James Sutherland History Lecture

To be held on 2 February 2006 at IStructE, 11 Upper Belgrave Street, London SW1X 8BH at 18.00h

Geometry and equilibrium: The gothic theory of structural design

Prof. Santiago Huerta
PhD
Departamento de Estructuras. ETS de Arquitectura. Universidad Politécnica de Madrid, Avda. Juan de Herrera, 4. 28040-Madrid. Spain

Keywords: Gothic architecture, Churches, Cathedrals, Design, History, Buttresses, Vaults, Masonry

Synopsis
Medieval builders did not have a scientific structural theory, however gothic cathedrals were not built without a theory. Gothic masters had a '*scientia*' and this *scientia* was firmly based on geometry. It is the form which guarantees a safe state of equilibrium.

In many gothic sources we find rules to design the structural elements, with special emphasis in buttress design. These rules lead in most cases to a proportional design, independent of scale (the depth of a buttress as a fraction of the span). The late-gothic Spanish architect Rodrigo Gil formulated arithmetical rules which lead to non-proportional designs (the buttresses become more slender as the general size grows).

Gothic structural rules were a means to register stable forms. Proportional rules are essentially correct and apply to most cases. Rodrigo Gil's rules express a finer adjustment to some non-proportional problems: buttress design for thin late-gothic vaults or wall design for towers.

Introduction
The building of gothic churches and cathedrals was not an amateur task. Medieval builders were 'masters'. Gothic structures justify this title and even today with a well developed structural theory very few architects or engineers, if any, would dare to put their name to similar projects (and this is a problem in restoration work and structural expertise).

The science of Statics was not sufficiently developed in the Middle Ages to allow a scientific structural design; in fact scientific structural theory originated in the 17th century (Galileo, Hooke), but began to be applied only in the second half of the 18th century. How is it possible, then, that the gothic masters built such magnificent structures? Was the design of gothic cathedrals a matter of pure chance, the result of a blind trial and error process? Is, therefore, the history of gothic architecture plagued with collapses and ruined buildings? The truth is that this was not so. There were collapses, but very few in comparison with the number of successes. Besides, there were so many variations in design, entirely new types of structures, as to invalidate completely a purely Darwinist theory based on the survival of the more apt designs. The development of gothic was revolutionary, an explosion of structural creativity.

The gothic master builders had a *scientia*, a theory, a body of knowledge which permitted them to design and build safe structures (Heyman 1995). This *scientia* was not 'scientific' in the sense we give today to this word; it was not deduced from general laws and scientific principles, it was not an 'applied science'. The set of rules and procedures were deduced empirically from the observation of existing buildings. This empirical approach is not altogether unscientific. Each building was a 'successful experiment' and the observation of ruins and collapsed structures was also very informative. Finally, during the building process the masonry structure moves and shakes, adapting itself to the different phases of construction. These movements suggest corrections to improve the stability of the work and may lead to new patterns of equilibrium.

What was then, precisely, the nature of this medieval *scientia* of structures? This is a difficult question to answer. It must have been a wide and complex body of knowledge. The construction of a gothic cathedral involved many different operations: surveying, soil mechanics, foundation design, centreing, buttress and vault design, stereotomy, carpentry, lifting devices, labour organisation, etc. These are the modern keywords for some of the activities involved. The master of the work had to make decisions on all these aspects which were probably intertwined in a complex way. The depth of understanding of all these aspects could be best judged from the results. Consider, for example, Beauvais cathedral. One can feel a security of design, an absence of doubts, a determination, which could arise only from a mastery of the building processes.

Buildings are, then, our primary source and any hypothesis concerning the nature of the medieval *scientia* of structures must account for the evidence of so many churches and cathedrals which have survived during centuries. Literary sources from the gothic period are scarce (Frankl 1960) and only very few gothic manuscripts about building design have survived, most of them from the late-gothic period. Not very much information to infer the nature of a knowledge which, as has been already said, was rich and complex.

Only the album of Villard de Honnecourt pertains to the classic gothic era, the age of wonder when the 'best' cathedrals were built. But Villard is silent on structural matters. However, a lot of information on structural matters can be found in certain late-gothic manuscripts. Some of them could be called 'a treatise' as they contain information about all aspects involved in the design of a gothic church. Others treat only particular aspects: the design of gablets or pinnacles, or the solution of certain geometrical problems. Finally, some knowledge concerning structural problems has also survived and this is an invaluable source for the understanding of gothic structural thinking. The special skills associated with the cathedrals of Milan, Chartres or Gerona have been analysed many times; nevertheless, many documents remain unpublished or unnoticed.

The structural knowledge was codified in the form of practical rules (Huerta 2004). There were rules to obtain, for example, the size of buttresses or the cross sections of the ribs. These rules were a mere register of right dimensions for different structural elements. By their very nature they are specific and pertain to certain structural types. The application of gothic rules to a Renaissance building, for example, will lead to disaster: the thrust of a gothic cross vault could be less than one half the thrust of a Renaissance barrel vault. Periods of transition were critical and, indeed, there is documentary evidence both in treatises and in the registers of many churches of pathologies associated with the use of the wrong rules.

