Performance Analysis of SVD-assisted Downlink Multiuser MIMO Systems

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Abstract: Multiuser multiple-input multiple-output (MIMO) downlink (DL) transmission schemes experience both multiuser interference as well as inter-antenna interference. Instead of treating all the users jointly as in zero-forcing (ZF) multiuser transmission techniques, the investigated singular value decomposition (SVD) assisted DL multiuser MIMO system takes the individual user’s channel characteristics into account. This translates to a choice of modulation constellation and transmitter power and, in our proposed system, to a choice of number of activated user-specific MIMO layers. The performed joint optimization of the number of activated MIMO layers and the number of bits per symbol along with the appropriate allocation of the transmit power shows that not necessarily all user-specific MIMO layers has to be activated in both frequency-selective and non-frequency selective MIMO channels in order to minimize the overall BER under the constraint of a given fixed data throughput.

1 Introduction

Multiple-Input Multiple-Output (MIMO) systems are capable of increasing the achievable capacity and integrity of wireless systems and hence, they may be expected to form an integral part of next generation wireless systems [ZT03, ZVDL05]. However, single-user MIMO transmission schemes for both non-frequency and frequency selective MIMO channels have attracted a lot of attention and reached a state of maturity [Kő6, AL08, ABP09b]. By contrast, MIMO-aided multiple-user systems require substantial further research where both multiuser as well as multi-antenna interferences has to be taken into account. Considering the entirety of the antennas of all mobile terminals at one end and the antennas of the base station at the other end of the communication link, state of the art interference cancellation is based on a central signal processing unit, e.g. a central unit at the base station, where joint detection can be applied in the uplink (UL) and joint transmission in the downlink (DL), respectively [MBW’00, WMSL02, WSLW03, CM04, JUN05]. Widely used linear preprocessing techniques such as Minimum Mean Square Error (MMSE) or Zero Forcing (ZF) have attracted a lot of research and have reached a state of maturity [CM03]. In this work, a singular value decomposition (SVD) assisted
downlink (DL) multiuser MIMO system is considered, which takes the individual user’s channel characteristics into account rather than treating all users channels jointly as in ZF multiuser transmission techniques [LYH08]. Treating all user independently, adaptive modulation is a promising technique to increase the spectral efficiency of wireless transmission systems by adapting the signal parameters, such as modulation constellation or transmit power, dynamically to changing channel conditions. Therein, the most beneficial choice of the number of activated user-specific MIMO layers together with the number of bits per symbol and the appropriate allocation of the transmit power offer a certain degree of design freedom, which substantially affects the performance of MIMO systems in both frequency-selective and non-frequency selective MIMO links. Existing bit loading and transmit power allocation techniques are often optimized for maintaining both a fixed transmit power and a fixed target bit-error rate while attempting to maximize the overall data-rate [KRJ00, FH96, ZVDL05]. However for fixed-rate applications, such as video transmission schemes, it is desirable to design algorithms, which minimize the bit-error rate (BER) at a given fixed data rate.

Against this background, in this paper a SVD-assisted multiuser MIMO scheme is investigated, where both multiuser interferences as well as multi-antenna interferences are perfectly eliminated. The novel contribution of this paper is that we demonstrate the benefits of amalgamating a suitable choice of activated MIMO layers and number of bits per symbol along with the appropriate allocation of the transmit power under the constraint of a given fixed data throughput.

The remaining part of this paper is organized as follows: Section 2 introduces the system model. The considered quality criteria are briefly reviewed in section 3. The proposed solutions of bit and power allocation are discussed in section 4, while the associated performance results are presented and interpreted in section 5. Finally, section 6 provides some concluding remarks.

2 Flat Fading multiuser MIMO system model

The system model considered in this work consists of a single base station (BS) supporting $K$ mobile stations (MSs). The BS is equipped with $n_T$ transmit antennas, while the $k$th (with $k = 1, \ldots, K$) MS has $n_{R_k}$ receive antennas, i.e. the total number of receive antennas including all $K$ MSs is given by $n_R = \sum_{k=1}^{K} n_{R_k}$. The $(n_{R_k} \times 1)$ user specific symbol vector $c_k$ to be transmitted by the BS is given by

$$c_k = (c_{k,1}, c_{k,2}, \ldots, c_{k,n_{R_k}})^T.$$  \hspace{1cm} (1)

