Multidimensional knapsack problem optimization using a binary particle swarm model with genetic operations

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Abstract Particle swarm optimization is a heuristic and stochastic technique inspired by the flock of birds when looking for food. It is currently being used to solve continuous and discrete optimization problems. This paper proposes a hybrid, genetic inspired algorithm that uses random mutation/crossover operations and adds penalty functions to solve a particular case: the multidimensional knapsack problem. The algorithm implementation uses particle swarm for binary variables with a genetic operator. The particles update is performed in the following way: first using the iterative process (*standard algorithm*) described in the PSO algorithm and then using the best particle position (*local*) and the best global position to perform a random crossover/mutation with the original particle. The mutation and crossover operations specifically apply to personal and global best individuals. The obtained results are promising compared to those obtained by using the probability binary particle swarm optimization algorithm.

Keywords Binary particle swarm optimization · Combinatory optimization · Multidimensional knapsack problem · Genetic operations

1 Introduction

Combinatory optimization research deals with problems that have a finite number of outcomes (Korte and Vygen 2007). As part of the optimization process, the paper focuses on the solution that maximizes or minimizes specific fitness functions according to the following:

$$\min/\max f(x), x \in \mathcal{S} \quad (1)$$

where $\mathcal{S}$ is a finite set of possible solutions.

The cost function (fitness function) f is an application from $\mathcal{S}$ into $\mathfrak{R}$

$$f : \mathcal{S} \rightarrow \mathfrak{R} \quad (2)$$

When the problem is minimization (or maximization), the solution consists of finding a value s^* that fulfills:

$$f(s^*) = \min_{s_i \in \mathcal{S}} f(s_i) \text{ or } f(s^*) = \max_{s_i \in \mathcal{S}} f(s_i) \quad (3)$$

where s^* is a global minimum.

In computational theory, a decision problem is a question in some formal system with a yes-or-no answer, depending on the values of some input parameters. Decision problems are typically defined in mathematical theory and computational complexity as decidable or non-decidable problems, that is to say, It is possible to find a method that determines the existence of set containing an instance of the problem;

some of the most important problems in mathematics are undecidable yet.

The easiest solution for this kind of decision problems is to choose between two alternatives: either 0 or 1. In this case, binary variables are used $x \in \{0, 1\}$ to design the process resolution. However, some issues emerge when using a high numbers of binary variables as this kind of decision problems are more difficult to deal with. In the simplest version of a linear decision model, the evaluation of decision process is basically to find a linear combination of the decisions assigned values (Kellerer et al. 2004).

For example, this occurs in the Boolean Satisfiability problem (Zhang and Malik 2002), commonly referred as SAT. The solution to SAT is to find an interpretation that satisfies a given Boolean formula. More in deep, the problem requires to determine whether the variables of a given boolean formula can be consistently replaced by the values *true* or *false*. By doing so, the formula evaluates to *true*. If this is the case, the formula is called satisfiable. On the other hand, if no such values assignment is found, the function expressed by the formula is *false* and unsatisfiable.

The 0–1 Knapsack problem (KP) is another example that falls into this category (Martello and Toth 1985). This consists of placing a finite number of objects in a knapsack, in which each object has an associated value and volume; the solution not only should not exceed the capacity of the container but must also obtain the highest possible benefit.

This paper propose a hybrid algorithm, based on the meta-heuristic particle swarm optimization (Kennedy and Eberhart 1995), that addresses problems in discrete models, such as the multidimensional and the multi restricted Knapsack problem. To show the robustness of the proposed algorithm in handling other problem types, in the further research, the algorithm must be tested and evaluated over other binary test problems. Moreover, statistical tests such as *Friedman test* must be employed to confirm the performance of the proposed methods statistically. Finally, testing against existing non-PSO state-of-the-art methods is helpful in establishing the algorithms for general purposes.

2 Knapsack problem

Knapsack problem (KP) is a combinatorial optimization problem. Given a set of items with weight and value, the problem consists of finding the number of items to be included in a knapsack so the total weight is less than or equal to a given threshold (knapsack capacity) and the total value (worth) is as large as possible. This is a common problem that a traveler faces when traveling with a fixed-size knapsack that must host the most valuable items. This kind of known problems often appear in fields such as combinatorics, com-

puter science, complexity theory, cryptography and applied mathematics (Martello and Toth 1990).

The way to model it is as follows:

Every decision, i , is equal to 0 or 1 and the benefit value $p_i > 0$. w_i represents the weight of the object. A set of decisions is feasible if the sum of all of them is not greater than the capacity C . This restriction can be checked in Eq. (4).

$$\sum_{i=1}^n w_i x_i \leq C \quad (4)$$

The binary Knapsack problem solution consists of finding the x_i values obtain the maximal benefit; see Eq. (5) (Kellerer et al. 2004).

$$\max \sum_{i=1}^n p_i x_i \quad (5)$$

where $\sum_{i=1}^n w_i x_i \leq C$ with $x_i \in \{0, 1\}$, $i = 1, \dots, n$.

There are many other problems that are modeled in a similar way to the binary knapsack problem such as the financial portfolio optimization, dynamic job programming, employees hiring, logistic operations.

There are also different variations of KP such as the multidimensional knapsack problem (MKP), which has several restrictions and is more complicated to solve. This problem is currently classified as NP-hard, see Labed et al. (2011), Wan and Nolle (2009) and it is formulated as follows:

$$\max \sum_{i=1}^n p_i x_i \quad (6)$$

where $\sum_{i=1}^n w_{ij} x_i \leq C_j$, $j = 1, \dots, m$ with $x_i \in \{0, 1\}$, $i = 1, \dots, n$ and $C_j > 0$, $w_{ij} \geq 0$, $p_i > 0$, $w_{ij} < C_j \leq \sum_{j=1}^m w_{ij}$, $\forall i, j$.

Both kinds of problems have been approached by using linear programming and stochastic algorithms. According to Wan and Nolle (2009), stochastic algorithms have used less computational resources than linear programming obtaining similar results. Furthermore, particle swarm optimization technique has been deployed in several optimization problems such as combinatory optimization, travel salesman problem (TSP) and flow shop problems, see Parsopoulos and Vrahatis (2002b).

