 & 2.49 & -0.5 \\ 45.83 & -31.04 & 8.9 & -1.77 & 0.22 \end{bmatrix} \text{ mm}$$

$$cfv^T = [1.0953 \ 1 \ 1 \ 1 \ 1]$$

$$v = \begin{bmatrix} 99.5 & 117.3 & 83.7 & 43.9 & 17.1 \\ 251.9 & 230.5 & 81.2 & -7.6 & -19.8 \\ 413.3 & 214.9 & -45.7 & -37.2 & 15.2 \\ 578.8 & 25.3 & -96 & 39.8 & -7.7 \\ 719.3 & -256.7 & 67.7 & -14.9 & 2.1 \end{bmatrix} \frac{mm}{s}$$

$$Y = \begin{bmatrix} 99.5 & 117.3 & 83.7 & 43.9 & 17.1 \\ 152.4 & 113.2 & -2.5 & -51.5 & -36.8 \\ 161.4 & -15.7 & -126.9 & -29.7 & 35 \\ 165.4 & -189.6 & -50.3 & 77 & -22.9 \\ 140.6 & -282 & 163.7 & -54.7 & 9.8 \end{bmatrix} \frac{mm}{s}$$

Static response at maximum displacement:

$$f_{mxd} = \begin{bmatrix} 56.8 & 282.8 & 408.3 & 360 & 205.1 \\ 143.8 & 555.7 & 396.3 & -62.2 & -237.6 \\ 236 & 517.9 & -222.9 & -305.1 & 182.5 \\ 330.5 & 61 & -468.3 & 325.6 & -92.1 \\ 385.3 & -580.4 & 309.6 & -114.7 & 24.1 \end{bmatrix} kN$$

$$V_{mxd} := T_U \cdot f_{mxd} = \begin{bmatrix} 1152.5 & 837 & 422.9 & 203.6 & 82 \\ 1095.7 & 554.3 & 14.7 & -156.4 & -123.1 \\ 951.8 & -1.4 & -381.6 & -94.2 & 114.5 \\ 715.8 & -519.4 & -158.6 & 210.9 & -68 \\ 385.3 & -580.4 & 309.6 & -114.7 & 24.1 \end{bmatrix} kN$$

Static response at maximum velocity:

$$V_{mxv} = \begin{bmatrix} 257.6 & 151.9 & 108.3 & 56.9 & 22.1 \\ 318.8 & 118.5 & -2.6 & -53.9 & -38.5 \\ 328.2 & -15.9 & -129 & -30.1 & 35.5 \\ 332.2 & -190.3 & -50.5 & 77.3 & -22.9 \\ 306.2 & -307.2 & 178.3 & -59.6 & 10.7 \end{bmatrix} kN$$

$$f_b = \begin{bmatrix} 286.5 & 168.9 & 120.5 & 63.3 & 24.6 \\ 354.5 & 131.7 & -2.9 & -60 & -42.8 \\ 364.9 & -17.7 & -143.4 & -33.5 & 39.5 \\ 369.4 & -211.6 & -56.1 & 86 & -25.5 \\ 340.5 & -341.6 & 198.2 & -66.2 & 11.9 \end{bmatrix} kN$$

Response at maximum acceleration

$$cf_1^T = [1 \ 1 \ 1 \ 1 \ 1]$$

$$cf_2^T = [0.546 \ 0.366 \ 0.404 \ 0.391 \ 0.371]$$

$$f_{mxa} = \begin{bmatrix} 90.2 & 295 & 453.1 & 403.3 & 227.6 \\ 148.9 & 604.8 & 447.3 & 71.5 & 265 \\ 238.2 & 581.8 & 254.6 & 347.1 & 204.2 \\ 344.7 & 103.7 & 560.6 & 379.1 & 104.6 \\ 552.5 & 692.7 & 381.6 & 137.9 & 28 \end{bmatrix} kN$$

$$V_{mxa} = \begin{bmatrix} 1293.1 & 892.6 & 466.7 & 225.8 & 90.2 \\ 1269.7 & 597.6 & 15.7 & 177.5 & 137.4 \\ 1131 & 7.2 & 433.7 & 106 & 127.6 \\ 897.2 & 589 & 179 & 241.1 & 76.6 \\ 552.5 & 692.7 & 381.6 & 137.9 & 28 \end{bmatrix} \text{ kN}$$

2.5.7.8 Combination of modal responses

Using the CQC rule, the following peak values are finally obtained (the corresponding values obtained by NLTHA analysis are also given for comparison):

Kinematic variables:

$$u_p^T = [36.94 \ 88.6 \ 138.71 \ 188.86 \ 233.7] \text{ mm}$$

(TH: 33.5775 75.6242 114.3981 148.6558 173.8777)

$$v_p^T = [212.89 \ 377.82 \ 479.64 \ 585.03 \ 740.97] \frac{1}{s} \cdot \text{mm}$$

(TH: 217.9313 415.1457 532.3443 602.7643 674.6770)

$$\Delta_p^T = [36.94 \ 52.26 \ 52.85 \ 56.26 \ 52.69] \text{ mm}$$

(TH: 33.5775 43.4420 42.8871 39.4988 31.5246)

$$Y_p^T = [212.89 \ 208.94 \ 210.62 \ 254.16 \ 310.01] \frac{1}{s} \cdot \text{mm}$$

(TH: 217.9313 242.2331 225.6572 221.8347 216.5647)

Damper stroke and velocity:

$$u_{b,P}^T = [33.2 \ 47 \ 47.5 \ 50.6 \ 47.4] \text{ mm}$$

(TH: 30.1075 38.5844 38.0695 35.2079 28.2174)

$$v_{b,P}^T = [191.5 \ 187.9 \ 189.4 \ 228.6 \ 278.8] \frac{\text{mm}}{s}$$

Unreduced forces in the structure (maximum displacement)

$$f_{s,P}^T = [847.5 \ 789.4 \ 653.9 \ 543.3 \ 674.6] \text{ kN}$$

$$V_{s,P}^T = [1661.3 \ 1286.5 \ 1027.4 \ 879.6 \ 674.6] \text{ kN}$$

(TH: 1,376 1,227 997 730 409)

Design forces and shear

$$f_{design,P}^T = [844.7 \ 774.4 \ 614.7 \ 465 \ 601.9] \text{ kN}$$

$$V_{design.P}^T = [1296.1 \ 848.3 \ 631.3 \ 656 \ 601.9] \text{ kN}$$

Damper shear and axial force (maximum velocity)

$$f_{b.P}^T = [401.3 \ 396.4 \ 392.9 \ 419 \ 468.2] \text{ kN}$$

(TH: 411 439 424 422 419)

$$V_{d.P}^T = [360.9 \ 356.5 \ 353.4 \ 376.8 \ 421.1] \text{ kN}$$

(TH: 370 395 382 380 377)

Total shear (maximum acceleration)

$$f_{t.P}^T = [937.7 \ 955.3 \ 960.2 \ 937.6 \ 1106.8] \text{ kN}$$

$$V_{t.P}^T = [1830 \ 1489.7 \ 1263.8 \ 1220.8 \ 1106.8] \text{ kN}$$

(TH 1,664 1,449 1,211 954 613)

2.6 Summary and conclusions

This Chapter presented the background to the NLRSA method for the analysis of structures with supplemental energy dissipation devices:

- A brief summary of the principles behind current earthquake design practice has been given.
- Viscous dampers have been presented and several studies characterizing their behavior both as independent elements and as added elements to structures have been summarized.
- The benefits of the technology were analyzed with reference to a practical example that forms part of the parametric study carried out. Practical considerations regarding their use have been pointed out.
- Different available constitutive models of the damper, with different levels of refinement, have been discussed; in particular, the pure viscous model, presented as the reference model for simplified analysis.
- Several aspects regarding construction of the capacity curve of MDOF systems have been discussed.
- The basis of non-linear static analysis has been summarized, and different improvements and alternative existing methods summarized.
- Finally, the NLRSA for structures with added dampers has been presented, including a numerical application example.

2.7 Original contributions in this Chapter

This Chapter contains a summary of current knowledge relating the NLRSA method applied to structures with added damping. The main contribution is the coherent organization and presentation of the material.

Regarding the capacity curve, a proposal has been made to substitute the ambiguous concept of displacement ductility by a clearer concept based on secant period. It has been shown that use of the energy capacity curve avoids bi-linearization of the capacity curve and permits exact determination of hysteretic energy dissipation and effective damping ratio as a function of spectral displacement (not ductility). The energy curve also allows determination of the exact onset of yielding in steel structures, which is an important point to characterize overstrength.

Chapter 3

Non-proportionality Indices as Error Predictors in Classical Modal Analysis

3.1 Introduction

As shown by different studies [Symans et al, 2008; Filiatrault & Christopoulos, 2006; Hanson & Soong, 2001; Sadek et al, 1999; Soon & Dargush, 1997; Sadek et al, 1996], careful addition of damping devices to a structure, improves considerably its dynamic performance. Among the different types of dampers, viscous dampers present the unique feature that their behavior is solely dependent on velocity, rather than displacement [Berquist et al, 2019; Seleemah & Constantinou, 1997; Aiken & Kelly, 1996; Reinhorn et al, 1995; Constantinou et al, 1993; Constantinou & Symans, 1992]. Under harmonic motion, the forces due to viscous dampers are in phase with the elastic restoring forces, making them very effective for drift reduction. If the damping matrix of the damped structure fulfils the commutability condition [Caughey & O’Kelly, 1965] (detailed below), the natural vibration modes of the system are real-valued and identical to those of the associated undamped system. This type of system is referred to as classically (or proportionally) damped, and the modal superposition method applied to it is known as ‘classical modal method’. To estimate the maximum response of a classically damped structure subjected to earthquake loading, the classical modal method is used in conjunction with the response spectrum concept. This is the basis of the well-known Response Spectrum Analysis (RSA), introduced by FEMA in 1997 [ASCE, 2016], and present in several current seismic codes around the world [ASCE, 2016; CEN, 2004; MLIT, 2007].

The addition of viscous dampers generally results in general in a non-proportional damping matrix, which does not satisfy the commutability condition. The modal method can also be applied to structures with non-classical damping in a generalized form due to Foss [1958], and referred to as ‘generalized modal method’ [Veletsos & Ventura, 1986]. This method, however, has found limited application in structural engineering practice, where classical modal analysis is preferred due to its conceptual clarity and simplicity. Seismic codes allow the use of classical modal analysis under certain restrictions, aimed at reducing non-proportionality effects. Typical conditions are structural regularity, a complete distribution

of dampers (dampers in every storey), with at least two damping devices in each principal direction (positioned to offer torsion resistance), a moderate amount of viscous damping ratio (less than or equal to 35% of critical), and spectral accelerations under certain threshold [ASCE, 2016, 18.2.3.1]. Under these conditions, the classical modal method is assumed to be reasonably exact (Constantinou & Symans, 1992; Ramirez et al, 2001).

Applying the classical modal method to systems with non-proportional damping involves ignoring the off-diagonal coupling coefficients of the exact modal coordinate damping matrix. The errors resulting from this approximation have been pointed out in previous studies [Clough & Penzien, 1995], yet there are, to this author's knowledge, no systematic investigations that quantify these errors; in particular, those related to properties of the system, or to the degree of non-proportionality of the damping matrix, or the amount of damping. The primary objective of the current study is to quantify this error for linear structures with viscous dampers having different number of stories, fundamental periods, levels of damping, distributions of viscous dampers and distributions of lateral stiffness.

In non-proportionally damped systems, free-vibration modal shapes fluctuate with time. This phenomenon is of great interest to modal analysis techniques. For this reason, several indices to measure the degree of non-proportionality of systems with added damping have been proposed. The most relevant indices are reviewed in this Chapter. Because the response errors incurred by use of classical modal analysis have their origin at damping non-proportionality, a certain relationship is expected between the values of indices and errors. This correlation is explored in this study, and found to be true for some indices. One index, proposed by Prater & Singh [1986], is chosen for its simplicity and good correlation with the expected maximum response error. Recommended design values for this index are given, to keep the maximum un-conservative errors (resulting from neglecting the off-diagonal terms of the modal coordinate damping matrix and applying classical modal methods) within acceptable bounds.

3.2 Background

3.2.1 Proportional and non-proportional damping

As shown in Chapter 2, the modal matrix $\Phi = [\phi_1, \dots, \phi_N]$ represents relative displacement of the natural modes of the system; for this reason, scaling by a uniform factor has no effect. In modal analysis, a customary scaling is the mass-normalization:

$$\hat{\Phi}^T \mathbf{m} \hat{\Phi} = \mathbf{I}, \quad (3.2-1)$$

where $\mathbf{I}$ is the identity $N \times N$ matrix; $\hat{\Phi} = [\hat{\phi}_1, \dots, \hat{\phi}_N]$ can be calculated as $\hat{\phi}_j = \phi_j M_j^{-1/2}$, where j is the relevant mode, and in that case the modal damping and modal stiffness matrices adopt especial forms,

$$\hat{\Phi}^T \mathbf{k} \hat{\Phi} = \Omega^2, \quad (3.2-2)$$

$$\hat{\mathbf{C}} = \hat{\Phi}^T \mathbf{c} \hat{\Phi} = \mathbf{M}^{-1/2} \mathbf{C} \mathbf{M}^{-1/2}, \quad (3.2-3)$$

where $\hat{\mathbf{C}}$ is referred to as normalized modal damping matrix, and Ω^2 is the spectral matrix, whose j^{th} diagonal term is the j^{th} mode squared undamped frequency ω_j^2 . The equation of motion in modal coordinates takes the form

$$\ddot{\mathbf{y}}(t) + \hat{\mathbf{C}} \dot{\mathbf{y}}(t) + \Omega^2 \mathbf{y}(t) = -\hat{\Phi}^T \mathbf{m} \mathbf{1} a_g(t). \quad (3.2-4)$$

Mass-normalized modes are often used in modal analysis; however, in structural design it is more common, for practical reasons, to normalize the modes to a unity roof modal displacement.

As shown by Caughey and O'Kelly [1965], $\hat{\mathbf{C}}$ will be diagonal if, and only if:

$$\mathbf{c} \mathbf{m}^{-1} \mathbf{k} = \mathbf{k} \mathbf{m}^{-1} \mathbf{c}. \quad (3.2-5)$$

This is known as commutability condition, and can be met by a Caughey damping matrix $\mathbf{c}$ [Caughey, 1960], where $\mathbf{c}$ is a combination of $\mathbf{m}$ and $\mathbf{k}$. Mass-proportional damping, stiffness-proportional damping, and Rayleigh damping [Chopra, 2007] are special cases of Caughey damping. If Eqn. (3.2-5) is fulfilled, $\mathbf{c}$ is referred to as ‘proportional damping matrix’, and Eqn. (3.2-4) is transformed into N uncoupled second-order differential equations:

$$\ddot{y}_j(t) + \hat{C}_j \dot{y}_j(t) + \omega_j^2 y_j(t) = -\hat{\Gamma}_j a_g, \quad (3.2-6)$$

where $\hat{\Gamma}_j (= \hat{\Phi}_j^T \mathbf{m} \mathbf{1})$ is the mass-normalized modal participation factor corresponding to mode j and $\hat{\Phi}_j$ is the j^{th} column of the mass-normal modal matrix. Each equation, corresponding to one mode, can be solved independently by time-history modal analysis (THMA) to find the complete modal response for each mode; addition of modal responses at every time step, produces the exact total system response; this is the classical modal method. On the other side, if $\mathbf{c}$ is not proportional, $\hat{\mathbf{C}}$ is not diagonal and Eqn. (3.2-6) reads

$$\ddot{y}_j + \sum_{n=1}^N \hat{C}_{j,n} \dot{y}_n + \omega_j^2 y_j = -\hat{\Gamma}_j a_g(t), \quad (3.2-7)$$

representing a coupled system of second-order differential equations, that cannot be solved by the classical modal method. An approximate solution to Eqn. (3.2-7) can be obtained neglecting the off-diagonal terms ($j \neq n$) in the second term; the resulting expression is identical to Eqn. (3.2-6) and can be solved applying time-history modal analysis to the non-classically damped system. By doing so, however, the solution is not exact. The errors associated with this simplification have been discussed by Clough and Penzien [1995]. This is the first type of error to be analyzed, namely, the error incurred by neglecting the diagonal terms in the modal damping matrix.

A second type of error is related to the way the modal response is found. If, instead of the complete time-history modal response, only the peak response is calculated for each mode j , the modal method is referred to as ‘response spectrum method’. As a general rule, peak values obtained for different modes are not simultaneous in time. To find the total response, the peak modal values are combined using a combination rule (SRSS or CQC, see Section 2.5.3.1). This method is referred to as response spectrum analysis (RSA). The approximation inherent in the SRSS rule is a second type of error.

Two other sources of error might appear in modal analysis: the third one is due to modal truncation, that is, when only a limited number of modes, $J < N$, is considered in the analysis. Estimation of significant number of modes, based on cumulative effective mass in the modes considered is no longer valid for non-classically damped systems. Truncation measures, developed specifically for this type of structures, exist in the literature [De Domenico & Ricciardi, 2019]. The fourth source of error occurs when peak modal responses are found using elastic spectral ordinates modified using damping correction factors. This error is examined in Chapter 4. This Chapter focuses only on under-estimation errors due to i) diagonalization of the modal damping matrix and ii) use of combination rules.

3.2.2 Modal concepts in non-proportionally damped systems

Prior to discussion of the existing indices, it is necessary to introduce the concept of complex modal shapes. For that, the general equation of motion must be transformed into $2N$ state-space [Foss, 1958; Veletsos & Ventura, 1986; Liang & Lee, 1991]:

$$[\mathbf{a}][\dot{\mathbf{z}}] + [\mathbf{b}][\mathbf{z}] = [\mathbf{p}], \quad (3.2-8)$$

where

$$[\mathbf{a}] = \begin{bmatrix} \mathbf{c} & \mathbf{m} \\ \mathbf{m} & \mathbf{0} \end{bmatrix}; [\mathbf{z}] = \begin{bmatrix} \mathbf{u} \\ \dot{\mathbf{u}} \end{bmatrix}; [\mathbf{b}] = \begin{bmatrix} \mathbf{k} & \mathbf{0} \\ \mathbf{0} & -\mathbf{m} \end{bmatrix}; [\mathbf{p}] = \begin{bmatrix} -\mathbf{m} \mathbf{1} a_g(t) \\ \mathbf{0} \end{bmatrix}; \quad (3.2-9)$$

here $[\mathbf{x}]$ indicates an extended matrix (a matrix with matrix components), and $\mathbf{0}$ is a null matrix of the required dimension and units. The eigenvalue problem stated by the homogeneous counterpart of Eqn. (3.2-8), leads to $2N$ eigenvectors and eigenvalues that, for under-critically damped systems, can be expressed as:

$$[\Psi_E] = \begin{bmatrix} \Psi & \bar{\Psi} \\ \Psi \Lambda & \bar{\Psi} \bar{\Lambda} \end{bmatrix}, \quad (3.2-10)$$

$$[\Lambda_E] = \begin{bmatrix} \Lambda & \\ & \bar{\Lambda} \end{bmatrix}, \quad (3.2-11)$$

where $\bar{x}$ indicates the complex conjugate of x , and:

$$\Psi = [\psi_1, \dots, \psi_N], \quad (3.2-12)$$

$$\Psi_j = [\psi_{1j}, \dots, \psi_{Nj}]^T = [\psi_{1j,R}, \dots, \psi_{Nj,R}]^T + i[\psi_{1j,I}, \dots, \psi_{Nj,I}]^T, \quad (3.2-13)$$

$$\Lambda = \text{diag}(\lambda_j) \quad (3.2-14)$$

$$\lambda_j = -\xi_j \omega_j + i \omega_j \sqrt{1 - \xi_j^2}. \quad (3.2-15)$$

In the former expressions, $\psi_{nj,R}$ and $\psi_{nj,I}$ denote the real and imaginary parts of ψ_{nj} , respectively. Correspondingly:

$$\Psi_{j+N} = \bar{\Psi}_j = [\psi_{1j,R}, \dots, \psi_{Nj,R}]^T - i[\psi_{1j,I}, \dots, \psi_{Nj,I}]^T, \quad (3.2-16)$$

$$\lambda_{j+N} = \bar{\lambda}_j = -\xi_j \omega_j - i \omega_j \sqrt{1 - \xi_j^2}. \quad (3.2-17)$$

$[\Psi_E]$ diagonalizes both $[\mathbf{a}]$ and $[\mathbf{b}]$, so that:

$$[\Psi_E]^T [\mathbf{a}] [\Psi_E] = [\mathbf{A}], \quad (3.2-18)$$

and the counterpart normalization of Eqn. (3.2-1) is:

$$[\hat{\Psi}_E]^T [\mathbf{a}] [\hat{\Psi}_E] = [\mathbf{I}], \quad (3.2-19)$$

which is achieved by making $[\hat{\Psi}_E] = [\Psi_E][\mathbf{A}]^{-1/2}$; then, using the transformation $[\mathbf{z}] = [\hat{\Psi}_E][\mathbf{q}]$ on the homogeneous form of expression (3.2-8) and pre-multiplying by $[\hat{\Psi}_E]^T$, a set of $2N$ uncoupled scalar equations is obtained (composed of N equations and their counterpart conjugates):

$$[\dot{\mathbf{q}}] + [\Lambda_E][\mathbf{q}] = [\mathbf{0}], \quad (3.2-20)$$

with solutions of the form $q_j = r \exp(\lambda_j t)$ for mode j . The imaginary components in $[\hat{\Psi}_E]$ are indicative of phase differences between the modes; the peak response is reached at a different time in every DOF. This is shown in Figure 3-1, where a part of the steady-state displacement response of a 3-storey steel moment-resisting frame described in the literature (Ramirez et al [2001]) is shown. The frame, with and without viscous dampers designed for a 30% added damping in the first mode, has been subjected to harmonic ground excitation at resonant frequency (1st mode period); the peaks in the response are coincident for modal damping, Figure 3-1a (which is proportional), but they differ for added damping, Figure 3-1b (non-classical)

Figure 3-1. Frame with non-proportional damping subjected to harmonic ground excitation ($T_g = T_1$) (a) 5% inherent damping; (b) 35% effective damping.

In the previous discussion, it has been assumed that the modes are under-critically damped; in this case every complex eigenvalue in expression (3.2-15) presents a conjugate counterpart, Eqn. (3.2-17), as happens with every complex eigenvector, expressions (3.2-13) and (3.2-16). These expressions hold true for systems with no damping (where the eigenvalues collapse to $\pm i\omega_j$) and proportional damping; in these two cases, complex eigenvectors are obtained, but they can be normalized to real eigenvectors. For over-critically damped systems, the eigenvalues of the over-damped modes are real numbers and their corresponding eigenvectors can again be normalized to real numbers. Expressions (3.2-10) and (3.2-11) are no longer valid, but using $[\Psi_E]$ and $[\Lambda_E]$, the system is uncoupled to expression (3.2-20). In all cases the system's frequency, damped frequency and modal damping can be calculated from Eqn. (3.2-15) as:

$$\omega_j = |\lambda_j|, \quad (3.2-21)$$

$$\omega_{d,j} = \text{Im}(\lambda_j), \quad (3.2-22)$$

$$\xi_j = -\text{Re}(\lambda_j) / \omega_j. \quad (3.2-23)$$

The dependence of λ/ω , Eqns. (3.2-15) and (3.2-17), with the value of ξ is shown in Figure 3-2; below the critical damping ratio, $\xi_{cr} = 1$, the equations render two complex solutions, that converge to one real solution for over-critical damping. For over-critically damped modes, $\xi_j = 1$ and $\omega_{d,j} = 0$, and there is motion decay instead of harmonic free vibration.

Figure 3-2. Evolution of eigenvalues with modal damping ratio ξ .

3.2.3 Indices to measure damping non-proportionality

Non-proportional damping results in modal displacements with imaginary components. At the same time, the modal damping matrix is populated. The indices present in the literature are generally based upon one of these features, namely, phase properties of the modes or measures of the relative importance of the off-diagonal terms in the damping matrix. In general, all indices attempt to establish deviations between a system with non-classical damping, and its counterpart system classically-damped. Some existing indices are reviewed briefly; emphasis is made on those indices with possibilities to be applied in a practical engineering context, leaving out indices, which require, for instance, evaluation of the system's response. The indices are divided in two families: those obtained from the modal damping matrix and those extracted from properties of modal displacements. The first group is more suitable to derive criteria for design practice, as complex modes are generally not used in this context.

In all cases, the zero value of the index is indicative of proportional damping; indices upper-bounded to unity are referred to as normal indices, and are of greater value; for this subset of indices, however, a question not answered in the literature is the type of correlation between the index value and the level of non-proportionality. The parametric study described in Section 0 addresses this question. When the index is mode-dependent, a practical global index can be based in all modes, a single mode, or a subset of the most relevant modes that could be selected by some criterion (for instance, mass participation ratio). In this study, the average throughout all modes is considered as representative. The indices considered are:

Indices based on components or properties of the modal damping matrix:

- Warburton & Soni [1977]: I_{WS1} , I_{WS2} .
- Prater & Singh [1986]: I_{PS5} , I_{PS6} .
- Claret and Venancio-Filho (1991), I_{CV1} .
- Tong, Liang & Lee [Tong et al, 1994]: I_{TLL1} .
- Bhaskar [1995]: I_{B1} .

Indices based on properties of the complex modal shapes or modal polygons:

- Prater & Singh [1986]: I_{PS1} , I_{PS2} , I_{PS3} , I_{PS4} .
- Bhaskar [1999]: I_{B2} , I_{B3} .
- Liu, Kujath & Zheng [2000; 2001]: I_{LKZ1} , I_{LKZ2} , I_{LKZ3} , I_{LKZ4} , I_{LKZ5} , I_{LKZ6} .
- Prells & Friswell [2000]: I_{PF1} .
- Adhikari [2004]: I_{A1} .

Other indices:

- Nair & Singh [1986] I_{NS1} .

The order in indices by the same authors is based on the order in which the indices were originally proposed. As presented later in the study, four of these indices (I_{PS5} , I_{LKZ4} , I_{TLL1} , I_{A1}) were found useful in predicting the error incurred by classical modal analysis of non-proportionally damped systems. Two (I_{LKZ4} , I_{A1}) are based on modal shapes, and two (I_{PS5} , I_{TLL1}) are based on the components of the modal damping matrix. The indices are briefly described in the next Section.

3.2.3.1 Analytical description of existing indices

Warburton and Soni

Warburton and Soni [1977] compared the resonant response for a MDOF with non-classical damping excited harmonically in the n^{th} DOF, and its approximate solution obtained neglecting off-diagonal terms, deriving the following condition:

$$|\hat{\phi}_{nj}| \gg \max_{\substack{s=1 \\ s \neq j}}^N \left(\left| \hat{C}_{j,s} \omega_j \hat{\phi}_{ns} / (\omega_s^2 - \omega_j^2) \right| \right), \quad (3.2-24)$$

to be fulfilled at every DOF n ; whereupon the following index can be defined for mode j :

$$I_{WS1,j} = \max_{j=1}^N \left(\max_{\substack{s=1 \\ s \neq j}}^N \left(\left| \hat{C}_{j,s} \omega_j \hat{\phi}_{ns} / (\omega_s^2 - \omega_j^2) \right| \right) / |\hat{\phi}_{nj}| \right), \quad (3.2-25)$$

and the global index I_{ws} is the average value of $I_{ws,j}$ throughout all modes. This index was not originally presented by the authors, but its rationale is based on expression (3.2-20). It presents several problems: first, it is not normal; second, it is sensitive to small values of modal displacements, which might happen in higher modes; third, it is not formally proved that an index derived for harmonic motion might be reasonably exact for other types of motion. Although cumbersome, this index might be valid in a design situation.

By assuming that the order of magnitude of the modal displacements is similar, the authors rewrite equation (3.2-24), and formulate the following condition:

$$\xi_j < \varepsilon \min_{\substack{s=1 \\ s \neq j}}^N \left(\left| (\hat{C}_{j,j} / 2\hat{C}_{j,s}) / (\omega_s^2 / \omega_j^2 - 1) \right| \right), \quad (3.2-26)$$

where ε is an error parameter, with a proposed value of 0.05; this expression might be reformulated as an index:

$$I_{WS2,j} = \xi_j / \min_{\substack{s=1 \\ s \neq j}}^N \left(\left| (\hat{C}_{j,j} / 2\hat{C}_{j,s}) / (\omega_s^2 / \omega_j^2 - 1) \right| \right) \quad (3.2-27)$$

Prater and Singh

Prater and Singh [1986] proposed several indices. After normalization of the modal displacements to the maximum of each mode (so that the modal displacements magnitudes are no larger than 1), the area of the polygon formed by the complex plane representation

of the modal displacements for a given mode j , $A_{1,j}$, is taken as representative of the phase differences between modes. This area is compared with the maximum possible area, corresponding to a regular polygon inscribed in a circle of unit radius $A_{1,\max}$. Thus:

$$I_{PS1,j} = A_{1,j} / A_{1,\max} \quad (3.2-28)$$

And I_{PS1} is the normal index obtained averaging throughout all modes. As discussed by Bhaskar [1999], the polygon area is ambiguous in the sense that it is dependent on the order of the DOFs, but this difficulty can be avoided considering only the convex polygon, as suggested by Ewins [1984]. Another difficulty is that this index is not defined for systems with only 2 DOFs; this difficulty can be overridden by adding the point (0; 0) to the set, giving rise to an alternative index, not discussed here.

Higher modes will always present a higher index because the presence of nodes ensures that the modal displacements will be in different quadrants of the complex plane. To override this problem, modal coordinates in the 2nd or 3rd quadrant can be rotated 180°, so that all polygons lie in the 1st and 4th quadrant. The polygon area in mode n is referred to as $A_{2,j}$, and the maximum area, $A_{2,\max}$ is half the formerly considered area; the index reads:

$$I_{PS2,j} = A_{2,j} / A_{2,\max} \quad (3.2-29)$$

And I_{PS2} is the normal index obtained averaging through the modes. Again, the area should be that of the convex polygon, and the index is undefined for 2 DOFs systems.

Next, considering the phase of the normalized rotated complex displacements, $\theta_{n,j} = \arg(\phi_{n,j})$, these authors define the simple index:

$$I_{PS3,j} = \sum_{n=1}^N 2|\theta_{n,j}| / j\pi, \quad (3.2-30)$$

normalized assuming the phase angle lies between $-\pi/2$ and $\pi/2$. This normal index does not account for differences of phase. A second, more meaningful normal index is given by:

$$I_{PS4,n} = (\max[\theta_{jn}] - \min[\theta_{jn}]) / \pi, \quad (3.2-31)$$

corresponding to the normalized angle spanned by the normalized-rotated displacements. Finally, two indices are derived based on the normalized modal damping matrix $\hat{\mathbf{C}}$; the relative magnitude of the non-diagonal terms can be used as a measure of the degree of non-proportionality:

$$I_{PS5} = \sum_{n=1}^N \sum_{j=1, j \neq n}^N |\hat{C}_{nj}| / \sum_{n=1}^N \sum_{j=1}^N |\hat{C}_{nj}|, \quad (3.2-32)$$

a normal index which is fairly simple to compute; a mode-dependent index can be calculated ignoring the first summation sign in the numerator and denominator; in this case the global value obtained by (3.2-32) is not the average of the different modes. Optionally, the determinant can be used to measure the importance of off-diagonal terms:

$$I_{PS6} = \det(\hat{\mathbf{C}}_o) / \det(\hat{\mathbf{C}}), \quad (3.2-33)$$

where $\hat{\mathbf{C}}_o$ is formed suppressing the diagonal terms in $\hat{\mathbf{C}}$. This index is not normal and cannot be applied to single modes.

A third index is proposed by this authors based on the response of the system to harmonic motion. Because it involves full evaluation of the system's response, it is not a practical option for design purposes and is left out of this review.

Nair and Singh

Nair & Singh [1986] proposed an index based on expression (3.2-5):

$$\mathbf{E} = \mathbf{c}\mathbf{m}^{-1}\mathbf{k} - \mathbf{k}\mathbf{m}^{-1}\mathbf{c}, \quad (3.2-34)$$

$$I_{NS1} = \sum_{n=1}^N \sum_{j=1}^N |E_{nj}| \quad (3.2-35)$$

For systems with proportional damping this index equals 0; however, it is not upper-bounded; moreover, it requires knowledge of the mass, stiffness and damping matrix, not always the case in practical situations. Another index proposed by the authors, requires computation of the system's frequency response, and is left out of this study.

Bellos and Inman

Using a frequency-domain expression of the equations of motion under resonant harmonic excitation, these authors [Bellos & Inman, 1990] derive a frequency-dependent index; to find a global index, the mean value of the index's integral over the whole frequency range is needed. This procedure is considered too cumbersome for design application to building structures, and the index is excluded from the present study.

Claret and Venancio-Filho

These authors proposed an index which is based on comparison of normalized modal damping matrix $\hat{\mathbf{C}}$ off-diagonal and diagonal terms [Claret & Venancio-Filho, 1991]:

$$I_{CV} = \max \left[\frac{\hat{C}_{nj}^2}{\hat{C}_{nn}\hat{C}_{jj}} \right]_{n \neq j}. \quad (3.2-36)$$

Tong, Liang and Lee

The index proposed by these authors [Tong et al, 1994] also relies in the normalized modal damping matrix $\hat{\mathbf{C}}$; the diagonal matrix $\hat{\mathbf{C}}_d = \hat{\mathbf{C}} - \hat{\mathbf{C}}_o$ is defined, and the index is expressed as:

$$I_{TLL} = (\hat{\sigma}_{\max} - \hat{\sigma}_{\min}) / (\hat{\sigma}_{\max} + \hat{\sigma}_{\min}), \quad (3.2-37)$$

where $\hat{\sigma}_{\max}$ and $\hat{\sigma}_{\min}$ are the maximum and minimum eigenvalues of $\hat{\mathbf{C}}_d^{-1}\hat{\mathbf{C}}$. Because of the properties of the normalized modal damping matrix, the index is normal. Moreover, the authors discuss three important properties of the index, namely its monotonic increase with

the non-diagonal terms of $\hat{\mathbf{C}}$, its independency from $\hat{\mathbf{C}}$ being scaled, and its convergence to 0 for proportional damping.

Bhaskar

This author proposed three different indices [Bhaskar, 1995; Bhaskar, 1999]. The first index is again based on the dominance of off-diagonal terms in the modal matrix:

$$I_{B1} = \max_{k=1}^N \left(\sum_{j=1, j \neq k}^N \left| \hat{C}_{kj} \right| / \left| \hat{C}_{kk} \right| \right) \quad (3.2-38)$$

The second index is based on the concept that complex representation of modal displacements for a proportionally damped system will radiate from the origin. For a generally damped structure and a given mode j , a certain rotation θ_j of the coordinate system can be found to minimize the sum of the squares of the ordinates of the modal displacements; in a proportionally damped system, this would be the orientation of modal displacements; deviation of each modal displacement from this direction is indicative of non-proportionality. Expression of the modal displacement in the new system is:

$$\phi'_{nj} = x'_{nj} + i y'_{nj} = |\phi_{nj}| \exp((\theta_{nj} - \theta_j)i) \quad (3.2-39)$$

and the sum of squared abscissas and ordinates is, respectively:

$$X'_j = \sum_{n=1}^N (x'_{nj})^2, \quad (3.2-40)$$

$$Y'_j = \sum_{n=1}^N (y'_{nj})^2. \quad (3.2-41)$$

Minimization of Y'_j (or maximization of X'_j), will render θ_j (a closed-form solution is given by the author, omitted here for brevity). Then, the index is expressed as:

$$I_{B2,j} = \sqrt{Y'_j / X'_j}. \quad (3.2-42)$$

The third index proposed by the author is formed by obtaining the area of the triangle formed by the modal displacement vector and its projection on the x' axis. This area is maximal when the projection is $|\phi_{nj}| \sqrt{2}$; therefore:

$$I_{B3,j} = \sum_{n=1}^N (x'_j y'_j / 2) / A'_{j,\max}, \quad (3.2-43)$$

where

$$A'_{j,\max} = (1/4) \sum_{n=1}^N |\phi_{nj}|^2 \quad (3.2-44)$$

I_{B2} and I_{B3} are normal; they are also invariant to scaling of the modal displacement vectors.

Liu, Kujath and Zheng

These authors proposed six different indices [Liu et al, 2000; Liu et al, 2001]; three of them are based on modal displacements and three on properties of the modal damping

matrix. To obtain indices 1 to 3, it is first necessary to manipulate the modes with a procedure somewhat similar to the one proposed by Bhaskar. The modal displacements Ψ_j of mode j are normalized to the maximum component, and rotated counterclockwise by 45° :

$$\tilde{\Psi}_j = (\Psi_j / \Psi_{j,\max}) \exp(i\pi / 4); \quad (3.2-45)$$

then the best fit angle of the rotated displacements is found with a least square method,

$$\theta_j = \arctan\left(\left(\tilde{\Psi}_{j,R}^T \tilde{\Psi}_{j,I}\right) / \left(\tilde{\Psi}_{j,R}^T \tilde{\Psi}_{j,R}\right)\right); \quad (3.2-46)$$

finally, the displacements are rotated clockwise by the best fit angle:

$$\hat{\Psi}_j = \tilde{\Psi}_j \exp(-i\theta_j). \quad (3.2-47)$$

The imaginary part of the modal vector $\hat{\Psi}_{j,I}$ is minimal (in the least square sense). The first index proposed by the authors is based on the increasing correlation (due to modal displacement alignment) between imaginary and real parts of $\hat{\Psi}_j$ as proportionality increases. Thus:

$$I_{LKZ1,j} = 1 - \left| \tilde{\Psi}_{j,R}^T \tilde{\Psi}_{j,I} \right| / \sqrt{\left[\tilde{\Psi}_{j,R}^T \tilde{\Psi}_{j,R} \right] \left[\tilde{\Psi}_{j,I}^T \tilde{\Psi}_{j,I} \right]}; \quad (3.2-48)$$

the second part in the right term is the expression of the correlation between real and imaginary parts of the modal displacement. The second index is a measure of the scatter of the imaginary parts of $\hat{\Psi}_j$:

$$I_{LKZ2,j} = \sum_{n=1}^N \left| \hat{\Psi}_{nj,I} \right| / N; \quad (3.2-49)$$

For the third mode, the normalized modal matrix is decomposed as:

$$\hat{\Psi} = \hat{\Psi}_R \hat{T} = \hat{\Psi}_R (\mathbf{I} + i \hat{T}_I), \quad (3.2-50)$$

where $\hat{T}$ is complex, $\hat{T}_R = \mathbf{I}$, $\hat{T}_I = \hat{\Psi}_R^{-1} \hat{\Psi}_I$. The magnitude of $\hat{T}_I$ is linked to modal coupling and therefore the following index is proposed:

$$I_{LKZ3,j} = \sum_{n=1}^N \left| \hat{T}_{I,nj} \right| / N; \quad (3.2-51)$$

The next three indices proposed by these authors are based on modal information. Using the expansion of the complex modes (normalized according to expression (3.2-19)):

$$\hat{\Psi} = \hat{\Phi} \hat{T} \Rightarrow \hat{T} = \hat{\Phi}^{-1} \hat{\Psi}. \quad (3.2-52)$$

The fourth index proposed is a counterpart of I_{PS5} based on $\hat{T}$ instead of $\hat{C}$:

$$I_{LKZ4} = \sum_{j=1}^N \sum_{n=1, n \neq j}^N \left| \hat{T}_{nj} \right| / \sum_{n=1}^N \sum_{j=1}^N \left| \hat{T}_{nj} \right|, \quad (3.2-53)$$

In the same way, a counterpart index to I_{TLL1} is proposed as:

$$I_{LKZ5} = (\sigma_{\max} - \sigma_{\min}) / (\sigma_{\max} + \sigma_{\min}), \quad (3.2-54)$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the maximum and minimum eigenvalues of $\hat{\mathbf{T}}_d^{-1} \hat{\mathbf{T}}$, and $\hat{\mathbf{T}}_d$ is the corresponding diagonal matrix formed from $\hat{\mathbf{T}}$. Finally, following the same rationale as in I_{LKZ3} , the sixth index is based on the off-diagonal elements of $\hat{\mathbf{T}}$:

$$I_{LKZ6,j} = \sum_{n=1, n \neq j}^N |\hat{\mathbf{T}}_{nj}| / (N-1); \quad (3.2-55)$$

Prells and Friswell

These authors [Prells & Friswell, 2000], propose decomposition of $\hat{\Psi}$ in the following form:

$$\hat{\Psi} = \hat{\Psi}_R (\mathbf{I} - i \Theta), \quad (3.2-56)$$

where $\Theta = -\hat{\Psi}_I \hat{\Psi}_R^{-1}$ is shown to be an orthonormal matrix; an important result proved by the authors is that damping is proportional if, and only if, $\Theta = \mathbf{I}$. Consequently, the degree of proportionality can be measured by:

$$I_{PF1} = \|\Theta - \mathbf{I}\| \quad (3.2-57)$$

where $\|\mathbf{x}\|$ indicates the spectral matrix norm of $\mathbf{x}$. This index, as given by the authors, is upper bounded to 2; it can be normalized by taking $I_{PF1}/2$. Unfortunately, calculation of this index is prone to numerical error, as the condition number of $\hat{\Psi}_R$ is, in many cases, high.

Adhikari

Scaling of complex modes by complex scalars changes significantly their geometric appearance; a certain scaling allows the complex modes to become as close as possible (again in the least square sense) to the natural normal modes; Adhikari [2004] referred to these as optimal complex modes, and derived an expression to find them; for mode j :

$$\tilde{\Psi}_j = \frac{\Psi_{R,j}(\Psi_{R,j}^T \hat{\Phi}_j) + \Psi_{I,j}(\Psi_{I,j}^T \hat{\Phi}_j)}{\|\Psi_{R,j}\|^2 + \|\Psi_{I,j}\|^2} + i \frac{\Psi_{I,j}(\Psi_{R,j}^T \hat{\Phi}_j) - \Psi_{R,j}(\Psi_{I,j}^T \hat{\Phi}_j)}{\|\Psi_{R,j}\|^2 + \|\Psi_{I,j}\|^2}. \quad (3.2-58)$$

Because optimal modes are closest to natural modes, the distance between them is a direct measure of non-proportionality; therefore:

$$I_{A1,j} = \|\tilde{\Psi}_j - \tilde{\Phi}_j\| / \|\tilde{\Phi}_j\| \quad (3.2-59)$$

This author suggests the use of both mean and standard deviation throughout modes to characterize the overall non-proportionality of the system. For compatibility with other indices, only the mean value is used in this study.

3.2.3.2 Application example of non-proportionality indices

The indices described in the previous section are applied to a 3 DOF lumped mass system with the following properties: $\mathbf{m} = 0.25 \cdot \mathbf{I}$ kN·s²/mm (where $\mathbf{I}$ is a 3×3 identity matrix); $\mathbf{k} = [99.6616, -49.8308, 0; -49.8308, 99.6616; -49.8308, 0, -49.8308, 49.8308]$ kN/mm. The natural undamped frequencies of the system are $\omega_n = [6.2832, 17.6051, 25.4401]$ Hz. The following cases are studied:

- no damping;
- (2-6), non proportional damping with $\mathbf{c}_{np} = c_1 \cdot [1, -0.4286, 0; -0.4286, 0.7143, -0.2857; 0, -0.2857, 0.2857]$, where the values of c_1 are adjusted in each case to target a certain modal damping ratio ξ_{a1} (10%, 20%, 30%, 50%, 120%) in the first mode.
- (7-11), non proportional damping with $\mathbf{c}_{np} = c_2 \cdot [1, 0, 0; 0, 0, 0; 0, 0, 0]$, where the values of c_1 are adjusted in each case to target a certain modal damping ratio ξ_{a1} (10%, 20%, 30%, 50%, 120%) in the first mode.

Calculated values of the indices are given in Table 3-1, along with the system's frequencies ω_n , damped frequency ω_{dn} and modal damping ξ_n obtained from Eqns. (3.2-21), (3.2-22) and (3.2-23). Empty entries for I_{PF1} could not be calculated due to numerical errors inherent to the index definition.

Systems 2 to 6 present increasing amounts of damping with the same relative values of the damping matrix. Indices I_{PS5} , I_{PS6} , I_{TLL1} , I_{B1} are dependent on relative components of the damping matrix and render the same value for all 5 systems, which is not consistent with the increasing level of damping. When ξ_{a1} increases from 10% to 20%, all other indices increase consistently but, with further increase of damping, indices I_{PS1} , I_{PS2} , I_{PS3} , I_{PS4} , I_{B2} , I_{B3} , I_{LKZ1} , I_{LKZ2} , I_{LKZ3} , I_{LKZ4} , I_{LKZ5} , I_{LKZ6} and I_{A1} decrease at some point, due to the presence of over-damped modes with non-complex modal shapes. This is consistent with a lower degree of modal uncoupling, because modal shapes for over-damped modes are proportional. In this respect, indices I_{LKZ4} , I_{LKZ5} , I_{LKZ6} , I_{A1} are only affected for high over-damping, and I_{WS1} , I_{WS2} , I_{NS1} are not affected at all.

Systems 7 to 11 present higher uncoupled terms in the modal damping matrix. Again, I_{PS5} , I_{PS6} , I_{TLL1} , I_{B1} render the same value for all systems. Focusing on the step from 20% to 30% damping, it can be noticed that I_{PS1} , I_{PS2} , I_{PS3} , I_{PS4} , I_{B2} , I_{B3} , I_{LKZ1} , I_{LKZ2} , I_{PF1} decrease value at this point, which is again not consistent and cannot be attributed to over-damping which does not exist in this system for these damping levels. In this case, I_{LKZ4} , I_{LKZ5} , I_{A1} are affected only at extreme values of over-damping, and I_{WS1} , I_{WS2} , I_{NS1} , I_{LKZ6} are not affected at all.

Finally, comparing cases 2 with 7, 3 with 8 and so on, an increase in index value is observed for all cases except I_{PS1} and I_{B3} . This is coherent with the higher level of non-proportionality in systems 7 to 11.

Table 3-1. Values of non-proportionality indices for 11 different cases.

Case	1	2	3	4	5	6	7	8	9	10	11
ξ_{a1}	0%	10%	20%	30%	50%	120%	10%	20%	30%	50%	120%
ω_1	6.28	6.29	6.31	6.34	6.45	9.50	6.41	6.84	7.61	8.40	8.68
ω_2	17.6	17.8	18.3	18.7	18.8	84.7	18.0	17.7	16.1	22.9	22.9
ω_3	25.4	25.2	24.4	33.8	78.5	204.4	24.4	23.3	23.0	50.8	137.3
ω_{d1}	6.28	6.26	6.18	6.05	5.61	0	6.38	6.72	7.43	8.31	8.66
ω_{d2}	17.6	17.3	16.4	14.6	0.66	0	17.6	15.1	5.5	22.9	22.9
ω_{d3}	25.4	23.8	17.9	0	0	0	24.4	23.2	23.0	0	0
ξ_1	0	0.10	0.20	0.30	0.49	1	0.10	0.18	0.21	0.14	0.06
ξ_2	0	0.23	0.44	0.62	1.00	1	0.20	0.52	0.94	0.02	0.01
ξ_3	0	0.33	0.68	1	1	1	0.06	0.05	0.03	1	1
I_{WS1}	0	0.23	0.47	0.70	1.16	2.79	0.48	0.96	1.44	2.39	5.74
I_{WS2}	0	0.11	0.22	0.34	0.56	1.34	1.95	3.91	5.86	9.77	23.45
I_{PS1}	0	0.14	0.20	0.13	0.01	0.00	0.19	0.13	0.07	0.04	0.02
I_{PS2}	0	0.03	0.04	0.01	0.01	0.00	0.08	0.11	0.04	0.02	0.01
I_{PS3}	0	0.08	0.17	0.08	0.03	0.00	0.15	0.27	0.26	0.20	0.22
I_{PS4}	0	0.11	0.19	0.07	0.03	0.00	0.23	0.39	0.33	0.29	0.32
I_{PS5}	0	0.21	0.21	0.21	0.21	0.21	0.64	0.64	0.64	0.64	0.64
I_{PS6}	0	0.00	0.00	0.00	0.00	0.00	2.00	2.00	2.00	2.00	2.00
I_{NS1}	0	736	1472	2209	3681	8834	1164	2328	3493	5821	13970
I_{TLL1}	0	0.31	0.31	0.31	0.31	0.31	1.00	1.00	1.00	1.00	1.00
I_{B1}	0	0.61	0.61	0.61	0.61	0.61	4.05	4.05	4.05	4.05	4.05
I_{B2}	0	0.12	0.18	0.09	0.03	0.00	0.25	0.29	0.20	0.11	0.05
I_{B3}	0	0.36	0.52	0.24	0.02	0.00	0.59	0.46	0.24	0.14	0.06
I_{LKZ1}	0	0.04	0.09	0.05	0.00	0.00	0.15	0.17	0.08	0.04	0.01
I_{LKZ2}	0	0.08	0.14	0.08	0.02	0.00	0.13	0.16	0.11	0.06	0.03
I_{LKZ3}	0	0.05	0.09	0.06	0.02	0.00	0.13	0.14	0.14	0.06	0.02
I_{LKZ4}	0	0.10	0.19	0.23	0.33	0.20	0.23	0.35	0.44	0.50	0.48
I_{LKZ5}	0	0.17	0.35	0.41	0.40	0.32	0.35	0.59	0.70	1.32	1.25
I_{LKZ6}	0.50	0.57	0.64	0.67	1.12	0.79	0.67	0.80	0.94	1.22	1.29
I_{PF1}	0	0.32	0.50				0.61	0.65	0.47		
I_{A1}	0	0.12	0.24	0.27	0.28	0.25	0.26	0.42	0.51	0.66	0.65

3.3 Description of parametric study

The equations of motion of a non-proportionally damped system can be solved using direct step-by-step integration (“exact” solution). Alternatively, an approximate solution may be obtained by ignoring the off-diagonal coupling coefficients of the modal coordinates damping matrix C , and solving the resulting uncoupled equations with the classical modal method [Clough & Penzien, 1995; Chopra, 2007]. This approximation involves errors. In

order to measure the deviation between exact and approximate response in MDOF systems with added damping, a parametric study is performed. The main target is to establish a range of maximum expected differences between the exact response of a MDOF with added damping (including off-diagonal modal damping matrix terms), and the approximate modal response (excluding such terms). The trend of these differences is expected to depend primarily on the vibration characteristics of the MDOF, damper distribution law, amount of damping, and excitation type. In this study all modes are included; therefore, the effect of modal truncation is not considered. Exact modal responses are obtained (within the accuracy of the numerical methods used), to avoid errors derived from statistically obtained damped response spectra.

3.3.1 Numerical models

The MDOF systems correspond to N -storey elastic frames, which can be described by a shear type model, with inherent damping ratio ξ_I in all modes, n^{th} storey mass m_n , n^{th} storey stiffness k_n and n^{th} storey added damping $c_{d,n}$ provided by linear viscous dampers installed in the storey, see Figure 3-3. Each mass presents one DOF (horizontal translation in the direction of the ground motion). The storey properties are characterized by: i) the storey distribution of mass, stiffness and added damping, that relates the n^{th} storey value to the 1st storey value; ii) the 1st storey value of mass (m_1), stiffness (k_1) and added damping ($c_{d,1}$). In this study the storey mass is assumed constant, with value $m_n = m_1$ (most actual buildings present approximately uniform height-wise mass distributions). The distribution for stiffness and damping, see Figure 3-3(c)(d), is assumed of the form:

$$\frac{k_n}{k_1} = 1 + (\alpha_k - 1) \left(\frac{n-1}{N-1} \right)^{\beta_k} \quad (3.3-1)$$

$$\frac{c_{d,n}}{c_{d,1}} = 1 + (\alpha_d - 1) \left(\frac{n-1}{N-1} \right)^{\beta_d} \quad (3.3-2)$$

Figure 3-3. Definition of MDOF systems.

The parameters α_k and α_d represent the proportion between the top storey and 1st storey values of stiffness and added damping, respectively; for instance, $\alpha_k = 1$ corresponds to a constant distribution of stiffness ($k_n = k_1$). The parameters β_k and β_d represent the variation law of stiffness and added damping in height; for instance, $\beta_k = 1$ corresponds to a linear variation of stiffness. A plot of several combinations for these parameters for a 5-storey frame is shown in Figure 3-4.

With the values of k_n and $c_{d,n}$ fixed according to the chosen parameters ($\alpha_k, \alpha_d, \beta_k, \beta_d$), the stiffness and added damping matrices, $\mathbf{k}$ and $\mathbf{c}_a$, are obtained. The modal matrix Φ and spectral matrix Ω^2 of the undamped system can be calculated from $\mathbf{m}$ and $\mathbf{k}$, and the first modal period T_1 is found as $2\pi/\omega_1$. Inherent damping is assumed as modal with constant damping ratio ξ_I in all modes; the corresponding inherent modal damping matrix $\mathbf{C}_h$ is diagonal, and its j -th term is calculated as:

$$C_{h,j,j} = 2\xi_h \omega_j M_j, \quad (3.3-3)$$

whereupon the inherent damping matrix is:

$$\mathbf{c}_h = (\Phi^T)^{-1} \mathbf{C}_h \Phi^{-1}. \quad (3.3-4)$$

In addition to inherent damping, the contribution of added viscous damping must be taken into account to obtain the total damping matrix of the system. Two different approaches are examined. The first approach is to use the exact damping matrix $\mathbf{c}$ calculated as:

$$\mathbf{c} = \mathbf{c}_a + \mathbf{c}_h \quad (3.3-5)$$

The second approach is to use an approximate diagonalizable damping matrix $\mathbf{c}_{approx}$, constructed retaining only the diagonal terms of the modal damping matrix. The modal damping matrix $\mathbf{C}$ is found from $\mathbf{c}$ as $\mathbf{C} = \Phi^T \mathbf{c} \Phi^{-1}$. The approximate damping matrix is constructed as:

$$\mathbf{C}_{approx} = \begin{cases} C_{approx,i,i} = C_{i,i} \\ C_{approx,i,j \neq i} = 0 \end{cases} \quad (3.3-6)$$

$$\mathbf{c}_{approx} = (\Phi^T)^{-1} \mathbf{C}_{approx} \Phi^{-1} \quad (3.3-7)$$

It is worth noting that the modal damping ratios $\xi_j (=C_{j,j}/2M_j\omega_j)$ are the same for both approaches, because the diagonal values of $\mathbf{C}$ and $\mathbf{C}_{approx}$ are identical.

3.3.2 Range of parameters

A system is defined by a certain combination of parameters. The range of parameter values examined in this study are: $N = [3, 5, 7, 9]$; $\alpha_k = [0.20; 0.40; 0.60; 0.80]$; $\beta_k = [0.5; 1; 2; 10]$; $\alpha_d = [0; 1/3; 2/3; 1]$; $\beta_d = [0.1; 0.5; 1; 2]$; $T_1 = [0.5; 1; 2; 4]$; $\xi_{a,1} = [10\%; 20\%; 30\%;$

40%]. The values chosen for N and T_1 cover the most usual cases in building structures equipped with linear viscous dampers, where a certain lateral flexibility is needed to render the damping devices useful. The 40% value for $\xi_{a,1}$ is assumed as the maximum realistic value that can be obtained within economic constraints, and is slightly above the current maximum value ($\xi_{l,1} + \xi_{a,1} = 35\%$) allowed in ASCE/SEI 7-16 [ASCE, 2016] for application of RSA. The values chosen for α_k correspond to realistic possibilities in a design situation; the values chosen for α_d , β_d , and β_k range between extreme ($\alpha_d = [0, 1/3]$, $\beta_d = 0.1$, $\beta_k = 10$) and usual ($\alpha_d = [2/3, 1]$, $\beta_d = [0.5, 1, 2]$, $\beta_k = [0.5, 1, 2]$). The extreme value ($\alpha_d = 0$, $\beta_d = 0.1$) represents approximately a structure with additional dampers only in the first storey. The total number of systems examined is 16384 (4^7).

Figure 3-4. Distributions adopted for stiffness and added damping.

3.3.3 System definition

For each system defined by a certain set of parameters $[\alpha_k, \beta_k, \alpha_d, \beta_d]$ and a certain number of storeys (N), the relative properties of storey stiffness, k_n / k_1 , and added damping, $c_{d,n} / c_{d,1}$, are determined using expressions (3.3-1) and (3.3-2). Storey mass is set as $m_n = 0.25 \text{ kN}\cdot\text{s}^2/\text{mm}$. The diagonal mass matrix $\mathbf{m}$ is built. The stiffness matrix of the frame for $k_1^* = 1 \text{ kN/mm}$ is calculated and is denoted by $\mathbf{k}^*$. From $\mathbf{m}$ and $\mathbf{k}^*$ the first undamped mode period of the system T_1^* can be readily calculated. A target period T_x is fixed, and k_1 is determined so that the MDOF period matches T_x ; this condition is met for $k_1 = (T_1^*/T_x) \cdot k_1^*$. The stiffness matrix $\mathbf{k}$ is then re-evaluated with the new value k_1 , and the modal properties obtained; inherent damping ratio ξ_I , constant for all modes, is fixed, and the inherent modal damping matrix $\mathbf{c}_h$ is evaluated using Eqns. (3.3-3) and (3.3-4).

The added damping matrix for $c_{d,1}^* = 1 \text{ kN}\cdot\text{s}/\text{mm}$ is calculated and denoted by $\mathbf{c}_a^*$. The corresponding added modal damping ratio $\xi_{a,1}^*$ in the first mode is evaluated using expression (3.3-3) specialized for $n = 1$. A target added damping ratio $\xi_{a,1}$ is fixed, and the added damping matrix is scaled to match this target damping ratio (i.e., scaling factor $\xi_{a,1}/\xi_{a,1}^*$) to obtain the added damping matrix $\mathbf{c}_a$. The total modal damping matrix $\mathbf{C}$ is obtained, and the approximate damping matrix $\mathbf{c}_{approx}$ calculated from expressions (3.3-6) and (3.3-7).

3.3.4 Ground motions

The selection of ground motions is described in Annex A, under the label ‘European scaled motions’. The motions are selected and pre-scaled to represent well the Eurocode 8 [CEN, 2004] elastic 5% pseudo-acceleration spectrum. Two pre-scaled sets of ground motions are used here: ‘ESB’, corresponding to site class B, and ‘ESC’, corresponding to site class C.

3.3.5 Cases examined

Each system defined by a combination of parameters is subjected to six different ‘cases’. A case is defined by combination of a pre-scaled set of ground motions (ESB, ESC) and a value of inherent damping. Inherent damping values considered in this study are $\xi_I = [0\%, 3\%, 5\%]$. The option with $\xi_I = 0\%$ is adopted to rule out of the conclusions the influence of this parameter; the second and third options correspond to usual values in design. Thus, six ‘cases’ are examined, corresponding to site class B or C, with 0%, 3% or 5% damping ratio. These cases are referred to B0%, B3%, B5%, C0%, C3%, C5%.

Table 3-2. Summary of procedures and notation for case nc .

Procedure	TH	THMA	RSA-SRSS (**)
Description	Time history step-by-step analysis	Time history modal analysis	Response spectrum analysis
System ns definition ($ns = 1$ to 4^7)	${}_{ns}\mathbf{m}, {}_{ns}\mathbf{k}, {}_{ns}\mathbf{c}$	${}_{ns}\mathbf{m}, {}_{ns}\mathbf{k}, {}_{ns}\mathbf{c}_{approx}$	${}_{ns}\mathbf{m}, {}_{ns}\mathbf{k}, {}_{ns}\mathbf{c}_{approx}$
Analysis method	Central difference applied to complete system	Central difference applied to complete system	Exact piecewise applied to each mode
Solution type (for register r in case nc)	Complete solution ${}^r_{ns}\mathbf{u}(t), {}^r_{ns}\mathbf{v}(t)$	Complete solution ${}^r_{ns}\mathbf{u}_1(t), {}^r_{ns}\mathbf{v}_1(t)$	Peak modal solution ${}^r_{ns}\mathbf{u}_{2,peak,s}, {}^r_{ns}\mathbf{v}_{2,peak,s}$
Damping forces (for register r in case nc)	${}^r_{ns}\mathbf{f}_d(t) = \mathbf{c} {}^r_{ns}\mathbf{v}(t)$	${}^r_{ns}\mathbf{f}_{d1}(t) = \mathbf{c} {}^r_{ns}\mathbf{v}_1(t)$	${}^r_{ns}\mathbf{f}_{d2,peak,n} = \mathbf{c} {}^r_{ns}\mathbf{v}_{2,peak,n}$
Variables derived from solution (for register r in case nc)	${}^r_{ns}\Delta(t), {}^r_{ns}\nabla(t), {}^r_{ns}\mathbf{f}_s(t), {}^r_{ns}\mathbf{f}_d(t), {}^r_{ns}\mathbf{f}_t(t), {}^r_{ns}\mathbf{V}_s(t), {}^r_{ns}\mathbf{V}_d(t), {}^r_{ns}\mathbf{V}_t(t).$	${}^r_{ns}\Delta_1(t), {}^r_{ns}\nabla_1(t), {}^r_{ns}\mathbf{f}_{s1}(t), {}^r_{ns}\mathbf{f}_{d1}(t), {}^r_{ns}\mathbf{f}_{t1}(t), {}^r_{ns}\mathbf{V}_{s1}(t), {}^r_{ns}\mathbf{V}_{d1}(t), {}^r_{ns}\mathbf{V}_{t1}(t).$	${}^r_{ns}\Delta_2(t), {}^r_{ns}\nabla_2(t), {}^r_{ns}\mathbf{f}_{s2}(t), {}^r_{ns}\mathbf{f}_{d2}(t), {}^r_{ns}\mathbf{f}_{t2}(t), {}^r_{ns}\mathbf{V}_{s2}(t), {}^r_{ns}\mathbf{V}_{d2}(t), {}^r_{ns}\mathbf{V}_{t2}(t).$
Peak solution (for register r in case nc and storey s)	${}^r_{ns}u_{peak,s} = \left {}^r_{ns}u_s(t) \right _{\max}$ Similar for other variables	${}^r_{ns}u_{1,peak,s} = \left {}^r_{ns}u_{1,s}(t) \right _{\max}$ Similar for other variables	${}^r_{ns}u_{2,peak,s} = SRSS({}^r_{ns}u_{2,peak,s,n})$ Similar for other variables
Error for register r in case nc and storey s	-	${}^r_{ns}u_{1,error,s} = \frac{{}^r_{ns}u_{peak,s} - {}^r_{ns}u_{1,peak,s}}{{}^r_{ns}u_{peak,s}}$ Similar for other variables	${}^r_{ns}u_{2,error,s} = \frac{{}^r_{ns}u_{peak,s} - {}^r_{ns}u_{2,peak,s}}{{}^r_{ns}u_{peak,s}}$ Similar for other variables
Significant storeys considered	-	First: ${}^r_{ns}u_{1,error,1}$ Roof: ${}^r_{ns}u_{1,error,N}$ Average: ${}^r_{ns}u_{1,error,av} = \sum_{s=1}^N {}^r_{ns}u_{1,error,s} / N$ Similar for other variables	First: ${}^r_{ns}u_{2,error,1}$ Roof: ${}^r_{ns}u_{2,error,N}$ Average: ${}^r_{ns}u_{2,error,av} = \sum_{s=1}^N {}^r_{ns}u_{2,error,s} / N$ Similar for other variables
Final error for system ns and case nc	-	${}^r_{ns}u_{1,error,*} = \sum_{r=1}^{NR} {}^r_{ns}u_{1,error,*} / NR$ where $*$ = 1, N , av. Similar for other variables	${}^r_{ns}u_{2,error,*} = \sum_{r=1}^{NR} {}^r_{ns}u_{2,error,*} / NR$ where $*$ = 1, N , av. Similar for other variables

Notes: peak = maxima of unsigned response. Variables: $\mathbf{u}$ = displacement; $\mathbf{v}$ = velocity; Δ inter-storey drift; ∇ interstorey drift-velocity; $\mathbf{f}_s$ restoring structural forces; $\mathbf{f}_d$ damping forces; $\mathbf{f}_t$ total forces; $\mathbf{V}_s$ restoring structural shear; $\mathbf{V}_d$ damping shear; $\mathbf{V}_t$ total shear; med = median; N total number of storeys; NR number of registers in set. (**) RSA-CQC is found with the same procedure as RSA-SRSS substituting $SRSS(\cdot)$ by $CQC(\cdot)$ at peak solution calculation, and replacing subscript '2' by '3'.

3.3.6 Procedures and response variables.

For a given case nc and a given system ns , three different procedures are used to find the solution for each register r in the case. Table 3-2 introduces the notation used to distinguish the three procedures, described herein:

Time History (TH): the complete response ${}^r_{ns}\mathbf{u}(t)$, ${}^r_{ns}\mathbf{v}(t)$ (relative displacement and velocity) of the system defined by ${}_{ns}\mathbf{m}$, ${}_{ns}\mathbf{k}$ and ${}_{ns}\mathbf{c}$ (non-classically damped system) to each scaled motion in the case, is calculated by the central difference step-by-step method. The corresponding restoring forces ${}^r_{ns}\mathbf{f}_s(t)$ and damping forces ${}^r_{ns}\mathbf{f}_d(t)$ are obtained as ${}_{ns}\mathbf{k} \cdot {}^r_{ns}\mathbf{u}(t)$ and ${}_{ns}\mathbf{c} \cdot {}^r_{ns}\mathbf{v}(t)$, respectively. Other variables are derived from these. The peak response for each variable and storey is preserved.

Time-history modal analysis (THMA): the complete displacement and velocity responses ${}^r_{ns}\mathbf{u}_1(t)$, ${}^r_{ns}\mathbf{v}_1(t)$, of the system defined by ${}_{ns}\mathbf{m}$, ${}_{ns}\mathbf{k}$ and ${}_{ns}\mathbf{c}_{approx}$ (approximate classically-damped system) to each scaled motion in the case, are calculated adding the responses obtained for each mode at each instant by the central difference step-by-step method. As in the previous procedure, restoring forces ${}^r_{ns}\mathbf{f}_{s1}(t)$ and damping forces ${}^r_{ns}\mathbf{f}_{d1}(t)$ are obtained as ${}_{ns}\mathbf{k}_{ns} \cdot {}^r_{ns}\mathbf{u}_1(t)$ and ${}_{ns}\mathbf{c}_{ns} \cdot {}^r_{ns}\mathbf{v}_1(t)$, respectively; it is worth noting that the forces obtained for every time-step are not in equilibrium, because ${}^r_{ns}\mathbf{f}_{d1}(t)$ is calculated using ${}_{ns}\mathbf{c}$ whereas ${}^r_{ns}\mathbf{u}_1(t)$ and ${}^r_{ns}\mathbf{v}_1(t)$ are calculated using ${}_{ns}\mathbf{c}_{approx}$. Peak responses for every variable and storey are preserved. THMA avoids the inaccuracies involved in the use of approximate model combination rules such as Square Root of the Sum of Squares (SRSS). This response is identified by subscript '1'.

Response spectrum analysis (RSA): for every mode n of system ns defined by ${}_{ns}\mathbf{m}$, ${}_{ns}\mathbf{k}$ and ${}_{ns}\mathbf{c}_{approx}$ (approximate classically-damped system), and for every register r in case nc , the modal peak displacement and modal peak velocity, ${}^r_{ns}\mathbf{u}_{2,peak,n}$, ${}^r_{ns}\mathbf{v}_{2,peak,n}$, are found using the piecewise step-by-step method. These vectors contain the peak values for every storey at mode n , i.e. ${}^r_{ns}\mathbf{u}_{2,peak,n}^T = [{}^r_{ns}u_{2,peak,1,n}, \dots, {}^r_{ns}u_{2,peak,N,n}]$, etc. Then, the modal peak values of restoring forces ${}^r_{ns}\mathbf{f}_{s2,peak,n}$ ($= {}_{ns}\mathbf{k}_{ns} \cdot {}^r_{ns}\mathbf{u}_{2,peak,n}$) and damping forces ${}^r_{ns}\mathbf{f}_{d2,peak,n}$ ($= {}_{ns}\mathbf{c}_{ns} \cdot {}^r_{ns}\mathbf{v}_{2,peak,n}$) are found. The modal peak values of all other variables are derived from these. The peak response for every variable and storey is obtained applying the SRSS rule (RSA-SRSS), identified with subscript '2', or CQC rule (RSA-CQC), identified with subscript '3'. This option corresponds closely with the Response Spectrum Procedure defined in Chapter 18 of ASCE-SEI 7/16 [ASCE, 2016], except that the systems analyzed remain elastic.

The response variables of interest are: storey displacement u , storey velocity v , inter-storey drift Δ (referred to herein simply as drift), inter-storey drift velocity ∇ (referred to herein simply as drift velocity); structural shear force V_s ; damper shear force V_d (calculated with the exact damping matrix); total storey shear force V_t (addition of V_d and V_l). As indicated

in Table 3-2, variables obtained from THMA, RSA-SRSS and RSA-CQC are marked off by adding subscripts 1, 2 or 3, respectively.

3.3.7 Measure of response errors

The deviation between TH results and approximate results (THMA, RSA) is referred to as ‘error’, and measured for every register r in case nc , response variable, storey s and system ns . For instance, for drift Δ and storey s of system ns in register r of the case:

$${}_{ns}^r \Delta_{1,error,s} = \left({}_{ns}^r \Delta_{peak,s} - {}_{ns}^r \Delta_{1,peak,s} \right) / {}_{ns}^r \Delta_{peak,s}, \quad (3.3-8)$$

Similar equations (with updated subscripts) are used for RSA-SRSS (${}_{ns}^r \Delta_{2,error,s}$) and RSA-CQC (${}_{ns}^r \Delta_{3,error,s}$). A negative value of ${}_{ns}^r \Delta_{1,error,s}$, ${}_{ns}^r \Delta_{2,error,s}$, or ${}_{ns}^r \Delta_{3,error,s}$ indicates that the approximate method (THMA, RSA-SRSS, RSA-CQC) overestimates the response (safe approximation). A positive value indicates that the approximate method underestimates the response (unsafe approximation). The storeys chosen as representative are: N^{th} storey (referred to as ‘roof’); 1^{st} storey (referred to as ‘first storey’ or simply ‘first’); finally, the mean value throughout storeys (referred to as ‘average storey’ or simply ‘average’) is also found.

The mean value throughout the NR registers in case nc is considered as significant for the system ns ; for instance, for drift Δ and storey s :

$${}_{ns} \Delta_{1,error,s} = \sum_{r=1}^{NR} {}_{ns}^r \Delta_{1,error,s} / NR, \quad (3.3-9)$$

where s can be 1, N or av , and a similar definition is used for other variables. Finally, the median (med), minimum (min) and maximum (max) values of ${}_{ns} \Delta_{1,error,s}$ and ${}_{ns} \Delta_{2,error,s}$ are found throughout all systems for each case. and also for ${}_{ns} \Delta_{2,error,s}$ and ${}_{ns} \Delta_{3,error,s}$. This procedure, applied to all variables, gives the study results examined in the next Section

3.3.8 Summary of study

16384 systems ($4^7 = 16384$) are defined using all possible combinations of 7 parameters (N ; α_k ; β_k ; α_d ; β_d ; T_1 ; ξ_{a1}), each of them taking 4 different representative values; for every system, 6 cases are studied, each case is comprised of a set of 40 motions binned by site class (site classes B or C) and a value of inherent damping ξ_h , (0%, 3% or 5%); these six cases are referred to as B0%, B3%, B5%, C0%, C3%, C5%. For each case nc , the relative deviation (error) of 3 analysis procedures (‘THMA’, ‘RSA-SRSS’ and ‘RSA-CQC’) compared to the exact (‘TH’) analysis, is found for each system ns , for 6 different variables (for THMA, $u_{1,error}$, $\Delta_{1,error}$, $\nabla_{1,error}$, $V_{s1,error}$, $V_{d1,error}$, $V_{t1,error}$; for RSA-SRSS, $u_{2,error}$, $\Delta_{2,error}$, $\nabla_{2,error}$, $V_{s2,error}$, $V_{d2,error}$, $V_{t2,error}$; for RSA-CQC, $u_{3,error}$, $\Delta_{3,error}$, $\nabla_{3,error}$, $V_{s3,error}$, $V_{d3,error}$, $V_{t3,error}$), at the first storey, roof, and average through storeys. A single value (mean value

throughout motions in the case) is taken as representative of the error for case nc , and system ns at storey s . The median (med), minimum (min) and maximum (max) values through all systems are found for each case nc and storey s .