In this paper only some specific structural rules are investi-

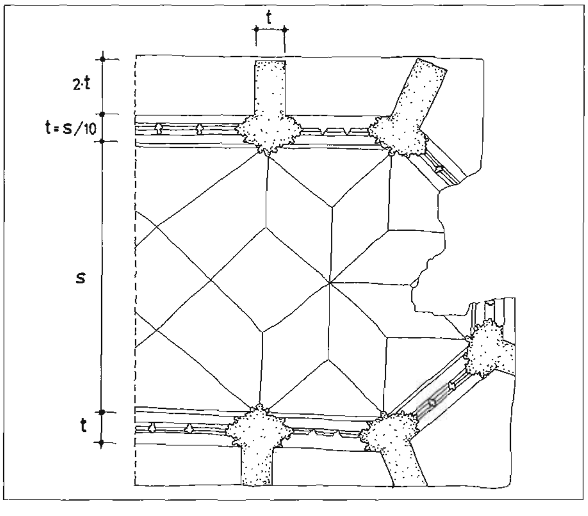


Fig 1. Proportions of wall and buttresses measured in a gothic 'riß' (redrawn after Koepf 1969: abb. 38)

gated, particularly those for vault and buttress design, with some comments also on tower design. We are going to consider, then, only one aspect of the whole process of vault design and construction. This separation is arbitrary; building is not the sum of several independent activities.

Late-gothic German rules

Coenen (1990) has published a diplomatic edition of the main late-gothic German treatises (Werkmeisterbücher), all written during the 15th century. Three of them are true architectural treatises and contemplate the whole process of church design: The *Unterweisungen* (Instructions) by Lorenz Lechler, *Von des Chores Maß und Gerechtigkeit* and the *Wiener Werkmeisterbuch*. Coenen has made an analysis of their contents and Shelby and Mark (1979) have studied the structural aspects in Lechler's treatise. However, a critical edition is lacking and they are difficult to read and interpret. The main structural rules refer to the design of buttresses, vaults and towers.

A buttress should have a depth c three times the thickness t of the wall, which is one-tenth of the span s ($t = s/10$); the buttress breadth is equal to the wall thickness. This leads to a dimension $c = 3\,t = s/3.33$ (at the base); this basic dimension could be diminished or increased depending on the quality of the masonry. Also, the cross section diminishes in height with taluses (*sic*) (*slope of a wall tapering with height*). The rule is cited several times in all the three treatises; the proportions could be found in many churches of this period and also in many of the surviving plans, as may be checked with the drawings of the Wiener Sammlung (Koepf 1968), Fig 1. Not all the plans adjust to the rules and it is evident that a true master felt free to deviate from the established rules.

A gothic vault is composed of ribs, keystones and webs (curved masonry that fills the voids between ribs). Only the ribs are mentioned. It is said specifically that the cross ribs are semicircular; other instructions referring to the geometry of the other ribs are difficult to interpret due to the absence of drawings. There are several rules for the transverse sections of the ribs. As an example, Lechler says that the depth of the cross rib should be one-third of the wall thickness (that is $s/30$), and the width was to be one-half of its depth. The dimensions of the transverse ribs were a function of the cross-rib. Transverse arches should be one-third larger than the cross-ribs ($s/22$ nearly).

High towers surmounted with spires are as typical of gothic architecture as flying buttresses and cross vaults. The relevant parameter, given the plan and general proportion of the tower (relation between the side and the height), is the wall thickness. Two of the treatises gave the same rule: the wall thickness of the tower should be $^1/_{20}$ of its height. If the tower has counterforts these were to have the same depth as the wall thickness and a breadth two-thirds the wall thickness. The first rule for the wall thickness must have been a common rule in Germany because Albrecht Dürer (1525) used it in his *Unterweisung der Messung* (Geometrical instructions) when he explains the design of a city tower of height 300ft; he gives 15ft to the wall (without citing any rule), i.e. $^1/_{20}$ of its height. Fig 2.

We may compare this rule with the rule proposed by Alberti (translation published in 1992) in his treatise (written *c.*1450): *cf.* $^1/_{15}$ of the height. It is no surprise that Renaissance rules are more conservative.

Geometrical rules for gothic buttresses

Other gothic rules have survived through Renaissance or Baroque treatises of standard form or architecture. Two of them are important for their wide use. Both rules refer to the dimensioning of gothic buttresses.

The first, which we will call Rule no 1, permits obtaining the buttress for a cross vault using the profile of the transverse arches, Fig 3. It was published for the first time by Derand (1643) in his *Architecture des voûtes*. But the rule can be traced back, at least to the first half of the 16th century in the lost treatise of Baccojani (Müller 1990). It appears again, in the second half of the same century in the unpublished manuscript on stonecutting of Martínez de Aranda of *c.*1590. The rule was well known in the 17th and 18th centuries and can be tracked in building manuals until the 1960s. Though since the beginning of the 17th century it has been applied to size the buttresses of simple arches, it is a gothic rule and applies to gothic buttresses. Derand (1643) said this explicitly and Ungewitter (1859) and Ungewitter/Mohrmann (1890) used it in this context applying the rule to the transverse arch of the gothic vaults.

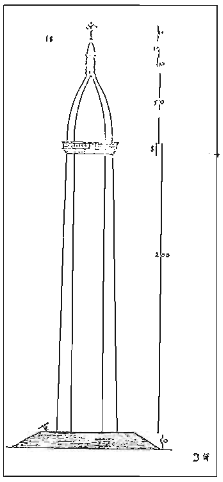


Fig 2. (above) Design for a city tower. The thickness of the wall is $^1/_{20}$ of its height. (Dürer 1525)

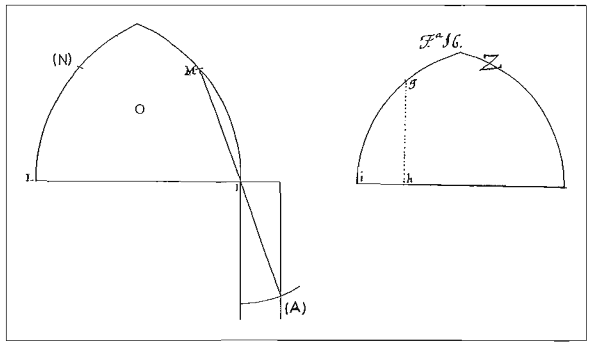


Fig 3. (top right) Geometrical rule no 1: left, Derand (1643); right, Martínez de Aranda (c. 1590)

