The role of the discount rate in tendering highway concessions under the LPVR approach

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A B S T R A C T

Flexible-term highway concessions are becoming quite popular around the world as a means of mitigating the traffic risk ultimately allocated to the concessionaire. The most sophisticated mechanism within flexible-term concession approaches is the least present value of the revenues (LPVR). This mechanism consists of awarding the concession to the bidder who offers the least present value of the revenues discounted at a discount rate fixed by the government in the contract. Consequently, the concession will come to an end when the present value of the revenues initially requested has been eventually reached. The aim of this paper is to evaluate the effect that the discount rate established by the government in the bidding terms has on the traffic-risk profile ultimately allocated to the concessionaire. To analyze this effect, a mathematical model is developed in order to obtain the results. I found that the lower the discount rate the larger will be the traffic risk allocated to the concessionaire. Moreover, I found that, if a maximum term is established in the contract, the lower the discount rate the less skewed towards the downside will be the traffic-risk profile allocated to the concessionaire.

1. Introduction

Investment in public infrastructure is a key factor for promoting economic growth (Aschauer, 1989; Aghion et al., 1999). Many countries around the world are seeking new means to involve the private sector in managing and financing infrastructure through public–private partnerships (PPPs). Three reasons lie behind this trend: first, the growing budgetary constraints experienced by many economies in the world, which led them to look for resources outside of the public budget; second, the search for greater productive efficiency in the provision of public goods; and third, the improvement of quality through a better allocation of risks and incentives (OECD, 2008).

One of the most common ways of implementing private participation in managing infrastructure is through the concession approach, which consists basically of transferring construction, maintenance, and operation of the infrastructure to a private consortium, in exchange for which that consortium receives the right to charge a user fee, for a period of time, fixed or variable, as contractually agreed upon in advance. Infrastructure concessions incorporate some features that distinguish them from other construction and maintenance contracts, and also from the basic asset privatization procedure (Vassallo and Gallego, 2005).

One of the major concerns regarding infrastructure concessions in the last few years is that of calculating how best to allocate traffic risk. On the one hand, traffic seems to depend to a great extent on factors that are beyond the control of both the concessionaire and the government. On the other hand, forecasting traffic accurately has proved to be a real challenge for both planners and private companies (Flyvbjerg et al., 2005; Bain and Polakovic, 2005).
In addition, concession contracts that fully allocate traffic risk to the concessionaire seem to be particularly prone to renegotiation. Based on the case study of highway concessions in Spain, Baeza and Vassallo (2010) show how concessionaires tend to put pressure on the government to renegotiate concession contracts when the real traffic turns out to be lower than originally calculated by the concessionaire. As Guasch et al. (2008) points out, renegotiations are unfortunately quite common in concession contracts, and they are initiated not only by the concessionaire, but also by mutual agreement between the government and the concessionaire.

In order to avoid future renegotiations and establish a more rational and fair traffic risk allocation, many governments around the world are introducing mechanisms to mitigate traffic risk in highway concessions (Vassallo, 2006). One of the most interesting mechanisms to mitigate traffic risk is the establishment of flexible-term contracts. The main characteristic of this approach is that the contract will end when a predetermined amount of accumulated revenues, as fixed by the terms of the contract, is ultimately reached.

The most sophisticated approach within the range of flexible term mechanisms is the "least present value of the revenues" (LPVR) mechanism, which has been developed and studied in detail by Engel et al. (1997, 2001). This approach introduces a discount rate to calculate the accumulated revenues. Even though this mechanism has been very well received by academics, its practical implementation has been infrequent, mostly because of the strong opposition to LPVR by concessionaires (Vassallo, 2006).

This paper discusses the influence that the LPVR discount rate \( \beta \), which is established by the government in the contract, has on the calculation of the traffic risk that is ultimately allocated to the concessionaire. In the first section of the paper, after the introduction, I describe the characteristics of the flexible-term concession contracts, and analyze the data on the implementation of this mechanism around the world. In the next section, I develop a mathematical model intended to estimate the sensitiveness of the return \( \eta \) obtained by the concessionaire in terms of \( \beta \) for different traffic scenarios. Then, I apply the model to a hypothetical case study, and analyze the results derived from it. In the last section, I discuss the results, and then offer some concluding remarks. My chief conclusion in this paper is that the lower the discount rate used by the government, the higher will be the traffic risk actually allocated to the concessionaire. This conclusion makes sense intuitively since the lower the discount rate the greater the value placed on future earnings, and consequently the greater the risk transferred to the concessionaire because more weight is now being assigned to future earnings.