3.4 Study results

3.4.1 General response

Table 3-3 and Table 3-4 present representative error results for the parametric study. Both tables show maximum (max), median (med) and minimum (min) errors for THMA ($\Delta_{1,error}$, etc.), RSA-SRSS ($\Delta_{2,error}$, etc.), and RSA-CQC ($\Delta_{3,error}$, etc.) listed binned by storey ('roof', 'average', 'first'); however, Table 3-3 shows detailed results for all cases (B0%, B3%, B5%, C0%, C3%, C5%) of one variable (Δ), whereas Table 3-4 presents sample mean and sample standard deviation errors throughout all cases for six different variables (u , v , Δ , ∇ , V_d , V_t); V_s is omitted because, in the systems considered, the error results for Δ are the same as those for V_s . The results include every combination of parameters N , α_k , β_k , α_d , β_d , T_1 , ξ_{a1} considered in the study. Negative values indicate a conservative prediction of the variable. Figure 3-5 displays the maximum, median and minimum error of the same variables averaged throughout storeys vs. the fundamental period of the system, T_1 . Similar plots were obtained for all variables, storeys and parameters. From Table 3-4 and Figure 3-5 it follows that:

- The results for each of the six cases are almost coincident; the overall maximum, median and minimum for each variable and storey in Table 3-4 are very similar. This is illustrated for the particular case of Δ in Table 3-3, where the median and maximum results for all cases are virtually identical, whereas the minimum presents differences between cases smaller than 19%; therefore, the influence of inherent damping ξ_l in the maximum (positive, un-conservative) error is negligible, and there are only small differences between errors for site classes B and C.
- Regarding the difference between RSA-SRSS and RSA-CQC, the trends are similar for both: The median errors for both methods are close to 0 for all variables and storeys, with SRSS more conservative at roof and very similar values at average and first storeys. The minimum errors (conservative predictions) are more conservative with SRSS at roof and average storey, and with CQC at first storey; in general, both methods can render very conservative values for all variables, particularly for ∇ , V_d and V_t . The maximum errors, which are the main concern when a simplified procedure is applied, are similar with both methods, with differences less than 11% between them; generally speaking, use of CQC rule is advisable for design purposes, as it strongly reduces response over-estimation, while keeping response under-estimation at levels similar to SRSS rule.

- Errors for THMA, RSA-SRSS and RSA-CQC are similar in trend, but different in value; the median error for THMA ($x_{1,error,*},med$ where x represents any of the variables considered) is very small (less than ± 0.02) for all variables, but varies significantly for RSA ($x_{2,error}$ and $x_{3,error}$); for instance, for $V_{d2,error}$, the median ranges from about -0.50 (safe) at roof to +0.13 (unsafe) at first (for $V_{d3,error}$, -0.28 and +0.08); that is, RSA can largely overestimate the maximum response (up to 50% of the exact value with SRSS and 28% with CQC), while the underestimation is relatively small (up to 13% of the exact value with SRSS and 8% with CQC); these differences are larger in static variables (shear forces) than kinematic variables (displacements) and are attributed to the inherent inaccuracy of both combination rules (SRSS and CQC).
- For THMA the median error is close to 0 for all storeys and variables; for RSA the median error is more erratic, but generally safe (negative) in the roof and average through storeys, and unsafe (positive) in the first storey; the minimum error (response overestimation) offers a large variability with analysis type and variable.
- As seen in Figure 3-5, for THMA the minimum, median and maximum of all variables averaged through all storeys ('average') remains basically constant, irrespective of T_1 ; however, when RSA is used, the median and minimum clearly tend to increase as T_1 increases, providing more conservative values.

Table 3-3. Summary of error results for Δ .

Variable	Case	Roof			Average			First		
		max	med	min	max	med	min	max	med	min
$\Delta_{1,error}$	B0%	0.79	0.00	-2.78	0.30	0.01	-0.25	0.41	-0.01	-3.00
	B3%	0.78	0.00	-2.81	0.28	0.01	-0.26	0.40	-0.01	-3.02
	B5%	0.78	0.00	-2.84	0.26	0.01	-0.26	0.40	-0.01	-3.02
	C0%	0.80	0.00	-2.81	0.35	0.01	-0.25	0.40	-0.01	-2.88
	C3%	0.79	0.00	-2.85	0.32	0.01	-0.26	0.40	-0.01	-2.88
	C5%	0.78	0.00	-2.87	0.31	0.01	-0.27	0.39	-0.01	-2.88
$\Delta_{2,error}$	B0%	0.59	-0.18	-4.36	0.30	-0.02	-0.52	0.57	0.05	-1.78
	B3%	0.56	-0.21	-4.47	0.27	-0.03	-0.55	0.57	0.06	-1.76
	B5%	0.53	-0.23	-4.55	0.25	-0.03	-0.56	0.57	0.07	-1.75
	C0%	0.64	-0.15	-4.10	0.33	-0.01	-0.48	0.55	0.05	-1.79
	C3%	0.60	-0.18	-4.21	0.31	-0.02	-0.50	0.55	0.06	-1.77
	C5%	0.58	-0.19	-4.29	0.29	-0.02	-0.52	0.55	0.06	-1.75
$\Delta_{3,error}$	B0%	0.68	-0.08	-3.16	0.30	0.00	-0.30	0.55	0.03	-1.97
	B3%	0.65	-0.09	-3.24	0.27	0.00	-0.32	0.55	0.04	-1.92
	B5%	0.64	-0.10	-3.29	0.26	0.00	-0.33	0.55	0.04	-1.91
	C0%	0.73	-0.06	-3.09	0.34	0.01	-0.29	0.50	0.02	-2.25
	C3%	0.71	-0.07	-3.15	0.32	0.00	-0.31	0.50	0.03	-2.28
	C5%	0.69	-0.08	-3.18	0.30	0.00	-0.33	0.50	0.03	-2.26

Table 3-4. Summary of error mean and standard deviation for all variables considered.

		Roof			Average			First		
		max	med	min	max	med	min	max	med	min
$u_{1,error}$	Mean	0.46	0.00	-0.06	0.30	0.00	-0.43	0.40	-0.01	-2.95
	Std	0.06	0.00	0.00	0.01	0.00	0.02	0.00	0.00	0.07
$u_{2,error}$	Mean	0.45	-0.01	-0.14	0.36	0.02	-0.20	0.56	0.06	-1.77
	Std	0.06	0.00	0.01	0.01	0.00	0.02	0.01	0.01	0.02
$u_{3,error}$	Mean	0.46	0.00	-0.11	0.36	0.02	-0.26	0.53	0.03	-2.09
	Std	0.06	0.00	0.01	0.01	0.00	0.04	0.03	0.01	0.18
$v_{1,error}$	Mean	0.47	0.00	-0.11	0.32	0.00	-0.44	0.41	-0.02	-2.98
	Std	0.04	0.00	0.00	0.01	0.00	0.03	0.01	0.00	0.16
$v_{2,error}$	Mean	0.45	-0.05	-0.36	0.37	0.04	-0.20	0.65	0.13	-1.69
	Std	0.04	0.01	0.01	0.01	0.00	0.01	0.00	0.01	0.05
$v_{3,error}$	Mean	0.46	-0.03	-0.34	0.37	0.03	-0.26	0.55	0.09	-2.16
	Std	0.04	0.01	0.02	0.01	0.00	0.02	0.01	0.01	0.16
$\Delta_{1,error}$	Mean	0.79	0.00	-2.82	0.31	0.01	-0.26	0.40	-0.01	-2.95
	Std	0.01	0.00	0.03	0.03	0.00	0.01	0.00	0.00	0.07
$\Delta_{2,error}$	Mean	0.59	-0.19	-4.31	0.30	-0.02	-0.52	0.56	0.06	-1.77
	Std	0.04	0.03	0.19	0.04	0.01	0.04	0.01	0.01	0.02
$\Delta_{3,error}$	Mean	0.69	-0.08	-3.17	0.30	0.00	-0.31	0.53	0.03	-2.09
	Std	0.04	0.02	0.08	0.04	0.00	0.02	0.03	0.01	0.18
$\nabla_{1,error}$	Mean	0.85	0.00	-3.75	0.30	0.01	-0.42	0.41	-0.02	-2.98
	Std	0.01	0.00	0.13	0.02	0.00	0.03	0.01	0.00	0.16
$\nabla_{2,error}$	Mean	0.60	-0.52	-10.45	0.29	-0.11	-1.59	0.65	0.13	-1.69
	Std	0.03	0.11	1.11	0.02	0.03	0.19	0.00	0.01	0.05
$\nabla_{3,error}$	Mean	0.70	-0.27	-5.41	0.30	-0.04	-0.83	0.55	0.09	-2.16
	Std	0.03	0.06	0.20	0.02	0.02	0.10	0.01	0.01	0.16
$V_{d1,error}$	Mean	0.70	-0.03	-3.72	0.27	0.00	-0.50	0.40	-0.02	-2.93
	Std	0.14	0.03	0.11	0.04	0.00	0.06	0.02	0.00	0.18
$V_{d2,error}$	Mean	0.62	-0.50	-10.31	0.28	-0.10	-1.58	0.64	0.13	-1.65
	Std	0.17	0.07	1.03	0.06	0.03	0.18	0.02	0.00	0.08
$V_{d3,error}$	Mean	0.65	-0.28	-5.40	0.28	-0.05	-0.83	0.54	0.08	-2.12
	Std	0.17	0.05	0.21	0.05	0.02	0.11	0.02	0.00	0.18
$V_{t1,error}$	Mean	0.80	0.00	-3.72	0.26	0.01	-0.41	0.39	-0.01	-2.92
	Std	0.01	0.00	0.11	0.01	0.00	0.04	0.01	0.00	0.18
$V_{t2,error}$	Mean	0.59	-0.16	-10.21	0.34	0.05	-1.43	0.67	0.21	-1.63
	Std	0.04	0.04	1.07	0.02	0.01	0.17	0.03	0.05	0.15
$V_{t3,error}$	Mean	0.69	-0.02	-5.26	0.33	0.08	-0.68	0.57	0.16	-2.13
	Std	0.03	0.01	0.19	0.01	0.02	0.08	0.03	0.04	0.25

Note: Errors for V_s are the same as those for Δ .

Figure 3-5. Average of errors through storeys for variables u , Δ , V_s , V_d as a function of parameter T_1 ; (a), (c), (e), (g) errors for THMA; (b), (d), (f), (h) errors for RSA.

3.4.2 Identification of significant parameters and error bound

Figure 3-6 displays the maximum, median and minimum errors for drift in THMA, averaged through all storeys ($\Delta_{1,error,average}$), vs. each of the parameters considered in this study (except T_1 , already shown in Figure 3-5), and for all cases analyzed. The plots show that the median of $\Delta_{1,error,average}$ is very close to zero in all cases (between 0 and +0.05), meaning that, on average, the response obtained with THMA is not affected by pruning (retaining only the diagonal elements) of the modal damping matrix $\mathbf{C}$. For THMA, the maximum and minimum average of errors is between -0.27 and +0.35, approximately (see Table 3-3). For the maximum $\Delta_{1,error,average}$, largest variability happens with α_d , Figure 3-6(m). For the minimum $\Delta_{1,error,average}$, the maximum variability happens with α_k , Figure 3-6(g). These results can be extended to other variables (see Table 3-4).

The variability of the maximum variable error with respect to a certain parameter can be measured by its Coefficient of Variation (CV) calculated as follows: the error results are sorted according to the chosen parameter, and the maximum value for each group is preserved; the sample standard deviation is then divided over the sample mean. As an example, the CV for site class B and $\xi_l = 0\%$ for the maximum and minimum $\Delta_{1,error}$ is given in Table 3-5, where the CV with respect to α_d is found to be 0.32; this value is obtained from the four points plotted in Figure 3-6(m) for B0%, namely 0.3045 ($\alpha_d = 0$), 0.1381 ($\alpha_d = 1/3$), 0.2018 ($\alpha_d = 2/3$) and 0.2404 ($\alpha_d = 1$); the mean of the values is 0.2212 and the sample standard deviation is 0.0697, whereupon the sample $CV = 0.3153$ (rounded to 0.32 in Table).

For each THMA response variable examined, Table 3-6 shows the parameters with the maximum and second maximum CV ; they are identified as ‘most relevant’ (column labeled ‘1st’) and ‘2nd most relevant’ (column labeled ‘2nd’), for the first storey (rows labeled ‘First’), roof (rows labeled ‘Roof’), and average through storeys (rows labeled ‘Average’). Similar tables, not shown for brevity, are produced for RSA-SRSS and RSA-CQC. In general terms, the most important parameters for all errors are α_d and α_k , followed closely by ξ_{a1} . In contrast, T , N , β_k and β_d are of little consequence for the error variability.

The maximum error is then binned according to the most relevant parameters for each variable and tabulated in Table 3-7 (THMA), Table 3-8 (RSA-SRSS) and Table 3-9 (RSA-CQC). For each variable and storey a sub-table is produced, using the relevant parameters identified as entries, but leaving out cases for which the chosen parameters were not relevant; for instance, for $\nabla_{1,error,roof}$, Table 3-6 indicates that the most relevant parameters are α_d (6 cases) and T_1 (4 cases), so these parameters are chosen; however, for two cases (B3% and B5%), the relevant parameters are α_d and ξ_{a1} ; for this reason, these two cases are left out for the calculation of the sub-table values; this is indicated with data in *italics* and a foot note specifying the cases left out. This situation happens for 2 of the 21 tables in THMA, 9 of the 21 tables in RSA-SRSS and 3 of the 21 tables in RSA-CQC; this is consistent with the larger expected variability in RSA-SRSS results. In these tables,

variable v is left out for brevity, and variable V_s is omitted because, as mentioned, the error results for this variable are identical to those for Δ . A negative value in the table indicates that the method is always conservative for the corresponding variable and values of parameters.

These tables can be directly used to approximate the underestimation errors incurred by THMA and RSA for each response variable investigated; for example, in a system with $\alpha_k = 0.8$ and $\xi_{a,1} = 10\%$ (10% added damping in the first mode), THMA produces a maximum displacement error in the first storey $u_{1,error,1,max} \approx 0\%$, whereas the same error for RSA-SRSS is $u_{2,error,1,max} \approx 16\%$, and for RSA-CQC $u_{3,error,1,max} \approx 10\%$. For a given system equipped with added viscous damping, these tables indicate if the THMA or RSA procedures are suitable to keep the underestimation error at a variable under a certain threshold. A typical acceptable value is 10-20% (Guyader & Iwan, 2006; Ramirez et al, 2001). For instance, if RSA-SRSS is used, a proportion of roof stiffness to first storey stiffness $k_N/k_1 \geq 0.8$ keeps the error in first storey below 0.26 for displacement, drift and shear force, 0.39 for drift-velocity, and 0.38 for damper force. Conversely, additional safety factors can be derived from the tables to correct the predicted deviations. For instance, in the case discussed, the first storey displacement obtained from RSA-SRSS could be factored by $1/(1-0.26) = 1.35$ to obtain a safe value.

The tables show that the THMA procedure generally renders acceptable results, except for very specific values of parameters that should be avoided; particularly an incomplete damper distribution in height (characterized by $\alpha_d = 0$) is prone to produce a larger underestimation in all variables for roof. For the first storey, however, more conservative results are obtained when the damper distribution is not complete, or if complete, when it follows the frame stiffness distribution (α_k similar in value to α_d). This proves the difficulty of proposing simple rules involving the parameters considered in order to keep the error under control, and suggests the need to find alternative criteria; this issue is addressed in the next Section.

As for the RSA-SRSS and RSA-CQC procedures, the values of variables at first storey are systematically under-predicted, more so when the frame reduction of stiffness with height is conspicuous (α_k small). For the roof response, incomplete damper distributions ($\alpha_d = 0$) produce the un-safest results. For the average response, the trends are more erratic; for drift and structural shear, incomplete distributions and high values of added damping ratio result in underestimated response; for drift-velocity and damper force, this happens with short periods and high values of added damping ratio.

On the other side, Table 3-7 to Table 3-9 show that very conservative estimates can be obtained; for instance, considering Δ : for THMA, up to -2.95 in the first storey, -2.82 in the roof and -0.26 in the average through storeys; for RSA-SRSS, up to -1.77 in the first storey, -4.31 in the roof and -0.52 in the average through storeys; for RSA-CQC, -2.09 in the first storey, -3.17 in the roof and -0.31 in the average through storeys; for other variables, even more conservative results can be obtained. Conservativeness is a minor

issue in simplified methods; for this reason a detailed discussion (which could be done with the same rationale used for the unsafe results) is left out of this study.

Table 3-5. Coefficient of variation (CV) of $\Delta_{1,error}$, for case B0%.

$\Delta_{1,error}$	Maximum error			Minimum error		
	Roof	Average	First	Roof	Average	First
T_1	0.11	0.08	0.09	0.25	0.31	0.37
ξ_{a1}	0.17	0.22	0.26	0.66	0.67	0.39
N	0.02	0.06	0.05	0.61	0.21	0.27
α_k	0.02	0.02	0.63	1.00	0.78	0.12
β_k	0.02	0.04	0.07	0.45	0.19	0.09
α_d	1.01	0.32	0.36	0.77	0.32	1.40
β_d	0.05	0.12	0.00	0.00	0.00	0.82

Table 3-6. Most and second most significant parameters for error variability.

Case		B0%		B3%		B5%		C0%		C3%		C5%	
Variable	Storey	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd	1st	2nd
$u_{1,error}$	roof	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}
	average	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d
	first	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d
$v_{1,error}$	roof	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}
	average	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d
	first	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d
$\Delta_{1,error}$	roof	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}
	average	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}
	first	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d
$V_{1,error}$	roof	α_d	T_1	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	T_1	α_d	T_1	α_d	T_1
	average	α_d	T_1	α_d	T_1	α_d	T_1	α_d	T_1	α_d	T_1	α_d	T_1
	first	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d
$V_{s1,error}$	roof	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}
	average	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}
	first	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d
$V_{d1,error}$	roof	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}
	average	α_d	T_1	ξ_{a1}	α_d	ξ_{a1}	α_d	T_1	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d
	first	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d
$V_{t1,error}$	roof	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}
	average	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}	α_d	ξ_{a1}
	first	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d	α_k	α_d

1st: most significant parameter for error variability; 2nd: 2nd most significant

Figure 3-6. Average of error through storeys for Δ_1 as a function of: (a) N ; (b) ξ_{a1} ; (c) α_k ; (d) β_k ; (e) α_d ; (f) β_d .

Table 3-7. Time-history modal analysis (THMA). Maximum error (error 1) sorted by relevant parameters.

	Roof					Average					First				
	ξ_{a1}					α_d					α_d				
	α_d	0.1	0.2	0.3	0.4	α_k	0	1/3	2/3	1	α_k	0	1/3	2/3	1
$u_{1,error}$	0	0.16	0.33	0.45	0.53	0.2	0.08	0.15	0.27	0.32	0.2	0.16	0.29	0.36	0.41
	1/3	0.02	0.05	0.07	0.08	0.4	0.10	0.04	0.09	0.14	0.4	0.06	0.15	0.22	0.27
	2/3	0.03	0.08	0.12	0.14	0.6	0.11	0.00	0.03	0.07	0.6	0.01	0.05	0.11	0.16
	1	0.04	0.11	0.15	0.17	0.8	0.12	0.00	0.00	0.02	0.8	0.00	0.00	0.03	0.07
$\Delta_{1,error}$	ξ_{a1}					ξ_{a1}					α_d				
	α_d	0.1	0.2	0.3	0.4	α_d	0.1	0.2	0.3	0.4	α_k	0	1/3	2/3	1
	0	0.55	0.69	0.76	0.80	0	0.18	0.25	0.31	0.35	0.2	0.16	0.29	0.36	0.41
	1/3	0.14	0.24	0.34	0.41	1/3	0.10	0.11	0.13	0.15	0.4	0.06	0.15	0.22	0.27
	2/3	0.04	0.08	0.12	0.15	2/3	0.11	0.15	0.20	0.22	0.6	0.01	0.05	0.11	0.16
	1	0.01	0.02	0.01	-0.01	1	0.13	0.18	0.24	0.27	0.8	0.00	0.00	0.03	0.07
$\nabla_{1,error}$	T_1 (s)					T_1 (s)					α_d				
	α_d	0.5	1	2	4	α_d	0.5	1	2	4	α_k	0	1/3	2/3	1
	0	0.7	0.8	0.8	0.9	0	0.33	0.28	0.26	0.30	0.2	0.15	0.29	0.37	0.42
	1/3	0.3	0.4	0.4	0.5	1/3	0.15	0.14	0.13	0.19	0.4	0.07	0.15	0.22	0.28
	2/3	0.1	0.1	0.2	0.2	2/3	0.22	0.20	0.14	0.18	0.6	0.00	0.05	0.11	0.17
	1	0	0	0	0.1	1	0.27	0.24	0.17	0.19	0.8	-0.01	0.00	0.02	0.08
<i>*B3%, B5% excluded</i>															
$V_{d1,error}$	ξ_{a1}					α_d					α_d				
	α_d	0.1	0.2	0.3	0.4	ξ_{a1}	0	1/3	2/3	1	α_k	0	1/3	2/3	1
	0	0.60	0.76	0.70	0.83	0.1	0.1	0.1	0.1	0.1	0.2	0.15	0.29	0.37	0.42
	1/3	0.20	0.32	0.41	0.47	0.2	0.2	0.1	0.1	0.2	0.4	0.07	0.15	0.22	0.28
	2/3	0.08	0.11	0.15	0.17	0.3	0.2	0.1	0.2	0.2	0.6	0.00	0.05	0.11	0.17
	1	0.06	0.02	0.01	-0.02	0.4	0.2	0.1	0.2	0.2	0.8	-0.01	0.00	0.02	0.08
<i>*B0%, C0% excluded</i>															
$V_{l1,error}$	ξ_{a1}					ξ_{a1}					α_d				
	α_d	0.1	0.2	0.3	0.4	α_d	0.1	0.2	0.3	0.4	α_k	0	1/3	2/3	1
	0	0.55	0.69	0.76	0.80	0	0.15	0.24	0.26	0.27	0.2	0.15	0.28	0.36	0.41
	1/3	0.15	0.28	0.38	0.44	1/3	0.12	0.12	0.13	0.14	0.4	0.06	0.14	0.22	0.27
	2/3	0.04	0.10	0.14	0.16	2/3	0.13	0.15	0.19	0.21	0.6	0.01	0.04	0.11	0.16
	1	0.02	0.02	0.01	-0.03	1	0.14	0.18	0.23	0.26	0.8	0.00	0.00	0.02	0.07

Table 3-8. Response Spectrum Analysis with SRSS rule (RSA-SRSS). Maximum error (error 2) sorted by relevant parameters.

	Roof					Average					First				
$u_{2,error}$	ξ_{a1}					α_d					ξ_{a1}				
	α_d	0.1	0.2	0.3	0.4	α_k	0	1/3	2/3	1	α_k	0.1	0.2	0.3	0.4
	0	0.15	0.31	0.44	0.51	0.2	0.14	0.23	0.32	0.37	0.2	0.39	0.48	0.53	0.57
	1/3	0.02	0.03	0.05	0.06	0.4	0.14	0.10	0.14	0.18	0.4	0.28	0.37	0.41	0.44
	2/3	0.03	0.05	0.07	0.09	0.6	0.14	0.06	0.08	0.10	0.6	0.21	0.28	0.32	0.34
	1	0.04	0.07	0.10	0.13	0.8	0.14	0.04	0.05	0.07	0.8	0.16	0.21	0.24	0.26
$\Delta_{2,error}$	ξ_{a1}					ξ_{a1}					ξ_{a1}				
	α_d	0.1	0.2	0.3	0.4	α_d	0.1	0.2	0.3	0.4	α_k	0.1	0.2	0.3	0.4
	0	0.39	0.51	0.59	0.64	0	0.19	0.25	0.30	0.33	0.2	0.39	0.48	0.53	0.57
	1/3	0.06	0.08	0.11	0.16	1/3	0.07	0.11	0.13	0.13	0.4	0.28	0.37	0.41	0.44
	2/3	0.03	-0.02	-0.04	-0.06	2/3	0.10	0.16	0.20	0.20	0.6	0.21	0.28	0.32	0.34
	1	0.04	-0.04	-0.14	-0.20	1	0.12	0.19	0.23	0.24	0.8	0.16	0.21	0.24	0.26
$V_{2,error}$	T_1 (s)					α_d					β_k				
	α_d	0.5	1	2	4	T_1 (s)	0	1/3	2/3	1	α_k	0.5	1	2	10
	0	0.62	0.61	0.54	0.59	0.5	0.29	0.25	0.18	0.21	0.2	0.65	0.58	0.59	0.65
	1/3	0.17	0.07	0.10	0.14	1	0.22	0.08	0.07	0.09	0.4	0.54	0.48	0.47	0.54
	2/3	0.02	0.0	-0.1	-0.1	2	0.20	0.11	0.02	0.0	0.6	0.45	0.42	0.39	0.45
	1	0.01	-0.1	-0.2	-0.2	4	0.22	0.14	0.04	-0.2	0.8	0.39	0.37	0.36	0.38
* C0%, C5% excluded					* B0%, C0%, C5% excluded					* C0%, C5% excluded					
$V_{d2,error}$	T_1 (s)					ξ_{a1}					T_1 (s)				
	α_d	0.5	1	2	4	T_1 (s)	0.1	0.2	0.3	0.4	α_k	0.5	1	2	4
	0	0.49	0.43	0.36	0.17	0.5	0.10	0.18	0.22	0.24	0.2	0.63	0.56	0.59	0.65
	1/3	0.14	0.10	-0.1	-0.5	1	0.09	0.15	0.18	0.19	0.4	0.52	0.46	0.47	0.54
	2/3	0.01	-0.1	-0.3	-0.8	2	0.08	0.11	0.12	0.12	0.6	0.43	0.39	0.37	0.45
	1	0.0	-0.1	-0.3	-0.8	4	0.0	0.02	0.05	0.08	0.8	0.36	0.34	0.33	0.38
* B0%, C0% excluded					* B0%, C0% excluded					* B0%, C5% excluded					
$V_{t2,error}$	ξ_{a1}					α_d					T_1 (s)				
	α_d	0.1	0.2	0.3	0.4	ξ_{a1}	0	1/3	2/3	1	α_k	0.5	1	2	4
	0	0.39	0.51	0.59	0.64	0.1	0.25	0.23	0.24	0.24	0.2	0.48	0.55	0.64	0.70
	1/3	0.14	0.17	0.22	0.28	0.2	0.30	0.26	0.30	0.32	0.4	0.40	0.46	0.55	0.60
	2/3	0.17	0.18	0.12	0.14	0.3	0.33	0.28	0.30	0.32	0.6	0.37	0.42	0.47	0.52
	1	0.17	0.19	0.12	0.01	0.4	0.35	0.25	0.30	0.31	0.8	0.35	0.40	0.40	0.45
* C0%, C5% excluded					* C0%, C5% excluded					* B0%, C0%, C5% excluded					

Table 3-9 Response Spectrum Analysis with CQC rule (RSA-CQC). Maximum error (error 3) sorted by relevant parameters.

	Roof					Average					First				
$U_{3,error}$	ξ_{a1}					α_d					ξ_{a1}				
	α_d	0.1	0.2	0.3	0.4	α_k	0	1/3	2/3	1	α_k	0.1	0.2	0.3	0.4
	0	0.11	0.26	0.38	0.45	0.2	0.13	0.22	0.32	0.36	0.2	0.30	0.42	0.48	0.53
	1/3	0.02	0.03	0.05	0.07	0.4	0.10	0.09	0.14	0.17	0.4	0.22	0.31	0.36	0.39
	2/3	0.02	0.04	0.08	0.10	0.6	0.09	0.06	0.07	0.10	0.6	0.15	0.22	0.26	0.28
	1	0.03	0.06	0.10	0.14	0.8	0.09	0.03	0.04	0.06	0.8	0.10	0.16	0.20	0.21
$\Delta_{3,error}$	ξ_{a1}					ξ_{a1}					ξ_{a1}				
	α_d	0.1	0.2	0.3	0.4	α_d	0.1	0.2	0.3	0.4	α_k	0.1	0.2	0.3	0.4
	0	0.41	0.56	0.62	0.68	0	0.16	0.22	0.27	0.30	0.2	0.30	0.42	0.48	0.53
	1/3	0.06	0.11	0.19	0.25	1/3	0.07	0.11	0.14	0.15	0.4	0.22	0.31	0.36	0.39
	2/3	0.01	0.02	0.05	0.07	2/3	0.10	0.16	0.19	0.21	0.6	0.15	0.22	0.26	0.28
	1	0.00	-0.04	-0.07	-0.10	1	0.11	0.18	0.22	0.24	0.8	0.10	0.16	0.20	0.21
$\nabla_{3,error}$	ξ_{a1}					ξ_{a1}					α_d				
	α_d	0.1	0.2	0.3	0.4	α_d	0.1	0.2	0.3	0.4	α_k	0	1/3	2/3	1
	0	0.41	0.56	0.64	0.69	0	0.16	0.23	0.27	0.30	0.2	0.38	0.47	0.51	0.55
	1/3	0.08	0.12	0.19	0.26	1/3	0.07	0.12	0.15	0.16	0.4	0.31	0.37	0.40	0.44
	2/3	0.02	0.02	0.03	0.09	2/3	0.09	0.16	0.20	0.22	0.6	0.27	0.30	0.33	0.35
	1	0.00	-0.1	-0.1	-0.1	1	0.11	0.19	0.23	0.25	0.8	0.22	0.25	0.27	0.29
$V_{d3,error}$	ξ_{a1}					ξ_{a1}					α_d				
	α_d	0.1	0.2	0.3	0.4	T_1 (s)	0.1	0.2	0.3	0.4	α_k	0	1/3	2/3	1
	0	0.29	0.44	0.50	0.60	0.5	0.09	0.17	0.21	0.24	0.2	0.38	0.47	0.51	0.54
	1/3	0.07	0.11	0.18	0.25	1	0.07	0.12	0.15	0.17	0.4	0.30	0.37	0.40	0.43
	2/3	0.02	0.02	0.03	0.08	2	0.06	0.09	0.10	0.11	0.6	0.26	0.29	0.32	0.34
	1	0.00	-0.1	-0.1	-0.1	4	0.0	0.04	0.07	0.08	0.8	0.21	0.24	0.26	0.28
* B0%, C0% excluded															
$V_{f3,error}$	ξ_{a1}					α_d					ξ_{a1}				
	α_d	0.1	0.2	0.3	0.4	ξ_{a1}	0	1/3	2/3	1	α_k	0.1	0.2	0.3	0.4
	0	0.41	0.56	0.63	0.68	0.1	0.16	0.17	0.16	0.22	0.2	0.37	0.42	0.44	0.46
	1/3	0.13	0.17	0.26	0.34	0.2	0.22	0.22	0.21	0.25	0.4	0.39	0.39	0.38	0.37
	2/3	0.12	0.12	0.17	0.22	0.3	0.26	0.30	0.20	0.30	0.6	0.40	0.39	0.37	0.36
	1	0.13	0.12	0.12	0.11	0.4	0.27	0.32	0.21	0.30	0.8	0.39	0.39	0.38	0.35
* B0%, B3%, B5% excluded															
* B5% excluded															

3.4.3 Correlation of error results with existing non-proportionality indices

In order to investigate the relationship between the damping non-proportionality indices described in Section 3.2.3 with the error estimates obtained in the previous Section, values of the indices are calculated for each system in the study. Index I_{PF1} is left out because of its numerical instability. Indices I_{A1} and I_{LKZ4} are very similar, showing a sample correlation coefficient $r(I_{LKZ4}; I_{A1}) = 0.9778$ (calculated including all systems analyzed in

this study). Two conditions indicate a good correlation of error and index: 1) the maximum error must increase steadily with increasing index value; 2) for index values close to 0 the error should be close to 0; limiting the index value will then result in an effective error limitation.

Figure 3-7 displays error $\Delta_{1,error,average}$ versus index value for all cases considered in the study. Figure 3-8 to Figure 3-15 display similar plots for $\Delta_{1,error}$, $\Delta_{2,error}$, $\Delta_{3,error}$ at average, first and roof. Every plot is composed of 21 subplots, one for each index analyzed; every subplot contains 6×16384 points, with different colors identifying different cases. All cases present similar trends. The plot shows that conditions 1) and 2) are met only by the following indices: I_{PS5} , I_{TLL1} , I_{LKZ4} , I_{AI} . Similar trends were identified for all THMA errors ($u_{1,error}$, $\Delta_{1,error}$, $\nabla_{1,error}$, $V_{s1,error}$, $V_{d1,error}$, $V_{t1,error}$) at average, first, and roof. For RSA errors ($u_{2,error}$, $\Delta_{2,error}$, $\nabla_{2,error}$, $V_{s2,error}$, $V_{d2,error}$, $V_{t2,error}$, $u_{3,error}$, $\Delta_{3,error}$, $\nabla_{3,error}$, $V_{s3,error}$, $V_{d3,error}$, $V_{t3,error}$) the results are similar, but not so clear in the trends; this is consistent with the inherent inaccuracies in response analysis, which are partially due to damping non-proportionality and partially due to use of combination rules. Focusing on the four mentioned indices, two of them (I_{PS5} , I_{TLL1}) are considered more suitable for practical design purposes, because they do not involve determination of complex modes; index I_{TLL1} involves one additional difficulty, namely determination of matrix eigenvalues; for this reason, index I_{PS5} is probably a more realistic choice for engineering practice. The envelope curve of the scatter plots shown in Figure 3-7 can be used to link the amount of error in a variable with the maximum index value; this is shown for $\Delta_{1,error}$ and indices I_{PS5} , I_{TLL1} , I_{LKZ4} , I_{AI} , in Figure 3-10; for $\Delta_{2,error}$ and $\Delta_{3,error}$ and index I_{PS5} , in Figure 3-17; and tabulated for all variables and index I_{PS5} in Table 3-10 (THMA), Table 3-11 (RSA-SRSS) and Table 3-12 (RSA-CQC). The 20% error threshold is highlighted.

From the data presented it follows that, if THMA (time-history modal analysis) is used:

- If all variables are examined, to keep the error under 20% in the roof response, the system should present a value of index $I_{PS5} \leq 0.11$; to keep the error under 20% in the average and first storey response, the system should present a value of index $I_{PS5} \leq 0.25$.
- If only drift Δ , structural shear V_s and damper shear V_d are examined (relevant variables for design), to keep the error under 20% for all variables in the roof response, the system should present a value of index $I_{PS5} \leq 0.12$; to keep the error under 20% in the average and first storey response, the system should present a value of index $I_{PS5} \leq 0.27$.

Likewise, if RSA (response history analysis) is used:

- If all variables except V_t are examined, to keep the error under 20% in the roof response, the system should present a value of index $I_{PS5} \leq 0.17$ for RSA-SRSS and 0.16 for RSA-CQC; to keep the error under 20% in the first storey and average response, the system should present a value of index $I_{PS5} \leq 0.11$ (both for SRSS and CQC).

- For variable V_{t2} , a maximum error of 20% can only be reached for proportionally-damped systems.
- If only drift Δ , structural shear V_s and damper shear V_d are examined (relevant variables for design), to keep the error under 20% for all variables in the roof response, the system should present a value of index $I_{PS5} \leq 0.16$; to keep the error under 20% in the average and first storey response, the system should present a value of index $I_{PS5} \leq 0.27$.

The results are consistent with RSA being a more conservative method than THMA, and RSA-CQC being less conservative than RSA-SRSS, whereupon restrictions on damping proportionality can be relaxed to keep the method conservative. This is true for almost all variables except velocity v , drift velocity ∇ and total shear V_t . It must be kept in mind that is not the accuracy of the method that is being examined, but its capacity to produce conservative results when a proportional simplification of the damping matrix is used.

3.5 Summary and conclusions

The addition of linear viscous dampers to frame structures, generally results in non-proportional damping matrices. A common practice to obtain the response of such non-proportionally damped systems is to use an approximate diagonalizable damping matrix constructed retaining only the diagonal terms of the modal damping matrix, and applying classical modal analysis. This study focuses on measurement of the un-conservative errors (underestimation of results) incurred with this approximation, and the relation with several indices proposed in the literature to measure the extent of damping non-proportionality. Two procedures based on modal analysis are considered: time-history modal analysis (THMA), where modal time history responses are added to obtain the total time-history response), and response spectrum analysis (RSA), where peak modal values are found and combined with either the SRSS rule (RSA-SRSS) or the CQC rule (RSA-CQC). The errors for both procedures are estimated through a parametric study that leads to the following conclusions:

1. The errors are mildly dependent on site-class and inherent damping, and depend mostly on the added damping and stiffness distributions through the height of the structure, and on the amount of added damping ratio in the first mode.
2. Complete distributions of dampers (dampers installed in all storeys) are not a safeguard against underestimation of the response with neither THMA nor RSA; incomplete distributions might lead to more accurate results of the methods for certain storeys, depending on other frame parameters (especially, but not only, the distribution of stiffness).
3. To keep the error under a certain threshold in THMA and RSA, simple criteria based on frame and damping system properties cannot be proposed from the data available, as parameters that are significant for the response averaged through storeys, first and roof,

are generally different. Moreover, values of parameters that reduce the response error in one storey might trigger a larger error in another.

4. As an alternative criterion to control error, four indices to measure the extent of damping non-proportionality are found to be significantly correlated with the maximum error discussed; among them, the one proposed by Prater and Singh, I_{PS5} , is recommended for practical use, due to its simplicity and reasonable correlation with errors. It is found that values of $I_{PS5} \leq 0.12$ in case of using THMA or $I_{PS5} \leq 0.16$ in case of RSA, keep the un-conservative error of the most relevant variables for design (i.e. inter-storey drift, structural storey shear and damper storey shear) under 20%.

These results are only valid if the system parameters (number of storeys, natural period, viscous damping ratio, stiffness and damping distributions), are kept within the bounds considered in this study. Particularly, only height-wise decreasing stiffness and damping configurations were considered. Only elastic systems have been considered. No near-fault or soft ground motions have been considered. No modal truncation was applied, and the spectral response was exact (within the accuracy of numerical methods), so these two possible sources of error were not considered.

3.6 Original contributions in this Chapter

- The influence of relevant structural parameters (number of storeys, natural period, added damping in the first mode, stiffness distribution, damper distribution) in the errors incurred using modal analysis (time history analysis THMA or response spectrum analysis RSA) has been quantified for common cases.
- The most relevant parameters for each response variable of interest (displacement, drift, velocity, drift velocity, design shear, damper shear, total shear) in three different storeys (roof, average and first) have been identified through statistical analysis for both types of analysis.
- The maximum expected error for each response variable of interest as a function of the two most relevant parameters for the variable considered, has been tabulated. These tables can be used directly to assess the maximum expected error for each variable.
- The inadequacy of a complete vertical damper distribution to limit the maximum error consistently in all storeys is demonstrated.
- A correlation of four non-proportionality indices present in the literature (among twenty-one indices examined) with the aforementioned maximum errors has been proved.
- Maximum values of one index (I_{PS5}) as a function of the desired maximum error are tabulated and can be used as a substitute of other simplified rules to keep the maximum error under a certain threshold.

Figure 3-7. Index values vs. error $\Delta I_{\text{error,average}}$ for all cases.

Figure 3-8. Index values vs. error $\Delta_{I,error,first}$ for all cases.

Figure 3-9. Index values vs. error $\Delta I_{\text{error,roof}}$ for all cases.

Figure 3-10. Index values vs. error $\Delta_{2,error,average}$ for all cases.

Figure 3-11. Index values vs. error $\Delta_{2,error,first}$ for all cases.

Figure 3-12. Index values vs. error $\Delta_{2,error,roof}$ for all cases.

Figure 3-13. Index values vs. error $\Delta_{3,error,average}$ for all cases.

Figure 3-14. Index values vs. error $\Delta_{3,error,first}$ for all cases.

Figure 3-15. Index values vs. error $\Delta_{3,error,roof}$ for all cases.

Figure 3-16. Indices I_{A1} (a-b-c), I_{LKZ4} (d-e-f), I_{PS5} (g-h-i), I_{TLL1} (j-k-l) vs. maximum values of $\Delta_{1,error}$ for roof, average and first.

Figure 3-17. Index I_{PS5} vs. maximum values of $\Delta_{2,error}$ and $\Delta_{3,error}$ for roof, average and first.

Table 3-10. Maximum values of index I_{PSS} for target error of all variables using THMA.

Storey	Error	u_{x1}	v_{x1}	Δ_{x1}	∇_{x1}	$V_{s,x1}$	$V_{d,x1}$	$V_{t,x1}$
Roof	0%	0.04	0.04	0.05	0.05	0.05	0.05	0.05
	5%	0.17	0.13	0.05	0.05	0.05	0.05	0.05
	10%	0.17	0.17	0.05	0.05	0.05	0.05	0.05
	15%	0.18	0.17	0.09	0.09	0.09	0.09	0.09
	20%	0.18	0.18	0.15	0.11	0.15	0.12	0.15
	25%	0.18	0.18	0.16	0.15	0.16	0.15	0.15
	30%	0.19	0.19	0.17	0.16	0.17	0.16	0.16
Average	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.10	0.10	0.11	0.11	0.11	0.11	0.11
	10%	0.11	0.11	0.13	0.13	0.13	0.13	0.13
	15%	0.24	0.24	0.25	0.24	0.25	0.24	0.26
	20%	0.25	0.25	0.27	0.27	0.27	0.28	0.28
	25%	0.28	0.27	0.34	0.34	0.34	0.35	0.35
	30%	0.34	0.34	0.59	0.61	0.59	0.70	0.77
First	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.10	0.10	0.11	0.11	0.11	0.11	0.11
	10%	0.11	0.11	0.13	0.13	0.13	0.13	0.13
	15%	0.24	0.24	0.25	0.24	0.25	0.24	0.26
	20%	0.25	0.25	0.27	0.27	0.27	0.28	0.28
	25%	0.28	0.27	0.34	0.34	0.34	0.35	0.35
	30%	0.34	0.34	0.59	0.61	0.59	0.70	0.77

Table 3-11. Maximum values of index I_{PSS} for target error of all variables using RSA-SRSS.

Storey	Error	u_{x2}	v_{x2}	Δ_{x2}	∇_{x2}	$V_{s,x2}$	$V_{d,x2}$	$V_{t,x2}$
Roof	0%	0.03	0.00	0.05	0.05	0.05	0.05	0.00
	5%	0.18	0.17	0.17	0.17	0.17	0.17	0.00
	10%	0.18	0.18	0.17	0.17	0.17	0.17	0.00
	15%	0.18	0.19	0.17	0.17	0.17	0.17	0.13
	20%	0.19	0.19	0.17	0.17	0.17	0.18	0.15
	25%	0.19	0.19	0.17	0.17	0.17	0.19	0.17
	30%	0.21	0.19	0.17	0.17	0.17	0.19	0.17
Average	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.03	0.03	0.10	0.10	0.10	0.10	0.00
	10%	0.10	0.10	0.13	0.13	0.13	0.13	0.00
	15%	0.11	0.11	0.23	0.26	0.23	0.26	0.00
	20%	0.13	0.11	0.33	0.34	0.33	0.34	0.00
	25%	0.25	0.25	0.53	0.57	0.53	0.54	0.06
	30%	0.26	0.26	0.57	0.61	0.57	0.60	0.30
First	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.03	0.03	0.10	0.10	0.10	0.10	0.00
	10%	0.10	0.10	0.13	0.13	0.13	0.13	0.00
	15%	0.11	0.11	0.23	0.26	0.23	0.26	0.00
	20%	0.13	0.11	0.33	0.34	0.33	0.34	0.00
	25%	0.25	0.25	0.53	0.57	0.53	0.54	0.06
	30%	0.26	0.26	0.57	0.61	0.57	0.60	0.30

Table 3-12. Maximum values of index I_{PS5} for target error of all variables using RSA-CQC.

Storey	Error	u_{x3}	v_{x3}	Δ_{x3}	∇_{x3}	$V_{s,x3}$	$V_{d,x3}$	$V_{t,x3}$
Roof	0%	0.00	0.00	0.04	0.00	0.04	0.00	0.00
	5%	0.17	0.17	0.06	0.05	0.06	0.05	0.00
	10%	0.17	0.18	0.09	0.09	0.09	0.09	0.00
	15%	0.18	0.19	0.16	0.16	0.16	0.16	0.00
	20%	0.19	0.19	0.17	0.16	0.17	0.16	0.06
	25%	0.19	0.19	0.17	0.17	0.17	0.18	0.06
	30%	0.21	0.19	0.17	0.17	0.17	0.19	0.17
Average	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.03	0.03	0.10	0.10	0.10	0.11	0.00
	10%	0.09	0.10	0.11	0.11	0.11	0.11	0.00
	15%	0.11	0.11	0.23	0.23	0.23	0.24	0.00
	20%	0.11	0.11	0.27	0.27	0.27	0.27	0.00
	25%	0.24	0.25	0.35	0.34	0.35	0.35	0.04
	30%	0.26	0.26	0.59	0.59	0.59	0.60	0.28
First	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.03	0.03	0.10	0.10	0.10	0.11	0.00
	10%	0.09	0.10	0.11	0.11	0.11	0.11	0.00
	15%	0.11	0.11	0.23	0.23	0.23	0.24	0.00
	20%	0.11	0.11	0.27	0.27	0.27	0.27	0.00
	25%	0.24	0.25	0.35	0.34	0.35	0.35	0.04
	30%	0.26	0.26	0.59	0.59	0.59	0.60	0.28

Table 3-13. Maximum values of index I_{A1} for target error of all variables using THMA.

Storey	Error	u_{x1}	v_{x1}	Δ_{x1}	∇_{x1}	$V_{s,x1}$	$V_{d,x1}$	$V_{t,x1}$
Roof	0%	0.02	0.02	0.02	0.02	0.02	0.02	0.02
	5%	0.12	0.12	0.05	0.02	0.05	0.02	0.05
	10%	0.21	0.21	0.06	0.06	0.06	0.06	0.05
	15%	0.21	0.24	0.07	0.07	0.07	0.07	0.07
	20%	0.26	0.26	0.17	0.14	0.17	0.15	0.15
	25%	0.27	0.27	0.17	0.17	0.17	0.17	0.17
	30%	0.31	0.31	0.21	0.22	0.21	0.23	0.21
Average	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.08	0.08	0.08	0.05	0.08	0.05	0.06
	10%	0.14	0.13	0.16	0.14	0.16	0.14	0.13
	15%	0.18	0.18	0.18	0.18	0.18	0.18	0.22
	20%	0.28	0.28	0.34	0.28	0.34	0.28	0.34
	25%	0.34	0.34	0.44	0.44	0.44	0.45	0.45
	30%	0.44	0.44	0.54	0.59	0.54	0.63	0.83
First	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.08	0.08	0.08	0.05	0.08	0.05	0.06
	10%	0.14	0.13	0.16	0.14	0.16	0.14	0.13
	15%	0.18	0.18	0.18	0.18	0.18	0.18	0.22
	20%	0.28	0.28	0.34	0.28	0.34	0.28	0.34
	25%	0.34	0.34	0.44	0.44	0.44	0.45	0.45
	30%	0.44	0.44	0.54	0.59	0.54	0.63	0.83

Table 3-14. Maximum values of index I_{A1} for target error of all variables using RSA-SRSS.

Storey	Error	u_{x2}	v_{x2}	Δ_{x2}	∇_{x2}	$V_{s,x2}$	$V_{d,x2}$	$V_{t,x2}$
Roof	0%	0.00	0.00	0.02	0.02	0.02	0.02	0.00
	5%	0.12	0.21	0.12	0.09	0.12	0.09	0.00
	10%	0.21	0.21	0.12	0.12	0.12	0.13	0.00
	15%	0.27	0.31	0.19	0.21	0.19	0.13	0.09
	20%	0.31	0.31	0.21	0.21	0.21	0.13	0.17
	25%	0.31	0.31	0.21	0.24	0.21	0.27	0.21
	30%	0.31	0.31	0.25	0.25	0.25	0.27	0.25
Average	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.03	0.02	0.04	0.04	0.04	0.04	0.00
	10%	0.08	0.06	0.09	0.11	0.09	0.11	0.00
	15%	0.13	0.13	0.18	0.18	0.18	0.18	0.00
	20%	0.17	0.15	0.34	0.34	0.34	0.34	0.04
	25%	0.28	0.28	0.44	0.48	0.44	0.44	0.06
	30%	0.34	0.34	0.50	0.59	0.50	0.58	0.28
First	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.03	0.02	0.04	0.04	0.04	0.04	0.00
	10%	0.08	0.06	0.09	0.11	0.09	0.11	0.00
	15%	0.13	0.13	0.18	0.18	0.18	0.18	0.00
	20%	0.17	0.15	0.34	0.34	0.34	0.34	0.04
	25%	0.28	0.28	0.44	0.48	0.44	0.44	0.06
	30%	0.34	0.34	0.50	0.59	0.50	0.58	0.28

Table 3-15. Maximum values of index I_{A1} for target error of all variables using RSA-CQC.

Storey	Error	u_{x3}	v_{x3}	Δ_{x3}	∇_{x3}	$V_{s,x3}$	$V_{d,x3}$	$V_{t,x3}$
Roof	0%	0.00	0.00	0.02	0.00	0.02	0.00	0.00
	5%	0.12	0.18	0.05	0.03	0.05	0.03	0.00
	10%	0.21	0.21	0.07	0.11	0.07	0.12	0.00
	15%	0.26	0.31	0.15	0.17	0.15	0.13	0.03
	20%	0.31	0.31	0.17	0.21	0.17	0.13	0.06
	25%	0.31	0.31	0.21	0.21	0.21	0.26	0.07
	30%	0.31	0.31	0.24	0.24	0.24	0.27	0.23
Average	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.03	0.02	0.04	0.04	0.04	0.04	0.00
	10%	0.07	0.06	0.09	0.11	0.09	0.11	0.00
	15%	0.12	0.09	0.18	0.18	0.18	0.18	0.00
	20%	0.14	0.15	0.28	0.28	0.28	0.28	0.03
	25%	0.28	0.28	0.45	0.44	0.45	0.42	0.04
	30%	0.34	0.34	0.53	0.54	0.53	0.58	0.28
First	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.03	0.02	0.04	0.04	0.04	0.04	0.00
	10%	0.07	0.06	0.09	0.11	0.09	0.11	0.00
	15%	0.12	0.09	0.18	0.18	0.18	0.18	0.00
	20%	0.14	0.15	0.28	0.28	0.28	0.28	0.03
	25%	0.28	0.28	0.45	0.44	0.45	0.42	0.04
	30%	0.34	0.34	0.53	0.54	0.53	0.58	0.28

Table 3-16. Maximum values of index I_{TLL1} for target error of all variables using THMA.

Storey	Error	THMA						
		u_{x1}	v_{x1}	Δ_{x1}	∇_{x1}	$V_{s,x1}$	$V_{d,x1}$	$V_{t,x1}$
Roof	0%	0.05	0.05	0.06	0.05	0.06	0.05	0.06
	5%	0.27	0.27	0.08	0.08	0.08	0.08	0.08
	10%	0.52	0.50	0.08	0.08	0.08	0.08	0.08
	15%	0.63	0.63	0.14	0.12	0.14	0.12	0.12
	20%	0.87	0.88	0.23	0.16	0.23	0.16	0.16
	25%	0.90	0.90	0.24	0.23	0.24	0.23	0.23
	30%	0.93	0.92	0.28	0.27	0.28	0.27	0.27
Average	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.19	0.19	0.22	0.20	0.22	0.20	0.20
	10%	0.23	0.23	0.32	0.30	0.32	0.32	0.30
	15%	0.44	0.43	0.48	0.48	0.48	0.49	0.50
	20%	0.47	0.46	0.55	0.54	0.55	0.56	0.58
	25%	0.56	0.53	0.65	0.65	0.65	0.68	0.68
	30%	0.63	0.63	0.97	0.98	0.97	0.99	0.99
First	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.19	0.19	0.22	0.20	0.22	0.20	0.20
	10%	0.23	0.23	0.32	0.30	0.32	0.32	0.30
	15%	0.44	0.43	0.48	0.48	0.48	0.49	0.50
	20%	0.47	0.46	0.55	0.54	0.55	0.56	0.58
	25%	0.56	0.53	0.65	0.65	0.65	0.68	0.68
	30%	0.63	0.63	0.97	0.98	0.97	0.99	0.99

Table 3-17. Maximum values of index I_{TLL1} for target error of all variables using RSA-SRSS.

Storey	Error	u_{x2}	v_{x2}	Δ_{x2}	∇_{x2}	$V_{s,x2}$	$V_{d,x2}$	$V_{t,x2}$
Roof	0%	0.04	0.00	0.07	0.07	0.07	0.07	0.00
	5%	0.42	0.40	0.28	0.27	0.28	0.27	0.00
	10%	0.63	0.61	0.36	0.36	0.36	0.36	0.00
	15%	0.88	0.88	0.40	0.39	0.40	0.39	0.25
	20%	0.90	0.90	0.79	0.81	0.79	0.88	0.26
	25%	0.92	0.91	0.86	0.86	0.86	0.89	0.36
	30%	0.94	0.93	0.86	0.87	0.86	0.91	0.86
Average	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.04	0.04	0.19	0.19	0.19	0.21	0.00
	10%	0.19	0.16	0.35	0.35	0.35	0.35	0.00
	15%	0.20	0.20	0.50	0.50	0.50	0.51	0.00
	20%	0.32	0.27	0.61	0.63	0.61	0.65	0.00
	25%	0.45	0.45	0.94	0.96	0.94	0.99	0.08
	30%	0.52	0.50	0.98	0.99	0.98		0.60
First	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.04	0.04	0.19	0.19	0.19	0.21	0.00
	10%	0.19	0.16	0.35	0.35	0.35	0.35	0.00
	15%	0.20	0.20	0.50	0.50	0.50	0.51	0.00
	20%	0.32	0.27	0.61	0.63	0.61	0.65	0.00
	25%	0.45	0.45	0.94	0.96	0.94	0.99	0.08
	30%	0.52	0.50	0.98	0.99	0.98		0.60

Table 3-18. Maximum values of index I_{TLL1} for target error of all variables using RSA-CQC.

Storey	Error	u_{x3}	v_{x3}	Δ_{x3}	∇_{x3}	$V_{s,x3}$	$V_{d,x3}$	$V_{t,x3}$
Roof	0%	0.00	0.00	0.05	0.00	0.05	0.00	0.00
	5%	0.37	0.37	0.08	0.08	0.08	0.08	0.00
	10%	0.56	0.54	0.15	0.10	0.15	0.10	0.00
	15%	0.68	0.68	0.24	0.24	0.24	0.24	0.00
	20%	0.90	0.90	0.29	0.27	0.29	0.27	0.08
	25%	0.91	0.91	0.39	0.34	0.39	0.39	0.08
	30%	0.93	0.93	0.86	0.42	0.86	0.90	0.28
Average	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.05	0.04	0.18	0.19	0.18	0.20	0.00
	10%	0.19	0.18	0.26	0.26	0.26	0.27	0.00
	15%	0.20	0.20	0.48	0.48	0.48	0.48	0.00
	20%	0.22	0.27	0.54	0.53	0.54	0.55	0.00
	25%	0.45	0.47	0.68	0.65	0.68	0.68	0.05
	30%	0.50	0.50	0.97	0.98	0.97	0.99	0.56
First	0%	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	5%	0.05	0.04	0.18	0.19	0.18	0.20	0.00
	10%	0.19	0.18	0.26	0.26	0.26	0.27	0.00
	15%	0.20	0.20	0.48	0.48	0.48	0.48	0.00
	20%	0.22	0.27	0.54	0.53	0.54	0.55	0.00
	25%	0.45	0.47	0.68	0.65	0.68	0.68	0.05
	30%	0.50	0.50	0.97	0.98	0.97	0.99	0.56

Chapter 4

Construction of Elastic Spectra for High Damping

4.1 Introduction

Damping has a beneficial effect on the dynamic performance of conventional structures: an increase in damping results in a reduction of structural displacements, which in turn represents smaller internal forces associated with the dynamic excitation [Clough & Penzien, 1995, Chopra, 2007]. The inherent dissipation of energy in building structures behaving elastically is quite stable and limited to about 5% in terms of equivalent viscous damping ratio. Under a severe earthquake excitation, an increase in energy dissipation takes place through inelastic action in the structural elements, at the cost of structural cumulative damage. Energy dissipation capacity can also be increased by installing damping devices, leading to the so-called structures with Passive Energy Dissipation systems (PEDs). Structures with PEDs typically feature overall equivalent damping ratios above 10% in the first mode, and much larger values in the higher modes [Soong & Dargush, 1997; Filiatrault & Christopoulos, 2006]. This is the case of structures equipped with viscous dampers (VD).

The energy dissipation in a building structure equipped with VD under earthquake excitation presents three basic sources, namely i) inherent damping, ii) added viscous damping due to added dampers and iii) hysteretic damping due to energy dissipation through macroscopic plastic deformations (when the structure presents non-linear excursions beyond yielding). Inherent damping presents several sources including friction, material viscosity, microscopic plastic effects, heat radiation, aerodynamic effects, etc; however the usual unifying engineering treatment is through linear viscous damping [Tamura, 2013]. Probably for this reason, it is widely accepted to unify the mathematical treatment for different sources of energy dissipation as a unique equivalent (or effective) viscous damping.

This procedure has found acceptance in the engineering world and is the basis for some simplified analysis methods. This is the case of the substitute structure method [Gulkan & Sozen, 1974; Shibata & Sozen, 1974], and the capacity spectrum method [Freeman, 1998], in which the behavior of a yielding structure is predicted using a series of single-degree of

freedom (SDOF) systems with effective modal properties (effective stiffness k_{eff} , effective period T_{eff} , and effective viscous damping ratio ξ_{eff}). The effective properties account for all energy dissipation sources (inherent damping, post-yielding structural hysteresis, energy dissipated by devices added to the structure), and other effects (lengthening of structural period). Stiffness is defined as secant stiffness at peak displacement, which generally requires definition of the non-linear force-displacement curve for the structure (pushover curve). The SDOF systems are used to predict the modal response of the original system, using elastic response spectrum analysis (RSA). The procedure described is implemented in the ASCE/SEI 7-16 standard [ASCE, 2017] and is being considered for inclusion in the next generation of Eurocode 8 for structures with velocity-dependent dampers, providing a robust and simple alternative or benchmark to time history analysis (THA) for regular structures.

Application of this procedure requires construction of spectra for values of viscous damping ratio higher than usual; in structures with VDs, for instance, the effective modal viscous damping ratio ξ_{eff} is typically above 10% in the first mode, and typically much larger in higher modes. Seismic action is generally defined through elastic spectra for a 5% value of viscous damping ratio. High damping spectra can be derived from these by scaling spectral ordinates; three spectral values are relevant, namely spectral displacement (S_d), spectral velocity (S_v), and spectral acceleration (S_a), defined, respectively, as [Lin & Chang, 2003]:

$$S_d(T, \xi) = |u(t, T, \xi)|_{\max}, \quad (4.1-1)$$

$$S_v(T, \xi) = |\dot{u}(t, T, \xi)|_{\max}, \quad (4.1-2)$$

$$S_a(T, \xi) = |\ddot{u}'(t, T, \xi)|_{\max} = |\ddot{u}(t, T, \xi) + \ddot{u}_g(t)|_{\max}, \quad (4.1-3)$$

where $u(t)$ is the time-history relative displacement response of a SDOF system with period T , and damping ratio ξ , subjected to the ground acceleration defined by $\ddot{u}_g(t)$. Damping correction factors (DCFs), can be defined for spectral displacement, velocity and acceleration, respectively, as:

$$\eta_d(T, \xi) = S_d(T, \xi) / S_d(T, 5\%), \quad (4.1-4)$$

$$\eta_v(T, \xi) = S_v(T, \xi) / S_v(T, 5\%), \quad (4.1-5)$$

$$\eta_a(T, \xi) = S_a(T, \xi) / S_a(T, 5\%); \quad (4.1-6)$$

Some authors and codes use damping correction coefficients rather than factors, with the basic concept otherwise unaltered. Current seismic codes such as Eurocode 8 [CEN, 2004] and ASCE/SEI 7-16 [ASCE, 2017] define 5% spectral pseudo-acceleration PS_a or 5% spectral displacement S_d ; pseudo-acceleration (PS_a) spectra and pseudo-velocity (PS_v) spectra were introduced by Newmark & Hall [1982]. These variables quickly became standard, mainly because they can be derived directly from each other and from the displacement spectrum, using basic modal considerations:

$$PS_v = \omega S_d, \quad (4.1-7)$$

$$PS_a = \omega^2 S_d, \quad (4.1-8)$$

where ω is the circular frequency $2\pi/T$. These relationships allow the use of four-way logarithmic figures to represent all spectra in a single plot, a convenient way for manual calculations. Spectral pseudo-values provide an approximation to spectral values; however, for single-degree-of-freedom systems (SDOF) it has been proved [Pekcan et al, 1999; Lin & Chang, 2003; Chopra, 2007] that velocity S_v and acceleration S_a differ considerably from pseudo-velocity and pseudo-acceleration, respectively. For acceleration, the proof is straightforward: for a system with frequency ω and damping ratio ξ , the total acceleration can be found from the normalized equation of motion, Eqn. (2.5-4), as:

$$\ddot{u}^t(t, T, \xi) = -\omega^2 u(t, T, \xi) - 2\omega \xi \dot{u}(t, T, \xi), \quad (4.1-9)$$

which can be transformed to

$$S_a(T, \xi) = \left| \ddot{u}^t(t, T, \xi) \right|_{\max} = \left| \omega^2 u(t, T, \xi) + 2\omega \xi \dot{u}(t, T, \xi) \right|_{\max} \quad (4.1-10)$$

Comparing Eqns. (4.1-8) and (4.1-10) it follows that $PS_a(T, \xi) = S_a(T, \xi)$, only if $\xi = 0$ or if the peak spectral acceleration takes place at an instant of no velocity, which again happens exactly only with $\xi = 0$. Thus, in general, $S_a \neq PS_a$, and similarly $S_v \neq PS_v$, not even for the undamped case, and more particularly for long periods ($T > 1$ s) and high values of damping.

This is of special relevance when VDs are used, as the behavior of these devices is velocity-dependent. Several authors [Pekcan et al, 1999; Sadek et al, 1999; Ramírez et al, 2001] have proposed to approximate the true spectral velocity through a correction factor, referred to as *correction factor for velocity* CF_v , defined as:

$$CF_v(T, \xi) = S_v(T, \xi) / PS_v(T, \xi). \quad (4.1-11)$$

Of particular interest are the CF_v s derived for 5% damping ratio, $CF_v(T, 5\%)$, that convert 5% spectral pseudo-velocity to 5% spectral velocity. Using these values, the spectral velocity can be expressed as:

$$S_v(T, \xi) = \eta_v(T, \xi) CF_v(T, 5\%) PS_v(T, 5\%), \quad (4.1-12)$$

Or, using the relationship between spectral pseudo-velocity and spectral displacement:

$$S_v(T, \xi) = \eta_v(T, \xi) CF_v(T, 5\%) \omega S_d(T, 5\%), \quad (4.1-13)$$

DCFs and CF_v s are referred hereby generally as spectral correction factors. The following relationship between η_v , η_d and CF_v is derived from (4.1-11):

$$\eta_v(T, \xi) = CF_v(T, \xi) \eta_d(T, \xi) / CF_v(T, 5\%). \quad (4.1-14)$$

4.1.1 Previous studies

4.1.1.1 Damping correction factors DCFs

There are several studies about DCFs and most of them use empirical procedures based on databases of motions, sometimes complemented with considerations from theory of dynamics. A comparative review can be found in Rezaeian et al [2012]. Only the most relevant contributions are reviewed hereby.

DCFs were first introduced by Newmark & Hall [1982] for elastic SDOF systems. Their study derived spectrum amplification factors relating peak values of ground motion to structural response for pseudo-acceleration, pseudo-velocity, and displacement, as median values from 28 records corresponding to events registered before 1973. Because these factors were a function of damping ratio, it was possible to relate the displacement response at a certain damping ratio to the 5% elastic response, thus rendering the relevant values of η_d . These are constant in every spectral region, but vary between spectral regions. Thus, the effect of period was considered between regions but not within each region. This study was the first to indicate the procedure for construction of a response spectrum and its results (extended for levels of damping ratio up to 50%) were implemented in FEMA 273 [FEMA, 1997a]. The expressions obtained (indirectly) from this study are:

$$\eta_{d,NH} = a + b \ln(100 \xi), \quad (4.1-15)$$

where a and b take region-dependent values: $a = 1.517$, $b = -0.321$, for the constant acceleration region ($T = 0.125\text{s}$ to 0.6s); $a = 1.400$, $b = -0.248$, for the constant velocity region ($T = 0.6\text{s}$ to 4s); and $a = 1.313$, $b = -0.195$ for the constant displacement region ($T = 4\text{s}$ to 10s). The study included a proposal to modify the spectrum for non-linear behavior and was limited to a 20% damping ratio.

Kawashima & Aizawa [1986] proposed an expression for DCFs for acceleration, valid for values of damping ratio ξ from 5% to 40%; the DCFs were derived using both components of 103 records, registered in Japan between 1966 and 1978 at 43 free-field sites, with magnitude ≥ 5.0 and focal depth less than 60km, and read:

$$\eta_{a,KA} = 0.5 + 1.5 / (1 + 40\xi). \quad (4.1-16)$$

the expression was adopted in Caltrans Seismic Design Criteria [CALTRANS, 2001].

The study by Newmark & Hall was extended to higher damping values up to 50% by Wu & Hanson [1989], who also included inelastic effects, with displacement ductility ratios in the range 1 to 6 for SDOF systems. Their study used 9 recorded earthquakes and one artificial earthquake to derive mean values of displacement DCFs; the formulae derived acknowledge the influence of period in the damping effectiveness, which is smaller in the constant acceleration region (identified in this study as below 0.5s) than in the velocity region (from 0.5s to 3.0s) or displacement region (from 3.0s to 10s). The authors also concluded that increasing yielding and damping reduced the dispersion in the results, and

that damping is less effective for short- and long- periods. The expression proposed for elastic systems is:

$$\eta_{d,WH} = a \ln(b\xi), \quad (4.1-17)$$

where a and b take the following period-dependent values: for $T = 0.1\text{s}$, $a = -0.35$, $b = 0.10$; for $T = 0.5\text{s}$, $a = -0.55$, $b = 0.42$; for T between 0.5s and 3s , $a = -0.47$, $b = 0.52$; for $T = 3\text{s}$, $a = -0.48$, $b = 0.48$; and for $T = 10\text{s}$, $a = -0.29$ and $b = 0.05$. These results were the base of the damping coefficients appearing in FEMA 222 [FEMA, 1995].

Sadek et al [1999, 2000] proposed damping correction factors η_d and η_a based on the mean response of SDOF systems. The motion set consisted of 72 horizontal components of accelerograms from 17 events, recorded at 36 stations in the Western U.S., with magnitudes 5.2 to 7.7, epicentral distances from 6 km to 127 km, PGA from 0.044g to 1.172g, and different soil conditions (not reported in the study). The values of ξ considered ranged between 0.02 and 0.6. The values tabulated by the authors are listed in Table 4-1, where the row corresponding to $\xi = 0.9$ (marked in *italics*) contains extrapolated data. The authors approximate η_a by the following expression:

$$\eta_{a,SMR} = \eta_d(1 + aT^b), \quad (4.1-18)$$

where a is given by $2.436\xi^{1.895}$ and b is given by $0.628 + 0.205\xi$. The study contained also values for CF_v , reported later in this Chapter.

Table 4-1. η_d and η_a after Sadek et al [1999].

ξ	$T (s)$									
	0.10	0.30	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00
η_d	0.02	1.17	1.35	1.26	1.27	1.27	1.23	1.22	1.17	1.15
	0.10	0.89	0.78	0.80	0.80	0.80	0.81	0.81	0.84	0.85
	0.15	0.85	0.67	0.68	0.68	0.69	0.71	0.71	0.75	0.77
	0.20	0.81	0.60	0.60	0.60	0.61	0.64	0.63	0.68	0.69
	0.30	0.76	0.51	0.49	0.49	0.51	0.54	0.53	0.58	0.60
	0.40	0.72	0.44	0.42	0.42	0.44	0.47	0.47	0.51	0.53
	0.50	0.70	0.40	0.37	0.36	0.39	0.42	0.42	0.46	0.48
	0.60	0.67	0.36	0.34	0.33	0.35	0.38	0.38	0.42	0.44
	0.90*	0.64	0.29	0.23	0.22	0.25	0.28	0.28	0.31	0.33
	0.02	1.16	1.35	1.25	1.26	1.22	1.21	1.15	1.14	1.14
η_a	0.10	0.90	0.79	0.81	0.82	0.82	0.84	0.84	0.89	0.90
	0.15	0.86	0.69	0.71	0.72	0.74	0.77	0.78	0.84	0.88
	0.20	0.83	0.63	0.65	0.67	0.71	0.75	0.76	0.84	0.88
	0.30	0.79	0.56	0.57	0.61	0.70	0.76	0.78	0.89	0.97
	0.40	0.77	0.52	0.54	0.61	0.73	0.82	0.86	0.99	1.10
	0.50	0.75	0.50	0.52	0.61	0.77	0.89	0.95	1.10	1.24
	0.60	0.74	0.48	0.51	0.63	0.82	0.96	1.05	1.23	1.39
	0.90*	0.67	0.38	0.43	0.63	0.94	1.21	1.41	1.71	2.01

* Extrapolated values

Bommer & Elnashai [1999] showed that spectral displacement ordinates obtained from accelerograms recorded by analogue accelerographs are very sensitive to filtering and correction options. Thus, Tolis & Faccioli [1999] used ground motion prediction equations (at the time referred to as attenuation relationships), and five European strong-motion accelerograms supplemented with digital recordings from the 1995 Hyogoken-Nanbu (Kobe) and 1997 Umbria-Marche earthquakes, in order to double-check the results for long periods, reaching the following expression:

$$\eta_{d,TF} = \sqrt{15 / (10 + 100\xi)} \geq 0.5. \quad (4.1-19)$$

Several expressions similar to (4.1-19) with different numerical correctors have also been proposed by Bommer and co-workers [Bommer et al, 2000; Bommer & Mendis, 2005] and one of these expressions has been adopted in the current version of Eurocode 8 [CEN, 2004], as reported later in this Section.

Ramirez et al [2001, 2002b] used a database prepared by Tsopelas et al [1997], consisting of a set of 20 horizontal components of 10 earthquake motions with magnitude larger than 6.5, epicentral distance between 10 and 20 km, and site class that can be described as firm (soft rock to stiff soil). The motions were scaled to represent a specific design response spectrum (as defined in FEMA 273 [FEMA, 1997a]). The study derived corresponding damping coefficients for a SDOF system, with a simple tri-linear form anchored to unity for zero period, and values tabulated to damping ratio for two control periods, damping coefficient being constant and equal for velocity and displacement spectral regions. Their results, somewhat simplified, were incorporated in FEMA 368 [FEMA, 2001] and with minor modifications, in subsequent editions of this code up to the present edition [FEMA, 2015]. The study included also the effect of damping in inelastic displacements. In this study, tabulated values were provided instead of a closed-form expression.

Naeim & Kircher [2001] studied 1046 records from North American, Alaskan and Hawaiian Island events, with Magnitudes ≥ 5 , Peak Ground Accelerations (PGA) $\geq 0.05g$, and distances between 10 and 20 km. The study, based on only nine representative periods, confirmed the FEMA 273 [FEMA, 1997a] damping coefficients for damping ratios up to 20%. These authors conclude that damping coefficients seem to be the same for acceleration and velocity domains of the spectra. These conclusions, however, are not in agreement with their own linear regression formulae, which show a tendency of damping coefficients to decrease with period, for damping ratios over 5%; moreover, the regression line negative slope doubles when damping ratio moves from 10% to 20%, showing clearly the dependency of decrease rate with damping ratio. This is coherent with the result of studies cited below which show that DCFs increase with period, although this trend is not so outstanding for damping ratios up to 20%. Interestingly, the authors justify the possible loss of effectiveness of damping with period, because damping is more effective with an increasing number of cycles. As, during earthquakes, structures tend to vibrate with their

fundamental period, structures with longer periods sustain less cycles, damping effect being thus reduced. Another contradictory result of this study is an increase in dispersion of the damping coefficients with increasing damping. The study provided tabulated values of damping coefficients (no analytical expression was derived). These values translate into $\eta_d = 0.807$ for $\xi = 10\%$, and $\eta_d = 0.618$ for $\xi = 20\%$, proposed as constant values over the acceleration and velocity regions of the spectrum. As indicated, the conclusions reached by this study are in frank disagreement with almost every other study cited.