The vector $c_k$ is preprocessed before its transmission by multiplying it with the $(n_T \times n_{R_k})$ DL preprocessing matrix $R_k$ and results in the $(n_T \times 1)$ user-specific transmit vector

$$s_k = R_k c_k.$$ \hspace{1cm} (2)
After DL transmitter preprocessing, the $n_T$-component signal $s$ transmitted by the BS to the $K$ MSs results in

$$s = \sum_{k=1}^{K} s_k = R \mathbf{c},$$  \hspace{1cm} (3)$$

with the $(n_T \times n_R)$ preprocessing matrix

$$R = (R_1, R_2, \ldots, R_K).$$  \hspace{1cm} (4)$$

In (3), the overall $(n_R \times 1)$ transmitted DL data vector $\mathbf{c}$ combines all $K$ DL transmit vectors $\mathbf{c}_k$ (with $k = 1, 2, \ldots, K$) and is given by

$$\mathbf{c} = (\mathbf{c}_1^T, \mathbf{c}_2^T, \ldots, \mathbf{c}_K^T)^T.$$  \hspace{1cm} (5)$$

At the receiver side, the $(n_R \times 1)$ vector $\mathbf{u}_k$ of the $k$th MS results in

$$\mathbf{u}_k = H_k s_k + n_k = H_k R \mathbf{c} + n_k,$$  \hspace{1cm} (6)$$

and can be expressed by

$$\mathbf{u}_k = H_k R_k \mathbf{c}_k + \sum_{i=1,i \neq k}^{K} H_k R_i \mathbf{c}_i + n_k,$$  \hspace{1cm} (7)$$

where the MSs received signals experience both multi-user and multi-antenna interferences. In (6), the $(n_{R,k} \times 1)$ channel matrix $H_k$ connects the $n_T$ BS specific transmit antennas with the $n_{R,k}$ receive antennas of the $k$th MS. It is assumed that the coefficients of the $(n_{R,k} \times n_T)$ channel matrix $H_k$ are independent and Rayleigh distributed with equal variance. The interference between the different antenna’s data streams, which is introduced by the off-diagonal elements of the channel matrix $H_k$, requires appropriate signal processing strategies. A popular technique is based on the SVD of the system matrix $H_k$. Upon carrying out the SVD of $H_k$ with $n_T \geq n_R$ and assuming that the rank of the matrix $H_k$ equals $n_{R,k}$, i.e., $\text{rank}(H_k) = n_{R,k}$, we get

$$H_k = U_k \cdot V_k \cdot D_k^H,$$  \hspace{1cm} (8)$$

with the $(n_{R,k} \times n_{R,k})$ unitary matrix $U_k$ and the $(n_T \times n_R)$ unitary matrix $D_k^H$, respectively\(^\dagger\). The $(n_{R,k} \times n_T)$ diagonal matrix $V_k$ can be decomposed into a $(n_{R,k} \times n_{R,k})$ matrix $V_{k,u}$ containing the non-zero square roots of the eigenvalues of $H_k^H H_k$, i.e.,

$$V_{k,u} = \begin{bmatrix} \sqrt{\zeta_{k,1}} & 0 & \cdots & 0 \\ 0 & \sqrt{\zeta_{k,2}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \sqrt{\zeta_{k,n_{R,k}}} \end{bmatrix},$$  \hspace{1cm} (9)$$

\(^\dagger\)The transpose and conjugate transpose (Hermitian) of $D_k$ are denoted by $D_k^T$ and $D_k^H$, respectively.
and a \((n_{R,k} \times (n_T - n_{R,k}))\) zero-vector \(V_{k,u}\) according to
\[
V_k = (V_{k,u} V_{k,n}) = (V_{k,u} 0).
\] (10)

Additionally, the \((n_T \times n_T)\) unitary matrix \(D_k\) can be decomposed into a \((n_T \times n_{R,k})\) matrix \(D_{k,u}\) constituted by the eigenvectors corresponding to the non-zero eigenvalues of \(H_k^H H_k\) and a \((n_T \times (n_T - n_{R,k}))\) matrix \(D_{k,n}\) constituted by the eigenvectors corresponding to the zero eigenvalues of \(H_k^H H_k\). The decomposition of the matrix \(D_k^H\) results in
\[
D_k^H = \begin{pmatrix} D_{k,u}^H \\ D_{k,n}^H \end{pmatrix}.
\] (11)

Finally, the received downlink signal \(u_k\) of the \(k\)th MS may be expressed as
\[
u_k = U_k V_k u D_k^H R_c + n_k.
\] (12)

Taking all MSs received DL signals \(u_k\) into account, the \((n_R \times 1)\) receive vector results in
\[
u = (u_1^T, u_2^T, \ldots, u_K^T)^T.
\] (13)