2. Flexible long-term concession contracts

2.1. Theoretical foundation

The theoretical foundation of flexible-term concession contracts is quite straightforward. The idea is to tie the duration of the concession contract to the achievement of a certain goal previously established in the contract. One approach is to tie the concession duration to the number of users or to the accumulated revenues obtained by the concessionaire. In other words, the concession contract will end when a certain number of vehicles have used the highway or when the concessionaire has received a certain amount of revenues from the users. Consequently, if traffic ultimately turns out to be higher than expected, the duration of the concession contract will be reduced from what had originally been estimated. And the reverse is also true. In cases where the actual traffic turns out to be lower than what had been estimated, the concession contract will be extended.

This approach means a substantial mitigation of the traffic risk that is actually allocated to the concessionaire compared to fixed-term contracts. However, traffic risk is not fully mitigated by using this approach for two reasons. First, the maintenance and operation costs accumulated throughout the life of the contract become larger for the concessionaire when the concession contract becomes longer and vice versa. And second, the revenues obtained at the beginning of the contract will have a higher value for the concessionaire than those obtained at the end of the contract. As a consequence of these two issues, if in the end the actual traffic is higher than expected, the return obtained by the concessionaire will be a little bit higher than the return that would have been obtained with a fixed-duration contract. In the opposite case, when the actual traffic turns out to be lower than expected, the return obtained by the concessionaire will be a little bit lower. This means that the ultimate return will go up and down with traffic fluctuations, but not as much as it would have, in either direction, if a fixed-term contract had been implemented. This traffic risk allocation profile makes sense from the standpoint of the theory of incentives since the concessionaire will still have an incentive to bring more traffic to the concession.

Engel et al. (1997, 2001) made two substantial contributions to the approach outlined above. First, they suggested discounting the revenues at a discount rate set up in the contract (present value of the revenues) in order to reflect the different value that revenues have for the concessionaire at different times. Second, and perhaps more important, they proposed to use the present value of the revenues (PVR), not only as a means of mitigating traffic risk, but also as the key variable to tender the concession contract. This way, the bidder who in the end requires the least present value of the revenues will be granted the concession contract. This is what Engel, Fischer, and Galeotivic called the "least present value of the revenues" (LPVR) mechanism.

Let me explain in greater detail how this mechanism works. Eq. (1) shows the net present value estimated by one bidder attending the tender in terms of the most relevant variables that determine the economic balance of a concession contract.
2.2. Practical experience around the world

An analysis of the procurement process of this highway can be found in Gómez-Lobo and Hinojosa (2000). The fixed rate was allowed the concessionaire to choose either a fixed or a variable discount rate. Since the auction mechanism based on the LPVR consists of granting the concession to the bidder that requires the lowest present value of the revenues to recover its costs. The concession will end when the real discounted flow of revenues reaches the level required by the concessionaire. If the real traffic is ultimately lower or higher than expected, the ultimate duration of the concession will be either extended or reduced.

In the last few years, some academics have proposed slight modifications of the LPVR mechanisms. Nombela and de Rus (2004) proposed using the least present value of the net revenues (LPVNR) instead of the LPVR to procure concession contracts. Vassallo (2004) suggested the possibility of using short-term concession contracts of fixed duration awarded under the LPVR approach. Once the contract has expired, the government will pay to the concessionaire the difference between the LPVR requested and the LPVR obtained at the end of the contract.

\[
\text{NPV}_0 = -l_0 + \sum_{i=0}^{n-1} \frac{p_i \cdot q_i(p_i) - m_i}{(1 + r)^i} \tag{1}
\]

where \(\text{NPV}_0\) is net present value calculated in year 0; \(l_0\) initial investment estimated by the bidder (capital cost); \(r\), weighted average cost of capital (WACC); \(n\), concession term; \(p_i\), price or toll in year \(i\); \(q_i(p_i)\), actual traffic estimated by the bidder in year \(i\) depending on \(p_i\); \(m_i\), maintenance and operation costs estimated by the bidder in year \(i\).

Each bidder will try to make his bid as competitive as possible in order to have the greatest chance of being awarded the concession. The most competitive bid under this restriction is always made when \(\text{NPV}_0 = 0\). Making Eq. (1) equal to zero and restating its terms, I obtain Eq. (2).

\[
l_0 + \sum_{i=1}^{n} m_i \frac{p_i}{(1 + r)^i} = \sum_{i=1}^{n} \frac{p_i \cdot q_i(p_i)}{(1 + r)^i} \tag{2}
\]

The left side of this equation shows the discounted costs that the bidder expects to bear in operating and constructing the concession for the duration \(n\), whereas the right side shows the present value of the revenues (PVR) that the bidder expects to obtain along the contract duration. The point at which the two terms of the equation are equal means that the concession has covered all its costs—according to a cost of capital equal to \(r\).

