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Application of the generalized finite difference method to seismic wave propagation in 2-D

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Abstract

This paper shows the application of generalized finite difference method (GFDM) to the problem of seismic wave propagation. We investigated stability and star dispersion in 2-D.
We obtained indepedent stability conditions and star dispersion for the *P* and *S* waves. Also, we are obtained *P* and *S*-wave group velocity.

1 Introducción

The Generalized finite difference method (GFDM) is evolved from classical finite difference method (FDM). GFDM can be applied over general or irregular clouds of points. The basic idea is to use moving least squares (MLS) approximation to obtain explicit difference formulae which can be included in partial differential equation to establish, together with an explicit method, a recursive relationship. The authors have made many contributions to the development of this method [1],[2], [3], [4] and [5].
In this paper, this meshless method is applied to seismic wave propagation . We derived stability conditions and grid dispersion relations in 2-D.

2 Explicit Generalized Differences Schemes for the seismic waves propagation problem for a perfectly elastic, homogeneous and isotropic medium

2.1 Equation of motion

The equation of motion and Hooke's law for a perfectly elastic, homogeneous, isotropic medium in 2-D are

$$\begin{cases} \dfrac{\partial^2 U(x,y,t)}{\partial t^2} = \alpha^2 \dfrac{\partial^2 U(x,y,t)}{\partial x^2} + \beta^2 \dfrac{\partial^2 U(x,y,t)}{\partial y^2} + (\alpha^2 - \beta^2) \dfrac{\partial^2 V(x,y,t)}{\partial x \partial y} \\ \dfrac{\partial^2 V(x,y,t)}{\partial t^2} = \beta^2 \dfrac{\partial^2 V(x,y,t)}{\partial x^2} + \alpha^2 \dfrac{\partial^2 V(x,y,t)}{\partial y^2} + (\alpha^2 - \beta^2) \dfrac{\partial^2 U(x,y,t)}{\partial x \partial y} \end{cases} \quad (1)$$

with the initial and the boundary conditions

$$\alpha = \sqrt{\frac{\lambda+2\mu}{\rho}}, \quad \beta = \sqrt{\frac{\mu}{\rho}}, \quad r = \frac{\alpha}{\beta}$$

ρ is the density, λ and μ are Lamé elastic coefficients and Γ is the boundary of Ω.

2.2 Explicit Generalized Differences Scheme

The replacement in the equation 1 of the explicit expressions obtained for the partial derivatives leads to

$$\begin{cases} u_0^{n+1} = 2u_0^n - u_0^{n-1} + (\triangle t)^2[\alpha^2(-m_0 u_0^n + \sum_1^N m_j u_j^n) + \beta^2(-\eta_0 u_0^n + \sum_1^N \eta_j u_j^n) \\ \qquad +(\alpha^2-\beta^2)(-\zeta_0 v_0^n + \sum_1^N \zeta_j v_j^n)] \\ v_0^{n+1} = 2v_0^n - v_0^{n-1} + (\triangle t)^2[\beta^2(-m_0 v_0^n + \sum_1^N m_j v_j^n) + \alpha^2(-\eta_0 v_0^n + \sum_1^N \eta_j v_j^n) \\ \qquad +(\alpha^2-\beta^2)(-\zeta_0 u_0^n + \sum_1^N \zeta_j u_j^n)] \end{cases} \tag{2}$$

3 Stability Criterion

For the stability analysis the first idea is to make a harmonic decomposition of the approximated solution at grid points and at a given time level (n). Following von Neumann method. The condition for stability of star is

$$\triangle t < \sqrt{\frac{4}{(\alpha^2+\beta^2)[(|m_0|+|\eta_0|)+\sqrt{(m_0+\eta_0)^2+\zeta_0^2}]}} \tag{3}$$

4 Star dispersion

4.1 Star-dispersion relations for the P and S waves

The phase velocity star-dispersion relationship are:

$$\frac{\alpha^{grid}}{\alpha} = \frac{\arccos\Phi}{2\pi sp} \tag{4}$$

$$\frac{\beta^{grid}}{\beta} = \frac{\arccos\Phi}{2\pi sp} \tag{5}$$

The group velocity star-dispersion for waves P and S are

$$\frac{\alpha^{grid}_{group}}{\alpha} = \frac{1}{2\sqrt{2}r}\frac{\Upsilon}{\sqrt{F-\left(\frac{pF}{\sqrt{(r^2+1)[(|m_0|+|\eta_0|)+\sqrt{(m_0+\eta_0)^2+\zeta_0^2}]}\sqrt{2}}\right)^2}} \tag{6}$$

$$\frac{\beta^{grid}_{group}}{\beta} = \frac{1}{2\sqrt{2}}\frac{\Upsilon}{\sqrt{F-\left(\frac{pF}{\sqrt{(r^2+1)[(|m_0|+|\eta_0|)+\sqrt{(m_0+\eta_0)^2+\zeta_0^2}]}\sqrt{2}}\right)^2}} \tag{7}$$

where Φ, Υ, F, s, p are functions of the coefficients of the explicit generalized finite difference formulae and α, β

5 Conclusiones

En este artículo se muestran las expresiones explíctas, en diferencias finitas generalizadas, para la propogación de ondas sísmicas en 2-D. Se estudia la estabilidad, obteniéndose un criterio en función de la razón de velocidades y los coeficientes de las fórmulas explícitas.
Se analiza la dispersión y se relaciona con la irregularidad de la estrella utilizando el índice de irregularidad de la malla. La utilización de mallas irregulares, según la geometría del problema, puede ocasionar que se produzcan dispersiones elevadas en algunas estrellas, lo cual va asociado con valores elevados del indicador de irregularidad de la malla (IIM). En este caso se redefine la malla mediante un proceso adaptativo hasta conseguir una malla con valores de dispersión e índice de irregularidad adecuados.

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