The rule is as follows: in Fig 2(a) from Derand the arc LI is divided in three equal parts by the points (N) and M. The line MI is then prolonged so that MI = I(A). The point (A) defines the outer edge of the buttress (points (A) and (N) have been added to help to explain the rule). In Fig 2(b), from Martínez de Aranda the same dimension is obtained by an alternative, more simple, construction. Again the arc is divided into three equal parts by two points. Trace a perpendicular from one of them, g, to the springing line to obtain point h. The distance ih is the thickness of the buttress. When applied to some single nave gothic buildings the rule gives good concordance, Fig 4. (This does not necessarily mean that precisely this rule was used; it only proves that the rule is gothic.)

The second geometrical rule for buttress design, Rule no 2, Fig 5, was discovered by the author in the architectural treatise of Hernán Ruiz el Joven, a Spanish architect of the 16th century (Navascués 1974). Hernán Ruiz gives the rule as a method to obtain the abutment for simple arches, but it is, again, a gothic rule for buttress design. The same construction is given by Ungewitter (1859) as a rule to size the buttresses of a polygonal gothic apse. Ungewitter says nothing of its origin, but it is very probable that both have the same gothic origin. The appearance of the same rule in so many different places and epochs is a demonstration of their importance and widespread use.

The rule is as follows: consider a drawing of half the transverse arch of a gothic vault with its thickness. Draw the chord of the semi-arc, then trace a parallel line tangent to the extrados; the point where this line cuts the horizontal line of the arch springings defines the thickness of the buttress. The results are similar to those obtained with the previous rule.

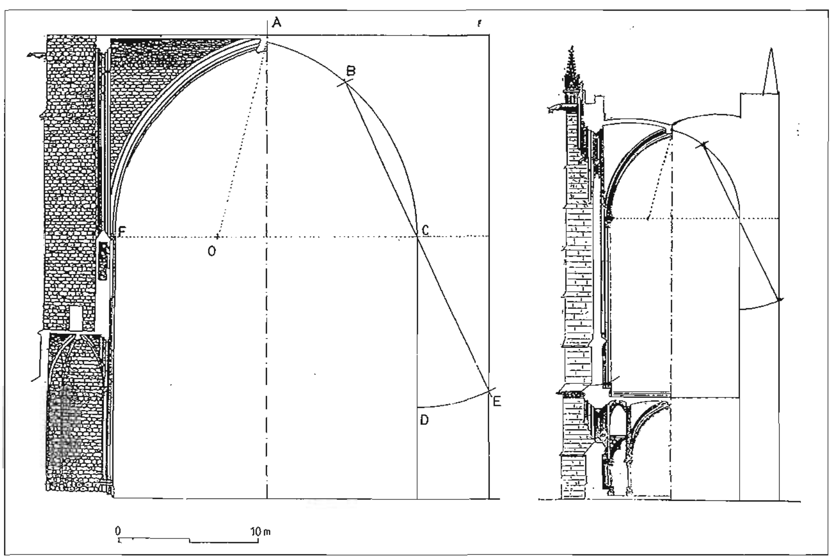


Fig 4. Application of geometrical rule no 1: left, Cathedral of Gerona; right, Sainte Chapelle

The structural rules of Rodrigo Gil de Hontañón

However, the most complete set of gothic rules appear in the manuscript of the architectural treatise of Rodrigo Gil de Hontañón (1500-1577), maybe the most important and prolific

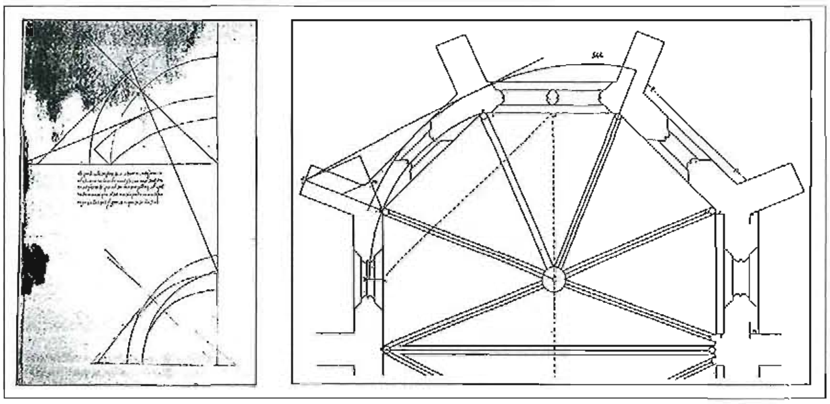


Fig 5. Geometrical rule no 2: left, manuscript of Hernán Ruiz; right, Ungewitter (1859)

Spanish architect of the 16th century. The son of a famous gothic master builder, Juan Gil de Hontañón, he inherited the tradition of gothic construction, but during his life he assimilated also the new vocabulary of the Renaissance. He participated to a greater or lesser degree in the construction of nine cathedrals (Astorga, Salamanca, Segovia, Plasencia, Santiago, etc.) and built many parish churches and civil buildings. Between 1544-1554 he wrote a treatise of architecture (Sanabria 1982) which was copied by Simón García in his *Compendio de Arquitectura* of 1681. There are two facsimile editions and an English translation by Sanabria (1984). In this paper all the English quotations to the manuscript are Sanabria's translation. References to the pages of the manuscript are in brackets.