The use of flexible-term concession contracts was reported for the first time in the case of the concession contract for the Second Severn Bridge in the United Kingdom, which was awarded in 1990. The length of the concession was pegged to a fixed target of "Required Cumulative Real Revenue" (Foice, 1998). This way, a figure was established, in 1989 prices which, once collected from toll income, would bring the concession to an end. Another similar experience is the Lusoponte concession in Portugal, which was awarded at the end of the 1990s. The concession agreement was designed so that the concession would expire no later than March 2028 or once a total cumulative traffic flow of 2250 million vehicles had been reached (Lemos et al., 2004).

Other countries have also implemented flexible-term concession approaches. The government of Colombia decided to move from fixed to flexible-term concession contracts at the end of the last decade. The first project to be awarded under this approach was the "Malla Vial del Valle del Cauca," which will expire when the concessionaire reaches the accumulated revenues—not discounted—requested in the tender, subject to a maximum duration (Benavides, 2008). The accumulated revenues required were also used as one of the key variables in the tender phase of the concession, though not the only one. From then on, many concession contracts have been awarded using this approach, particularly in the last few years. Right now, it is still too early to properly assess how this mechanism actually works since most of the contracts are either in the construction phase or moved in the first few years of operation.

In the last few years, due to budgetary constraints, Portugal has moved from shadow toll concessions to real toll concessions. This fact encouraged the government to implement traffic risk mitigation mechanisms. For this reason, the Litoral Central Highway in Portugal was awarded under the LPVR approach. The concession will come to an end when the net present value of the total revenues reaches €784 million, subject to a minimum period of 22 years and a maximum period of 30 years. The concession ends after 30 years, regardless of whether the consortium reaches the PVR initially requested or not. Since the award of this concession, however, use of the LPVR has been discontinued in Portugal owing to the opposition of potential concessionaires.

Undoubtedly, Chile is the country where there has been the greatest experience in the implementation of the LPVR mechanism in the way that Engel, Fischer and Galetovic designed it. The Chilean Public Works Concession Law defined the possibility of using the sum of total revenues—discounted or not—to be required by the concessionaire as the main economic variable for tendering concessions. From then on, the LPVR has been used as a procurement mechanism in some highway and airport concessions in Chile. The bidding terms of the concessions awarded in Chile on the basis of the LPVR approach allowed the concessionaire to choose either a fixed or a variable discount rate.

The first concession using LPVR in Chile, and also the most successful, was the Santiago-Valparaiso highway (Route 68). An analysis of the procurement process of this highway can be found in Gómez-Lobo and Hinojosa (2000). The fixed rate was set in the bidding terms as a risk-free rate of 6.5% plus a risk premium of 4%. The variable rate was set as the monthly average...
3. The role of the discount rate

The objective of this section is to know how the discount rate $\beta$ used by the government in tendering a concession contract under the LPVR approach influences the return $\eta$ ultimately achieved by the concessionaire for different traffic scenarios. To that end, in this section, a mathematical model is developed to relate $\beta$ to $\eta$ when traffic estimates turn out to be inaccurate. Once the model has been defined, I apply the model to a hypothetical case in order to visualize and assess the results.

3.1. Definition of the model

The net present value of a concession business is given by Eq. (1). This equation is defined in terms of discrete time units. This means that all the revenues and costs produced in a specific year are supposed to be concentrated at a particular point. In order to carry out a more thorough analysis, it is useful to express this equation on the basis of continuous time units by using the continuous interest formula. This way, the length of the compounding period is reasoned to be infinitely small. This approach will enable the model to obtain more accurate results in determining the ultimate length of the contract. Eq. (3) is obtained by replacing the discrete time interest formula by the continuous time one. This way, the former variable $t$, used to refer to a discrete number of years, is replaced by the variable $t$, which refers to continuous time units.

$$NPV_0 = -I_0 + \int_0^T \left[ p(t) \cdot q(p(t),t) - c(t) \right] \cdot e^{-\beta t} dt$$

(3)

To simplify the model, I make two assumptions. First, I assume that inflation is either non-existent or grows at a constant rate, which enables me to use real, instead of nominal, monetary units from the present, and going forward. Second, I consider that the price or toll does not vary in real terms throughout the life of the contract, so $p$ does not depend on $t$. This assumption is not far from the reality since most of toll road concession contracts incorporate price caps indexed so as to rise at the same rate as inflation.