Lin & Chang [2003, 2004], used a larger database (1053 U.S. ground motions) to show the dependence of S_d on damping up to 50% and periods up to 10s. Moreover, they concluded that the effect of site classes on η_d is scarcely relevant for damping ratios below 20% (less than 5% error) and not very relevant for damping ratios up to 50% (less than 10% error). For η_a , however, the influence of site class becomes more relevant as the damping ratio increases, and the authors recommend considering it for systems with damping ratios over 10%. They also found that FEMA-273 values are conservative for all damping ratios below a period of 2 s, and unsafe for higher periods and damping ratios over 30%. Their study was limited to site classes with average shear-wave velocities above 180 m/s. These authors also proved that for system with energy dissipation devices the forces in the primary structure are better approximated by the pseudo-acceleration than by the real acceleration. Thus, the relevant DCF for force reduction in structures with PEDs is η_d , not η_a . The following expressions were obtained in the second study [Lin & Chang, 2004] from a non-linear regression analysis:

$$\eta_{d,LC} = 1 - a T^b / (1 + T)^c, \quad (4.1-20)$$

$$\eta_{a,LC} = d + eT, \quad (4.1-21)$$

where a, b, c, d, e , are parameters dependent on site class, listed in Table 4-2.

Table 4-2. Parameters in Lin & Chang [2004], Eqns. (4.1-19).

Site Class (FEMA 2000) [EC8]	a	b	c	d	e
(A,B) [A]	$1.1637 + 0.3885\ln(\xi)$	0.229	0.505	$0.391\xi^{0.304}$	$0.0057 + 0.383\xi - 1/15.929\xi^2$
(C) [B]	$1.4532 + 0.4872\ln(\xi)$	0.354	0.810	$0.309\xi^{0.392}$	$0.0151 + 0.474\xi - 1/10.241\xi^2$
(D) [C]	$1.3243 + 0.4426\ln(\xi)$	0.311	0.664	$0.326\xi^{0.371}$	$0.0348 + 0.248\xi - 1/8.250\xi^2$
All	$1.303 + 0.436\ln(\xi)$	0.30	0.65	$0.342\xi^{0.354}$	$0.0186 + 0.368\xi - 1/10.644\xi^2$

Four of the expressions reviewed so far (Newmark & Hall $\eta_{d,NH}$, Wu & Hanson $\eta_{d,WH}$, Ramirez et al $\eta_{d,R}$, Lin & Chang $\eta_{d,LC}$) were verified by Lin et al [2005] using 216 well-documented acceleration time histories from 12 different Californian earthquakes with M_s between 5.7 and 7.7, and $PGA > 0.045g$. These authors concluded that the maximum error for all methods happens in the short period region. $\eta_{d,NH}$ rendered reasonable results for $\xi \leq 20\%$ and $T > 0.2s$. $\eta_{d,WH}$ lead to overestimation of displacements for $T < 0.5s$ and

underestimation for $2s < T < 3.5s$. $\eta_{d,R}$ produced good results for $T > 2s$, but lead to overestimation of displacements for $T < 2s$ (up to 30% for $\xi = 50\%$). Finally, $\eta_{d,LC}$ produced the most accurate results for all damping ratios and periods.

Pavlou & Constantinou [2004], examined the response of SDOF systems to near-field, pulse-like and soft-soil motions (shear-wave velocity below 180 m/s), using three sets, two for near-field motions with 10 and 13 registers and one for soft-soil motions with 14 registers. They concluded that FEMA values are conservative for soft-soil ground motions and periods up to 5s, and for near-field ground motions and periods up to 3s, although this is strongly dependent on the proper choice of the transition period between acceleration and velocity regions.

Bommer & Mendis [2005], used four sets of ground-motion prediction equations (attenuation equations) to render spectral ordinates for different levels of damping. The four models, derived by different authors, were based on different sets of ground-motions, two of them from western North American data, and two of them from European data. The study shows that DCFs decrease with increasing magnitude and distance; also that they increase slightly for softer site conditions and decrease with duration of ground motion (that is, damping is more effective in long duration motions). The significant duration $D_{5-95\%}$ (defined as duration between instants where the Arias Intensity is 5% and 95% of the total record value) was chosen as representative duration parameter. The dependence of DCFs on ground motion duration is a key finding of the study, demonstrated by dynamic considerations (comparing the strong-motion to a harmonic oscillation with varying number of cycles, in the same line of thought as Naeim & Kircher, [2001]), and empirical models. This finding was confirmed and quantified in a subsequent study by Stafford et al [2008], and Rezaeian et al [2012, 2014]. Moreover, these studies established that the influence of ground motion duration can be captured through the dependence on magnitude and distance, showing that the rate of increase of the DCFs towards unity at very long periods is magnitude and source-to-site distance dependent, converging faster for smaller magnitudes and closer earthquakes.

The dependence on duration was used by Mendis & Bommer [2006] to propose a modification of the Eurocode 8 damping reduction factors suitable for the Type 2 Eurocode 8 spectrum; using attenuation equations and a significant $D_{5-95\%}$ duration of 5s, the following expression was proposed by these authors:

$$\eta_{d,BM} = \sqrt{35 / (30 + 100 \xi)}. \quad (4.1-22)$$

Cameron & Green [2007] also used simple dynamic considerations to explain the results for η_d of a numerical study, whose database comprised 1268 records from active seismic regions (676 records) and scaled motions from stable continental regions (592 records), allowing conclusions for both sets to be obtained independently. The records were also binned by magnitude, site-to-source distance and site class. Damping ratios up to 50% were considered, including 1% and 2% values. Their study shows that, for damping ratios

over 5%, displacement DCFs depend primarily on the earthquake content of frequencies, which in turn depends primarily on earthquake magnitude and tectonic features; site-to-source distance, however, has little effect for damping ratios over to 2%. Accordingly, the authors tabulate the mean and standard deviation of the natural logarithm of the DCFs as a function of damping ratio, period, tectonic setting, site class and magnitude. The dependency on duration (quantified as number of cycles) seems to diminish with increasing damping. Interestingly, the authors show that ground motion duration strongly depends on site-to-source distance up to approximately 50km. Consequently, the authors conclude that duration is the prime contributor to differences in DCFs at all damping levels.

Cardone et al [2009] reviewed some of the existing DCFs, including code-based expressions such as the Eurocode 8, Japanese and Chinese. They used both components of 60 free-field records with magnitude between 6 and 8, epicentral distance between 0.1 and 100 km, and $PGA > 0.1g$. Their fundamental conclusion is that DCFs cannot be accepted for damping ratios over 10% for individual records. For representative groups of records, the most accurate results were obtained for $\eta_{d,WH}$ and $\eta_{d,LC}$; the first of these expressions was found particularly good for the long period range (between 2 and 4s). The study acknowledges the need for different DCFs for acceleration and displacement.

Hatzigeorgiou [2010] considered three groups of accelerograms (far-fault, near-fault earthquake and artificial accelerograms compatible with Type 1 spectrum of Eurocode 8), each one further divided in four bins according to site-class, and proposed an expression for DCFs comprising periods from 0 to 5 s and damping ratios below 50%. This author also concluded that, from a practical point of view, class site can be properly acknowledged by making a difference between rock and soil. According to this author, near-fault and far-fault records lead to similar DCFs and therefore the influence of source-to-site distance may be ignored. This conclusion, as discussed in the next paragraph, has been later refuted by other authors based on an inadequate selection of ground motions. The author indicates that caution is necessary at long periods, where the DCFs are close to 1, and that pseudo-acceleration spectra may lead to considerable underestimation of design seismic forces for highly damped systems. The proposed empirical expression is:

$$\eta_{d,H} = 1 + (100\xi - 5) [1 + c_1 \ln(100\xi) + c_2 (\ln(100\xi))^2] [c_3 + c_4 \ln(T) + c_5 (\ln(T))^2], \quad (4.1-23)$$

where, to obtain an approximate result for all site classes, the following parameters can be taken $c_1 = -0.29910$, $c_2 = 0.02074$, $c_3 = -0.09003$, $c_4 = 0.00837$, $c_5 = 0.01018$. Specific parameters for site class A, B, C and D are listed in the referenced study. The same expression with different parameters is used for velocity ($c_1 = -0.32225$, $c_2 = 0.02619$, $c_3 = -0.07720$, $c_4 = 0.02588$, $c_5 = 0.00637$) and acceleration ($c_1 = -0.36113$, $c_2 = 0.03459$, $c_3 = -0.04614$, $c_4 = 0.03316$, $c_5 = -0.00037$).

The observations derived by Hatzigeorgiou in relation with near-fault motions, are in conflict with those of Hubbard & Mavroeidis [2011a]. These authors calculated DCFs for a large range of periods and damping levels, studying SDOF systems subjected to near-fault

ground motions. The database was formed by 38 records from 19 events, with magnitudes M_w between 5.6 and 7.6, fault distances between 0.1 and 16.4 km, and peak ground velocities PGV between 15.3 and 380 cm/s, collected by the authors in previous studies. These motions, characterized by exhibiting velocity pulses, were carefully rotated to fault-normal and fault-parallel directions, retaining only the significant (fault-normal) component. The authors noticed that the DCFs derived for the motions exhibited large differences when binned by magnitude. Three groups were proposed: moderate ($M_w = 5.6-6.3$), medium ($M_w = 6.4-6.7$) and large ($M_w = 6.8-7.6$). After comparison, they concluded that DCFs proposed in design codes and previous studies, based primarily on far-field ground motion records, are unsafe for near-fault seismic ground motions (referring to fault-normal components). For this type of motion, the authors propose to normalize the period axis by the duration of the ground velocity pulse T_p , a variable that they defined as rise time on the fault plane and is directly related to the earthquake magnitude by the following expression:

$$\log T_p = -2.9 + 0.5M_w, \quad (4.1-24)$$

where M_w is the magnitude of the event. Using this value, the following expressions are proposed for η_d :

$$\eta_{d, HM} = \left[1 + a(\xi + b)^c / (T / T_p)^{1.3} \right]^{-1}, \quad (4.1-25)$$

where a , b , c are parameters depending on ξ ; for $10\% \leq \xi \leq 50\%$, $a = 3.4$, $b = 0$, $c = 1.3$; for $50\% < \xi \leq 100\%$, $a = 2$, $b = 0.3$, $c = 1.5$). These expressions acknowledge that damping is more effective for structures with periods near T_p . The results of this study have been questioned [Hatzigeorgiou & Papagiannopoulos, 2011], among other issues, by the small number of motions included.

Arakawa & Kobayashi [2012] proposed a modification of the Japanese DCFs to account for high damping and long periods. Their proposal, dependent on period, damping ratio, soil predominant periods and equivalent duration of ground motion, was based on the theory of random processes and verified using time history analysis with 30 artificial ground motions based on real earthquake characteristics. Although the authors acknowledge that further study is needed to include the duration of ground motion, their main conclusion is that the Japanese code DCFs for long periods is unsafe, especially in pulse-like motions. Their expression reads:

$$\eta_{d, AK} = 1.5R / (1 + 10\xi), \quad (4.1-26)$$

where R is dependent on the structure fundamental period in relation with the spectral region: For the central region ($T_1 \leq T \leq T_2$), $R = 1$. For the short period region ($T < T_1$), $R = 1 + (20\xi - 1) \cdot (1 - T/T_1)/3$. For the displacement region ($T > T_2$), $R = 1 + 20T\xi (1 - T_2/T)/T_g$. Here T_g is the ground motion duration, and T_1 and T_2 seem to depend on the ground characteristics; however, these parameters are not defined accurately in the study.

Rezaeian et al [2012, 2014] used the large NGA-West2 database (over 8000 three-component recordings, corresponding to active tectonic regions, of which 2250 records for horizontal component and 2229 records for vertical component were selected) to analyze the influence of different variables to DCFs for displacement: damping ratio, spectral period, duration, earthquake magnitude, source-to-site distance and site conditions. The motion component used is the rotated component referred to as RotD50 [Boore, 2010], a component extracted combining the two horizontal components for a number of angles between 0° and 180° and choosing the angle with minimum distance from the median spectral ordinate for a range of periods. This procedure has the advantage of removing the recorder orientation random variable, but its practical importance is not clear. The authors conclude that duration is correlated to magnitude and distance and therefore can be accounted for indirectly by these two parameters. This solution is deemed realistic, as duration is not included in current earthquake scenarios. As a result, they propose a statistical model dependent on damping, period, magnitude and distance to rupture plane, valid for shallow crustal earthquakes in active tectonic regions up to 30% damping ratio. The model has a functional form dependent on period, damping ratio, magnitude and closest distance to rupture plane, with up to eight parameters tabulated for 21 values of period ranging from 0.01s to 10s. A modified model is also proposed for vertical component. The model is valid up to 50km of distance, but preliminary validation by the authors suggests that it can be extended beyond this range.

As a summary, since 1982 several studies have been carried that considered the influence of several parameters in DCFs and derived expressions to approximate them; contradictory results appear in some of the studies. The following facts are backed by the studies cited:

- All studies acknowledge that DCFs are dependent on period and damping ratio.
- Some studies conclude that the influence of site class is mild [Lin & Chang, 2004; Cameron & Green, 2007]; others just considered relevant the difference between soft and stiff soils [Hatzigeorgiou, 2010]; finally others found the incidence insignificant [Rezaeian et al, 2012];
- Most modern studies consider as very significant the influence of ground motion duration [Bommer & Mendis, 2005; Cameron & Green, 2007; Stafford et al, 2008; Rezaeian et al, 2012];
- The influence of magnitude and source-to-site distance is considered relevant by all studies that included them as possible parameters [Cameron and Green, 2007; Rezaeian et al, 2012].
- The influence of the tectonic setting is highlighted in some studies [Cameron & Green, 2007; Rezaeian et al, 2012]. DCFs are expected to be different in stable continental areas than in active seismic areas.
- According to Rezaeian et al [2012], the influence of duration can be approximately captured through dependence on magnitude and source-to-site distance.
- Some authors [Hubbard & Mavroeidis, 2011a] argue that the DCFs for near-fault motions are completely different than those for far-field motions, and that magnitude

seems to be the most important factor for this type of motions; other authors deny this difference [Hatzigeorgiou, 2010].

All DCFs (η_d , η_v , η_a) tend to unity as the period gets closer to 0 s. η_d , η_v also approach unity for long periods and present smaller values for larger damping ratios.

4.1.1.2 Correction factors for velocity CF_v

The concept of CF_v was proposed almost simultaneously by Sadek et al [1999] and Pekcan et al [1999].

Sadek et al [1999, 2000] tabulated a set of values obtained from the same study reported in Section 4.1.1.1. The values, obtained for damping ratios between 0.02 and 0.60, are listed in Table 4-3, where additional values for $\xi = 0.90$ (in *italics*) have been extrapolated from the data. The authors propose also an analytical expression to approximate these values:

$$CF_v(T, \eta) = aT^b, \quad (4.1-27)$$

where $a = 1.095 + 0.647\xi - 0.328\xi^2$, and $b = 0.193 + 0.838\xi - 0.621\xi^2$.

Table 4-3. CF_v after Sadek et al [1999].

T	ξ									
	0.02	0.05	0.10	0.15	0.20	0.30	0.40	0.50	0.60	0.90*
0.10	0.70	0.63	0.57	0.53	0.51	0.47	0.45	0.43	0.41	<i>0.37</i>
0.30	0.96	0.94	0.91	0.88	0.86	0.82	0.79	0.78	0.77	<i>0.71</i>
0.50	0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	<i>1.01</i>
1.00	1.06	1.08	1.11	1.13	1.15	1.22	1.28	1.32	1.36	<i>1.51</i>
1.50	1.13	1.20	1.27	1.31	1.36	1.46	1.54	1.61	1.65	<i>1.83</i>
2.00	1.20	1.28	1.39	1.47	1.53	1.63	1.73	1.82	1.90	<i>2.10</i>
2.50	1.23	1.33	1.47	1.55	1.64	1.78	1.89	1.99	2.09	<i>2.33</i>
3.00	1.39	1.47	1.56	1.64	1.72	1.88	2.02	2.15	2.26	<i>2.62</i>
3.50	1.51	1.60	1.70	1.79	1.87	2.01	2.16	2.29	2.42	<i>2.76</i>
4.00	1.61	1.74	1.88	2.00	2.09	2.23	2.36	2.50	2.62	<i>2.89</i>

* *Extrapolated values.*

Pekcan et al [1999] used 36 ground motions (mainly Western U.S., but including some records from 1995 Kobe, Japan; 1978 Tabas, Iran and 1992 Erzincan, Turkey), with PGA from 0.156g to 1.778g, and PGV from 0.175 m/s to 1.787 m/s. The records were all scaled to $v_{g,max} = 1$ m/s. Periods of 0 to 10s and damping ratios for 5% to 50% were considered. Elastic SDOF with these characteristics were analyzed. The results for velocity and pseudo-velocity were used to derive expressions based on the different behavior of these variables in the three spectral ranges. The authors found that values for S_v , compared to the corresponding values for PS_v , are lower in the low period range; similar in the medium period range; and considerably higher in the long period range. This is explained by the fact that for long periods (low stiffness), structural peak relative velocity equals the peak

ground velocity whereas the pseudo-velocity approaches 0. The expression derived by the authors is:

$$CF_v(T, \eta) = \begin{cases} (T/T_1)^{a_1} (T/T_m)^{a_m} & , 0 < T < T_1 \\ (T/T_m)^{a_m} & , T_1 < T < T_u \\ (T/T_m)^{a_m} (T/T_u)^{a_u} & , T_u < T, \end{cases} \quad (4.1-28)$$

where T_1 , T_m and T_u are damping-dependent corner periods between spectral regions, and a_1 , a_m , a_u , damping-dependent numerical exponents, obtained from numerical regression as: $T_1 = 0.042 \cdot \xi + 0.113$; $T_m = 0.064 \cdot \xi + 0.756$; $T_u = 3.149 \cdot \xi + 3.112$; $a_1 = 0.450 \cdot \xi + 0.729$; $a_m = 0.455 \cdot \xi + 0.132$; $a_u = 0.607 \cdot \xi + 0.489$.

Ramirez et al [2001] derived CF_v in the same study reported in Section 4.1.1.1., where different non-linear SDOF systems were considered. Regarding the hysteretic behavior, perfect bilinear hysteretic (with post-yield stiffness ranging between 0.05 and 1) and bilinear elastic systems (with post-elastic stiffness of 0.05) were considered, both of them with no strength or stiffness degradation. Regarding the damping devices, linear viscous devices, non-linear viscous devices, and yielding devices were considered. The periods considered ranged between 0.3 s and 2 s. The study acknowledges that the greatest errors incurred by taking the pseudo-velocity as predictor of the velocity happen at low post-elastic stiffness ratio and large ductility. The values obtained in the study were tabulated and are shown in Table 4-4.

Table 4-4. CF_v after Ramirez et al [2001].

T (s)	ξ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
0.3	0.72	0.70	0.69	0.67	0.63	0.60	0.58	0.58	0.54	0.49
0.5	0.75	0.73	0.73	0.70	0.69	0.67	0.65	0.64	0.62	0.61
1.0	0.82	0.83	0.86	0.86	0.88	0.89	0.90	0.92	0.93	0.95
1.5	0.95	0.98	1.00	1.04	1.05	1.09	1.12	1.14	1.17	1.20
2.0	1.08	1.12	1.16	1.19	1.23	1.27	1.30	1.34	1.38	1.41
2.5	1.05	1.11	1.17	1.24	1.30	1.36	1.42	1.48	1.54	1.59
3.0	1.00	1.08	1.17	1.25	1.33	1.42	1.50	1.58	1.67	1.75
3.5	1.09	1.15	1.22	1.30	1.37	1.45	1.52	1.60	1.67	1.75
4.0	0.95	1.05	1.15	1.24	1.38	1.49	1.60	1.70	1.81	1.81

4.1.2 Spectral correction factors defined in codes

4.1.2.1 Spanish code, NCSE-02

The current version of the Spanish seismic code [Ministerio de Fomento de España, 2002] presents a simple expression for η_a :

$$\eta_{NCSE} = (5/100 \xi)^{0.4}, \quad (4.1-29)$$

that is dependent on damping-ratio, and constant for periods $T > T_A$ (corner period at beginning of spectral constant acceleration region, equivalent to T_B in Eurocode 8). For $T < T_A$, the damping factor must be interpolated between 1.0 (at $T = 0$) and the value given by expression (4.1-30) (at $T = T_A$). The expression has its origin in the French code AFPS-1990 (not cited here).

4.1.2.2 Eurocode 8, 2004 version

The current version of Eurocode 8, officially known as EN-1998:2004 [CEN, 2004], proposes a simple DCF based on the work of Bommer et al [2008]:

$$\eta_{EC8} = \sqrt{10/(5 + 100\xi)} \geq 0.55. \quad (4.1-30)$$

For periods over T_B (corner period of constant spectral acceleration region), this factor is used on the elastic spectral pseudo-acceleration to obtain the damped value; therefore, the expression is a displacement DCF. For periods below T_B , a correction is applied to the pseudo-acceleration spectrum definition to guarantee that the damped spectral value at $T = 0$ equals the 5% value:

$$PS_a(T \leq T_B) = a_g S [1 + T(2.5\eta_{EC8} - 1) / T_B] \quad (4.1-31)$$

where T_B is the corner period for beginning of the constant acceleration range, a_g is the design ground acceleration for site class A, and S the soil factor; this expression varies linearly between $a_g \cdot S$ at $T = 0$, and $2.5a_g \cdot S$ at $T = T_B$. For periods over T_B , η_{EC8} simply multiplies the spectral ordinates. Similarly, in the displacement spectrum, a linear expression is provided for periods above T_E (the period for which the defined displacement spectrum begins to decay), to achieve a unity value at periods longer than 10s.

Eurocode 8 does not provide any expression for CF_v s.

4.1.2.3 Eurocode 8, current draft of new version

The current draft of the new release of Eurocode 8 contains the following proposal for displacement DCF:

$$\begin{cases} T \leq 0.5s \Rightarrow \eta_{DEC8} = \sqrt{\frac{10 + T_C(100\xi - 5)/(T_C + 30T)}{5 + 100\xi}} \\ T > 0.5s \Rightarrow \eta_{DEC8} = \sqrt{10/(5 + 100\xi)} \end{cases} \quad (4.1-32)$$

This expression is a displacement DCF based on interpolation over the values suggested in Ramirez et al [2001]. The draft contains also the following expression for CF_v :

$$CF_{v,DEC8} = 0.88 + 1.25\xi - (0.29 + 1.63\xi) \exp\left[-\frac{T}{0.75 + 1.53\xi}\right] \geq 1 \quad (4.1-33)$$

This expression was obtained with numerical regression performed on values obtained by Ramirez et al [2001].

4.1.2.4 ASCE-SEI 7/16

DCF's have been included in U.S. codes since 1990 based on the research discussed in the previous section. The proposal by Ramirez et al [2001] was the basis for a revision of the NEHRP 1997 provisions, [FEMA, 1997b], to include passive energy dissipation systems in the code. A chapter was included in the NEHRP 2000 provisions, [FEMA, 2000] and the damping correction coefficients listed there (Chapter 13) were included later in the ASCE provisions. Chapter 18 of ASCE-SEI 7/16 [ASCE, 2016] contains tabulated data for coefficients referred as B , intended to divide (rather than factor) spectral ordinates. These coefficients (listed in Table 4-5), are damping-ratio dependent and constant for periods $T > T_0$, where T_0 is the corner period at the beginning of the constant acceleration region of the design spectrum (equivalent to T_B in Eurocode 8). For $T < T_0$, the damping coefficient must be interpolated between 1.0 (at $T = 0$) and the table value (at $T = T_0$). Additionally, Chapter 17 contains DCF's to be applied to base isolation systems, unchanged since 1997 [FEMA, 1997b]. For values of damping over 30% these DCF's are more conservative than the ones listed above,

Table 4-5. DCF's in Chapter 18 of ASCE-SEI 7/16 [ASCE, 2016].

ξ		≤ 0.02	0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	≥ 1
Base isolation	$B=1/\eta$	0.80	1.00	1.20	1.50	1.70	1.90				2.00		
	η	1.25	1.00	0.83	0.67	0.59	0.53				0.50		
PEDs	$B=1/\eta$	0.80	1.00	1.20	1.50	1.80	2.10	2.40	2.70	3.00	3.30	3.60	4.00
	η	1.25	1.00	0.83	0.67	0.56	0.48	0.42	0.37	0.33	0.30	0.28	0.25

The code contains no expression for CF_v 's.

4.1.2.5 Japanese Code

The Japanese code [JMC, 2000] contemplates a DCF for systems with seismic isolation systems, to be applied on pseudo-acceleration:

$$\eta_{JC} = 1.5 / (1 + 10\xi) \geq 0.40. \quad (4.1-34)$$

The expression is given in the context of base-isolation systems.

4.2 Development of new spectral correction factors

The DCF expression in Eurocode 8 [CEN, 2004], Eqn. (4.1-15), renders constant values (0.55) for $\xi \geq 28\%$. This practical limit renders the expression inadequate for calculations of structures with PED systems, where effective damping can easily exceed 30% in the first mode and approach 100% in the higher modes, and contrast with ASCE-SEI 7/16 [ASCE, 2016] (see Table 4-5), where lower values (up to 0.25) are allowed. However, when the restriction $\eta \geq 0.55$ is removed, Expression (4.1-15) captures well the general trends of η_d for higher values of damping in the velocity region, as will be proved later on in this chapter. A disadvantage of the Expression is to render values dependent only on ξ ;

thus, the dependence of spectral ordinates with period has to be included in the spectrum definition. From a conceptual point of view, it seems more correct to define the 5% spectrum independently from its correction factors, but, as pointed out by Cardone et al [2009], the Eurocode 8 DCF is a remarkable compromise between simplicity and accuracy. The code offers no definition for η_v and η_a .

It would be possible to apply existing DCFs on the Eurocode 8 spectrum; however, one of the main conclusions derived from existing literature [Cameron & Green, 2007; Rezaeian et al, 2012] is the influence of the tectonic setting in the DCFs, due to the frequency content of the signal. For this reason, it seems more logical and correct to derive DCFs using sets of motions generated in the European continent. On the other side, although magnitude and site-to-source distance seem to be relevant parameters for DCFs, these parameters are not included in the code; instead, a uniform hazard spectrum (UHS) anchored at mapped ground accelerations and modified to account for site class, is used.

Another parameter clearly marked as relevant is duration. The possibility to indirectly include duration has been proposed by Mendis & Bommer [2006], by assuming representative significant duration values for regions of high seismicity (which correspond to the Type 1 spectrum in current Eurocode 8, associated with $D_{5-95\%} > 16s$), and for regions where magnitudes are unlikely to exceed Ms 5.5 (Type 2 spectrum, associated with $D_{5-95\%} \leq 16s$).

The main target of this study is to propose expressions for η_d , η_v and η_a , based on modifications to Eqn. (4.1-30), that extend its range of use to high damping ratios (over 28%) and long periods, accounting for the different factors listed above. At the same time, the study will be used to derive CF_v s, currently absent from the code proposal. The new correction factors, or simplified expressions derived from them, are intended for use with the current Eurocode 8 spectrum.

4.2.1 Description of study

4.2.1.1 Description of ground motions

Both horizontal components of far-field records with epicentral distance larger or equal than 10 km and moment magnitude $M_w \geq 5$ were chosen from the European Strong Motion Database [Ambraseys et al, 2002, 2004], adding up to 890 accelerograms. The records are binned according to site class (as defined in Eurocode 8 [CEN, 2004]) and significant duration $D_{5-95\%}$, (as defined in Bommer & Mendis [2005]) in eight 'groups'. The main features of the ground motions are listed in Table 4-6, and displayed graphically in Figure 4-1. The small number of records, events and stations, as well as the uncertainty about the soil classification in site class D, advice caution in the results obtained for this group. This stands to a lesser extent, for site class A. Figure 4-2 presents scatter plots of $D_{5-95\%}$ vs. magnitudes M_w and M_s , showing a positive correlation of both types of magnitude and $D_{5-95\%}$. Figure 4-3 presents scatter plots of magnitude and $D_{5-95\%}$ against epicentral distance. Again, positive correlation exists between $D_{5-95\%}$ and epicentral distance.

Information about motion processing can be found in Ambraseys et al [2004]. Due to instrument limits, and processing process (the band-pass filters cut-off frequencies were 0.25Hz and 25 Hz), caution must be exerted for results obtained for periods below 0.04s and over 4s. Bommer & Elnashai [1998] indicate that spectral displacement ordinates show a high sensitivity to filtering procedure at long periods. For these two reasons, the highest period range is bound to 4s in this study, with increasing uncertainty in spectral values as the period moves from 2s to 4s.

Table 4-6. Ground motions used in the study.

Significant Duration D5-95%			$\leq 16s$				$> 16s$			
Site Class			A	B	C	D	A	B	C	D
number of records			107	341	123	20	31	141	87	40
Events contributing			42	90	50	6	8	42	28	14
Stations contributing			24	76	27	6	15	46	27	7
$v_{s,30}$	mean	m/s	1253	514	246	148	1063	477	265	171
	median	m/s	1021	490	240	130	958	471	263	172
<i>PHA</i>	mean	mm/s/s	599	844	873	1263	945	656	416	383
	median	mm/s/s	253	382	427	553	588	318	245	239
	mean	g	0.061	0.086	0.089	0.129	0.096	0.067	0.042	0.039
	median	g	0.026	0.039	0.044	0.056	0.060	0.032	0.025	0.024
<i>PHV</i>	mean	mm/s	36.4	51.5	71.8	191.9	103.1	65.5	59.4	42.8
	median	mm/s	13.4	19.6	29.7	68.7	43.6	28.5	26.1	22.5
<i>PHD</i>	mean	mm	7.3	8.9	14.7	55.1	34.2	20.6	19.5	11.7
	median	mm	2.7	3.5	5.1	18.5	13.1	7.5	6.2	5.4
<i>Mw</i>	mean		5.7	5.7	5.9	6.8	6.8	6.3	6.5	5.9
	median		5.5	5.6	5.7	6.9	6.9	6.5	6.6	5.7
<i>Ms</i>	mean		5.5	5.5	5.7	6.7	6.8	6.2	6.4	5.8
	median		5.3	5.4	5.6	7.0	6.9	6.3	6.5	5.6
Epic. Dist.	mean	km	44.1	41.1	55.0	141.9	64.7	81.5	135.2	99.2
	median	km	31.0	28.0	42.0	149.0	50.0	58.0	111.0	87.5
$D_{5-95\%}$	mean	s	8.8	9.2	10.9	10.9	26.4	25.5	27.2	30.4
	median	s	8.5	8.7	11.2	11.0	21.4	23.3	24.2	30.5

Figure 4-1: Distribution of parameters: (a) motions with $D_{5-95\%} \leq 16s$; (b) motions with $D_{5-95\%} > 16s$.

Figure 4-2: Scatter plot of duration $D_{5-95\%}$ vs. magnitude: (a) M_w , $D_{5-95\%} \leq 16s$; (b) M_w , $D_{5-95\%} > 16s$; (c) M_s , $D_{5-95\%} \leq 16s$; (d) M_s , $D_{5-95\%} > 16s$.

Figure 4-3: Scatter plots of: M_w vs epicentral distance (a) $D_{5-95\%} \leq 16s$; (b) $D_{5-95\%} > 16s$. M_s vs epicentral distance (c) $D_{5-95\%} \leq 16s$; (d) $D_{5-95\%} > 16s$. Significant duration $D_{5-95\%}$ vs. epicentral distance (e) $D_{5-95\%} \leq 16s$; (f) $D_{5-95\%} > 16s$.

4.2.1.2 Description of analysis

In this study, SDOF systems of different natural period, T , and viscous damping ratio, ξ , were subjected to recorded accelerograms using step-by-step time-history (TH) elastic analysis (exact piecewise method [Nigam & Jennings, 1969; Wilson, 2002]). T ranged from 0s to 4s in 0.01s increments, and ξ from 10% to 90% in 10% increments. The response for $\xi = 5\%$ was also calculated for each record. From TH peak values, the displacement, velocity and acceleration spectra were found for every record r , and damping ratio ξ .

Figure 4-4 shows with dotted lines the 5% damped pseudo-acceleration spectral response ${}^rPS_a(T, 5\%)$ of every record r in every group, normalized to its peak horizontal acceleration rPHA . It was obtained multiplying the spectral relative displacement ${}^rS_d(T, 5\%)$, obtained from the time history analysis, by ω^2 . As discussed in the previous section, for ξ as low as 5%, PS_a is a reasonable approximation to S_a , i.e. ${}^rPS_a(T, 5\%) \approx {}^rPS_a(T, 5\%)$. The figure also shows the average normalized curve for all records (dashed line) and the normalized elastic

response spectrum of the current Eurocode 8, defined for the spectrum type associated with the group (for $D_{5-95\%} \leq 16s$, spectrum type 2; for $D_{5-95\%} > 16s$, spectrum type 1).

Figure 4-4. 5% spectra by site class and duration $D_{5-95\%}$.

η_d for record r , period T and damping ratio ξ , ${}^r\eta_d(T, \xi)$, is obtained as:

$${}^r\eta_d(T, \xi) = {}^rS_d(T, \xi) / {}^rS_d(T, 5\%), \quad (4.2-1)$$

The same rationale is applied to velocity (η_v) and acceleration (η_a). Figure 4-5 shows the DCFs obtained considering both mean and median values over the records in two groups (site class B for both durations) and two values of damping ratio. Similar results were obtained for other groups and damping ratios. For η_d and η_v , both statistics are very close,

with the mean being less erratic and slightly more conservative. For this reason, the mean is chosen as representative in this study, i.e.:

$$\eta_d(T, \xi) = \sum_{r=1}^{NR} \eta_d(T, \xi) / NR, \quad (4.2-2)$$

and similarly for η_v and η_a and also for CF_v . For η_a , the mean is taken for coherence with the other variables, but in this case, the differences between mean and median values are more conspicuous.

Figure 4-5. Mean and median DCFs.

4.2.2 Study results and proposal of approximation

In this Section, the results of the study are presented, and expressions to approximate them as a function of the relevant parameters (period, damping ratio, site-class, duration) are proposed, using the current Eurocode 8 expression, (Eqn. (4.1-30) without the 0.55 limit) as kernel. This function is hereby referred simply as η :

$$\eta = \sqrt{10 / (5 + 100\xi)}. \quad (4.2-3)$$

The expressions proposed contemplate the change of tendency of the DCFs between the short and long period ranges, with the minimum possible number of parameters; for this reason, the mathematical functions are cumbersome, but nonetheless it is advantageous to

handle a single expression for the complete period range. In every case, the parameters were adjusted by least-square fit, and the resulting coefficient of determination R^2 and Root Mean Square Error $RMSE$, were obtained.

Due to the small amount of records, results for site class D with duration $D_{5-95\%} \leq 16s$ were found erratic, and excluded from the regression process.

4.2.2.1 Damping correction factors for displacement

The results for spectral relative displacement DCF η_d , are shown in Figure 4-6, where the nine curves (one for each damping ratio) for each group (each combination of site class and $D_{5-95\%}$) are plotted in order. The irregularity of results for groups D and, to a lesser extent, A, are due to the reduced number of registers; it is also a feature in the DCFs for velocity and acceleration. Based on these results the following expression is proposed:

$$\eta_d(T, \xi) = \left[1 + \left(\frac{1}{\eta} - 1 \right) \left(\frac{T_Z}{T} \right)^{\alpha \frac{T - T_Z}{T}} \right]^{-1}, \quad (4.2-4)$$

where T_Z is a control period (approximately the point where the curve slope changes sign) and α a coefficient; both values are adjusted by nonlinear regression to the data for every group individually (Best Fit) and for all groups as a set (Simple Fit). The coefficient of determination R^2 and the Root Mean Squared Error (RMSE) for every group are listed in Table 4-7. Figure 4-7 and Figure 4-8 show a comparison between the proposed simple fit expression and the data for the different groups.

4.2.2.2 Damping correction factors for velocity

The results for the spectral relative velocity DCFs η_v are shown in XXXXX. To match these results the following expression is proposed:

$$\eta_v(T, \xi) = \left[1 + \beta \left(\frac{1}{\eta} - 1 \right) \left(\frac{T_Y}{T} \right)^{\gamma \frac{T - T_Y}{T}} \right]^{-1} \quad (4.2-5)$$

Again, parameters β , γ , and the control period T_Y are adjusted by nonlinear regression for each group individually (Best Fit), and for all groups as a set (Simple Fit); this is done in all groups except group D with $D_{5-95\%} \leq 16s$, where the scattering of results makes the regression senseless. These values and the corresponding R^2 and $RMSE$ are listed in Table 4-7. Comparative plots are given in Figure 4-10 and Figure 4-11.

4.2.2.3 Damping correction factors for acceleration

Results for spectral absolute acceleration DCFs, η_a , are shown in Figure 4-12, whereupon the following expression is proposed:

$$\eta_a(T, \xi) = \eta_d(T, \xi) + \varepsilon \xi^\lambda T \quad (4.2-6)$$

where η_d is given by expression (4.2-4). Because two options existed for η_d ('best fit' and 'simple fit'), both are contemplated in Table 4-7, where parameters ε and λ are regressed according to the value of η_d . The parameters listed for ' η_a Best Fit' correspond to a regression where η_d is calculated using Eqn. (4.2-4) and the parameters listed under ' η_d Best Fit' scenario in Table 4-7. The parameters listed for ' η_a Simple Fit' correspond to a regression where η_d is calculated with the parameters listed under ' η_d Simple Fit' scenario in Table 4-7. This simple adjustment is reasonably good for all groups. Comparative plots are shown in Figure 4-13 and Figure 4-14.

4.2.2.4 Correction factors for velocity

The values of CF_v obtained from the numerical study are presented in Figure 4-15. The proposed approximation is based on the following observations: i) the different curves tend to 0 as T approaches 0; ii) at every group, for a certain period T_1 , all curves are approximately coincident at a value of $CF_v = 1$. Adjustment was performed in two steps: first, the following expression was proposed for the 5% case:

$$CF_v(T, 5\%) = \frac{\exp(bT) - \exp(cT)}{\exp(bT_1) - \exp(cT_1)}, \quad (4.2-7)$$

with parameters b , c and T_1 ; the expression equals 1 for period T_1 , and 0 for $T = 0$. Least-square fit of Expression (4.2-7) to the available data resulted in the parameter values in, featuring a good adjustment to data in the complete period range. Then, a modified expression was proposed for other values of ξ :

$$CF_v(T, \xi) = CF_v(T, 5\%) + a(T - T_1)(1 - 1/\eta) \geq 0, \quad (4.2-8)$$

with parameter a . The resulting values for all groups are listed in Table 4-8, and shown graphically for a sample of ξ values in Figure 4-16 and Figure 4-17.

Table 4-7. Parameters for Eqns. (4.2-4), (4.2-5) and (4.2-6).

SIGNIFICANT DURATION D5-95%									
		≤ 16 s				> 16 s			
Site class		A	B	C	D	A	B	C	D
η_d Best Fit Eqn. (4.2-4)	T_Z (s)	0,792	0,423	0,461	0,789	1,180	1,365	1,158	1,054
	α	0,036	0,137	0,198	0,145	0,024	0,021	0,053	0,056
	R^2	0,982	0,996	0,979	0,898	0,900	0,877	0,896	0,840
	RMSE	0,022	0,011	0,024	0,056	0,056	0,060	0,057	0,073
η_d Simple Fit Eqn. (4.2-4)	T_Z (s)	0,44				1,30			
	α	0,13				0,03			
	R^2	0,931	0,977	0,955	0,855	0,908	0,900	0,900	0,832
	RMSE	0,045	0,025	0,035	0,063	0,053	0,056	0,055	0,071
η_v Best Fit Eqn. (4.2-5)	T_Y (s)	0,149	0,178	0,257	0,718	0,227	0,254	0,547	0,668
	β	1,591	1,589	1,572	1,564	1,898	1,847	1,695	1,748
	γ	0,650	0,791	0,845	1,167	0,535	0,613	0,928	1,201
	R^2	0,976	0,991	0,989	0,980	0,975	0,973	0,988	0,955
	RMSE	0,024	0,016	0,018	0,026	0,026	0,027	0,020	0,039
η_v Simple Fit Eqn. (4.2-5)	T_Y (s)	0,20				0,33			
	β	1,56				1,80			
	γ	0,80				0,70			
	R^2	0,919	0,949	0,923	-	0,879	0,899	0,921	0,872
	RMSE	0,051	0,040	0,048	-	0,066	0,060	0,053	0,068
η_a Best Fit with η_d from 'Best-Fit' Eqn. (4.2-6)	ε	0,826	1,047	0,901	0,474	0,387	0,474	0,474	0,457
	λ	1,240	1,204	1,185	1,110	1,235	1,301	1,237	1,315
	R^2	0,963	0,993	0,991	0,870	0,964	0,975	0,912	0,868
	RMSE	0,105	0,066	0,066	0,146	0,050	0,051	0,107	0,126
η_a Simple Fit Eqn. (4.2-6)	ε	1,00				0,47			
	λ	1,20				1,27			
	R^2	0,868	0,987	0,974	-	0,891	0,973	0,870	0,803
	RMSE	0,272	0,084	0,119	-	0,101	0,049	0,107	0,134

Table 4-8. Parameters for Eqns. (4.2-7) and (4.2-8).

$D_{5-95\%}$		$\leq 16s$				$> 16s$			
Site Class		A	B	C	D	A	B	C	D
$CF_v(T, 5\%)$ Best Fit Eqn. (4.2-7)	T_1	0.219	0.233	0.334	1.101	0.698	0.494	0.858	0.872
	b	0.175	0.259	0.258	0.239	0.150	0.151	0.193	0.172
	c	-9.793	-9.731	-8.686	-11.441	-14.406	-12.308	-9.750	-10.378
	R^2	0.984	0.998	0.997	0.997	0.985	0.996	0.997	0.974
	$RMSE$	0.048	0.028	0.031	0.020	0.030	0.018	0.018	0.048
$CF_v(T, 5\%)$ Simple Fit Eqn. (4.2-7)	T_1		0.248		-		0.656		
	b		0.245		-		0.165		
	c		-9.566		-		-11.614		
	R^2	0.515	0.972	0.984	-	0.946	0.989	0.977	0.959
	$RMSE$	0.086	0.033	0.023	-	0.019	0.009	0.017	0.020
$CF_v(T, \xi)$ Best Fit Eqn. (4.2-8)	a	-0.363	-0.362	-0.273	-0.184	-0.222	-0.269	-0.233	-0.235
	R^2	0.990	0.987	0.990	0.986	0.985	0.996	0.991	0.988
	$RMSE$	0.100	0.131	0.096	0.077	0.071	0.044	0.063	0.071
$CF_v(T, \xi)$ Simple Fit Eqn. (4.2-8)	a		-0.343		-		-0.254		
	R^2	0.943	0.970	0.912	-	0.933	0.975	0.982	0.961
	$RMSE$	0.233	0.201	0.289	-	0.152	0.112	0.090	0.125

Figure 4-6. Damping correction factors for displacement η_d .

Figure 4-7. η_d for $D_{5-95\%} \leq 16s$. Actual values (solid line) vs. 'simple fit' (dashed line).

Figure 4-8. η_d for $D_{5-95\%} > 16s$. Actual values (solid line) vs. 'simple fit' (dashed line).

Figure 4-9. Damping correction factors for velocity η_v .

Figure 4-10. η_v for $D_{5-95\%} \leq 16s$. Actual values (solid line) vs. 'simple fit' (dashed line).

Figure 4-11. η_v for $D_{5-95\%} > 16s$. Actual values (solid line) vs. 'simple fit' (dashed line).

Figure 4-12. Damping correction factors for acceleration η_a .

Figure 4-13. η_a for $D_{5-95\%} \leq 16s$. Actual values (solid line) vs. 'simple fit' (dashed line).

Figure 4-14. η_a for $D_{5-95\%} > 16s$. Actual values (solid line) vs. 'simple fit' (dashed line).

Figure 4-15. Correction factors for velocity CF_v .

Figure 4-16. CF_v for $D_{5-95\%} \leq 16s$. Actual values (solid line) vs. 'simple fit' (dashed line).

Figure 4-17. CF_v for $D_{5.95\%} > 16s$. Actual values (solid line) vs. 'simple fit' (dashed line).

4.2.3 Comparison with other studies

This Section compares several of the existing expressions (discussed in Section 4.1.1) and those proposed here (section 4.2.2), with the empirical results obtained from the study.

Regarding DCFs, the expressions proposed for η_d by Newmark & Hall [1982], Kawashima & Aizawa [1986], Wu & Hanson [1989], Tolis & Faccioli [1999], Sadek et al [1999], Ramirez et al. [2001], Lin & Chang [2004] and Hatzigeorgiou [2010], are considered, along with the current Spanish code NCSE-02 [Ministerio de Fomento de España, 2002], Eurocode 8 [CEN, 2004], current draft of the new Eurocode 8 (Expression (4.1-32)), and ASCE/SEI 7-16 [ASCE, 2016] proposals. The study by Hatzigeorgiou [2010] also contains expressions for η_a and η_v and both are included in the corresponding comparison, as are those for η_a derived by Lin & Chang [2004]. Finally, the proposal for η_a by Sadek et al [1999] and the values of η_v implied by their study, derived using Eqn. (4.1-14), are also included. Regarding CF_v , the proposals by Pekcan et al [1999], Sadek et al [1999] and Ramirez et al. [2001] are included.

Comparison between different proposals is shown in Figure 4-18 (η_d), Figure 4-19 (η_v) Figure 4-20 (η_a) and Figure 4-21 (CF_v), where the correction factors calculated in this study are compared with the expressions proposed here (Section 4.2.2) and in other studies (cited in the previous paragraph). Each figure presents a scatter plot in which the x-value of each point is the actual result from this study (Section 4.2.1), and the y-value is the approximation obtained from some expression. A perfect fit is indicated by the black diagonal line. Conservative values (prediction larger than actual value) are located in the triangular area upper left of the black diagonal line; un-conservative values (prediction smaller than actual value) are located in the lower right triangular area. Red points correspond to duration $D_{5-95\%}$ above 16s, whereas blue points correspond to duration below or equal to 16s. Grey points indicate that the values of damping ratio ξ are outside the scope of the corresponding study. For instance, referring to Figure 4-18(c), the study by Hatzigeorgiou [2010] is intended for values of $\xi \leq 0.5$, so values of $\xi = 0.6$ to 0.9 were extrapolated and are therefore displayed in gray color. Each sub-plot contains 7 cases: site class A, B, C with two durations ($D_{5-95\%} \leq 16s$ or $D_{5-95\%} > 16s$) and site class D, with $D_{5-95\%} > 16s$. The number of points in each sub-plot is $7 \times 3609 = 25263$ points. It must be acknowledged that the term ‘conservative’ here refers to the predicted variable, not to the effect of this variable in the structure.

The plots show that:

- The expressions proposed here for displacement η_d produce results very similar to those obtained using Hatzigeorgiou [2010], Lin & Chang [2004] and Sadek et al [1999] expressions.
- The un-conservative results obtained in some approximations for values of η_d close to 1 correspond to small values of T , for which many of the expressions pay no close

attention, but that in this study are accounted for. This spectral region is relevant for higher modes.

- The expression proposed in the draft of the Eurocode 8 for η_d renders reasonable results, in agreement with Ramirez et al (2001), except for short periods.
- The expressions in this study give slightly better approximations to η_v than other existing studies, with large deviations happening only in the conservative side. For other expressions un-conservative results are obtained for duration below 16s.
- The expressions in this study give a closer approximation to η_a , than all existing studies.
- For duration over 16s, the results for η_a reported here are in good agreement with Sadek et al [1999], but not with Hatzigeorgiou [2010] or Lin & Chang [2004]. For duration below 16s, all other studies fail to capture the trends in η_a , which shows increasing values up to about 4; for these cases, the expressions by other studies render un-conservative results.
- The approximation for CF_v proposed in this study gives the best results for both durations; the results here are in good agreement with Sadek et al [1999] for duration above 16s, for which the results obtained by Pekcan [1999] and Ramirez et al [2001] are slightly un-conservative; for duration below 16s all other studies give very un-conservative results.

These results are proved qualitatively using two indices referred to as deviation signed (DS) and deviation unsigned (DU), defined as:

$$DS(\xi) = \frac{1}{P} \sum_{i=1}^P \frac{y(T_i, \xi) - x(T_i, \xi)}{x(T_i, \xi)}, \quad (4.2-9)$$

$$DU(\xi) = \frac{1}{P} \sum_{i=1}^P \left| \frac{y(T_i, \xi) - x(T_i, \xi)}{x(T_i, \xi)} \right|, \quad (4.2-10)$$

Where P are the total number of periods T_i considered (for a single site class and duration, $P = 400$, excluding $T = 0$), y is the value of the expression, x is the corresponding value obtained from the numerical study. For both indices 0 indicates a perfect match between expression and data. DS indicates the overall distance of the expression to the actual values including sign: a positive value indicates a conservative result ($y > x$). DU is an indicator of the overall distance of the expression to the data. The value of DS and DU can be found for any particular site class and duration, but also for a set of groups. Here the indices are found for two sets: i) site classes A, B, C with $D_{5-95\%} \leq 16s$ (listed in Table 4-9 and Table 4-10) and ii) site classes A, B, C, D with $D_{5-95\%} > 16s$ (listed in Table 4-11 and Table 4-12). For DS , red color indicates un-conservative results ($x > y$) and green color indicates conservative results ($y > x$). For DU , a graded color scale is used to indicate the expression that gives the best approximation to the data for every spectral correction factor at every value of ξ . The following conclusions are drawn from the data:

- The simple fit expressions proposed in this study give conservative results (positive values of DS) for all durations and site classes.
- For $D_{5-95\%} \leq 16s$, the best fit and simple fit expressions proposed here give the best and second best results for all variables (S_d , S_v , S_a , CFV) and all levels of damping.
- For $D_{5-95\%} > 16s$, the best fit and simple fit expressions proposed here give the best results for S_v , S_a , and CFV in all levels of damping over 10%.
- For $D_{5-95\%} > 16s$, and S_d , the best expression (if un-conservativeness is not an issue), is that given by Lin & Chang [2004], followed by Sadek et al [1999], Wu & Hanson [1989], Spanish Code, ASCE/SEI 7 [2016] and finally the one proposed in this study. The difference, however, is very small; for Lin & Chang the overall DU considering all levels of damping is 7.9% and for the expression in this study 13.4%.
- For $D_{5-95\%} > 16s$, and S_d , if the conservativeness of the expression is considered, Lin & Chang [2004], Wu & Hanson [1989], and the Spanish Code render un-conservative results for some ranges of damping. The expression proposed by Sadek et al [1999], ASCE/SEI 7 [2016], and this study are conservative and therefore a better option for design purposes.

As a summary, the best fit and simple fit expressions provide the best set of SCF to Eurocode Spectrum type 2. For Eurocode Spectrum type 1, they provide the best set of η_v , η_a and CF_v , and a reasonable (although not the best) η_d .

Figure 4-18. η_d . Comparison of different expressions with this study results.

Figure 4-19. η_v . Comparison of different expressions with this study results.

Figure 4-20. η_a . Comparison of different expressions with study results.

Figure 4-21. CF_v . Comparison of different expressions with study results.

Table 4-9. Deviation Signed (DS) for $D_{5-95\%} \leq 16s$ (site classes A, B, C).

		ξ								
Expression		10%	20%	30%	40%	50%	60%	70%	80%	90%
η_d	Proposed Best Fit	1.2%	1.2%	1.3%	1.7%	2.4%	3.1%	3.9%	4.8%	5.7%
	Proposed Simple Fit	1.4%	1.7%	1.9%	2.5%	3.2%	4.0%	4.8%	5.8%	6.7%
	Hatzigeorgiou [2010]	1.5%	1.4%	1.8%	3.9%	8.4%	15%	25%	37%	52%
	Lin & Chang [2004]	-0.4%	-1.8%	-3.8%	-6.3%	-9.2%	-12%	-16%	-20%	-24%
	Ramirez et al [2000]	0.7%	1.3%	4.5%	5.5%	0.8%	-6.8%	-9.6%	-15%	-19%
	Tolis & Faccioli [1999]	4.6%	7.3%	8.6%	9.6%	11%	21%	30%	39%	48%
	Sadek et al [1999]	-0.9%	-1.8%	-2.8%	-3.8%	-4.8%	-5.4%	-6.9%	-9.9%	-14%
	Wu & Hanson [1989]	-1.3%	-3.8%	-6.6%	-9.8%	-13%	-17%	-21%	-26%	-30%
	Kaw. & Aiz. [1986]	-3.4%	1.2%	9.1%	18%	26%	35%	44%	52%	60%
	Newmark & Hall [1982]	3.8%	9.7%	28%	45%	60%	74%	88%	102%	114%
	NCSE02 [2002]	-8.3%	-13%	-13%	-13%	-12%	-10%	-9.2%	-7.8%	-6.4%
	EC8 [2004]	-1.4%	-4.0%	-2.5%	10%	22%	33%	43%	53%	63%
	ASCE SEI 7 [2016]	0.7%	1.3%	-1.3%	-4.5%	-7.6%	-10%	-13%	-15%	-17%
	Draft EC8 [2019]	-1.1%	-3.2%	-4.0%	-4.0%	-3.6%	-2.9%	-2.1%	-1.1%	0.0%
η_v	Proposed Best Fit	2.0%	2.2%	1.7%	1.2%	0.8%	0.6%	0.5%	0.5%	0.7%
	Proposed Simple Fit	2.4%	3.2%	3.1%	2.8%	2.7%	2.6%	2.7%	2.9%	3.1%
	Hatzigeorgiou [2010]	-3.5%	-7.7%	-10.0%	-12%	-13%	-14%	-15%	-18%	-22%
	Sadek et al [1999]	-1.3%	-3.8%	-5%	-5.5%	-4.8%	-3.2%	-0.7%	2.4%	6.2%
η_a	Proposed Best Fit	7.7%	5.1%	1.2%	-1.3%	-2.4%	-2.3%	-1.4%	0.1%	2.0%
	Proposed Simple Fit	9.2%	8.2%	5.4%	3.6%	3.1%	3.6%	4.8%	6.7%	8.9%
	Sadek et al [1999]	-3.6%	-13%	-21%	-25%	-29%	-31%	-33%	-35%	-36%
	Hatzigeorgiou [2010]	6.5%	1.1%	-11%	-22%	-31%	-38%	-44%	-49%	-53%
	Lin & Chang [2004]	-0.2%	-11%	-21%	-29%	-35%	-40%	-43%	-46%	-49%
CF_v	Proposed Best Fit	-1.0%	-1.7%	-1.9%	-1.9%	-1.9%	-1.7%	-1.5%	-1.3%	-1.0%
	Proposed Simple Fit	2.3%	1.6%	1.4%	1.4%	1.5%	1.8%	2.1%	2.4%	2.7%
	Ramirez et al [2000]	-47%	-51%	-52%	-54%	-54%	-55%	-55%	-55%	-55%
	Sadek et al [1999]	-24%	-27%	-28%	-29%	-29%	-30%	-30%	-30%	-30%
	Pekcan et al [1999]	-33%	-38%	-41%	-43%	-44%	-45%	-45%	-46%	-46%
	Draft EC8 [2019]	-32%	-34%	-35%	-35%	-35%	-35%	-35%	-36%	-36%

Table 4-10. Deviation Unsigned (DU) for $D_{5.95\%} \leq 16s$ (site classes A, B, C).

Expression		ξ								
		10%	20%	30%	40%	50%	60%	70%	80%	90%
η_d	Proposed Best Fit	1.6%	2.5%	3.2%	3.8%	4.5%	5.2%	6.1%	6.9%	7.8%
	Proposed Simple Fit	1.6%	3.1%	4.5%	5.8%	6.9%	7.8%	8.8%	9.8%	11%
	Hatzigeorgiou [2010]	2.1%	5.0%	7.3%	9.2%	11%	16%	25%	37%	52%
	Lin & Chang [2004]	2.0%	4.8%	7.2%	9.6%	12%	15%	18%	21%	24%
	Ramirez et al [2000]	2.1%	4.6%	7.4%	9.4%	9.4%	12%	14%	18%	21%
	Tolis & Faccioli [1999]	5.0%	8.4%	10%	12%	14%	23%	32%	41%	50%
	Sadek et al [1999]	2.2%	4.9%	7.2%	8.9%	11%	12%	14%	16%	20%
	Wu & Hanson [1989]	2.9%	7.0%	11%	14%	18%	22%	27%	31%	36%
	Kaw. & Aiz. [1986]	3.8%	4.7%	11%	19%	28%	36%	45%	53%	61%
	Newmark & Hall [1982]	4.6%	11%	30%	46%	61%	76%	89%	103%	116%
	NCSE02 [2002]	8.3%	13%	13%	14%	14%	14%	14%	14%	14%
	EC8 [2004]	2.5%	5.9%	7.0%	12%	23%	34%	44%	54%	64%
	ASCE SEI 7 [2016]	2.1%	4.6%	6.6%	8.9%	11%	14%	16%	18%	20%
	Draft EC8 [2019]	2.4%	5.7%	7.8%	9.2%	10%	11%	12%	13%	14%
η_v	Proposed Best Fit	2.4%	3.0%	2.8%	2.6%	2.6%	2.8%	3.3%	3.9%	4.5%
	Proposed Simple Fit	2.8%	4.1%	4.4%	4.5%	4.7%	5.1%	5.5%	5.9%	6.4%
	Hatzigeorgiou [2010]	3.7%	8.4%	11%	13%	15%	16%	18%	21%	25%
	Sadek et al [1999]	2.0%	4.6%	6.1%	6.6%	6.7%	6.8%	7.1%	7.7%	9.2%
η_a	Proposed Best Fit	7.7%	5.6%	3.8%	4.4%	5.3%	5.6%	5.6%	5.8%	6.5%
	Proposed Simple Fit	9.2%	8.3%	6.9%	7.3%	8.0%	8.8%	9.6%	11%	12%
	Sadek et al [1999]	4.2%	14%	22%	27%	30%	33%	34%	36%	37%
	Hatzigeorgiou [2010]	7.8%	9.3%	16%	25%	33%	39%	45%	49%	53%
	Lin & Chang [2004]	2.4%	11%	21%	29%	35%	40%	43%	46%	49%
CF_v	Proposed Best Fit	2.8%	3.5%	3.9%	4.4%	4.8%	5.2%	5.6%	5.9%	6.1%
	Proposed Simple Fit	6.4%	6.6%	7.1%	7.8%	8.4%	8.9%	9.4%	9.8%	10%
	Ramirez et al [2000]	47%	51%	52%	54%	54%	55%	55%	55%	55%
	Sadek et al [1999]	24%	27%	28%	29%	29%	30%	30%	30%	30%
	Pekcan et al [1999]	33%	38%	41%	43%	44%	45%	45%	46%	46%
	Draft EC8 [2019]	48%	53%	55%	57%	58%	59%	60%	61%	62%

Table 4-11. Deviation Signed (DS) for $D_{5-95\%} > 16s$ (site classes A, B, C, D).

		ξ								
Expression		10%	20%	30%	40%	50%	60%	70%	80%	90%
η_d	Proposed Best Fit	3.6%	6.2%	8.1%	10%	12%	14%	16%	19%	21%
	Proposed Simple Fit	3.5%	6.0%	7.9%	9.8%	12%	14%	16%	18%	21%
	Hatzigeorgiou [2010]	5.5%	9.9%	13%	18%	24%	34%	48%	64%	84%
	Lin & Chang [2004]	3.6%	6.4%	6.9%	6.0%	4.2%	1.9%	-1.0%	-4.4%	-8.2%
	Ramirez et al [2000]	4.9%	10%	17%	20%	17%	10%	8.7%	4.3%	0.8%
	Tolis & Faccioli [1999]	8.8%	16%	21%	24%	28%	41%	55%	68%	81%
	Sadek et al [1999]	3.2%	6.7%	8.3%	9.1%	9.5%	10%	10.0%	7.6%	3.3%
	Wu & Hanson [1989]	2.6%	4.3%	3.7%	1.8%	-0.9%	-4.2%	-8.0%	-12%	-17%
	Kaw. & Aiz. [1986]	0.5%	9.8%	22%	34%	46%	58%	71%	83%	96%
	Newmark & Hall [1982]	7.5%	18%	41%	63%	83%	103%	122%	141%	160%
	NCSE02 [2002]	-4.3%	-4.6%	-2.7%	-0.1%	2.7%	5.7%	8.7%	12%	15%
	EC8 [2004]	2.6%	4.2%	8.6%	25%	40%	56%	70%	85%	99%
	ASCE SEI 7 [2016]	4.9%	10%	10%	8.8%	7.0%	5.3%	3.8%	2.3%	1.1%
	Draft EC8 [2019]	3.2%	5.8%	8.1%	11%	13%	16%	19%	23%	26%
η_v	Proposed Best Fit	2.8%	3.3%	3.1%	3.0%	3.2%	3.6%	4.2%	4.9%	5.7%
	Proposed Simple Fit	3.1%	3.9%	3.7%	3.6%	3.6%	3.9%	4.4%	5.0%	5.7%
	Hatzigeorgiou [2010]	3.5%	6.7%	8.9%	10%	12%	13%	13%	12%	8.0%
	Sadek et al [1999]	5.7%	11%	14%	18%	22%	27%	33%	40%	47%
η_a	Proposed Best Fit	6.2%	7.0%	5.2%	3.7%	3.2%	3.6%	4.9%	6.9%	9.5%
	Proposed Simple Fit	6.4%	7.7%	6.4%	5.5%	5.4%	6.2%	7.8%	10%	13%
	Sadek et al [1999]	3.8%	7.5%	8.8%	11%	12%	13%	14%	14%	15%
	Hatzigeorgiou [2010]	15%	25%	22%	16%	8.2%	1.1%	-5.4%	-11%	-16%
	Lin & Chang [2004]	7.7%	10%	8.8%	5.6%	2.2%	-1.1%	-3.9%	-6.4%	-8.6%
CF_v	Proposed Best Fit	0.0%	-1.2%	-2.1%	-2.8%	-3.3%	-3.7%	-4.1%	-4.4%	-4.6%
	Proposed Simple Fit	1.9%	1.5%	1.3%	1.0%	0.8%	0.7%	0.6%	0.6%	0.6%
	Ramirez et al [2000]	-28%	-31%	-32%	-33%	-33%	-33%	-34%	-33%	-33%
	Sadek et al [1999]	2.2%	1.4%	1.7%	2.1%	2.6%	2.9%	3.3%	4.0%	4.8%
	Pekcan et al [1999]	-10%	-14%	-17%	-18%	-19%	-20%	-20%	-20%	-20%
	Draft EC8 [2019]	-10.0%	-10%	-9.8%	-9.1%	-8.6%	-8.2%	-8.0%	-7.8%	-7.8%

Table 4-12. Deviation Unsigned (DU) for $D_{5-95\%} > 16s$ (site classes A, B, C, D).

		ξ								
Expression		10%	20%	30%	40%	50%	60%	70%	80%	90%
η_d	Proposed Best Fit	3.8%	6.8%	9.1%	11%	13%	16%	18%	20%	22%
	Proposed Simple Fit	3.8%	6.9%	9.2%	11%	14%	16%	18%	21%	23%
	Hatzigeorgiou [2010]	5.7%	10%	14%	19%	25%	35%	48%	64%	84%
	Lin & Chang [2004]	4.1%	7.8%	9.0%	8.9%	8.1%	7.3%	7.3%	8.3%	11%
	Ramirez et al [2000]	5.3%	11%	18%	22%	19%	15%	15%	14%	14%
	Tolis & Faccioli [1999]	9.3%	18%	23%	27%	30%	44%	57%	70%	83%
	Sadek et al [1999]	3.5%	7.3%	9.2%	10%	11%	12%	12%	10%	9.1%
	Wu & Hanson [1989]	3.6%	6.6%	7.7%	8.6%	9.3%	10%	13%	16%	20%
	Kaw. & Aiz. [1986]	2.9%	12%	23%	36%	48%	60%	73%	85%	97%
	Newmark & Hall [1982]	8.7%	21%	44%	65%	85%	105%	124%	143%	161%
	NCSE02 [2002]	4.5%	6.3%	7.3%	8.7%	11%	13%	15%	18%	20%
	EC8 [2004]	3.8%	7.1%	12%	27%	43%	57%	72%	86%	101%
	ASCE SEI 7 [2016]	5.3%	11%	12%	12%	13%	13%	14%	14%	15%
	Draft EC8 [2019]	3.9%	7.6%	10%	13%	16%	19%	22%	25%	28%
	η_v	Proposed Best Fit	3.9%	5.9%	6.4%	6.8%	7.5%	8.2%	8.8%	9.5%
Proposed Simple Fit		4.3%	6.6%	7.1%	7.5%	8.0%	8.5%	9.0%	9.6%	10%
Hatzigeorgiou [2010]		3.7%	7.0%	9.2%	11%	12%	13%	13%	12%	10.0%
Sadek et al [1999]		6.1%	12%	16%	20%	24%	29%	35%	42%	49%
η_a	Proposed Best Fit	6.2%	7.5%	7.4%	8.6%	9.3%	9.8%	10%	11%	12%
	Proposed Simple Fit	6.4%	8.2%	8.5%	9.4%	10%	11%	12%	13%	15%
	Sadek et al [1999]	4.0%	8.0%	9.7%	12%	13%	14%	15%	17%	19%
	Hatzigeorgiou [2010]	16%	28%	26%	24%	24%	25%	26%	27%	28%
CF_v	Lin & Chang [2004]	8.1%	11%	10%	9.6%	10%	12%	13%	15%	16%
	Proposed Best Fit	2.5%	2.9%	3.1%	3.6%	4.1%	4.7%	5.4%	6.1%	6.7%
	Proposed Simple Fit	4.4%	5.0%	5.6%	6.1%	6.5%	6.9%	7.2%	7.6%	7.9%
	Ramirez et al [2000]	28%	31%	32%	33%	33%	33%	34%	33%	33%
	Sadek et al [1999]	5.3%	6.5%	7.4%	7.8%	7.9%	7.8%	7.9%	8.2%	8.6%
	Pekcan et al [1999]	12%	16%	18%	19%	20%	20%	20%	20%	20%
	Draft EC8 [2019]	34%	39%	41%	43%	44%	45%	47%	48%	49%

4.3 Summary and conclusions

Spectral analysis methods for yielding structures equipped with dampers require definition of response spectra for high values of viscous damping ratio. These can be obtained scaling 5%-damped elastic response spectra with damping correction factors (DCFs); independent values of correction factors are necessary for displacement (η_d), velocity (η_v) and acceleration (η_a).

DCFs are dependent on the properties of the earthquake input. Past studies show that the most relevant ground parameters are site-class and ground motion duration, which captures

the dependence on magnitude and source-to-site distance. DCFs are also dependent on structural properties: period and damping ratio; for very short and very long periods the factors converge to unity.

The current version of Eurocode 8 presents only an expression for displacement DCF, which renders constant values for viscous damping over 28%; no DCFs for velocity or acceleration are given. Within this framework, application of spectral methods for structures with dampers, neglect the important effect of high viscous damping ratio in flexible structures. For this reason, development of a set of DCFs to be used with the Eurocode 5% elastic spectrum is desirable.

In this study, DCFs are derived from a database of 880 far-field European records, binned according to site class (as defined in Eurocode 8) and significant ground motion duration $D_{5-95\%}$ (duration between instants of 5% and 95% of total Arias Intensity I_A). A total of 8 groups are created, corresponding to 4 site classes (A, B, C and D) and 2 durations ($D_{5-95\%} \leq 16s$ and $D_{5-95\%} > 16s$), which can be identified with spectrum types 2 (moderate seismicity) and 1 (high seismicity) of Eurocode 8, respectively. Expressions to approximate the values obtained as a function of period, damping ratio, site class and spectrum type are suggested, using the current expression in Eurocode 8 as kernel. The results are compared to other expressions proposed in the literature, showing in general a better fit to the data obtained in the study, particularly for $D_{5-95\%} \leq 16s$.

The behavior of viscous and visco-elastic devices is velocity-dependent. Analysis of these devices requires an accurate estimate of maximum velocity, which at 5% damping differs considerable from the 5% pseudo-spectral velocity. The previous motion database is used to derive Correction Factors for Velocity (CF_v) based on European motions. These factors relate true spectral velocity with spectral pseudo-velocity at a certain period and damping ratio. Approximate expressions are suggested and the values are compared to other expressions proposed in the literature, showing a better fit to the data in the study for all cases.

The spectral correction factors derived are intended to be used with the Eurocode 8 spectrum. The possible application in other tectonic settings must be studied. This study does not contemplate impulsive or near-source earthquakes. Due to the small amount of motions, the results obtained for soft soils are not conclusive.

4.4 Original contributions in this Chapter

- New expressions for displacement, velocity and acceleration damping correction factors (DCFs) are proposed.
- New correction factors for velocity CF_v were derived in the same study.
- These spectral correction factors (SCFs) were derived for the first time exclusively from European earthquakes and using the Eurocode 8 site class classification, which is coherent with their intended use with this code spectra.

- Additionally, the SCFs were binned for the first time by significant duration, to derive specific factors for Eurocode 8 Spectrum type 1 and 2.

Chapter 5

Influence of Maxwell Stiffness in the Response of Systems with Added Damping

5.1 Introduction

Basic mathematical models to represent the behavior of damping systems were presented in Chapter 2. Four different models were examined: pure damper, Kelvin model, Maxwell model and the Standard Solid model. The pure damping model offers a simple representation of the viscous component of the device. The Kelvin model is a combination of pure damper and pure spring in parallel, suitable for approximate representation of a SDOF structure with added dampers, where no influence of the latter's stiffness is considered. The Maxwell model includes the pure viscous and stiffness components of the damping system and offers a more realistic description of the damper behavior. Finally, the Standard Solid model uses the Maxwell element in parallel with the storey spring to describe in a more accurate way the behavior of a damped structure.

Neglecting the stiffness of the damping system is a source of error in numerical results, which is usually considered small. Recent tests [Kasai et al, 2008] reported in Chapter 2, indicate that the error might be larger than expected. A primary objective of this Chapter is to derive a practical rule to determine if auxiliary elements stiffness is significant enough to be included in the procedure formulation. Some approximate rules of thumb exist for SDOF systems, which are discussed hereby, but a more general expression applicable to MDOF systems is desirable. Because the NLRSA method presented in Chapter 2 is based on viscous energy dissipation, it seems logical that the relevance of auxiliary elements is assessed according to their influence on the energy dissipation of the Maxwell system. This concept is referred to as energy dissipation efficiency. Reasonable thresholds for this parameter result in small deviations between the pure damping system and the Maxwell system behavior.

For those cases where energy dissipation efficiency is low, the influence of auxiliary elements is relevant, and a method to include such influence in NLRSA is needed. Hereby, a numerical approximation to the Maxwell model is derived and used to include the influence of damping system stiffness components in a simplified Kelvin model representation of the damped structure.

5.1.1 Stiffness conditions of damping systems

Current seismic codes do not impose conditions on the stiffness of auxiliary elements of viscous damping systems. However, they require inclusion of these auxiliary elements in the mathematical models of the damped structure [ASCE, 2016, cf. Section 18.3.1].