2.1 Multidimensional KP and PSO

The 0–1 multidimensional knapsack problem (MKP) is categorized as a difficult NP-hard combinatorial optimization problem. Some population search algorithms try to solve these problems. Specifically, the particle swarm optimiza-

tion (PSO) model and its variants have proved to be a fairly good technique to deal with the MKP.

The essential particle swarm optimization queen (EPSOq) is one of the recent discrete PSO versions that further simplifies the PSO principles and improves its optimization ability. Hybridization is a principle in combining two (or more) approaches. By doing so, the resulting algorithm includes positive features of both (or all) the algorithms. [Ktari and Chabchoub \(2013\)](#) propose a new heuristic approach that is inspired from the Tabu Search and incorporated in the EPSOq algorithm in order to obtain an enhanced discrete PSO version. Experimentally, this approach is able to optimally choose a large-scale of strongly correlated binary values representing multidimensional knapsack problem (MKP) instances. Computational results show that TEPSOq has outperforms not only the EPSOq, but also other existing PSO-based approaches and some other meta-heuristics in solving the 0–1 MKP. Furthermore, this algorithm is able to find extremely closed solutions and even to reach the best known results available in the literature.

A more recent research ([Haddar et al. 2016a](#)) proposes a new hybrid heuristic approach that combines the quantum particle swarm optimization technique ([Haddar et al. 2016b](#)) with a local search method to solve the multidimensional knapsack problem. This approach also incorporates a heuristic repair operator that uses problem-specific knowledge instead of the *standard* penalty function technique commonly used for constrained problems. Experimental results obtained on a wide set of benchmark problems clearly demonstrate the competitiveness of the proposed method compared to the state-of-the-art heuristic methods.

Chin-Jung Lin proposes a novel PSO algorithm, the binary PSO based on surrogate information with proportional acceleration coefficients (BPSOSIPAC) ([Lin et al. 2016](#)). The particles movement of BPSOSIPAC just reuses the proportional acceleration coefficients from the traditional PSOs and incorporate cognition and social learning factors, which reduces the possibilities of influencing the particles movement and the time of computing the velocity of the particles at each iteration. Simultaneously, it stably drives the quality of the MKPs benchmark problems toward the optimal direction in every experiment. The simulation and evaluation results showed that BPSOSIPAC is superior to the other methods in any case for low-dimensional or high-dimensional knapsack problems. Thus, BPSOSIPAC is definitively a promising algorithm in solving multidimensional knapsack problems.

3 Particle swarm optimization

Particle swarm optimization (PSO) is a heuristic optimization technique introduced by [Kennedy and Eberhart \(1995\)](#). It is based on the evolution of an initial random set of points

in a multidimensional space. This set is addressed as population. The population is moving over a searching space by an algorithm that looks for a solution. A particle is an element of the population and represents a candidate solution to the problem. [Kennedy and Eberhart \(1997\)](#) introduced a new algorithm with binary and discrete variables. This approach is inspired by flocks of birds when moving to obtain food. The initial population and the evolutionary process are generated in the same stochastic way as the one in evolutionary algorithms; every single individual of the population is a particle with a speed and location (multidimensional coordinates).

Particles are evolutionary solutions that move based on the combination of their best personal solution (*pbest*—personal best) and the best solution reached by their neighbor's particles (*lbest*—local best). When the neighborhood is designed following the standards of a star topology, the *lbest* term is referred as *gbest* (global best) since all particles are neighbors (see Fig. 2). Furthermore, the performance (*fitness* function) of every particle is computed in every evolutionary step.

In the binary problem, the particles are strings with 0's and 1's so they can be used in a similar way than genetic algorithm individuals.

Initially, particles are randomly distributed in the problem domain, and they move along the searching space by using specific rules for each evolutionary step. The goal consists of obtaining the best solution (max or min fitness value) moving (evolving) every particle closer to the neighbor that has the best fitness so far; in that way, it influences the other particles too; in that way, the algorithm updates the swarm of particles in every evolutionary step by updating the position and velocity of every particle.

3.1 Standard PSO model

Let us define the following symbols to represent properties of a particle:

- x_i is the current position of particle i
- v_i is the current velocity of particle i
- p_{Best} is the personal best position of the particle
- g_{Best} is the global best particle
- $c_1 \in R$ is the personal influence (acceleration coefficient)
- $c_2 \in R$ is the global influence (acceleration coefficient)
- r_1 and r_2 are random numbers distributed using a uniform pattern on interval $[0, 1]$.

With these notations, the formula to calculate a particle's velocity at time $t + 1$ (time is simulated using iteration number) is:

$$v_i(t + 1) = v_i(t) + c_1 * r_1 * (p_{Best} - x_i) + c_2 * r_2 * (g_{Best} - x_i) \quad (7)$$

Fig. 1 Particle swarm optimization model: position update using a star topology (g_{Best}) and an inertia w term

Fig. 2 Particle swarm topology: g_{Best} (star) topology and l_{Best} (ring) topology (from left to right)

where, r_1 and r_2 are randomly generated for every velocity update and $0 \leq r_1, r_2 \leq 1$. Such parameters should be set with a different value in each iteration. And c_1, c_2 are user-defined values called acceleration coefficients where $0 \leq c_1, c_2 \leq 2$. Their value depends on the problem to be optimized.

New particle position at time $t + 1$ just adds the newly calculated velocity to its current position at time t . In other words, the position now is the previous one adding its velocity, see Fig. 1.

$$x_i(t + 1) = x_i(t) + v_i(t + 1) \quad (8)$$

Note that g_{Best} refers to a star topology, see Fig. 2.

Algorithm 1 Standard particle swarm optimization algorithm: update process.