Traffic flows depend on variables such as the population growth in the area around the infrastructure, the income per capita of the population, the competition with other infrastructure facilities, the price of gasoline, and so on. Traffic depends also on the toll $p$ imposed, which is a variable explicitly included in the model. However, according to the previous hypotheses, $p$ will remain constant in real terms throughout the life of the contract so this variable will no longer influence traffic. Consequently, traffic will be a function of several exogenous variables (variables which are outside the model).

The objective of this paper is not to dig into the causes that explain traffic growth, but rather to evaluate the influence of the ultimate profitability for the concessionaire in terms of an exogenous traffic growth. For this reason, I assume that traffic in time $t$ will depend only on two exogenous variables: the traffic $q_0$ in the first year of operation (year 0), and the annual traffic growth $\alpha$. For the sake of simplification I consider that $\alpha$ is constant throughout the lifespan of the concession. Consequently, the traffic volume for year $t$ is calculated according to Eq. (4) in the following way:

$$q(t) = q_0 \cdot e^{\alpha t}$$

where $q(t)$, annual traffic in year $t$; $q_0$, annual traffic in year 0 (first year of operation); $\alpha$, annual traffic growth rate.

Regarding the incorporation of the maintenance and operation costs in the model, I establish that those costs can be split into a fixed and a variable part. The fixed part does not depend on traffic. For instance, the concessionaire will have to cut the real risk-free rate plus a 4% risk premium. Four bidders attended the tender. Three of them chose the fixed discount rate whereas only one of them chose the variable one.

The second attempt to tender a highway concession in Chile under the LPVR mechanism took place at the beginning of 1999. The highway selected was the Costanera Norte, an urban expressway in Santiago, which was a very risky project for several reasons. First, it was located in an urban area, thus competing with other roads and means of transportation. Second, part of the highway was built on a subterranean level. And third, there was public opposition to the project by residents of some city neighborhoods. Only one consortium presented an offer and it was ultimately disqualified because the guarantee bond offered was below the level established in the bidding documents. This experience proved that the LPVR was not a magic wand, able to get very risky projects off the ground without public support. From then on, some road and airport projects have been awarded under the LPVR approach. However, in spite of the interest in this mechanism, in the last fifteen years, only four road concessions out of the 28 presently granted were successfully awarded under the LPVR. The main reason why this attractive mechanism has not been implemented more often in Chile, Portugal and other countries is the strong opposition from concessionaires (Vassallo, 2006). The main reason put forth by the concessionaires to explain their opposition to implementation of this mechanism is that LPVR sets a cap to the upside while it does not, at the same time, set a floor to the downside. The downside is caused by the fact that concession legislations and contracts (such as the ones in Spain, Portugal and Chile) tend to establish a maximum duration for concession contracts. Consequently, while the duration of a contract can be always reduced to mitigate the upside, the necessary extension to avoid the downside in the case of a traffic shortfall is not guaranteed beyond the maximum contract duration. As Brealey et al. (1996) claim, private shareholders expect a large upside that compensates for the possibility of losing all their capital in very risky and highly leveraged projects. The traffic-risk profile resulting from reliance on the LPVR approach turns out to be just the opposite of the profile desired by would-be sponsors since the upside is very small while the downside remains significant.
grass, and keep the embankments in good repair, irrespective of the number of vehicles that ultimately will use the road. The variable part will depend on the ultimate number of vehicles that will use the road every year. This second part mostly reflects the wear and tear of the road pavement, costs that will vary depending on the size of the vehicles that travel. Eq. (5) shows how maintenance and operation costs are modeled.

\[
m(t) = m_0 + \theta \cdot q_0 \cdot e^{\alpha t}
\]

where \(m(t)\), total maintenance cost in year \(t\); \(m_0\), annual fixed maintenance and operation cost; \(\theta\), variable maintenance and operation cost per vehicle; \(\alpha\), annual traffic growth.

Introducing Eqs. (4) and (5) in Eq. (3), and adopting the simplifications explained before, I obtain Eq. (6a), from which I easily obtain Eq. (6b).

\[
NPV_0 = -I_0 + \int_0^T [p \cdot q_0 \cdot e^{\alpha t} - m_0 - \theta \cdot q_0 \cdot e^{\alpha t} \cdot e^{\gamma t}] \cdot e^{-\gamma t} dt
\]

\[
NPV_0 = -I_0 + \int_0^T (p - \theta) \cdot q_0 \cdot e^{e^{(\gamma - \alpha) t} \cdot e^{-\gamma t}} \cdot d\gamma - \int_0^T m_0 \cdot e^{\gamma t} \cdot d\gamma
\]

Eq. (6b) is easy to integrate. As a result of that, I obtain Eq. (7), which enables me to calculate the net present value in terms of the variables of the model.

\[
NPV_0 = -I_0 + (p - \theta) \cdot q_0 \cdot \frac{1}{r - \alpha} [1 - e^{e^{(\gamma - \alpha) t} \cdot e^{-\gamma t}}] - m_0 \cdot \frac{1}{r} [1 - e^{-r t}]
\]

The internal rate of return (IRR) obtained by the concession is the value of the discount rate that makes the net present value of Eq. (7) equal to zero.