The manuscript treats in a systematic way the different aspects of the design of a late-gothic church. In particular, in chapter 6, he treats specifically the sizing of structural elements using certain general rules (*'reglas generales'*). It is this last part which converts the manuscript into something unique. In no other gothic source appears such a conscientious separation of the structural skeleton. In spite of this, the rules have not received great attention: only Kubler (1944), Sanabria (1982, 1984) and Huerta (2004) have studied them in detail.

The rules could be divided in two groups:

1) rules for the design of the structural elements of a gothic church;
2) rules to investigate the buttress for an arch in a Renaissance arcade.

It is important to make this distinction which is justified by their location in the manuscript and, above all, by their different goals: practical in the first case, of research in the second. (Kubler and Sanabria make no distinction between the rules).

In the 16th century most of the churches built in Spain were covered by a special type of gothic vault, the *'bóvedas baídas'*. These vaults are of domical form and the ribs are very nearly disposed in the surface of a sphere which has as diameter the diagonal of the bay (cross ribs are perfect semicircles). All the examples in the manuscript correspond to this type of vault. Rodrigo explains the process of construction of the vaults, the only description from a gothic master which has survived, Fig 6 left. However, he notes that '...these things may be difficult to understand if one lacks experience and practice, or if one is not a stone mason, or has never been present at the closing of a rib vault'.(24r)

First, a platform is built at level of the *tas-de-charge* (a little above the springings). There the plan of the vault is drawn over it and the keystones are placed in position above wooden struts. Then, centrings between the keystones were constructed, the ribs were built and finally the masonry web between the ribs was laid. The rib skeleton functions as a permanent centring and ribs and keystones should have certain dimensions so that this skeleton would be in equilibrium, not only at the end, but during the whole building process.

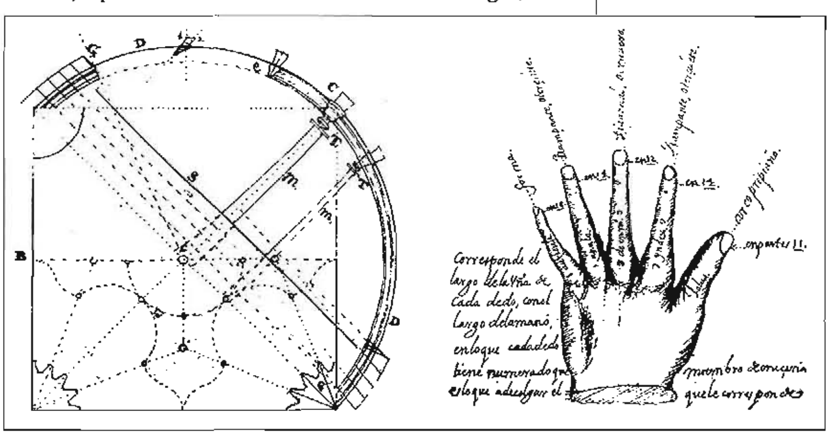


Fig 6. Right: construction of a cross vault. Left: sizing of the ribs by analogy with the hand

After defining the general proportions of the church, Rodrigo expresses his general rules. They refer to the sizing of piers, buttresses, ribs and keystones of the vault, and the walls of towers.

Piers

Rodrigo gives a rule to obtain the diameter (piers were usually cylindrical) of the interior piers. The rule is arithmetical and contains a square root but it is expressed discursively, by writing: 'Returning to the thickness of the piers, I say that the width of a nave bay, 40ft, should be added to the length, 30, which is 70. To this should be added the height of the column, 40ft, which is 110. The square root of 110 is 10-10/21, half of this is 5-5/21, and this should be the diameter of the column on the lower part. This is the closest to what is right.' (17r). The rule can be expressed algebraically:

$$d = \frac{1}{2}\sqrt{h + w + s} \qquad ...(1)$$

where, h is the height of the pier, and w and s are the width and span of that bay. The rule is not dimensionally correct and to obtain good results the data should be introduced in Castilian feet (0.28m); if we introduce the dimensions in metres the results are multiplied nearly by a factor of two. This rule is easy to verify in actual buildings; the author has checked the rule in the Church of Villacastín, near Madrid, and the agreement is perfect. In general, it can be said that the dimensions obtained by the rule agree quite well with those seen in published plans.

Buttresses

Another arithmetical rule is given to determine the size of the vault buttresses. Rodrigo gives first the rule and then applies it to a vault of certain dimensions. It is an important rule and he wanted, possibly, that no error could be committed. The text says: 'To find the necessary projection of the pier buttress, add up the feet of circumference (i.e. the perimeter) of the ribs supported by the buttress. By this is to be understood half of the length of the ribs, which is the lengths of the tiercerons to their keystones, the lengths of the diagonal ribs to their central bosses and half of the length of the transverse arch. Having added up all these dimensions, subtract one third, which is what is normally taken up by the mouldings. Should the mouldings take up more or less subtract more or less accordingly. Now measure the height of the buttress, and add it to the remainder of the previous operation. Take the square root, and divide it by three. One of these thirds will be the width of the buttress, and the remaining two thirds its length, including the engaged half column, the wall thickness, and the external projection'. (17v). Algebraically:

$$c = \frac{2}{3}\sqrt{h + \frac{2}{3}\sum N_i} \qquad ...(2)$$

where c is the total thickness of the buttress (including the wall) at the level of the springings of the vault, h is the height of the buttress and ΣN_i is the sum of the lengths of the ribs converging on the buttress, measured from the springing to their respective keystones. The breadth of the buttress is $c/2$. After giving a detailed numerical example Rodrigo affirms: 'This is the right size to hold the thrust of the arches. The workman can add somewhat more, because it is better to have too much than too little, although this size will be sufficient, as was stated.' (18r)