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1: for Each time step  $t$  do
2:   for Each particle  $i$  in the swarm do
3:     update position  $x_i(t + 1)$  using equations (7, 8)
4:     calculate particle fitness  $f(x_i(t + 1))$ 
5:     update  $p_{Best}$  and  $g_{Best}$ 
6:   end for
7: end for

```

3.1.1 Swarm topology

g_{Best} is used because the particles reside in a star topology neighborhood. This means that all particles are connected to each other, see Fig. 2. Another topology, the l_{Best} swarm model, often referred to as a local topology, see Fig. 2 (right), constitutes perhaps the most significant variation in the original PSO algorithm, and was in fact proposed in one of the very first PSO publications (Eberhart and Kennedy 1995). For one reason or another, this topology has not been widely used in the community up to this point. Original investigations into this model using the original PSO algorithm showed inferior performance when compared to the global g_{Best} model, or global topology, shown in Fig. 2 (left), but more recent research has revealed that l_{Best} swarms return improved results across many standard problem sets when used in conjunction with other improvements to the algorithm (Kennedy and Mendes 2006). It is important to note that the local topology only affects Eq. (7), where:

$$v_i(t + 1) = v_i(t) + c_1 * r_1 * (p_{Best} - x_i) + c_2 * r_2 * (l_{Best} - x_i) \quad (9)$$

Much of the PSO literature uses the term local topology to describe not just a single swarm model, but applies it to any swarm model without global communication. A number of different limited communication topologies have been tested with varying results; the l_{Best} model shown is perhaps the simplest form of a local topology, what is known as the ring model, shown in Fig. 2 (right). The l_{Best} ring model connects each particle to only two other particles in the swarm, in contrast to the g_{Best} model where every particle is able to obtain information from the very best particle in the entire swarm population.

The advantage of the l_{Best} model appears to lie in its slower convergence rate relative to the g_{Best} model. Having convergence take place when the swarm has found the global optimum is obviously beneficial, but when the converged-upon location is suboptimal, it is referred to as *premature* and is undesirable as it prevents the algorithm from escaping from an inferior local optimum.

Ironically, it is the slower rate of convergence of the l_{Best} model that is most responsible for the general disregard of it as an alternative up to this point. The much faster convergence of the g_{Best} model seems to indicate that it produces superior performance, but this is misleading. While results for the g_{Best} model are indeed superior for many problems relatively early in the optimization process, the best found fitness for the l_{Best} model quite often surpasses that of the g_{Best} after some number of function evaluations have been performed, particularly on multimodal problems.

Despite the advantages of a local topology, it is important to note that it should not always be considered to be the optimal choice in all situations. The faster convergence rate of a global topology will usually result in better performance on simple unimodal problems than that of any local topology due to the lack of any danger of convergence on a suboptimal local minima. Even on some very complex multimodal problems, a g_{Best} model swarm can deliver performance competitive with the l_{Best} model, given proper circumstances. For thorough, rigorous results, tests should be run using both topologies, but in the interest of having a single standard PSO algorithm, the superior performance of the l_{Best} model over the majority of benchmarks qualifies it as the better choice for cases where a straightforward means of comparison is desired. In any case, modern research performed using only swarms with a global topology is incomplete at best (Eberhart and Kennedy 1995; Kennedy and Mendes 2006).

The inclusion of the local ring topology as part of a standard algorithm for particle swarm optimization comes with a caveat, however. Given the slower convergence of the l_{Best} model, more function evaluations are required for the improved performance to be seen. This is especially important on unimodal functions, where the fast convergence of the g_{Best} model combined with a single minima in the feasible

search space results in quicker performance than that of the g_{Best} swarm with its limited communication.

3.2 Binary PSO model

Kennedy and Eberhart (1997) proposed a discrete binary version of PSO for binary problems. In their model, a particle will decide on *yes* or *no*, also this binary values can be a representation of a real value in binary search space. The binary model is based on the probability of making decisions. Making a decision is modeled as 1 (yes) or 0 (no). In the end, a particle can decide yes or no (1 or 0) based on its fitness value and some probability rules.

The probability P of getting a 1 in particle i in dimension d at time t (represented by $x_{id}(t)$) depends on:

$$P(x_{id}(t+1) = 1) = f(x_{id}(t), v_{id}(t), p_{id}, p_{gd}) \quad (10)$$

where:

- $x_{id}(t)$ is the status of the particle i in the position d
- $v_{id}(t)$ is the velocity of the individual
- p_{id} is the best position of the particle i , found in the dimension d
- p_{gd} is the best position of all its neighbors, with dimension d

In the binary PSO, the particles personal best and global best is updated as in continuous version. The major difference between binary PSO with continuous version is that velocities of the particles are rather defined in terms of probabilities that a bit will change to one. Using this definition a velocity must be restricted within the range $[0, 1]$. So a map is introduced to map all real-valued numbers of velocity to the range $[0, 1]$. New velocity v'_{id} is computed as:

$$v'_{id}(t) = \text{sigmoid}(v_{id}(t)) = \frac{1}{1 + e^{-v_{ij}(t)}} \quad (11)$$

In order to update $v_{id}(t)$, the following formula is used:

$$\begin{aligned} v_{id}(t+1) = & v_{id}(t) \\ & + \varphi_1 \epsilon_1 (p_{id} - x_{id}(t)) \\ & + \varphi_2 \epsilon_2 (p_{gd} - x_{id}(t)) \end{aligned} \quad (12)$$

where φ_1 y φ_2 represent cognitive and social factors fulfilling $\varphi_1 + \varphi_2 = 4$ and (ϵ_1, ϵ_2) are independent random uniform numbers within $(0, 1)$.

The following equation updates the particle's position:

$$x_{id}(t) = \begin{cases} 1 & \text{if } \rho_{id} < s(v_{id}(t)) \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

where ρ_i is an array of random numbers and s is a function that returns values within $[0,1]$, which makes the function act as a threshold [similar to the sigmoid function, see Eq. (11)], which is widely used in neural networks.

3.3 Model variants

Regarding the PSO algorithm, different variants have been developed, aimed at speeding up the convergence of it. In addition to the unconstrained optimization problem in discrete or continuous variable, the multitarget problem and the constrained problem have been addressed. The new guidelines are aimed at avoiding PSO stagnation of the local optimal solutions.