The procurement mechanism based on LPVR consists in awarding the concession contract to the bidder who offers the least present value of the revenues—that is, the lowest value of the accumulated revenues discounted at a certain discount rate \(\beta\) established by the government in the contract. The rate \(\beta\) is not necessarily have to coincide with the WACC \(r\) estimated by each bidder. In order to win the tender, each bidder will try to offer the least present value of the revenues under the restriction that its expected NPV should be always \(\geq 0\). This condition is achieved when Eq. (7) is equal to 0 and consequently the weighted average cost of capital \(r\) estimated by each bidder coincides with the internal rate of return (IRR) that each bidder expects to attain. Making NPV equal to 0, I obtain Eq. (8), which is equivalent to Eq. (2) but using continuous time units instead of discrete time units. The left side of the equation shows the present value of the costs and the right side of the equation shows the net present value of the revenues PVRLD(r).

\[
l_0 + \theta \cdot q_0 \cdot \frac{1}{r - \alpha} [1 - e^{e^{(\gamma - \alpha) t} \cdot e^{-\gamma t}}] + m_0 \cdot \frac{1}{r} [1 - e^{-r t}] = \theta \cdot q_0 \cdot \frac{1}{r - \alpha} [1 - e^{e^{(\gamma - \alpha) t} \cdot e^{-\gamma t}}] - m_0 \cdot \frac{1}{r} [1 - e^{-r t}]
\]

where \(PVRLD(r)\), Present value of the revenues discounted at \(r\).

As many bidders can attend the tender, and each one of them can have different estimates of traffic, costs, and WACC, they can offer different present values of the revenues depending on their estimates. Eq. (9) shows the same relationship as Eq. (8), but this time, the equation is made specific for the estimates conducted by each bidder. In Eq. (9), the subscript \(E\) means "expected" and the superscript \(j\) refers to each one of the bidders. For instance, \([q_0]_E\) means the level of traffic in year \(0\) expected by bidder \(j\). I have added the subscript \(E\) to the variables that the bidder has to estimate in the tender. \(l_0, q_0, \theta, m_0, \alpha, \beta\).

\[
[q_0]_E + \beta \cdot [q_0]_E \cdot \frac{1}{r - \alpha} [1 - e^{e^{(\gamma - \alpha) t} \cdot e^{-\gamma t}}] + [m_0]_E \cdot \frac{1}{r} [1 - e^{-r t}] = \theta \cdot [q_0]_E \cdot \frac{1}{r - \alpha} [1 - e^{e^{(\gamma - \alpha) t} \cdot e^{-\gamma t}}] - m_0 \cdot \frac{1}{r} [1 - e^{-r t}]
\]

where \(PVRLD(r)\), present value of the revenues needed by bidder \(j\) discounted at \(r\); \(\alpha\), WACC estimated by bidder \(j\); \([q_0]_E\), annual traffic in year \(0\) expected by bidder \(j\); \(\alpha\), annual traffic growth expected by bidder \(j\); \(T_n\), term needed by bidder \(j\) to obtain a rate of return equal to \(r\).

From Eq. (9), it is possible to calculate for each bidder the contract duration \(T_n\) which, according to their estimates, would be necessary to achieve an internal rate of return (IRR) equal to the cost of capital \(r\). The period \(T_n\) is, then, the contract duration that, according to the estimates of each bidder, is necessary for the bidder to achieve a rate of return \(r\) equal to its cost of capital.

However, as has already been mentioned, the discount rate \(\beta\) imposed by the government on the bidders for their calculation of the present value of the revenues does not have to coincide with the WACC \(r\) estimated by each bidder. Consequently, the present value of the revenues offered by each bidder \(PVRLD(r)\) does not have to coincide necessarily with the value \(PVRLD(r)\) from Eq. (9). Even though the government intends to make \(\beta\) coincide with \(r\), this is a real challenge due to asymmetrical information problems.