Then, the overall DL signal vector \(\nu\) including the received signals of all \(K\) MSs can be expressed by
\[
u = U V_u D_u^H R_c + n,
\] with the overall \((n_R \times 1)\) noise vector
\[
n = (n_1^T, n_2^T, \ldots, n_K^T)^T.
\] (15)

the \((n_R \times n_R)\) block diagonal matrix \(U\)
\[
U = \begin{bmatrix}
U_1 & 0 & \cdots & 0 \\
0 & U_2 & \ddots & \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & U_K
\end{bmatrix},
\] (16)

the \((n_R \times n_R)\) block diagonal matrix \(V_u\)
\[
V_u = \begin{bmatrix}
V_{1,u} & 0 & \cdots & 0 \\
0 & V_{2,u} & \ddots & \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & V_{K,u}
\end{bmatrix},
\] (17)

and the \((n_T \times n_R)\) matrix \(D_u\) which is given by
\[
D_u = (D_{1,u}, D_{2,u}, \ldots, D_{K,u}).
\] (18)
In order to suppress the DL multi-user interferences (MUI) perfectly, the DL preprocessing matrix $\mathbf{R}$ has to be designed to satisfy the following condition

$$\mathbf{D}_u^H \mathbf{R} = \mathbf{P}, \quad (19)$$

with the real-valued $(n_R \times n_R)$ diagonal matrix $\mathbf{P}$ taking the transmit-power constraint into account. In order to satisfy (19), $\mathbf{R}$ can be defined as follows

$$\mathbf{R} = \mathbf{D}_u \left( \mathbf{D}_u^H \mathbf{D}_u \right)^{-1} \mathbf{P}. \quad (20)$$

Taking the ZF design criterion for the DL preprocessing matrix into account, the matrix $\mathbf{P}$ simplifies to an $(n_R \times n_R)$ diagonal matrix, i.e.

$$\mathbf{P} = \sqrt{\beta} \mathbf{I}_{n_R \times n_R},$$

with the parameter $\sqrt{\beta}$ taking the transmit-power constraint into account. When taking the DL preprocessing matrix, defined in (20), into account, the overall received vector of all $K$ MSs, defined in (14), can be simplified to

$$\mathbf{u} = \mathbf{U} \mathbf{V}_u \mathbf{P}_c + \mathbf{n}. \quad (21)$$

Therein, the $(n_R \times n_R)$ block diagonal matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{bmatrix}
\mathbf{P}_1 & 0 & \cdots & 0 \\
0 & \mathbf{P}_2 & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & \mathbf{P}_K
\end{bmatrix}. \quad (22)$$

In (21), the user-specific $(n_R \times 1)$ vector $\mathbf{u}_k$ can be expressed as

$$\mathbf{u}_k = \mathbf{U}_k \mathbf{V}_u \mathbf{P}_k \mathbf{c}_k + \mathbf{n}_k, \quad (23)$$

with the user-specific $(n_R \times n_{R,k})$ power allocation matrix

$$\mathbf{P}_k = \begin{bmatrix}
\sqrt{p_{k,1}} & 0 & \cdots & 0 \\
0 & \sqrt{p_{k,2}} & \cdots & \vdots \\
\vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & \sqrt{p_{k,n_{R,k}}}
\end{bmatrix}. \quad (24)$$

As long as the transmit power is uniformly distributed over the number of activated MIMO layers, the matrix $\mathbf{P}_k$ simplifies to $\mathbf{P}_k = \sqrt{\beta} \mathbf{I}_{n_{R,k} \times n_{R,k}}$. After postprocessing of the received signal vectors $\mathbf{u}_k$ with the corresponding unitary matrix $\mathbf{U}_k^H$, the user-specific decision variables result with $\mathbf{U}_k^H \mathbf{n}_k = \mathbf{w}_k$ in

$$\mathbf{y}_k = \mathbf{U}_k^H \mathbf{u}_k = \mathbf{V}_k \mathbf{u} \mathbf{P}_k \mathbf{c}_k + \mathbf{w}_k, \quad (25)$$

or alternatively with $\mathbf{U}^H \mathbf{n} = \mathbf{w}$ in

$$\mathbf{y} = \mathbf{U}^H \mathbf{u} = \mathbf{V}_u \mathbf{P} \mathbf{c} + \mathbf{w}, \quad (26)$$

where interferences between the different antenna data streams as well as MUI imposed by the other users are avoided. The resulting system model is depicted in Fig. 1.
3 Theoretical Analysis of System Performance

In general, the user-specific quality of data transmission can be informally assessed by using the signal-to-noise ratio (SNR) at the detector’s input defined by the half vertical eye opening and the noise power per quadrature component according to

\[
\varrho = \frac{\text{(Half vertical eye opening)}^2}{\text{Noise Power}} = \frac{(U_A)^2}{(U_R)^2},
\]

which is often used as a quality parameter [AL08]. The relationship between the signal-to-noise ratio \( \varrho = U_A^2 / U_R^2 \) and the bit-error probability evaluated for AWGN channels and \( M \)-ary Quadrature Amplitude Modulation (QAM) is given by [Pro00]

\[
P_{\text{BER}} = \frac{2}{\log_2(M)} \left( 1 - \frac{1}{\sqrt{M}} \right) \text{erfc} \left( \frac{\sqrt{\varrho}}{\sqrt{2}} \right).
\]