Implications of the brace axial stiffness have been discussed by Lin & Chopra [2003]; these authors studied the influence of the ratio λ / T_1 , in the earthquake response of SDOF systems, where λ is the Maxwell relaxation time (defined as c/k_d for linear dampers in Section 2.4.10.4) and T_1 the first undamped period. The authors extended the Maxwell relaxation time λ to non-linear dampers. First, an equivalent linear damper is defined. This is done by equating the energy dissipation (Eqn. (2.4-20)) of a non-linear damper (defined by c and a) to the energy dissipation of a linear damper (defined by $c_{1,eq}$) (Eqn. (2.4-18)) under the same excitation frequency ω and amplitude u_0 :

$$c_{1,eq} = b c \omega^{a-1} u_0^{a-1}. \quad (5.1-1)$$

$c_{1,eq}$ has units of kN·s/mm. The definition of relaxation time ($\lambda = c_1/k_d$) is applied to this linear damper to obtain an expression for non-linear dampers:

$$\lambda_a = \frac{c}{k_d} b \omega^{a-1} u_0^{a-1}. \quad (5.1-2)$$

Using Eqns. (2.4-20) and (2.5-6), this equation can be transformed to:

$$\frac{k_s}{k_d} = \frac{\pi}{\xi} \frac{\lambda_a}{T}. \quad (5.1-3)$$

Lin & Chopra propose a practical range of $k_d / k_s \geq 5$; with a 30% damping ratio, this expression is transformed into ($\lambda_a/T_1 \leq 0.02$). The authors conclude that, if this condition is met, the influence of brace stiffness on response is negligible, regardless of the damper exponent.

Fu & Kasai [1998] studied the behavior of viscoelastic and viscous SDOF systems, suggesting a higher proportion $k_d / k_s \geq 10$ to ensure an effective damping behavior. The fact that different authors propose such different ranges of stiffness is indicative of the absence of a universally accepted criterion. Moreover, in both cases, this condition is only meaningful for SDOF systems, because a direct relationship between system stiffness and period, $T = 2\pi(m/k_s)^{0.5}$ is used in the derivation, which is no longer valid in MDOF systems. One objective of this work is to find a substitute condition for the brace stiffness, valid for MDOF systems.

5.1.2 Brace and damper stiffness

The damper element is generally mounted diagonally using a brace extender and attached to the structure by strong gusset plates. Figure 5-1 shows this type of mounting arrangement. A series of auxiliary elements (gusset plates, cleats, bolts, clevises, end plates, etc.), are needed in addition to the brace extender and damper. The damper element presents a certain stiffness k_{bd} which in many cases is assumed as infinite. The stiffness of other elements is difficult to assess with accuracy; some rules regarding the flexibility of joint elements are given in EN-1993-1-8: 2005, Clause 6.3 [CEN, 2005]. A simple approach is to estimate the stiffness of the brace extender on the basis of the total centerline length [Berquist et al., 2019]; this stiffness is assumed to represent the combined stiffness of the brace extender and other auxiliary elements, except the damper.

Figure 5-1. Stiffness of damper and auxiliary elements.

The total damping system stiffness k_b can be calculated as:

$$\frac{1}{k_b} = \frac{1}{k_{bo}} + \frac{1}{k_{bb}} + \frac{1}{k_{bd}}, \quad (5.1-4)$$

Regarding the damping system stiffness, there is little experimental data available. The data shown in Table 5-1 has been adapted from Akcelyan *et al.* [2016], and is taken from the E-Defense test data reported in Chapter 2. It shows the stiffness of different components measured during experiments. The value of k_{bo} has been obtained from Eqn. (5.1-4). k_{bb}^* is the reference value of brace stiffness obtained using the total length L instead of the brace driver length L_b . According to this data, the actual stiffness is 0.13 to 0.28 times this value, which is a very considerable reduction.

Most manufacturers recommend neglecting the damper stiffness (that is, assuming $k_{bd} = \infty$). Table 2-3 gives some experimental results reported by Kasai *et al.* [2008] for 12 non-linear dampers in the E-Defense project. For these dampers, with capacities between 500 kN and 2000 kN, the relationship c_b/k_{bd} (relaxation time λ in linear dampers) takes values around 0.5 s/mm^{1-a} , with $a = 0.38$. Constantinou & Symans [1992] reported values of $\lambda =$

0.006s for linear dampers with $c_b = 15.45$ N·s/mm, resulting in $k_{bd} = 2.58$ kN/mm. Reinhorn *et al.* [1995] obtained values of $\lambda =$ of 0.014s for dampers with $c_b = 201.4$ N·s/mm, resulting in $k_{bd} = 14.39$ kN/mm. Finally, Seleemah and Constantinou [1997] report $\lambda = 0.008$ s for $c_b = 17.7$ N·s/mm, corresponding to $k_{bd} = 2,21$ kN/mm.

Table 5-1. Stiffness data of E-Defense test damping system (adapted from Akcelyan *et al.*, 2016).

L mm	L_b mm	A_b mm ²	c_b kN·(s/mm) ^{0.38}	k_{bd} kN/mm	k_{bb} kN/mm	k_{bo} kN/mm	k_b kN/mm	k_{bb}^* kN/mm	k_b/k_{bb}^*
4025	2429	9121	49	119	751	144	60	453	13%
4025	2429	9121	49	119	751	144	60	453	13%
3947	2104	8380	98	193	797	315	104	425	24%
4706	2864	8380	98	193	585	332	101	356	28%
3947	2104	8380	98	193	797	315	104	425	24%
3947	2104	8380	98	193	797	315	104	425	24%
3849	1542	15323	196	438	1987	357	179	796	22%
4629	2322	15323	196	438	1320	356	171	662	26%

k_{bb} calculated with L_b . k_{bb}^* calculated with L . Damper exponent $a = 0.38$.

Original data in italics.

The difference in c_b/k_{bd} between the values reported by Kasai *et al.* and those of the latter three studies can be attributed to many different factors; the most obvious are the differences in damper exponent a and the sheer scale of the damper element, which obviously exerts an influence on the damper components. In the study described in Chapter 7, the dampers present forces which are similar to those reported by Kasai. In order to approximately evaluate the effect of the auxiliary mounting elements, Table 5-2 and Table 5-3 offer the same data as Table 5-1, adapted to frame 05D, (described in Annex B) with added damping systems Q.1.5 and Q.1.1 (both described in Annex D), respectively. The brace elements have been designed as described in Annex D. Values shown in blue correspond to specific parameters for the first storey of the frame. In calculating the values, a relaxation time $\lambda = 0.006$ s has been assumed for the linear dampers in order to estimate k_{bd} . For the non-linear dampers, k_{bd} has been taken from Table 5-1 and a displacement value $u_b = 30.25$ mm and frequency $\omega = 3.14$ s has been adopted.

Table 5-2. Stiffness data for Frame 05D with added damping system Q.1.5.

L mm	L_b mm	A_b mm ²	c_b kN· (s/mm) ^{0.5}	k_{bd} kN/mm	k_{bb} kN/mm	k_{bo} kN/mm	k_b kN/mm	k_{bb}^* kN/mm	k_b/k_{bb}^*	λ_b s
8006	6410	5440	30.28	119	170	144	47	136	35%	0.073
8006	6410	5440	30.28	119	170	144	47	136	35%	0.073
8006	6163	5440	30.28	193	177	315	71	136	52%	0.048
8006	6164	5440	30.28	193	177	332	72	136	53%	0.048
8006	6163	5440	30.28	193	177	315	71	136	52%	0.048
8006	6163	5440	30.28	193	177	315	71	136	52%	0.048
8006	5699	5440	30.28	438	191	357	97	136	71%	0.036
8006	5699	5440	30.28	438	191	356	97	136	71%	0.036

Table 5-3. Stiffness data for Frame 05D with added damping system Q.1.1.

L mm	L_b mm	A_b mm ²	c_b kN·(s/mm)	k_{bd} kN/mm	k_{bb} kN/mm	k_{bo} kN/mm	k_b kN/mm	k_{bb}^* kN/mm	k_b/k_{bb}^*	λ_b s
8006	6410	6690	2.33	388	209	144	70	167	42%	0.033
8006	6410	6690	2.33	388	209	144	70	167	42%	0.033
8006	6163	6690	2.33	388	217	315	97	167	58%	0.024
8006	6164	6690	2.33	388	217	332	98	167	59%	0.024
8006	6163	6690	2.33	388	217	315	97	167	58%	0.024
8006	6163	6690	2.33	388	217	315	97	167	58%	0.024
8006	5699	6690	2.33	388	235	357	104	167	62%	0.022
8006	5699	6690	2.33	388	235	356	104	167	62%	0.022

These tables are purely speculative, as the accurate values of k_{bd} , k_{bo} and L_b are unknown for the frames analyzed. The tables are only intended to provide an order of magnitude of the influence of the auxiliary elements stiffness. Assuming that k_{bd} and k_{bo} can take different values in both lists, the proportion k_b/k_{bb}^* takes values between 35% and 71% for non-linear dampers, and 42% to 62% for linear dampers. Thus, neglecting the influence of the damper and other mounting elements stiffness (i.e., assuming $k_{bd} = k_{bo} = \infty$) considerably over-estimates the total damping system stiffness. This over-estimation is larger for short brace elements, because k_b in Eqn. (5.1-4) is dominated by the lowest stiffness in the right term. The effect of this over-estimation is quantitatively discussed in Chapter 7. However, treatment of this effect seems at present difficult due to lack of accurate stiffness data corresponding to i) the damper itself; ii) the auxiliary mounting elements. The simple approach followed here is to calculate the net stiffness with the brace extender area and total length, and apply a reduction that, for the cases studied, has been taken of 50%.

5.2 Influence of stiffness in linear Maxwell model

The linear Maxwell model was presented in Chapter 2. It was found that the energy dissipated in the damping system can be expressed as a fraction of the energy dissipated in the pure viscous system. In the linear system this fraction β_W is coincident with the reduction of damper force β_d , Eqn. (2.4-59) or (2.4-61), and is defined by expression Eqn. (2.4-56), repeated here for convenience:

$$\beta_W = \frac{W}{W_{k_d=\infty}} = \frac{\pi c \omega u_0^2}{\pi c' \omega u_0^2} = \beta_d = \frac{1}{1 + \lambda^2 \omega^2}, \quad (5.2-1)$$

where λ is the relaxation time ($\lambda = c/k_d$), and all other variables were introduced in Chapter 2. β_W is referred here as *energy dissipation efficiency* (or simply efficiency) of the Maxwell system, and represents the amount of energy actually dissipated with respect to a theoretical maximum, i.e., pure viscous damping behavior. For the linear system, β_W is coincident with β_d (presented in Chapter 2). A minimum value $\beta_{W,\min}$ can be prescribed to achieve a minimum system efficiency:

$$\beta_W = \frac{1}{1 + (\omega \lambda)^2} \geq \beta_{W,\min}. \quad (5.2-2)$$

A reasonable value of $\beta_{W,\min}$ must be established on the basis of different reasons. First, the damper is a highly industrialized element, and consequently, it is much more expensive than the brace elements. Second, because of the relationship between strength and stiffness, flexible brace elements will be prone to instability problems resulting in unpredictable behavior of the damping system. Current engineering practice is that all auxiliary elements must remain elastic, not only for design earthquake level but also for maximum earthquake level, with an adequate safety factor.

Figure 5-2 shows the relationship between λ , ω and β_W according to Eqn. (5.2-2) (T is used instead of ω). For long periods (2-4s) a 90% efficiency requires values of λ below 0.1 to 0.2s. For the short period range, to keep a certain threshold of efficiency, increasingly larger values of λ are needed as T approaches 0. This means that, in a MDOF structure, the efficiency of the damping system will decrease for high periods. This phenomenon has been already described by some researchers [Makris & Constantinou, 1990; Constantinou & Symans, 1992] based on experimental results. Reduction of β_W implies increase of β_S (see Eqn. (2.4-57) in Chapter 2), which results in increased effective stiffness for the Maxwell system and reduced period of the damped system, further reducing the damping efficiency for higher modes.

Figure 5-2. Energy dissipation efficiency of a linear Maxwell system for different excitation periods. (a) 0-4s period range; (b) detail of 0-0.5s period range.

Minimum brace extender stiffness

Using the definition of the relaxation time, the minimum auxiliary element stiffness to reach a certain efficiency $\beta_{W,\min}$ can be derived from Eqn. (5.2-2):

$$k_{d,\min} = c\omega \sqrt{\frac{\beta_{w,\min}}{1-\beta_{w,\min}}} \quad (5.2-3)$$

If the spring contribution of the damper device is neglected, k_d derives solely from the brace component. For a damper with constant c_b mounted on a diagonal of cross-section area A_b , elastic modulus E_b and diagonal length L_b , Eqn. (5.2-3) can be transformed to the following condition, assuming that the structure is vibrating at mode j :

$$A_b \geq c_b \frac{2\pi L_b}{T_j E_b} \sqrt{\frac{\beta_{w,\min}}{1-\beta_{w,\min}}} \quad (5.2-4)$$

As an example, for frame 05D (defined in Annex B), the periods corresponding to the first to fifth mode are $\mathbf{T}^T = [2.0098; 0.6916; 0.3417; 0.2035; 0.1387] \cdot \text{s}$; a damper coefficient of 2.333 kN·s/mm (used in damping system Q.1.1, see Annex D) provides approximate elastic viscous damping ratios $\xi_a^T = [0.114; 0.317; 0.347; 0.321; 0.281]$. For a 90% effective stiffness ($\beta_w = 0.9$), the bracing area needed at each mode is $A \geq [8.8; 25.5; 51.5; 86.5; 126.9] \cdot \text{cm}^2$. If a square hollow section is adopted, (SHSH 180.10) with an area of 66.9 cm², loss of efficiency happens only at the 4th and 5th mode. The corresponding β_w coefficients, calculated from (5.2-2) for all modes are: $\beta_w^T = [0.998; 0.984; 0.938; 0.843; 0.714]$. Because the influence of higher modes in response is generally small (the cumulative effective weight ratio for the second mode is 91.4%), the conclusion is that the pure viscous damping model is suitable for representation of this particular system.

Relationship between energy efficiency and force response

An alternative measure of the efficiency of the Maxwell system is by comparison of the relative spring and damper force responses of the equivalent Kelvin system. For efficient behavior, the damper response should dominate. The relationship between this force efficiency and the energy efficiency previously defined is given by:

$$\frac{f'_{d0}}{f'_{s0}} = \frac{\beta_d c \omega}{\beta_s k} = \frac{\omega \lambda}{\omega^2 \lambda^2} = \frac{1}{\omega \lambda} = \sqrt{\frac{\beta_{w,\min}}{1-\beta_{w,\min}}} \quad (5.2-5)$$

A 90% energy efficiency results in a damper force 3 times the spring force in the equivalent Kelvin system.

5.3 Influence of stiffness in non-linear Maxwell model

5.3.1 Energy dissipation and force-displacement loops

Equations for the non-linear Maxwell model were presented in Chapter 2. The differential equation for harmonic motion excitation is repeated here for convenience:

$$\dot{f} = k_d u_0 \omega \cos(\omega t) - \frac{k_d}{c^{1/a}} |f|^{1/a} \text{sgn}(f). \quad (5.3-1)$$

Some interesting features of this equation are displayed in Figure 5-3, that shows the numerical solution for Maxwell systems with $a = 0.3$, different values of k_d , with constant values of $u_0 = 19.5 \text{ mm}$ and $\omega = 3.13 \text{ s}^{-1}$. First, at the peak force response f_0 , f is maximum or minimum, therefore $\dot{f} = 0$, and Eqn. (5.3-1) takes the form:

$$c^{1/a} \dot{u}_{f=f_0} = |f_0|^{1/a} \text{sgn}(f_0), \quad (5.3-2)$$

which corresponds to the equation for pure non-linear damping, see Eqn. (2.4-67), meaning that the point of maximum force is the intersection of the hysteresis loop of the system with the hysteresis loop for pure damping; these points are marked in Figure 5-3; in them, for the positive range, the following identity is fulfilled:

$$f_0 = c \omega^a u_{f=f_0}^a, \quad (5.3-3)$$

Second, at the intersection with the x axis, $f = 0$; therefore:

$$\dot{f} = k_d u_0 \omega \cos(\omega t) = k_d \dot{u}, \quad (5.3-4)$$

and, after recalling that $\dot{f} = df / dt$, $\dot{u} = du / dt$, Eqn. (5.3-3) can be rewritten as:

$$\left(\frac{df}{du} \right)_{f=0} = k_d, \quad (5.3-5)$$

that is, the slope of the tangent to the curve at this point ($f = 0$) is k_d . Third, at the intersection with the y axes, $u = 0$, and $\dot{u} = \omega u_0$; therefore, at the positive range:

$$\dot{f} = k_d u_0 \omega - \frac{k_d}{c^{1/a}} f_{d0}^{1/a}; \quad (5.3-6)$$

this equation can be transformed to:

$$\frac{df}{du} = k_d \left[1 - \left(\frac{f_{d0}}{c \omega^a u_0^a} \right)^{1/a} \right] = k_d \left[1 - \left(\frac{f_{d0}}{f_{d0, k_d=\infty}} \right)^{1/a} \right]. \quad (5.3-7)$$

The second term inside the bracket can be seen as a reduction factor for the slope at this point. If the damping force equals the pure damping force, the problem reduces to the pure damping problem and the tangent at this point is horizontal.

The spring stiffness of the damper response k_{eff} can be obtained from f_{s0} (force at maximum displacement) and u_0 , $k_{eff} = f_{s0} / u_0$. The spring component of the response is then given by $u \cdot k_{eff}$. Figure 5-3 shows the Maxwell model hysteresis loop (black line) superimposed to the loop obtained subtracting the spring part (red line) (in all loop figures

presented in this Chapter, the x-axis corresponds to u in mm and the y-axis to f in kN). Both curves are equivalent in terms of dissipated energy. From this plot, it is clear that the shape of the dissipative loop does not follow the shape of the non-linear pure viscous model (blue curve).

Figure 5-3 Force-displacement loops for Maxwell models with non-linear dampers, $a = 0.3$; (a) $k_d = 100$ kN/mm; (b) $k_d = 20$ kN/mm; (c) $k_d = 5$ kN/mm; (d) $k_d = 2$ kN/mm;. Units are mm, kN.

Figure 5-4 shows the force-displacement loops for six different values of damping exponent a (1, 0.8, 0.5, 0.3, 1.2, 2). The damping coefficient c_1 , corresponding to case $a = 1$, is fixed as 2.333 kN·s/mm. Values of damping coefficient c_a for other cases ($a \neq 1$), have been obtained to obtain the same dissipated energy as in the linear case (c_1), assuming pure damping in both ($k_d \rightarrow \infty$):

$$c_a = c_1 \frac{\omega^{1-a} u_0^{1-a}}{b} \quad (5.3-8)$$

where c_a is the damping coefficient for exponent a , and c_1 is the damping coefficient for the linear case ($a = 1$). This expression was previously presented by Lin & Chopra [2003]. Numerical values of interest are shown in Table 5-4. The non-dimensional values

presented in the table are referred to as *relative peak response* values, because they represent the peak value of a variable referred to a relevant reference value. These reference values are: for dissipated energy W_d , the maximum possible dissipated energy $W_{d,kd=\infty}$. For force at maximum displacement f_{s0} , the maximum spring force $k_d \cdot u_0$. For force at maximum velocity f_{d0} and peak force f_0 , the maximum damping force $f_{d0,kd=\infty}$. For displacement at peak force u_{f0} , maximum displacement u_0 .

$$\beta_W = \frac{W_d}{W_{d,kd=\infty}} = \frac{W_d}{b\pi c \omega^a u_0^{1+a}}, \quad (5.3-9)$$

$$\beta_s = \frac{f_{s0}}{f_{s0,c=0}} = \frac{f_{s0}}{k_d u_0}, \quad (5.3-10)$$

$$\beta_d = \frac{f_{d0}}{f_{d0,kd=\infty}} = \frac{f_{d0}}{c \omega^a u_0^a}, \quad (5.3-11)$$

$$\beta_f = \frac{f_0}{f_{0,kd=\infty}} = \frac{f_0}{c \omega^a u_0^a}. \quad (5.3-12)$$

$$\beta_u = \frac{u_{f0}}{u_0}. \quad (5.3-13)$$

These values are the counterpart to the ones defined for the linear case, except that β_W , β_f and β_u are now included.

Data show how the effectiveness of damping can be strongly affected by the stiffness component k_d . Lower values of a are more sensitive to this parameter; for instance, for $k_d = 5$ kN/mm, the relative values of W for $a = 1$ and $a = 0.3$ are 0.319 and 0.185; a similar situation happens with the pure damper force f_{d0} , and maximum force f_0 .

Table 5-4 shows how the energy dissipated by the same damping system is strongly influenced by k_d . With very high stiffness ($k_d = 10000$ kN/mm), a maximum value of dissipated energy is reached. The stiffness component is negligible, and the behavior is that of a pure damper. The fraction of this energy dissipation actually achieved is dependent on k_d and other parameters of the damping system (a , c , ω , u_0). It can be seen in the table that a value of $k_d = 10$ kN/mm results in 61% of the pure damping energy for $a = 0.3$; this is indicative of a poor design of the brace extender, i.e., lack of brace stiffness. f_0 and f_{d0} present a similar dependency, whereas f_{s0} presents an inverse dependency. These values are small and have been selected so that the effect of the stiffness component is more evident.

Figure 5-4 Force-displacement loops for Maxwell models with $a = 2, 1.2, 0.8, 0.5, 0.3$.

Table 5-4. Relative peak response for Maxwell models subjected to forced harmonic excitation.

a	c	k_d	β_W	β_f	β_d	β_s	u_{f0}/u_0	f_0/f_{d0}	δ/π
	$\text{kN}\cdot(\text{s}/\text{mm})^a$	kN/mm							
1	2.33	10000	1.000	1.000	1.000	0.000	0.000	1.000	0.639
		20	0.882	0.939	0.882	0.118	0.345	1.065	0.036
		10	0.652	0.807	0.652	0.348	0.588	1.239	0.064
		5	0.319	0.565	0.319	0.681	0.824	1.771	0.098
0.8	5.12	10000	1.000	1.000	1.000	0.000	0.000	1.000	0.000
		20	0.887	0.965	0.928	0.133	0.291	1.040	0.030
		10	0.657	0.856	0.703	0.367	0.567	1.217	0.061
		5	0.302	0.609	0.317	0.726	0.844	1.920	0.102
0.5	16.51	10000	1.000	1.000	1.000	0.000	0.000	1.000	0.639
		20	0.872	0.991	0.980	0.163	0.187	1.011	0.019
		10	0.647	0.928	0.790	0.400	0.509	1.175	0.054
		5	0.251	0.688	0.278	0.806	0.879	2.474	0.109
0.3	35.86	10000	1.000	1.000	1.000	0.000	0.000	1.000	0.000
		20	0.842	0.998	0.997	0.193	0.100	1.002	0.010
		10	0.613	0.969	0.850	0.434	0.437	1.140	0.046
		5	0.185	0.755	0.202	0.877	0.920	3.740	0.119
1.2	1.06	10000	1.000	1.000	1.000	0.000	0.000	1.000	0.639
		20	0.871	0.909	0.835	0.106	0.386	1.090	0.040
		10	0.641	0.761	0.606	0.330	0.603	1.257	0.066
		5	0.329	0.526	0.313	0.641	0.809	1.682	0.096
2	0.04	10000	1.000	1.000	1.000	0.000	0.000	1.000	0.639
		20	0.792	0.777	0.660	0.069	0.471	1.177	0.050
		10	0.579	0.604	0.461	0.262	0.628	1.310	0.069
		5	0.333	0.408	0.266	0.512	0.771	1.536	0.089

Note: $u_0 = 19.8 \text{ mm}$, $\omega = 3.132 \text{ Hz}$.

5.3.2 Numerical approximation to the Maxwell system behavior

In reference to the Maxwell system defined by c , a , k_d , the following non-dimensional variable is introduced:

$$q = \omega^a u_0^{a-1} \lambda, \quad (5.3-14)$$

where λ is c/k_d , although in this case the units are $\text{s}^a \cdot \text{mm}^{1-a}$. The variable q is the non-linear counterpart to $\lambda \cdot \omega$ defined for the linear case.

The physical meaning of q can be clarified by modifying Eqn. (5.3-14):

$$q = \frac{c_d \omega^a u_0^a}{k_d u_0}, \quad (5.3-15)$$

where the numerator is the maximum force for a pure damping system defined by c_d , a , and subjected to harmonic motion of frequency ω and amplitude u_0 , and the denominator is

the pure spring force for a pure spring system defined by k_d . Thus q can be seen as the proportion between maximum forces of these two pure systems.

In order to study the variation of these values with brace properties, a parametric study has been carried out, for the following combination of parameters:

- damper exponent $a = [1; 0.8; 0.5; 0.4; 0.3; 0.2]$,
- damping coefficients $c = [1; 5; 10; 25; 50] \text{ kN} \cdot (\text{s/mm})^a$,
- maximum displacements $u_0 = [10; 20; 30] \text{ mm}$,
- excitation periods $T = [0.1; 0.25; 0.5; 1; 2; 4; 8] \text{ s}$,
- diagonal stiffness k_d between 1000 and 1 kN/mm, with more points in the range 100 and 10 kN/mm.

For every combination of values, differential equation (5.3-1) has been solved numerically for 5 cycles of motion, in order to reach the steady-state condition. The final point was taken as a set of initial conditions and the equation was solved again for 1 cycle, retaining the maximum force f_0 , damping force f_{d0} , force at maximum displacement f_{s0} , and dissipated energy W . 630 cases (out of 12,600) were removed from the set because numerical convergence was not achieved. Results of the study, displayed in Figure 5-5, show the relationship between peak values of different variables (expressed in non-dimensional form $\beta_W, \beta_d, \beta_f, \beta_s$ as indicated above) with parameters q and a . A value of $q = 3$ corresponds (approximately) to $\beta_W = 0.1$ for the linear damper and has been taken as a maximum practical limit. The behavior for $q > 3$ is essentially that of a pure spring.

Figure 5-5 Non-linear Maxwell model. Relationship between $\beta_W, \beta_d, \beta_s, \beta_f$ and parameter q .

Relationships between non-dimensional forms of some variables are also explored in Figure 5-6.

Figure 5-6 Non-linear Maxwell model. Relationship between non-dimensional variables.

The following trends can be identified:

- The relationship between β_s and β_w is only mildly dependent on a ; this is also the case with the proportion between β_u and β_d , and between β_d and $\beta_f^{a/2}$.
- There is an almost linear relationship between β_w and $1-\beta_s$. This is an important relationship showing that the spring component diminishes as the energy dissipating efficiency increases.
- There is an almost linear relationship between β_d and $\beta_f^{a/2}$.

- For low values of β_w (below 0.3), an almost linear relationship exists between β_w and β_d .
- For low values of β_f ($q > 3$), f_0/f_{s0} is practically one, indicating that the behavior is essentially that of a spring.

These trends are subsequently used to find numerical approximations to the different variables. q can be used to approximate all variables, but in some cases due to the relationships shown in Figure 5-6, it is simpler to first derive the value of β_w , and then other variables from this. Two types of expressions are proposed: i) simple expressions for high damping energy efficiency; ii) high order polynomial approximations to cover a wider range of damping energy efficiency.

5.3.2.1 Polynomial regression

All relative peak values are approximated with a single polynomial expression

$$\begin{aligned} \beta(x, a) = & r_{00} + r_{10}x + r_{01}a + r_{20}x^2 + r_{11}xa + r_{02}a^2 + r_{30}x^3 + r_{21}x^2a + r_{12}xa^2 + \\ & r_{40}x^4 + r_{31}x^3a + r_{22}x^2a^2 + r_{50}x^5 + r_{41}x^4a + r_{32}x^3a^2, \end{aligned} \quad (5.3-16)$$

where β is the relative peak value approximated ($\beta_w, \beta_f, \beta_s, \beta_d, \beta_u$), x can be q or β_w , and a is the damper exponent. Coefficients are listed in Table 5-5, along with the application range of the approximation. A polynomial approximation of q from β_w is also included. The goodness of fit is expressed using the coefficient of determination R^2 , and the *RMSE* (root-mean-square error), also listed in the table. The number of terms of the approximation has been chosen to ensure a good fit in the whole application range. The approximations are displayed in Figure 5-7 (solid lines), along with the actual values (markers). For β_f two approximations are given, one dependent on q , valid for a larger range, and one dependent on β_w , valid for $\beta_w \geq 0.1$. For β_d , the approximation proposed is only valid for $\beta_w \geq 0.3$. To extend both expressions to the whole range, linear interpolation can be performed:

$$\beta_f \approx \beta_w \frac{\beta_f[\beta_w = 0.1]}{0.1}. \quad (5.3-17)$$

$$\beta_d \approx \beta_w \frac{\beta_d[\beta_w = 0.3]}{0.3}. \quad (5.3-18)$$

Table 5-5. Maxwell model under force harmonic excitation. Coefficients for polynomial approximation to relative peak response values.

	β_w	β_f	β_f	β_s	β_d	β_u	q
x	q	q	β_w	β_w	β_w	β_w	β_w
Range	$0 \leq q \leq 3$	$0 \leq q \leq 3$	$0.1 \leq \beta_w \leq 1$	$0 \leq \beta_w \leq 1$	$0.3 \leq \beta_w \leq 1$	$0 \leq \beta_w \leq 1$	$0.1 \leq \beta_w \leq 1$
R^2	0.9998	0.9990	0.9998	0.9999	0.9993	0.9994	0.9994
$RMSE$	0.0044	0.0057	0.0017	0.0038	0.0043	0.0101	0.01276
r_{00}	1	1	0.8141	1	0.3595	1	1.206
r_{10}	-0.5712	0.09663	1.759	-0.4531	-5.986	-0.4381	-4.92
r_{01}	-	-0.02101	-1.21	-	3.941	-0.01223	3.314
r_{20}	-1	-0.07388	-4.753	-1.8810	29.75	0.5641	24.44
r_{11}	1.4350	0.04175	0.9131	0.2845	-14.05	-0.2363	-15.77
r_{02}	-	0.03184	0.5232	-	-5.103	-0.01153	0.03144
r_{30}	1	-0.2052	6.32	2.2790	-40.76	-9.574	-57.85
r_{21}	-0.8081	-0.4772	2.164	0.2032	-0.6124	4.83	33.35
r_{12}	-0.8960	-0.2622	-0.3809	-0.6618	25.98	0.1075	-0.07981
r_{40}	-0.3105	0.1094	-4.595	-0.9489	18.73	12.24	58.05
r_{31}	0.0507	0.3193	-1.545	-0.4473	30.33	0.8085	-28.14
r_{22}	0.6695	0.3073	-1.91	0.6195	-39.53	-4.661	-2.464
r_{50}	0.0323	-0.01506	1.455	-	-1.085	-3.769	-20.95
r_{41}	0.0211	-0.05144	-0.3194	-	-19.62	-5.519	7.371
r_{32}	-0.1229	-0.08718	1.766	-	18.65	4.709	2.47

Figure 5-7 Non-linear Maxwell model. Polynomial approximation to peak relative response values (solid lines) vs. actual values (markers): (a) β_W vs. q ; (b) q vs. β_W ; (c) β_f vs. q ; (d) β_f vs. β_W ; (e) β_s vs. β_W ; (f) close-up of β_s vs. β_W ; (g) β_d vs. β_W ; (h) β_u vs. β_W .

5.3.2.2 Simple expressions

Simple expressions are also derived for the variables, with a more limited application range.

Simple estimate of β_w

An extension of Eqn. (5.2-1) is proposed for the range $0.8 \leq \beta_w \leq 1$:

$$\beta_w \approx \frac{1}{1+q^{1+a}}, \quad (5.3-19)$$

with $a \leq 1$.). If $a = 1$, this expression reduces to Eqn. (5.2-2). For the application range indicated, $R^2 = 0.9294$ and $RMSE = 0.0134$. The expression is generally conservative in this range (it produces a value slightly below the actual one. Values obtained with Eqn. (5.3-19) are compared to the actual values in Figure 5-8.

Simple estimate of β_s

Based on the almost linear relationship shown in Figure 5-6, the following expression is proposed:

$$\beta_s \approx 1 - \beta_w, \quad (5.3-20)$$

with $R^2 = 0.9967$, $RMSE = 0.02396$ in the whole range of β_w . This linear approximation is exact for $a = 1$, Eqn. (2.4-55). Using β_s an equivalent spring stiffness of the Maxwell system can be defined as:

$$k_{eq} \approx \beta_s k_d. \quad (5.3-21)$$

Simple estimate of β_d

A simple expression for the range $0.8 \leq \beta_w \leq 1$ is:

$$\beta_d \approx 1 - a^2 + \beta_w a^2, \quad (5.3-22)$$

with $R^2 = 0.9632$, $RMSE = 0.004756$ in the mentioned interval. This expression slightly overestimates β_d . Values obtained with Eqn. (5.3-22) are compared to the actual values in Figure 5-8. This expression is consistent with β_d for $a = 1$, defined in Eqn. (2.4-56).

Simple estimate of β_f

An approximation can be obtained from the quasi-linear relationship between $\beta_f^{2/a}$ and β_d , shown in Figure 5-6:

$$\beta_f \approx \beta_d^{a/2}, \quad (5.3-23)$$

with $R^2 = 0.9947$, $RMSE = 0.0263$ in the whole range. Values obtained with Eqn. (5.3-22) are compared to the actual values in Figure 5-8.

Simple estimate of β_u

The value of β_u can be correlated with β_d , as shown in Figure 5-6:

$$\beta_u \approx \sqrt{1 - \beta_d} \quad (5.3-24)$$

with $R^2 = 0.9974$, $RMSE = 0.02178$ in the whole interval. Values obtained with Eqn. (5.3-24) are compared to the actual values in Figure 5-8.

Figure 5-8 Non-linear Maxwell model. Simple approximations: (a) β_W vs. q ; (b) β_d vs. β_W ; (c) β_f vs. β_d ; (d) β_u vs. β_d .

5.3.2.3 Estimate of viscous damping ratio

The viscous damping ratio can be estimated by Jacobsen's expression. The energy dissipation is obtained from Eqn. (5.3-9); the equivalent elastic stiffness is given by Eqn. (5.3-21):

$$\xi = \frac{W_d}{4\pi W_s} = \frac{\beta_W W_{d,kd=\infty}}{4\pi 0.5\beta_s k_d u_0^2} = \frac{\beta_W}{\beta_s} \frac{b\pi c \omega^a u_0^{1+a}}{2\pi k_d u_0^2} = \frac{b}{2} \frac{\beta_W}{\beta_s} q, \quad (5.3-25)$$

with β_W and β_s calculated using either the simple or polynomial expressions given in previous sections. Once β_W and β_f are determined, the response of the damping system is characterized both in terms of energy and force:

$$W_d = \beta_W b\pi c \omega^a u_0^{1+a}, \quad (5.3-26)$$

$$f_0 = \beta_f c \omega^a u_0^a. \quad (5.3-27)$$

From equilibrium considerations, the force response in the Maxwell system is the same for the damper and the bracing element; therefore, the maximum response of the damping system in terms of force, both for damper and brace, is given by f_0 .

5.3.3 Condition for minimum diagonal stiffness

In order to achieve a minimum damper efficiency, a fraction of the dissipated pure damper energy must be preserved in the Maxwell configuration. This condition corresponds to a minimum value $\beta_{W,\min}$. For values of $0.8 \leq \beta_{W,\min} \leq 1$, Eqn. (5.3-19) is approximately valid and therefore:

$$q \approx \left(\frac{1 - \beta_{W,\min}}{\beta_{W,\min}} \right)^{\frac{1}{1+a}}, \quad (5.3-28)$$

with $R^2 = 0.958$ and $RMSE = 0.0235$ in the mentioned interval. This approximation is shown in Figure 5-9.

Figure 5-9 Non-linear Maxwell model. Simple approximation β_W vs. q , for $0.8 \leq \beta_W \leq 1$.

From this equation follows that:

$$k_d \geq c \omega^a u_0^{a-1} \left(\frac{\beta_{W,\min}}{1 - \beta_{W,\min}} \right)^{\frac{1}{1+a}}. \quad (5.3-29)$$

For $a = 1$ this expression simplifies to Eqn. (5.2-3).

To extend the expression for values of $0.1 \leq \beta_{W,\min} \leq 1$, a two-step procedure is suggested. First, the value of q should be found using the polynomial approximation. After q is determined, the value of k_d can be found from Eqn. (5.3-14) as:

$$k_d = \frac{c}{q} \omega^a u_0^{a-1}. \quad (5.3-30)$$

The effective viscous damping ratio can be estimated from (5.3-25). The brace element should also withstand f_0 with the adequate level of safety. This element should be designed to remain elastic for this force, as no yielding was assumed when deriving this approximate model.

5.4 Influence of damping system stiffness in standard solid model

5.4.1 Energy dissipation and force-displacement loops

The basic equations for the standard model (Maxwell model representing the damping system in parallel with linear spring representing the structural stiffness) were given in Section 2.4.10.7. The basic equation is repeated here for convenience:

$$\dot{f} = k_d \dot{u} - \frac{k_d}{c^{1/a}} |f_M|^{1/a} \text{sgn}(f_M) + \dot{f}_s, \quad (5.4-1)$$

where $f_s = k_s \cdot u$ is the constitutive equation of the spring component. A plot of this expression is shown in Figure 5-10(a). Again, some interesting features can be discussed. First, at $u = 0$, it reduces to:

$$\left(\frac{df}{du} \right)_{u=0} = k_s + k_d \left[1 - \left(\frac{f_{d0}}{c \omega^a u_0^a} \right)^{1/a} \right], \quad (5.4-2)$$

and therefore:

$$\left(\frac{df}{du} \right)_{u=0} = k_s + k_d (1 - \beta_d^{1/a}); \quad (5.4-3)$$

therefore the slope at this point is given by k_s and a fraction of k_d . Second, at the point of maximum total force ($f = f_{t,0}$), Eqn. (5.4-1) equals 0 and after some modifications:

$$f_{t,0} = c \left(1 + \frac{k_s}{k_d} \right)^a \omega^a u_{f0}^a + k_s u_{f0}. \quad (5.4-4)$$

This expression can be seen as the maximum force in an equivalent Kelvin system with stiffness k_s and a modified damping coefficient c' :

$$c' = c \left(1 + \frac{k_s}{k_d} \right)^a. \quad (5.4-5)$$

Third, at the intersection of the loops with the straight line representing the spring with stiffness k_s , it is $f = k_s \cdot u$; replacing in Eqn. (5.4-1) and after some algebraic modifications:

$$\left(\frac{df}{du}\right)_{f=k_s u} = k_s + k_d; \quad (5.4-6)$$

that is, the slope of the tangent to the loop at this point is $k_s + k_d$ (point marked g in Figure 5-10). At the point of maximum force in the Maxwell system (point c) the slope of the tangent to the loop is equal to k_s .

In this expression $f = f_M + k_s \cdot u$; deriving and replacing in the first term, the equation reduces to the Maxwell system equation. For forced harmonic excitation the solution to Eqn. (5.4-1) can be found solving the equivalent Maxwell system with parameters a , c , ω , u_0 , k_d , and adding the term $k_s \cdot u$ due to the storey stiffness. The Maxwell system can be solved with the numerical methods discussed in the previous section. The complete response can be found adding the spring response due to k_s .

Based on this concept, the behavior of the SD-S model is shown schematically in Figure 5-10, where the relevant variables are defined. The figure shows the response for three related systems: a) complete system (Maxwell + spring); b) Maxwell system obtained subtracting the spring component from the total response; c) Maxwell system where the effective spring component due to $\beta_s \cdot k_d$ has been removed, and a non-elliptical shape is evident. Systems (a), (b) and (c), dissipate the same amount of energy, and present the same damper force, f_{d0} , at peak velocity. The bracing system is subjected to a peak force $f_{M,0}$, which is not dependent on k_s . The storey spring is subjected to a peak force $f_{s,s0}$. The peak force in the complete system $f_{t,0}$ is participated from f_{d0} and $f_{t,s0} = f_{M,s0} + f_{s,s0}$; however, each one of the forces in the damping system and storey at this point are not necessarily at their peak. Finally, the damping ratio must be calculated with respect to the complete system stiffness k_t .

Figure 5-10: Standard Solid Model under forced harmonic excitation: (a) force-displacement loop of complete system including storey spring; (b) loop of Maxwell system without spring component vs. pure damper loop.

Numerical solution of the Standard Model equations is shown in Figure 5-11 for $a = 0.5$, $c = 100 \text{ kN} \cdot (\text{s/mm})^{0.5}$, $u_0 = 19.5 \text{ mm}$, $\omega = 6.28 \text{ Hz}$, two values of k_d (1000 kN/mm and 40 kN/mm) and two values of k_s (100 kN/mm and 10 kN/mm). For high values of k_d (left column) the shape of the hysteresis loop without effective spring component (red curves) resembles that of pure damping (blue curves). For low values of k_d (right column) the shapes are distorted and there is a pronounced loss of efficiency. The proportion of dissipated energy to pure damping energy does not depend on k_s . This result is consistent with the Maxwell model already discussed. Storey stiffness (blue dotted line) influences only the slope of the loop, not its area. The maximum force $f_{t,0}$ achieved is dependent on its value.

Figure 5-11: Standard Solid Model, forced harmonic excitation with different combinations of k_d and k_s . Units: kN, mm.

5.4.2 Numerical approximation

As in the Maxwell model, a parametric study was carried out in order to approximate numerically the peak system response. The same set of damping exponents (a), damping coefficients (c), maximum displacements (u_0), excitation frequencies (ω) and diagonal

stiffnesses (k_d) as in the previous study were adopted. A new parameter k_s was taken as [100, 50, 20, 10, 1] kN/mm; the solution procedure was the same, except that the differential equation to solve was given by Eqn. (5.4-1). The numerical relationships between β_w , q and β_d derived in Section 5.3.2 are unchanged. The other relative peak values are affected by the spring stiffness and can be estimated as follows:

Estimate of peak force in Maxwell system

The peak force in this system was discussed in the previous section. It is repeated here with updated notation:

$$f_{M,0} = \max\{\beta_f c \omega^a u_0^a; f_{d0}; f_{M,s0}\}. \quad (5.4-7)$$

The max function is necessary because β_f is only approximate, and for some cases of high stiffness k_d and high exponent ($a = 1$) the first expression inside the curly brackets renders values slightly smaller than f_{d0} . The displacement corresponding to this maximum force is given by:

$$u_{M,f0} = \beta_u u_0. \quad (5.4-8)$$

Estimate of force at peak displacement $f_{t,s0}$

According to and, $f_{t,s0}$ is the sum of the storey spring part $f_{s,s0}$ and the Maxwell system spring part $f_{M,s0}$; the Maxwell component can be estimated using β_d ; therefore $f_{t,s0}$ can be estimated as:

$$f_{t,s0} = \beta_s k_d u_0 + k_s u_0, \quad (5.4-9)$$

where β_s is the value obtained from the Maxwell model. A consistent total stiffness k_t can be defined as:

$$k_t = \frac{f_{t,s0}}{u_0} = \beta_s k_d + k_s, \quad (5.4-10)$$

The prediction obtained with Eqn. (5.4-5) is shown in Figure 5-12, where the value of β_s was obtained using the polynomial approximations derived in the previous section. For this reason, the plot is not a perfect straight line.

Figure 5-12: Standard Solid Model, estimation of force at maximum displacement.

Estimation of peak total force $f_{t,0}$

There is no closed-form expression to find the peak total force $f_{t,0}$. Four procedures are suggested here, three existing ones and a new one.

Force coefficient method: this is an existing procedure [Tsopelas et al, 1997; Ramirez *et al*, 2001] implemented in ASCE/SEI 7 [ASCE, 2016]. The value of $f_{t,0}$ is calculated from an equivalent Kelvin system with the same energy dissipation W_d and equivalent stiffness k_t , as discussed in Section 2.4.10.3. first, the viscous damping ratio is calculated with reference to this modified stiffness, in any of the following forms:

$$\xi = \frac{W_d}{2\pi f_{t,s0} u_0} = \frac{W_d}{2\pi k_t u_0^2} \quad (5.4-11)$$

With this value, the phase lag angle δ is estimated from (2.4-38), and the maximum force $f_{t,0}$ is given by Eqn. (2.4-41), replacing f_{s0} with $f_{t,s0}$.

A comparison between the values of $f_{t,0}$ obtained using this procedure vs. the actual values obtained from the parametric study, is shown in Figure 5-13, segregated by k_s , Figure 5-13(a), and a , Figure 5-13(b). The estimated values are very close to the actual ones, and are generally conservative. Only for values of $a \geq 0.8$ and low values of the storey stiffness ($k_s \leq 1$) the procedure results in unsafe values; this is because the inherent assumption (δ small) is clearly violated in those cases, as also shown in Figure 5-13(d). From the practical point of view, this problem can be solved imposing the following condition upon calculating $f_{t,0}$:

$$f_{t,0} = \max\{f_{d0}; f_{t,s0}\} \quad (5.4-12)$$

The values obtained, shown in Figure 5-13(d), are conservative. It must be emphasized that $f_{t,0}$ is of no importance for the structural system, because this instant is not the instant of maximum force in the spring storey.

Lin-Chopra: this simple expression was proposed by Lin & Chopra [2002, 2003].

$$f_{t,0} \approx \sqrt{f_{d0}^2 + f_{t,s0}^2}. \quad (5.4-13)$$

Results of this expression are shown in Figure 5-13(e), proving that its accuracy is superior to the force coefficient method. In addition, application is much simpler.

Kasai *et al.* [2018]: These authors proposed the following expression:

$$f_{t,0} \approx \sqrt{a f_{d0}^2 + f_{t,s0}^2}. \quad (5.4-14)$$

Results of this expression are shown in Figure 5-13(e). The accuracy is similar to the one obtained with Lin-Chopra, but with this expression the results are consistently under-conservative, whereas the Lin-Chopra expression gives systematically conservative results.

Figure 5-13: Standard Solid model. Estimate of peak force $f_{t,0}$ from three different methods: (a-d) equivalent Kelvin system; (e) Lin-Chopra; (f) Kasai.

Estimation of $f_{t,0}$ based on a Bézier curve: an alternative procedure is given here, based on geometrical properties of the hysteresis curve; referring to Figure 5-10, points a (maximum displacement) and c (maximum bracing response) are completely defined; for point a, $u_a = u_0$ and $f_a = f_{t,0}$; for point c:

$$u_c = u_{M,f_0} = \beta_u u_0, \quad (5.4-15)$$

$$f_c = k_s u_c + f_{M,0} = k_s \beta_u u_0 + \beta_f c \omega^a u_0^a; \quad (5.4-16)$$

in addition, the slope of the tangent lines at c and a are known:

$$\frac{df}{du}(u_a) = \infty; \quad (5.4-17)$$

$$\frac{df}{du}(u_c) = k_s; \quad (5.4-18)$$

using these properties, a quadratic Bézier curve can be constructed, defined by points a, c and e, where e is the intersection of the tangents at c and a, and is defined by:

$$u_e = u_0, \quad (5.4-19)$$

$$f_e = f_c + k_s(u_0 - u_c). \quad (5.4-20)$$

The parametric equation of the Bézier curve is:

$$u = (1-x)^2 u_c + 2x(1-x)u_e + x^2 u_a; \quad (5.4-21)$$

$$f = (1-x)^2 f_c + 2x(1-x)f_e + x^2 f_a; \quad (5.4-22)$$

where x is a parameter that takes values between 0 (point c) and 1 (point a). Deriving both expressions, the slope of the hysteresis loop is expressed as:

$$\frac{df}{du} = \frac{2(1-x)f_c - 2xf_e + 2(1-x)f_e + 2xf_a}{2(1-x)u_c - 2xu_e + 2(1-x)u_e + 2xu_a}. \quad (5.4-23)$$

The maximum response is defined by:

$$\frac{df}{du}(x_0) = 0. \quad (5.4-24)$$

Solving Eqn. (5.4-24), an explicit solution is found for x_0 ; upon application to Eqns. (5.4-21) and (5.4-22), the following solution for point d is found:

$$f_{t,0} \approx f_d = f_c + \frac{k_s^2 (u_0 - \beta_u u_0)^2}{f_c - f_{t,s0} + 2k_s u_0 - 2k_s \beta_u u_0} \geq f_{t,s0} \quad (5.4-25)$$

$$u_{f,0} \approx u_d = \dots$$

$$\dots u_{Mf0} + \frac{2k_s (u_0 - \beta_u u_0)^2}{f_c - f_{t,s0} + 2k_s u_0 - 2k_s \beta_u u_0} - \frac{k_s^2 (u_0 - \beta_u u_0)^3}{(f_c - f_{t,s0} + 2k_s u_0 - 2k_s \beta_u u_0)^2} \geq u_{M,f0} \quad (5.4-26)$$

The approximation obtained with these expressions is compared to the actual values obtained from the parametric study in Figure 5-14; it is clear that expression (5.4-25) gives a very accurate estimate of $f_{t,0}$, better even than Eqn. (5.4-15). The scatter in $u_{f,0}$ is larger due to the small slope of the curve in this region; however, $u_{f,0}$ is not a relevant variable. Although the results are excellent, the resulting expressions are cumbersome, and for practical application Eqns. (5.4-15) and (5.4-16) are accurate enough.

Alternative estimation of $f_{t,0}$ based on a Bézier curve: a simpler expression can be obtained taking as control points a, b and h (see points marked a', b' and h' in the negative range of

Figure 5-10(a)). At point b the force is $f_b = f_{d0}$ and the displacement is 0; the slope of the tangent at this point k_1 was determined in Eqn. (5.4-2):

$$k_1 = k_s + k_d(1 - \beta_d^{1/a}) \quad (5.4-27)$$

Using the same procedure, the following expressions are derived:

$$f_{t,0} \approx f_{d0} + \frac{k_1^2 u_0^2}{f_{d0} - f_{t,s0} + 2k_1 u_0} \geq f_{t,s0} \quad (5.4-28)$$

$$u_{f0} \approx \frac{k_1 u_0^2 (2f_{d0} - 2f_{t,s0} + 3k_1 u_0)}{(f_{d0} - f_{t,s0} + 2k_1 u_0)^2} \geq u_{M,f0} \quad (5.4-29)$$

The results, shown in Figure 5-14(c) and (d), are of similar accuracy to the Lin-Chopra expression. The analytical procedure, however, is much more complicated and does not justify its practical use.

Figure 5-14: Standard Solid model. Approximation to $f_{t,0}$ (left) and $u_{f,0}$ (right) using a quadratic Bézier curve.

5.4.3 Summary of solution procedure for the standard solid model

The steps to solve the model for the set of parameters a , c , k_s , k_d , w , u_0 under harmonic excitation are summarized hereby. First, the Maxwell part must be solved independently:

- 1) q is found using Eqn. (5.3-15).

- 2) $\beta_W(q)$ is found using the numerical approximation given by Eqn. (5.3-16) with the coefficients listed in Table 5-5.
- 3) If $q > 3$ or $\beta_W < 0.1$: damping should be dismissed. In that case, $\beta_W = 0$, $\beta_d = 0$, $\beta_s = 1$, $\beta_f = 1$.
- 4) If $q \leq 3$ and $\beta_W \geq 0.1$: β_s , β_d , β_f and β_u are estimated with Eqn. (5.3-16) and Table 5-5.
- 5) The force at peak displacement $f_{M,s0}$, force at peak velocity $f_{M,d0}$, dissipated energy W_d , peak total force of the brace system $f_{M,0}$, and displacement at peak total force u_{fM0} are obtained using Eqns. (5.3-10), (5.3-11), (5.3-12) and (5.3-13), correspondingly.

Once the Maxwell system is determined, the results for the complete model are addressed:

- 6) Total system stiffness is given by:

$$k_t = k_s + \beta_s k_d. \quad (5.4-30)$$

- 7) Total force at peak displacement (storey + brace participation) is given by:

$$f_{t,s0} = k_t u_0. \quad (5.4-31)$$

- 8) Structural force at peak displacement is the part of $f_{t,s0}$ due to storey stiffness:

$$f_{t,s0} = k_t u_0. \quad (5.4-32)$$

- 9) The viscous damping ratio is estimated as:

$$\xi = \frac{W_d}{4\pi W_s} = \frac{W_d}{2\pi f_{t,s0} u_0} \quad (5.4-33)$$

- 10) The peak total force can be found by different methods: equivalent Kelvin system, Bézier curve, or simplified Bézier curve. If the last approach is taken, Eqn. (5.4-28) approximates $f_{t,0}$.

5.5 Influence of damping system stiffness in standard solid model with yielding spring

In this Section a special case of the standard model is treated, where the spring component representing the structural stiffness (k_s) presents yielding. The yielding system can be characterized using the Bouc-Wen hysteresis model [Tsopelas *et al*, 1997]:

$$f = \alpha \frac{f_y}{u_y} u + (1-\alpha) f_y z = \alpha k_s u + (1-\alpha) k_s u_y z \quad (5.5-1)$$

$$\dot{z} = \frac{1}{u_y} \left(\dot{u} - \frac{1}{2} |\dot{u}| |z|^{r-1} - \frac{1}{2} \dot{u} |z|^r \right), \quad (5.5-2)$$

where f_y is the yielding force, u_y is the yielding displacement, α is the post-yield stiffness, r is the yielding transition exponent, and z is an auxiliary non-dimensional function. The hysteresis loop for this model is shown in Figure 5-15; the left plot displays a case with moderate value of exponent $r = 5$, corresponding to a smooth yielding transition; the right plot displays a case with high value of $r = 100$, corresponding to an almost pure bilinear behavior.

Figure 5-15: Yielding system: Bouc-Wen hysteresis model; (a) smooth yield transition, $r = 5$; (b) sharp yield transition, $r = 100$.

The hysteresis loop is often described in terms of displacement ductility μ ; the case of bilinear behavior was treated in Chapter 2, 2.5.2.3; referring to Figure 5-16(a), point a corresponds to the hysteresis loop skeleton, described as u_y, f_y ; the maximum displacement u_0 and maximum force f_{s0} (point b) are given by:

$$u_0 = \mu u_y, \quad (5.5-3)$$

$$f_{s0} = k_s u_y + \alpha k_s (\mu - 1) u_y = (1 + \alpha(\mu - 1)) f_y, \quad (5.5-4)$$

and the equivalent viscous damping ratio is given by Eqn. (2.5-52), repeated here for convenience:

$$\xi_H = q_H \frac{2}{\pi} \frac{(\mu - 1)(1 - \alpha)}{\mu(1 + \alpha(\mu - 1))}, \quad (5.5-5)$$

and, for the case of smooth yielding transition, q_H can be interpreted as:

$$q_H = \frac{W_{h,r}}{W_{h,\infty}}, \quad (5.5-6)$$

where $W_{h,r}$ is the energy dissipated in the actual hysteresis loop, and $W_{h,\infty}$ the energy dissipated in the bilinear hysteresis loop.

Figure 5-16: Yielding system: parameters (a) bilinear simplification; (b) soft transition.

The differential equation of the complete yielding standard model is again given by (5.4-1), but with f_s described by Eqns. (5.5-1) and (5.5-2). As in the previous case, both systems can be solved independently to find the combined response.

Figure 5-17: Yielding system: parameters (a) bilinear simplification; (b) soft transition.

The solution process is made clear by Figure 5-17. For the yielding system, Figure 5-17(a), the peak force $f_{s,s0}$, is given by Eqn. (5.5-4); the relevant points are defined in the figure, and the energy dissipated by hysteresis in the yielding system, W_h , is given by Eqn. (5.5-5).

Considering only the Maxwell system, Figure 5-17(b), the force at maximum displacement $f_{M,s0}$, force at maximum velocity f_{d0} , maximum force $f_{M,0}$, displacement at maximum force $u_{M,f0}$, and energy dissipated by viscous damping W_d , are independent of the properties of the yielding system and can be found using the equations in Section 5.3.

The total system is considered in Figure 5-17(c). The total response can be found in the following way:

Response at peak displacement

The total force at peak displacement, $f_{t,s0}$, point a in Figure 5-17 (c), is given by:

$$f_{t,s0} = f_{s,s0} + f_{M,s0} = \beta_s k_d \mu u_y + (1 + \alpha(\mu - 1)) k_s u_y; \quad (5.5-7)$$

which allows definition of the total effective stiffness as:

$$k_t = \frac{f_{t,s0}}{u_0} = \beta_s k_d + \frac{1 + \alpha(\mu - 1)}{\mu} k_s; \quad (5.5-8)$$

Response at peak velocity

The response at peak velocity, point b, has two components, due to the bracing system response and the yielding system response. Thus:

$$f_{t,mxv} = \beta_d c \omega^a u_0^a + (\mu - 1)(1 - \alpha) f_y; \quad (5.5-9)$$

Energy dissipation and equivalent viscous damping ratio

The total dissipated energy is the addition of the hysteretic and viscous energy:

$$W_t = 4(\mu - 1)(1 - \alpha) k_s u_y^2 + \beta_w \pi b c_d \omega^a \mu^{a+1} u_y^{a+1}; \quad (5.5-10)$$

or, as a function of u_0 :

$$W_t = 4 \frac{(\mu - 1)(1 - \alpha)}{\mu^2} k_s u_0^2 + \beta_w \pi b c_d \omega^a u_0^{a+1}. \quad (5.5-11)$$

The maximum elastic energy stored in the system is defined by the effective stiffness:

$$W_s = \frac{1}{2} k_t u_0^2. \quad (5.5-12)$$

An equivalent total viscous damping ratio can be defined using Jacobsen's formula; after simplification:

$$\xi_t = \frac{2 k_s}{\pi k_t} \frac{(\mu - 1)(1 - \alpha)}{\mu^2} + \beta_w \frac{b c_d \omega^a u_0^{a-1}}{2 k_t}. \quad (5.5-13)$$

The first term is the equivalent viscous damping ratio due to the hysteresis loop. It can be modified to include the quality factor:

$$\xi_h = q_H \frac{2 k_s}{\pi k_t} \frac{(\mu - 1)(1 - \alpha)}{\mu^2}; \quad (5.5-14)$$

the second term is the equivalent viscous damping ratio due to the added viscous damping:

$$\xi_a = \beta_w \frac{b c_d \omega^a u_0^{a-1}}{2 k_t}. \quad (5.5-15)$$

The total viscous damping ratio is then expressed as:

$$\xi_t = \xi_h + \xi_a. \quad (5.5-16)$$

Peak total force

To find the maximum force $f_{t,0}$ and corresponding displacement $u_{f,t,0}$, the approximate procedures proposed in Section 5.4.2 can be used. The Bézier curve procedure requires to be adapted as follows: The displacement for peak Maxwell system response u_{fM0} is given by Eqn. (5.3-13). This displacement must be compared with the corner point of the yielding system,

$$u_{c,\lim} = (2 - \mu)u_y \quad (5.5-17)$$

Two different cases are possible:

Case 1: $u_{M,f0} > u_{c,\lim}$

As shown in Figure 5-17(c), the displacement at maximum bracing system force (point c) is larger than the displacement at the loop corner (point f); point c is defined as:

$$u_c = u_{M,f0} \quad (5.5-18)$$

$$f_c = f_{M,0} + (1 - \alpha(\mu - 1))f_y + (u_{M,f0} - (2 - \mu)u_y)\alpha k_s; \quad (5.5-19)$$

Case 2: $u_{M,f0} \leq u_{c,\lim}$

The displacement at maximum bracing system force is smaller than the displacement at the loop corner; the approximate, conservative solution is found using the corner point in combination with the maximum damper response:

$$u_c \approx u_{c,\lim} \quad (5.5-20)$$

$$f_c \approx f_{M,0} + (1 - \alpha(\mu - 1))f_y \quad (5.5-21)$$

After determination of u_c and f_c , Eqns. (5.4-28) and (5.4-29) can be reformulated, recalling that, due to yielding, the tangent to the loop at point c is parallel to αk_s . The final expressions for maximum force and corresponding displacement are:

$$f_{t,0} \approx f_d = f_c + \frac{\alpha^2 k_s^2 (u_0 - u_c)^2}{f_c - f_{t,s0} + 2\alpha k_s u_0 - 2\alpha k_s u_c} \geq f_{t,s0} \quad (5.5-22)$$

$$u_{f0} \approx u_d = u_c + \frac{2\alpha k_s (u_0 - u_c)^2}{f_c - f_{t,s0} + 2\alpha k_s u_0 - 2\alpha k_s u_c} \dots$$

$$- \frac{\alpha^2 k_s^2 (u_0 - u_c)^3}{(f_c - f_{t,s0} + 2\alpha k_s u_0 - 2\alpha k_s u_c)^2} \geq u_{M,f0} \quad (5.5-23)$$

5.6 Numerical examples

5.6.1 Elastic system with moderate extender stiffness

System properties

The following properties are adopted for the SD-S model: storey stiffness $k_s = 30$ kN/mm; damping system stiffness (corresponding to the brace extender stiffness) $k_d = 60$ kN/mm; damping exponent $a = 0.5$; damping coefficient $c = 50$ kN·(s/mm)^a. The coefficient b is taken from Table 2-5, $b = 1.1128$.

Excitation properties

The system is subjected to forced harmonic motion with excitation frequency $\omega = 6.28$ Hz (excitation period $T = 1$ s), and excitation amplitude $u_0 = 20$ mm.

Peak response of Maxwell system

The q parameter is found using Eqn. (5.3-15) as $q = 0.4671$. With q , the following relative peak values are calculated using the approximations in Sections 5.3.2.1 and 5.3.2.2:

- Simple expressions: $\beta_W = 0.758$; $\beta_d = 0.94$; $\beta_s = 0.242$; $\beta_f = 0.985$; $\beta_u = 0.246$.
- Polynomial expressions: $\beta_W = 0.782$; $\beta_d = 0.929$; $\beta_s = 0.262$; $\beta_f = 0.967$; $\beta_u = 0.320$.

The polynomial approximations are taken for solution. β_W measures the efficiency of this bracing system, which is 78% of the maximum possible energy dissipation. The relevant peak values are obtained using Eqns. (5.3-9) to (5.3-13):

Energy dissipation in one cycle:

$$W_d := \beta_W \cdot (\pi \cdot b \cdot c_d \cdot \omega^a \cdot u_0^{a+1}) = 30634.6 \text{ J}$$

Maxwell system force at peak velocity:

$$f_{M,d0} := \beta_d \cdot c_d \cdot \omega^a \cdot u_0^a = 520.7 \text{ kN}$$

Maxwell system force at peak displacement:

$$f_{M,s0} := \beta_s \cdot k_d \cdot u_0 = 314.9 \text{ kN}$$

Peak force in Maxwell system:

$$f_{M0} := \beta_f \cdot c_d \cdot \omega^a \cdot u_0^a = 542.1 \text{ kN}$$

Displacement at peak force in Maxwell system:

$$u_{fM0} := \beta_u \cdot u_0 = 6.4 \text{ mm}$$

Damping ratio of Maxwell system alone:

$$\xi_M := \frac{W_d}{2 \cdot \pi \cdot f_{M,s0} \cdot u_0} = 77.4\%$$

This damping ratio is an alternative measure of the efficiency of damping achieved by the brace extender stiffness. The actual values obtained by solution of the differential equations are: $W = 30679 \text{ J}$, $f_{M,d0} = 521.5 \text{ kN}$, $f_{M,s0} = 310.4 \text{ kN}$, $f_{M,0} = 545.5 \text{ kN}$, $u_{Mf0} = 6.42 \text{ mm}$.

Properties of the complete system:

The solution to the complete system is found using Eqns. (5.4-30) to (5.4-33).

Modified stiffness:

$$k_t := k_s + \beta_s \cdot k_d = 45.7 \frac{\text{kN}}{\text{mm}}$$

Total force at peak displacement:

$$f_{t,s0} := k_t \cdot u_0 = 914.9 \text{ kN}$$

Maximum storey force:

$$f_{s,s0} := k_s \cdot u_0 = 600 \text{ kN}$$

Damping ratio of total system:

$$\xi := \frac{W}{2 \cdot \pi \cdot k_t \cdot u_0^2} = 25.8\%$$

Maximum force calculated using method 1 (based on phase lag angle).

Phase lag angle:

$$\delta := \left(\frac{2 \cdot a \cdot \xi}{b} \right)^{\frac{1}{2-a}} = 0.378$$

Displacement for maximum force:

$$u_{f0} := \cos(\delta) \cdot u_0 = 18.59 \text{ mm}$$

Maximum force:

$$f_{t,0} := f_{t,s0} \cdot \cos(\delta) + f_{M,d0} \cdot (\sin(\delta))^a = 1166.7 \text{ kN}$$

Maximum force calculated using the proposed method (based on quadratic Bézier curve).

Displacement of maximum bracing system force:

$$u_c := u_{M,d0} = 6.4 \text{ mm}$$

Total force at u_c :

$$f_c := k_s \cdot u_c + f_{M,0} = 734 \text{ kN}$$

Maximum total force:

$$f_{t,0} := f_c + \frac{k_s^2 \cdot (u_0 - u_c)^2}{f_c - f_{t,s0} + 2 \cdot k_s \cdot u_0 - 2 \cdot k_s \cdot u_c} = 996.2 \text{ kN}$$

$$f_{t,0} := \max(f_{t,0}, f_{t,s0}) = 996.2 \text{ kN}$$

Displacement at maximum total force:

$$u_{f,t0} := u_c + \frac{2 \cdot k_s \cdot (u_0 - u_c)^2}{f_c - f_{t,s0} + 2 \cdot k_s \cdot u_0 - 2 \cdot k_s \cdot u_c} - \frac{k_s^2 \cdot (u_0 - u_c)^3}{(f_c - f_{t,s0} + 2 \cdot k_s \cdot u_0 - 2 \cdot k_s \cdot u_c)^2} = 18.3 \text{ mm}$$

$$u_{f,t0} := \max(u_{f,t0}, u_c) = 18.3 \text{ mm}$$

The actual values are $f_{t,s0} = 910.5 \text{ kN}$; $f_{s,s0} = 600 \text{ kN}$; $u_{f,t0} = 18.25 \text{ mm}$; $f_{t,0} = 986.4 \text{ kN}$. The proposed method improves the prediction of $f_{t,0}$. As an alternative, the simpler expressions (5.4-27) and (5.4-28) can be used to obtain $f_{t,0} = 1035.7 \text{ kN}$, a more inaccurate result.

Figure 5-18 shows the force-displacement loops for the Maxwell part (left) and the complete system (right).

5.6.2 Elastic system with low damping system stiffness.

The same system is now analyzed with a reduced brace extender stiffness of 15 kN/mm , whereupon $q = 1.8683$. Using the polynomial expressions, the following relative peak values are obtained: $\beta_W = 0.101$; $\beta_d = 0.123$; $\beta_s = 0.937$; $\beta_f = 0.510$; $\beta_u = 0.949$. The value of β_W is very low, confirming the poor efficiency of this damping system. From these values it follows that $W_d = 3953 \text{ J}$, $f_{M,d0} = 68.7 \text{ kN}$, $f_{M,s0} = 281.2 \text{ kN}$; (actual values $W_d = 4197 \text{ J}$, $f_{M,d0} = 63.4 \text{ kN}$, $f_{M,s0} = 280.6 \text{ kN}$).

The total stiffness is $k_t = 44.1 \text{ kN/mm}$, leading to a force at maximum displacement $f_{t,s0} = 881.2 \text{ kN}$. The damping ratio is 3.6% . With the force coefficient method, the phase angle $\delta = 0.101$; therefore $u_{f,t0} = 19.9 \text{ mm}$ and $f_{t,0} = 898.5 \text{ kN}$; with Method 2, $u_c = 18.99 \text{ mm}$; $f_c = 855.2 \text{ kN}$; $f_{t,0} = 881.8 \text{ kN}$, and $u_{f,t0} = 20 \text{ mm}$; (actual values $f_{t,s0} = 880.6 \text{ kN}$, $u_{f,t0} = 19.9 \text{ mm}$

and $f_{t,0} = 883.9$ kN). Figure 5-19 shows the force-displacement loops for the Maxwell part (left) and the complete system (right).

Figure 5-18: Force-displacement loops for standard Solid model with moderate brace stiffness, under harmonic excitation. Maxwell model (left). Complete model (right).

Figure 5-19: Force-displacement loops for standard solid model with low brace stiffness, harmonic excitation. Maxwell model (left). Complete model (right).

5.6.3 Elastic system with infinite bracing stiffness:

The same system is analyzed with infinite brace stiffness. $q = 0$, thus $\beta_w = 1$, $\beta_d = 1$, $\beta_s = 0$, $\beta_f = 1$, $\beta_u = 0$. The Maxwell component reduces to a pure damping system with $W_d = 39189.7$ J, $f_{M,d0} = 560.5$ kN, $f_{M,s0} = 0$ kN, $f_{M,0} = 560.5$ kN; $u_{M,0} = 0$ mm. The complete

system reduces to a Kelvin system. The stiffness is $k_t = 30$ kN/mm; the force at maximum displacement is $f_{t,s0} = 600$ kN, which includes only the storey component. The damping ratio is $\xi = 52\%$, whereupon using Method 1 the phase lag δ is 0.602 rad and correspondingly $u_{t,f0} = 16.48$ mm and $f_{t,0} = 916.3$ kN; using Method 2, $f_{t,0} = 869.8$ kN and $u_{f,t0} = 15.35$ mm; the actual values are $f_{t,0} = 916.95$ kN and $u_{f,t0} = 16.89$ mm. For this case, the force method is more exact because it is based on an equivalent Kelvin system.

5.6.4 Elastic system with no bracing stiffness:

The same system with $k_d = 0$ kN/mm degenerates to a pure spring; the maximum force is $f_{t,s0} = 600$ kN. There is no damping and no energy dissipation.

5.6.5 Yielding system SD-Y with small ductility:

The system defined in the first numerical example ($k_s = 30$ kN/mm; $k_d = 60$ kN/mm; $a = 0.5$; $c = 50$ kN·(s/mm)^a; $b = 1.1128$; $\omega = 6.28$ Hz; $u_0 = 20$ mm) is now analyzed with yielding behavior. The yield force is assumed to be $f_y = 480$ kN, and the post-yield stiffness coefficient is $\alpha = 0.25$. The yield displacement $u_y = f_y/k_s = 16$ mm, and therefore the ductility demand is $u_0/u_y = 1.25$. The approximate solution for the Maxwell system has been found in Section 5.6.1. The total solution is found in the following way:

Response at peak displacement

The total effective stiffness is found by Eqn. (5.5-8):

$$k_t := \beta_s \cdot k_d + \frac{1 + \alpha \cdot (\mu - 1)}{\mu} k_s = 41.2 \frac{\text{kN}}{\text{mm}}$$

And the force at peak displacement is:

$$f_{t,s0} := k_t \cdot u_0 = 824.9 \text{ kN}$$

The actual value of $f_{t,s0}$ is 820.5 kN. The part corresponding to the yielding spring is:

$$f_{s,s0} := f_y \cdot (1 + \alpha \cdot (\mu - 1)) = 510 \text{ kN}$$

Response at peak velocity

The total force is:

$$f_{t,d0} := f_{M,d0} + (\mu - 1) \cdot (1 - \alpha) \cdot f_y = 610.7 \text{ kN}$$

The actual value is 611.5 kN.

Energy dissipation and equivalent viscous damping ratio

The hysteretic viscous damping ratio is given by Eqn. (5.5-14):

$$\xi_h := \frac{2}{\pi} \cdot \frac{k_s}{k_t} \cdot \frac{(\mu - 1) \cdot (1 - \alpha)}{\mu^2} = 5.6\%$$

And the added viscous damping ratio is given by Eqn. (5.5-15):

$$\xi_a := \beta_w \cdot \frac{b}{2} \cdot \frac{c_d \cdot \omega^a \cdot u_0^{a-1}}{k_t} = 29.6\%$$

Finally, the total viscous damping ratio is:

$$\xi_t := \xi_h + \xi_a = 35.1\%$$

Peak force response

Using the force method with the total viscous damping ratio, the phase lag is 0.463 rad, whereupon $u_{f,t0} = 17.89\text{mm}$ and $f_{t,0} = 1086.1\text{ kN}$.