Shi and Eberhart (1998) proposed adjustments to the velocities of the particles by using a factor w called *inertial weight*. This factor utilizes the inertia of the particles in the process of friction when they are moving. This modification in the algorithm is done to control the search space. In order to do that, it must change (14). The large inertia weight makes the global search easier; however, small inertia weight does not improve local search. That is why was the initial value is greater than 1.0 to promote global exploration and then gradually decreases to obtain more refined solutions. The algorithm decreases linearly at each iteration. Moreover, the use of inertial weight removes the restriction $V_{\max}$ on the velocity.

$$v_d^{(i)} = wv_d^{(i)} + c_1\epsilon_1(p_d^{(i)} - x_d^{(i)}) + c_2\epsilon_2(g_d^{(i)} - x_d^{(i)}) \quad (14)$$

In each iteration, inertia weight decrease linearly through the following expression:

$$w = w_{\max} - (w_{\max} - w_{\min}) \frac{g}{G} \quad (15)$$

g is the index of the generation, G is the maximum number of iterations previously determined, $w_{\max}$ is a value greater than 1, and $w_{\min}$ a value under 0.5. This variation of the method has proven to accelerate convergence.

Term *velocity clamping* has not been previously included, but it is necessary to restrain velocity updates. Without doing so, particles would move too far from the search space, ignoring the current solution. If the search space has a range $[-x_{\max}, x_{\max}]$, then the velocity should also have a range $[-v_{\max}, v_{\max}]$. Our proposal for $v_{\max}$ is $k * x_{\max}$, k being the clamping factor between 0.1 and 1. The maximum velocity works as a constraint to control the global exploration of a swarm. If the value is too high, particles might bypass good solution and the exploration would be poor. On the other hand, slow particles might only be searching locally which would be good for local solutions but not for finding a global one; this means that the exploiting process becomes

poor too. Balancing between exploration and exploitation considerably improves PSO. Introducing an inertia weight, for example, would sort that out.

Clerc and Kennedy (2002) obtain another variation in the speed calculation. A constriction factor χ is introduced with that purpose. This factor depends on the constants that are used when calculating speed. This factor affects the formula (14). The aim is to avoid the explosion of velocity:

$$v_d^{(i)} = \chi \left[v_d^{(i)} + c_1\epsilon_1(p_d^{(i)} - x_d^{(i)}) + c_2\epsilon_2(g_d^{(i)} - x_d^{(i)}) \right] \quad (16)$$

χ is:

$$\chi = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|}, \quad \varphi = c_1 + c_2 = 4.1$$

The results are: $\chi = 0.729$ and $c_1 = c_2 = 2.05$. These parameters were obtained performing several tests.

χ factor is similar to the inertial weight. This means that controlling the velocity with $V_{\max}$ is not required when χ is used. Bratton and Kennedy (2007), analyzed the stability of this algorithm by using these values and by following a comparative study of both PSO algorithms (inertial weight and χ factor). Both of them are mathematically equivalent, in particular the algorithm with constriction factor is a special case of the inertial weight. Moreover, Parsopoulos and Vrahatis (2002a) combined both for problems with constraints and they obtained equally good results in several tests.

We observe that the convergence always becomes slower when problem size increase, so when it comes to high-dimensional problems, a larger number of iterations occurs. Korenaga et al. (2007) developed a PSO model, where velocity values are updated, by considering the rotation of the coordinate system. This model is aimed at problems of high dimensionality and it showed good results when applied to all functions of Potter and Jong (1994).

4 Adapting PSO to the MKP

One of the constraints The MKP problem (6) has is that variables can only be equal to either 0 or 1. When running PSO, every particle p are set to either 0 or 1. According to Wang et al. (2008), there are different ways to solve the constraint problem such as the use of penalty functions method, the *greedy* algorithm or the *Surrogate Duality* method (Li and Sun 2006). When working with constraints $\{g_j(x)\}$, the penalty functions φ are the most commonly used. These functions return some degree of solution unfeasibility when the violation of a constraint occurs.

$$\varphi = \sum_{j=1}^m \max(0, g_j(x)) \quad (17)$$

where the *fitness* function of a given particle p_i is:

$$\text{fitness}(p_i) = f(x)^i + \varphi^i \quad (18)$$

If there is no constraint violation, then φ^i is equal to zero. Michalewicz and Schoenauer (1996) show several types of penalty-based algorithms such as static ones, dynamic ones and adaptative ones. Each one of them has advantages and disadvantages.

As this problem involves linear restrictions and binary variables, the most simple method is proposed in this paper which is the one that uses the penalty function (17). The functions output is multiplied by a big constant number—KMAX, and then, the result is evaluated as shown in (18).

In order to solve MKP with PSO this work has been inspired by Hembecker et al. (2007) who used PSO without optimized parameters to deal with the multirestriction problem with a penalty method. Tests were carried out for a sample of data: (Shizuo Senju 1968; Weingartner and Ness 1967; Shih 1979), and the results were fairly closed to the optimal ones. Furthermore, Wan and Nolle (2009) introduced a hybrid algorithm which used a known operator based on heritage (random bits of the “parent” particle g_{best} y l_{best} were copied over to the “children” particles). On top of this, they used a mutation operator which updated a bit randomly by using a fix mutation rate equal to 0.33. The particles then were forced to fulfill all the constraint by deploying a repairing operator. With all these updates, they also reached optimal results for high-dimensional types (Chu and Beasley 1998). A different approach was tried by Wang et al. (2008), who presented a new strategy to update the particle position by using a linear function $L(x)$, with parameters $R_{\min}$ and $R_{\max}$. Wang also used random numbers generation processes to transform a vector of real numbers into a vector of binary values.

4.1 Proposed algorithm (HPSOGO)

The proposed implementation, see Algorithm 2, uses PSO for binary variables, although the particles update is performed in three different ways:

- (a) An iterative process that uses (12) y (13) to upgrade all the components of the particles. The vector components ρ_i are random numbers which are generated in the program.
- (b) A random number $rand_1$ within the interval $(1, M)$ is generated; this value is the number of components to be replaced by the optimal ones (best personal particle) in

the current solution (note that M is the dimension of a given particle).

- (c) Another random number $rand_2$ within $(1, M)$ is created; this value is the number of components to be replaced by the optimal ones (best particle among the whole swarm) in the current solution.

The update process in item (a) is what is normally used in PSO. Updates items (b) and (c) are similar to the *crossover/mutation* operations which are commonly used in genetic algorithms. Figure 3 shows operations (b) and (c).

Algorithm 2 Hybrid particle swarm optimization algorithm using binary variables.