Rodrigo Gil remarks that this is the depth of the buttress at the level of the springing of the vaults, but that downwards it will increase by forming 'steps' at intervals. (The mean inclination of the outside face of a gothic buttress is, after Ungewitter (1890) 1:20). In Fig 7 left, is represented the way to use the rule; at the right, the relationship c/s has been plotted for different relations h/s, and different spans (the figures within the squares, in

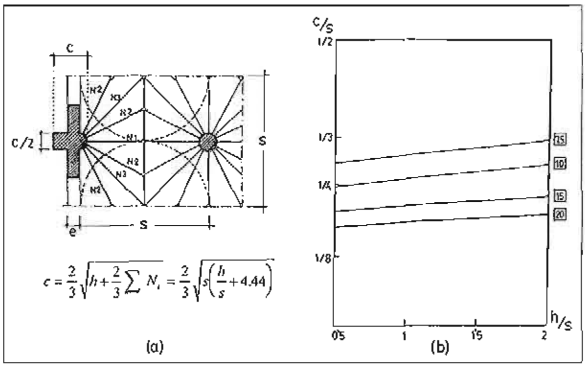


Fig 7.
Rodrigo Gil's rule for buttress design. Left, the rule expressed algebraically; right; the slenderness of the buttresses c/s, for different proportions height/span, for spans (7.5-20m)

metres). The buttresses become more slender as the span grows.

The rule is cited again twice in other parts of the manuscript. The first time at the beginning of Chapter 2 where he discusses several church designs; here he applies simply the rule without explanation, as a routine calculation (5r). It appears again at the end of chapter 6 where Rodrigo remarks strongly on the validity of the rule: 'Thus seeking the *intrinsic reasons and irreproachable causes*, it is necessary first to study the elevation of the temple to determine which members are thrusting against the buttress ... Having followed all the various instructions discussed above *the result will be strong, safe, beautiful, and proper*.' (22r, 22v, author's italics).

Vaults: ribs and keystones

The sizing of ribs and keystones is treated together. Rodrigo stresses the importance of the problem: 'It is good to know the correct size and thickness of the ribs and bosses of rib vaults, since we have seen many ruined either because their bosses were too heavy and thus much larger than what the ribs could hold, or else much too light so that the weight of the ribs lift them causing cracks to open in the walls.' (22v) Rodrigo alludes, probably, not only to the completed vault but also to the vault under construction

For the ribs he gives simple arithmetical formulae. It is interesting that he tries to reconcile older gothic geometrical rules with the design by analogy with the human body: 'Now in order to have a general rule, which is what we want, we must understand that the thumb may be viewed as the transverse arch, the index and ring fingers as tiercerons, the middle finger as the diagonal rib, and the little finger as the formeret. To determine the proportions of the fingers to the hands, take half the ounces of these fingers, which is the length of each fingernail ... dividing the length, or side, or a bay in 20 parts, one part shall be the height of the voussoirs of the transverse rib. The length of the bay divided in 24 parts shall be the height of the diagonal rib. The tiercerons will be one twenty eighth, and the formeret one thirtieth. Thus shall they be proportioned, in accordance with the work they do'. (23r) See Fig 6.

The thickness of the ribs in function of the span s are:

- transverse ribs $s/20$
- cross ribs $s/24$
- tiercerons $s/28$
- formerets $s/30$

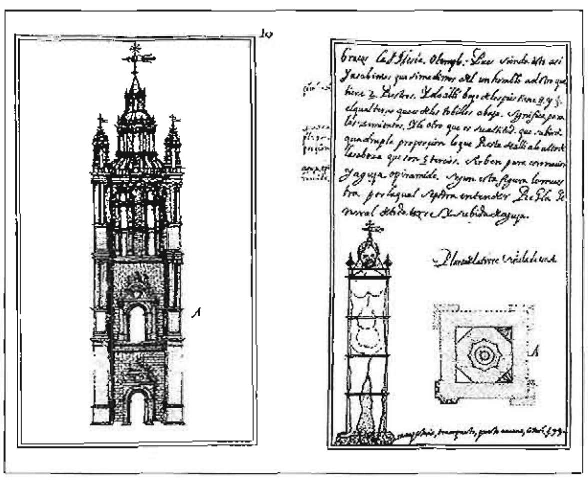


Fig 8.
Design of towers in the Treatise of Rodrigo Gil de Hontañón (García 1681)

When the bay is rectangular 'do not take either the long or the short sides but add them and divide by two.' (23v).

For the keystones the rule is again arithmetical. It is one of the most difficult rules to interpret. The rule gives the weight of the keystones in 'quintales' (a quintal = 46kg or, approximately, the weight of a cubic foot of a medium stone). In the formula enter again the lengths of the ribs, but a distinction should be made between those members that 'sustain' and those that 'are sustained': 'Those that are sustained must be subtracted from those that sustain. They can be told apart because those that sustain spring from the tas-de-charge, and those that are sustained spring from bosses. There are also sustaining and sustained bosses. Those found along the lengths of the diagonal rib or tiercerons are sustained. Those that are on the ends of the diagonal ribs or tiercerons sustain all others.' (23v, 24r) Then Rodrigo gives his formula, which can be written algebraically:

$$Q = P\sqrt{\sum Ri - \sum Si} \qquad ...(3)$$

where Q = weight of the boss in quintales; P = weight of the cross rib (quintales/foot); ΣRi = sum of the lengths of the ribs that sustain; ΣSi = sum of the lengths of the ribs that are sustained. The rule is, again, dimensionally incorrect. To use the rule correctly we should enter the data in Castilian feet and quintales, and the result will be in quintales. The keystones serve, obviously, to solve a complicated stereotomic problem (the union of different ribs), but they play also a fundamental role stabilising the rib skeleton during the construction of the masonry webs, as we shall see later.