Using the Bézier curve approach:

$$u_{c,lim} := (2 - \mu) \cdot u_y = 12 \text{ mm}$$

Because u_{Mf0} is 6.3mm, point c is conservatively taken as the corner point of the yielding system hysteresis loop; then by Eqns. (5.5-20) and (5.5-21):

$$u_c := \max(u_{Mf0}, u_{c,lim}) = 12 \text{ mm}$$

$$f_c := f_{M0} + (1 - \alpha \cdot (\mu - 1)) \cdot f_y = 992.1 \text{ kN}$$

And, using Eqns. (5.5-22) and (5.5-23):

$$f_{t,0} := f_c + \frac{\alpha^2 \cdot k_s^2 \cdot (u_0 - u_c)^2}{f_c - f_{t,s0} + 2 \cdot \alpha \cdot k_s \cdot u_0 - 2 \cdot \alpha \cdot k_s \cdot u_c} = 1004.6 \text{ kN}$$

$$f_{t,0} := \max(f_{t,0}, f_{t,s0}) = 1004.6 \text{ kN}$$

$$u_{f,t0} := u_c + \frac{2 \cdot \alpha \cdot k_s \cdot (u_0 - u_c)^2}{f_c - f_{t,s0} + 2 \cdot \alpha \cdot k_s \cdot u_0 - 2 \cdot \alpha \cdot k_s \cdot u_c} - \frac{\alpha^2 \cdot k_s^2 \cdot (u_0 - u_c)^3}{(f_c - f_{t,s0} + 2 \cdot \alpha \cdot k_s \cdot u_0 - 2 \cdot \alpha \cdot k_s \cdot u_c)^2} = 15 \text{ mm}$$

$$u_{f,t0} := \max(u_{f,t0}, u_c) = 15 \text{ mm}$$

The true value happens at $u_{f,t0} = 12.75\text{mm}$ and $f_{t,0} = 978.3\text{ kN}$. The system behavior is represented in Figure 5-20(a).

5.6.6 Yielding system SD-Y with larger ductility:

Finally, the same system is analyzed with a smaller yielding force, $f_y = 300\text{kN}$, which results in a larger ductility $\mu = 2$. The total response is:

Response at maximum displacement

Total stiffness:

$$k_t := \beta_s \cdot k_d + \frac{1 + \alpha \cdot (\mu - 1)}{\mu} k_s = 34.5 \frac{\text{kN}}{\text{mm}}$$

Force at maximum displacement:

$$f_{t,s0} := k_t \cdot u_0 = 689.9 \text{ kN}$$

Maximum storey force:

$$f_{s,s0} := f_y \cdot (1 + \alpha \cdot (\mu - 1)) = 375 \text{ kN}$$

The actual value of $f_{t,s0}$ is 685.5 kN.

Response at maximum velocity:

$$f_{t,d0} := f_{d0} + (\mu - 1) \cdot (1 - \alpha) \cdot f_y = 745.7 \text{ kN}$$

The actual value of $f_{t,d0}$ is 745 kN.

Effective viscous damping ratio

Effective hysteretic damping ratio:

$$\xi_h := \frac{2}{\pi} \cdot \frac{k_s}{k_t} \cdot \frac{(\mu - 1) \cdot (1 - \alpha)}{\mu^2} = 10.4\%$$

Effective added viscous damping ratio:

$$\xi_a := \beta_w \cdot \frac{b}{2} \cdot \frac{c_d \cdot \omega^a \cdot u_0^{a-1}}{k_t} = 35.3\%$$

Total equivalent viscous damping ratio:

$$\xi_t := \xi_h + \xi_a = 45.7\%$$

Peak force response

Using the force method, $\delta = 0.553$ rad, whereupon $u_{f0} = 17.02\text{mm}$ and $f_{f0} = 964.5$ kN.

Using the Bézier curve method:

Displacement at corner point of yielding loop:

$$u_{c.lim} := (2 - \mu) \cdot u_y = 0 \text{ mm}$$

Because $u_{fM0} > u_{c.lim}$, definition of point c is given by Eqns. (5.5-18) and (5.5-19):

$$u_c := \max(u_{Mf0}, u_{c.lim}) = 6.3 \text{ mm}$$

$$f_c := f_{M0} + (1 - \alpha \cdot (\mu - 1)) \cdot f_y + (u_{Mf0} - (2 - \mu) \cdot u_y) \cdot \alpha \cdot k_s = 813.9 \text{ kN}$$

Peak force response:

$$f_{t.0} := f_c + \frac{\alpha^2 \cdot k_s^2 \cdot (u_0 - u_c)^2}{f_c - f_{t.s0} + 2 \cdot \alpha \cdot k_s \cdot u_0 - 2 \cdot \alpha \cdot k_s \cdot u_c} = 846.1 \text{ kN}$$

$$f_{t.0} := \max(f_{t.0}, f_{t.s0}) = 846.1 \text{ kN}$$

$$u_{f.t0} := u_c + \frac{2 \cdot \alpha \cdot k_s \cdot (u_0 - u_c)^2}{f_c - f_{t.s0} + 2 \cdot \alpha \cdot k_s \cdot u_0 - 2 \cdot \alpha \cdot k_s \cdot u_c} - \frac{\alpha^2 \cdot k_s^2 \cdot (u_0 - u_c)^3}{(f_c - f_{t.s0} + 2 \cdot \alpha \cdot k_s \cdot u_0 - 2 \cdot \alpha \cdot k_s \cdot u_c)^2} = 13.5 \text{ mm}$$

$$u_{f.t0} := \max(u_{f.t0}, u_c) = 13.5 \text{ mm}$$

The actual maximum response is $f_{t.0} = 843.4 \text{ kN}$; $u_{f.t0} = 12.65 \text{ mm}$. The system response is displayed in Figure 5-20(b).

Figure 5-20: Force-displacement loops for standard solid model with yielding spring, harmonic excitation. Low ductility (left). Moderate ductility (right).

5.7 Summary and conclusions

The Maxwell model is a suitable mathematical representation for the behavior of a viscous damping system formed by a spring component (dominated by the extender stiffness) and a

pure viscous component acting in series. The behavior of this model is more complex than the pure damper or the Kelvin model (damper and spring in parallel), and it is highly influenced by damping, stiffness and excitation frequency.

The concept of damping efficiency β_W for the Maxwell system has been introduced. It represents the amount of energy dissipated by cycle relative to the maximum dissipation capability of the pure damping system (with infinite stiffness). High values of efficiency (close to 1) indicate a pure viscous behavior. Low values of efficiency indicate a poor performance of the damping system, in which case the spring component of the damping system dominates the behavior.

At a given frequency, a minimum brace stiffness requirement can be derived to reach a certain threshold of dissipation efficiency. Here, the practical range 0.9-0.95 has been proposed. This efficiency can be converted into a minimum stiffness condition for the damping system operating at natural frequencies of the system.

Damping efficiency is dependent on the frequency of excitation. This means that, for the same system, increasing frequencies (decreasing periods) result in increasing stiffness requirements to achieve the same damping efficiency. If design of bracing elements is based on the first mode, most likely their behavior in higher modes will be dominated by their spring component. In that case, the pure viscous representation usually adopted is not valid for higher modes.

A numerical approximation to the response of the Maxwell model under forced harmonic excitation has been derived, using high order polynomial expressions. The numerical approximation has been used to find the response of two important systems, i) the standard solid model and ii) the standard solid model with yielding spring; these systems represent the damping system inserted in an elastic or inelastic structural system.

Each model response has been discussed under the assumption of forced harmonic excitation, and the response examined in terms of dissipated energy W_d , pure damping force f_{d0} , maximum damping system force f_{M0} , maximum storey force $f_{s,s0}$, maximum total force at maximum displacement $f_{t,s0}$, and maximum total force $f_{t,0}$. The energy dissipation capacity and maximum damper force of these systems (under forced harmonic excitation) are not dependent on the storey stiffness, and can be calculated independently using the numerical Maxwell model. The system's total stiffness is composed of the storey component and a fraction of the damping system stiffness; this fraction is inversely dependent on the energy dissipation efficiency; for systems with low energy dissipation efficiency β_W , the participation of the damper stiffness will be large; conversely, for high values of β_W , the participation of the damper stiffness will be negligible. Based on these observations, approximate expressions for $f_{t,s0}$, $f_{s,s0}$ are given.

An approximate expression based on interpolation of a Bézier quadratic curve is proposed for $f_{t,0}$, and shown to render more accurate values than the current procedure implemented in codes. Two very simple expressions proposed by other authors (Lin & Chopra [2001;

2002]; Kasai *et al.* [2018]) are also examined. All expressions are shown to be more accurate than the current ASCE-SEI 7/16 expression based on force coefficients.

5.8 Original contributions in this Chapter

- Quantification of the influence of the brace extender stiffness in the response of the Maxwell system. A new non-dimensional parameter q is introduced. The dependence of the response on this parameter is proved and quantified through accurate polynomial approximations in terms of energy, force and displacement.
- A measure of the efficiency of the damping system (brace + damper) is introduced using energy concepts.
- Conversely, a rational minimum design criterion for the brace extender stiffness is derived to ensure a minimum damping efficiency.
- The damping ratio for higher modes is shown to be strongly influenced by the brace stiffness; it is shown that quasi-elastic, non-dissipative behavior is to be expected for high frequencies.
- New, accurate equations, to predict the response of the standard solid model (with or without yielding) are derived making use of the equations obtained for the Maxwell system; these equations apply directly to structures equipped with viscous dampers.
- Finally, an alternative method to predict the maximum force response of these systems is derived using a Bézier curve interpolation. This method is proved more accurate than the existing method implemented in ASCE/SEI 7/16.

Chapter 6

Modification of RSA to include the Influence of Auxiliary Elements

6.1 Introduction

In this Chapter, existing RSA and NLRSA procedures proposed for viscoelastic damping systems, are used to calculate the response of systems with added viscous damping systems to earthquake excitation. The influence of auxiliary element stiffness is taken into account using the simplified expressions derived in Chapter 5, allowing response prediction of the Maxwell system formed by bracing element and damper. As shown, the damping system contributes to both stiffness and viscous damping, and both contributions are taken into account to find the global frame response. Because the Maxwell system response depends on vibration frequency and amplitude, an iterative process is needed to find an adjusted solution within a certain error margin. The procedure is also adapted for inelastic systems, for which pushover properties must be re-evaluated due to the contribution of the stiffness part. Numerical examples of the modified procedure both for elastic and inelastic cases are given.

Two fundamental assumptions are made: i) the modal damping matrix of the frame with added damping system is quasi-diagonal; ii) during earthquake excitation the frame response at mode j can be assimilated to forced harmonic vibration at frequency ω_j . The first assumption ensures that the vibration modes are not complex. The second assumption allows the fundamental properties of the frame to be obtained by the simplified numerical procedure derived for harmonic motion.

6.2 Modified RSA procedure for elastic systems

6.2.1 Properties of frame with damping system

A frame with added dampers vibrating in its mode j is considered. Vibration properties are not the same as those of the original frame, due to the contribution of the added damping system to stiffness and damping. The following fundamental simplification is made: the

modal damping matrix of the damped frame is almost diagonal, so vibration modes are not complex. Under this simplification, every storey vibrates harmonically at mode j . The modal properties at this mode are $\omega_{e,j}$ (or $T_{e,j}$) and $\phi_{e,j}$ (modal displacement, normalized to roof). The corresponding properties of the undamped frame are ω_j , T_j and ϕ_j .

Figure 6-1: Frame with damping system vibrating at mode j ; (a) actual frame; (b) analysis model formed by original frame + damping system stiffness contribution.

The n -th storey of the frame vibrating harmonically at mode j can be represented by a Standard Model in which the brace stiffness and damping are known, and the storey stiffness can be found by the procedure indicated in Section 2.5.3.2. To obtain this stiffness for mode j the modified modal properties (ϕ_j, ψ_j) must be used. The response of this SDOF model can be approximated by the Equations derived in Chapter 05, which are summarized here with the notation introduced in Figure 2-27. First, the q parameter for the Maxwell damping system representing storey n vibrating at mode j is found:

$$q_{n,j} = \frac{c_{d,n} |\dot{\Delta}_{n,j}|^{a_n}}{k_{d,n} |\Delta_{n,j}|}. \quad (6.2-1)$$

This expression can be considered an extension of Eqn. (5.3-15), and is convenient if correction factors are introduced in either velocity or drift.

Once q_{nj} is known, the values of q_{nj} , $\beta_{W,n,j}$, $\beta_{d,n,j}$ and $\beta_{s,n,j}$ can be estimated using the relationships derived in Chapter 5; the relevant equations are repeated here:

Stiffness contribution of damping system

The Maxwell system contribution to the n -th storey stiffness at mode j , $k_{sa,n,j}$, can be estimated as:

$$k_{sa,n,j} = \beta_{s,n,j} k_{d,n}. \quad (6.2-2)$$

The total updated stiffness matrix for mode j , $\mathbf{k}_e^{(j)}$ can be found considering the damped system, Figure 6-1(a), as the addition of the original frame storey stiffness plus the storey

stiffness contribution of the damping system, Figure 6-1(b); using basic matrix analysis, this can be expressed as:

$$\mathbf{k}_e^{(j)} = \mathbf{k} + \mathbf{H} \text{diag}[\mathbf{k}_{sa}^{(j)}] \mathbf{B}, \quad (6.2-3)$$

where $\text{diag}[\mathbf{x}]$ indicates the diagonal matrix constructed from vector $\mathbf{x}$, and superscript (j) indicates that the matrix $\mathbf{k}_e^{(j)}$ differs for every mode j . $\mathbf{k}_{sa}^{(j)}$ is the vector whose n -th term is $k_{sa,n,j}$; all other terms have been introduced in Section 2.5.3.2. This matrix is different for each mode, because the components $k_{sa,n,j}$ are mode-dependent. The second term in (6.2-3) is a banded matrix of order 2, whereas the first term is generally a fully populated matrix. Modal properties must fulfill the eigenvalue equation for mode j :

$$(\mathbf{k}_e^{(j)} - \omega_{e,j}^2 \mathbf{m}) \boldsymbol{\phi}_{e,j} = \mathbf{0}, \quad (6.2-4)$$

where $\mathbf{m}$ is the mass matrix.

Solution of Eqn. (6.2-4) renders a consistent set of eigenvectors and eigenvalues for all modes, for the modified system whose total stiffness is dependent on the frequency of mode j . Modal analysis is then possible and renders a modal solution for each mode. From these solutions, only the solution for mode j is preserved. The process is repeated for all modes. Thus, the final set of eigenmodes is not consistent, due to the frequency-dependency of the stiffness contribution of the Maxwell system. However, the particular solution for each mode is consistent because the set of modal shapes derived from Eqn. (6.2-4) fulfils the orthogonality condition.

Effective damping ratio

The energy dissipated by the added damping system at storey n in one cycle of motion at mode j is:

$$W_{d,n,j} = \beta_{W,n,j} b_n \pi c_{d,n} \omega_{e,j}^{a_n} |\Delta_{n,j}|^{1+a_n} = \beta_{W,n,j} b_n \pi c_{d,n} |\dot{\Delta}_{n,j}|^{a_n} |\Delta_{n,j}|, \quad (6.2-5)$$

and the total energy dissipated at mode j is:

$$W_{d,j} = \sum_{n=1}^N \beta_{W,n,j} b_n \pi c_{d,n} \omega_{e,j}^{a_n} |\Delta_{n,j}|^{1+a_n} = \sum_{n=1}^N \beta_{W,n,j} b_n \pi c_{d,n} |\dot{\Delta}_{n,j}|^{a_n} |\Delta_{n,j}|. \quad (6.2-6)$$

the maximum strain energy stored in storey n during one cycle of motion at mode j is:

$$W_{se,n,j} = \frac{k_{se,n,j} \Delta_{n,j}^2}{2}, \quad (6.2-7)$$

where $k_{se,n,j}$ is the storey stiffness matrix obtained from $\mathbf{k}_e^{(j)}$ using the procedure discussed in Section 2.5.3.2. The effective modal damping ratio of the whole system at mode j can be expressed using Jacobsen's equation as:

$$\xi_{a,j} = \frac{W_{d,j}}{4\pi W_{se,j}}, \quad (6.2-8)$$

and, substituting:

$$\xi_{a,j} = \frac{1}{2} \frac{\sum_{n=1}^N \beta_{W,n,j} b_n c_{d,n} \omega_{e,j}^{a_n} |\Delta_{n,j}|^{a_n+1}}{\sum_{n=1}^N k_{se,n,j} \Delta_{n,j}^2} = \frac{1}{2} \frac{\sum_{n=1}^N \beta_{W,n,j} b_n c_{d,n} |\dot{\Delta}_{n,j}|^{a_n} |\Delta_{n,j}|}{\sum_{n=1}^N k_{se,n,j} \Delta_{n,j}^2}. \quad (6.2-9)$$

It is, however, simpler to assimilate the maximum strain energy in the whole structure to the maximum kinetic energy:

$$W_{se,j} = \frac{1}{2} \mathbf{u}_j^T \mathbf{m} \mathbf{u}_j = \frac{1}{2} \sum_{n=1}^N m_n u_{n,j}^2. \quad (6.2-10)$$

Then $\xi_{a,j}$ can be expressed as:

$$\xi_{a,j} = \frac{1}{2} \frac{\sum_{n=1}^N \beta_{W,n,j} b_n c_{d,n} \omega_{e,j}^{a_n} |\Delta_{n,j}|^{a_n+1}}{\omega_{e,j}^2 \sum_{n=1}^N m_n u_{n,j}^2} = \frac{1}{2} \frac{\sum_{n=1}^N \beta_{W,n,j} b_n c_{d,n} |\dot{\Delta}_{n,j}|^{a_n} |\Delta_{n,j}|}{\omega_{e,j}^2 \sum_{n=1}^N m_n u_{n,j}^2}. \quad (6.2-11)$$

For practical application, viscous damping should be calculated using the velocity, right term in Eqn. (6.2-11), resulting in more exact values than the use of pseudo-velocity, second term in Eqn. (6.2-11), [Pekcan *et al.*, 1999]. The expression can be rephrased as:

$$\xi_{a,j} = \frac{1}{2} \frac{\sum_{n=1}^N \beta_{W,n,j} b_n c_{d,n} \omega_{e,j}^{a_n} |\Delta_{n,j}|^{a_n+1} cfv_j^{a_n}}{\omega_{e,j}^2 \sum_{n=1}^N m_n u_{n,j}^2}, \quad (6.2-12)$$

where cfv_j is the corresponding Correction Factor for Velocity, calculated with frequency and roof amplitude at mode j .

For elastic behavior, the total damping at mode j , $\xi_{e,j}$, is expressed as:

$$\xi_{e,j} = \xi_{a,j} + \xi_{I,j}, \quad (6.2-13)$$

where $\xi_{I,j}$ is the inherent damping at mode j , adjusted for period $T_{e,j}$. The adjustment depends on the damping type considered. For modal damping no adjustment is performed.

Approximate solution assuming constant modal shapes

An approximate solution can be obtained if the modal vector $\boldsymbol{\phi}_{e,j}$ is assumed approximately equal to the original modal vector $\boldsymbol{\phi}_j$. If the following notation is used:

$$\mathbf{k}_a^{(j)} = \mathbf{H} \text{diag}[\mathbf{k}_{sa}^{(j)}] \mathbf{B}, \quad (6.2-14)$$

Eqn. (6.2-3) can be rewritten as:

$$(\mathbf{k} + \mathbf{k}_a^{(j)} - (\omega_j^2 + \omega_{a,j}^2) \mathbf{m}) \boldsymbol{\phi}_j = \mathbf{0}, \quad (6.2-15)$$

and, applying modal properties:

$$(\mathbf{k}_a^{(j)} - \omega_{a,j}^2 \mathbf{m}) \boldsymbol{\phi}_j = \mathbf{0}. \quad (6.2-16)$$

If the expression is pre-multiplied by $\boldsymbol{\phi}_j^T$, the value $\omega_{a,j}$ is found as:

$$\omega_{a,j}^2 = \boldsymbol{\phi}_j^T \mathbf{k}_a^{(j)} \boldsymbol{\phi}_j / M_j, \quad (6.2-17)$$

and

$$\omega_{e,j}^2 \approx \omega_j^2 + \omega_{a,j}^2 \quad (6.2-18)$$

This simple method is similar to the one proposed by Ramirez *et al.* [2001] for viscoelastic dampers. The expression can be derived alternatively using Rayleigh's quotient:

$$\omega_{e,j}^2 = \frac{\boldsymbol{\phi}_j^T (\mathbf{k} + \mathbf{k}_a^{(j)}) \boldsymbol{\phi}_j}{M_j}, \quad (6.2-19)$$

Stiffness matrix

The condensed stiffness matrix is needed to apply the procedure. This is not always directly available from commercial analysis programs, but it can readily be obtained using the eigenvalue properties:

$$\mathbf{k} = \mathbf{m} \boldsymbol{\Phi} \boldsymbol{\Omega}^2 \boldsymbol{\Phi}^{-1} \quad (6.2-20)$$

Because modal shapes obtained from analysis are approximate, the previous expression renders a slightly non-symmetrical matrix. A symmetrical matrix can be obtained adding the matrix to its transpose and dividing over 2, $(\mathbf{k}^T + \mathbf{k})/2$.

6.2.2 Modal solution

6.2.2.1 Iterative procedure

Modal analysis can be used to obtain the kinematic response at mode j of the frame defined in the previous section. The spectral displacement response $D_{e,j}$ is:

$$D_{e,j} = S_d(T_{e,j}, \xi_{e,j}, a_g), \quad (6.2-21)$$

where S_d is the spectral function the gives the peak spectral displacement of the SDOF system with period $T_{e,j}$, viscous damping ratio $\xi_{e,j}$ and peak ground acceleration a_g . Therefore:

$$u_{n,j} = \Gamma_{e,j} \phi_{e,n,j} S_d(T_{e,j}, \xi_{e,j}, a_g), \quad (6.2-22)$$

$$\Delta_{n,j} = \Gamma_{e,j} \psi_{e,n,j} S_d(T_{e,j}, \xi_{e,j}, a_g), \quad (6.2-23)$$

$$\dot{\Delta}_{n,j} = \text{CFV}(T_{e,j}, \xi_{e,j}) \omega_{e,j} \Delta_{n,j}, \quad (6.2-24)$$

where $\psi_{e,n,j}$ is the n -th component of the modal drift vector at mode j ; $\Gamma_{e,j}$ is the modal participation factor of the frame at mode j ; and CFV is a function that gives the Correction Factor for Velocity to convert pseudo-velocity to true velocity. $\Psi_{e,j}$ can be calculated as $\mathbf{B} \cdot \boldsymbol{\phi}_{e,j}$, with $\boldsymbol{\phi}_{e,j}$ normalized to roof. $\Gamma_{e,j}$ is calculated using conventional modal analysis, Eqn. (2.5-102), repeated here with updated notation:

$$\Gamma_{e,j} = \frac{\boldsymbol{\phi}_{e,j}^T \mathbf{m} \mathbf{1}}{\boldsymbol{\phi}_{e,j}^T \mathbf{m} \boldsymbol{\phi}_{e,j}}, \quad (6.2-25)$$

To determine the peak drift and drift velocity response of the damped system in mode j , definition of the period $T_{e,j}$, modal drift $\Psi_{e,j}$ and damping ratio $\xi_{e,j}$ are needed. As shown by Eqns. (6.2-3) and (6.2-11), these properties are dependent on drift and drift velocity. Therefore, an iterative procedure is needed, which is described hereby:

Elastic solution, exact method

For every mode j :

1. Fix initial values $\xi_{e,j}^{(0)} = \xi_{I,j}$, $T_{e,j}^{(0)} = T_j$, $\Psi_{e,j}^{(0)} = \Psi_j$, $\Gamma_{e,j}^{(0)} = \Gamma_j$.
2. Calculate roof displacement $u_{N,j}^{(0)}$ based on $\xi_{e,j}^{(0)}$, $\Psi_{e,j}^{(0)}$, $\Gamma_{e,j}^{(0)}$, $T_{e,j}^{(0)}$, using Eqn. (6.2-22).
3. Calculate drift $\Delta_{n,j}$ with Eqn. (6.2-23), and drift velocity $\dot{\Delta}_{n,j}$ with Eqn. (6.2-24).
4. For all storeys ($n = 1$ to N) calculate $q_{n,j}$ using Eqn. (6.2-1).
5. Calculate $\beta_{w,n,j}$ and $\beta_{s,n,j}$ from $q_{n,j}$ using the approximations given by Eqn. (5.3-16).
6. Calculate the contribution of bracing to frame stiffness at mode j , $k_{sa,n,j}$, using Eqn. (6.2-2).
7. Recalculate the modal properties of the frame with added damping ($\Psi_{e,j}, T_{e,j}$), solving the eigenvalue problem stated by Eqn. (6.2-3). Find $\Gamma_{e,j}$ with Eqn. (6.2-25).
8. Calculate the added viscous damping ratio $\xi_{e,j} = \xi_{a,j} + \xi_{I,j}$, where $\xi_{a,j}$ is found with Eqn. (6.2-11), and $\xi_{I,j}$ is adjusted (if necessary) according to inherent damping type and updated modal properties.
9. Using Eqn. (6.2-23) and the updated modal properties, find the new values of $u_{N,j}$.
10. If the roof displacement $u_{N,j}$ is similar within tolerance to the initial one $u_{N,j}^{(0)}$ for all $u_{N,j}^{(0)}$ storeys, the procedure has converged and can be stopped. Otherwise, make $\Delta_{n,j}^{(0)} = u_{N,j}$, $\xi_{e,j}^{(0)} = \xi_{e,j}$, $\Psi_{e,j}^{(0)} = \Psi_{e,j}$, $\Gamma_{e,j}^{(0)} = \Gamma_{e,j}$, and go to step 3.

Optionally, the convergence criterion can be based on drift, frequency or period.

For the simplified method, change step 7 of the previous procedure as follows: modal shapes ($\Phi_{e,j}$, $\Psi_{e,j}$) and derived properties ($\Gamma_{e,j}$) are those of the undamped frame; find $\omega_{a,j}$ with Eqn. (6.2-17); find $\omega_{e,j}$ with Eqn. (6.2-18), and $T_{e,j}$ from $\omega_{e,j}$.

6.2.2.2 Modal kinematic response

Modal displacement and velocity matrices are calculated from drift and velocity as:

$$\mathbf{u} = \mathbf{T}_L \Delta, \quad (6.2-26)$$

$$\dot{\mathbf{u}} = \mathbf{T}_L \dot{\Delta}, \quad (6.2-27)$$

where $\mathbf{T}_L$ was defined in Section 2.5.3.2. The values for damper stroke $\mathbf{u}_b$ and damper velocity $\dot{\mathbf{u}}_b$, can be found from Δ and $\dot{\Delta}$ taking into account the mounting configuration; for diagonally mounted dampers, Eqns. (2.4-4) and (2.4-7).

6.2.2.3 Modal static response

The static response is calculated with the procedures of Chapter 5; the following notation is used: V_t indicates total shear (structure + damping system), V_s indicates structural shear; V_d indicate damping system shear. Subscript 'mxd' is used to indicate the point of maximum drift (no velocity). Subscript 'mxv' indicates the point of maximum velocity (no drift). Subscript '0' indicates a point of maximum of the variable considered.

Response at peak drift. Maximum storey shear

Total forces at maximum displacement can be obtained as:

$$f_{t,mxd,n,j} = \omega_{e,j}^2 m_n u_{n,j}. \quad (6.2-28)$$

Shear for the total system (structure+ brace extender participation) is obtained at maximum drift (no velocity) from these forces as:

$$\mathbf{V}_{t,mxd} = \mathbf{T}_U \mathbf{f}_{t,mxd}. \quad (6.2-29)$$

The shear force $V_{t,mxd,n,j}$ is composed of the pure structural shear $V_{s,mxd,n,j}$ (maximum storey shear) and the damper spring contribution $V_{d,mxd,n,j}$; the latter can be calculated as:

$$V_{d,mxd,n,j} = k_{sa,n,j} \Delta_{n,j}. \quad (6.2-30)$$

Subsequently, structural shear is given as:

$$\mathbf{V}_{s,mxd} = \mathbf{V}_{t,mxd} - \mathbf{V}_{d,mxd}. \quad (6.2-31)$$

Response at peak drift velocity. Pure damper shear.

The shear force at the damping system at maximum drift velocity (zero drift) is obtained as:

$$V_{d,mxv,n,j} = \beta_{d,n,j} c_{d,n} \left| \dot{\Delta}_{n,j} \right|^{a_n}. \quad (6.2-32)$$

Pure damper force at maximum velocity can be calculated from this value taking into account the type of mounting configuration. For diagonally mounted dampers,

$$f_{d,mxv,n,j} = \frac{V_{d,mxv,n,j}}{\cos \theta_n}, \quad (6.2-33)$$

Peak damping system force

Peak shear (and corresponding force) in the damping system (brace + damper) is obtained as:

$$V_{d,0,n,j} = \max \left\{ \beta_{f,n,j} c_{d,n} \left| \dot{\Delta}_{n,j} \right|^{a_n}; V_{d,mxv,n,j}; V_{d,mxd,n,j} \right\}, \quad (6.2-34)$$

and, for diagonally mounted dampers,

$$f_{d,0,n,j} = \frac{V_{d,0,n,j}}{\cos \theta_n}. \quad (6.2-35)$$

Peak shear in the damping system occurs at the following value of drift:

$$\Delta_{d,0,n,j} = \beta_{u,n,j} \left| \Delta_{n,j} \right|, \quad (6.2-36)$$

Peak total shear

As discussed in Chapter 5, the peak shear force can be found using different approaches:

Force coefficients: [Ramirez et al, 2001; ASCE, 2016] discussed in Section 2.4.10.3; the maximum shear response is calculated by combining the responses at maximum drift and maximum velocity weighted by force coefficients obtained from harmonic response.

$$V_{t,0,n,j} = CF_1 \left| V_{s,mxd,n,j} \right| + CF_2 \left| V_{d,mxv,n,j} \right| \quad (6.2-37)$$

Bézier curve approximation: discussed in Chapter 05, the maximum shear response is obtained at harmonic response by approximating the hysteresis loop between the maximum velocity response and the maximum drift response to a quadratic Bézier curve. Drift and total shear force responses at the point of maximum response of the damping system are:

$$\Delta_{c,n,j} = \Delta_{d,0,n,j}; \quad (6.2-38)$$

$$V_{c,n,j} = k_{sf,n,j} \Delta_{d,0,n,j} + V_{d,0,n,j}; \quad (6.2-39)$$

total shear response is given by the following approximations:

$$\Delta_{t,0,n,j} \approx \Delta_{c,n,j} + \frac{2k_{sf,n,j} (\Delta_{n,j} - \Delta_{c,n,j})^2}{V_{c,n,j} - V_{t,s0,n,j} + 2k_{sf,n,j} \Delta_{n,j} - 2k_{sf,n,j} \Delta_{c,n,j}} \dots$$

$$- \frac{k_{sf,n,j}^2 (\Delta_{n,j} - \Delta_{c,n,j})^3}{(V_{c,n,j} - V_{t,s0,n,j} + 2k_{sf,n,j} \Delta_{n,j} - 2k_{sf,n,j} \Delta_{c,n,j})^2} \geq \Delta_{c,n,j} \quad (6.2-40)$$

$$V_{t,0,n,j} \approx V_{c,n,j} + \frac{k_{sf,n,j}^2 (\Delta_{n,j} - \Delta_{c,n,j})^2}{V_{c,n,j} - V_{t,s0,n,j} + 2k_{sf,n,j}\Delta_{n,j} - 2k_{sf,n,j}\Delta_{c,n,j}} \geq V_{t,s0,n,j} \quad (6.2-41)$$

Simplified Bézier curve approximation: this is a simplified version of the previous procedure, discussed in Section 5.4.3, that can be adapted to the MDOF system as follows: first, the stiffness at the point of maximum velocity has been shown to be:

$$k_{mxv,n,j} = k_{sf,n,j} + k_{d,n} \left[1 - \beta_{d,n,j}^{1/a_n} \right] \quad (6.2-42)$$

And applying Eqn. (5.4-28), the total shear force can be approximated as:

$$V_{t,0,n,j} \approx V_{d,mxv,n,j} + \frac{k_{mxv,n,j}^2 \Delta_{n,j}^2}{V_{d,mxv,n,j} - V_{t,mxd,n,j} + 2k_{mxv,n,j}\Delta_{n,j}} \quad (6.2-43)$$

Lin & Chopra [2002, 2003]: this is a very simple method, which makes it convenient for practical application:

$$V_{t,0,n,j} = \left(V_{t,s0,n,j}^2 + V_{d,0,n,j}^2 \right)^{0.5}. \quad (6.2-44)$$

Kasai *et al.*, [2018]: another very simple method based on adjustment of the Lin-Chopra expression:

$$V_{t,0,n,j} = \left(V_{t,s0,n,j}^2 + a_n V_{d,0,n,j}^2 \right)^{0.5}. \quad (6.2-45)$$

From maximum total shear calculated with either procedure, total inertial forces can be found:

$$\mathbf{f}_{t,0} = \mathbf{H} \mathbf{V}_{t,0}. \quad (6.2-46)$$

Modal acceleration can be found dividing the inertial forces over the storey mass:

$$a_{n,j} = \frac{f_{t,0,n,j}}{m_n} \quad (6.2-47)$$

6.2.2.4 Total response

Peak static and kinematic values can be obtained by application of combination rules. For any storey n , the relevant values obtained are:

Kinematic variables

Drift $\Delta_{P,n}$; drift velocity $\dot{\Delta}_{P,n}$; relative displacement $u_{P,n}$; relative displacement velocity $v_{P,n}$; absolute acceleration $a_{P,n}$.

Damper response

Damper axial displacement $u_{b,P,n}$; damper axial velocity $v_{b,P,n}$; damper shear $V_{d,P,n}$; damper axial force $f_{d,P,n}$.

Static variables

Structural shear $V_{s,P,n}$; total shear $V_{t,P,n}$.

Common combination rules (SRSS and CQC) have been discussed in Chapter 2.

6.2.2.5 Optional procedure for damper response

In this Section, an optional method to find the total damper response is developed. The storey is assumed to vibrate harmonically in a fictitious mode, defined by its peak drift velocity $\dot{\Delta}_{P,n}$ and peak drift $\Delta_{P,n}$. Once these values are calculated by their SRSS, the overall q parameter of the damping system for storey n can be recalculated as:

$$q_{P,n} = \frac{c_{d,n} |\dot{\Delta}_{P,n}|^{a_n}}{k_{d,n} |\Delta_{P,n}|}, \quad (6.2-48)$$

whereupon $\beta_{d,P,n}$ and $\beta_{f,P,n}$ can be re-evaluated as:

$$V_{d,P,n} = \max \left\{ \beta_{d,P,n} c_{d,n} |\dot{\Delta}_{P,n}|^{a_n}, \beta_{f,P,n} c_{d,n} |\dot{\Delta}_{P,n}|^{a_n} \right\}, \quad (6.2-49)$$

$$f_{d,P,n} = \frac{V_{d,P,n}}{\cos \theta_n}. \quad (6.2-50)$$

6.3 Modified NLRSA procedure for inelastic systems

The procedure outlined for elastic systems can be extended to inelastic systems using the capacity curve obtained from static pushover analysis. For generality, yielding is considered at the first JY modes, with $JY \leq N$, for which the pushover curve must be available. However, for practical application, $JY = 1$ is generally acceptable. As in the elastic case, the procedure is iterative and it involves solution of an eigenproblem in order to update the modal properties. A simplified solution using constant modal shapes is also proposed.

6.3.1.1 Inelastic properties of frame with added damping system

Figure 6-2(a) shows the frame pushover curve (0p), labeled as ‘frame contribution’ obtained under the j -th mode lateral force pattern of the frame with added dampers. The frame lateral tangent stiffness κ_{fj} is increased by the stiffness contribution of the added damping system $\kappa_{a,j}$ thereby obtaining the modified pushover curve (0p) with total tangent stiffness $\kappa_{e,j}$. Notice that i) the stiffness contribution of the damping system $\kappa_{a,j}$ must be obtained at the displacement amplitude and velocity corresponding to point p (performance point); ii) the modal stiffness of the undamped frame κ_{fj} must be obtained with the total system (frame + added damping system) modal properties and therefore differs from the modal stiffness κ_j of the bare frame at first mode. Bi-linearization of the pushover curve

results in effective yield point of the frame with added damping ($V_{ey,1,j}$; $u_{y,N,j}$), obtained with an equal energy criterion, as discussed in Section 2.5.4.5. The performance point ($V_{e,1,j}$; $u_{N,j}$) lies on the modified pushover curve. Using modal properties of the first mode ($\Phi_{e,j}$, $\Psi_{e,j}$, $T_{e,j}$, $\omega_{e,j}$, $W_{eff,j}$, $L_{e,j}$ and $\Gamma_{e,j}$) the stiffness contribution of the different elements can be calculated as:

$$\kappa_{e,j} = \omega_{e,j}^2 \frac{M_{e,eff,j}}{\Gamma_{e,j}} = \omega_{e,j}^2 L_{e,j}, \quad (6.3-1)$$

$$\kappa_{a,j} = \psi_{e,1,j} k_{sa,1,j}, \quad (6.3-2)$$

$$\kappa_{f,j} = \kappa_{e,j} - \kappa_{a,j}. \quad (6.3-3)$$

κ_j is given by Eqn. (2.5-123), rewritten here as:

$$\kappa_j = \omega_j^2 L_j, \quad (6.3-4)$$

which is only dependent on first mode properties of the bare frame.

Figure 6-2: (a) bilinear global pushover curve for j -th mode lateral load pattern of frame with added damping system; (b) normalized curve and hysteresis loop.

The updated pushover curve can be approximated multiplying every ordinate of the original curve by the factor $\kappa_{e,j}/\kappa_j$. If the base shear corresponding to $u_{N,j}$ in the original curve is notated as $V_{1,j}$, see Figure 6-2(a), the value of base shear of the total system $V_{e,1,j}$ is given as:

$$V_{e,1,j} = V_{1,j} \frac{\kappa_{e,j}}{\kappa_j}. \quad (6.3-5)$$

The effective stiffness at the performance point p ($V_{e,1,j}$, $u_{N,j}$) is given by:

$$\kappa_{eff,j} = \frac{V_{e,1,j}}{u_{N,j}}. \quad (6.3-6)$$

And the effective period and frequency at the performance point p are:

$$T_{eff,j} = T_{e,j} \sqrt{\frac{\kappa_{e,j}}{\kappa_{eff,j}}}. \quad (6.3-7)$$

$$\omega_{eff,j} = \omega_{e,j} \sqrt{\frac{\kappa_{eff,j}}{\kappa_{e,j}}}. \quad (6.3-8)$$

The contributions to $V_{e,1,j}$ due to added damping ($V_{a,j}$) and frame stiffness ($V_{f,j}$) can be obtained replacing $\kappa_{e,j}$ in Eqn. (6.3-5) by the corresponding values $\kappa_{a,j}$ and $\kappa_{f,j}$. By the same rationale, the work $W_{e,j}$ in the modified pushover curve (area below the modified pushover curve) at point $u_{N,j}$, see Figure 6-2(d), can be calculated from the work W_j at $u_{N,j}$ in the original pushover curve:

$$W_{e,j} = W_j \frac{\kappa_{e,j}}{\kappa_j}. \quad (6.3-9)$$

The hysteretic energy dissipated in one-quarter cycle at maximum displacement is shown in Figure 6-2(c), and through energy bi-linearization criterion can be expressed using $W_{e,j}$ as:

$$W_{h,j} = 4 \cdot (2W_{e,j} - V_{e,1,j} u_{N,j}) \quad (6.3-10)$$

The elastic strain energy at p is given by $V_{e,j} \cdot u_{N,j}$ whereby the hysteretic damping ratio $\xi_{h,j}$ at the performance point can be expressed as (see Section 2.5.4.8):

$$\xi_{h,j} = q_{H,j} \frac{2}{\pi} \frac{2W_{e,j} - V_{e,1,j} u_{N,j}}{V_{e,1,j} u_{N,j}}, \quad (6.3-11)$$

and the period ductility is given by:

$$\mu_{T,j} = \frac{\kappa_{e,j}}{\kappa_{eff,j}}. \quad (6.3-12)$$

A more conventional approach uses bi-linearization of the pushover curve, that results in values of effective yield displacement $u_{y,N,j}$, effective yield base shear $V_{ey,l,j}$, ductility demand μ_j and effective post-yield stiffness $\alpha_{e,j}$, see Figure 6-2(c):

$$u_{y,N,j} = \frac{2W_{e,j} - V_{e,l,j} u_{N,j}}{\kappa_{e,j} u_{N,j} - V_{e,l,j}}, \quad (6.3-13)$$

$$V_{ey,l,j} = \kappa_{e,j} u_{y,N,j}, \quad (6.3-14)$$

$$\mu_j = \frac{u_{N,j}}{u_{y,N,j}}, \quad (6.3-15)$$

$$\alpha_{e,j} = \frac{V_{e,l,j} - V_{ey,l,j}}{u_{N,j} - u_{y,N,j}} \frac{1}{\kappa_{e,j}}, \quad (6.3-16)$$

Whereupon Eqn. (2.5-52) (repeated here for convenience) can be used:

$$\xi_{h,j} = q_{H,j} \frac{2}{\pi} \left(\frac{1}{1 + \alpha_{e,j}(\mu_j - 1)} - \frac{1}{\mu_j} \right). \quad (6.3-17)$$

Eqns. (6.3-11) and (6.3-17) give the same results. It must be noticed that the definition of ductility based the effective yield point gives meaningless results for the first portion of the pushover curve; for small values of displacement, the numerator in Eqn. (6.3-13) takes a value close to 0 resulting in unbounded values of μ . However, Eqn. (6.3-12) is robust and gives a stable value of 1 for the same range.

Viscous damping ratio must be referred to the properties of the performance point c, resulting in the following modification to Eqn. (6.2-11):

$$\xi_{a,eff,j} = \frac{1}{2} \frac{\sum_{n=1}^N \beta_{w,n,j} b_n c_{d,n} |\dot{\Delta}_{n,j}|^{a_n} |\Delta_{n,j}|}{\omega_{eff,j}^2 \sum_{n=1}^N m_n u_{n,j}^2}, \quad (6.3-18)$$

where the subscript ‘eff’ is used to indicate that the added viscous damping ratio is based on the properties at the effective performance point. The total effective viscous damping ratio $\xi_{eff,j}$ is the addition of three terms:

$$\xi_{eff,j} = \xi_{l,eff,j} + \xi_{a,eff,j} + \xi_{h,j}, \quad (6.3-19)$$

where $\xi_{l,eff,j}$ is the inherent damping adjusted at the frequency corresponding to point p. This adjustment is dependent on the type of damping considered.

As in the elastic case, the values of $q_{n,j}$ and the added stiffness $k_{sa,n,j}$ of the added damping system are given by Eqns. (6.2-1) and (6.2-2), the stiffness matrix $k_{e,j}$ is given by Eqn. (6.2-3), and the modal properties are obtained solving the eigenproblem stated by Eqn. (6.2-4).

The simplified solution assuming constant modal shapes used for elastic analysis, Eqns. (6.2-14) to (6.2-18), is also possible in this case.

6.3.2 Modal solution

The corresponding response at the effective point is given by:

$$u_{n,j} = \Gamma_{e,j} \phi_{e,n,j} S_d(T_{eff,j}, \xi_{eff,j}, a_g), \quad (6.3-20)$$

$$\Delta_{n,j} = \Gamma_{e,j} \psi_{e,n,j} S_d(T_{eff,j}, \xi_{eff,j}, a_g), \quad (6.3-21)$$

$$\dot{\Delta}_{n,j} = CFV(T_{eff,j}, \xi_{eff,j}) \omega_{eff,j} \Delta_{n,j}. \quad (6.3-22)$$

As in the elastic case, the effective properties $T_{eff,j}$ and $\xi_{eff,j}$ needed for solution are dependent on the resulting values of drift and drift velocity, so an iterative procedure, described hereby, is needed. Prior to the procedure, the pushover curve of the frame must be determined, and the tangent modal stiffness κ_j found using Eqn. (6.3-4).

If the inelastic solution is started with very low values of damping (for instance, inherent damping only) like in the elastic case, high values of displacement are obtained, that can be larger than those available in the pushover curve. For this reason, it is advisable to start the procedure with a cycle of the elastic procedure, to obtain an approximation to the actual values of damping. It is also possible to run the elastic procedure first for all modes, and use the final elastic values as seed for the inelastic procedure.

Inelastic solution

For every mode j where inelastic mode is considered:

1. Fix initial values performing a cycle of elastic solution, obtaining $\xi_{eff,j}^{(0)} = \xi_{e,j}$, $T_{eff,j}^{(0)} = T_{e,j}$, $\psi_{e,j}^{(0)} = \psi_{e,j}$, $\Gamma_{e,j}^{(0)} = \Gamma_{e,j}$, $W_{e,eff,j}^{(0)} = W_{e,eff,j}$.
2. Calculate the roof displacement $u_{N,j}^{(0)}$ using Eqn. (6.3-20).
3. Calculate drift $\Delta_{n,j}$ using Eqn. (6.3-21).
4. Calculate drift velocity, $\dot{\Delta}_{n,j}$, using Eqn. (6.3-22).
5. Calculate $q_{n,j}$ by Eqn. (6.2-1).
6. Calculate $\beta_{w,n,j}$ and $\beta_{s,n,j}$ from $q_{n,j}$ using Eqn. (5.3-16).
7. Calculate the contribution of the added damping system to the frame stiffness at mode 1, $k_{sa,n,j}$ using Eqn. (6.2-2).
8. Recalculate the modal properties ($\psi_{e,j}, T_{e,j}$) solving the eigenvalue problem stated by Eqn. (6.2-3). Find $\Gamma_{e,j}$ with Eqn. (6.2-25).
9. Find the modal stiffnesses κ_{ej} , κ_{aj} , κ_{fj} , using Eqns. (6.3-1), (6.3-2) and (6.3-3), respectively.

10. Find base shear $V_{1,j}$ and work W_j corresponding to displacement $u_{N,j}^{(0)}$ in the bare frame pushover curve.
11. Find base shear $V_{e,1,j}$ and work $W_{e,j}$ in the total system pushover curve using Eqns. (6.3-5) and (6.3-9), respectively.
12. Find the effective frequency $\omega_{eff,j}$ and period $T_{eff,j}$ using Eqns. (6.3-7) and (6.3-8).
13. Find the hysteretic damping ratio $\xi_{h,j}$ using Eqn. (6.3-11). Optionally, find $u_{y,N,j}$, $V_{ey,1,j}$, μ_j , $\alpha_{e,j}$, and $\xi_{h,j}$ using Eqns. (6.3-13), (6.3-14), (6.3-15), (6.3-16), (6.3-17), respectively.
14. Find the effective added viscous damping ratio $\xi_{a,eff,j}$ using Eqn. (6.3-18).
15. Find the total effective viscous damping ratio $\xi_{eff,j}$ using Eqn. (6.3-19) (adjusting, if necessary, the inherent damping ratio $\xi_{I,j}$).
16. Using Eqn. (6.3-20) and the updated modal properties, find the new value of $u_{N,j}$.
17. If $u_{N,j}$ is similar to the initial value $u_{N,j}^{(0)}$, the procedure has converged and can be stopped. Otherwise, make $u_{N,j}^{(0)} = u_{N,j}$, $\xi_{e,j}^{(0)} = \xi_{e,j}$, $\Psi_{e,j}^{(0)} = \Psi_{e,j}$, $\Gamma_{e,j}^{(0)} = \Gamma_{e,j}$, and go to step 3.

Optionally, the convergence criterion can be based on variation of drift, frequency or period.

For the simplified method, change step 8 of the previous procedure as follows: modal shapes ($\Phi_{e,j}$, $\Psi_{e,j}$) and derived properties ($\Gamma_{e,j}$) are those of the undamped frame; find $\omega_{a,j}$ with Eqn. (6.2-17); find $\omega_{e,j}$ with Eqn. (6.2-18), and $T_{e,j}$ from $\omega_{e,j}$.

6.3.3 Kinematic and static solution

Solution for the kinematic and static variables is found by exactly the same procedure as in elastic case, by application of a suitable combination rule to the modal solution for the different variables. When the CQC rule is applied, the frequency (or period) for the first mode should be consistently taken as the effective value $\omega_{eff,1}$.

For the static variables, the procedures discussed in the elastic case are valid, with some minor modifications. First, for the response at maximum drift in yielding modes, the effective period should be used:

$$f_{t, mxd, n, j} = \omega_{eff, j}^2 m_n u_{n, j}. \quad (6.3-23)$$

Second, if the Bézier curve approximation is used to find the maximum response, the equations need to be modified as indicated in Section 5.5.

Approximation of maximum total shear by Bézier curve:

The equations are given for the usual case of yielding in the first mode. First, the corner point corresponding to yielding is found as:

$$\Delta_{c, lim, n, 1} = (2 - \mu_n) \Delta_{y, n, 1}; \quad (6.3-24)$$

the corner point is compared with the point of maximum brace displacement, and V_c is calculated in consequence:

Case 1: $\Delta_{d,0,n,1} > \Delta_{c,lim,n,1}$; in this case:

$$\Delta_{c,n,1} = \Delta_{d,0,n,1}; \quad (6.3-25)$$

$$V_{c,n,1} = V_{d,0,n,1} + (1 - \alpha_e(\mu - 1))V_{y,n,1} + (\Delta_{c,n,1} - \Delta_{c,lim,n,1})\alpha_e k_{s,n,j}; \quad (6.3-26)$$

Case 2: $\Delta_{d,0,n,1} \leq \Delta_{c,lim,n,1}$; in this case:

$$\Delta_{c,n,1} = \Delta_{c,lim,n,1}; \quad (6.3-27)$$

$$V_{c,n,1} = V_{d,0,n,1} + (1 - \alpha_e(\mu - 1))V_{y,n,1}. \quad (6.3-28)$$

The total shear response in the first mode is given by the following approximation:

$$V_{t,0,n,1} \approx V_{c,n,1} + \frac{k_{s,n,j}^2 (\Delta_{n,j} - \Delta_{c,n,j})^2}{V_{c,n,1} - V_{t,s0,n,1} + 2k_{s,n,j}\Delta_{n,j} - 2k_{s,n,j}\Delta_{c,n,j}} \geq V_{t,msd,n,1} \quad (6.3-29)$$

6.3.4 Estimation of 1st mode storey yield properties including damping system contribution

In Chapter 2, Section 2.5.4.6, a simplified procedure based on limit analysis was presented to find the pushover properties of the frame without damping systems. It was shown that the internal work W_{int} for a complete collapse mechanism can be expressed as $X_1\theta$, where X_1 is a constant with units of moment, independent of the force pattern applied as long as the collapse mechanism is unchanged, and θ is the overall drift ratio, see Eqn. (2.5-199). It was also shown that the external work can be expressed as the product of base shear $V_{y,1,1}$, roof drift ratio θ , and a constant dependent only on geometrical properties, Eqn. (2.5-197), repeated here for convenience:

$$W_{ext} = \mathbf{f}_{y1}^T \mathbf{z} \theta = \frac{\boldsymbol{\phi}_1^T \mathbf{m} \mathbf{z}}{\boldsymbol{\phi}_1^T \mathbf{m} \mathbf{1}} V_{y,1,1} \theta. \quad (6.3-30)$$

Equating internal and external work, the approximate value of $V_{y,1,1}$ is derived. The same rationale can be applied to the structure with added damping systems, as long as it is assumed that the collapse mechanism is the same as in the undamped structure. This is a reasonable assumption for frames designed with capacity criteria and moderate values of added damping.

First, the yield shear of the frame without added damping system with new modal properties $\boldsymbol{\phi}_{e,1}$ is found as:

$$V_{fy,1,1} = \frac{\boldsymbol{\phi}_{e,1}^T \mathbf{m} \mathbf{1}}{\boldsymbol{\phi}_{e,1}^T \mathbf{m} \mathbf{z}} X_1. \quad (6.3-31)$$

The 1st storey yield drift $\Delta_{y,1,1}$ can be related to the yield roof displacement as:

$$\Delta_{ey,1,1} = \psi_{e,n,1} u_{y,N,1}, \quad (6.3-32)$$

and, multiplying by the first storey stiffness, the following relationship between yield base shear and yield roof displacement is obtained:

$$u_{ey,N,1} = \frac{V_{fy,1,1}}{\psi_{e,1,1} k_{sf,1,1}}. \quad (6.3-33)$$

The total yield base shear can be obtained adding the (elastic) stiffness contribution of damping system:

$$V_{ey,1,1} = V_{fy,1,1} + \psi_{e,1,1} k_{sa,1,1} u_{ey,N,1}. \quad (6.3-34)$$

Further simplification is obtained if the original pushover properties are considered unchanged, which is reasonable for the first mode.

6.4 Convergence of the proposed procedures

The procedures presented can be considered to converge when the input (seed) value of an iteration step is not significantly different from the output value. The convergence criterion can be based on different variables. A traditional control variable is roof displacement $u_{N,j}$. Another possibility is to use the value of drift $\Delta_{n,j}$. Finally, frequency or period are also reasonable candidates. When evaluating the relative change of a variable X , an expression of the type

$$\left| \frac{X - X^{(0)}}{X^{(0)}} \right| < tol, \quad (6.4-1)$$

can be used, where tol is a predefined tolerance, X is the variable value at the end of the step and $X^{(0)}$ the variable value at the beginning of the step. Because both drift and roof displacement can be close to 0 for some modes and storeys, the maximum value of the variables throughout storeys $X = \max(X_n)$ should be used in the previous expression. If frequency is used, because the values are always positive this precaution is not necessary.

In this study, the convergence of both maximum modal drift and modal frequency are examined; convergence is considered achieved if the pre-fixed tolerance is reached for any of these variables.

Table 6-1 shows the maximum number of iterations needed to reach a certain convergence tolerance using the ‘consistent’ method with updated modal properties. For a reasonable tolerance of 1% the procedure is always convergent, the maximum number of iterations at any mode is 8 and the average number of iterations is 5. For a tolerance of 0.1%, up to 11 iterations are needed and the average is 7. At a smaller tolerance of 0.01%, convergence is not reached for one case among those examined.

Table 6-2 shows the maximum number of iterations needed to reach a certain convergence tolerance using the ‘simple’ method, with constant modal properties. The method is always convergent. The number of iterations is smaller than in the RSAB method.

Table 6-1. Convergence of proposed ‘consistent’ method (updated modal shapes).

MRF	ADS	ξ_I	α	PHA/g	ξ_e	μ_T	Iterations to reach tolerance of				
							1E-01	1E-02	1E-03	1E-04	1E-05
03D	Q.1.1	5%	0%	0.10	20%	1.00	2	2	3	4	5
03D	Q.1.1	5%	0%	0.40	21%	1.45	2	2	3	4	5
03D	Q.1.1	2%	0%	0.10	17%	1.00	2	3	3	4	5
03D	Q.1.1	2%	0%	0.40	17%	1.52	2	3	3	5	6
03D	Q.1.1	2%	3%	0.10	17%	1.00	2	3	3	4	5
03D	Q.1.1	2%	3%	0.40	17%	1.41	2	3	3	4	5
03D	Q.2.1	2%	0%	0.15	34%	1.00	2	3	4	6	7
03D	Q.2.1	2%	0%	0.40	34%	1.20	2	3	4	6	7
03D	Q.3.1	2%	0%	0.20	50%	1.00	2	4	5	7	8
03D	Q.3.1	2%	0%	0.40	50%	1.05	2	4	5	7	8
03D	Q.1.5	2%	0%	0.15	26%	1.00	4	6	9	12	15
03D	Q.1.5	2%	0%	0.40	16%	1.48	3	5	7	9	11
03D	Q.2.5	2%	0%	0.20	44%	1.00	4	7	10	13	16
03D	Q.2.5	2%	0%	0.40	33%	1.14	4	7	11	14	17
03D	Q.3.5	2%	0%	0.20	56%	1.00	4	6	8	10	12
03D	Q.3.5	2%	0%	0.40	46%	1.02	5	8	11	14	17
05D	Q.1.1	5%	0%	0.10	22%	1.00	2	3	4	5	6
05D	Q.1.1	5%	0%	0.40	22%	1.19	3	6	9	13	16
05D	Q.1.1	2%	0%	0.10	18%	1.00	2	3	4	5	6
05D	Q.1.1	2%	0%	0.40	19%	1.23	3	7	11	15	19
05D	Q.1.1	2%	3%	0.10	18%	1.00	2	3	4	5	6
05D	Q.1.1	2%	3%	0.40	19%	1.20	3	5	8	11	14
05D	Q.2.1	2%	0%	0.15	35%	1.00	2	4	5	7	8
05D	Q.2.1	2%	0%	0.40	35%	1.08	2	4	5	7	8
05D	Q.3.1	2%	0%	0.15	47%	1.00	3	5	6	8	10
05D	Q.3.1	2%	0%	0.40	47%	1.01	3	5	6	8	10
05D	Q.1.5	2%	0%	0.10	34%	1.00	4	7	10	13	15
05D	Q.1.5	2%	0%	0.40	19%	1.22	4	6	8	11	14
05D	Q.2.5	2%	0%	0.15	46%	1.00	4	7	9	12	14
05D	Q.2.5	2%	0%	0.40	35%	1.04	4	7	11	100	100
05D	Q.3.5	2%	0%	0.20	50%	1.00	4	6	8	10	12
05D	Q.3.5	2%	0%	0.40	46%	1.00	5	8	11	14	16

Table 6-2. Convergence of proposed ‘simple’ method (constant modal shapes).

MRF	ADS	ξ_l	α	PHA/g	ξ_e	μ_T	iterations to reach tolerance of				
							1E-01	1E-02	1E-03	1E-04	1E-05
03D	Q.1.1	5%	0%	0.10	20%	1.00	2	2	3	4	5
03D	Q.1.1	5%	0%	0.40	21%	1.45	2	2	3	4	5
03D	Q.1.1	2%	0%	0.10	17%	1.00	2	3	3	4	5
03D	Q.1.1	2%	0%	0.40	17%	1.52	2	3	3	5	6
03D	Q.1.1	2%	3%	0.10	17%	1.00	2	3	3	4	5
03D	Q.1.1	2%	3%	0.40	17%	1.41	2	3	3	4	5
03D	Q.2.1	2%	0%	0.15	34%	1.00	2	3	4	5	7
03D	Q.2.1	2%	0%	0.40	34%	1.20	2	3	4	5	7
03D	Q.3.1	2%	0%	0.20	50%	1.00	2	4	5	7	8
03D	Q.3.1	2%	0%	0.40	50%	1.05	2	4	5	7	8
03D	Q.1.5	2%	0%	0.15	26%	1.00	3	6	8	11	13
03D	Q.1.5	2%	0%	0.40	16%	1.48	3	5	7	9	10
03D	Q.2.5	2%	0%	0.20	44%	1.00	4	7	10	13	16
03D	Q.2.5	2%	0%	0.40	33%	1.14	4	6	9	12	15
03D	Q.3.5	2%	0%	0.20	56%	1.00	4	6	9	11	13
03D	Q.3.5	2%	0%	0.40	46%	1.02	4	7	10	13	16
05D	Q.1.1	5%	0%	0.10	22%	1.00	2	3	4	5	6
05D	Q.1.1	5%	0%	0.40	22%	1.19	3	6	9	13	16
05D	Q.1.1	2%	0%	0.10	18%	1.00	2	3	4	5	6
05D	Q.1.1	2%	0%	0.40	19%	1.23	3	7	11	15	19
05D	Q.1.1	2%	3%	0.10	18%	1.00	2	3	4	5	6
05D	Q.1.1	2%	3%	0.40	19%	1.20	3	6	9	12	15
05D	Q.2.1	2%	0%	0.15	35%	1.00	2	4	5	7	8
05D	Q.2.1	2%	0%	0.40	35%	1.08	2	4	5	7	8
05D	Q.3.1	2%	0%	0.15	47%	1.00	3	5	6	8	10
05D	Q.3.1	2%	0%	0.40	47%	1.01	3	5	6	8	10
05D	Q.1.5	2%	0%	0.10	34%	1.00	3	6	9	12	15
05D	Q.1.5	2%	0%	0.40	19%	1.22	3	6	8	11	15
05D	Q.2.5	2%	0%	0.15	46%	1.00	4	7	9	11	14
05D	Q.2.5	2%	0%	0.40	35%	1.04	3	6	10	13	16
05D	Q.3.5	2%	0%	0.20	50%	1.00	4	6	7	9	10
05D	Q.3.5	2%	0%	0.40	46%	1.00	4	7	10	13	16

6.5 Numerical examples

This Section presents a series of Numerical Examples to show the application of the proposed method. In the calculation of response, spectral correction factors (as described in Chapter 4) have been used consistently. This is, however, not relevant to the procedures presented. In inelastic cases, the hysteresis loop factor q_H has been taken as described by .

6.5.1 Example 1: elastic frame equipped with non-linear viscous dampers

The proposed procedure is applied to frame 05D with 2% inherent damping and 3% post-yield stiffness (defined in Annex B) equipped with non-linear viscous dampers in configuration Q.1.5, as defined in Annex D. The frame is located at site class B. The Peak Horizontal Acceleration is taken as $PHA = 0.1g$. In site class B for spectrum Type 1 (soil parameter $S = 1.2$ according to CEN [2004], Table 3.2) corresponds to a moderate rock acceleration $a_{gR} = 0.0833g$. For this acceleration, the mean and median responses obtained from THNLA are elastic.

This example was solved using the conventional NLRSA method for a larger intensity ($PHA = 0.4g$) in Section 2.5.7, where frame and damping system properties are listed, along with undamped modal properties. The bracing elements of this structure have been designed for damper forces obtained for that intensity, with a rather large stiffness.

Inherent damping is of the Rayleigh type, fixed as 2% in modes 1 and 3. Eqn. (2.5-128) can be used to adjust the inherent damping to modified frequencies.

6.5.1.1 Bracing properties

$$k_b^T = \begin{bmatrix} 135.9 & 167.1 & 167.1 & 173.6 & 173.6 \end{bmatrix} \frac{kN}{mm}$$

$$k_d^T = \begin{bmatrix} 109.9 & 135.2 & 135.2 & 140.4 & 140.4 \end{bmatrix} \frac{kN}{mm}$$

6.5.1.2 Initialization of iterative procedure

Initial assumed value of total damping ratio is ξ_l . The initial modal properties are those of the undamped frame. Displacement and drift are found using Eqns. (6.2-22) and (6.2-23):

$$u = \begin{bmatrix} 13.99 & 6.19 & 2.12 & 0.59 & 0.14 \\ 35.43 & 12.17 & 2.06 & -0.1 & -0.16 \\ 58.13 & 11.34 & -1.16 & -0.5 & 0.12 \\ 81.41 & 1.34 & -2.43 & 0.54 & -0.06 \\ 101.18 & -13.55 & 1.71 & -0.2 & 0.02 \end{bmatrix} mm$$

$$\Delta = \begin{bmatrix} 13.99 & 6.19 & 2.12 & 0.59 & 0.14 \\ 21.43 & 5.98 & -0.06 & -0.69 & -0.3 \\ 22.71 & -0.83 & -3.21 & -0.4 & 0.29 \\ 23.27 & -10.01 & -1.27 & 1.04 & -0.19 \\ 19.77 & -14.89 & 4.14 & -0.74 & 0.08 \end{bmatrix} \text{ mm}$$

The velocity $\mathbf{Y}$ (the symbol $\mathbf{Y}$ is used in this example instead of $\dot{\mathbf{\Delta}}$) is found using (6.2-24), with the correction factor for velocity described in Chapter 4, which is a function of period and viscous damping:

$$cfv^T = [1.193 \quad 1.022 \quad 0.972 \quad 0.891 \quad 0.784]$$

Whereupon:

$$Y = \begin{bmatrix} 52.2 & 57.48 & 37.88 & 16.3 & 4.95 \\ 79.95 & 55.48 & -1.11 & -19.12 & -10.68 \\ 84.71 & -7.67 & -57.45 & -11 & 10.13 \\ 86.81 & -92.89 & -22.76 & 28.56 & -6.62 \\ 73.75 & -138.19 & 74.07 & -20.28 & 2.84 \end{bmatrix} \frac{\text{mm}}{\text{s}}$$

The Maxwell parameters can be calculated from Eqn. (6.2-1) and the numerical approximations in Chapter 5, Eqn. (5.3-16):

$$q = \begin{bmatrix} 0.121 & 0.288 & 0.682 & 1.601 & 3.753 \\ 0.08 & 0.238 & 3.234 & 1.202 & 2.077 \\ 0.077 & 0.64 & 0.451 & 1.585 & 2.132 \\ 0.074 & 0.177 & 0.689 & 0.947 & 2.539 \\ 0.08 & 0.145 & 0.382 & 1.124 & 3.877 \end{bmatrix}$$

$$\beta_w = \begin{bmatrix} 0.974 & 0.897 & 0.627 & 0.156 & 0 \\ 0.986 & 0.924 & 0 & 0.302 & 0.08 \\ 0.987 & 0.657 & 0.793 & 0.16 & 0.075 \\ 0.988 & 0.953 & 0.622 & 0.446 & 0.047 \\ 0.986 & 0.966 & 0.839 & 0.342 & 0 \end{bmatrix}$$

$$\beta_s = \begin{bmatrix} 0.04 & 0.1348 & 0.4228 & 0.8936 & 1 \\ 0.0238 & 0.1028 & 1 & 0.7569 & 0.9527 \\ 0.023 & 0.3915 & 0.2503 & 0.89 & 0.956 \\ 0.0218 & 0.0673 & 0.4277 & 0.6097 & 0.974 \\ 0.0238 & 0.051 & 0.2003 & 0.7163 & 1 \end{bmatrix}$$

Therefore, the initial value of contribution of the added damping system to the frame stiffness is given by Eqn. (6.2-2):

$$k_{sa} = \begin{bmatrix} 4.4 & 14.81 & 46.48 & 98.23 & 109.92 \\ 3.22 & 13.9 & 135.19 & 102.32 & 128.78 \\ 3.11 & 52.93 & 33.83 & 120.32 & 129.24 \\ 3.06 & 9.46 & 60.07 & 85.63 & 136.79 \\ 3.35 & 7.16 & 28.13 & 100.6 & 140.44 \end{bmatrix} \frac{\text{kN}}{\text{mm}}$$

Where every column corresponds to a mode and every row corresponds to a storey. Observation of this matrix makes clear that the stiffness contribution increases for higher modes. The points located close to nodal points of modal displacements present also a large contribution due to their small value of drift.

6.5.1.3 Response in the first mode

Initial iteration

At the frame first natural frequency, the added damping system contributes to the total storey stiffness with:

$$k_{sa.mode}^T = [4.396 \ 3.216 \ 3.112 \ 3.057 \ 3.349] \frac{kN}{mm}$$

This storey stiffness contributes to the global stiffness at first mode, according to the second term of Eqn. (6.2-3):

$$k_{a.mode} = \begin{bmatrix} 7.6 & -3.2 & 0 & 0 & 0 \\ -3.2 & 6.3 & -3.1 & 0 & 0 \\ 0 & -3.1 & 6.2 & -3.1 & 0 \\ 0 & 0 & -3.1 & 6.4 & -3.3 \\ 0 & 0 & 0 & -3.3 & 3.3 \end{bmatrix} \frac{kN}{mm}$$

According to Eqn. (6.2-3), this term is added to the frame stiffness matrix $\mathbf{k}$, to give the total first-mode matrix $\mathbf{k}_{e,1}$.

$$k_{e.mode} = \begin{bmatrix} 288 & -183.5 & 56 & -11.6 & 1.9 \\ -183.5 & 254.3 & -161.7 & 46.9 & -7.6 \\ 56 & -161.7 & 215.4 & -125.8 & 27.8 \\ -11.6 & 46.9 & -125.8 & 145.8 & -57.7 \\ 1.9 & -7.6 & 27.8 & -57.7 & 35.9 \end{bmatrix} \frac{kN}{mm}$$

Then, the eigenvalue stated by Eqn. (6.2-4) is solved to obtain the new modal properties $T_{e,1}$; $\omega_{e,1}$; $\phi_{e,1}$; $\psi_{e,1}$ consistent with $\mathbf{k}_{e,1}$; additionally, $\Gamma_{e,1}$ is found with Eqn. (6.2-25). The values are $T_{e,1} = 1.895$ s; $\omega_{e,1} = 3.316$ Hz; $\Gamma_{e,1} = 1.353$, and:

$$\phi_{e,1}^T = [0.138 \ 0.35 \ 0.575 \ 0.805 \ 1]$$

$$\psi_{e,1}^T = [0.138 \ 0.212 \ 0.224 \ 0.23 \ 0.195]$$

The added viscous damping ratio is found with Eqn. (6.2-13), $\xi_{a,1} = 20.49\%$. Inherent damping for frequency $\omega_{e,1}$ is found $\xi_{i,1} = 1.92\%$. Total damping ratio in the first mode is found adding viscous and inherent damping ratios as $\xi_{e,1} = 22.41\%$. By using (6.2-22), (6.2-23) and (6.2-24):

$$u_I^T = [7 \ 17.6 \ 28.5 \ 39.4 \ 48.1] \text{ mm}$$

$$\Delta_I^T = [6.99 \ 10.57 \ 10.96 \ 10.84 \ 8.78] \text{ mm}$$

$$Y_I^T = [34.4 \ 52.1 \ 54 \ 53.4 \ 43.2] \frac{\text{mm}}{\text{s}}$$

Comparing the drift values with the initial ones, the maximum variation is 53%; the variation in frequency is -6%. Thus, further iteration is needed.

Second iteration

With the drift and drift velocity values for the mode, the Maxwell parameters are updated using (6.2-1) and the corresponding Eqn. (5.3-16) In Chapter 05:

$$q_I^T = [0.197 \ 0.13 \ 0.128 \ 0.124 \ 0.138]$$

$$\beta_{W,I}^T = [0.944 \ 0.971 \ 0.972 \ 0.973 \ 0.968]$$

$$\beta_{s,I}^T = [0.078 \ 0.044 \ 0.043 \ 0.041 \ 0.047]$$

The contribution of damping system braces to stiffness is given by Eqn. (6.2-3):

$$k_{sa,mode}^T = [8.63 \ 5.95 \ 5.81 \ 5.78 \ 6.66] \frac{\text{kN}}{\text{mm}}$$

The contribution to the total stiffness is found, the corresponding eigenproblem solved, and the updated modal properties are found to be: $T_{e,1} = 1.809 \text{ s}$; $\omega_{e,1} = 3.474 \text{ Hz}$; $\Gamma_{e,1} = 1.344$, and:

$$\phi_{e,1}^T = [0.15 \ 0.376 \ 0.607 \ 0.829 \ 1]$$

$$\psi_{e,1}^T = [0.15 \ 0.226 \ 0.231 \ 0.221 \ 0.171]$$

The added viscous damping ratio, inherent damping ratio and total damping ratio are $\xi_{a,1} = 29.7\%$, $\xi_{l,1} = 1.86\%$, $\xi_{e,1} = 31.6\%$.

And, by application of (6.2-22), (6.2-23) and (6.2-24), the displacement, drift and drift-velocity solution are:

$$u_I^T = [5.9 \ 14.9 \ 24 \ 32.7 \ 39.5] \text{ mm}$$

$$\Delta_I^T = [5.94 \ 8.93 \ 9.12 \ 8.75 \ 6.77] \text{ mm}$$

$$Y_I^T = [31.8 \ 47.9 \ 48.9 \ 46.9 \ 36.3] \frac{\text{mm}}{\text{s}}$$

The maximum variation in drift is 17.9% and in frequency -4.8%. The iterations are continued with exactly the same process.