```

1: Init population  $\{x^i\}$  in the domain interval
2: for Each particle  $i$  in the swarm do
3:   if ( $f(x) > f(p)$ ) then
4:     for ( $d = 1$  until  $D$ ) do
5:        $p_{id} = x_{id}$ 
6:     end for
7:   end if
8:    $g = i$ 
9:   for  $j \in J$  do
10:    if ( $f(p_j) > f(p_g)$ ) then
11:      ( $g = j$ )
12:    end if
13:  end for
14:  for  $d = 1$  until  $D$  do
15:     $v_{id}(t) = v_{id}(t-1) + c_1 \epsilon_1 (p_{id} - x_{id}(t-1)) +$ 
16:       $+ c_2 \epsilon_2 (g_d - x_{id}(t-1))$ 
17:    if ( $\rho_{id} < s(v_{id}(t))$ ) then
18:       $x_{id} = 1$ 
19:      if not  $x_{id} = 0$ 
20:        end if
21:    end if
22:     $nmut = rand(1, M)$ 
23:    for  $j = 1$  until  $nmut$  do
24:       $k = rand(1, M)$ 
25:       $x_{ik} = p_{ik}$ 
26:    end for
27:     $nmut = rand(1, M)$ 
28:    for  $j = 1$  until  $nmut$  do
29:       $k = rand(1, M)$ 
30:       $x_{ik} = g_k$ 
31:    end for
32:  end for

```

5 Experiment and results using ORLib benchmarks

Benchmark data samples were obtained from known problems (Shizuo Senju 1968; Weingartner and Ness 1967; Shih 1979). This section attempts to apply benchmark data sets to model evaluation. Proposed algorithm (HPSOGO) was tested using several groups of MKP benchmarks selected from OR-Library (Beasley 1990). The first group contains 10

Fig. 3 Genetic mutation over a current particle: 2 random bits from p_{Best} and 1 random bit from g_{Best} are copied to the current particle

Table 1 Results after applying the algorithm to the Weing data set

Prob. <i>weing</i>	<i>n</i>	<i>m</i>	Optimal	It	Best obtained	gap
1	28	2	141.278	360	141.278	0
2	28	2	130.883	360	130.883	0
3	28	2	95.677	2.000	95.677	0
4	28	2	119.337	2.000	119.337	0
5	28	2	98.796	539	98.796	0
6	28	2	130.623	539	130.623	0
7	105	2	1095.445	7.000	1094.192	0.11
8	105	2	624.319	7.000	615.927	1.34

Table 2 Results after applying the algorithm to the Sento data set

Prob. <i>sento</i>	<i>n</i>	<i>m</i>	Optimal	It	Best obtained	gap
1	60	30	7.772	3.605	7.772	0
2	60	30	8.722	5.000	8.722	0

benchmark problems and corresponds to “SENTO” (Shizuo Senju 1968) and “WEING” (Weingartner and Ness 1967). The second group contains 30 benchmark problems and corresponds to “WEISH” (Shih 1979).

The algorithm was built-in wxDev-C++ and executed 20 times. The swarms had 15 particles. Results are described in Tables 1, 2 and 3. It also shows the number of variables (n), number of constraints (m), the optimal known value, the average number of iterations (it), the optimal value obtained after 20 executions and the *gap*. Indeed, for the set of the instances tested in this paper the optimal value is already known. This last value is used to measure the error (optimal value known, optimal value found) and it is calculated according to the following expression.

$$gap = \frac{\text{optimal value} - \text{best found}}{\text{optimal value}} \quad (19)$$

Table 3 Results after applying the algorithm to the Weish data set

Prob. <i>weish</i>	<i>n</i>	<i>m</i>	Optimal	It	Best obtained	gap
1	30	5	4.554	1.000	4.554	0
2	30	5	4.536	1.000	4.536	0
3	30	5	4.115	1.000	4.115	0
4	30	5	4.561	1.000	4.561	0
5	30	5	4.514	500	4.514	0
6	40	5	5.557	1.000	5.557	0
7	40	5	5.567	500	5.567	0
8	40	5	5.605	1.000	5.605	0
9	40	5	5.246	500	5.246	0
10	50	5	6.339	1.000	6.339	0
11	50	5	5.643	2.000	5.643	0
12	50	5	6.339	1.000	6.339	0
13	50	5	6.159	1.000	6.159	0
14	60	5	6.954	1.500	6.954	0
15	60	5	7.486	1.500	7.486	0
16	60	5	7.289	2.000	7.289	0
17	60	5	8.633	2.000	8.633	0
18	70	5	9.580	1.500	9.580	0
19	70	5	7.698	2.000	7.698	0
20	70	5	9.450	4.000	9.450	0
21	70	5	9.074	7.000	9.074	0
22	80	5	8.947	5.400	8.947	0
23	80	5	8.344	5.850	8.344	0
24	80	5	10.220	7.000	10.220	0
25	80	5	9.939	7.000	9.939	0
26	80	5	9.584	8.094	9.563	0.21
27	90	5	9.819	7.197	9.819	0
28	90	5	9.492	7.197	9.398	0.99
30	90	5	11.191	7.000	11.191	0

If the value is 0, the optimal solution is found. Note that *weing7* and *weing8* did not obtain the best solution after 20 executions of their algorithm. However in different times, the algorithm was able to find the optimal solutions after

Table 4 Results after applying the proposed algorithm (HPSOGO) to four problems, in comparison with PBPSO

Prob.	Optimal	Algorithm	Best obtained	Success rate (%)	Average fitness	Average execution time
Sento1	7.772	PBPSO	7.772	5	7.6959	3.158
$n = 60$		HPSOGO	7.772	25	7.7339	3.605
$m = 30$						
Sento2	8.722	PBPSO	8.772	5	8.651	2.221
$n = 60$		HPSOGO	8.722	10	8.6893	5.000
$m = 30$						
Weish12	6.339	PBPSO	6.339	45	6.33175	1.436
$n = 50$		HPSOGO	6.339	35	6.30505	1.000
$m = 5$						
Weish20	9.450	PBPSO	9.450	5	9.36205	3.652
$n = 50$		HPSOGO	9.450	10	9.39835	4.000
$m = 5$						

Bold values represent best obtained result

20 executions. This occurred due to the inherent stochastic nature of the technique.

Table 4 shows results using HPSOGO with benchmark problems *sento1*, *sento2*, *weish12* and *weish20*, so it is possible to compare them with the ones obtained by Wang et al. (2008) after executing the probability binary particle swarm optimization algorithm (PBPSO). The entries are: problem's name, optimal known value, applied algorithm, optimal obtained solution, success rate after 20 independent executions, optimal values average after 20 executions and average number of iterations after 20 executions.