Towers

Rodrigo treats also the structural design of towers. First he discusses the general proportions of the tower using the analogy with the human body, Fig 8. Then, he gives rules to size the wall thickness and the counterforts of the towers. The rules are arithmetical and were given discursively in the manuscript. Expressed algebraically:

$$t = \frac{1}{2}\sqrt{h} \qquad ...(4)$$

$$b = \frac{1}{2}\sqrt{h + a} \qquad ...(5)$$

where t is the wall thickness and b the buttress thickness at the top of the tower; h = height of the tower; a = height of the element (spire, dome, etc) on top of the tower.

In the manuscript we find evidence of the practical application of these rules. Chapter 75 of García's *Compendio* has the title 'General conditions to rebuild a ruined building'. The ruined building in question is a tower and the text is a report written by Rodrigo describing carefully the demolition of the ruin and the construction of a new tower (the elevation in Fig 8, left). The tower was to have a height of 120ft. Rodrigo does not cite any rule but recommends a wall thickness of 5ft and a buttress thickness of 7ft. If we use his rules the wall thickness should be 5.5ft and the buttress (for a side of 30ft) 6.1ft. There is no doubt that Rodrigo is using his rules in the structural design of the new tower.

Rules for the buttresses of Renaissance arcades

Rodrigo manifests no doubts in designing gothic vaults, buttresses and towers. His rules were an empirical adjustment of the data of many buildings, data which he would have inherited from his father and obtained in the archives of the many cathedrals and churches in which he worked. But when it comes to design the buttress for a single arch, Rodrigo confesses himself at a loss. He commences the corresponding section saying:

'I have tried many times to account for the buttress that any arch may need, but I have never found any rule to be sufficient. I have also discussed this with both Spanish and foreign architects, and none seems to have been able to verify such a rule: but all follow their own judgment. When I ask how do we know that so much is sufficient for a buttress, the answer is that it needs that much, but no reason is given.' (18v)

The word 'reason' here does not refer to a certain scientific theory; reason, '*razón*' in Spanish means also 'the order and

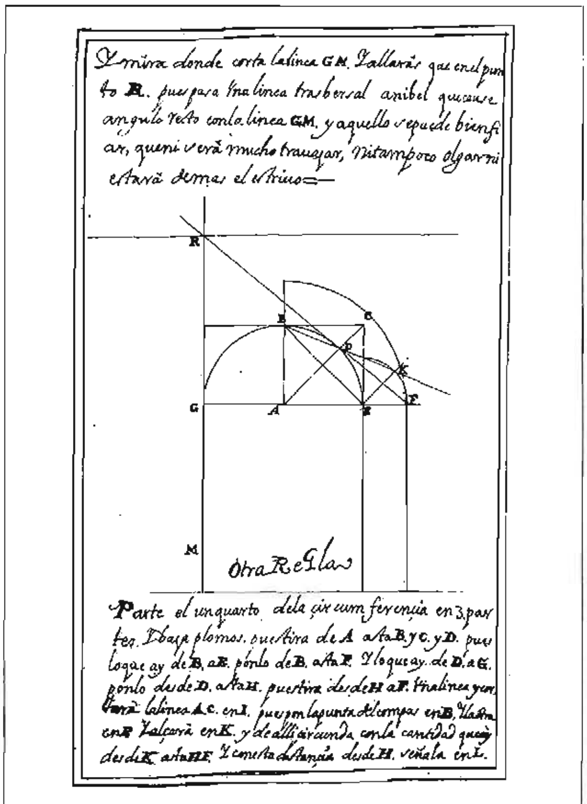


Fig 9. Geometrical rule by Rodrigo Gil to obtain the abutment pier in a Renaissance arcade (García 1681)

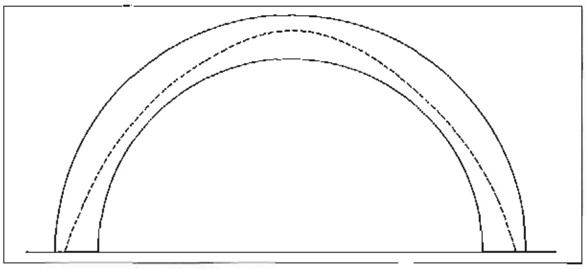


Fig 10. Semicircular masonry arch in a safe state of equilibrium

method to do something'. Rodrigo wanted a set of verified procedures, like those he used in the design of gothic structures. A simple barrel vault was an alien structure to him (as far as I know he built none) and he was perplexed.

The section, then, has an experimental nature. Rodrigo gives four different geometrical rules and an arithmetical rule. One of them is reproduced in Fig 9: the complicated geometrical construction determines the depth of the buttress and the height of the wall which can be supported. There is no space here to discuss the types and evolution of the rules (see Sanabria 1982, Huerta 2004) but their experimental character is evident. Sanabria has even suggested that the rules may be a register of actual experiments with real arches, and there are many arguments in favour of this hypothesis. In any case, it is evident that Rodrigo knew the specific character of the gothic rules and he does not even try to apply them to the new structural type.