Final solution for the first mode:

After two more iterations, a good convergence is obtained, with a maximum variation of -1.2% in drift and -0.6% in frequency. One more iteration results in 0.22% variation in drift and -0.13% in frequency. The iterative process for the first mode is stopped, and the following final solution values are retrieved: $T_{e,1} = 1.761\text{s}$; $\omega_{e,1} = 3.567\text{s}$; $\Gamma_{e,1} = 1.337$. And:

$$k_{sa,1}^T = [10.83 \ 7.45 \ 7.4 \ 7.62 \ 9.35] \frac{kN}{mm}$$

$$\phi_{e,1}^T = [0.154 \ 0.386 \ 0.619 \ 0.837 \ 1]$$

$$\psi_{e,1}^T = [0.154 \ 0.231 \ 0.233 \ 0.218 \ 0.163]$$

$$u_I^T = [5.7 \ 14.2 \ 22.9 \ 30.9 \ 36.9] \text{ mm}$$

$$\Delta_I^T = [5.7 \ 8.54 \ 8.62 \ 8.06 \ 6] \text{ mm}$$

$$Y_I^T = [31.5 \ 47.2 \ 47.6 \ 44.5 \ 33.1] \frac{mm}{s}$$

6.5.1.4 Response in higher modes

The iterative procedure described in the previous Section is repeated for all relevant modes. In this Example, all modes were included in the analysis. The convergence criterion for each mode was a maximum absolute variation of 1% in either drift or frequency. The final kinematic solution reached is:

$$\Delta = \begin{bmatrix} 5.702 & 1.287 & 0.319 & 0.115 & 0.034 \\ 8.54 & 0.994 & -0.123 & -0.173 & -0.081 \\ 8.615 & -0.243 & -0.47 & -0.02 & 0.087 \\ 8.06 & -2.01 & -0.014 & 0.206 & -0.065 \\ 6.001 & -2.74 & 0.586 & -0.191 & 0.032 \end{bmatrix} \text{ mm}$$

$$Y = \begin{bmatrix} 31.48 & 17.64 & 8.73 & 4.16 & 1.42 \\ 47.15 & 13.62 & -3.37 & -6.26 & -3.37 \\ 47.57 & -3.33 & -12.87 & -0.71 & 3.64 \\ 44.5 & -27.55 & -0.37 & 7.47 & -2.71 \\ 33.13 & -37.54 & 16.04 & -6.9 & 1.34 \end{bmatrix} \frac{mm}{s}$$

The final values are given hereby, along with those of the undamped frame for comparison:

Table 6-3. Properties of frame 05D with and without added damping system Q.1.5, elastic level ($PHA = 0.1g$).

Frame with added damping system	Frame without added damping system
$T_e^T = [1.761 \ 0.448 \ 0.19 \ 0.129 \ 0.101] \text{ s}$	$T^T = [2.006 \ 0.704 \ 0.361 \ 0.226 \ 0.163] \text{ s}$

$$\omega_e^T = [3.57 \ 14.04 \ 33.08 \ 48.7 \ 62.35] \frac{1}{s} \quad \omega^T = [3.13 \ 8.92 \ 17.42 \ 27.84 \ 38.57] \frac{1}{s}$$

$$\Gamma_e^T = [1.331 \ -0.467 \ 0.183 \ -0.073 \ 0.017] \quad \Gamma^T = [1.361 \ -0.506 \ 0.203 \ -0.075 \ 0.016]$$

$$\Phi_e = \begin{bmatrix} 0.154 & -0.475 & 1.07 & -1.852 & 4.327 \\ 0.386 & -0.841 & 0.657 & 0.937 & -5.905 \\ 0.619 & -0.752 & -0.918 & 1.254 & 5.148 \\ 0.837 & -0.01 & -0.964 & -2.076 & -3.071 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \Phi = \begin{bmatrix} 0.132 & -0.44 & 1.198 & -2.963 & 8.275 \\ 0.346 & -0.902 & 1.228 & 0.395 & -9.223 \\ 0.573 & -0.849 & -0.651 & 2.532 & 7.031 \\ 0.806 & -0.1 & -1.424 & -2.65 & -3.553 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\Psi_e = \begin{bmatrix} 0.154 & -0.475 & 1.07 & -1.852 & 4.327 \\ 0.231 & -0.367 & -0.412 & 2.789 & -10.233 \\ 0.233 & 0.09 & -1.576 & 0.317 & 11.054 \\ 0.218 & 0.741 & -0.046 & -3.329 & -8.22 \\ 0.163 & 1.01 & 1.964 & 3.076 & 4.071 \end{bmatrix} \quad \Psi = \begin{bmatrix} 0.132 & -0.44 & 1.198 & -2.963 & 8.275 \\ 0.215 & -0.462 & 0.031 & 3.357 & -17.498 \\ 0.227 & 0.052 & -1.88 & 2.137 & 16.254 \\ 0.233 & 0.749 & -0.773 & -5.182 & -10.584 \\ 0.194 & 1.1 & 2.424 & 3.65 & 4.553 \end{bmatrix}$$

Notice that modal shapes listed for the structure with added dampers do not form a compatible set, as they do not fulfill the orthogonality condition. For a certain mode (for instance, second mode) the modal shape listed (second column in Φ_e) is coherent with the rest of modal shapes obtained for that particular mode in the procedure (i.e., by solution of the eigenvalue problem $(\mathbf{k}_{e,2} - \omega_{e,r}^2 \mathbf{m})\boldsymbol{\phi}_{e,r}$ for all values of r (including $r = 2$). This set of modal shapes fulfils the orthogonality condition, and thus the motion equation is uncoupled and can be solved with conventional modal techniques. Finally, only the solution for mode 2 is preserved.

Modal shapes and frequencies are slightly changed in the first mode, and increasingly so in higher modes, due to the larger contribution of brace stiffness. Additionally, the added viscous damping increases for the second mode but then decreases with increasing mode number; in the fifth mode, no added damping occurs. The final values of added stiffness are:

$$k_{sa} = \begin{bmatrix} 10.8 & 53.2 & 105.4 & 109.9 & 109.9 \\ 7.4 & 59.8 & 132.9 & 132.6 & 135.2 \\ 7.4 & 115 & 115.9 & 135.2 & 135.2 \\ 7.6 & 38.2 & 140.4 & 136.2 & 140.4 \\ 9.4 & 31.1 & 109.9 & 136.8 & 140.4 \end{bmatrix} \frac{kN}{mm}$$

To put these values in perspective, the storey stiffness of the bare frame is given hereby:

$$k_s = \begin{bmatrix} 52.6 & 64.8 & 92.9 & 143 & 217.6 \\ 32.6 & 44.5 & -109.6 & 93.7 & 151.4 \\ 26.8 & 0.8 & 55.3 & 98.1 & 148.3 \\ 19.6 & 24.9 & 58 & 84.6 & 134.9 \\ 12.4 & 18.7 & 34.8 & 64.8 & 111.2 \end{bmatrix} \frac{kN}{mm}$$

Added viscous damping, inherent damping and total damping ratios are:

$$\xi_a^T = [32.4\% \quad 41.9\% \quad 13.7\% \quad 3.7\% \quad 0]$$

$$\xi_I^T = [1.83\% \quad 1.69\% \quad 3.24\% \quad 4.64\% \quad 5.88\%]$$

$$\xi_e^T = [34.3\% \quad 43.6\% \quad 16.9\% \quad 8.3\% \quad 5.9\%]$$

The values of k_{sa} show that the contribution of damping system elements to global stiffness in the 3rd, 4th and 5th mode is very large. This is confirmed by the small amount of added viscous damping provided at these modes. At the 5th mode $\xi_{a,5} = 0\%$.

For the same frame, the added viscous damping if the brace stiffness is not considered is:

$$\xi_a^T = [32.9\% \quad 39.6\% \quad 43.6\% \quad 39.5\% \quad 34\%]$$

6.5.1.5 Total kinematic solution:

The total kinematic solution is found applying a combination rule to the modal solution. Eqns. (6.2-26) and (6.2-27) are used to obtain the storey displacement and velocity ($\mathbf{u}$ and $\mathbf{v}$); then the peak values $\mathbf{u}_p$, $\mathbf{v}_p$, are found using the CQC rule. Finally, Eqns. (2.4-4) and (2.4-7) are applied to obtain the damper stroke and velocity ($\mathbf{u}_{b,p}$, $\mathbf{v}_{b,p}$). Values obtained from direct TH analysis are given below in a different typo for comparison.

$$u_p^T = [6.1 \quad 14.8 \quad 23.3 \quad 30.9 \quad 36.5] \text{ mm}$$

$$[6.8876 \quad 16.0951 \quad 24.5072 \quad 31.7862 \quad 36.6616] \text{ mm}$$

$$v_p^T = [41.4 \quad 90.4 \quad 134 \quad 170.9 \quad 200.3] \frac{\text{mm}}{\text{s}}$$

$$[51.8352 \quad 101.4862 \quad 133.6312 \quad 154.9470 \quad 170.7497] \text{ mm/s}$$

$$\Delta_p^T = [6.11 \quad 8.78 \quad 8.58 \quad 7.94 \quad 6.1] \text{ mm}$$

$$[6.8876 \quad 9.5531 \quad 9.2729 \quad 8.3782 \quad 6.0718] \text{ mm}$$

$$Y_p^T = [41.4 \quad 51.9 \quad 48.9 \quad 48.2 \quad 45.5] \frac{\text{mm}}{\text{s}}$$

$$[51.8352 \quad 60.2719 \quad 55.3198 \quad 48.7699 \quad 38.2717] \text{ mm/s}$$

Damper stroke and velocity:

$$u_{b,p}^T = [5.49 \quad 7.89 \quad 7.72 \quad 7.14 \quad 5.48] \text{ mm}$$

$$[6.2301 \quad 8.4409 \quad 8.1532 \quad 7.4060 \quad 5.4447] \text{ mm}$$

$$v_{b,p}^T = [37.2 \quad 46.7 \quad 44 \quad 43.4 \quad 40.9] \frac{\text{mm}}{\text{s}}$$

6.5.1.6 Response at maximum drift

The total modal forces at maximum drift are obtained using Eqn. (6.2-28). The total modal shear forces at maximum drift are calculated using Eqn. (6.2-29). The added damping system contribution to the shear forces at maximum drift is given by Eqn. (6.2-30), and the structure contribution is given by Eqn. (6.2-31). After modal combination, the following peak values are obtained for the structure contribution:

$$V_{s,P}^T = [340 \quad 292.2 \quad 236.7 \quad 159.8 \quad 70.2] \text{ kN}$$

$$[432.3 \quad 368.2 \quad 291.3 \quad 193.7 \quad 71.7] \text{ kN}$$

The response of the damping system at maximum displacement is:

$$V_{d.mxd,P}^T = [113.2 \quad 98 \quad 88.3 \quad 92.3 \quad 102.5] \text{ kN}$$

This spring response is considerable when compared with the structural response.

6.5.1.7 Response at maximum drift velocity

The shear response at maximum drift velocity is given by Eqn. (6.2-32). However, the actual damper maximum response is given by Eqn. (6.2-34). The corresponding peak values after modal combination are:

$$V_{d.mvx,P}^T = [175 \quad 203 \quad 182.4 \quad 230.9 \quad 238.4] \text{ kN}$$

$$V_{M,P}^T = [195.3 \quad 216.2 \quad 198.2 \quad 237.4 \quad 259.8] \text{ kN}$$

$$V_{d0,P}^T = [196.8 \quad 216.2 \quad 198.9 \quad 237.4 \quad 260.1] \text{ kN}$$

$$[171.4922 \quad 191.8887 \quad 184.9913 \quad 174.9903 \quad 151.5443] \text{ kN}$$

It is seen that the total damper contribution is larger than the pure viscous component (maximum velocity).

6.5.1.8 Approximate maximum damper response

When compared to the actual TH response, the values obtained by the modal approach for the damper shear are conservative. The approximate procedure presented in Section 6.2.2.5 can be used instead. The values of the Maxwell system are re-evaluated using the peak velocity and peak drift for every storey:

$$q_P^T = [0.248 \quad 0.157 \quad 0.156 \quad 0.161 \quad 0.203]$$

$$\beta_{WP}^T = [0.919 \quad 0.961 \quad 0.962 \quad 0.959 \quad 0.941]$$

$$\beta_{s,P}^T = [0.109 \quad 0.057 \quad 0.056 \quad 0.059 \quad 0.082]$$

$$\beta_{d,P}^T = [0.991 \quad 0.998 \quad 0.999 \quad 0.998 \quad 0.996]$$

$$\beta_{fP}^T = [0.995 \ 1 \ 1 \ 0.999 \ 0.998]$$

And, using Eqns. (6.2-49) and (6.2-50):

$$V_{dP}^T = [165.3 \ 186 \ 180.6 \ 179.3 \ 173.9] \text{ kN}$$

$$[171.4922 \ 191.8887 \ 184.9913 \ 174.9903 \ 151.5443] \text{ kN}$$

$$f_{bP}^T = [183.8 \ 206.8 \ 200.8 \ 199.3 \ 193.3] \text{ kN}$$

$$[190.6808 \ 213.3596 \ 205.6904 \ 194.5704 \ 168.5009] \text{ kN}$$

6.5.1.9 Maximum total shear

The maximum shear response can be found using several procedures. In this Example, the Lin & Chopra method is used for its simplicity. Applying Eqn. (6.2-44) to the values of $V_{d,P}$ and $V_{t,msd,P}$:

$$V_{tP}^T = [471.7 \ 421.7 \ 356.2 \ 298.5 \ 241.3] \text{ kN}$$

$$[556.2 \ 483.0 \ 408.4 \ 311.4 \ 179.5] \text{ kN}$$

The same variable using the force coefficient method renders:

$$V_{tP}^T = [420.5 \ 390.5 \ 326.8 \ 299.8 \ 221] \text{ kN}$$

6.5.2 Example 2: elastic frame, simple method

The same Example is solved by the simple method. Modal shapes are assumed as constant, therefore avoiding solution of the eigenproblem stated by Eqn. (6.2-4). The initial steps are the same as in the previous example; the contribution of the damping system to the global stiffness in the first mode was found to be:

$$k_{sa,mode}^T = [4.396 \ 3.216 \ 3.112 \ 3.057 \ 3.349] \frac{\text{kN}}{\text{mm}}$$

Applying Eqns. (6.2-17) and (6.2-18), $\omega_{e,1} = 3.318$ Hz, and $T_{e,1} = 1.894$ s. Then, $\omega_{e,1}$ is used on Eqn. (6.2-12) to obtain $\xi_{a,1} = 20.46$ %; the inherent damping at $\omega_{e,1}$ is given by Eqn. (2.5-128), $\xi_{l,1} = 1.92$ %. With these values displacement, drift and drift velocity are updated as:

$$u_I^T = [6.7 \ 17 \ 27.9 \ 39 \ 48.5] \text{ mm}$$

$$\Delta_I^T = [6.71 \ 10.27 \ 10.89 \ 11.16 \ 9.48] \text{ mm}$$

$$Y_I^T = [33 \ 50.6 \ 53.6 \ 54.9 \ 46.7] \frac{\text{mm}}{\text{s}}$$

The maximum change in drift is 52.1%, and in frequency -6.1%.

With the new values the Maxwell parameters and stiffness contribution are updated, and a new frequency is found, whereupon viscous damping ratio is refreshed and new response values are obtained. The procedure continues until convergence is achieved, and is repeated for all storeys. The final results are listed in Table 6-4 along with the corresponding results for the consistent method obtained with updated modal shapes.

$$\xi_e^T = [36.4\% \ 45\% \ 20.4\% \ 9.1\% \ 5.9\%]$$

$$\xi_a^T = [34.6\% \ 43.3\% \ 17.3\% \ 4.5\% \ 0]$$

$$\xi_J^T = [1.82\% \ 1.65\% \ 3.14\% \ 4.59\% \ 5.86\%]$$

$$T_e^T = [1.75 \ 0.466 \ 0.196 \ 0.131 \ 0.101] \text{ s}$$

$$\omega_e^T = [3.59 \ 13.47 \ 32.02 \ 48.14 \ 62.11] \frac{1}{\text{s}}$$

Table 6-4. Comparison between elastic results using the consistent and simple method.

Consistent method (modified modal shapes)	Simple method (constant modal shapes)
$T_e^T = [1.761 \ 0.448 \ 0.19 \ 0.129 \ 0.101] \text{ s}$	$T_e^T = [1.75 \ 0.466 \ 0.196 \ 0.131 \ 0.101] \text{ s}$
$\omega_e^T = [3.57 \ 14.04 \ 33.08 \ 48.7 \ 62.35] \frac{1}{\text{s}}$	$\omega_e^T = [3.59 \ 13.47 \ 32.02 \ 48.14 \ 62.11] \frac{1}{\text{s}}$
$\xi_e^T = [34.3\% \ 43.6\% \ 16.9\% \ 8.3\% \ 5.9\%]$	$\xi_e^T = [36.4\% \ 45\% \ 20.4\% \ 9.1\% \ 5.9\%]$
$\xi_a^T = [32.4\% \ 41.9\% \ 13.7\% \ 3.7\% \ 0]$	$\xi_a^T = [34.6\% \ 43.3\% \ 17.3\% \ 4.5\% \ 0]$
$u_P^T = [6.1 \ 14.8 \ 23.3 \ 30.9 \ 36.5] \text{ mm}$	$u_P^T = [5.6 \ 13.7 \ 21.6 \ 29.4 \ 35.9] \text{ mm}$
$\Delta_P^T = [6.11 \ 8.78 \ 8.58 \ 7.94 \ 6.1] \text{ mm}$	$\Delta_P^T = [5.59 \ 8.13 \ 8.16 \ 8.22 \ 7.19] \text{ mm}$
$Y_P^T = [41.4 \ 51.9 \ 48.9 \ 48.2 \ 45.5] \frac{\text{mm}}{\text{s}}$	$Y_P^T = [41.7 \ 51.3 \ 48.7 \ 52.2 \ 53.9] \frac{\text{mm}}{\text{s}}$
$V_{s,P}^T = [340 \ 292.2 \ 236.7 \ 159.8 \ 70.2] \text{ kN}$	$V_{s,P}^T = [367 \ 294.1 \ 233.7 \ 153.5 \ 67.9] \text{ kN}$
$V_{d,P}^T = [165.3 \ 186 \ 180.6 \ 179.3 \ 173.9] \text{ kN}$	$V_{d,P}^T = [165.7 \ 184.9 \ 180.2 \ 186.5 \ 189.4] \text{ kN}$
$V_{t,P}^T = [471.7 \ 421.7 \ 356.2 \ 298.5 \ 241.3] \text{ kN}$	$V_{t,P}^T = [510.2 \ 431.2 \ 357.6 \ 304.1 \ 253.1] \text{ kN}$

6.5.3 Example 3: yielding frame equipped with non-linear viscous dampers

The frame discussed in the previous Section is now subjected to a larger intensity ($PHA = 0.4g$) resulting in inelastic action. The fundamental properties and damping system have already been defined in previous sections. The pushover properties of this frame without

added damping system were presented in Section 2.5.7; the pushover curve for the first mode was displayed in Figure 2-40. In this example, higher modes are considered as elastic and inelastic action is restricted to the first mode.

The analysis is initialized with $\xi_e = \xi_I$, $T_e = T$, resulting in the following large values of displacement, drift and velocity:

$$u = \begin{bmatrix} 55.98 & 24.77 & 8.48 & 2.37 & 0.56 \\ 141.71 & 48.68 & 8.23 & -0.41 & -0.65 \\ 232.54 & 45.37 & -4.63 & -2.01 & 0.5 \\ 325.62 & 5.34 & -9.72 & 2.14 & -0.25 \\ 404.71 & -54.2 & 6.85 & -0.8 & 0.07 \end{bmatrix} \text{ mm}$$

$$\Delta = \begin{bmatrix} 55.98 & 24.77 & 8.48 & 2.37 & 0.56 \\ 85.73 & 23.91 & -0.25 & -2.78 & -1.2 \\ 90.83 & -3.31 & -12.85 & -1.6 & 1.14 \\ 93.08 & -40.03 & -5.09 & 4.15 & -0.75 \\ 79.09 & -59.55 & 16.58 & -2.95 & 0.32 \end{bmatrix} \text{ mm}$$

$$Y = \begin{bmatrix} 208.8 & 229.9 & 151.5 & 65.2 & 19.8 \\ 319.8 & 221.9 & -4.5 & -76.5 & -42.7 \\ 338.8 & -30.7 & -229.8 & -44 & 40.5 \\ 347.2 & -371.6 & -91 & 114.2 & -26.5 \\ 295 & -552.7 & 296.3 & -81.1 & 11.4 \end{bmatrix} \frac{\text{mm}}{\text{s}}$$

If the value of roof displacement ($u_{N,1}$) is larger than the maximum roof displacement value used in the pushover curve, one cycle of the elastic procedure can be run in order to obtain a better approximation to the actual damping. In this case, the value of 404 mm obtained is large ($\delta = 0.023$) but within the limits of the pushover value, so this step is not necessary.

The values of q , β_W and β_s are calculated using Eqns. (6.2-1) and (5.3-16):

$$q = \begin{bmatrix} 0.061 & 0.144 & 0.341 & 0.801 & 1.876 \\ 0.04 & 0.119 & 1.617 & 0.601 & 1.039 \\ 0.039 & 0.32 & 0.225 & 0.793 & 1.066 \\ 0.037 & 0.089 & 0.345 & 0.473 & 1.269 \\ 0.04 & 0.073 & 0.191 & 0.562 & 1.938 \end{bmatrix}$$

$$\beta_W = \begin{bmatrix} 0.991 & 0.966 & 0.865 & 0.543 & 0.103 \\ 0.995 & 0.975 & 0.152 & 0.686 & 0.39 \\ 0.995 & 0.878 & 0.93 & 0.549 & 0.374 \\ 0.996 & 0.984 & 0.863 & 0.777 & 0.271 \\ 0.995 & 0.988 & 0.947 & 0.714 & 0.095 \end{bmatrix}$$

$$\beta_s = \begin{bmatrix} 0.0178 & 0.0503 & 0.1714 & 0.5091 & 0.936 \\ 0.0124 & 0.039 & 0.8969 & 0.3623 & 0.6676 \\ 0.0121 & 0.1568 & 0.095 & 0.5032 & 0.684 \\ 0.0117 & 0.0269 & 0.1737 & 0.2671 & 0.7878 \\ 0.0124 & 0.0214 & 0.075 & 0.3329 & 0.9421 \end{bmatrix}$$

6.5.3.1 Response in the first mode

First iteration

The initial roof displacement is 404.7mm. With the values of β_s obtained, the stiffness contribution due to the added damping system in the first mode is calculated with Eqn. (6.2-2):

$$k_{sa,1}^T = [1.955 \ 1.672 \ 1.637 \ 1.641 \ 1.74] \frac{kN}{mm}$$

The modal properties are obtained solving the eigenproblem stated by Eqn. (6.2-3): $T_{e,1} = 1.948$ s, $\omega_{e,1} = 3.226$ Hz, $\Gamma_{e,1} = 1.358$ s, and:

$$\phi_{e,1}^T = [0.143 \ 0.359 \ 0.585 \ 0.812 \ 1]$$

$$\psi_{e,1}^T = [0.143 \ 0.216 \ 0.226 \ 0.227 \ 0.188]$$

The following stiffness values are obtained using Eqns. (6.3-1), (6.3-2) and (6.3-3): $\kappa_{e,1} = 7.831$ kN/mm, $\kappa_{a,1} = 0.279$ kN/mm, $\kappa_{f,1} = 7.552$ kN/mm. The undamped stiffness is found with Eqn. (6.3-4) as $\kappa_1 = 7.275$ kN/mm.

For the roof displacement $u_{N,1} = 404.7$ mm, the corresponding base shear is $V_{1,1} = 1290.6$ kN. This value is modified using Eqn. (6.3-5) to find $V_{e,1,1} = 1389.3$ kN. The Vu pushover work (area under the Vu pushover curve) is $W = 390,867$ J, and using Eqn. (6.3-9), $W_{e,1} = 420,751$ J. With these values, the effective stiffness from Eqn. (6.3-6) is $\kappa_{eff,1} = 3.433$ kN/mm, whereupon the effective period and frequency from Eqns. (6.3-7) and (6.3-8) are $T_{eff,1} = 2.942$ s, and $\omega_{eff,1} = 2.136$ Hz.

The hysteretic damping ratio is found using Eqn. (6.3-11); q_H is given by Eqn. (2.5-229), $q_H = 0.5$, $\xi_h = 31.62\%$. The effective added viscous damping ratio is calculated with Eqn. (6.3-18), as $\xi_{a,eff,1} = 24.9\%$ (the elastic added viscous damping using Eqn. (6.2-12) is $\xi_{a,1} = 10.9\%$). The inherent damping is adjusted to the effective period as $\xi_{l,eff,1} = 2.7\%$. The total effective viscous damping ratio is calculated with Eqn. (6.3-19) as $\xi_{eff,1} = 43.43\%$ (the total elastic damping is $\xi_{e,1} = 12.88\%$).

Displacement, drift and drift velocity can be updated using the effective period and damping ratio, Eqns. (6.3-20), (6.3-21) and (6.3-22):

$$u_I^T = [22 \ 55.3 \ 90.1 \ 125.1 \ 154] \text{ mm}$$

$$\Delta_I^T = [22 \ 33.3 \ 34.8 \ 35 \ 29] \text{ mm}$$

$$Y_I^T = [105.1 \ 159.4 \ 166.6 \ 167.2 \ 138.5] \frac{mm}{s}$$

Second iteration

The process is repeated using the updated values as initial values. First the Maxwell parameters are found:

$$q_I^T = [0.11 \ 0.072 \ 0.071 \ 0.068 \ 0.075]$$

$$\beta_{W,I}^T = [0.978 \ 0.988 \ 0.989 \ 0.989 \ 0.988]$$

$$\beta_{s,I}^T = [0.035 \ 0.021 \ 0.021 \ 0.02 \ 0.022]$$

The stiffness contribution of the added damping system is:

$$k_{sa,I}^T = [3.85 \ 2.89 \ 2.82 \ 2.81 \ 3.11] \frac{kN}{mm}$$

And the updated modal properties found by solution of Eqn. (6.2-15): $T_{e,1} = 1.905$ s, $\omega_{e,1} = 3.299$ Hz, $\Gamma_{e,1} = 1.354$ s, and:

$$\phi_{e,I}^T = [0.145 \ 0.364 \ 0.592 \ 0.817 \ 1]$$

$$\psi_{e,I}^T = [0.145 \ 0.219 \ 0.228 \ 0.225 \ 0.183]$$

The updated stiffness values are obtained using Eqns. (6.3-1), (6.3-2) and (6.3-3): $\kappa_{e,1} = 8.245$ kN/mm, $\kappa_{a,1} = 0.559$ kN/mm, $\kappa_{f,1} = 7.686$ kN/mm. For the roof displacement $u_{N,1} = 154$ mm, the corresponding base shear and work are $V_{1,j} = 1055.9$ kN, $W = 85,858$ J. These values are modified using Eqns. (6.3-5) and (6.3-9) to find $V_{e,1,1} = 1196.8$ kN, $W_{e,1} = 97,310$ J. With these values, the effective stiffness from Eqn. (6.3-6) is $\kappa_{eff,1} = 7.770$ kN/mm, whereupon the effective period and frequency from Eqns. (6.3-7) and (6.3-8) are $T_{eff,1} = 1.962$ s, and $\omega_{eff,1} = 3.202$ Hz. The hysteretic damping ratio is found using Eqns. (6.3-11) and (2.5-229), $q_H = 0.5$, $\xi_h = 1.78\%$. The effective added viscous damping ratio is calculated with Eqn. (6.3-18), as $\xi_{a,eff,1} = 19.9\%$. The inherent damping is adjusted to the effective period as $\xi_{l,eff,1} = 1.97\%$. The total effective viscous damping ratio is calculated with Eqn. (6.3-19) as $\xi_{eff,1} = 23.62\%$. Displacement, drift and drift velocity are updated using the effective period and damping ratio, Eqns. (6.3-20), (6.3-21) and (6.3-22):

$$u_I^T = [28.3 \ 71.1 \ 115.5 \ 159.5 \ 195.2] \text{ mm}$$

$$\Delta_I^T = [28.3 \ 42.8 \ 44.4 \ 44 \ 35.7] \text{ mm}$$

$$Y_I^T = [138.1 \ 208.9 \ 216.8 \ 214.5 \ 174.2] \frac{mm}{s}$$

The maximum change in drift is -21.1%, and the change in frequency is -2.21%. The process is continued. With two more iterations the change in drift is -1.37% and the change

in frequency is -0.13%. From this point, two additional iterations are run; in the last iteration the change in drift is -0.1% and the change in frequency is -0.008%.

Final iteration

The following kinematic results are obtained:

$$u_I^T = [26.9 \ 67.6 \ 109.9 \ 151.9 \ 186.1] \text{ mm}$$

$$\Delta_I^T = [26.9 \ 40.7 \ 42.3 \ 42 \ 34.2] \text{ mm}$$

$$Y_I^T = [131.1 \ 198.4 \ 206.1 \ 204.4 \ 166.5] \frac{\text{mm}}{\text{s}}$$

The modal properties are: $T_{e,1} = 1.913\text{s}$; $\omega_{e,1} = 3.28\text{s}$; $\Gamma_{e,1} = 1.354$. The modal vectors:

$$\phi_{e,1}^T = [0.145 \ 0.364 \ 0.591 \ 0.816 \ 1]$$

$$\psi_{e,1}^T = [0.145 \ 0.219 \ 0.227 \ 0.225 \ 0.184]$$

The effective period and frequency are $T_{eff,1} = 2.096\text{s}$, and $\omega_{eff,1} = 2.998\text{ Hz}$. The values of damping ratio are: $\xi_{l,eff,1} = 2.06\%$; $\xi_{a,eff,1} = 20.69\%$; $\xi_h = 5.04\%$; $\xi_{eff,1} = 27.79\%$.

The final contribution of the added damping system to the frame stiffness in the first mode is:

$$k_{sa,1}^T = [3.43 \ 2.62 \ 2.57 \ 2.58 \ 2.87] \frac{\text{kN}}{\text{mm}}$$

The base shear in the modified pushover curve is $V_e = 1264.4\text{ kN}$, and the stiffness properties are $\kappa_{e,1} = 8.164\text{ kN/mm}$, $\kappa_{a,1} = 0.496\text{ kN/mm}$, $\kappa_{f,1} = 7.668\text{ kN/mm}$, $\kappa_{eff,1} = 6.802\text{ kN/mm}$.

The ductility demand of the frame can be measured in different ways. As proposed in this study, the period ductility is found with Eqn. (6.3-12) as $\mu_{T,1} = 1.20$. A more common possibility is to refer the ductility to an effective yield displacement obtained by bi-linearization of the pushover curve. Using the energy criterion, Eqns. (6.3-13), (6.3-14), (6.3-15), (6.3-16), $u_{y,N} = 147.1\text{ mm}$, $V_y = 1200.5\text{ kN}$, $\mu = 1.264$ and $\alpha_e = 0.201$.

6.5.3.2 Response in higher modes

The response in higher modes is elastic and can be obtained by the same procedure discussed in the previous example. Only the final results are given here. A comparison between the modal properties obtained for the frame without and with added damping system are given in Table 6-5.

Table 6-5. Properties of frame 05D with and without added damping system Q.1.5, inelastic ($PHA = 0.4g$).

Frame with added damping system					Frame without added damping system				
$T_e^T = [1.913 \ 0.605 \ 0.258 \ 0.139 \ 0.102] \text{ s}$					$T^T = [2.006 \ 0.704 \ 0.361 \ 0.226 \ 0.163] \text{ s}$				
$\omega_e^T = [3.28 \ 10.39 \ 24.4 \ 45.16 \ 61.79] \frac{1}{s}$					$\omega^T = [3.13 \ 8.92 \ 17.42 \ 27.84 \ 38.57] \frac{1}{s}$				
$\Gamma_e^T = [1.354 \ -0.492 \ 0.187 \ -0.077 \ 0.017]$					$\Gamma^T = [1.361 \ -0.506 \ 0.203 \ -0.075 \ 0.016]$				
$\Phi_e = \begin{bmatrix} 0.145 & -0.445 & 1.254 & -2.076 & 4.235 \\ 0.364 & -0.858 & 1.124 & 0.77 & -5.718 \\ 0.591 & -0.801 & -0.862 & 1.532 & 4.987 \\ 0.816 & -0.074 & -1.295 & -2.148 & -3.014 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$					$\Phi = \begin{bmatrix} 0.132 & -0.44 & 1.198 & -2.963 & 8.275 \\ 0.346 & -0.902 & 1.228 & 0.395 & -9.223 \\ 0.573 & -0.849 & -0.651 & 2.532 & 7.031 \\ 0.806 & -0.1 & -1.424 & -2.65 & -3.553 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$				
$\Psi_e = \begin{bmatrix} 0.145 & -0.445 & 1.254 & -2.076 & 4.235 \\ 0.219 & -0.413 & -0.129 & 2.845 & -9.953 \\ 0.227 & 0.057 & -1.986 & 0.762 & 10.705 \\ 0.225 & 0.728 & -0.433 & -3.68 & -8.002 \\ 0.184 & 1.074 & 2.295 & 3.148 & 4.014 \end{bmatrix}$					$\Psi = \begin{bmatrix} 0.132 & -0.44 & 1.198 & -2.963 & 8.275 \\ 0.215 & -0.462 & 0.031 & 3.357 & -17.498 \\ 0.227 & 0.052 & -1.88 & 2.137 & 16.254 \\ 0.233 & 0.749 & -0.773 & -5.182 & -10.584 \\ 0.194 & 1.1 & 2.424 & 3.65 & 4.553 \end{bmatrix}$				

The final displacements, drifts and drift velocities are:

$$u = \begin{bmatrix} 26.91 & 8.32 & 2.12 & 0.54 & 0.13 \\ 67.63 & 16.06 & 1.9 & -0.2 & -0.18 \\ 109.94 & 15 & -1.46 & -0.4 & 0.16 \\ 151.89 & 1.38 & -2.19 & 0.56 & -0.09 \\ 186.06 & -18.71 & 1.69 & -0.26 & 0.03 \end{bmatrix} \text{ mm}$$

$$\Delta = \begin{bmatrix} 26.908 & 8.324 & 2.122 & 0.544 & 0.133 \\ 40.725 & 7.732 & -0.219 & -0.746 & -0.311 \\ 42.305 & -1.058 & -3.362 & -0.2 & 0.335 \\ 41.953 & -13.619 & -0.733 & 0.965 & -0.25 \\ 34.167 & -20.091 & 3.884 & -0.825 & 0.126 \end{bmatrix} \text{ mm}$$

$$Y = \begin{bmatrix} 131.11 & 90.65 & 44.87 & 17.97 & 5.37 \\ 198.44 & 84.2 & -4.62 & -24.63 & -12.63 \\ 206.13 & -11.53 & -71.11 & -6.6 & 13.59 \\ 204.42 & -148.31 & -15.5 & 31.86 & -10.15 \\ 166.48 & -218.8 & 82.16 & -27.25 & 5.09 \end{bmatrix} \frac{\text{mm}}{s}$$

Elastic modal damping ratios:

$$\xi_e^T = [19.2\% \ 34.8\% \ 33.2\% \ 18.4\% \ 9.6\%]$$

$$\xi_a^T = [17.2\% \ 33.4\% \ 30.7\% \ 14.1\% \ 3.8\%]$$

$$\xi_I^T = [1.93\% \ 1.48\% \ 2.49\% \ 4.32\% \ 5.83\%]$$

And, as indicated, $\xi_{l,eff,1} = 2.06\%$; $\xi_{a,eff,1} = 20.69\%$; $\xi_h = 5.04\%$; $\xi_{eff,1} = 27.79\%$.

6.5.3.3 Kinematic solution

The final kinematic solution is obtained applying the CQC rule to the modal solutions:

$$u_P^T = [29.8 \ 72.2 \ 113.2 \ 152 \ 184] \text{ mm}$$

$$[33.5775 \ 75.6242 \ 114.3981 \ 148.6558 \ 173.8777] \text{ mm}$$

$$\Delta_P^T = [29.8 \ 42.63 \ 42.19 \ 42.03 \ 36.36] \text{ mm}$$

$$[33.5775 \ 43.4420 \ 42.8871 \ 39.4988 \ 31.5246] \text{ mm}$$

$$Y_P^T = [189 \ 228.6 \ 216 \ 236 \ 243.5] \frac{\text{mm}}{\text{s}}$$

$$[217.9313 \ 242.2331 \ 225.6572 \ 221.8347 \ 216.5647] \text{ mm/s}$$

And the stroke and velocity in the damping devices:

$$u_{b,P}^T = [26.8 \ 38.34 \ 37.94 \ 37.8 \ 32.71] \text{ mm}$$

$$[30.1075 \ 38.5844 \ 38.0695 \ 35.2079 \ 28.2174] \text{ mm}$$

$$v_{b,P}^T = [169.9 \ 205.6 \ 194.3 \ 212.3 \ 219] \frac{\text{mm}}{\text{s}}$$

6.5.3.4 Static solution

Application of the corresponding equations lead to the following values:

Total shear at maximum displacement:

$$V_{t,msd,P}^T = [1628.6 \ 1347.3 \ 1082.5 \ 863.6 \ 587.2] \text{ kN}$$

Storey shear at maximum displacement:

$$V_{s,P}^T = [1433.5 \ 1194.3 \ 949.1 \ 709 \ 434.3] \text{ kN}$$

$$[1375.7 \ 1226.5 \ 996.5 \ 730.0 \ 408.7] \text{ kN}$$

Peak damper shear obtained through the optional procedure described in Section 6.2.2.5:

$$V_{d,P}^T = [355 \ 390.5 \ 379.6 \ 396.8 \ 403] \text{ kN}$$

$$[369.7115 \ 394.7808 \ 381.5522 \ 379.7971 \ 377.2121] \text{ kN}$$

Peak total shear obtained by the Lin-Chopra method:

$$V_{t,P}^T = [1666.8 \ 1402.8 \ 1147.1 \ 950.4 \ 712.2] \text{ kN}$$

[1663.6 1448.9 1210.7 954.1 613.3] kN

6.5.4 Example 4: inelastic frame, simple method

Hereby the solution for the same frame is found using the simplified inelastic method, in which modal shapes are assumed unchanged. The starting values are obtained for the undamped structure; the contribution of added damping system is found as usual with the Eqn. (6.2-2), and is listed in the previous Example. Then, Eqn. (6.2-17) and (6.2-18) are used to find the updated frequency and period $\omega_{e,1} = 3.227$ Hz, $T_{e,1} = 1.947$ s.

Using the original frame unchanged modal properties Γ_1 , $W_{eff,1}$, the stiffness values of the pushover curve are obtained through Eqns. (6.3-1), (6.3-2) and (6.3-3) as $\kappa_{e,1} = 7.749$ kN/mm, $\kappa_{a,1} = 0.27$ kN/mm, $\kappa_{f,1} = 7.479$ kN/mm. The undamped frame value is $k_1 = 7.275$ kN/mm.

Using these values, the corresponding base shear in the modified pushover curve found using Eqn. (6.3-5) is $V_{e,1,1} = 1374.8$ kN, and the corresponding work $W_{e,1} = 416,376$ J. The effective stiffness from Eqn. (6.3-6) is $\kappa_{eff,1} = 3.397$ kN/mm, whereupon the effective period and frequency from Eqns. (6.3-7) and (6.3-8) are $T_{eff,1} = 2.941$ s, and $\omega_{eff,1} = 2.136$ Hz.

The hysteretic damping ratio is found using Eqns. (6.3-11) and (2.5-229), $q_H = 0.5$, $\xi_h = 15.8\%$. The effective added viscous damping ratio is calculated with Eqn. (6.3-18), as $\xi_{a,eff,1} = 24.9\%$. The inherent damping is adjusted to the effective period as $\xi_{l,eff,1} = 2.7\%$. The total effective viscous damping ratio is calculated with Eqn. (6.3-19) as $\xi_{eff,1} = 43.4\%$. Displacement, drift and drift velocity are updated using the effective period and damping ratio, Eqns. (6.3-20), (6.3-21) and (6.3-22):

$$u_I^T = [21.4 \quad 54.2 \quad 88.9 \quad 124.5 \quad 154.7] \text{ mm}$$

$$\Delta_I^T = [21.4 \quad 32.8 \quad 34.7 \quad 35.6 \quad 30.2] \text{ mm}$$

$$Y_I^T = [102.4 \quad 156.8 \quad 166.1 \quad 170.2 \quad 144.6] \frac{\text{mm}}{\text{s}}$$

The maximum change in drift is 161.6%, and the change in frequency is -3.1%. The process is continued. With three more iterations the change in drift is -1.48% and the change in frequency is -0.15%. From this point, two additional iterations are run; in the last iteration the change in drift is -0.1% and the change in frequency is -0.01%.

Final iteration

The following kinematic results are obtained:

$$u_I^T = [25.7 \quad 65.1 \quad 106.8 \quad 149.6 \quad 185.9] \text{ mm}$$

$$\Delta_I^T = [25.7 \ 39.4 \ 41.7 \ 42.8 \ 36.3] \text{ mm}$$

$$Y_I^T = [125.8 \ 192.7 \ 204.2 \ 209.2 \ 177.8] \frac{\text{mm}}{\text{s}}$$

The modal properties are: $T_{e,1} = 1.912\text{s}$; $\omega_{e,1} = 3.29\text{s}$. The effective period and frequency are $T_{eff,1} = 2.094\text{s}$, and $\omega_{eff,1} = 3.001\text{ Hz}$. The values of damping ratio are: $\xi_{l,eff,1} = 2.06\%$; $\xi_{a,eff,1} = 21.2\%$; $\xi_h = 5.03\%$; $\xi_{eff,1} = 28.3\%$. The final contribution of the added damping system to the frame stiffness in the first mode is:

$$k_{sa,1}^T = [3.54 \ 2.68 \ 2.6 \ 2.56 \ 2.79] \frac{\text{kN}}{\text{mm}}$$

The base shear in the modified pushover curve is $V_e = 1244.6\text{ kN}$, and the stiffness properties are $\kappa_{e,1} = 8.039\text{ kN/mm}$, $\kappa_{a,1} = 0.489\text{ kN/mm}$, $\kappa_{f,1} = 7.55\text{ kN/mm}$, $\kappa_{eff,1} = 6.702\text{ kN/mm}$.

The ductility demand of the frame can be measured in different ways. As proposed in this study, the period ductility is found with Eqn. (6.3-12) as $\mu_{T,1} = 1.199$. Ductility referred to an effective yield displacement obtained by bi-linearization of the pushover curve is obtained using the energy criterion, Eqns. (6.3-13), (6.3-14), (6.3-15), (6.3-16), $u_{y,N} = 147.1\text{ mm}$, $V_y = 1182.6\text{ kN}$, $\mu = 1.262$ and $\alpha_e = 0.200$.

Response for higher modes is assumed as elastic, and is found by the procedure indicated in Example 6.5.2, assuming unchanged modal shapes. The final values are compared to those obtained with the ‘consistent’ method in Table 6-6:

Table 6-6. Comparison between inelastic results using the consistent and simple method.

Consistent method (modified modal shapes)					Simple method (constant modal shapes)				
$T_e^T = [1.913 \ 0.605 \ 0.258 \ 0.139 \ 0.102] \text{ s}$					$T_e^T = [1.912 \ 0.609 \ 0.261 \ 0.143 \ 0.103] \text{ s}$				
$T_{eff,1} = 2.096 \text{ s}$					$T_{eff,1} = 2.094 \text{ s}$				
$\omega_e^T = [3.28 \ 10.39 \ 24.4 \ 45.16 \ 61.79] \frac{1}{\text{s}}$					$\omega_e^T = [3.29 \ 10.32 \ 24.06 \ 43.85 \ 61.15] \frac{1}{\text{s}}$				
$\xi_e^T = [19.2\% \ 34.8\% \ 33.2\% \ 18.4\% \ 9.6\%]$					$\xi_e^T = [19.6\% \ 34\% \ 33.7\% \ 20\% \ 10.7\%]$				
$\xi_{eff,1} = 27.8\%$					$\xi_{eff,1} = 28.3\%$				
$\xi_a^T = [17.2\% \ 33.4\% \ 30.7\% \ 14.1\% \ 3.8\%]$					$\xi_a^T = [17.7\% \ 32.5\% \ 31.2\% \ 15.8\% \ 5\%]$				
$u_P^T = [29.8 \ 72.2 \ 113.2 \ 152 \ 184] \text{ mm}$					$u_P^T = [29 \ 70.4 \ 110.5 \ 149.8 \ 183.8] \text{ mm}$				
$\Delta_P^T = [29.8 \ 42.63 \ 42.19 \ 42.03 \ 36.36] \text{ mm}$					$\Delta_P^T = [29.04 \ 41.64 \ 41.6 \ 43.01 \ 38.74] \text{ mm}$				

$$Y_P^T = [189 \ 228.6 \ 216 \ 236 \ 243.5] \frac{mm}{s} \quad Y_P^T = [193.4 \ 229.7 \ 216.2 \ 247.8 \ 260] \frac{mm}{s}$$

$$V_{s,P}^T = [1433.5 \ 1194.3 \ 949.1 \ 709 \ 434.3] \text{ kN} \quad V_{s,P}^T = [1501.9 \ 1213.7 \ 945 \ 710.2 \ 442.3] \text{ kN}$$

$$V_{d,P}^T = [355 \ 390.5 \ 379.6 \ 396.8 \ 403] \text{ kN} \quad V_{d,P}^T = [359.2 \ 391.5 \ 379.8 \ 406.6 \ 416.4] \text{ kN}$$

$$V_{t,P}^T = [1666.8 \ 1402.8 \ 1147.1 \ 950.4 \ 712.2] \text{ kN} \quad V_{t,P}^T = [1752.3 \ 1429.6 \ 1146.9 \ 965.4 \ 728.5] \text{ kN}$$

6.5.5 E-Defense 5-steel building with added dampers

In this Section, the proposed elastic method is applied to the 5 storey steel structure with added viscous dampers, tested in the E-Defense shaking table and reported in Section 2.4.5.2. Only the X direction is considered. Plan and elevation of the structure is shown in Figure 2-8. The basic joint is of the type shown in Figure 2-7. Description of materials, dimensions and sections are taken from Kasai *et al.* [2010], and additional information about viscous dampers is taken from Akcelyan *et al.* [2016]. These references do not include detailed information about mode shapes; these are obtained from a SAP2000 model, including the variable section of the elements and the seismic weight. The model neglects two important sources of stiffness: i) influence of the spurious composite action with the concrete slab; ii) contribution of constructive elements (according to the authors, approximately 30% of the structural stiffness). For this reason, the periods obtained are longer than expected. The authors report $T_{1,x} = 0.650s$ at low amplitude; in this model, a value of 0.7276s is obtained, implying that about 21% of stiffness is missing. The values of period at all modes are corrected by the proportion (0.65/0.7276) to match the ones reported. The modal shapes are taken directly from the model, Table 6-7.

Figure 6-3. E-Defense 5-storey steel building. View of SAP2000 model.

Table 6-7. E-Defense 5-storey steel building. Modal info obtained from SAP2000 in X direction.

n	$z(m)$	$h(m)$	$W(kN)$	$T(s)$	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5
1	3.485	3.485	841.3	0.7276	0.2095	-0.8326	2.0656	-3.7654	8.7018
2	6.485	3.000	822.0	0.2306	0.4451	-1.3500	1.3346	1.7055	-12.2610
3	9.485	3.000	816.7	0.1167	0.6606	-1.1210	-1.5016	2.8310	10.4757
4	12.485	3.000	798.8	0.0734	0.8564	-0.1447	-2.0735	-4.1936	-5.9581
5	15.485	3.000	1454.8	0.0524	1	1	1	1	1

Dampers are installed as shown in Figure 2-8, in storeys 1 to 4. The damper exponent is $\alpha = 0.38$ in all cases. In the X direction, the damper coefficients are $\mathbf{c}_b = [98, 98, 49, 49, 0]$ $\text{kN} \cdot (\text{s}/\text{mm})^{0.38}$, and the added damping system axial stiffness including all components is $\mathbf{k}_b = [104, 104, 60, 60, 0]$ kN/mm . The damper bay is 5m long in all cases, and with the height reported in Table 6-7, it leads to $\mathbf{c}_d = [32.6, 34.6, 17.3, 17.3, 0]$ $\text{kN} \cdot (\text{s}/\text{mm})^{0.38}$, and $\mathbf{k}_d = [68, 76.5, 44.1, 44.1, 0]$ kN/mm .

The original structure was subjected to a recorded accelerogram. Exact spectral data are not available in the reports. In this preliminary analysis, the Eurocode 8 spectrum is used, scaled to match roughly the spectral data available. At very moderate $PHA = 0.05g$, the added damping obtained is 4.5%, and the period is 0.482s. The values reported by the authors are 4.3% and 0.536s.

For $PHA = 0.5g$ (estimated for equivalent PS_a from spectral data) using the Eurocode spectrum, the method results in added damping of 14.8% and period of 0.58s; the (approximate) values reported are 12% and 0.53s. There is approximate coincidence with drift ratios, damper forces and storey shear, however these data are given only graphically in the cited reports so no quantitative evaluation can be performed. However, the method captures the modification trends with intensity in period and damping ratio due to the damping system. The influence of constructive elements on stiffness seems to play an important role, which is difficult to quantify, but seems to be in the order of 20% or higher, even for the type of steel construction used. Inherent damping ratios are on the order of 1%, much smaller than usually considered for steel buildings (3%). The important conclusion is that additional variability in fundamental periods and damping ratios should be considered in design.

6.6 Influence of bracing stiffness on modal properties

The model proposed is dependent on two fundamental non-dimensional variables: β_w and β_s . The first one represents the energy efficiency of the damping system. The second one is indicative of the stiffness contribution of the added damping system. The behavior of the proposed model can be explained considering the physical meaning of both variables, and their dependency from the damping system properties. The energy dissipation of a certain storey n vibrating harmonically at mode j was found, to be a fraction β_w of the pure

viscous damper energy dissipation, see Eqn. (6.2-5). β_s represents the contribution of the stiffness of the added damping system to the overall stiffness, as expressed by Eqn. (6.2-2). Both variables are dependent on a single parameter q , given by Eqn. (6.2-1). The behavior of q is discussed hereby according to damper exponent.

Linear dampers

For linear dampers, q can be expressed as:

$$q_{n,j} = \frac{c_{d,n}}{k_{d,n}} \omega_{eff,j} c f v_j. \quad (6.6-1)$$

For small increases of $\omega_{eff,j}$, $c f v_j$ is almost constant. This proves that q is neither dependent on modal drift Ψ_e , nor on drift amplitude $\Delta_{n,j}$. Consequently, β_w and β_s are not dependent on Ψ_e or $\Delta_{n,j}$, but they are dependent on frequency $\omega_{eff,j}$. This is illustrated in Figure 6-4, displaying the predictions for frame 05D with added damping system Q.1.1 ($c_{b,n} = 2.33$ kN·s/mm), with different levels of brace stiffness (high, $k_{b,1} = 10000$ kN/mm; medium $k_{b,1} = 167.13$ kN/mm; low $k_{b,1} = 54.63$ kN/mm). If the frame remains elastic, the effective damping ratio $\xi_{eff,j}$, Figure 6-4(a)(c)(e), and effective period $T_{eff,j}$ Figure 6-4(b)(d)(f), are constant, regardless of the excitation intensity. After yielding, $T_{eff,1}$ and $\xi_{eff,1}$ increase slowly with intensity, but higher modes maintain the same period and effective damping ratio for all intensities. Damping ratio is moderately affected by the damper stiffness; this is clear comparing Figure 6-4(a)(c)(e). Modal shapes are constant for all intensities. For linear cases the predictions using RSAB and RSABS are similar.

Non-linear dampers

For non-linear dampers, Eqn. (6.2-1) can be expressed as:

$$q_{n,j} = \frac{c_{d,n}}{k_{d,n}} \omega_{eff,j}^{a_n} |\Delta_{n,j}|^{a_n-1} c f v_j^{a_n}, \quad (6.6-2)$$

which is strongly dependant on amplitude (and therefore on intensity) and frequency. Figure 6-5 displays the variation of ξ_{eff} with intensity for frame 05D + Q.1.5 with different values of stiffness (high, $k_{b,1} = 10000$ kN/mm; medium $k_{b,1} = 135.9$ kN/mm; low $k_{b,1} = 54.63$ kN/mm) and both methods (RSAB, RSABS). Results are very similar for both methods, and show the importance of the added damping system stiffness in the behavior of systems equipped with non-linear dampers. For instance, considering $PHA = 0.3g$, for high brace stiffness, $\xi_{eff,1} = 10\%$ and $\xi_{eff,2} = 30.8\%$, Figure 6-5(a); for low brace stiffness, $\xi_{eff,1} = 20.7\%$ and $\xi_{eff,2} = 25.7\%$, Figure 6-5(e). Predictions using the simple method RSABS are very similar to those obtained using the consistent method RSAB.

The influence of brace stiffness in the first mode effective period is also remarkable, as shown by comparison of Figure 6-5(g), (j) and (l); the difference is small between

moderate and low brace stiffness. However, the periods are fairly constant for all intensities. Again, the prediction with both methods is very similar.

Finally, the modal shapes predicted by the method, are virtually identical for all intensities. This is proved in Figure 6-6, that shows the changes with intensity in the first three modal shapes, for high and low bracing stiffness, estimated with the RSAB method.

Figure 6-4: Prediction of effective damping, effective period and hysteretic damping using RSAB for linear added damping system with different values of brace stiffness.

Figure 6-5: Prediction of effective damping and effective period using RSAB and RSABS for non-linear added damping systems with high, medium and low (realistic) stiffness.

Figure 6-6: Prediction of modal shapes using RSAB for high and low damping system stiffness.

Added modal damping ratios $\xi_{a,n}$ and periods $T_{e,n}$ calculated for frame 05D+Q.1.5 assuming elastic behavior with different brace stiffness values are shown in Table 6-8. These values prove that, for the first mode, the most important effect is the reduction of period due to the increase stiffness, whereas for higher modes the modification of damping ratios is more substantial. Again, the predictions with RSAB and RSABS are very similar. As a consequence of these observations, it is clear that the predictions using the simple method, RSABS, should be accurate enough for design purposes.

Table 6-8. Elastic added damping ratio ξ_a and period T_e for frame 05D+Q.1.5 subjected to $PHA = 0.35g$.

Model	k_b	ξ_a for mode					$T_e(s)$ for mode				
		1	2	3	4	5	1	2	3	4	5
RSA	∞ (*)	0.15	0.20	0.22	0.20	0.17	2.01	0.69	0.34	0.20	0.14
	1000	0.16	0.38	0.52	0.54	0.22	1.82	0.60	0.29	0.15	0.06
	500	0.17	0.39	0.48	0.41	0.19	1.88	0.61	0.28	0.14	0.08
RSAB	100	0.17	0.31	0.20	0.06	0.01	1.90	0.58	0.25	0.15	0.11
	50	0.16	0.22	0.08	0.02	0.00	1.87	0.56	0.26	0.17	0.12
	10	0.06	0.02	0.00	0.00	0.00	1.83	0.61	0.32	0.20	0.14
RSABS	1000	0.17	0.37	0.50	0.54	0.45	1.82	0.61	0.29	0.16	0.09
	500	0.18	0.38	0.47	0.42	0.24	1.88	0.62	0.28	0.14	0.09
	100	0.18	0.30	0.21	0.07	0.02	1.90	0.58	0.25	0.15	0.11
RSABS	50	0.16	0.22	0.09	0.02	0.00	1.87	0.57	0.27	0.17	0.12
	10	0.06	0.03	0.00	0.00	0.00	1.83	0.61	0.32	0.20	0.14

(*) implicit in method formulation

6.7 Summary and conclusions

In this Chapter, the Equations derived in Chapter 5 to approximate the response of a SDOF Maxwell system under harmonic motion, have been used to find the brace stiffness contribution of frames equipped with added viscous damping systems. Using this added stiffness, existing RSA and NLRSA procedures for frames equipped with viscoelastic systems have been adapted to estimate the response of frames equipped with purely viscous systems mounted on auxiliary flexible elements.

Frames equipped with purely viscous damping systems are traditionally analyzed neglecting the auxiliary element contribution, which can be a reasonable assumption in the first mode if the brace elements are designed to remain elastic under high intensity levels. However, for higher modes, the stiffness contribution is increasingly relevant, leading to reduced viscous damping ratios, and increased frame stiffness.

The stiffness contribution and modal response are mutually dependent, thus iterative procedures are needed in both elastic and inelastic cases to find the response. For elastic cases, iterative analysis leads to a solution with consistent frequencies, modal properties and stiffness contribution. This procedure is well-established for viscoelastic dampers and has been reformulated for non-linear viscous dampers.

For inelastic cases, conventional procedures for viscoelastic dampers operate in two steps: first the elastic response is found, and then it is modified to account for ductility. This is possible because viscoelastic dampers are linear and therefore their response shows no dependency on amplitude, only on frequency. For non-linear viscous dampers, an iterative method is proposed here, which is a natural extension of the elastic method, but uses the pushover curve of the undamped frame, modified according to the added stiffness, to calculate the inelastic modal response. The method is presented in a general way and allows inelastic action in several modes simultaneously, if the corresponding pushover curves are available.

The concept of ductility based on displacement and effective yield point is shown to be unclear, especially for values of displacement in the elastic range, where it becomes meaningless. The concept can be effectively substituted by the period-based ductility, a much more effective way of measuring the global inelastic level of one frame, especially for low values of inelastic action. Work calculated from the pushover curve is used to assess hysteretic damping for different levels of ductility.

Use of the approximate Maxwell equations remove the need to consider the dampers as linear in higher modes. In the modified procedure, because the contribution in damping and stiffness corresponding to each storey are individually considered, it is possible to estimate the response of systems with unequal height-wise damper exponents.

Modal methods have been shown to offer better predictions for kinematic variables than for static variables. Hereby, an alternative procedure to find the damper response, based on the overall peak kinematic response, is proposed. At every storey, a harmonic SDOF

system is defined by its peak drift velocity and peak drift, and the approximate Maxwell equations can be used to estimate the damper response.

6.8 Original contributions in this Chapter

- Existing RSA and NLRSA methods used for structures with viscoelastic dampers are used to approximate iteratively the earthquake response of elastic or inelastic frames equipped with viscous damping systems, by using the Maxwell approximations derived in previous Chapters.
- For the NLRSA, an alternative procedure based on effective modal properties at performance point is proposed. This procedure treats the damping properties at each storey individually, so it can be used in frames with non-homogeneous height-wise damper exponents.
- Application of the simplified Maxwell model is shown to render unnecessary the usual hypothesis of linear damper exponent at higher modes, due to realistic consideration of system stiffness in higher modes.
- A simplified procedure to obtain the static damper response from the kinematic response (found by the proposed method) is given.

Chapter 7

Numerical validation

7.1 Introduction

In this Chapter, a numerical study is carried out in order to verify the method proposed in Chapter 6 (hereby referred to as RSAB). The method allegedly improves the response prediction obtained with the RSA method, taking into account in a simplified manner the flexibility of brace elements through a numerical Maxwell model dependent on damping properties, drift and drift velocity. Different options and coefficients are examined individually, to find out if they improve the accuracy of predictions in both the RSA and RSAB methods. The simplified alternative with unchanged modal shapes (referred to as RSABS) is also examined.

To this end, a numerical study is carried out. Several steel frames are equipped with added viscous damping systems and analyzed for different sets of seismic excitations. The same cases are analyzed with the simplified methods proposed, and the differences in prediction are obtained. Valuable information about the accuracy of the current NLRSA method implemented in ASCE/SEI 7-16 is also derived.

7.2 Description of the study

7.2.1 Study target

The target of the study is to find out if a certain modification of the RSA method gives a prediction of variables which is closer to the time history results (TH) than the unmodified RSA method. Incidentally, the study is used to assess the accuracy of the RSA method as currently formulated.

7.2.2 Systems considered

In this study, a system is formed by:

1. A Moment Resisting Frame (MRF).
2. An Added Damping System (ADS).

The MRFs are flexible moment resisting frames 3, 5, or 9 storeys high, that provide lateral strength and stiffness to a building. A full description of the frames and their dynamic properties is given in Annex B.

The ADSs are formed by viscous dampers, mounted on brace elements. Different configurations of dampers have been explored: linear and non linear, with different decreasing vertical distributions of damper coefficient, and providing different amounts of added viscous damping, generally higher than 10%; damping ratios for non-linear viscous dampers are dependent on the excitation intensities. Details of the ADSs considered, including definition of brace elements, are given in Annex D.

All models considered in the numerical comparisons of this Chapter are Type 4 models as defined in Annex C, except otherwise indicated. These 2D models include distributed plasticity, axial force – bending moment interaction, P-Delta effects, and panel zone flexibility, but they do not include strength and stiffness deterioration. As moderate ductility is expected in the frames with added dampers, and frame collapse is not discussed, this simplification is reasonable, as proved by Akcelyan *et al.* [2016]. This study also showed that, for regular steel buildings, no particular advantage is achieved using 3D models, as long as the model element used for the damping system is capable of reproducing the Maxwell model non-linear features dependent on the interaction between damping and stiffness, as is the case in this study.

A unique label is used to identify every system. It is composed of two parts; the first one refers to the MRF, including the inherent damping considered; the second one to the ADS; as an example, case ‘05D2+Q.1.1’ is the label for frame 05D, with 2% inherent damping and added damping system Q.1.1.

7.2.3 Cases considered

In this study, a ‘case’ is formed by:

1. A system (formed by a MRF and an ADS).
2. A ground excitation, formed by a Ground Motion Set (GMS).
3. An intensity level or series of intensity levels.

The systems have been subjected to a variety of GMSs, including a single accelerogram (set EUK) and sets of accelerograms (sets ES, comprised of four independent sets: ESA, ESB, ESC and ESD) representing different site-class conditions. Motion sets are described in Annex A. MRFs are described in Annex B. ADSs are listed in Annex D.

7.2.4 Design scenario

The design scenario consists of MRFs acting as seismic force-resisting systems providing lateral strength and stiffness to a building; added viscous damping systems are included for enhanced earthquake performance and structural protection at design level. Moderate

levels of ductility ($\mu_T \leq 1.5$) are expected at this intensity level. In many cases the frames will simply remain elastic for the design level. High levels of ductility are unusual in frames equipped with viscous added dampers at this intensity.

7.2.4.1 Relationship between wind loads and lateral stiffness

The frames chosen for this study are MRFs that provide lateral strength and stiffness to the building, not only during earthquake action, but also for persistent situations such as those derived from wind and accidental imperfections. The need to resist wind actions with a certain roof drift limit ($H/500$, where H is the total frame height) implies a certain lateral stiffness and, given the tributary mass, a certain value of period. The required minimum level of lateral stiffness can be reached with different relative values of beams and columns. In this case, beams and columns are designed according to the weak-beam strong-column, so that a full collapse mechanism is formed at the first mode lateral pushover. The frame is flexible enough to take advantage of the viscous damping system, whose contribution is dependent on drift velocity and therefore on drift and period.

Figure 7-1: Relationship between wind load, earthquake load and lateral stiffness.

Strength and stiffness derived from wind loads present a clear relationship with earthquake behavior. The MRF in Figure 7-3(a), with total height H , is subjected to wind loads following a certain height-wise profile whose resultant V_w is located at a certain height H_w , Figure 7-3(b). In this study the wind load has been taken as constant, which is reasonable for moderate height; the total wind load (pressure plus suction) is taken as $q_w = 1.4 \text{ kN/m}^2$, b is the contributory width of the frame (in this study $b = 18.75 \text{ m}$, see Annex B), $V_w = H \cdot 26.25 \text{ kN/m}$, and thus $H_w = H/2$. The roof deflection limit for wind load is $u_w = H/500$, thus the frame lateral stiffness can be calculated as $\kappa_w = V_w / u_w = 13125 \text{ kN/m} = 13.13 \text{ kN/mm}$. This stiffness is the same for all frames regardless of their height, as long as they strictly fulfill the drift limit for wind. The actual stiffness fluctuates around this number. Smaller drift under wind loads would result in higher lateral stiffness.

The relationship between modal stiffness and lateral stiffness can be found by a simple beam analogy, Figure 7-3(b)(c). From basic structural considerations, the lateral deflection of the beam in the wind-loaded structure is given by:

$$u_w = V_w \frac{H_w^2}{8(EI)_{eq}} \left(H - \frac{H_w}{3} \right). \quad (7.2-1)$$

where $(EI)_{eq}$ is the stiffness of an equivalent constant flexural beam element with the same roof deflection. Likewise, the lateral deflection in the earthquake-loaded structure is:

$$u_s = V_s \frac{H_s^2}{8(EI)_{eq}} \left(H - \frac{H_s}{3} \right). \quad (7.2-2)$$

The stiffness in both cases is found dividing total shear over displacement:

$$\kappa_w = \frac{8(EI)_{eq}}{H_w^2(H - H_w/3)}, \quad (7.2-3)$$

$$\kappa_s = \frac{8(EI)_{eq}}{H_s^2(H - H_s/3)}, \quad (7.2-4)$$

And the proportion between both stiffnesses is:

$$\frac{\kappa_s}{\kappa_w} = \frac{H_w^2(H - H_w/3)}{H_s^2(H - H_s/3)}. \quad (7.2-5)$$

For the constant wind distribution assumed in this study, the previous expressions simplify to:

$$u_w = V_w \frac{5H^3}{48(EI)_{eq}}, \quad (7.2-6)$$

$$\kappa_w = \frac{48(EI)_{eq}}{5H^3}, \quad (7.2-7)$$

$$\frac{\kappa_w}{\kappa_s} = \frac{24}{5} \left(\frac{H_s^2}{H^2} - \frac{H_s^3}{3H^3} \right). \quad (7.2-8)$$

The relative value of H_s/H can be found from dynamic considerations, assuming first-mode vibration:

$$H_s = \frac{\mathbf{z}^T \mathbf{m} \boldsymbol{\phi}_1}{\mathbf{1}^T \mathbf{m} \boldsymbol{\phi}_1}, \quad (7.2-9)$$

where all vectors and variables were discussed in Chapter 2. The values obtained for the study frames using the previous considerations are listed in Table 7-1, where $\kappa_{S,approx}$ is found using Eqn. (7.2-8) and $\kappa_{S,actual}$ is derived from modal properties as $\kappa_1 = \omega^2 \cdot L_1$. The table shows a good coincidence of both values. Moreover, the establiity of the parameter

H_S/H (0.78 – 0.73) is indicative that the first modal stiffness κ_1 is determined by lateral wind stiffness, as long as the design is governed by wind deformation.

The table also lists the value of $PHA_{W=S}$, for which seismic base shear V_S equals the design wind base shear $\gamma_W V_W$, where γ_W is the partial factor for wind action, $\gamma_W = 1.5$ according to EN-1990-1-1 [CEN, 2002]. The seismic base shear is obtained using the lateral force method of Eurocode 8 [CEN, 2004], Eqn. (2.2-7), with a behavior factor $q = 6.5$.

The value of PHA for which $\mu_{av} < 1$, obtained from Table B-11 (Annex B) is also listed at the table bottom. It can be seen that $PHA_{W=S}$ is generally below the elastic range, except at frame 05C. For values of $PHA \leq PHA_{W=S}$, the wind load dominates the design from the point of view of strength,. For values above, the earthquake load dominates the design.

Table 7-1. Relationship between lateral wind stiffness and modal stiffness.

Frame		03D	05D	09D	05C
H	m	10.5	17.5	31.5	17.5
H_W/H		0.500	0.500	0.500	0.500
H_S/H		0.782	0.746	0.733	0.778
$\kappa_{W,approx}$	kN/mm	13.13	13.13	13.13	13.13
$\kappa_{S,approx}$	kN/mm	6.05	6.54	6.73	6.10
$\kappa_{S,actual}$	kN/mm	7.82	7.27	6.58	6.24
$PHA_{W=S}$ (site B, spectrum 1)	g	0.14	0.13	0.10	0.14
$PHA \mu_{av} < 1$	g	0.15	0.20	0.25	0.10

7.2.4.2 Earthquake resistance of bare frame

The bare frame is capable of withstanding a certain value of earthquake intensity, which is limited by strength or drift. This verification is presented in Annex B, Table B-12 (frame 03D), Table B-13 (frame 05D) and Table B-14 (frame 09D) for site class B conditions. In all cases the limiting value of PHA is 0.35g, and the critical condition is drift limit. The design level intensity PHA_D for the frames with added damping is determined so that the maximum drift ratio of the frame without added damping exceeds this limit value of 2%. For the three frames in site class B, this level is taken as 0.40g, corresponding to $S_d(T_1, 5\%) = 0.274g$ (frame 03D), 0.22g (frame 05D), 0.129g (frame 09D); see also Table B-8 in Annex B, presenting details for other site classes. The added damping system reduces the drift demand to acceptable values, minimizing damage in the steel frame, which is a usual design criterion.

Once PHA_D is determined, the frame with damping system is subjected to the corresponding set of accelerograms (ESA, ESB, ESC, ESD) scaled to this intensity, with a very high value of brace stiffness, and models type 1, which are simpler and faster to analyze, but do not include P-Delta effects and do not consider P-M interaction in columns. In this analysis, P-Delta effects are considered in a simplified way using the drift

sensitivity, Section 2.5.4.7. Axial forces in the damping system are determined and the brace extender is designed accordingly.

Finally, the frame with the added damping system and brace elements properly designed is modeled using a more advanced analysis model (model type 4, described in Annex C) and is subjected to:

1. A set of accelerograms (ESA, ESB, ESC or ESD) with two levels of intensity: PHA_D (the design level) and PHA_E (a level for which the frame remains totally elastic under all motions in the set).
2. A single accelerogram (set EUK) which is scaled progressively from 0.05g in 0.05g intervals, up to an intensity where numerical non-convergence of the analysis is no longer possible.

7.2.4.3 Bracing stiffness

Three different models are built, depending on the bracing stiffness:

- HB: in this type of model the Maxwell stiffness is calculated using only the brace stiffness k_{bb} , see Section 5.1.2. The centerline length is used on the calculation of the brace stiffness.
- LB: in this type of model the Maxwell stiffness is calculated considering the brace stiffness k_{bb} as in type HB, but including the damper stiffness in a simplified way. The approximation used for the study is to multiply the brace stiffness by a reduction factor of 50%. This value was shown to be realistic in Section 5.1.2., where reduction factors for some examples were shown to oscillate between 30% and 70%.
- RB: in this type of model the Maxwell stiffness is assumed as very large. A value of 1000 kN/mm has been used. Larger values have been found to produce numerical problems in the analytical models.

7.2.4.4 Damping level

Three different damping levels are provided for each MRF+ADS; the damping coefficients in level 2 and 3 are two and three times those of level 1. The damping ratio for linear dampers in levels 2 and 3 is two and three times that of level 1. For non-linear dampers this proportion is approximately kept.

7.2.4.5 Label of a set of cases

According to all options discussed, the label used to identify a set of cases is of the form:

EUK.05D2.QRSU.LOW.LINEAR.HB

This label indicates GMS EUK, frame type 05D with 2% damping, ADS type Q, R, S, U, low damping, linear dampers, and HB type diagonal stiffness. If no inherent damping indication is given, the set contains cases with 2% and 5% inherent damping. Additionally, 'Y3' after inherent damping descriptor identifies frames with a 3% post-yield stiffness. For example, 'EUK.05D2Y3.QRSU.LOW.LINEAR.HB'.

7.2.5 Calculation of TH response

Calculation of TH response is performed as described in Annex C. For the case of ground motion EUK (comprised of a single motion) the value obtained is unique. For ground motion sets ESA, ESB, ESC and ESD, considering a single system subjected to a certain intensity, every accelerogram i of the set produces an individual response x_i . The response of the system for the corresponding intensity and site class can be approximated using different statistically defined central values. Clause 18.3.3 of ASCE/SEI 7-16 [ASCE, 2016] accepts the mean response $\bar{x}$ across the suite of accelerograms. However, most existing studies use the median based on the assumption of lognormal distribution of sample data:

$$\hat{x} = \exp\left(\frac{1}{N} \sum_{i=1}^{NR} \ln x_i\right), \quad (7.2-10)$$

where NR is the number of accelerograms in the suite. The dispersion of this distribution is given by the standard deviation of the natural logarithm of the data. The 16% and 84% fractiles are given by:

$$x^{16} = \hat{x} / \exp(\sigma(\ln x)), \quad (7.2-11)$$

$$x^{84} = \hat{x} \exp(\sigma(\ln x)). \quad (7.2-12)$$

$\hat{x}$ is hereby referred to as ‘logMedian’ to distinguish it from the sample median. The difference between mean and logMedian is examined for ground motions sets ESA, ESB, ESC, ESD, all frames type D, all added damping systems, and both intensities PHA_D and PHA_E in Figure 7-2. A regression line has also been plotted. It is seen that the mean value is systematically larger than the logMedian value for all variables and storeys. Using the mean implies over-estimating the response, which is acceptable from the point of view of design, but introduces bias when TH is used as a reference value. logMedian is used as the main central value in this study, but some results are also presented with the mean value.

Figure 7-2: comparison of mean and logMedian.