After applying the proposed algorithm (HPSOGO) in all the cases, the optimal value was returned (same as with PBPSO). It is noticeable a good performance in the proposed algorithm, although the number of iterations get higher than the ones of PBPSO (mainly because this worked with a 60 particles population). If each particle is evaluated for every iterative step, 2.221 iterations of PBPSO make 133.260 functions be evaluated; when 5.000 iterations of the proposed algorithm with 15 particles are evaluated, 75.000 functions are checked. Furthermore, it showed a better performance than Kennedy and Eberhart algorithms Kennedy and Eberhart (1997), Shen et al. (2004) and Hemberger et al. (2007), for the same particular problems.

5.1 Comparison with recent research

These benchmark instances were likewise solved using other existing PSO algorithms: BPSOTVAC (Chih et al. 2014), CBPSOTVAC (Chih et al. 2014), BPSOSIPAC (Lin et al. 2016) and PBPSO (Wang et al. 2008). Previous methods have considered 0–1 MKP benchmark instances where the optimal solution is known. In order to fairly compare the proposed algorithm with that in the previous studies, we mainly followed the measures and parameter settings which are the

same as the different existing algorithms used in their original works. Thus, the comparison among these PSO algorithms was performed based on five performance measures used in Chih et al. (2014): success rate (SR)—the value enclosed in parentheses is the quality measure (standard error) of the corresponding estimate—, mean absolute percentage error (MAPE), least error (LE) and standard deviation (SD). SR is the number of runs out of all of the executions that produce the optimum solution within the termination criterion. MAD is the average of the absolute difference between the simulation data and the given optimal solution. MAPE is obtained by dividing MAD by the corresponding optimal solution and usually expresses the accuracy as a percentage. LE is the least error obtained by the minimum of absolute difference between the optimum solution and final solution. SD is the standard deviation of final solutions over runs.

Based on the results of Table 5, the average 85.9% SR of HPSOGO is superior to the average below 64.8% SR of MBPSO, CBPSO1, BPSOTVAC and CBPSOTVAC for the *sento* and *weish*. Based on the results, the average 0.4% SR of HPSO is superior to the average 0% SR of BPSOTVAC and CBPSOTVAC. In addition, HPSOGO is superior to MBPSO, CBPSO1, BPSOTVAC and CBPSOTVAC on the measure of MAD, MAPE, LE and SD. Therefore, BPSOSIPAC outperforms the other four PSOs in terms of the four criteria.

HPSOGO was compared with the PSO algorithms for the *weing* problems. Simulation results are shown in Table 5. Based on the results, the average 100% SR of HPSOGO is superior to the average below 74.8 SR of BPSO, SCPSO and CBPSOTVAC. In addition, HPSOGO is superior to BPSO, SCPSO and CBPSOTVAC on the measure of SR, MAPE, LE and SD. Therefore, HPSOGO outperforms the other PSOs in terms of the four criteria.

HPSOGO and BPSOSIPAC have a similar performance.

Table 5 Comparing proposed algorithm with the MBPSO, CBPSO1, BPSOTVAC, CBPSOTVAC and BPSOSIPAC algorithms proposed in Chih et al. (2014)

Problem	#Knapsacks	#Items	Algorithm	SR	MAPE	LE	SD
Sento1	30	60	MBPSO	0.16 (0.04)	0.0058 (0.0006)	229	43.23
			CBPSO1	0 (0)	0.026 (0.001)	302	49.92
			BPSOTVAC	0.57 (0.05)	0.0011 (0.0001)	34	11.52
			CBPSOTVAC	0.39 (0.05)	0.021 (0.007)	3146	357.78
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	0.25 (0.01)	0 (0)	35	9.21
Sento2	30	60	MBPSO	0.03 (0.02)	0.0029 (0.0002)	81	18.8
			CBPSO1	0 (0)	0.012 (0.0003)	150	25.78
			BPSOTVAC	0.27 (0.04)	0.001 (0.0001)	38	7.04
			CBPSOTVAC	0.2 (0.04)	0.0063 (0.0012)	633	101.03
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	0.1 (0.02)	0 (0)	52	8.01
Weing1	2	28	MBPSO	0.82 (0.04)	0.0008 (0.0002)	801	250.43
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.92 (0.03)	0.0004 (0.0002)	1961	281.98
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weing2	2	28	MBPSO	0.65 (0.05)	0.0009 (0.0002)	1700	314.08
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.88 (0.03)	0.0009 (0.0004)	3341	545.5
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weing3	2	28	MBPSO	0.11 (0.03)	0.0112 (0.0009)	3500	876.78
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	0.92 (0.03)	0.00007 (0.00003)	160	25.53
			CBPSOTVAC	0.75 (0.04)	0.0019 (0.0007)	3789	672.42
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weing4	2	28	MBPSO	0.76 (0.04)	0.0049 (0.0011)	4001	1270.8
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.97 (0.02)	0.0004 (0.0003)	3774	378.58
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weing5	2	28	MBPSO	0.52 (0.05)	0.017 (0.002)	4778	1923.5
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.94 (0.02)	0.0009 (0.0006)	4728	572.82