Validity of the rules

As we have seen the gothic master builders used empirical rules for the design of the structural elements of their buildings. The rules were only a part of a more complex body of knowledge, and could not be used safely but by a master builder. These rules were widely known both geographically and chronologically and there is abundant evidence of their use throughout Europe.

Proportional rules

A great majority of the structural rules for masonry are 'proportional', that is to say, they produce 'similar' forms in a geometrical sense. They give, for example, the depth of the buttress for an arch depending on its curve of intrados but regardless of its size. In other words, they implicitly believe in the existence of a 'law of similitude': a valid structural form continues to be correct independently of its size (Fig 4).

Galileo argued the impossibility of the existence of this kind of principle: in structures supporting as the main load their own weight the dead load rises as the cube of the linear dimensions while the section of the structural members rise as the square; the tensions rise, therefore, linearly with the size (the so-called 'square-cube law'). Galileo's argument is valid only when the criterion of strength governs the design. The point has been made many times by Professor Heyman (1995) that this is not the case with masonry structures: the most restricted condition is that of stability. A masonry structure will be safe if it is possible to find a system of compressive internal forces in equilibrium with the loads. This is a geometrical condition, which depends on the form of the structure but not on its size. The case of a simple arch may be used as an example: in Fig 10 the semicircular arch is in a state of safe equilibrium with the line of thrust comfortably within the middle half of its thickness, and this leads to $t = s/18$. (This state is independent of the scale and the rule will be valid for arches, say, up to 1km span, when Galileo's law will begin to govern the design.) It is this kind of rule which was used in gothic rib design.

Proportional rules are therefore of the correct form and the old master builders possessed this all-important knowledge. The same property applies to much more complex structures and, for example, in a gothic cathedral the forms and dimensions of his elements allow a system of internal compressive forces which transmit the loads within the masonry, in the same way as it occurs with the simple arch. Therefore scaling up and down does not affect the safety of a masonry building. The rules for buttress design register the proportion between the buttress and the span. Some rules, the known geometrical rules, consider the fact that the thrust grows with the relation span/height of the vault. Surbaissé arches and vaults thrust more than semicircular or pointed arches or vaults. Of course, the rules can only be applied within the whole context of building; its deep meaning is understood only by the masters, who sometimes decide to deviate from them (compensating with other changes in the geometry).

Non-proportional rules

Many of the rules of Rodrigo Gil de Hontañón are non-proportional, not even dimensionally correct. They have been considered therefore incorrect and nonsensical (Sanabria 1982). In fact they refer to non-proportional problems. For reasons of space we will consider only three cases: vault buttress design, wall design in towers and boss design.

In late gothic Spanish vaults the thickness of the webs is very often constant: the minimum that can be practically built (150-200mm of stone). In this situation, the weight, and therefore the thrust, of a gothic vault rises with the square of its linear dimensions; the weight of the buttresses on the contrary rises with the cube, as is evident looking at Fig 12. If we scale up a building it will need buttresses proportionally more slender. Just the contrary of Galileo's square-cube law. The matter has been studied with modern details by the author elsewhere, by computing the buttress following the rule and comparing it with the result obtained by the calculated thrust of the vault, but here we want only to point out the essentially correct character of the rule.

The same occurs with high towers. Here the main load is the action of wind. The total thrust of the wind rises with the cross