The present value of the revenues offered by each bidder will be given by Eq. (10) where, again, the subscript \(E\) means "expected" and the superscript \(j\) refers to each one of the bidders.

\[
PVRLD(r) = p \cdot [q_0]_E \cdot \frac{1}{\beta - q_0} [1 - e^{e^{(\gamma - \alpha) t} \cdot e^{-\gamma t}}]
\]
where \( \text{PVR}_j(\beta) \), present value of the revenues offered by the bidder \( j \): \( [q_0]^j \), annual traffic in year 0 expected by the bidder \( j \); \( a_j \), annual traffic growth expected by the bidder \( j \).

As the interaction among different bidders in the tender has now been explained, from now on I will focus my attention on the bidder who offers the least present value of the revenues and consequently will end up becoming the concessionaire. The present value of the revenues offered by the concessionaire will be given by Eq. (11), which is the same as Eq. (10) without superscript \( j \). For the sake of simplicity, I consider that the values without superscript \( j \) are the values estimated by the bidder who ultimately becomes the one selected to be the concessionaire. Eq. (11) shows how the lower the discount rate adopted by the government in the contract, the larger will be the LPVR requested by the concessionaire.

\[
\text{PVR}_0(\beta) = p \cdot [q_0]_0 \cdot \frac{1}{\beta - a} \cdot [1 - e^{(\beta - a)T_R}] \\
\text{PVR}_0(\beta) = \min(\text{PVR}_j(\beta))
\] (11)

According to the LPVR procurement mechanism, the concession will come to an end at the time \( T_R \) when the LPVR requested is ultimately reached. This condition is shown by Eq. (12). This equation is similar to Eq. (11), but the expected traffic variables \([q_0]_0\) and \( a \) are replaced by the real traffic variables \([q_0]_R\) and \( a_R \). Consequently, the value \( T_R \) that satisfies Eq. (12) will be the ultimate duration of the contract. Depending on how \([q_0]_R\) and \( a_R \) ultimately behave compared to \([q_0]_0\) and \( a \) the contract duration can be either extended \((T_R > T_0)\) or reduced \((T_R < T_0)\).

\[
\text{PVR}_0(\beta) = p \cdot [q_0]_R \cdot \frac{1}{\beta - a} \cdot [1 - e^{(\beta - a)T_R}] \\
\text{PVR}_0(\beta) = \min(\text{PVR}_j(\beta))
\] (12)

Once the real term \( T_R \) that will be determined in accordance with LPVR is calculated for different traffic scenarios, it is quite straightforward to estimate the net present value ultimately reached by the concessionaire by introducing all the real variables—including the value of \( T_R \) previously calculated—in Eq. (7). This gives Eq. (13). The ultimate rate of return \( \eta \) obtained by the concessionaire will be the value of the discount rate \( \beta \) that makes Eq. (13) equal to 0 (see Eq. (14)).

\[
[NPv]_0 = -[a]_R \cdot (p - \theta) \cdot [q]_R \cdot \frac{1}{\eta - a} \cdot [1 - e^{(\eta - a)T_R}] - [m]_R \cdot \frac{1}{\eta} \cdot [1 - e^{-\eta T_R}] \\
[NPv]_R = 0 = -[a]_R \cdot (p - \theta) \cdot [q]_R \cdot \frac{1}{\eta - a} \cdot [1 - e^{(\eta - a)T_R}] - [m]_R \cdot \frac{1}{\eta} \cdot [1 - e^{-\eta T_R}]
\] (13)

The goal of the model that I develop in this paper is to estimate the actual rate of return \( \eta \) obtained by the concessionaire as a function \( \Phi(\beta) \) of the discount rate \( \beta \) adopted by the government, for different traffic scenarios (see Eq. (15)).

\[
\eta = \Phi(\beta, [q_0]_R, [q_0]_R, a_R - a)
\] (15)

3.2 Results of the model

Unfortunately, some of the equations I have developed in the previous section do not have an algebraic solution. For instance, it is not possible to obtain an algebraic solution for \( T_R \) from Eq. (9). Similarly, it is not possible to obtain an algebraic solution to obtain \( \eta \) from Eq. (14). Consequently, it is not feasible to calculate in algebraic terms the derivative expression \( \partial \Phi/\partial \beta \) that shows whether \( \eta \) increases or decreases with \( \beta \). In spite of this, it is still possible to obtain a numerical solution for these equations. For this reason, I can assess the different solutions for different scenarios implemented in a hypothetical case study. This way, the lack of an algebraic solution can be circumvented.