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weing6	2	28	MBPSO	0.36 (0.05)	0.0023 (0.0002)	1340	322.4
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	0.97 (0.02)	0.00009 (0.00005)	390	66.86
			CBPSOTVAC	0.87 (0.03)	0.0007 (0.0003)	2460	343.45
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0

Table 5 continued

Problem	#Knapsacks	#Items	Algorithm	SR	MAPE	LE	SD
Weing7	2	105	MBPSO	0.02 (0.01)	0.0006 (0.0001)	6111	1130.6
			CBPSO1	0 (0)	0.0308 (0.0005)	42,425	5002
			BPSOTVAC	0 (0)	0.00026 (0.00004)	2069	383.74
			CBPSOTVAC	0 (0)	0.011 (0.003)	154,486	30,020
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0.11 (0.3)	0	0
Weing8	2	105	MBPSO	0.03 (0.02)	0.0095 (0.0008)	44,731	4704.3
			CBPSO1	0 (0)	0.234 (0.004)	160,402	15,988
			BPSOTVAC	0.35 (0.05)	0.0030 (0.0003)	6463	2000.9
			CBPSOTVAC	0.20 (0.04)	13.7 (13.6)	623,862	75,169
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	1.34 (1.36)	0	0
Weish1	5	30	MBPSO	0.82 (0.04)	0.0024 (0.0006)	114	26.34
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.94 (0.02)	0.0012 (0.0008)	248	32.81
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish2	5	30	MBPSO	0.55 (0.05)	0.0018 (0.0004)	123	18.01
			CBPSO1	0.79 (0.04)	0.00023 (0.00005)	5	2.04
			BPSOTVAC	0.64 (0.05)	0.00040 (0.00005)	5	2.41
			CBPSOTVAC	0.66 (0.05)	0.0009 (0.0005)	231	23.12
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish3	5	30	MBPSO	0.63 (0.05)	0.0051 (0.0009)	141	34.98
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	0.99 (0.01)	0.00015 (0.00015)	63	6.3
			CBPSOTVAC	0.95 (0.02)	0.0024 (0.0014)	394	52.69
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish4	5	30	MBPSO	0.96 (0.02)	0.0004 (0.0002)	56	8.99
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.99 (0.01)	0.0023 (0.0023)	859	85.9
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish5	5	30	MBPSO	0.99 (0.01)	0.00012 (0.00012)	54	5.4
			CBPSO1	1 (0)	0 (0)	0	0
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.98 (0.01)	0.0021 (0.0019)	742	74.45
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish6	5	40	MBPSO	0.32 (0.05)	0.0028 (0.0003)	56	14.39
			CBPSO1	0.65 (0.05)	0.00098 (0.00014)	34	7.92
			BPSOTVAC	0.59 (0.05)	0.00121 (0.00015)	18	8.19
			CBPSOTVAC	0.53 (0.05)	0.0044 (0.0016)	518	79.28
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0

Table 5 continued

Problem	#Knapsacks	#Items	Algorithm	SR	MAPE	LE	SD
Weish7	5	40	MBPSO	0.64 (0.05)	0.0018 (0.0003)	122	18.92
			CBPSO1	0.83 (0.04)	0.0006 (0.0001)	25	7.79
			BPSOTVAC	0.96 (0.02)	0.00013 (0.00006)	18	3.45
			CBPSOTVAC	0.78 (0.04)	0.0036 (0.0014)	511	71.95
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish8	5	40	MBPSO	0.44 (0.05)	0.0013 (0.0002)	72	13.07
			CBPSO1	0.64 (0.05)	0.00013 (0.00002)	2	0.96
			BPSOTVAC	0.79 (0.04)	0.00008 (0.00001)	2	0.82
			CBPSOTVAC	0.68 (0.05)	0.0016 (0.0008)	418	42.81
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish9	5	40	MBPSO	0.78 (0.04)	0.0021 (0.0005)	200	25.65
			CBPSO1	0.96 (0.02)	0.0003 (0.0001)	34	6.69
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.85 (0.04)	0.0027 (0.0014)	641	65.7
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish10	5	50	MBPSO	0.56 (0.05)	0.0017 (0.0004)	83	22.17
			CBPSO1	0.07 (0.03)	0.0068 (0.0005)	141	33.65
			BPSOTVAC	0.91 (0.03)	0.0002 (0.0002)	68	9.56
			CBPSOTVAC	0.67 (0.05)	0.0102 (0.0037)	1394	188.63
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish11	5	50	MBPSO	0.4 (0.05)	0.0053 (0.0008)	167	43.95
			CBPSO1	0.05 (0.02)	0.0144 (0.0008)	191	47.06
			BPSOTVAC	0.88 (0.03)	0.0013 (0.0005)	113	25.72
			CBPSOTVAC	0.62 (0.05)	0.028 (0.011)	2245	403.03
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish12	5	50	MBPSO	0.65 (0.05)	0.0026 (0.0006)	226	35.68
			CBPSO1	0.09 (0.03)	0.0092 (0.0007)	191	42.35
			BPSOTVAC	0.89 (0.03)	0.00005 (0.00003)	19	1.91
			CBPSOTVAC	0.71 (0.04)	0.020 (0.006)	1497	304.43
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	0.35 (0.01)	0 (0)	34	3.83
Weish13	5	50	MBPSO	0.87 (0.03)	0.0014 (0.0004)	155	25.19
			CBPSO1	0.15 (0.04)	0.0098 (0.0007)	159	40.97
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.85 (0.04)	0.0075 (0.0039)	1725	180.04
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish14	5	60	MBPSO	0.66 (0.05)	0.0023 (0.0004)	100	25.95
			CBPSO1	0 (0)	0.031 (0.001)	347	66.79
			BPSOTVAC	0.98 (0.01)	0.00089 (0.00006)	31	4.36
			CBPSOTVAC	0.79 (0.04)	0.021 (0.007)	2127	364.66
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0

Table 5 continued

Problem	#Knapsacks	#Items	Algorithm	SR	MAPE	LE	SD
Weish15	5	60	MBPSO	0.72 (0.05)	0.0014 (0.0002)	70	18.64
			CBPSO1	0 (0)	0.0266 (0.0008)	359	59.99
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.8 (0.04)	0.030 (0.011)	2978	554.35
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish16	5	60	MBPSO	0.44 (0.05)	0.0011 (0.0002)	87	17.49
			CBPSO1	0 (0)	0.0195 (0.0007)	259	47.07
			BPSOTVAC	0.54 (0.05)	0.00016 (0.00002)	8	1.71
			CBPSOTVAC	0.43 (0.05)	0.023 (0.006)	1931	367.29
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish17	5	60	MBPSO	0.56 (0.05)	0.0007 (0.0001)	41	7.38
			CBPSO1	0 (0)	0.0107 (0.0004)	170	32.79
			BPSOTVAC	1 (0)	0 (0)	0	0
			CBPSOTVAC	0.72 (0.05)	0.011 (0.003)	1035	227.16