7.2.6 Calculation of approximate response

Approximate response is calculated with the procedures described in previous Chapters: RSA is applied as described in Section 2.5.6. RSAB and RSABS, as described in Section 6.2 (elastic cases) or Section 6.3 (inelastic cases), but with the hysteretic damping ratio calculated according to the conservative expressions proposed in Chapter 2, Eqns. (2.5-70), (2.5-72) and (2.5-74) and based on FEMA-440, with $\xi_4 = 0.11$. It has been proved [Akcelyan et al., 2008] that hysteretic damping derived from energy-based linearization is over-estimated. A robust expression was derived for consistency in Section 6.3.1.1 based on the pushover curve. However, for practical application a conservative expression is preferred. Because the pushover curve of the steel frames in the study is basically bi-linear, the EP option is adopted.

All simplified methods (RSA, RSAB, RSABS) rely on spectral response for their predictions. Spectral values obtained from statistically obtained spectra (soft spectra) are not exact, and therefore introduce a source of variability which is difficult to control. To avoid it, the comparison between methods is performed using the ‘exact’ elastic spectral response, found using the same central value used for the TH (logMedian or mean) of

‘exact’ elastic spectral responses calculated for all the accelerograms in the set. This is consistent with the accelerogram scaling as defined in Eurocode 8 [CEN 2004] and performed in this study (Annex A). The spectral response for accelerogram r in the set, $D_{e,r,j}$ is obtained as the peak value of the solution to the following differential equation:

$$\ddot{D}_{e,r,j}(t) + 2\xi_{eff,j} \omega_{eff,j} \dot{D}_{e,r,j}(t) + \omega_{eff,j}^2 D_{e,r,j}(t) = a_{g,r}(t), \quad (7.2-13)$$

where $a_{g,r}(t)$ is the ground acceleration corresponding to accelerogram r in the set (scaled to the corresponding intensity level); $\omega_{eff,j}$ and $\xi_{eff,j}$ are the effective frequency and viscous damping ratio of the system with added damping system at mode j ; for elastic behavior, $\omega_{eff,j} = \omega_{e,j}$, $\xi_{eff,j} = \xi_{e,j}$. Pseudo-velocity and pseudo-acceleration are derived from displacement. Solution to Eqn. (7.2-13) is found using the exact piecewise method [Nigam & Jennings, 1969; Wilson, 2002].

7.2.7 Ductility demand

The results of the study are binned according to their degree of inelasticity. As mentioned in Section 6.3, ductility is traditionally defined as the quotient between displacement and yield displacement. However, yield displacement is an ambiguous concept, as it depends on both performance point and a bi-linearization criterion which is not unique.

In this study, the definition of ductility is based on stiffness relaxation, measured as the secant stiffness in the pushover curve at the first mode. The procedure to determine the ductility is:

- The tangent stiffness at origin κ_1 is determined using Eqn. (6.3-4).
- For every point $(u_{N,l}, V_{1,l})$ in the pushover curve of the bare frame, the corresponding effective stiffness $\kappa_{eff,1}$ is found using Eqn. (6.3-6), and its ductility is found using:

$$\mu_{T,1} = \frac{\kappa_1}{\kappa_{eff,1}}. \quad (7.2-14)$$

The value of ductility for any roof displacement $u_{N,P}$ is then obtained by interpolation. The relationship between displacement-based ductility μ and period-based ductility μ_T can be obtained from Figure 7-3 as:

$$\mu_{T,1} = \frac{1 + \alpha_1(\mu_1 - 1)}{\mu_1}. \quad (7.2-15)$$

This definition can be extended to any mode j .

Figure 7-3: Period-based ductility and relationship with displacement-based ductility.

7.2.8 Variables and storeys considered

The comparison is performed for the following relevant variables: displacement u , drift Δ , drift velocity ∇ , structural shear V_s , damper shear V_d , total shear V_t .

The response is examined at:

- all storeys (referred to as ‘all’), where the result at each storey is a data point;
- first storey (referred to as ‘first’), where only the first storey values are considered;
- roof (referred to as ‘roof’) where only the roof values are considered.

7.2.9 Acceptability of prediction results

7.2.9.1 Error definition

A certain method is used to approximate the response of the system. The prediction of a structural variable x at a certain storey of a certain frame produces a data point. For instance, such value calculated according to method 1 is referred to as $x_{1,i}$, where i is the order of the data point in the total set of data points comprising P points. Likewise, the prediction for the same storey and frame by another method 2 would be $x_{2,i}$, etc. These predictions are normalized by the mean value of TH results for the same variable and storey (referred to as $x_{TH,n}$), which is taken here as ‘true’ response. The error is defined subtracting 1 to the normalized prediction. For instance, for $x_{1,i}$:

$$x_{\text{error},1,i} = \frac{x_{1,i}}{x_{TH,i}} - 1 \quad (7.2-16)$$

$x_{\text{error},1,i} = 0$ indicates a perfect match between $x_{1,i}$ and $x_{TH,i}$. Negative values of $x_{\text{error},1,i}$ indicate un-conservative (unsafe) prediction. Positive values of $x_{\text{error},1,i}$ indicate conservative (safe) prediction. This criterion for the error sign is not coincident with the one presented in Chapter 3; the reason is that here both conservative and un-conservative errors are examined, whereas in Chapter 3 the focus was only on un-conservative errors.

7.2.9.2 Engineering acceptability range.

An engineering acceptability range EAR [Guyader & Iwan, 2006] is defined for the error as $[E_u, E_c]$, where $E_c (> 0)$ is the acceptable maximum conservative error, and $E_u (< 0)$ is the acceptable minimum un-conservative error.

There is no general consensus about this acceptability range. As a first step to establish approximate values based on the opinion of the engineering community, a survey was conducted among both structural engineering professionals (15 answers) and academy staff (5 answers) specialized in structural engineering. The engineering professionals were practitioners from three top companies in Madrid. The academy staff were teachers from a structural department in Universidad Politécnica de Madrid. Four simple questions were formulated as follows:

A method is available to calculate the response (displacements, internal forces, etc.) of a structure subjected to earthquake loading. The true response is referred to as R . The estimated response is referred to as E . Indicate the range $[(E/R)_{\min}; (E/R)_{\max}]$ suitable for practical application of the method. As an example, $[0.80; 1.25]$ indicates that the method is acceptable if its predictions lie between 80% and 125% of the actual values.

1. Indicate the acceptable range for storey displacements:
2. Indicate the acceptable range for internal forces:
3. ¿Do you usually work on the field of earthquake engineering?
4. Indicate your activity: (a) structural engineering; (b) academy/research; (c) others.

In the notation of this study, E/R corresponds to x/x_{TH} , and it was used to simplify the questions and avoid misinterpretation. A summary of the survey results is shown in Table 7-2, and Figure 7-4. It can be observed that:

- the ranges are asymmetrical, acknowledging a preference for conservative errors in structural engineering estimates;
- except outliers, results show a remarkably small dispersion, generally larger for the conservative (upper) limit; median and mean are very close;
- acceptable ranges are narrower for forces than displacements, indicating that forces are perceived as more relevant for safety than displacements;
- no remarkable differences are observed if the results are binned by profession or by involvement with earthquake engineering.

Figure 7-4: results of survey for acceptability range; (a) displacements; (b) forces.

Based on all data, the reasonable ranges $[(x/x_{TH})_{\min}; (x/x_{TH})_{\max}]$ are [0.86; 1.25] for displacements and [0.89; 1.23] for forces; the equivalent $EARs$ are [-0.14, +0.25] and [-0.11, +0.23].

The previous results can be compared to the values offered by Guyader & Iwan [2006]; in the context of errors derived from linearization procedures, these authors consulted with several prominent engineers and found out an $EAR = [-0.1; +0.2]$, which can be expressed as $[(x/x_{TH})_{\min}; (x/x_{TH})_{\max}] = [0.90; 1.20]$. This range was determined for displacements.

Considering the large dispersion generally obtained in TH analysis under suites of accelerograms, and for the sake of simplicity, the final acceptability range adopted in this study is $EAR_1 = [-0.15, +0.25]$. It can be interpreted as the range of results expected for a verification method used in the final stage of a project.

A wider range $EAR_2 = [-0.30, +0.50]$ is also presented for some cases. It has been obtained doubling the length of the acceptability range, and can be interpreted as the range of results expected for a verification method used on initial stages of a project.

Table 7-2. Survey about simplified methods.

		N	Storey displacement		Internal forces		
			(E/R) _{min}	(E/R) _{max}	(E/R) _{min}	(E/R) _{max}	
Results considering all data		22	mean	0.86	1.25	0.89	1.23
			median	0.85	1.20	0.90	1.15
			std	0.11	0.21	0.10	0.23
Profession	Academy / research	5	mean	0.91	1.39	0.91	1.32
			median	0.80	1.25	0.90	1.20
			std	0.19	0.34	0.17	0.28
	Structural engineering	15	mean	0.86	1.19	0.89	1.21
			median	0.90	1.15	0.90	1.15
			std	0.06	0.12	0.07	0.23
	Both	2	mean	0.76	1.33	0.85	1.18
			median	0.76	1.33	0.85	1.18
			std	0.13	0.25	0.07	0.11
Involvement in earthquake engineering	Yes	9	mean	0.85	1.20	0.89	1.23
			median	0.90	1.15	0.90	1.10
			std	0.09	0.17	0.07	0.30
	No	10	mean	0.87	1.29	0.88	1.24
			median	0.80	1.20	0.85	1.18
			std	0.14	0.26	0.13	0.21
	Occasionally	3	mean	0.88	1.25	0.95	1.20
			median	0.90	1.20	0.95	1.20
			std	0.03	0.13	0.05	0.10

7.2.9.3 Error estimate

Once the $EAR = [E_u, E_c]$, has been fixed, the error trends inherent in a simplified method can be identified with the following procedure: the errors $x_{\text{error},i}$ are sorted in ascending

order. The empirical cumulative distribution function (CDF) is obtained, and the empirical cumulative density values $\varepsilon_c = P(x_{\text{error}} < E_c)$, $\varepsilon_0 = P(x_{\text{error}} \leq 0)$ and $\varepsilon_u = P(x_{\text{error}} \leq E_u)$, are obtained. In this process no data distribution is assumed. Then:

- $1 - \varepsilon_c$ indicates the fraction of results that are unacceptably conservative.
- ε_u indicates the fraction of results that are unacceptably unconservative.
- $\varepsilon_a = \varepsilon_c - \varepsilon_u$ indicates the fraction of results that are within acceptability range.
- $1 - \varepsilon_0$ indicates the fraction of results that are conservative or exact.
- ε_0 indicates the fraction of results that are unconservative.

Figure 7-5: Quantification of errors based on Engineering Acceptability Range (EAR).

An example is shown in Figure 7-5(b), based on a set of error results (MRF type D equipped with ADS type P, Q, R, S, U of linear and non-linear type calculated with RSA), which is treated here as method 1.

Figure 7-6 shows the error results obtained for a particular set (GMS EUK, frame 05, 2% inherent damping ratio, ADS type QRSU, non-linear dampers, high brace stiffness), presented for one variable (V_t) and three methods, RSA, RSAB, RSABS. Explanation of the figures follows:

Figure 7-6(a) shows the empirical cumulative density function CDF of the error for total shear obtained with the RSA method, $V_{t,\text{error,RSA}}$; three ranges are marked with different colors: too conservative (blue), within range (green) and too un-conservative (red). For this data set, $\varepsilon_c = 0.675$; $\varepsilon_0 = 0.408$; $\varepsilon_u = 0.192$; thus, 32.5% of the results ($1 - 0.675$) are too conservative, 19.2% are too un-conservative, and 48.4% ($0.675 - 0.192$) are within range. The maximum and minimum value in the X-axis are indicative of the maximum and minimum range of results (around -0.5 to 3.0 in this case). The dashed lines (red and blue)

indicate the lower and upper limits of the range $EAR_1 = [-0.15; + 0.25]$. Similar plots are presented for RSAB and RSABS.

These results are summarized in Figure 7-6(e); three lines are presented, one for each method. The position of the line corresponds to the interval $[\varepsilon_u, \varepsilon_c]$, and the circular marker is located at ε_0 . Focusing on the third line (RSA method), the red number to the left is the proportion of too un-conservative results (19.2%); the blue number to the right is the proportion of too conservative results (32.5%). The numbers to the right of the method descriptor indicate the proportion of results in range, 48.4% and the value ε_0 (40.8%). Notice the correspondence of these values with those shown in Figure 7-6(a). Additional information about the set is given in the label at the figure top left 'EUK.05D2.QRSU.NL.HB'. This tag provides information about GMS (EUK), MRF (05D2), ADS (QRSU), type of damper (NL non-linear) and bracing stiffness (HB). The label contains also information about the EAR considered, ductility, number of data points included in the graph, storey. For multiple ground motions, it also indicates the central value considered ('mean' or 'logMedian'). Figure 7-6(f) contains the same information calculated for the wider range EAR_2 .

The bar length and position provides valuable first-sight information about the method accuracy and allows direct comparison between methods. For example, in the plot presented, RSAB and RSABS are more conservative than RSA. However, RSA renders a larger proportion of un-conservative results below the acceptable threshold. Overall, using RSAB 56% of the results are in EAR, versus 48.4% using RSA. Finally, using RSAB, only 12.7% of the results are un-conservative, whereas RSA renders 40.8% of un-conservative results.

Additionally, Figure 7-6(d) shows summary statistics for the error, in a modified whisker-and-box format. Three lines correspond to the three methods used. The third line corresponds to RSA. The extension of the thin line corresponds to the extreme error values obtained in this set (-0.50 to +2.93). The thick line covers the 25% to 75% quartile. The numbers following the method descriptor RSA indicate the mean 0.04, median [0.28], inter-quartile range IQR (0.49) and total range {3.42}.

7.2.9.4 Acceptability criterion

Hereby, a method is considered acceptable for a certain range EAR if at least 67% ($\approx 2/3$) of the error results obtained from the study lie within the acceptability range. The value of 67% is taken as an approximation to the traditional rule of mean $\pm$ standard deviation, often adopted for studies in earthquake engineering. This rule implies that, under normal distribution, approximately 68.2% of results lie within this interval.

Another important criterion is the proportion on unacceptable un-conservative results (un-conservative results out of range). Fulfillment of the first criterion automatically implies that the proportion of un-conservative results is smaller than 37%. Both criteria are examined below.

7.2.9.5 Relative distance between predictions

The relative distance between TH and method predictions is measured using the root-mean-square error (RMSE), defined as:

$$\text{RMSE}_{\text{error},1} = \sqrt{\frac{\sum_{i=1}^P x_{\text{error},1,i}^2}{P}}, \quad (7.2-17)$$

where P is the number of points in the sets. RMSE does not take into account the error sign, so it will be used in conjunction with other statistics.

Figure 7-6: Example of error quantification for EAR_1 (left) and EAR_2 (right).

7.3 Study results

7.3.1 Influence of damping system stiffness on TH results

As indicated in Section 7.2.4.3, regarding the influence of the damping system stiffness, three different types of model (RB, HB, LB) have been used. In model type RB the influence of the damping system stiffness is negligible. It has been found that analysis for this type of system takes much longer and presents more convergence problems than for any of the other two types (using the openSees viscous damper element). Model type HB represents realistic brace stiffness conditions. Model type LB represents in a simple way the influence of the damper stiffness (see Chapter 5, Section 5.1.2). For GMS EUK (one motion) the three types of model have been analyzed. For GMS ESB (set of motions), only models HB and LB have been used.

7.3.1.1 Influence of damping system stiffness on TH results. One motion (set EUK)

Figure 7-7 shows the analysis results for GMS EUK, MRF 05D2, with complete ADS types Q, R and S; all damping levels and damper exponents are presented. Every plot shows the quotient between variable responses calculated with two different types of models. For instance, the top-left plot shows the quotient between displacement calculated with model type HB (u_{HB}) and model type RB (u_{RB}). Every line represents the variation of this quotient with intensity (represented in the x-axis). Thus, every row of plots deals with a different variable. The left column of plots shows the variations obtained in analysis from model type RB to model type HB. The central column shows the variations from model type HB to model type LB. The right column shows the variations from model type RB to model type LB. The intensities presented cover a range of *PHA* from 0.1g to 2g. From the data presented it seems that:

- The influence of diagonal and damper bracing on the response is relevant and should not be dismissed, although it is dependent on excitation intensity and variable.
- The amount of variation from RB to HB models is of a similar order to the variation from HB to LB models.
- For u , Δ and V_s , the influence is large at low intensities, but it tends to decrease with increasing intensities.
- For ∇ , V_d , and to a lesser extent V_r , the influence is larger a low intensity levels, but at large intensity levels it is still considerable and presents large variability.
- V_d is the only variable for which the reduction of brace stiffness may result in reduction of response.

From these observations it seems clear that modelization of these damping systems assuming pure viscous damper of the damping system may result in considerable error. In

order to identify systems for which this could be critical, the trends for different types of ADS is examined.

Figure 7-8 presents the previous results binned according to low and high damping. It is clear that the influence of the damping system stiffness is larger for systems with high damping. This is particularly outstanding for ∇ , V_d and V_t throughout the whole intensity range.

Figure 7-9 shows the results binned according to damper exponent as linear ($a = 1$) or non-linear ($a = 0.5$). The influence of the damping system stiffness is much larger for non-linear systems. Also, for non-linear dampers the influence is dependent on intensity, with more moderate values for larger intensities, whereas for linear dampers the influence is similar throughout intensities.

Finally, Figure 7-10 shows the results for first storey and roof. For the first storey the influence tends to diminish with intensity. This is also the case for the roof and variables u , Δ and V_s . For ∇ , V_d and V_t at roof the variability is kept throughout intensity levels. The influence at roof is much larger for all variables.

As a summary, neglecting the Maxwell stiffness of the damping system is un-conservative for basically all variables, in a degree which is generally larger for smaller intensities, and more pronounced for non-linear dampers, and high damping levels. The roof is more affected than the first storey. In non-linear TH modeling the brace stiffness should not be neglected, especially when a intensity-dependent analysis is carried out (such as IDA). The influence of the damper stiffness should also be taken into account. If no accurate data is available, a 50% reduction of bracing stiffness might be used as an approximation. Further experimental and analytical research to characterize the stiffness of non-linear dampers is needed.

7.3.1.2 Influence of damping system stiffness on TH results. Results for sets of motions (set ESB)

A similar analysis is presented for GMS ESB and MRF 05D with the same ADSs. In this case only models HB and LB are compared, at both intensity levels considered in the study (PHA_E and PHA_D). Comparative plots are presented in Figure 7-11 to Figure 7-22. In each figure, an intensity level and variable is represented. A single plot within the figure compares the results for the variable of interest x at every storey n , calculated with both model types ($x_{TH,HB,n}$ and $x_{TH,LB,n}$), and normalized to the logMedian value calculated with model HB at storey n ($\hat{x}_{TH,HB,n}$). The extension of the line covers the interval $[x^{16}; x^{84}]$, normalized as indicated. The black solid line corresponds to results with model type HB, the blue dashed line corresponds to results with model type LB. In both cases, the central tick indicates the normalized logMedian (which, for model type HB will always be one).

From data presented, the following observations are made:

- As with individual motions, the relative influence of the Maxwell system stiffness is dependent on variable, intensity, amount of damping, damping exponent and storey of interest. The trends are similar, but milder.
- The relative influence of Maxwell stiffness is larger at small intensities, with larger relative variation of logMedian and data dispersion.
- The influence of Maxwell stiffness is larger for non-linear dampers and higher damping.
- The influence is relatively small for displacement; more relevant for drift and damper shear; and large for drift velocity, storey shear and total shear.
- The influence is more relevant at the top half of the frame, particularly at roof. The bottom half of the frame is not very sensitive to changes in the Maxwell system stiffness.

In Chapter 5, a conditions for minimum diagonal stiffness was suggested, based on reaching a minimum value of energy dissipation efficiency β_W . Figure 7-23 and Figure 7-24 show scatter plots of $\beta_{W,n}$ versus variation of variables between models with HB and LB stiffness. The values of $\beta_{W,n}$ have been calculated using the equations derived in Chapter 5, with q_n obtained using $c_{d,n}$, $k_{d,n}$, a_n , Δ_n and ∇_n corresponding to the LB system. The plots show a positive correlation of β_W with the modification in variables u , Δ , ∇ and V_s , and no clear correlation with V_d and V_t . It is clear that high values of β_W result in moderate changes of the variables. A minimum limit $\beta_W = 0.9$ has been marked in the plots, as orientation. The value of $k_{d,n}$ used to obtain β_W should consider the stiffness of all auxiliary elements, including the damper.

Figure 7-7: Influence of damping system stiffness on TH results. One motion.

Figure 7-8: Influence of damping system stiffness. High vs low damping systems. One motion.

Figure 7-9: Influence of damping system stiffness. Linear vs non-linear dampers. One motion.

Figure 7-10: Influence of damping system stiffness. First storey vs roof. One motion.

Figure 7-11: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_E . Displacement.

Figure 7-12: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_D . Displacement.

Figure 7-13: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_E . Drift.

Figure 7-14: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_D . Drift.

Figure 7-15: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_E . Drift velocity.

Figure 7-16: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_D . Drift velocity.

Figure 7-17: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_E . Storey shear.

Figure 7-18: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_D . Storey shear.

Figure 7-19: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_E . Damper shear.

Figure 7-20: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_D . Damper shear.

Figure 7-21: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_E . Total shear.

Figure 7-22: Influence of damping system stiffness. Storey distribution. Set of motions (ESB). Intensity level PHA_D . Total shear.

Figure 7-23: Influence of damping system stiffness. Set of motions (ESB). Intensity level PHA_E . Relationship between β_W and variable change from HB to LB stiffness.

Figure 7-24: Influence of damping system stiffness. Set of motions (ESB). Intensity level PHA_D . Relationship between β_W and variable change from HB to LB stiffness.

7.3.2 Results with exact spectral response

In this Section, results of the simplified methods are obtained using the exact spectral response described in Section 7.2.6. No spectral correction factors are used. The options considered for application of the methods are listed in Table 7-3. The results are presented for different sets, including one accelerogram (EUK) or suites of accelerograms (ES). In this last case, the logMedian is adopted as central value, except otherwise indicated. The results are presented segregated by ductility levels: $\mu_T \leq 3$ (elastic), $\mu_T \leq 1.05$ (elastic), $1.05 < \mu_T \leq 1.5$ (moderate ductility), $1.5 < \mu_T \leq 3$ (high ductility).

Table 7-3. Options considered .

	Method 1 (RSA)	Method 2 (RSAB)	Method 3 (RSABS)
Spectral solution	Exact (piecewise)	Exact (piecewise)	Exact (piecewise)
Damping Correction Factors	None	None	None
Correction Factor for Velocity	None	None	None
Hysteretic damping	ASCE	Eqns. (2.5-69) ff.	Eqns. (2.5-69) ff.
Combination Rule	CQC	CQC	CQC
Method for Damper Shear	Modal	Modal	Modal
Method for Total Shear	Force Coefficients	Force Coefficients	Force Coefficients
Minimum elastic displacement	No	No	No

7.3.2.1 Results with exact spectral response: general overview

Systems included: MRF 05D with 5% inherent damping, complete vertical damping systems (ADS Q, R, S), all added damping ratio levels, bracing stiffness type HB. 5% inherent damping is a high value for steel frames, for which a more usual assumption is 2%. This value has been selected because scaling of the ES sets was done for 5% inherent damping. The influence of the inherent damping ratio in the results has been examined and found to be negligible, as reported below.

The frame selected is representative of mid-height buildings. The ADS selected covers three different possibilities: a constant vertical distribution (sometimes adopted for simplicity), and two decreasing vertical distributions.

The Maxwell model stiffness (HB) takes into account the brace stiffness, but not the damper contribution, because there is no accurate information available. The influence of this parameter is considered below using an approximation.

The number of data points included in each case are:

- Set EUK: 610 ($\mu_T \leq 1.05$), 425 ($1.05 < \mu_T \leq 1.5$), 705 ($1.5 < \mu_T \leq 3$).
- Set ESB: 155 ($\mu_T \leq 1.05$), 170 ($1.05 < \mu_T \leq 1.5$), 375 ($1.5 < \mu_T \leq 3$).

However, the most representative results are those obtained for elastic or moderate ductility ($1.05 < \mu_T \leq 1.5$), which is the reasonable target when dampers are installed.

The error statistics are presented in the following figures:

- For $\mu_T \leq 3$ (all levels) in Figure 7-25.
- For $\mu_T \leq 1.05$ (elastic levels) in Figure 7-29.
- For $1.05 < \mu_T \leq 1.5$ in Figure 7-33.
- For $1.5 < \mu_T \leq 3$ in Figure 7-35.
- Summary of mean and standard deviation of errors including different ductility levels are given in Table 7-5 (set EUK), Table 7-6 (set ESB), and Table 7-7 (differences between sets).

The method acceptability as defined in Section 7.2.9, is presented in the following figures:

- For $\mu_T \leq 3$ (all levels) in Figure 7-26.
- For $1.05 < \mu_T \leq 1.5$ in Figure 7-34.
- For $1.5 < \mu_T \leq 3$ in Figure 7-36.
- Summary of method acceptability is given in Table 7-8 (set EUK, acceptability for EAR_1), Table 7-9 (set EUK, acceptability for EAR_2), Table 7-10 (set ESB, acceptability for EAR_1), and Table 7-11 (set ESB, acceptability for EAR_2).

Scatter plots comparing the results of RSAB vs. RSA are shown in the following figures:

- For $\mu_T \leq 3$ (all levels) in Figure 7-27.
- For $\mu_T \leq 1.05$ (elastic levels) in Figure 7-31.

Scatter plots comparing the results of RSABS vs. RSA are shown in the following figures:

- For $\mu_T \leq 3$ (all levels) in Figure 7-28.
- For $\mu_T \leq 1.05$ (elastic levels) in Figure 7-32.

Differences in results for one or several accelerograms

The methods present consistent results when applied to one single accelerogram (EUK) or several accelerograms (ESB). Table 7-5 and Table 7-6 present the mean and standard deviation of errors for set EUK and ESB, respectively; mean positive values have been shadowed. Most of mean error results are un-conservative.

Table 7-7 presents the difference between the corresponding values in the previous two tables; positive values have been shadowed, showing clearly that the error dispersion obtained for the ESB set is generally smaller than for set EUK, but a smaller mean error is obtained for EUK.

Scatter plots show a positive correlation between methods, larger for u , Δ , and V_t .

Results statistics

When results statistics (Figure 7-29, Figure 7-30, Figure 7-35) are examined, the following trends are clear:

- The results for one motion and a set of motions present similar trends for the same ductility level.
- Results for one motion present larger variability than those for a set of motions. This is consistent with a softer spectral dependency from period in the latter. In addition, results for a set of motions present a more marked tendency to under-prediction, except for V_s and V_t at high ductility levels.
- Any of the three methods can largely under-predict or over-predict the response for all variables.
- There is a general tendency to under-estimation, with mean and median values systematically negative; the only exception is V_t , a variable for which the trend is over-conservative.
- In general (with a few exceptions), the mean value obtained with RSAB and RSABS methods is closer to 0 than the corresponding mean value obtained with RSA. Exceptions are V_s and V_t .
- When only elastic or moderate ductility levels are considered, the RSAB and RSABS methods improve the predictions for all variables.

Acceptability of the method for EAR_1 :

- None of the methods are clearly acceptable for interval EAR_1 , all ductility levels and all storeys. The best prediction is obtained for V_s , but even for this variable the results are unacceptable in the elastic range.
- The results are more acceptable for static variables than for kinematic variables.
- Within kinematic variables, the best results are obtained for Δ , and the worst for ∇ .
- Acceptability of the methods is slightly better for one motion (set EUK) than for a set of motions (set ESB).
- RSAB and RSABS present larger acceptability than RSA for all variables. Proportion of results within range is larger and the proportion of unacceptable un-conservative results is smaller.
- The proportion of un-conservative results is larger than the proportion of conservative results. This is proved by the position of ε_0 (dot in plots), always above 50%, except for V_t , where very conservative results are obtained.
- For all variables, the acceptability worsens remarkably for elastic levels ($\mu_T \leq 1.05$), and improves for large ductility levels ($1.5 < \mu_T \leq 3$).

Acceptability of the method for EAR_2 :

- Considering EAR_2 , the three methods offer acceptable results for variables u , Δ , V_s , and V_t .
- For V_d , the results are acceptable only with methods RSAB and RSABS.

- For ∇ the results are unacceptable with either method.
- For all variables, the acceptability worsens for elastic levels ($\mu_T \leq 1.05$), and improves for large ductility levels ($1.5 < \mu_T \leq 3$).
- All methods are biased towards un-conservative results in all variables.

Error by storey:

Plots of error statistics and acceptability for the first storey and roof are given in Section F.1.1 (Annex F), binned by ductility levels. The following trends are observed:

- The error variability at first storey is smaller than the variability at roof, where larger extreme errors are observed.
- The results for one accelerogram (EUK) and a set of accelerograms (ESB) present similar trends but larger differences than those obtained considering all storeys.
- The results for u , Δ and ∇ at first storey are unacceptable, both for interval EAR_1 and EAR_2 .
- The results for V_t are unacceptable at roof with either method and for both intervals.

Conclusion:

Overall, for all variables, the accuracy of the three methods is poor, with a clear tendency to under-estimation of results, both when using a single accelerogram or when using a set of accelerograms. This contradicts the general belief that RSA is conservative. According to results in this study, the RSA method can render both over-conservative or under-conservative results. This conclusion is coincident with the results presented in Chapter 3, and has been pointed out for specific cases by previous studies (Goel & Chopra, 2004; Akcelyan *et al.* 2016; FEMA, 2005).

From the results presented, it is obvious that none of the three methods (RSA, RSAB, RSABS) produces accurate results. Large under- or over-predictions can be obtained.

For initial stages of design, where accuracy is not so demanding an issue, any of the three methods offer reasonable results. The following exceptions must be accounted for:

- Errors out of range are obtained for drift velocity at all storeys except at roof using RSABS.
- The results for damper shear are more accurate using the RSAB or RSABS methods. Damper shear at roof is correctly predicted only by RSAB or RSABS.
- Drift and displacement at first storey are only correctly predicted by the RSA method.
- Total shear at roof is strongly over-predicted.

For practical application, the most important results are those obtained in the range of ductility below 1.5, which is a reasonable target for structures with added viscous damping systems. In this range (Figure 7-34) for initial stages of design, the proposed methods (RSAB or RSABS) give better results for ∇ and V_d , and similar results for V_s and V_t ; for

this ductility range, the RSA method gives unacceptable results for ∇ and V_d . However, kinematic results for the first storey will be strongly under-predicted with either method.

The influence of different parameters on the error is now considered. To avoid overloading the main text, graphical presentation of results is presented in Annex F.

7.3.2.2 Results with exact spectral response. Influence of damper stiffness.

In order to assess the influence of damper stiffness modeling on results, two types of bracing stiffness model are considered: The first one (HB) models the Maxwell element with the stiffness due to the brace extender. The second one (LB) halves this value, to take into account (in a simplified manner) the stiffness of the damper element. Graphical results are presented in Section F.1.2 (Annex F):

The plots show that results of the methods are not particularly sensitive to the inclusion of the damper stiffness in the model, except that:

- the error for Δ and V_s with RSA is substantially larger with low stiffness models, whereas the error with RSAB or RSABS remains basically unaffected;
- the error for ∇ and V_t for RSAB or RSABS reduces with low stiffness models.

7.3.2.3 Results with exact spectral response. Influence of damper exponent.

The errors are binned according the damper exponent, linear ($a = 1$) or non-linear ($a = 0.5$). Graphical results are presented in Section F.1.3 (Annex F).

These plots show that the prediction obtained with RSA is sensitive to the damper exponent. The results also show discrepancies between one motion (EUK) and a set of motions (ESB), particularly for ∇ and a . In general:

- The errors for u , Δ , V_s are smaller for linear dampers.
- The errors for V_d , are smaller for non-linear dampers.

The results are sensitive to the level of ductility and damper linearity:

Linear dampers

- For EAR_1 and elastic levels ($\mu_T \leq 1.05$), the three methods are acceptable for u and Δ , and unacceptable for all other variables.
- For EAR_1 and moderate to high ductility levels ($1.05 < \mu_T \leq 3$), only RSA is acceptable for Δ . However, all methods are acceptable for V_s .
- For EAR_2 and all ductility levels, results are acceptable with all three methods for u , Δ , V_s and V_t , but not acceptable for ∇ , a and V_d . The best results for V_d are obtained with RSABS.

Non-linear dampers

- For EAR_1 and elastic levels ($\mu_T \leq 1.05$), the three methods are unacceptable for all variables.

- For EAR_1 and moderate ductility levels ($1.05 < \mu_T \leq 1.5$), V_s and V_d are acceptable with RSAB and RSABS. All other variables are unacceptable.
- For EAR_1 and high ductility levels ($1.5 < \mu_T \leq 3$), V_s is acceptable with either method. All other variables are unacceptable.
- For EAR_2 and elastic levels ($\mu_T \leq 1.05$), only RSAB is acceptable for u and Δ . RSAB and RSABS are acceptable for a , V_d and V_t . RSA is unacceptable.
- For EAR_2 and inelastic levels ($\mu_T > 1.05$), u , Δ , V_s , V_d and V_t are acceptable with all three methods, except V_d that cannot be calculated with RSA.

7.3.2.4 Results with exact spectral response. Influence of damping level.

For every damper distribution, three different damping levels are analyzed, where levels 2 and 3 provide 2 and 3 times the damping ratio in level 1, respectively. The errors are binned according to damping level between high (level 3) and low (level 1). The results are shown in Section F.1.4. (Annex F). The trends can be summarized as follows:

Low damping

- Considering EAR_1 and elastic ductility levels, u and Δ are acceptable for low damping, but unacceptable for high damping; all other variables are unacceptable in either case.
- Considering EAR_1 and medium ductility levels, u , Δ , V_s , V_t are acceptable. V_d is acceptable only with RSAB and RSABS, and ∇ and a are unacceptable.
- Considering EAR_1 and high ductility levels, u , Δ , V_s , V_t are acceptable. V_d , ∇ and a are unacceptable.
- Considering EAR_2 , u , Δ , V_s , and V_t are acceptable with all three methods, V_d is acceptable only with RSAB and RSABS, and ∇ and a are unacceptable.

High damping

- Considering EAR_1 and elastic ductility levels, all variables are unacceptable with either method.
- Considering EAR_1 and moderate ductility levels, V_s and V_d are acceptable with RSAB or RSABS. Other variables are unacceptable.
- Considering EAR_1 and high ductility levels, V_s is acceptable with RSAB or RSABS. All other variables are unacceptable.
- Considering EAR_2 , u , Δ , V_s , and V_t are acceptable with all three methods, V_d is acceptable only with RSAB and RSABS, and ∇ are unacceptable. Acceptable predictions are obtained for a using the RSAB or RSABS methods for elastic ductility levels.

7.3.2.5 Results with exact spectral response. Influence of site class.

Analysis are performed for frame 05D5+Q and site classes A, B, C and D. The error results are presented in Section F.1.5 (Annex F), and are virtually identical for all site classes,

showing that this parameter has no influence on results when the exact spectral response is used.

7.3.2.6 Results with exact spectral response. Influence of inherent damping.

Result errors for frames type D with 5% and 2% inherent damping are presented in Section F.1.6 (Annex F). The results presented are basically identical, showing that this parameter has no influence on the errors produced by the method.

7.3.2.7 Results with exact spectral response. Influence of damper distribution.

Three different damper distributions are examined independently: Q, R, and S. The results, presented in Section F.1.7 (Annex F) show very similar error results for all three distributions, with a slight improvement in predictions at most variables for ADS type R, indicating a mild dependency of prediction error on this parameter. This dependency can be associated to the different levels of non-proportionality of the distributions.

7.3.2.8 Results with exact spectral response. Influence of combination rule.

Two different combination rules (SRSS and CQC) are examined. The rules are used to determine the RSA results, with sets EUK and ESB, for MRF 05D5 with complete ADS Q, R and S. The complete results are displayed in Section F.1.9 (Annex F), where results are offered for all storeys, first storey and roof. A summary is offered in Table 7-4. These data prove that both combination rules offer very similar results when all the storeys are considered; however, the CQC method generally improves the accuracy for the roof and first storey results, with the only exception of V_t at roof where CQC renders a large –albeit conservative- error. In summary, the CQC combination rule improves slightly the error results.

Table 7-4. Results with exact spectral response. Influence of combination rule. Statistical summary (mean, std). Sets EUK and ESB.

		u		Δ		∇		V_s		V_d		V_t	
		mean	std	mean	std	mean	std	mean	std	mean	std	mean	std
ductility		EUK.05D5.QRS.ALL.ALL.HB											
All	CQC	-0.18	0.15	-0.07	0.26	-0.15	0.32	0.03	0.35	-0.11	0.21	0.32	0.68
	SRSS	-0.21	0.17	-0.02	0.35	-0.03	0.49	0.14	0.57	-0.01	0.34	0.04	0.48
First	CQC	-0.26	0.17	-0.26	0.17	-0.09	0.16	0.17	0.30	0.02	0.12	-0.04	0.19
	SRSS	-0.35	0.17	-0.35	0.17	-0.36	0.17	-0.07	0.25	-0.26	0.14	-0.26	0.19
Roof	CQC	-0.10	0.12	0.22	0.34	-0.02	0.52	0.28	0.55	-0.07	0.31	1.42	0.74
	SRSS	-0.07	0.12	0.47	0.37	0.60	0.62	0.94	0.83	0.50	0.35	0.77	0.55
		ESB.05D5.QRS.ALL.ALL.HB											
All	CQC	-0.19	0.12	-0.11	0.22	-0.34	0.20	0.03	0.26	-0.25	0.17	0.27	0.56
	SRSS	-0.23	0.15	-0.04	0.34	-0.22	0.38	0.16	0.50	-0.15	0.30	-0.03	0.38
First	CQC	-0.29	0.10	-0.29	0.10	-0.28	0.07	0.20	0.24	-0.11	0.11	-0.09	0.16
	SRSS	-0.39	0.11	-0.39	0.11	-0.52	0.09	-0.07	0.20	-0.38	0.11	-0.31	0.16
Roof	CQC	-0.11	0.11	0.11	0.28	-0.23	0.30	0.15	0.37	-0.23	0.23	1.19	0.52
	SRSS	-0.08	0.10	0.43	0.33	0.38	0.33	0.89	0.66	0.35	0.22	0.55	0.36

Cells shadowed in green indicate best result.

7.3.2.9 Results with exact spectral response. Minimum elastic displacement.

ASCE-SEI 7/16 imposes the roof displacement to be no less than the minimum elastic displacement, calculated with damping ratio corresponding to the added damping and inherent damping ($\xi_I + \xi_a$). This rule is examined by comparing the error results of RSA calculated with (labeled as ‘Yes uel’) and without (labeled as ‘No uel’) application of the elastic minimum displacement. Complete results are given in Section F.1.10 (Annex F), for MRF 05D5 with ADS QRS, sets EUK and ESB, all storeys, first storey and roof.

The results prove (as expected) that imposition of the minimum elastic displacement reduces the un-conservativeness of the results in u and Δ , but it does so to the cost of a considerable over-estimation of Δ . For instance, as shown in Figure F-115 (Annex F), by use of the rule, the proportion of un-conservative results for all storeys in Δ for set ESB is reduced from 41.4% to 33.7%, but the over-conservative results change from 6.4% to 13.1%, so the acceptability moves only from 52.2% of the values to 53.1%, less than 1%. Overall, the total acceptability of all other variables is basically unchanged by use of the rule, which suggests that it could be removed, leading to more consistent results between forces and displacements.

Figure 7-25: Results with exact spectral values, all ductility levels, $\mu_T \leq 3$. Comparison of method statistics.

Figure 7-26: Results with exact spectral values, all ductility levels, $\mu_T \leq 3$. Method acceptability..

Figure 7-27: Results with exact spectral values, all ductility levels, $\mu_T \leq 3$. Scatter plots RSAB vs RSA.

Figure 7-28: Results with exact spectral values, all ductility levels, $\mu_T \leq 3$. Scatter plots RSABS vs RSA.

Figure 7-29: Results with exact spectral values, elastic levels, $\mu_T \leq 1.05$. Comparison of method statistics.

Figure 7-30: Results with exact spectral values, elastic levels, $\mu_T \leq 1.05$. Method acceptability..

Figure 7-31: Results with exact spectral values, elastic levels, $\mu_T \leq 1.05$. Scatter plots RSAB vs RSA.

Figure 7-32: Results with exact spectral values, elastic levels, $\mu_T \leq 1.05$. Scatter plots RSABS vs RSA.

Figure 7-33: Results with exact spectral values, moderate ductility levels $1.05 < \mu_T \leq 1.5$.
 Comparison of method statistics.

Figure 7-34: Results with exact spectral values, moderate ductility levels, $1.05 < \mu_T \leq 1.5$. Method acceptability.

Figure 7-35: Results with exact spectral values, high ductility levels, $1.5 < \mu_T \leq 3$. Comparison of method statistics.

Figure 7-36: Results with exact spectral values, high ductility levels, $1.5 < \mu_T \leq 3$. Method acceptability.

Table 7-5. Summary of statistical results (mean, std) with exact spectral values, set EUK.

		u		Δ		∇		V_s		V_d		V_t	
		mean	std	mean	std	mean	std	mean	std	mean	std	mean	std
storey	method	EUK.05D5.QRS.ALL.ALL.HB - $\mu \leq 3.0$											
all	RSA	-0.18	0.15	-0.07	0.27	-0.15	0.33	0.03	0.36	-0.12	0.21	0.31	0.69
	RSAB	-0.14	0.17	-0.01	0.28	-0.09	0.26	-0.05	0.22	0.03	0.22	0.30	0.58
	RSABS	-0.15	0.18	-0.01	0.30	-0.05	0.27	-0.03	0.22	0.06	0.24	0.35	0.63
first	RSA	-0.26	0.18	-0.26	0.18	-0.09	0.16	0.17	0.30	0.02	0.12	-0.05	0.19
	RSAB	-0.27	0.17	-0.27	0.17	-0.19	0.14	-0.03	0.20	-0.03	0.19	-0.06	0.16
	RSABS	-0.29	0.18	-0.29	0.18	-0.17	0.13	0.03	0.20	-0.02	0.19	-0.02	0.15
roof	RSA	-0.11	0.12	0.22	0.35	-0.03	0.53	0.29	0.56	-0.07	0.31	1.42	0.76
	RSAB	-0.03	0.12	0.29	0.32	0.19	0.33	0.20	0.29	0.24	0.25	1.27	0.56
	RSABS	-0.03	0.12	0.36	0.34	0.26	0.36	0.19	0.30	0.31	0.30	1.40	0.62
EUK.05D5.QRS.ALL.ALL.HB - $\mu \leq 1.05$													
all	RSA	-0.17	0.16	-0.16	0.18	-0.32	0.25	-0.18	0.23	-0.21	0.19	0.03	0.39
	RSAB	-0.15	0.13	-0.12	0.14	-0.18	0.12	-0.16	0.16	-0.06	0.13	0.13	0.37
	RSABS	-0.17	0.15	-0.11	0.19	-0.15	0.14	-0.15	0.15	-0.04	0.17	0.17	0.42
first	RSA	-0.16	0.17	-0.16	0.17	-0.09	0.17	-0.15	0.18	0.06	0.08	-0.24	0.12
	RSAB	-0.19	0.14	-0.19	0.14	-0.13	0.09	-0.22	0.15	-0.04	0.10	-0.14	0.12
	RSABS	-0.22	0.18	-0.22	0.18	-0.11	0.09	-0.17	0.13	-0.04	0.12	-0.11	0.12
roof	RSA	-0.16	0.16	-0.08	0.20	-0.39	0.30	-0.03	0.35	-0.31	0.16	0.70	0.29
	RSAB	-0.11	0.11	-0.02	0.14	-0.08	0.16	-0.01	0.21	0.06	0.17	0.76	0.27
	RSABS	-0.11	0.11	0.09	0.20	-0.01	0.18	-0.01	0.21	0.15	0.25	0.89	0.33
EUK.05D5.QRS.ALL.ALL.HB - $1.05 < \mu \leq 1.5$													
all	RSA	-0.10	0.13	-0.02	0.18	-0.13	0.21	0.04	0.23	-0.11	0.14	0.29	0.53
	RSAB	-0.04	0.18	0.06	0.21	-0.10	0.16	-0.03	0.12	0.02	0.17	0.30	0.49
	RSABS	-0.04	0.18	0.06	0.24	-0.07	0.18	-0.02	0.11	0.05	0.19	0.33	0.53
first	RSA	-0.18	0.15	-0.18	0.15	-0.02	0.14	0.17	0.09	0.04	0.10	-0.02	0.12
	RSAB	-0.17	0.17	-0.17	0.17	-0.16	0.12	-0.01	0.06	0.01	0.19	-0.02	0.13
	RSABS	-0.19	0.19	-0.19	0.19	-0.13	0.11	0.03	0.07	0.03	0.19	0.01	0.11
roof	RSA	-0.03	0.09	0.20	0.19	-0.06	0.34	0.21	0.42	-0.11	0.19	1.23	0.42
	RSAB	0.07	0.13	0.29	0.17	0.10	0.20	0.12	0.19	0.17	0.21	1.15	0.38
	RSABS	0.07	0.14	0.34	0.23	0.15	0.26	0.10	0.16	0.22	0.27	1.25	0.46
EUK.05D5.QRS.ALL.ALL.HB - $1.5 < \mu \leq 3$													
all	RSA	-0.24	0.12	-0.03	0.34	-0.02	0.37	0.21	0.40	-0.03	0.22	0.58	0.85
	RSAB	-0.20	0.16	0.04	0.36	0.00	0.34	0.03	0.26	0.11	0.27	0.45	0.73
	RSABS	-0.20	0.16	0.04	0.38	0.04	0.36	0.06	0.27	0.15	0.29	0.51	0.78
first	RSA	-0.40	0.07	-0.40	0.07	-0.13	0.16	0.45	0.16	-0.02	0.14	0.10	0.14
	RSAB	-0.41	0.08	-0.41	0.08	-0.27	0.15	0.11	0.16	-0.06	0.23	-0.01	0.18
	RSABS	-0.42	0.08	-0.42	0.08	-0.24	0.13	0.19	0.16	-0.03	0.23	0.04	0.16
roof	RSA	-0.10	0.06	0.48	0.31	0.32	0.56	0.61	0.61	0.15	0.30	2.14	0.50
	RSAB	-0.02	0.07	0.56	0.25	0.48	0.28	0.42	0.26	0.44	0.20	1.79	0.37
	RSABS	-0.02	0.07	0.61	0.30	0.55	0.33	0.41	0.29	0.50	0.25	1.93	0.46

Table 7-6. Summary of statistical results (mean, std) with exact spectral values, set ESB.

storey	method	u		Δ		∇		V_s		V_d		V_t	
		mean	std	mean	std	mean	std	mean	std	mean	std	mean	std
ESB.05D5.QRS.ALL.ALL.HB - $\mu \leq 3.0$													
all	RSA	-0.19	0.12	-0.11	0.22	-0.34	0.20	0.03	0.26	-0.25	0.17	0.27	0.56
	RSAB	-0.15	0.14	-0.04	0.24	-0.27	0.19	-0.02	0.17	-0.12	0.20	0.27	0.50
	RSABS	-0.16	0.15	-0.03	0.27	-0.24	0.20	-0.01	0.17	-0.09	0.21	0.32	0.54
first	RSA	-0.29	0.10	-0.29	0.10	-0.28	0.07	0.21	0.23	-0.11	0.11	-0.08	0.16
	RSAB	-0.32	0.10	-0.32	0.10	-0.37	0.07	-0.01	0.17	-0.17	0.19	-0.08	0.16
	RSABS	-0.33	0.11	-0.33	0.11	-0.34	0.06	0.05	0.18	-0.15	0.19	-0.04	0.15
roof	RSA	-0.11	0.11	0.11	0.28	-0.22	0.31	0.15	0.37	-0.22	0.23	1.20	0.52
	RSAB	-0.02	0.11	0.22	0.26	0.00	0.17	0.14	0.20	0.06	0.17	1.11	0.40
	RSABS	-0.02	0.11	0.27	0.29	0.05	0.19	0.12	0.22	0.11	0.20	1.22	0.46
ESB.05D5.QRS.ALL.ALL.HB - $\mu \leq 1.05$													
all	RSA	-0.21	0.14	-0.22	0.16	-0.50	0.17	-0.24	0.20	-0.37	0.17	0.02	0.44
	RSAB	-0.19	0.11	-0.16	0.13	-0.34	0.13	-0.19	0.15	-0.21	0.16	0.10	0.42
	RSABS	-0.21	0.14	-0.15	0.18	-0.30	0.15	-0.19	0.16	-0.18	0.18	0.15	0.47
first	RSA	-0.22	0.13	-0.22	0.13	-0.31	0.09	-0.15	0.14	-0.08	0.11	-0.27	0.07
	RSAB	-0.28	0.11	-0.28	0.11	-0.35	0.06	-0.26	0.12	-0.22	0.11	-0.20	0.11
	RSABS	-0.31	0.14	-0.31	0.14	-0.32	0.05	-0.20	0.12	-0.21	0.11	-0.15	0.11
roof	RSA	-0.19	0.16	-0.18	0.20	-0.54	0.19	-0.19	0.26	-0.46	0.12	0.78	0.35
	RSAB	-0.11	0.09	-0.06	0.15	-0.14	0.12	-0.07	0.21	-0.04	0.17	0.82	0.30
	RSABS	-0.12	0.10	0.05	0.21	-0.08	0.13	-0.12	0.26	0.03	0.22	0.96	0.36
ESB.05D5.QRS.ALL.ALL.HB - $1.05 < \mu \leq 1.5$													
all	RSA	-0.17	0.09	-0.11	0.14	-0.35	0.14	0.04	0.15	-0.27	0.14	0.20	0.47
	RSAB	-0.16	0.12	-0.08	0.18	-0.29	0.15	0.02	0.11	-0.14	0.18	0.26	0.44
	RSABS	-0.17	0.13	-0.07	0.21	-0.26	0.16	0.03	0.10	-0.11	0.19	0.31	0.49
first	RSA	-0.26	0.09	-0.26	0.09	-0.26	0.07	0.17	0.07	-0.11	0.10	-0.13	0.10
	RSAB	-0.31	0.09	-0.31	0.09	-0.36	0.07	-0.01	0.06	-0.16	0.20	-0.07	0.14
	RSABS	-0.32	0.11	-0.32	0.11	-0.33	0.06	0.04	0.09	-0.14	0.20	-0.03	0.13
roof	RSA	-0.10	0.08	0.06	0.15	-0.29	0.21	0.12	0.23	-0.29	0.15	1.01	0.41
	RSAB	-0.05	0.06	0.14	0.16	-0.07	0.13	0.14	0.17	0.01	0.15	1.02	0.33
	RSABS	-0.05	0.06	0.18	0.22	-0.02	0.16	0.12	0.15	0.05	0.17	1.13	0.40
ESB.05D5.QRS.ALL.ALL.HB - $1.5 < \mu \leq 3$													
all	RSA	-0.20	0.11	-0.05	0.24	-0.26	0.20	0.15	0.24	-0.20	0.16	0.41	0.60
	RSAB	-0.13	0.16	0.03	0.28	-0.24	0.21	0.04	0.14	-0.07	0.22	0.35	0.54
	RSABS	-0.14	0.17	0.03	0.30	-0.21	0.22	0.05	0.15	-0.05	0.22	0.40	0.58
first	RSA	-0.34	0.07	-0.34	0.07	-0.29	0.06	0.37	0.10	-0.12	0.11	0.02	0.11
	RSAB	-0.34	0.09	-0.34	0.09	-0.39	0.08	0.10	0.11	-0.16	0.21	-0.04	0.16
	RSABS	-0.35	0.10	-0.35	0.10	-0.36	0.06	0.16	0.11	-0.13	0.21	0.01	0.15
roof	RSA	-0.08	0.09	0.26	0.24	-0.07	0.27	0.31	0.35	-0.10	0.20	1.46	0.48
	RSAB	0.04	0.10	0.37	0.23	0.09	0.15	0.23	0.13	0.13	0.14	1.28	0.39
	RSABS	0.03	0.10	0.41	0.28	0.13	0.18	0.23	0.13	0.17	0.18	1.38	0.47

Table 7-7. Summary of difference between statistical results (mean, std) with exact spectral values obtained for ESB and EUK.

storey	method	u		Δ		∇		V_s		V_d		V_t	
		mean	std	mean	std	mean	std	mean	std	mean	std	mean	std
ESB.05D5.QRS.ALL.ALL.HB - $\mu \leq 3.0$													
all	RSA	-0.01	-0.03	-0.04	-0.05	-0.18	-0.12	0.00	-0.09	-0.14	-0.04	-0.04	-0.12
	RSAB	-0.01	-0.02	-0.03	-0.03	-0.19	-0.07	0.03	-0.05	-0.15	-0.02	-0.03	-0.08
	RSABS	-0.01	-0.02	-0.03	-0.03	-0.19	-0.08	0.03	-0.05	-0.15	-0.03	-0.03	-0.09
first	RSA	-0.03	-0.08	-0.03	-0.08	-0.20	-0.09	0.04	-0.07	-0.13	-0.01	-0.03	-0.04
	RSAB	-0.04	-0.07	-0.04	-0.07	-0.18	-0.06	0.03	-0.02	-0.14	0.00	-0.03	0.00
	RSABS	-0.04	-0.07	-0.04	-0.07	-0.18	-0.07	0.03	-0.02	-0.13	0.00	-0.02	0.00
roof	RSA	-0.01	-0.01	-0.10	-0.07	-0.20	-0.22	-0.14	-0.19	-0.15	-0.08	-0.22	-0.23
	RSAB	0.01	-0.01	-0.08	-0.06	-0.19	-0.16	-0.05	-0.10	-0.18	-0.09	-0.16	-0.17
	RSABS	0.01	-0.01	-0.09	-0.04	-0.21	-0.17	-0.06	-0.08	-0.19	-0.10	-0.17	-0.16
ESB.05D5.QRS.ALL.ALL.HB - $\mu \leq 1.05$													
all	RSA	-0.04	-0.02	-0.06	-0.02	-0.17	-0.08	-0.06	-0.03	-0.15	-0.01	-0.01	0.05
	RSAB	-0.04	-0.01	-0.04	-0.01	-0.16	0.01	-0.04	-0.01	-0.15	0.03	-0.03	0.05
	RSABS	-0.04	-0.02	-0.04	0.00	-0.15	0.01	-0.05	0.00	-0.14	0.02	-0.02	0.05
first	RSA	-0.07	-0.04	-0.07	-0.04	-0.22	-0.08	0.00	-0.04	-0.14	0.03	-0.04	-0.05
	RSAB	-0.09	-0.03	-0.09	-0.03	-0.22	-0.03	-0.05	-0.03	-0.18	0.01	-0.06	-0.01
	RSABS	-0.08	-0.04	-0.08	-0.04	-0.21	-0.03	-0.03	-0.01	-0.17	-0.01	-0.04	0.00
roof	RSA	-0.03	0.00	-0.10	0.00	-0.15	-0.10	-0.16	-0.08	-0.15	-0.04	0.08	0.06
	RSAB	-0.01	-0.02	-0.04	0.01	-0.07	-0.04	-0.07	0.00	-0.10	0.00	0.06	0.02
	RSABS	-0.01	-0.01	-0.04	0.01	-0.07	-0.05	-0.11	0.05	-0.11	-0.03	0.07	0.04
ESB.05D5.QRS.ALL.ALL.HB - $1.05 < \mu \leq 1.5$													
all	RSA	-0.07	-0.03	-0.09	-0.04	-0.22	-0.07	0.00	-0.08	-0.16	0.00	-0.09	-0.06
	RSAB	-0.13	-0.06	-0.14	-0.04	-0.19	-0.01	0.05	-0.01	-0.17	0.00	-0.03	-0.04
	RSABS	-0.12	-0.06	-0.13	-0.03	-0.19	-0.03	0.05	-0.01	-0.17	-0.01	-0.02	-0.05
first	RSA	-0.08	-0.06	-0.08	-0.06	-0.24	-0.07	0.00	-0.02	-0.14	0.00	-0.11	-0.03
	RSAB	-0.13	-0.08	-0.13	-0.08	-0.20	-0.05	0.00	0.00	-0.17	0.01	-0.06	0.02
	RSABS	-0.13	-0.08	-0.13	-0.08	-0.20	-0.05	0.01	0.02	-0.17	0.01	-0.05	0.01
roof	RSA	-0.07	-0.02	-0.14	-0.04	-0.23	-0.13	-0.10	-0.18	-0.18	-0.03	-0.22	-0.01
	RSAB	-0.12	-0.07	-0.16	-0.01	-0.17	-0.06	0.02	-0.02	-0.17	-0.06	-0.13	-0.05
	RSABS	-0.12	-0.07	-0.16	-0.01	-0.17	-0.10	0.02	0.00	-0.17	-0.10	-0.12	-0.06
ESB.05D5.QRS.ALL.ALL.HB - $1.5 < \mu \leq 3$													
all	RSA	0.05	-0.01	-0.03	-0.10	-0.25	-0.17	-0.06	-0.16	-0.17	-0.07	-0.17	-0.25
	RSAB	0.07	0.00	-0.01	-0.08	-0.24	-0.13	0.00	-0.12	-0.19	-0.06	-0.10	-0.19
	RSABS	0.06	0.01	-0.01	-0.08	-0.25	-0.14	0.00	-0.12	-0.19	-0.07	-0.11	-0.20
first	RSA	0.07	-0.01	0.07	-0.01	-0.16	-0.10	-0.08	-0.06	-0.10	-0.04	-0.08	-0.03
	RSAB	0.07	0.01	0.07	0.01	-0.12	-0.07	-0.01	-0.05	-0.10	-0.02	-0.03	-0.02
	RSABS	0.07	0.02	0.07	0.02	-0.12	-0.07	-0.03	-0.05	-0.10	-0.02	-0.03	-0.01
roof	RSA	0.02	0.02	-0.23	-0.06	-0.38	-0.29	-0.30	-0.25	-0.25	-0.10	-0.69	-0.01
	RSAB	0.05	0.03	-0.20	-0.02	-0.39	-0.13	-0.18	-0.13	-0.30	-0.05	-0.52	0.02
	RSABS	0.05	0.03	-0.20	-0.02	-0.42	-0.14	-0.19	-0.15	-0.33	-0.08	-0.55	0.02

Shadowed cells represent values > 0 (ESB values larger than EUK values).

Table 7-8. Summary of acceptability for EAR_1 with exact spectral values for EUK.

		u		Δ		∇		V_s		V_d		V_t	
		$\epsilon_{u,1}$	$\epsilon_{a,1}$	$\epsilon_{u,1}$	$\epsilon_{a,1}$	$\epsilon_{u,1}$	$\epsilon_{a,1}$	$\epsilon_{u,1}$	$\epsilon_{a,1}$	$\epsilon_{u,1}$	$\epsilon_{a,1}$	$\epsilon_{u,1}$	$\epsilon_{a,1}$
storey	method	EUK.05D5.QRS.ALL.ALL.HB - $\mu \leq 3.0$											
all	RSA	0.53	0.47	0.35	0.56	0.59	0.32	0.24	0.59	0.49	0.46	0.17	0.51
	RSAB	0.44	0.55	0.28	0.57	0.49	0.41	0.26	0.65	0.20	0.64	0.13	0.53
	RSABS	0.46	0.53	0.29	0.55	0.44	0.44	0.26	0.64	0.20	0.62	0.10	0.55
first	RSA	0.70	0.30	0.70	0.30	0.36	0.61	0.14	0.45	0.10	0.89	0.32	0.63
	RSAB	0.72	0.28	0.72	0.28	0.62	0.38	0.22	0.70	0.27	0.64	0.35	0.62
	RSABS	0.74	0.26	0.74	0.26	0.55	0.45	0.19	0.68	0.27	0.61	0.21	0.76
roof	RSA	0.24	0.76	0.08	0.52	0.52	0.23	0.17	0.44	0.48	0.35	0.00	0.01
	RSAB	0.12	0.85	0.04	0.46	0.11	0.53	0.03	0.60	0.04	0.49	0.00	0.01
	RSABS	0.13	0.84	0.03	0.41	0.11	0.45	0.03	0.62	0.06	0.42	0.00	0.00
EUK.05D5.QRS.ALL.ALL.HB - $\mu \leq 1.05$													
all	RSA	0.43	0.57	0.39	0.61	0.77	0.23	0.53	0.44	0.69	0.31	0.45	0.35
	RSAB	0.40	0.60	0.33	0.67	0.65	0.34	0.45	0.52	0.26	0.71	0.20	0.55
	RSABS	0.43	0.57	0.33	0.63	0.59	0.40	0.44	0.55	0.28	0.66	0.17	0.55
first	RSA	0.43	0.57	0.43	0.57	0.28	0.72	0.40	0.60	0.00	0.99	0.79	0.21
	RSAB	0.52	0.48	0.52	0.48	0.48	0.52	0.62	0.38	0.13	0.86	0.48	0.52
	RSABS	0.57	0.43	0.57	0.43	0.35	0.65	0.52	0.48	0.19	0.80	0.37	0.63
roof	RSA	0.39	0.61	0.21	0.77	0.86	0.12	0.32	0.53	0.85	0.15	0.00	0.02
	RSAB	0.31	0.69	0.11	0.87	0.27	0.70	0.09	0.78	0.11	0.74	0.00	0.02
	RSABS	0.33	0.67	0.10	0.70	0.29	0.65	0.09	0.83	0.16	0.56	0.00	0.00
EUK.05D5.QRS.ALL.ALL.HB - $1.05 < \mu \leq 1.5$													
all	RSA	0.34	0.66	0.22	0.71	0.59	0.35	0.07	0.81	0.45	0.55	0.04	0.62
	RSAB	0.27	0.69	0.16	0.66	0.44	0.53	0.04	0.92	0.14	0.76	0.05	0.62
	RSABS	0.28	0.68	0.19	0.63	0.37	0.57	0.04	0.92	0.12	0.76	0.00	0.65
first	RSA	0.56	0.44	0.56	0.44	0.21	0.72	0.00	0.80	0.05	0.95	0.16	0.84
	RSAB	0.56	0.44	0.56	0.44	0.54	0.46	0.00	1.00	0.24	0.64	0.25	0.75
	RSABS	0.56	0.44	0.56	0.44	0.47	0.53	0.01	0.99	0.21	0.60	0.02	0.98
roof	RSA	0.14	0.86	0.01	0.62	0.51	0.28	0.14	0.48	0.42	0.56	0.00	0.00
	RSAB	0.04	0.87	0.00	0.41	0.05	0.80	0.00	0.80	0.01	0.68	0.00	0.00
	RSABS	0.05	0.86	0.00	0.40	0.06	0.66	0.00	0.80	0.02	0.64	0.00	0.00
EUK.05D5.QRS.ALL.ALL.HB - $1.5 < \mu \leq 3$													
all	RSA	0.74	0.26	0.40	0.42	0.44	0.38	0.09	0.57	0.33	0.54	0.00	0.57
	RSAB	0.58	0.42	0.32	0.43	0.39	0.40	0.22	0.60	0.19	0.50	0.13	0.46
	RSABS	0.59	0.41	0.32	0.42	0.35	0.40	0.23	0.57	0.17	0.51	0.10	0.48
first	RSA	1.00	0.00	1.00	0.00	0.51	0.46	0.00	0.10	0.21	0.77	0.02	0.86
	RSAB	1.00	0.00	1.00	0.00	0.79	0.21	0.00	0.80	0.40	0.45	0.30	0.62
	RSABS	1.00	0.00	1.00	0.00	0.77	0.23	0.00	0.66	0.37	0.47	0.18	0.73
roof	RSA	0.18	0.82	0.00	0.24	0.23	0.29	0.06	0.33	0.19	0.40	0.00	0.00
	RSAB	0.01	0.99	0.00	0.14	0.00	0.23	0.00	0.32	0.00	0.15	0.00	0.00
	RSABS	0.01	0.99	0.00	0.16	0.00	0.16	0.00	0.33	0.00	0.16	0.00	0.00

Cells shadowed in green indicate a proportion of unacceptably un-conservative results smaller than 34%.

Cells shadowed in blue indicate a proportion of results in the acceptability range larger than 67%.

Table 7-9. Summary of acceptability for EAR_2 with exact spectral values for EUK.

		u				Δ		∇		V _s		V _d		V _t	
		ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}
storey	method	EUK.05D5.QRS.ALL.ALL.HB - μ ≤ 3.0													
all	RSA	0.21	0.79	0.16	0.80	0.29	0.66	0.09	0.83	0.14	0.84	0.04	0.71		
	RSAB	0.19	0.81	0.14	0.81	0.12	0.83	0.07	0.90	0.03	0.93	0.01	0.76		
	RSABS	0.22	0.78	0.16	0.78	0.07	0.86	0.06	0.91	0.02	0.91	0.01	0.74		
first	RSA	0.49	0.51	0.49	0.51	0.10	0.90	0.06	0.80	0.01	0.99	0.12	0.88		
	RSAB	0.55	0.45	0.55	0.45	0.21	0.79	0.11	0.89	0.08	0.92	0.05	0.95		
	RSABS	0.57	0.43	0.57	0.43	0.16	0.84	0.07	0.92	0.06	0.94	0.03	0.97		
roof	RSA	0.05	0.95	0.05	0.76	0.34	0.50	0.06	0.66	0.24	0.70	0.00	0.11		
	RSAB	0.03	0.97	0.01	0.70	0.03	0.78	0.03	0.82	0.00	0.83	0.00	0.05		
	RSABS	0.03	0.97	0.00	0.67	0.00	0.76	0.03	0.82	0.00	0.71	0.00	0.03		
EUK.05D5.QRS.ALL.ALL.HB - μ ≤ 1.05															
all	RSA	0.18	0.82	0.20	0.80	0.48	0.52	0.24	0.74	0.29	0.71	0.13	0.73		
	RSAB	0.17	0.83	0.12	0.88	0.16	0.84	0.19	0.81	0.00	1.00	0.03	0.79		
	RSABS	0.23	0.77	0.16	0.84	0.07	0.93	0.17	0.83	0.00	0.97	0.02	0.78		
first	RSA	0.20	0.80	0.20	0.80	0.10	0.90	0.18	0.82	0.00	1.00	0.34	0.66		
	RSAB	0.30	0.70	0.30	0.70	0.01	0.99	0.31	0.69	0.00	1.00	0.13	0.87		
	RSABS	0.34	0.66	0.34	0.66	0.00	1.00	0.20	0.80	0.01	0.99	0.09	0.91		
roof	RSA	0.16	0.84	0.15	0.85	0.60	0.40	0.16	0.77	0.48	0.52	0.00	0.30		
	RSAB	0.08	0.92	0.03	0.97	0.08	0.92	0.07	0.92	0.00	1.00	0.00	0.11		
	RSABS	0.08	0.92	0.00	0.98	0.00	0.98	0.07	0.91	0.00	0.88	0.00	0.06		
EUK.05D5.QRS.ALL.ALL.HB - 1.05 < μ ≤ 1.5															
all	RSA	0.07	0.93	0.05	0.94	0.17	0.82	0.00	0.96	0.09	0.91	0.00	0.79		
	RSAB	0.08	0.92	0.06	0.92	0.02	0.96	0.00	0.99	0.00	0.97	0.00	0.78		
	RSABS	0.10	0.90	0.08	0.88	0.01	0.97	0.00	1.00	0.00	0.96	0.00	0.76		
first	RSA	0.22	0.78	0.22	0.78	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00		
	RSAB	0.28	0.72	0.28	0.72	0.11	0.89	0.00	1.00	0.01	0.99	0.00	1.00		
	RSABS	0.33	0.67	0.33	0.67	0.06	0.94	0.00	1.00	0.00	1.00	0.00	1.00		
roof	RSA	0.00	1.00	0.00	0.93	0.31	0.61	0.00	0.78	0.19	0.81	0.00	0.02		
	RSAB	0.00	1.00	0.00	0.87	0.00	0.93	0.00	0.95	0.00	0.88	0.00	0.05		
	RSABS	0.00	1.00	0.00	0.78	0.00	0.91	0.00	0.98	0.00	0.81	0.00	0.05		
EUK.05D5.QRS.ALL.ALL.HB - 1.5 < μ ≤ 3															
all	RSA	0.33	0.67	0.21	0.71	0.21	0.70	0.00	0.83	0.05	0.92	0.00	0.65		
	RSAB	0.28	0.72	0.19	0.68	0.15	0.73	0.02	0.91	0.06	0.84	0.00	0.72		
	RSABS	0.30	0.70	0.20	0.66	0.11	0.74	0.01	0.92	0.04	0.82	0.00	0.69		
first	RSA	0.91	0.09	0.91	0.09	0.16	0.84	0.00	0.66	0.01	0.99	0.00	1.00		
	RSAB	0.91	0.09	0.91	0.09	0.45	0.55	0.00	0.99	0.19	0.81	0.00	1.00		
	RSABS	0.91	0.09	0.91	0.09	0.37	0.63	0.00	0.98	0.15	0.85	0.00	1.00		
roof	RSA	0.00	1.00	0.00	0.57	0.14	0.52	0.00	0.49	0.07	0.78	0.00	0.00		
	RSAB	0.00	1.00	0.00	0.37	0.00	0.57	0.00	0.66	0.00	0.66	0.00	0.00		
	RSABS	0.00	1.00	0.00	0.33	0.00	0.47	0.00	0.65	0.00	0.51	0.00	0.00		

Cells shadowed in green indicate a proportion of unacceptably un-conservative results smaller than 34%.

Cells shadowed in blue indicate a proportion of results in the acceptability range larger than 67%.

Table 7-10. Summary of acceptability for EAR_1 with exact spectral values for ESB.

		u		Δ		∇		V _s		V _d		V _t	
		ε _{u,1}	ε _{a,1}	ε _{u,1}	ε _{a,1}	ε _{u,1}	ε _{a,1}	ε _{u,1}	ε _{a,1}	ε _{u,1}	ε _{a,1}	ε _{u,1}	ε _{a,1}
storey	method	ESB.05D5.QRS.ALL.ALL.HB - μ ≤ 3.0											
all	RSA	0.61	0.39	0.40	0.53	0.86	0.13	0.18	0.65	0.74	0.25	0.17	0.49
	RSAB	0.49	0.50	0.33	0.55	0.78	0.20	0.17	0.75	0.46	0.51	0.16	0.49
	RSABS	0.50	0.49	0.34	0.53	0.75	0.22	0.18	0.74	0.43	0.52	0.12	0.50
first	RSA	0.89	0.11	0.89	0.11	0.99	0.01	0.09	0.40	0.36	0.64	0.36	0.63
	RSAB	0.96	0.04	0.96	0.04	1.00	0.00	0.19	0.76	0.56	0.44	0.39	0.60
	RSABS	0.94	0.06	0.94	0.06	1.00	0.00	0.17	0.71	0.55	0.45	0.34	0.64
roof	RSA	0.29	0.71	0.13	0.59	0.59	0.34	0.19	0.49	0.61	0.37	0.00	0.01
	RSAB	0.14	0.84	0.07	0.51	0.20	0.72	0.09	0.58	0.12	0.79	0.00	0.01
	RSABS	0.15	0.83	0.04	0.47	0.15	0.71	0.09	0.63	0.10	0.69	0.00	0.01
ESB.05D5.QRS.ALL.ALL.HB - μ ≤ 1.05													
all	RSA	0.57	0.43	0.56	0.44	1.00	0.00	0.70	0.30	0.86	0.14	0.57	0.23
	RSAB	0.61	0.39	0.53	0.47	0.92	0.08	0.69	0.30	0.70	0.30	0.32	0.43
	RSABS	0.61	0.39	0.50	0.47	0.85	0.15	0.69	0.30	0.65	0.32	0.26	0.47
first	RSA	0.65	0.35	0.65	0.35	1.00	0.00	0.42	0.58	0.32	0.68	0.97	0.03
	RSAB	0.87	0.13	0.87	0.13	1.00	0.00	0.84	0.16	0.68	0.32	0.68	0.32
	RSABS	0.84	0.16	0.84	0.16	1.00	0.00	0.71	0.29	0.68	0.32	0.52	0.48
roof	RSA	0.52	0.48	0.52	0.48	1.00	0.00	0.61	0.35	1.00	0.00	0.00	0.06
	RSAB	0.42	0.58	0.32	0.68	0.61	0.39	0.39	0.58	0.42	0.58	0.00	0.03
	RSABS	0.42	0.58	0.19	0.68	0.35	0.65	0.42	0.55	0.26	0.55	0.00	0.03
ESB.05D5.QRS.ALL.ALL.HB - 1.05 < μ ≤ 1.5													
all	RSA	0.56	0.44	0.38	0.62	0.92	0.08	0.06	0.86	0.83	0.17	0.15	0.55
	RSAB	0.49	0.51	0.31	0.62	0.85	0.15	0.00	0.94	0.46	0.52	0.12	0.52
	RSABS	0.52	0.48	0.34	0.58	0.84	0.14	0.02	0.93	0.44	0.53	0.08	0.55
first	RSA	0.88	0.12	0.88	0.12	0.94	0.06	0.00	0.91	0.38	0.62	0.41	0.59
	RSAB	0.97	0.03	0.97	0.03	1.00	0.00	0.00	1.00	0.53	0.47	0.38	0.62
	RSABS	0.94	0.06	0.94	0.06	1.00	0.00	0.06	0.94	0.53	0.47	0.32	0.68
roof	RSA	0.26	0.74	0.06	0.91	0.71	0.29	0.12	0.59	0.76	0.24	0.00	0.00
	RSAB	0.09	0.91	0.00	0.68	0.24	0.76	0.00	0.71	0.09	0.82	0.00	0.00
	RSABS	0.15	0.85	0.00	0.59	0.24	0.68	0.00	0.74	0.12	0.74	0.00	0.00
ESB.05D5.QRS.ALL.ALL.HB - 1.5 < μ ≤ 3													
all	RSA	0.65	0.35	0.35	0.53	0.77	0.20	0.02	0.69	0.65	0.34	0.02	0.58
	RSAB	0.45	0.55	0.26	0.55	0.70	0.27	0.03	0.86	0.36	0.60	0.10	0.50
	RSABS	0.45	0.54	0.27	0.53	0.67	0.29	0.03	0.84	0.33	0.60	0.09	0.50
first	RSA	1.00	0.00	1.00	0.00	1.00	0.00	0.00	0.09	0.36	0.64	0.09	0.89
	RSAB	0.99	0.01	0.99	0.01	1.00	0.00	0.00	0.91	0.53	0.47	0.28	0.71
	RSABS	0.99	0.01	0.99	0.01	1.00	0.00	0.00	0.77	0.51	0.49	0.28	0.68
roof	RSA	0.21	0.79	0.00	0.49	0.37	0.49	0.05	0.51	0.37	0.59	0.00	0.00
	RSAB	0.04	0.92	0.00	0.36	0.01	0.84	0.00	0.52	0.01	0.85	0.00	0.00
	RSABS	0.04	0.92	0.00	0.33	0.03	0.76	0.00	0.61	0.03	0.72	0.00	0.00

Cells shadowed in green indicate a proportion of unacceptably un-conservative results smaller than 34%.