To that end, I have decided to apply the model to a hypothetical case study. The characteristics of the hypothetical case study are based on average values of highway concession contracts in Spain. The variables adopted are \( I = 300 \) millions, \( p = 10 \) €/trip, \( q_0 = 3650 \) million trips/year, \( \alpha = 2\% \) annual, \( m = 3 \) million, \( \theta = 2 \) €/trip, \( r = 7.5\% \). In addition, the maximum legal duration of the concession contract is assumed to be 30 years.

Moreover, in order to simplify the results, I assume that all the variables behave as expected except the traffic growth \( \alpha \). In this respect, traffic deviation is defined as \( \alpha_R = \alpha - \alpha \).

Fig. 1 graphically shows how the model behaves for two different discount rate scenarios: first, \( \beta = r = WACC = 7.5\% \), and second, \( \beta = 0\% \). The first scenario assumes that, even though the government is not able to know the private cost of capital, the government adopts a discount rate equal to the WACC of the project. The second scenario assumes that the government decides not to discount the revenues, as was the case for example with the Second Severn Bridge in the UK. The left graph within Fig. 1 shows the process that the concessionaire would have followed in order to estimate the present value of the revenues to offer in the tender, according to the expected behavior of the variables of the concession.

The curve with square-shaped data points represents the evolution during the years of the accumulated present value of the revenues. According to Eq. (2), the equilibrium point of the concession will be the intersection of those two curves. The present value of the revenues requested by the concessionaire, if the \( \beta \) adopted by the government equals the WACC, will be €411.65 million. The expected duration of the concession will be 17.6 years.
However, if the government decides to adopt a discount rate $\beta$ equal to 0%, the curve representing the evolution of the accumulated revenues will be that shown with circle shapes. If the government decides not to discount the revenues, the concessionaire will require in the tender a level of accumulated revenues that would correspond to 17.6 years of concession, which is the number of years necessary for the concessionaire to recoup its costs according to its cost of capital $r$. The amount to be required will be the intersection of the yellow curve with the vertical line corresponding to 17.6 years. The accumulated revenues that the concessionaire will require for this scenario will be €770.32 million.

The left graph in Fig. 1 shows what would have happened if the annual traffic growth $a$ had been the expected annual traffic growth plus 4% (i.e. if the annual traffic growth expected was 2%, the real growth would be $2\% + 4\% = 6\%$). In this scenario the revenue curve, discounted at the WACC (curve with square-shaped data points), and the revenue curve, not discounted (curve with circle-shaped data points), will be steeper than they were in the expected scenario due to a greater traffic increase. Moreover, the maintenance and operation costs throughout the years will be higher than expected since more vehicles will accelerate the deterioration of the infrastructure. This fact explains why the curve with triangle-shaped data points is a little bit steeper in this scenario than it is in the original scenario.

If the government has adopted a discount rate equal to 7.5%, the concession will expire when the curve with square-shaped data points reaches the €410.65 million requested in the tender. This will occur after 12.36 years. However, if the government has chosen not to use any discount rate (that is, a discount rate equal to 0), the concession will expire when the curve with circle-shaped data points reaches the €770.32 million requested in the tender. This will occur after 13.64 years. The upside to be obtained by the concessionaire will be higher if the government does not discount the revenues rather than if the government chooses to discount the revenues at the WACC, since, for a specific traffic scenario, the ultimate rate of return $\eta$ will depend only on the duration of the concession.

Fig. 2 shows a sensitivity analysis around the discount rate $\beta$. This analysis displays the ultimate rate of return $\eta$ obtained by the concessionaire in terms of the discount rate $\beta$ fixed by the government in the tender for different traffic growth deviation scenarios. The continuous thick curve shows the ultimate rate of return $\eta$ obtained by the concessionaire when $a = 0\%$, which means that the real traffic growth turned out to be exactly as expected. For this case, $\eta$ will be the same regardless of the value of the discount rate $\beta$ adopted by the government in the tender.

The set of curves made up of discontinuous long dashes shows the outcome when the real traffic growth turns out to be higher than expected. It is notable that the lower the value of $\beta$ the higher will be the return for the concessionaire $\eta$. I have not included in this graph the outcome when $\beta$ is higher than the weighted average cost of capital $r$. This is because, when this happens, the higher the traffic growth, the lower will be the return obtained by the concessionaire, which does not make any sense from the perspective of the theory of incentives. Consequently, a first conclusion of this analysis is that the discount rate $\beta$ used to procure a highway under the LPVR approach should never be higher than the WACC. Of course, because of asymmetrical information problems, the government is not able to know the private WACC adopted by the bidders. However, the government is still able to approach this value either through experience or through financial models such as the Capital Asset Pricing Model (Sharpe, 1964). In any case, the government knows that the value of the WACC adopted by the private sector will be no greater than the return on the risk-free public securities.