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish18	5	70	MBPSO	0.38 (0.05)	0.0015 (0.0002)	94	18.4
			CBPSO1	0 (0)	0.0278 (0.0006)	367	54.01
			BPSOTVAC	0.75 (0.04)	0.00029 (0.00005)	15	5.25
			CBPSOTVAC	0.53 (0.05)	0.011 (0.003)	1595	275.53
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish19	5	70	MBPSO	0.55 (0.05)	0.0027 (0.0004)	149	33.67
			CBPSO1	0 (0)	0.059 (0.001)	615	83.65
			BPSOTVAC	0.65 (0.05)	0.0006 (0.0001)	35	7.13
			CBPSOTVAC	0.62 (0.05)	0.028 (0.009)	3060	489.37
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish20	5	70	MBPSO	0.53 (0.05)	0.0011 (0.0002)	69	15.99
			CBPSO1	0 (0)	0.037 (0.001)	528	89.41
			BPSOTVAC	0.78 (0.04)	0.0004 (0.00008)	20	7.53
			CBPSOTVAC	0.69 (0.05)	0.015 (0.005)	2482	410.74
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	0.1 (0.01)	0 (0)	52	15.01
Weish21	5	70	MBPSO	0.61 (0.05)	0.0019 (0.0003)	88	24.97
			CBPSO1	0 (0)	0.039 (0.001)	499	84.42
			BPSOTVAC	0.74 (0.04)	0.0007 (0.0001)	24	10.41
			CBPSOTVAC	0.67 (0.05)	0.016 (0.005)	2574	378.38
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish22	5	80	MBPSO	0.33 (0.05)	0.0033 (0.0004)	112	31.55
			CBPSO1	0 (0)	0.082 (0.001)	935	94.69
			BPSOTVAC	0.16 (0.04)	0.00169 (0.00007)	18	6.63
			CBPSOTVAC	0.17 (0.04)	0.024 (0.007)	3063	486.71
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0

Table 5 continued

Problem	#Knapsacks	#Items	Algorithm	SR	MAPE	LE	SD
Weish23	5	80	MBPSO	0.24 (0.04)	0.0036 (0.0004)	126	35.43
			CBPSO1	0 (0)	0.087 (0.002)	902	119.64
			BPSOTVAC	0.85 (0.04)	0.00013 (0.00006)	36	5.11
			CBPSOTVAC	0.58 (0.05)	0.026 (0.007)	3114	437.23
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish24	5	80	MBPSO	0.27 (0.04)	0.0017 (0.0002)	70	18.09
			CBPSO1	0 (0)	0.0373 (0.0006)	499	56.96
			BPSOTVAC	0.7 (0.05)	0.00029 (0.00006)	31	6.44
			CBPSOTVAC	0.55 (0.05)	0.012 (0.003)	1841	295.79
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish25	5	80	MBPSO	0.29 (0.05)	0.0015 (0.0001)	61	13.39
			CBPSO1	0 (0)	0.0475 (0.0009)	648	86.21
			BPSOTVAC	0.49 (0.05)	0.00045 (0.00007)	24	7.09
			CBPSOTVAC	0.32 (0.05)	0.013 (0.004)	2321	361.88
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish26	5	90	MBPSO	0.31 (0.05)	0.0028 (0.0002)	114	24.27
			CBPSO1	0 (0)	0.103 (0.001)	1122	126.39
			BPSOTVAC	0.36 (0.05)	0.0012 (0.0001)	47	12.81
			CBPSOTVAC	0.28 (0.04)	0.04 (0.01)	4084	710.77
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0.21 (0.0005)	0	0
Weish27	5	90	MBPSO	0.65 (0.05)	0.0024 (0.0005)	203	48.62
			CBPSO1	0 (0)	0.109 (0.001)	1205	120.34
			BPSOTVAC	0.99 (0.01)	0.00004 (0.00004)	39	3.9
			CBPSOTVAC	0.83 (0.04)	0.028 (0.009)	3915	640.43
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0
Weish28	5	90	MBPSO	0.64 (0.05)	0.0016 (0.0003)	144	26.72
			CBPSO1	0 (0)	0.115 (0.002)	1266	122.32
			BPSOTVAC	0.87 (0.03)	0.00031 (0.00008)	23	7.77
			CBPSOTVAC	0.62 (0.05)	0.06 (0.02)	4387	887.33
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0.99 (0.43)	0	0
Weish30	5	90	MBPSO	0.38 (0.05)	0.001 (0.0001)	57	14.48
			CBPSO1	0 (0)	0.0516 (0.0009)	760	87.58
			BPSOTVAC	0.87 (0.03)	0.00005 (0.00001)	4	1.35
			CBPSOTVAC	0.63 (0.05)	0.021 (0.005)	2836	491.81
			BPSOSIPAC	1 (0)	0 (0)	0	0
			HPSOGO	1 (0)	0 (0)	0	0

Bold values represent best obtained result

6 Conclusion

This paper proposes a new binary algorithm, HPSOGO, to solve the multidimensional knapsack problem. This algorithm has proven to be effective in terms of performance in comparison with other PSO-based algorithms. The proposed algorithm finds the optimal solution by evaluating the functions a smaller number of times. All benchmarks are based on multidimensional constraints. Furthermore, it is a very manageable algorithm, easy to implement, adaptable to discrete structures and it does not require parameters' adjustment.

In this study, a novel PSO algorithm, named HPSOGO, has been used to solve the MKPs. Several benchmark problems (*Sento*, *Weing*, *Weish*) from OR-Library were tested, and the results were compared with other existing PSO algorithms. Benchmarks used in HPSOGO are based on the multidimensional constraints.

The major conclusion is that this binary algorithm has proven to be effective in terms of performance in comparison with the other PSO-based algorithms. Our proposal finds the optimal solution by evaluating the functions a fairly small number of times. Furthermore, it is a very manageable algorithm, easy to implement, adaptable to discrete structures and it does not require parameters' adjustment.

Although the proposed algorithm works very well for MKP, further research needs to be conducted when dealing with other problems not MKP related before speaking about universality and robustness, the proposal must be tested and evaluated in other binary problems. In addition, the algorithm should increase the success rate in high-dimensional knapsack problems. Finally, comparing with the existing non-PSO state-of-the-art methods is helpful in establishing the algorithm for general purposes.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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