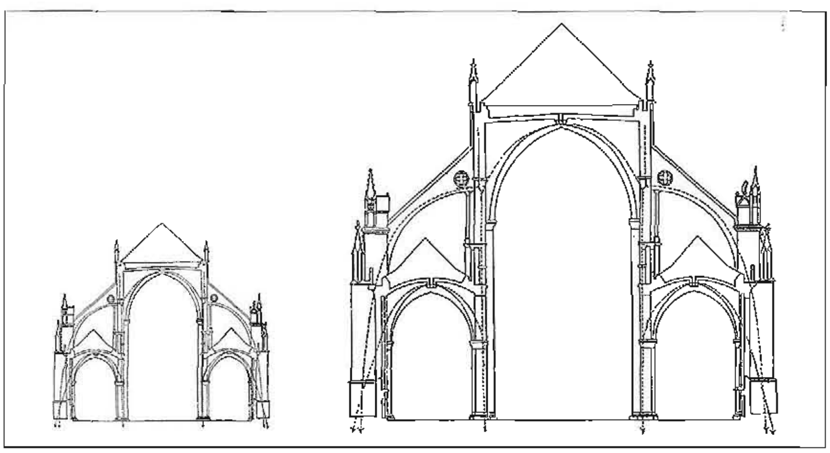


Fig 11. Scaling up and down does not affect the safety of a masonry structure

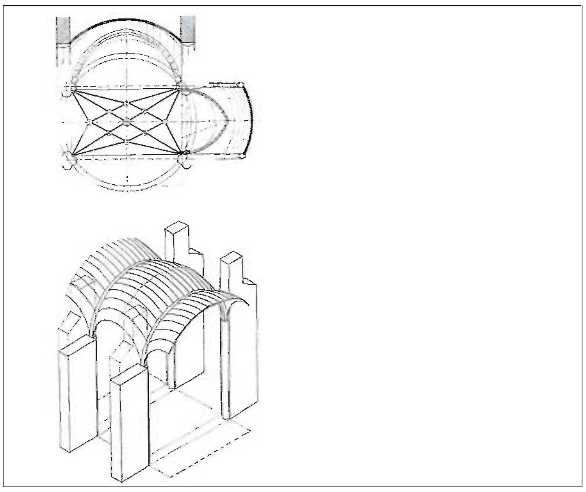
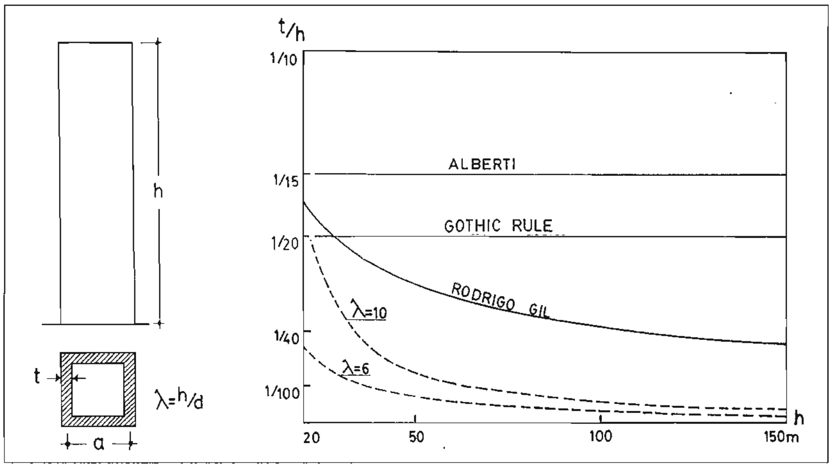


Fig 12. Vault-buttress system of a late gothic Spanish church (Huerta 2004)

Fig 13. Design of masonry towers. In solid lines the traditional rules; in dotted line the results of scientific calculation of typical values for masonry and wind (Huerta 2004)



sectional surface of the tower, but its weight grows with the volume. Again, greater towers could have proportionally lesser thickness, and this property could easily be seen if we compare similar towers of different sizes. In this case, the calculations are quite easy. In Fig 13, the relationship t/h has been calculated for different heights. The dotted lines have been calculated so that the whole section is in compression (the resultant within the central nucleus of inertia at the base; masonry of specific weight $20kN/m^3$ and $1.5kN/m^2$ unit wind pressure) leading to thicknesses so thin as to be impossible to use in normal masonry building (only in gothic spires may we find such orders of magnitude). It is evident, that Rodrigo's rule gives a much better adjustment than the proportional rules. The tower of the cathedral at Segovia has a height, nearly, of h = 90m or 322 Castilian feet; the gothic rule (eqn 4) gives t = 90/20 = 4.5m and Rodrigo's rule t = 9 Castilian feet or 2.52m. The actual thickness at the base is 10ft or 2.8m. (Scientific calculation t = 0.17m) Rodrigo's rule represents a much finer adjustment than the traditional proportional rules, and remains safe and practical.

Finally, Rodrigo Gil stressed the importance of a correct size for the heavy gothic keystones. The skeleton of ribs must be stable during construction. Arch rib design is proportional and the rules are a fraction of the span. Web construction would have progressed from the perimeter to the centre of the bay. In this situation it is possible that the skeleton of ribs, loaded mainly in the haunches, could collapse by rising of its central keystone. Keystones placed on top of wooden struts were a passive weight which was used, if necessary, to stabilise the rib skeleton during construction. The statics are evident and are explained in Fig 14 (the ribs are supposed 'weightless'). The dotted line, completely outside the ribs, represents the situation without a keystone, and the ribs will collapse inwards by raising the keystone.

In summary, non-proportional rules represent a finer adjustment to a non-linear relationship of the involved variables. They were deduced empirically from a close observation of existing structures or of structures under construction. What is significant is that they point to important design aspects. An unprejudiced critical reading of the old gothic treatises has served to disclose some properties of the design of masonry structures that are usually not noticed.

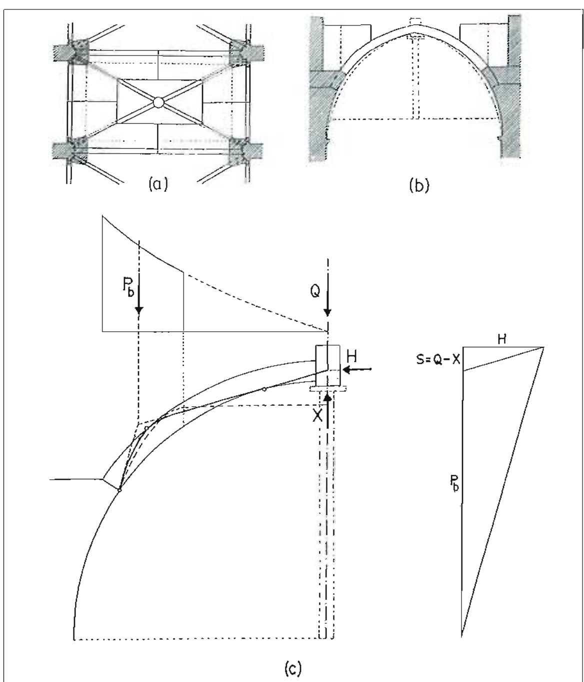


Fig 14. Stabilising function of the central keystone during the building of the vault (Huerta 2004)

Santiago Huerta became an architect in 1981 following study at the School of Architecture, Polytechnic University of Madrid (Escuela Tecnica Superior de Arquitectura, Universidad Politecnica de Madrid). He was in professional practice as an architect between 1982-89. In 1989 he became Assistant Professor in the School of Architecture of Madrid. He gained a PhD in 1990 with a dissertation on 'Structural design of arches and vaults in Spain: 1500–1800'. From1992 he has been Professor of Structural Design at the School of Architecture of Madrid. Since 2003 he has beed the President of the Spanish Society for Construction History and has written a book, *Arcos, bóvedas y cúpulas*, (Madrid, 2004).

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