The set of curves made up of discontinuous short dashes shows the outcome when the real traffic growth turns out to be lower than expected. In this case, the lower the discount rate $\beta$ adopted by the government the higher will be the return $\eta$ obtained by the concessionaire. However, in this case, I note a substantial difference. For the scenarios with large traffic deviations, the reduction of the return $\eta$ is outstanding even for discount rates close to the WACC $r$. The reason for this behavior is...
that a legal maximum term of 30 years is established in the contract, which limits the extension of the contract to the point where the present value of the revenues initially requested is ultimately achieved. This explains the uneven shape of the lowest of the short-stroke curves in Fig. 2.

This figure graphically justifies the criticism made by the concessionaires to the LPVR mechanism—that the upside and the downside are not symmetric. On the one hand, the upside in cases of a positive deviation of the traffic growth is a little bit smaller than the downside that would occur when the traffic growth expected experiences the symmetric negative deviation. Furthermore, the maximum term of the concession contract accentuates the asymmetry, particularly when the discount rates adopted are close to the weighted average cost of capital.

Fig. 3 shows the same results from a different point of view. Now the curves display the ultimate return $r_\eta$ for the concessionaire in terms of the annual traffic growth deviation for three different procurement mechanisms adopted: first, a fixed-term contract, which is depicted by the thick continuous curve in black; second, the LPVR mechanisms with $\beta = 0\%$,
depicted with square-shaped data points, which means that the revenues are not discounted—such as was the case with the Second Severn Bridge Concession in the UK; and third, the LPVR mechanisms with \( \beta = WACC = 7.5\% \), depicted with circle-shaped data points, which is basically the LPVR mechanism as it has been implemented in Chile.

Several conclusions result from the analysis of Fig. 3. First, the traffic risk allocated to the concessionaire in a fixed-term contract is much higher than it is in the two scenarios of flexible-term contracts. Second, whereas \( \frac{\partial^2 R}{\partial \alpha^2} < 0 \) for a fixed-term contract, \( \frac{\partial^2 R}{\partial \alpha^2} > 0 \) for flexible-term contracts based on LPVR, which means that for the same traffic growth deviation, the upside obtained by the concessionaire will be lower than the downside. And third, the downside is even more accentuated when the discount rate \( \beta \) approaches the value of the weighted average cost of capital \( r \) due to the maximum limit established for the contract’s duration.

4. Discussion and conclusions

This research shows that, even though LPVR is a very interesting approach in both the procurement of highway concessions and in the mitigation of traffic risk, there is still a lot of work ahead to evaluate the consequences of its application. This paper presents a first step in this analysis. To that end, I have developed a model that sheds some light on the effect that the discount rate \( \beta \) fixed by the government in the contract has on the traffic-risk profile ultimately allocated to the concessionaire. The conclusions of this research could be extremely useful for decision makers who opt for using LPVR as a procuring mechanism to grant highway concessions. The conclusions are the following:

- The discount rate \( \beta \) to be used in the LPVR should never be higher than the WACC estimated by the concessionaire. Otherwise, the concessionaire will not have an incentive to attract more traffic to the concession. The government does not know the private WACC even though it is still able to make a reasonable estimate. For this reason, it is advisable that the government choose a conservative discount rate close to the return on the risk-free public securities.
- The lower the value of the discount rate \( \beta \), the higher will be the traffic risk allocated to the concessionaire. Despite this, any kind of flexible term contract allocates much less traffic risk to the concessionaire than a fixed-term contract.
- The establishment of a maximum concession term causes an accentuated asymmetry between the upside and the downside in terms of \( \eta \) when the concession is awarded on the basis of the LPVR approach. This asymmetry becomes more pronounced when the discount rate \( \beta \) adopted by the government is close to the WACC and the contract sets up a maximum duration for the concession.

The theoretical analysis conducted in this paper demonstrates that the opposition of the concessionaires to the implementation of the LPVR mechanism is justified, since LPVR substantially limits the upside but not the downside. This effect is particularly accentuated when \( \beta \) is close to the WACC.

The main policy implication of this research is that using low discount rates in the LPVR mechanism could have important advantages. First, the traffic-risk profile allocated to the concessionaire is still substantially mitigated compared to using fixed-term contracts. Second, unlike the case in which a discount rate \( \beta \) close to the WACC is used, using a low discount rate allows the concessionaire to enjoy a certain upside if traffic eventually becomes higher than expected. And third, the downside caused by a maximum term established in the contract in case that traffic ultimately becomes lower than expected is not substantially different across different discount rates \( \beta \).

References