Cells shadowed in blue indicate a proportion of results in the acceptability range larger than 67%.

Table 7-11. Summary of acceptability for EAR_2 with exact spectral values for ESB.

		u		Δ		∇		V _s		V _d		V _t	
		ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}	ε _{u,2}	ε _{a,2}
storey	method	ESB.05D5.QRS.ALL.ALL.HB - m ≤ 1.05											
all	RSA	0.20	0.80	0.16	0.82	0.62	0.38	0.07	0.88	0.41	0.59	0.04	0.73
	RSAB	0.16	0.84	0.12	0.85	0.56	0.44	0.06	0.94	0.25	0.75	0.01	0.75
	RSABS	0.19	0.81	0.14	0.81	0.51	0.49	0.05	0.94	0.21	0.78	0.01	0.74
first	RSA	0.49	0.51	0.49	0.51	0.39	0.61	0.03	0.91	0.02	0.98	0.09	0.91
	RSAB	0.56	0.44	0.56	0.44	0.82	0.18	0.09	0.91	0.33	0.67	0.05	0.95
	RSABS	0.56	0.44	0.56	0.44	0.74	0.26	0.04	0.96	0.28	0.72	0.02	0.98
roof	RSA	0.08	0.92	0.06	0.84	0.42	0.58	0.08	0.75	0.39	0.61	0.00	0.09
	RSAB	0.00	1.00	0.01	0.83	0.01	0.99	0.03	0.96	0.00	0.99	0.00	0.04
	RSABS	0.00	1.00	0.00	0.75	0.00	0.98	0.03	0.95	0.01	0.96	0.00	0.04
ESB.05D5.QRS.ALL.ALL.HB - μ ≤ 1.05													
all	RSA	0.31	0.69	0.32	0.68	0.86	0.14	0.33	0.67	0.74	0.26	0.17	0.69
	RSAB	0.17	0.83	0.13	0.87	0.70	0.30	0.28	0.72	0.30	0.70	0.06	0.77
	RSABS	0.24	0.76	0.19	0.81	0.58	0.42	0.24	0.76	0.28	0.72	0.03	0.78
first	RSA	0.26	0.74	0.26	0.74	0.45	0.55	0.13	0.87	0.00	1.00	0.39	0.61
	RSAB	0.48	0.52	0.48	0.52	0.77	0.23	0.42	0.58	0.26	0.74	0.19	0.81
	RSABS	0.48	0.52	0.48	0.52	0.58	0.42	0.19	0.81	0.23	0.77	0.10	0.90
roof	RSA	0.35	0.65	0.29	0.71	0.87	0.13	0.32	0.68	0.87	0.13	0.00	0.29
	RSAB	0.00	1.00	0.06	0.94	0.06	0.94	0.13	0.87	0.00	1.00	0.00	0.13
	RSABS	0.00	1.00	0.00	0.97	0.00	1.00	0.13	0.87	0.03	0.97	0.00	0.10
ESB.05D5.QRS.ALL.ALL.HB - 1.05 < μ ≤ 1.5													
all	RSA	0.11	0.89	0.06	0.94	0.68	0.32	0.01	0.99	0.42	0.58	0.01	0.81
	RSAB	0.13	0.87	0.10	0.90	0.56	0.44	0.00	1.00	0.26	0.74	0.00	0.78
	RSABS	0.18	0.82	0.12	0.86	0.52	0.48	0.00	1.00	0.24	0.76	0.00	0.77
first	RSA	0.29	0.71	0.29	0.71	0.32	0.68	0.00	1.00	0.00	1.00	0.03	0.97
	RSAB	0.50	0.50	0.50	0.50	0.76	0.24	0.00	1.00	0.29	0.71	0.00	1.00
	RSABS	0.53	0.47	0.53	0.47	0.68	0.32	0.00	1.00	0.26	0.74	0.00	1.00
roof	RSA	0.00	1.00	0.00	1.00	0.56	0.44	0.03	0.94	0.41	0.59	0.00	0.09
	RSAB	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	1.00	0.00	0.06
	RSABS	0.00	1.00	0.00	0.91	0.00	1.00	0.00	1.00	0.00	0.97	0.00	0.06
ESB.05D5.QRS.ALL.ALL.HB - 1.5 < μ ≤ 3													
all	RSA	0.20	0.80	0.14	0.82	0.49	0.51	0.00	0.91	0.27	0.73	0.00	0.70
	RSAB	0.17	0.83	0.13	0.82	0.49	0.51	0.00	0.99	0.22	0.78	0.00	0.73
	RSABS	0.18	0.82	0.13	0.78	0.47	0.52	0.00	0.99	0.18	0.81	0.00	0.71
first	RSA	0.68	0.32	0.68	0.32	0.39	0.61	0.00	0.88	0.04	0.96	0.00	1.00
	RSAB	0.61	0.39	0.61	0.39	0.87	0.13	0.00	1.00	0.37	0.63	0.01	0.99
	RSABS	0.61	0.39	0.61	0.39	0.83	0.17	0.00	1.00	0.31	0.69	0.00	1.00
roof	RSA	0.00	1.00	0.00	0.83	0.17	0.83	0.00	0.69	0.17	0.83	0.00	0.00
	RSAB	0.00	1.00	0.00	0.71	0.00	1.00	0.00	0.97	0.00	0.99	0.00	0.00
	RSABS	0.00	1.00	0.00	0.59	0.00	0.96	0.00	0.96	0.00	0.95	0.00	0.00

Cells shadowed in green indicate a proportion of unacceptably un-conservative results smaller than 34%.

Cells shadowed in blue indicate a proportion of results in the acceptability range larger than 67%.

7.3.3 Results with code spectrum

In this Section, results of the simplified methods are obtained using the Eurocode 8 spectral response (described analytically in Section 2.5.1.1). Spectral correction factors are used to improve the relative displacement and velocity response. The TH results are obtained only from suites of accelerograms (ES) pre-scaled to represent the elastic Eurocode 8 spectrum. The logMedian is adopted as central value, except otherwise indicated. The results are presented segregated by ductility levels: $\mu_T \leq 3$ (elastic), $\mu_T \leq 1.05$ (elastic), $1.05 < \mu_T \leq 1.5$ (moderate ductility), $1.5 < \mu_T \leq 3$ (high ductility).

Table 7-12. Initial options considered for results with soft spectrum.

	Method 1 (RSA)	Method 2 (RSAB)	Method 3 (RSABS)
Spectral solution	EC8 elastic spectrum	EC8 elastic spectrum	EC8 elastic spectrum
Hysteretic damping	ASCE	Eqns. (2.5-69) ff.	Eqns. (2.5-69) ff.
Combination Rule	CQC	CQC	CQC
Method for Damper Shear	Modal	Modal	Modal
Method for Total Shear	Force Coefficients	Force Coefficients	Force Coefficients
Minimum elastic displacement	No	No	No

Systems included: MRF 05D with 2% and 5% inherent damping, complete vertical damping systems (ADS Q, R, S), all added damping ratio levels, bracing stiffness type HB. 5% inherent damping is a high value for steel frames, for which a more usual assumption is 2%. It was shown in the previous section that this parameter does not influence the error results. The ADS are the same selected for the previous Section.

Intensity levels: For every system, only two levels are considered: PHA_D (design level) and PHA_E (elastic level). For these excitation intensities, the systems behave mostly elastically or with moderate ductility demand. This situation is typical of frames with added viscous dampers. The case of damped frames with large ductility demands is also examined independently.

Spectral adjustment: Sets of pre-scaled motions (ESA, ESB, ESC, ESD) are used to simulate the Eurocode 8 spectrum. Pre-scaling of the motions was performed to approximate the elastic spectrum in a certain period interval; therefore, for a specific period the adjustment between the average spectral value of the set and the Eurocode spectrum (both scaled at the same PHA) cannot be expected to be perfect. This can be taken into account through the following a spectral adjustment factor λ_{SA} :

$$\lambda_{SA}(T_x) = \frac{\bar{S}_d(T_x, 5\%)}{S_{d,EC8}(T_x, 5\%)}, \quad (7.3-1)$$

where $\bar{S}_d(T_x, 5\%)$ is the average of spectral ordinates of the set, and $S_{d,EC8}$ is the spectral ordinate of the Eurocode 8 spectrum. This spectral adjustment is period-dependent. When modal response for mode j is found, the spectral adjustment factor should be applied to

every period, using $PHA \cdot \lambda_{SA}(T_{eff,j})$ as ground acceleration. Hereby the results of the study are presented with and without this spectral adjustment.

7.3.3.1 Results with code spectrum. Influence of damping correction factors

Two different DCFs for displacement (η_d) are examined: those proposed by ASCE SEI 7/16 (referred to as ‘ASCE’) and those proposed in Chapter 4 (referred to as ‘PROPOSED’). Only η_d has been considered, i.e., no correction factors for velocity (CFVs) have been applied. η_d is applied to the three methods, so the results obtained can be directly compared.

Comparisons are shown in Figure 7-37 (method statistics), Figure 7-38 (acceptability for EAR_1) and Figure 7-39 (acceptability for EAR_2) with no spectral adjustment. The corresponding results with spectral adjustment are shown in Figure 7-40 (method statistics) Figure 7-41 (acceptability for EAR_1) Figure 7-42 (acceptability for EAR_2). These results indicate that η_d presented in this study produce only a marginal improvement in variable prediction, on the order of 5%. The improvement is larger for the RSA method than for the RSAB and RSABS methods. The results of RSAB and RSABS present a slightly larger acceptability than those of RSA, but the difference is very small, except for V_d , where a larger acceptability is observed with η_d from this study. The improvement is smaller if spectral adjustment is used. Therefore, it can be concluded that there is no significant difference between the η_d proposed in this study and the one proposed in ASCE.

7.3.3.2 Results with code spectrum. Influence of correction factors for velocity

The Correction Factors for Velocity derived in Chapter 4 are considered. The CFVs are applied consistently in conjunction with η_d derived in Chapter. The results are compared to those obtained without application of the CFV; in this last case, to rule out the small variability due to η_d , this factor has been applied to both sets.

Comparisons without spectral adjustment are shown in Figure 7-37 (method statistics), Figure 7-38 (acceptability for EAR_1) and Figure 7-39 (acceptability for EAR_2). Corresponding results with spectral adjustment are shown in Figure 7-40, Figure 7-41 and Figure 7-42. These results indicate that the CFVs presented in this study improve the method predictions. This is true with or without spectral adjustment. The results of RSAB and RSABS with the proposed CFVs present a much larger acceptability than those calculated without them. Because velocity is a relevant variable for the Maxwell system behavior, the response is improved in all variables. In fact, by combined use of the RSAB (or RSABS) methods with the spectral correction factors derived in this study (CFV and η_d), the error results for all variables (except V_t) fall within acceptability interval EAR_1 . The results are virtually identical for RSAB and RSABS, suggesting that this last method, which is less computationally demanding, can be used as a codified verification method.

7.3.3.3 Results with code spectrum. Improvement of damper shear prediction.

Improvement of damper shear prediction can be achieved by use of the procedure proposed in Section 6.2.2.5, where the damper forces are derived from kinematic peak values (drift and drift velocity) using the Maxwell parameters for every storey. The method is applied to both RSAB and RSABS methods; this solution, referred to as ‘peak’, is compared with the modal solution, referred to as ‘modal’. The results, shown in Figure 7-43 (without spectral adjustment) and Figure 7-44 (with spectral adjustment), indicate improvement of damper force prediction when the forces are calculated using peak kinematic values instead of using modal combination.

7.3.3.4 Results with code spectrum. Improvement of total shear prediction.

Different methods were discussed in Section 6.2.2.4 to obtain the maximum total shear: ‘force coefficients’ (as proposed in ASCE SEI 7/16), ‘Bézier’, ‘Kasai’ and ‘Lin-Chopra’. The simple Bézier method is left out because it was shown in Chapter 5 to render similar results to the ‘Kasai’ and ‘Lin-Chopra’ methods, with a larger computational effort.

The comparison of results is offered in Figure 7-45, showing that the Bézier method is superior to all others. If simplicity is considered, both the ‘Kasai’ and ‘Lin-Chopra’ methods are of much simpler application the force coefficient method currently proposed in ASCE-SEI 7/16, and render more accurate results. The Bézier method is adopted hereinafter as reference method. It is of interest to note that all methods considered are conservative.

7.3.3.5 Results with code spectrum. Options adopted.

In the following sections, the influence of different parameters is examined. The ‘best’ method options adopted for comparison are listed in Table 7-13. The plots for set ESB.05D2.ALL.ALL.ALL with these options are given in Figure 7-47, including spectral adjustment. The plots show how a very reasonable level of prediction can be obtained using any of the three methods, except for variable V_t , where results are too conservative. The best acceptability considering all variables is achieved with RSAB method.

Table 7-13. Best method options.

	Method 1 (RSA)	Method 2 (RSAB)	Method 3 (RSABS)
Spectral solution	EC8 elastic spectrum*	EC8 elastic spectrum*	EC8 elastic spectrum*
Spectral Correction Factors	This study	This study	This study
Hysteretic damping	ASCE	Eqns. (2.5-69) ff.	Eqns. (2.5-69) ff.
Combination Rule	CQC	CQC	CQC
Method for Damper Shear	Modal	Peak	Peak
Method for Total Shear	Kasai	Bézier	Bézier
Minimum elastic displacement	No	No	No

* spectral ordinates are considered with and without spectral adjustment

7.3.3.6 Results with code spectrum. Influence of brace stiffness.

Analysis results are binned according to the type of bracing stiffness model, as explained in Section 7.3.2.2, HB and LB. A summary is presented in Figure 7-48 (without spectral adjustment) and Figure 7-49 (with spectral adjustment). The acceptability increases for the LB systems and variables u , Δ , ∇ and V_t , but reduces for V_s and V_d , although in all cases the results are well within the EAR_1 range. With spectral adjustment, the results are similar but less conspicuous.

7.3.3.7 Results with code spectrum. Influence of damper exponent.

Results are binned according to the damper exponent, linear ($a = 1$) or non-linear ($a = 0.5$). A summary is presented in Figure 7-50 (without spectral adjustment) and Figure 7-51 (with spectral adjustment). The methods present a larger acceptability for linear dampers and variables u , Δ , ∇ and V_s ; for variables V_d and V_t , the acceptability is larger for non-linear dampers. These results are consistent with those obtained for exact spectral response, Section 7.3.2.3. However, in this case, all results obtained with RSAB and RSABS are within the acceptability limits for EAR_1 , except V_t for linear dampers, where the results are out of range, albeit on the over-conservative side. Again, if spectral adjustment is considered, similar conclusions are obtained.

7.3.3.8 Results with code spectrum. Influence of damping level.

Results are binned according to damping level, low (level 1) or high (level 3). A summary is presented in Figure 7-52 (without spectral adjustment) and Figure 7-53 (with spectral adjustment). In general, the methods present a larger acceptability for low damping in all variables except u , whose acceptability is much larger for high damping, but only when no spectral adjustment is considered. All results obtained with RSAB and RSABS are within the acceptability limits for EAR_1 , except u for low damping if no spectral adjustment is done, and V_t for high damping with or without spectral adjustment, with results are out of range albeit on the over-conservative side.

7.3.3.9 Results with code spectrum. Influence of site class.

Results for frames type D2 with ADS type Q are binned by site class. A summary without spectral adjustment is presented in Figure 7-54 (method statistics) and Figure 7-55 (method acceptability); corresponding figures with spectral adjustment are given in Figure 7-56 and Figure 7-57. These figures can be compared with those in Section F.1.5 (Annex F). In that case (exact spectral response) the input value of spectral displacement was calculated accurately, and results were unaffected by site class. In this Section, the spectral displacement is taken from the Eurocode 8 spectrum, and the GMS is pre-scaled to approximate this response, therefore a difference exists between both values, which reflects on a larger variability of results with site class. Additional variability is introduced by the unequal accuracy of the spectral correction factors. This variability is, therefore, not due to the method but to the motion selection. The variability is partially reduced when spectral adjustment is performed.

7.3.3.10 Results with code spectrum. Relationship with TH dispersion.

Figure 7-58 to Figure 7-81 display an overview of comparative results for MRF 05D2 with different ADS (Q, R, S) for GMS ESB, at the two intensity levels studied (PHA_E , PHA_D) and for brace stiffness LB. Figure 7-58 to Figure 7-69 are traced without spectral adjustment; the corresponding plots considering spectral adjustment are given in Figure 7-70 to Figure 7-81. These plots presents the logMedian ($\hat{x}$, solid black line, circular marker), and the 16th and 84th fractiles (x^{16} , x^{84} , dashed black line) for the set, along with the RSAB (red line, square marker), RSABS (blue line, triangular marker) and RSA (green line, rhomboidal marker). For the three methods, the options adopted are indicated in Table 7-13. These options include use of η_d and CFV derived in this study.

The figures show how the three methods render good results. Generally speaking, a better approximation is systematically obtained by use of RSAB and RSABS methods, with small differences between them. The three methods tend to be conservative and their predictions are generally within the logMedian $\pm$ logStd dispersion limits. The difference between RSAB and RSA reduces for higher intensity levels; this is coincident with the conclusions derived in Section 7.3.1. Focusing on each of the variables, the following observations can be made:

Displacement:

- For linear dampers, the RSA method is especially conservative, more so for higher damping.

Drift:

- The difference between predictions is larger at the bottom half of the frame, and reduces at the top storey.
- For the Q and R ADS with non-linear dampers, predictions at the top half worsen considerably: for low intensity levels and high damping, un-conservative predictions (slightly below dispersion limits) might be obtained.

Drift velocity:

- Drift velocity predictions present more variability than drift predictions.
- The predictions are unconservative for large damping and top half of the frame. This is more so for PHA_E . In this range, RSA renders results well out of dispersion limits, but RSAB and RSABS generally stay within limits.

Storey shear:

- The predictions for PHA_E with RSAB and RSABS tend to be slightly un-conservative, but within limits. This tendency increases for high damping
- The predictions with RSA can be very conservative for the bottom half of the frame, especially first floor.

Damper shear:

- At the bottom half of the frame the predictions are conservative with all methods, but more so for the RSA method, which for some cases is out of the dispersion limits on the conservative side.
- At the top half of the frame the predictions are very conservative with RSABS and conservative with RSAB, more so for the Q distributions, less for the R distribution. For the S distribution the results are within dispersion limits.

Total shear:

- Results with RSAB and RSABS present a consistent approximation for the bottom half of the building, and are conservative (and slightly out of dispersion limits) for the top half.
- Results with RSA are more erratic and can be either conservative or un-conservative; in both cases, for high damping, the results can be out of dispersion limits.

7.3.3.11 Results with code spectrum. Relative distance between solutions

The relative distance between TH solution and method predictions is discussed in this Section. Table 7-14 lists the RMSE for GMS ESB, MRF 05D, ADS QRS with all levels, damper exponents and model type LB. Table 7-15 lists the RMSE for GMS ESB, MRF 05D, ADS QRS with results binned by damping level, damper exponent, and type of brace stiffness model. The shadowed cells indicate the method giving the minimum distance between TH result and prediction. Overall, best results are obtained for RSAB method. In particular, the drift prediction with RSAB is much more accurate than the prediction with any of the other two methods, regardless of the subset of data considered.

Table 7-15 offers valuable information about the influence of different parameters on predictions. This is made clear in Table 7-16, that shows the quotient between the left and right sides of Table 7-15. Values over 1 are highlighted in green and values below 1 are highlighted in red. A value over 1 (green) indicates that the relative distance between TH and method prediction is larger for the group at the left side in Table 7-15, that is, the prediction of the left side is worse than that of the right side. The table shows clearly how the three methods give better predictions for linear dampers, with low damping levels, and with bracing stiffness type LB. More extreme differences in predictions are observed for static variables than for kinematic variables.

7.3.3.12 Results with code spectrum. Influence of damper coefficient distribution.

The error results for GMS ESB, MRF 05D2 binned by ADS (Q, R, S, U) are shown in Figure 7-82 and Figure 7-83. Results for RMSE are given in Table 7-17. As in the analysis with exact spectral response, the errors are only mildly dependent on the damper exponent distribution, with a tendency for better results with distributions that resemble the stiffness distribution of the frame. This is expected, as the non-proportionality of these systems is smaller. A reduction in dispersion is observed from S to R and from R to Q distributions.

The smaller un-conservative values are obtained with the U distribution, the one which presents the smallest non-proportionality. Minimum errors are generally obtained for the S distribution, as shown in Table 7-17, where the minimum values across sets Q, R, S are shadowed. U-type has been left out of the comparison because it only contains linear dampers.

7.3.3.13 Results with code spectrum. 3-storey frame.

Results for GMS ESB, MRF 03D2 with ADS type Q are shown in Figure 7-86. This type of vertical distribution of dampers (constant) is usual for a 3-storey frame. Additional plots of predictions vs. TH results are given in Figure 7-85 and Figure 7-86. The results are coincident with those obtained for frame 05D, and show excellent performance of all three methods for this frame.

7.3.3.14 Results with code spectrum. 9-storey frame.

Results for GMS ESB, MRF 09D2 with linear ADS type Q and U are shown in Figure 7-87. Type Q (constant vertical distribution of damper constant) is not usual for a 9-storey frame, and presents a large non-proportionality ($I_{PS5} = 0.30$ for Q.1; $I_{A1} = 0.42, 0.78, 0.81$ for Q.1, Q.2, Q.3), resulting in large errors of modal analysis. As a result, the errors in prediction are large. For type U ADS, which is optimized for small non-proportionality ($I_{PS5} = 0.14$ for U.1; $I_{A1} = 0.10, 0.31, 0.47$ for U.1, U.2, U.3) the errors are smaller. The data available is not enough to reach conclusions, and further research is needed for frames in this range of height. However, acceptability results for EAR_2 show that the methods are still very good for preliminary design. Comparative plots for all variables and ADS type Q and U are given in Figure 7-89, Figure 7-90 and Figure 7-91.

7.3.3.15 Results with code spectrum. Frame without capacity rules.

Results for a 5 storey frame designed without capacity rules (05C5) are shown in Figure 7-92, for GMS ESB, and different ADS type Q, R, S, with different damping levels. All the results are for brace stiffness model type HB. These results should be compared with those shown in Figure 7-47. For this frame the acceptability of the methods reduces drastically, and it is out of EAR_1 range for most variables and methods. It must be pointed out that all data points analyzed correspond to the elastic range. For the inelastic range, worst results are expected, due to possible soft storey collapse modes (see Annex B). As a conclusion, caution must be exerted when using these simplified methods for frames designed without capacity rules.

Figure 7-37: Comparison of results with η_d and CFV from ASCE and this study. Method statistics. Without spectral adjustment.

Figure 7-38: Comparison of results with η_d and CFV from ASCE and this study. Method acceptability for EAR_1 . Without spectral adjustment.

Figure 7-39: Comparison of results with η_d and CFV from ASCE and this study. Method acceptability for EAR_2 . Without spectral adjustment.

Figure 7-40: Comparison of results with η_d and CFV from ASCE and this study. Method statistics. With spectral adjustment.

Figure 7-41: Comparison of results with η_d and CFV from ASCE and this study. Method acceptability for EAR_1 . With spectral adjustment.

Figure 7-42: Comparison of results with η_d and CFV from ASCE and this study. Method acceptability for EAR_2 . With spectral adjustment.

Figure 7-43: Improvement of damper shear prediction without spectral adjustment.

Figure 7-44: Improvement of damper shear prediction with spectral adjustment.

Figure 7-45: Improvement of total shear prediction. Without spectral adjustment.

Figure 7-46: Improvement of total shear prediction. With spectral adjustment.

Figure 7-47: Results for GMS ESB, MRF 05D2 with different types of ADS for best method options (see Table 7-13). With spectral adjustment.

Figure 7-48: Results with code spectrum. Influence of damper stiffness. Method statistics and acceptability for EAR_1 . Without spectral adjustment.

Figure 7-49: Results with code spectrum. Influence of damper stiffness. Method statistics and acceptability for EAR_1 . With spectral adjustment.

Figure 7-50: Results with code spectrum. Influence of damper exponent. Method statistics and acceptability for EAR_1 . Without spectral adjustment.

Figure 7-51: Results with code spectrum. Influence of damper exponent. Method statistics and acceptability for EAR_1 . With spectral adjustment.

Figure 7-52: Results with code spectrum. Influence of damping level. Method statistics and acceptability for EAR_1 . Without spectral adjustment.

Figure 7-53: Results with code spectrum. Influence of damping level. Method statistics and acceptability for EAR_1 . With spectral adjustment.

Figure 7-54: Results with code spectrum. Influence of site class. Method statistics. Without spectral adjustment.

Figure 7-55: Results with code spectrum. Influence of site class. Method acceptability for EAR_1 . Without spectral adjustment.

Figure 7-56: Results with code spectrum. Influence of site class. Method statistics. With spectral adjustment.

Figure 7-57: Results with code spectrum. Influence of site class. Method acceptability for EAR_1 .
 With spectral adjustment.

Table 7-14. RMSE for ESB.05D2.QRS.ALL.ALL.LB (with spectral adjustment).

Storey	Method	u	Δ	∇	V_s	V_d	V_t
		ESB.05D2.QRS.ALL.ALL.LB					
All	RSA	0.16	0.16	0.19	0.18	0.18	0.37
	RSAB	0.10	0.11	0.10	0.13	0.14	0.10
	RSABS	0.13	0.15	0.12	0.15	0.20	0.14
First	RSA	0.13	0.13	0.08	0.23	0.33	0.14
	RSAB	0.12	0.12	0.05	0.13	0.17	0.07
	RSABS	0.18	0.18	0.05	0.15	0.17	0.10
Roof	RSA	0.19	0.22	0.30	0.23	0.12	0.72
	RSAB	0.11	0.13	0.12	0.10	0.19	0.18
	RSABS	0.10	0.19	0.19	0.18	0.34	0.26

Shadowed cells correspond to minimum RMSE across methods

Table 7-15. RMSE for GMS ESB, MRF 05D, different ADSs and model stiffnesses (with spectral adjustment).

Storey	Method	u	Δ	∇	V_s	V_d	V_t	u	Δ	∇	V_s	V_d	V_t
		ESB.05D2.QRS.ALL.LINEAR.LB						ESB.05D2.QRS.ALL.NONLINEAR.LB					
All	RSA	0.20	0.15	0.15	0.13	0.21	0.46	0.11	0.16	0.23	0.23	0.13	0.23
	RSAB	0.10	0.10	0.08	0.12	0.15	0.09	0.10	0.12	0.11	0.14	0.11	0.12
	RSABS	0.12	0.12	0.11	0.14	0.21	0.11	0.15	0.17	0.13	0.16	0.18	0.16
First	RSA	0.15	0.15	0.06	0.22	0.39	0.07	0.10	0.10	0.10	0.24	0.24	0.19
	RSAB	0.10	0.10	0.03	0.13	0.20	0.07	0.13	0.13	0.06	0.13	0.11	0.07
	RSABS	0.15	0.15	0.04	0.17	0.21	0.10	0.21	0.21	0.07	0.13	0.11	0.09
Roof	RSA	0.23	0.16	0.24	0.13	0.12	0.95	0.13	0.27	0.36	0.31	0.12	0.34
	RSAB	0.11	0.11	0.08	0.10	0.18	0.13	0.11	0.15	0.15	0.09	0.19	0.22
	RSABS	0.10	0.15	0.18	0.18	0.34	0.19	0.11	0.23	0.20	0.17	0.34	0.32
Storey	Method	ESB.05D2.QRS.LOW.ALL.LB						ESB.05D2.QRS.HIGH.ALL.LB					
		u	Δ	∇	V_s	V_d	V_t	u	Δ	∇	V_s	V_d	V_t
All	RSA	0.17	0.16	0.12	0.16	0.11	0.23	0.14	0.14	0.27	0.20	0.22	0.47
	RSAB	0.13	0.12	0.10	0.12	0.10	0.10	0.09	0.11	0.10	0.14	0.16	0.09
	RSABS	0.13	0.13	0.11	0.13	0.12	0.11	0.16	0.17	0.12	0.18	0.25	0.15
First	RSA	0.14	0.14	0.08	0.23	0.21	0.11	0.12	0.12	0.08	0.23	0.41	0.18
	RSAB	0.07	0.07	0.05	0.15	0.13	0.08	0.15	0.15	0.05	0.11	0.20	0.06
	RSABS	0.09	0.09	0.04	0.16	0.14	0.09	0.25	0.25	0.07	0.15	0.18	0.12
Roof	RSA	0.21	0.22	0.21	0.21	0.09	0.43	0.16	0.18	0.43	0.23	0.15	0.91
	RSAB	0.17	0.16	0.13	0.10	0.13	0.14	0.06	0.12	0.07	0.11	0.22	0.16
	RSABS	0.16	0.17	0.17	0.12	0.17	0.18	0.06	0.21	0.16	0.24	0.46	0.28
Storey	Method	ESB.05D2.QRS.ALL.ALL.HB						ESB.05D2.QRS.ALL.ALL.LB					
		u	Δ	∇	V_s	V_d	V_t	u	Δ	∇	V_s	V_d	V_t
All	RSA	0.19	0.23	0.18	0.28	0.14	0.57	0.16	0.16	0.19	0.18	0.18	0.37
	RSAB	0.11	0.11	0.13	0.14	0.11	0.13	0.10	0.11	0.10	0.13	0.14	0.10
	RSABS	0.14	0.17	0.16	0.15	0.17	0.18	0.13	0.15	0.12	0.15	0.20	0.14
First	RSA	0.16	0.16	0.15	0.26	0.22	0.13	0.13	0.13	0.08	0.23	0.33	0.14
	RSAB	0.13	0.13	0.09	0.16	0.10	0.08	0.12	0.12	0.05	0.13	0.17	0.07
	RSABS	0.19	0.19	0.09	0.16	0.11	0.07	0.18	0.18	0.05	0.15	0.17	0.10
Roof	RSA	0.22	0.39	0.32	0.51	0.15	1.18	0.19	0.22	0.30	0.23	0.12	0.72
	RSAB	0.12	0.13	0.21	0.15	0.20	0.26	0.11	0.13	0.12	0.10	0.19	0.18
	RSABS	0.11	0.26	0.28	0.19	0.31	0.36	0.10	0.19	0.19	0.18	0.34	0.26

Shadowed cells correspond to minimum RMSE across methods.

Table 7-16. Influence of different parameters on RMSE for GMS ESB, MRF 05D (with spectral adjustment).

		Quotient of two groups.					
		u	Δ	∇	V_s	V_d	V_t
Storey	Method	ESB.05D2.QRS.ALL.LINEAR/NONLINEAR.LB					
All	RSA	1.82	0.94	0.65	0.57	1.63	1.96
	RSAB	0.98	0.79	0.73	0.82	1.36	0.73
	RSABS	0.78	0.71	0.84	0.92	1.19	0.71
First	RSA	1.52	1.52	0.65	0.94	1.64	0.35
	RSAB	0.77	0.77	0.45	0.98	1.84	0.93
	RSABS	0.71	0.71	0.57	1.28	1.99	1.18
Roof	RSA	1.72	0.58	0.67	0.41	1.05	2.77
	RSAB	1.04	0.75	0.56	1.08	0.97	0.57
	RSABS	0.95	0.69	0.87	1.06	0.99	0.60
		ESB.05D2.QRS.LOW/HIGH.ALL.LB					
All	RSA	1.20	1.09	0.47	0.78	0.51	0.49
	RSAB	1.39	1.08	1.00	0.91	0.66	1.05
	RSABS	0.78	0.72	0.88	0.74	0.49	0.72
First	RSA	1.12	1.12	0.92	1.02	0.51	0.60
	RSAB	0.50	0.50	1.09	1.40	0.64	1.42
	RSABS	0.38	0.38	0.58	1.09	0.78	0.79
Roof	RSA	1.30	1.21	0.49	0.89	0.58	0.47
	RSAB	2.92	1.33	1.95	0.88	0.61	0.91
	RSABS	2.56	0.81	1.04	0.48	0.36	0.64
		ESB.05D2.QRS.ALL.ALL.HB/LB					
All	RSA	1.18	1.48	0.94	1.51	0.77	1.57
	RSAB	1.10	1.03	1.30	1.06	0.85	1.28
	RSABS	1.04	1.15	1.34	1.01	0.84	1.28
First	RSA	1.25	1.25	1.82	1.12	0.67	0.94
	RSAB	1.13	1.13	1.79	1.23	0.62	1.21
	RSABS	1.03	1.03	1.77	1.05	0.67	0.74
Roof	RSA	1.19	1.78	1.04	2.19	1.21	1.64
	RSAB	1.05	1.04	1.82	1.48	1.05	1.44
	RSABS	1.01	1.35	1.48	1.09	0.92	1.39

Cells shadowed in green indicate best performance of second group.

Cells shadowed in red indicate best performance of first group.

Figure 7-58: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Displacement. Intensity level PHA_E .

Figure 7-59: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Displacement. Intensity level PHA_D .

Figure 7-60: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Drift. Intensity level PHA_E .

Figure 7-61: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Drift. Intensity Level PHA_D .

Figure 7-62: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Drift velocity. Intensity Level PHA_E .

Figure 7-63: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Drift velocity. Intensity Level PHA_D .

Figure 7-64: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Storey shear. Intensity level PHA_E .

Figure 7-65: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Storey shear. Intensity level PHA_D .

Figure 7-66: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Damper shear. Intensity level PHA_E .

Figure 7-67: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Damper shear. Intensity level PHA_D .

Figure 7-68: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Total shear. Intensity level PHA_E .

Figure 7-69: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Total shear. Intensity level PHA_D .

Figure 7-70: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Displacement. Intensity level PHA_E with spectral adjustment.

Figure 7-71: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Displacement. Intensity level PHA_D with spectral adjustment.

Figure 7-72: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Drift. Intensity level PHA_E with spectral adjustment.

Figure 7-73: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Drift. Intensity level PHA_D with spectral adjustment.

Figure 7-74: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Drift Velocity. Intensity level PHA_E with spectral adjustment.

Figure 7-75: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Drift Velocity. Intensity level PHA_D with spectral adjustment.

Figure 7-76: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Storey Shear. Intensity level PHA_E with spectral adjustment.

Figure 7-77: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Storey Shear. Intensity level PHA_D with spectral adjustment.

Figure 7-78: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Dumper Shear. Intensity level PHA_E with spectral adjustment.

Figure 7-79: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Damper Shear. Intensity level PHA_D with spectral adjustment.

Figure 7-80: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Total Shear. Intensity level PHA_E with spectral adjustment.

Figure 7-81: Comparison of TH results vs RSAB, RSABS and RSA methods for GMS ESB, frame 05D2, different ADS and low brace stiffness (LB). Total Shear. Intensity level PHA_D with spectral adjustment.

Table 7-17. RMSE for ESB.05D2. Influence of damper coefficient distribution (with spectral adjustment).

Storey	Method	$\mathbf{u}$	Δ	∇	V_s	V_d	V_t	$\mathbf{u}$	Δ	∇	V_s	V_d	V_t
		ESB.05D2.Q.ALL.ALL.LB						ESB.05D2.R.ALL.ALL.LB					
All	RSA	0.14	0.18	0.21	0.19	0.13	0.45	0.15	0.15	0.19	0.17	0.15	0.37
	RSAB	0.10	0.12	0.10	0.13	0.14	0.12	0.10	0.11	0.09	0.13	0.13	0.10
	RSABS	0.16	0.19	0.15	0.18	0.24	0.18	0.13	0.14	0.11	0.14	0.18	0.14
First	RSA	0.10	0.10	0.04	0.19	0.23	0.17	0.10	0.10	0.06	0.22	0.29	0.14
	RSAB	0.13	0.13	0.07	0.13	0.12	0.08	0.12	0.12	0.04	0.13	0.14	0.07
	RSABS	0.24	0.24	0.07	0.18	0.10	0.13	0.18	0.18	0.04	0.15	0.14	0.07
Roof	RSA	0.18	0.29	0.34	0.28	0.09	0.91	0.19	0.22	0.31	0.20	0.08	0.73
	RSAB	0.11	0.16	0.11	0.11	0.22	0.20	0.11	0.13	0.11	0.10	0.19	0.18
	RSABS	0.11	0.27	0.26	0.25	0.47	0.34	0.10	0.16	0.19	0.16	0.33	0.26
	Method	ESB.05D2.S.ALL.ALL.LB						ESB.05D2.U.ALL.LINEAR.LB					
		$\mathbf{u}$	Δ	∇	V_s	V_d	V_t	$\mathbf{u}$	Δ	∇	V_s	V_d	V_t
All	RSA	0.19	0.14	0.17	0.18	0.23	0.24	0.22	0.15	0.14	0.14	0.24	0.34
	RSAB	0.11	0.10	0.10	0.13	0.13	0.09	0.11	0.09	0.08	0.11	0.16	0.08
	RSABS	0.10	0.10	0.08	0.13	0.14	0.08	0.10	0.08	0.08	0.11	0.18	0.09
First	RSA	0.17	0.17	0.12	0.28	0.43	0.11	0.19	0.19	0.08	0.26	0.49	0.07
	RSAB	0.11	0.11	0.03	0.13	0.22	0.07	0.09	0.09	0.03	0.13	0.26	0.06
	RSABS	0.10	0.10	0.04	0.12	0.24	0.07	0.09	0.09	0.04	0.14	0.28	0.06
Roof	RSA	0.18	0.11	0.27	0.22	0.18	0.41	0.24	0.10	0.23	0.12	0.09	0.70
	RSAB	0.11	0.08	0.13	0.08	0.13	0.14	0.13	0.10	0.08	0.10	0.16	0.11
	RSABS	0.11	0.10	0.08	0.08	0.10	0.11	0.11	0.09	0.12	0.11	0.23	0.14

Shadowed cells indicate the minimum RMSE for all methods and groups, excluding U.

Figure 7-82: Results with code spectrum. Influence of damper coefficient distribution. Method statistics. With spectral adjustment.

Figure 7-83: Results with code spectrum. Influence of damper coefficient distribution. Method acceptability for EAR_1 . With spectral adjustment.

Figure 7-84: Results for GMS ESB, MRF 03D2 with ADS type Q for best method options (see Table 7-13). Including spectral adjustment.

Figure 7-85: Comparative results for GMS ESB, MRF 03D2 with different types of ADS, stiffness model LB, for best method options (see Table 7-13). Displacement, drift and drift velocity. Including spectral adjustment.

Figure 7-86: Comparative results for GMS ESB, MRF 03D2 with different types of ADS, stiffness model LB, for best method options (see Table 7-13). Intensity level PHA_D . Storey shear, damper shear, total shear. Including spectral adjustment.

Figure 7-87: Results for GMS ESB, MRF 09D2 with ADS type Q and U for best method options (see Table 7-13). Including spectral adjustment.

Figure 7-88: Results for GMS ESB, MRF 09D2 with ADS type Q and U for best method options (see Table 7-13). Including spectral adjustment.

Figure 7-89: Comparative results for GMS ESB, MRF 09D2 with ADS type Q with stiffness LB, for best method options (see Table 7-13). Displacement, drift, drift velocity. Including spectral adjustment.

Figure 7-90: Comparative results for GMS ESB, MRF 09D2 with ADS type Q, with stiffness model LB, for best method options (see Table 7-13). Storey shear, damper shear, total shear. Including spectral adjustment.

Figure 7-91: Comparative results for GMS ESB, MRF 09D2 with ADS type U, stiffness type LB, for best method options (see Table 7-13). Including spectral adjustment.

Figure 7-92: Results for GMS ESB, MRF 05C5 with different types of ADS for best method options (see Table 7-13). Including spectral adjustment.

7.3.4 Influence of non-proportionality.

This Section examines the relationship of errors in the RSA method with the non-proportionality indices presented in Chapter 3. A significant correlation between 4 indices (I_{TLL1} , I_{PS5} , I_{A1} , I_{LKZ4}) and the un-conservative error was found for the shear-type systems analyzed. This correlation is now verified for the frame systems in this numerical study.

Figure 7-93 and Figure 7-94 display the error for all variables in the RSA method for type D frames (03D, 05D, 09D) with linear ADS at elastic levels ($\mu_T < 1.05$); the relative error is plotted versus three indices (I_{LKZ4} has been left out because it is virtually identical to I_{A1}). The errors are shown for set ESB; similar results for set EUK are plotted in Section F.1.8 (Annex F).

From the data presented, it is clear that there exists a correlation of the three indices to displacement error. A good correlation appears also for drift error; for this variable, the indices are also related to the maximum conservative error. The indices, however, fail to correlate with errors for any other variable, except, maybe, design shear and total shear for indices I_{A1} and I_{PS5} . The trends presented here for set ESB are also observed for set EUK.

The figures in Section F.1.8 (Annex F) compare the errors obtained for frames D5 with linear ADS type Q and U. ADS type Q presents constant damper coefficients, whereas type U has been obtained minimizing the I_{PS5} index (see Annex D). By comparing the results it is clear that the acceptability of type U for method RSA improves for all variables, but is more conspicuous for Δ and u . Because Δ is precisely the most important variable from the point of view of damage control and performance, keeping indices I_{A1} below 0.4, or I_{PS5} below 0.15, still provide a useful tool to ascertain whether the simplified RSA method can be successfully applied to a system for this variable. However, the indices are not successful in preventing errors for other variables.

The amount and relative weight of off-diagonal terms in the reduced stiffness matrix of a frame increases with height. The added damping matrix is always a banded matrix with fixed bandwidth of 2. This indicates that the differences between both matrices increase with height, resulting in larger non-proportionality. This is clear examining Table 7-19, where cells containing minimum values across frames have been shadowed, showing that the 03D frame presents smaller errors than the rest. If the errors for a certain method across groups are considered, as shown in Table 7-18, it is clear that the error increases with height (except for u in RSAB and u , Δ in RSA). This trend could not be detected in the simplified analysis performed in Chapter 3, because the stiffness matrix corresponded to a shear type system. Shear type systems present fundamental differences in their dynamic behavior when compared to frame type systems.

The relationship between errors and non-proportionality indices has been explored also for systems with inelastic behavior. Figure 7-95 shows the relationship between drift errors and the three mentioned indices, for set ESB with moderate ductility levels ($1.05 < \mu_T < 1$).

5) and large ductility levels ($1.5 < \mu_T < 3$). The correlation is weaker as the ductility level increases, as expected. For moderate ductility levels the correlation still stands.

Figure 7-93: Relationship of non-proportionality indices with errors. Elastic levels. Set ESB. Kinematic variables.

Figure 7-94: Relationship of non-proportionality indices with errors. Elastic levels. Set ESB. Static variables.

Table 7-18. RMSE for frames with distribution type U (with spectral adjustment) binned by method.

	RSAB						RSA					
	u	Δ	∇	V_s	V_d	V_t	u	Δ	∇	V_s	V_d	V_t
03D2.U	0.04	0.04	0.04	0.09	0.08	0.07	0.15	0.12	0.08	0.14	0.15	0.19
05D2.U	0.11	0.09	0.08	0.11	0.16	0.08	0.22	0.15	0.14	0.14	0.24	0.34
09D2.U	0.07	0.10	0.15	0.16	0.19	0.11	0.17	0.12	0.33	0.19	0.48	0.52

Cells shadowed in green indicate minimum RMSE, cells shadowed in red indicate maximum RMSE.

Table 7-19. RMSE for frames with distribution type U (with spectral adjustment).

Storey	Method	u	Δ	∇	V_s	V_d	V_t	u	Δ	∇	V_s	V_d	V_t
		ESB.03D2.U.ALL.LINEAR.LB						ESB.05D2.U.ALL.LINEAR.LB					
All	RSA	0.15	0.12	0.08	0.14	0.15	0.19	0.22	0.15	0.14	0.14	0.24	0.34
	RSAB	0.04	0.04	0.04	0.09	0.08	0.07	0.11	0.09	0.08	0.11	0.16	0.08
	RSABS	0.04	0.04	0.05	0.10	0.09	0.07	0.10	0.08	0.08	0.11	0.18	0.09
First	RSA	0.15	0.15	0.08	0.22	0.24	0.02	0.19	0.19	0.08	0.26	0.49	0.07
	RSAB	0.04	0.04	0.01	0.10	0.12	0.04	0.09	0.09	0.03	0.13	0.26	0.06
	RSABS	0.05	0.05	0.01	0.10	0.13	0.04	0.09	0.09	0.04	0.14	0.28	0.06
Roof	RSA	0.14	0.08	0.08	0.08	0.05	0.33	0.24	0.10	0.23	0.12	0.09	0.70
	RSAB	0.03	0.05	0.04	0.09	0.07	0.07	0.13	0.10	0.08	0.10	0.16	0.11
	RSABS	0.03	0.04	0.04	0.09	0.08	0.08	0.11	0.09	0.12	0.11	0.23	0.14
	Method	ESB.09D2.U.ALL.LINEAR.LB						ESB.09D2.Q.ALL.LINEAR.LB					
		u	Δ	∇	V_s	V_d	V_t	u	Δ	∇	V_s	V_d	V_t
All	RSA	0.17	0.12	0.33	0.19	0.48	0.52	0.23	0.33	0.41	0.38	0.21	0.36
	RSAB	0.07	0.10	0.15	0.16	0.19	0.11	0.06	0.13	0.13	0.18	0.14	0.09
	RSABS	0.08	0.10	0.12	0.16	0.22	0.10	0.18	0.22	0.15	0.22	0.25	0.14
First	RSA	0.19	0.19	0.12	0.16	1.27	0.12	0.18	0.18	0.17	0.20	0.54	0.24
	RSAB	0.07	0.07	0.07	0.08	0.41	0.05	0.04	0.04	0.01	0.07	0.19	0.04
	RSABS	0.10	0.10	0.09	0.08	0.40	0.06	0.26	0.26	0.08	0.18	0.15	0.14
Roof	RSA	0.21	0.14	0.47	0.28	0.28	1.35	0.27	0.44	0.57	0.49	0.10	0.85
	RSAB	0.08	0.11	0.09	0.16	0.12	0.07	0.10	0.21	0.11	0.16	0.18	0.17
	RSABS	0.09	0.07	0.04	0.26	0.26	0.05	0.10	0.32	0.24	0.29	0.57	0.32

Shadowed cells indicate minimum RMSE across sets and methods.

Figure 7-95: Relationship of non-proportionality indices with errors. Inelastic levels. Set ESB.

7.4 Summary and conclusions

A numerical study has been carried out in order to evaluate the RSA method and two modifications (RSAB, RSABS) proposed to include the influence of the bracing stiffness through a numerical approximation to the Maxwell behavior of the damping system. The study consisted on TH analyses of a set of moment resisting frames (MRFs) equipped with different added damping systems (ADS) and subjected to different ground motion sets (GMS). The GMSs were formed by a single representative motion and by suites of pre-scaled motions, both scaled to different intensities. The damping system stiffness was modeled as unrealistically high, realistic without consideration of damper stiffness, and realistic with consideration of damper stiffness. Especial emphasis was placed on a 5-storey frame with different ADS, which is considered representative of mid-height buildings for which simplified procedures are usually applied.

Regarding the stiffness of the added damping systems, the following observations have been made:

- Damping system stiffness affects TH results to a considerable degree.
- The influence is larger at low intensity levels, for variables velocity $\dot{V}$, damper shear V_d and total shear V_t .
- Non-linear dampers and systems with high damping are more affected.
- The top half of the building is more affected.
- The influence is larger for a single motion than for a set of motions.
- The response variability (measured by the interval $x^{84}-x^{16}$) is affected to a lesser degree than the logMedian response.
- A positive correlation has been observed between the storey energy dissipation efficiency of the damping system $\beta_{W,n}$ (proposed in Chapter 5) and the proportion in which the damping system stiffness affects the response.
- Keeping the values of efficiency $\beta_{W,n}$ above a certain proportion (90-95% is suggested), appears as a reasonable criterion to determine a minimum stiffness for the damping system.

Damping system stiffness is influenced by all elements conforming the damping system; the main contributor for diagonal mounting is the brace extender; the damper stiffness and auxiliary mounting elements contribution has traditionally been neglected, and few experimental attempts to accurately measure their actual values exist. However, this study shows that the relative response error when this contribution is neglected is of the same order that the relative response error when the brace stiffness is altogether ignored. Further study is needed, particularly regarding characterization of commercial damper stiffnesses including their auxiliary mounting elements, which are often out of the designer's control.

The three simplified methods, RSA, RSAB and RSABS, were subsequently applied to obtain an approximate solution for the frames in the study. The spectral values used in the methods were found in three different ways: i) 'exact', obtained by direct integration of a

SDOF oscillator with effective period and damping; ii) ‘code’, obtained from Eurocode spectrum; iii) ‘code with spectral adjustment’, in which period-dependent correction was applied to match the scaled Eurocode 8 and the scaled average motion spectrum.

The relative distance between TH and simplified response was measured for six relevant variables, and treated statistically. In addition to conventional statistics, the concept of engineering acceptability range (EAR) was used to define the ‘acceptability’ of results, that is, the proportion of total results within acceptable limits. Two different EARs were used, EAR_1 comprising a small acceptability range intended for accurate verification, and EAR_2 , with a wider acceptability range intended for preliminary design purposes.

For results obtained with ‘exact’ spectral ordinates, the following observations apply:

- The response is only approximate. Large un-conservative errors are obtained, well outside the EAR_1 range, but generally within EAR_2 range.
- Trends in errors are very similar for the case of one motion and a set of motions.
- The errors are sensitive to the level of ductility, damper exponent, amount of damping and type of bracing stiffness model. Site class, inherent damping and the type of damper distribution exert only a mild influence.
- The CQC combination rule is found to render slightly better results than the SRSS rule.
- The minimum elastic displacement condition of ASCE-SEI 7/16 increases the conservativeness of the predictions without changing their acceptability. Application of this rule is deemed unnecessary.

For results obtained with ‘soft’ spectral ordinates, the following observations apply:

- The three methods are suitable to find an accurate enough response of structures with added viscous damping systems for soft spectra (such as the Eurocode 8 spectrum or equivalent Uniform Hazard Spectrum), if and only if, spectral correction factors η_d and CFV are applied.
- The damping correction factors η_d proposed in this study give results similar to those in ASCE/SEI 7-16.
- The Correction Factors for Velocity CFV are essential to obtain accurate results. The CFVs derived in this study have been used throughout the study and shown to provide accurate predictions when used with the Eurocode 8 spectrum and consistent η_d .
- The predictions are better for linear dampers, low damping and systems modeled with LB bracing stiffness, with all three methods.
- Results for the first storey and roof are generally acceptable, although the results for the roof present a larger dispersion.
- The accuracy of results is better for kinematic variables than for static variables.
- For damper forces, the procedure suggested in Section 6.2.2.5, based on the Maxwell equations applied to peak drift and drift velocity, gives more accurate results than conventional modal combination.

- For total forces, the method proposed in this study, based on a Bézier quadratic approximation, gives results of higher accuracy than any other method.
- The force coefficient method implemented in ASCE-SEI 7/16 has been found to render poor results. The alternative methods proposed by Kasai or Lin-Chopra are of simpler application and give better results than the force coefficient method. However, the accuracy of the three methods is well below that of the method proposed in this study, for all the cases analyzed.
- For frames 03D and 05D the acceptability is larger than 67% in all variables and methods except for total shear V_t calculated with RSA, for which the result is over-conservatively out of range.
- For frame 09D the RSAB and RSABS give acceptable predictions except in V_s . For this frame RSA gives unacceptable predictions. This is probably due to the large non-proportionality of this frame, and is indicative that a height limit should be imposed in its use; further research is needed in this direction.
- For ADS type U, the error increases with frame height. This is explained considering the increasing difference between stiffness and damping matrices with height, consistent with larger values of non-proportionality indices. This results could not be detected with the shear type models used in the analyses performed in Chapter 3.
- The predictions for frame 05C designed without capacity rules are less accurate and acceptable than those of 05D (designed with capacity rules), even if the frame remained elastic for the excitation intensities analyzed; for higher intensities this type of frames might develop a soft storey collapse mechanism; therefore the application of these methods to frames with no capacity conditions is questionable and requires further research.
- With respect to the TH dispersion limit interval (x^{16} ; x^{84}), for specific conditions (high damping and high non-proportionality hereby represented by ADS type Q), the following results can be un-conservative: drift velocity at roof with RSA; storey shear at roof with RSA; damper shear at roof with RSA; total shear at mid-height with RSA. The following results can be over-conservative: drift at roof with RSABS or RSA; storey shear at first with RSA; damper shear at first with RSA; damper shear at roof with RSABS and RSAB; total shear at roof with RSA method (even if calculated with the Kasai method).

The RSAB is more accurate than RSA and RSABS methods. It gives the minimum relative distance between TH response and method response for most of the variables, and the largest acceptability in a general sense. The vertical distribution of variable response with this method follows closely the logMedian TH response height wise distribution. Conceptually speaking, the method avoids the need to assume damper linearization in higher modes, maybe the most confusing part of the RSA method in its current formulation.

The RSAB method, however, is computationally much more demanding than RSA. This is not so relevant if a distinction between ‘simplified analysis’ and ‘manual analysis’ is made

clear. In the context of earthquake analysis, ‘simplified’ implies avoiding the complexities of TH analysis, but it is obvious that, nowadays, a computer is going to be involved in the design process at some point. The software developed for this study can be used to support these initial analyses. It must be acknowledged, though, that for initial stages of design, the additional accuracy provided by RSAB is not meaningful. In this context, the RSA method, probably applied with a reduced number of modes, is enough. Once the structure and damping system are designed, the RSAB method can be used for verification. The RSAB method should always be used in cases where the brace flexibility is relevant. After RSA analysis, this can be assessed estimating $\beta_{W,n}$ with Δ and ∇ obtained from the RSA analysis. If this value is under a certain threshold, the RSAB method should be applied.

7.5 Original contributions in this Chapter

- A systematic verification of an existing simplified method (RSA), applied to frames with viscous dampers, under several intensity levels and damping systems for single motions and sets of motions has been presented. The verification included two proposed improvements (RSAB and RSABS) and comparison of the performance of the three methods.
- A procedure to measure the acceptability of results in engineering terms has been proposed and used to establish the acceptability of the three methods. Suggested acceptability ranges have been proposed based on engineering judgement.
- The influence of different parameters on the accuracy of results has been established.
- The proposed method RSAB has been proved to be more accurate than RSA or RSABS, but in general any of the three methods suffice for practical applications.
- The influence of the damping system stiffness on the response variation has been quantified. A positive correlation has been shown between the energy dissipation efficiency β_W of the damping system and the variation of response with damping system stiffness.
- A positive correlation has been shown between displacement and drift error in RSA method and the non-proportionality indices I_{PS5} and, especially, I_{A1} , for actual systems with added damping.

Chapter 8

Conclusions and Guidelines for Future Research

8.1 Summary and conclusions

This study examines the prediction of seismic response of structures equipped with viscous damping systems, using simplified methods based on effective properties. The research was motivated by the possibility of inclusion of these methods in the future version of Eurocode 8. Chapter 2 contains a summary of the background. The relevant findings and conclusions are discussed hereby.

8.1.1 Overview

Fluid viscous dampers are mounted on structures using auxiliary elements. Among others: brace extenders, connection elements, hinges, gusset plates, bolts, frontal plates, etc. All these elements are connected in series, therefore the overall Maxwell stiffness of the damping element is participated by all components including the damper, and strongly dependent on the most flexible one. This study has shown how inclusion of these components results in dramatic reduction of stiffness. Further experimental and analytical study is needed regarding determination of stiffness contribution of the different components.

Current practice is to neglect the influence of the auxiliary elements and assimilate the Maxwell stiffness to the brace extender stiffness. The study has proved that errors associated with this simplification can be quite large, particularly for high levels of damping and in the top half of the building.

When using RSA analysis, no method exists to consider the damping system stiffness. This study proposes a numerical approximation to the peak response of the Maxwell element, which is subsequently introduced in the RSA method in order to consider the aforementioned stiffness. The stiffness contribution modifies the vibration properties of the structure, thus the solution is iterative. Two procedures are proposed, one (RSAB) in which modal shapes and frequencies are updated, and one (RSABS) in which the original modal shapes are kept and only frequencies are updated.

The procedures are tested by means of a large numerical study. Results indicate that, for soft spectra (codified spectra) modified with suitable spectral correction factors, the accuracy of the three methods (RSA, RSAB and RSABS) is acceptable for design purposes. The best results are obtained with the RSAB method, particularly for a 9 storey frame included in the study. The methods do not perform well in a frame designed with no capacity criteria, even in the elastic range.

In order to improve the method performance, a set of spectral correction factors has been derived from European motions, in order to be used with the Eurocode 8 elastic spectrum.

The maximum un-conservative errors in the methods have been correlated to limit values of four existing non-proportionality indices. The correlation has been proved for elastic shear-type frames. For moment-resisting frames, the correlation has been shown to exist for kinematic variables, but is weaker or non-existing for static variables.

The contents of each Chapter are summarized hereby.

8.1.2 Chapter 3: Non-proportionality Indices as Error Predictors in Classical Modal Analysis

This Chapter focuses on errors derived from non-proportionality of the damping matrix in systems with added viscous damping, when modal analysis is used to find their seismic response. Several indices have been proposed by other authors to measure non-proportionality. Here, the correlation of these existing indices with the actual prediction errors of different relevant variables is studied. To this purpose, a large numerical study is performed, where different shear systems representing actual elastic frames are equipped with a variety of added linear viscous damping systems. The frames are subjected to pre-scaled sets of motions representing approximately the Eurocode spectrum. The exact TH response of the systems is found and used as benchmark to determine the relative error in the approximate modal response (obtained with TH analysis and a diagonalized damping matrix), and in the response spectrum method (obtained combining modal peak values with the SRSS or CQC combination rules). These errors are found for six relevant variables, three kinematic (displacement, drift, drift velocity) and three static (storey shear, damper shear, total shear). The relationship of the errors with several descriptive parameters of the structure is established. The correlation of errors with the non-proportionality indices is also studied. Some relevant conclusions are:

1. The predictions of the three methods can be over- or under-conservative, with a tendency to the former. The errors depend primarily on the added damping distribution, stiffness distribution, and amount of added damping.
2. Complete distributions are not a safeguard against underestimation of response with RSA.
3. Simple criteria based on frame and damping system properties in order to keep the error systematically below a threshold cannot be formulated.

4. The maximum error in the study seems to be well-correlated with the values of four non-proportionality indices. Keeping the value of the indices below a threshold can limit the error, at least in the most significant variable (drift).
5. As an example, practical indications regarding one of the indices, proposed by Prater and Singh (I_{PS5}) are given. Keeping this index below 0.12 (THMA) or 0.16 (RSA) keeps the prediction error below 20% in most variables.

The conclusions are limited to shear type frames, regular and within the boundaries of the study in height, period, added damping ratio, stiffness distribution and damping distribution. However, the study comprised a very wide range for all these parameters, and it is expected that most real systems will be represented. Some important limitations to the study are: i) it did not include non-linear viscous dampers, as their behavior is dependent on the excitation intensity and not amenable to the kind of analysis performed; ii) the systems are shear-type, which are different to actual MRF in several important aspects; iii) the systems are linear; iv) soft-soil or near-fault motions were not include.

8.1.3 Chapter 4: Spectral Correction Factors

Application of the RSA method to structures with added dampers requires evaluation of modal response for high damping. This is performed by application of spectral correction factors to elastic spectral response. The current Eurocode 8 expression includes a damping correction factor for displacement ($\eta_{d,EC8}$), but its application is limited to damping ratios below 28%. The code does not contain correction factors to approximate velocity from pseudo-velocity (CFVs), which has shown to be critical for correct response prediction in structures with added viscous dampers. Thus, derivation of spectral correction factors is needed to use the RSA method within the Eurocode 8 framework.

To obtain these factors, a numerical study is carried out, using 880 far-field European records. These are binned by site class and significant ground motion duration, in order to derive specific expressions for spectral shapes type 1 or 2. Damped spectra are found, and by statistical and numerical treatment, a set of parametric expressions is proposed, in which the current Eurocode 8 DCF $\eta_{d,EC8}$ is used as kernel. The expressions are compared with existing ones.

The study does not contemplate near-fault records. The results for soft soils are not conclusive, due to the small amount of data available and its reduced variability.

8.1.4 Chapter 5: Influence of Stiffness in the Response of Systems with Added Damping

In this Chapter a numerical approximation to the behavior of the Maxwell system subjected to forced harmonic vibration is derived. The model allows estimation of the peak Maxwell system parameters as a function of a single significant parameter q that represents the quotient of the peak pure viscous force and the peak pure spring force. Another

relevant parameter introduced is the energy dissipation efficiency of the Maxwell system, defined as the energy actually dissipated over the energy dissipated at pure viscous damping. The minimum system stiffness is naturally obtained when a minimum energy dissipation efficiency is targeted for the first mode. Values of 90% to 95% are suggested here as reasonable. For higher modes the model predicts that quasi-elastic, non-dissipative behavior is to be expected. This is in good agreement with available experimental data.

The Maxwell model is included then in a broader model, the standard solid model, that represents the behavior of a structural system equipped (elastic or yielding) with a damping system. The peak response of the standard solid model under forced harmonic vibration is found using the numerical approximations. Several alternatives are considered for calculation of the peak total force. It is shown that two existing simpler expressions (Lin-Chopra, Kasai) render better results than the current ASCE-SEI 7/16 procedure (force coefficients). However, a more accurate formulation is proposed, based on a Bézier quadratic approximation. The expression is more complex, but presents a superior performance. The numerical procedures are illustrated with some examples.

8.1.5 Chapter 6: Modification of RSA to include the Influence of Auxiliary Elements

The numerical model derived in Chapter 5 is implemented in the RSA method. The model considers the flexibility and viscosity components of the damping system at every storey. The stiffness contribution modifies the vibrational properties of the structure, so iterative solution is required. Two procedures are suggested, a consistent one (RSAB) in which all modal properties of the system are recalculated at each step, and a simple one (RSABS) in which the modal shapes are considered constant but the period is updated at every step. The proposed procedures are similar to the RSA applied to viscoelastic dampers, with the significant difference of non-linearity of the dampers for all modes, which makes the response dependent of frequency and amplitude. The method proposed removes the need for linearization of non-linear dampers in the upper modes, so it leads to a more consistent solution. In this study, inelastic action is restricted to the first mode, but the formulation of the method allows for inelastic action in as many modes as required, with the only condition of availability of modal pushover analysis for the mode considered. Alternative expressions for total shear and damper shear are proposed.

The period and damping ratio predicted by the proposed methods are analyzed. With RSAB and RSABS, the values of these two parameters are dependent on the intensity, both for linear and non-linear cases, but following different patterns. For non-linear cases the mode shapes are also dependent on intensity, but this dependency is proved to be very mild. In contrast, the RSA method predicts constant periods and modal shapes for all excitation intensities.

Finally, numerical examples illustrate the practical application of both proposed methods.

8.1.6 Chapter 7: Numerical Validation

TH analyses are performed for four moment resisting frames (MRF) equipped with different viscous added damping systems (ADS), under a single motion (set EUK) or sets of motions (sets ESA, ESB, ESC, ESD). Each analysis is performed for different damping ratios, and at different intensity levels. The brace extenders for each system are designed for a certain targeted intensity, with additional safety factors and design considerations. Then, different models are created in which the stiffness of the damping system is modeled as very large (RB); with a realistic value based on the brace extender stiffness (HB); and half the previous value (LB) to consider in a simple way the influence of auxiliary elements.

Comparison of results with the three types of models show that the stiffness of the damping system affects the TH results to a considerable degree, with a larger influence at low intensity levels, for non-linear dampers and for systems with high damping ratios. The top half of the building seems to be more affected, and the influence is larger for a single motion than for a set of motions. A positive correlation appears between energy dissipation efficiency and the proportion in which the damping system affects the response, which leads to a minimum stiffness criterion as suggested in Chapter 5.

Results of TH analyses are compared to results obtained using the existing (RSA) and new (RSAB, RSABS) procedures. The conclusions are different if the response to actual motions or to a soft spectrum is considered:

For seismic excitation defined by a motion or a set of motions, the response prediction is poor. In this case the method is not suitable for verification but it can be used for preliminary design.

For seismic excitation defined by a soft spectrum, such as the Eurocode 8 spectrum, the three methods render good results, as long as suitable spectral correction factors are applied. The accuracy is good for all three methods, for the 3-storey and 5-storey frames with stiffness model type LB. The RSAB method is generally the most accurate, but good results are obtained for RSA and RSABS, except for total shear in the RSA method.

For the 9-storey frame the accuracy of the RSA method is unacceptable. The two other methods render good results for all variables except V_s . This may be indicative that a height limit should be set in the method application. Further study is needed.

The three methods offer reasonable results for frames designed with capacity criteria. The accuracy for a frame designed without capacity criteria was found to be lower, suggesting that the method should be used with caution when applied to this type of frames.

Regarding the different options of the methods, the approximation of total shear using the expressions proposed in this study produces more accurate results than other existing alternatives. The approximation of damper forces using the Maxwell model with the peak kinematic drift and drift velocity gives the best results for this variable. The CQC rule

gives better results than the SRSS rule. The imposition of a minimum elastic displacement does not improve the accuracy of the results and seems unnecessary.

8.1.7 Summary of Original Contributions

- The relationship of four existing non-proportionality indices with errors inherent to classical modal analysis has been established.
- A new set of spectral correction factors has been derived for application with Eurocode 8 spectrum. The set includes damping correction factors for displacement and correction factors for velocity, which are needed for application of simplified verification methods to structures with viscous dampers.
- A numerical approximation to the peak response of the Maxwell model under forced harmonic vibration has been proposed.
- An improved expression to obtain the peak total force in the Standard Model has been proposed and tested. This expression is more accurate than other existing expressions.
- An improved expression is proposed to derive the peak damper force, based upon peak kinematic values rather than modal combination.
- A modification to the existing RSA procedure has been proposed, in order to consider the stiffness of the added viscous damping system. Two different methods are considered, consistent (RSAB) and simplified (RSABS).
- The range of accuracy and acceptability in relevant response variables expected from the three methods (RSA, RSAB, RSABS) has been established, and proved to be acceptable when the Eurocode 8 spectrum is used, provided that suitable spectral correction factors are applied.

8.2 Guidelines for future studies

8.2.1 Extension of non-proportionality indices to non-linear dampers

Current formulation of non-proportionality indices is restricted to linear dampers. It would be of interest to extend the four relevant indices identified in this study to the case of non-linear dampers. This could be done through equivalent linearization. The results are expected to be intensity-dependent.

8.2.2 Verification of spectral correction factors for low seismicity

Verification of spectral correction factors proposed for high seismicity has been carried out in this work, but a similar verification has not been performed for the factors proposed for low seismicity. This verification implies selection and scaling of a ground motion set representative of such conditions, followed by numerical analysis of a set of frames.

8.2.3 Improvement of approximate expressions

The polynomial expressions suggested in Chapter 5 have proved to be accurate. However, it is likely that simpler expressions can be derived using different mathematical functions. The expressions should also be expanded for damper exponents larger than unity. To improve the expressions, the practical range for application should be established, and a larger number of points within this range should be included in the numerical fit.

Also, as a practical option, instead of the complicated general expressions derived, simpler but more accurate approximations could be obtained if a single value of the damper exponent, corresponding (for instance) to a certain model of a manufacturer, is considered.

8.2.4 Identification of damping system stiffness

In this study, the relevance of the damping system stiffness on the seismic response of frames equipped with viscous dampers has been proved. A key line of research is the accurate identification of stiffness properties of commercial dampers. This could be done in two phases: The first phase would include testing of damper units, independent auxiliary elements (particularly clevises and spherical bearings) and complete damping units. The second phase would be the analysis of experimental data and proposal of approximate expressions to estimate the damping system stiffness as a function of the elements included in the unit.

Because manufacturers generally provide some key elements contributing to the Maxwell system flexibility, a reasonable option is that they establish ‘predefined’ complete bracing sets (with all the elements needed for their direct implementation in the structure) and include in their catalogs the Maxwell information about such units.

8.2.5 Improvement of shear prediction

As shown in the study, RSA renders better predictions of kinematic variables than static variables. In this study, an expression was derived to obtain damper shear directly from peak kinematic variables. The prediction of structural shear with the methods proposed has room for improvement. This could be done deriving structural shear directly from peak drift values.

8.2.6 Additional verification of the proposed method

The verification carried out in this study for frames with LB stiffness and code spectrum contemplated only two intensity levels, with limited inelastic action of the frames. It would be of great interest to add a higher intensity level, corresponding approximately to the Near Collapse intensity level, with more considerable inelastic action.

The verification presented in this study is limited to steel MRFs. The methods need verification and adjustment for concrete MRFs.

It would be of interest to extend the investigation to steel and concrete frames designed without strict capacity conditions.

In ASCE-SEI 7/16 the use of RSA is not limited to a certain height. In this study, response prediction of a 9-storey frame with three different damping systems and RSA proved of dubious accuracy. Use of the proposed methods improved the prediction considerably. Further study is needed. First, the accuracy obtained for different damping distributions in the 9-storey frame should be analyzed, in a similar way to that carried out for the 5-storey frame. Second, taller frames should be included, probably up to 20-25 storeys.

The RSAB method accounts for the damping contribution of each storey independently. It would be possible to use the method for frames with incomplete vertical damper distributions, or with mixed distributions, in which different exponents could be used at different levels. This last strategy has not been explored and it might be advantageous in some respects.

The study presented in Section 7.3.1.2 omitted the important case of frames with rigid brace (RB). This case should be added to complete the study. The LB type stiffness model has been obtained multiplying the HB stiffness by 50%. In Section 5.1.2 it was shown that lower values can be expected. The study could be supplemented with a third type with 25% stiffness. This would also provide valuable data about the performance of the RSAB method for very low bracing stiffness.

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