When applying the proposed system structure for the \( k \)th user, depicted in Fig. 1, the applied signal processing leads to different eye openings per activated MIMO layer \( \ell \) (with \( \ell = 1, 2, \ldots, L \) and \( L \leq n_{R,k} \) describing the number of activated user-specific MIMO layers) and per transmitted symbol block \( m \) according to

\[
U_{A_k}^{(\ell,m)} = \sqrt{p_{k,\ell}^{(m)}} \cdot \sqrt{\xi_{k,\ell}^{(m)}} \cdot U_{s_k}^{(\ell)},
\]

where \( U_{s_k}^{(\ell)} \) denotes the half-level transmit amplitude assuming \( M_\ell \)-ary QAM, \( \sqrt{\xi_{k,\ell}^{(m)}} \) represents the corresponding positive square roots of the eigenvalues of the matrix \( HH_k^H \) and \( \sqrt{p_{k,\ell}^{(m)}} \) represents the corresponding power allocation weighting parameters (Fig. 1). Together with the noise power per quadrature component, introduced by the additive, white Gaussian noise (AWGN) vector \( w_k = U_k^H n_k \) in (25), the \( k \)th user-specific SNR per MIMO layer \( \ell \) at the time \( m \) results in

\[
\varrho_{(\ell,m)} = \frac{U_{A_k}^{(\ell,m)}^2}{U_R^2}.
\]

Using the parallel transmission over \( L \) MIMO layers, the overall mean transmit power becomes \( P_{s,k} = \sum_{\ell=1}^L P_{s_k}^{(\ell)} \). Considering QAM constellations, the average user-specific
Table 1: Investigated user-specific QAM transmission modes

<table>
<thead>
<tr>
<th>throughput</th>
<th>layer 1</th>
<th>layer 2</th>
<th>layer 3</th>
<th>layer 4</th>
</tr>
</thead>
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<tr>
<td>8 bit/s/Hz</td>
<td>256</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8 bit/s/Hz</td>
<td>64</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8 bit/s/Hz</td>
<td>16</td>
<td>16</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8 bit/s/Hz</td>
<td>16</td>
<td>4</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>8 bit/s/Hz</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Transmit power $P_{s,k}^{(\ell)}$ per MIMO layer $\ell$ may be expressed as [Pro00]

$$P_{s,k}^{(\ell)} = \frac{2}{3} \left( \frac{U_{s,k}^{(\ell)}}{U_{s,k}^{(\ell)}} \right)^2 (M_k \ell - 1).$$

(31)

Combining (30) and (31) together with (29), the layer-specific SNR at the time $m$ results in

$$\varphi_{(\ell,m)} = p_{k,\ell}^{(m)} c_{k,\ell}^{(m)} \frac{3}{2(M_k \ell - 1)} \frac{P_{s,k}^{(\ell)}}{U_{R}^2}. \quad (32)$$

Assuming that the transmit power is uniformly distributed over the number of activated MIMO layers, i.e., $P_{s,k}^{(\ell)} = P_{s,k}/L$, the layer-specific signal-to-noise ratio at the time $m$, defined in (32), results with the ratio of symbol energy to noise power spectral density $E_s/N_0 = P_{s,k}/(2 U_{R}^2)$ in

$$\varphi_{(\ell,m)} = p_{k,\ell}^{(m)} c_{k,\ell}^{(m)} \frac{3}{L(M_k \ell - 1)} \frac{E_s}{N_0}. \quad (33)$$

In order to transmit at a fixed data rate while maintaining the best possible integrity, i.e., bit-error rate, an appropriate number of user-specific MIMO layers has to be used, which depends on the specific transmission mode, as detailed in Table 1 for the exemplarily investigated two-user multiuser-system ($n_R = 4$ with $k = 1, 2$, $K = 2$, $n_T = 8$). In general, the BER per spatial division multiplexing (SDM) MIMO data vector is dominated by the specific transmission modes and the characteristics of the singular values, resulting in different BERs for the different QAM configurations in Table 1. An optimized adaptive scheme would now use the particular transmission modes, e.g., by using bit auction procedures [WCLM99], that results in the lowest BER for each SDM MIMO data vector. This would lead to different transmission modes per SDM MIMO data vector and a high signaling overhead would result. However, in order to avoid any signalling overhead, fixed transmission modes are used in this contribution regardless of the channel quality. The $k$th user MIMO layer specific bit-error probability at the time $m$ is given by

$$P_{e_k}^{(\ell,m)} = \frac{2}{\log_2(M_k \ell)} \text{erfc} \left( \frac{\varphi_{(\ell,m)}^{(m)}}{\sqrt{2}} \right). \quad (34)$$
The resulting average $k$th user bit-error probability per transmitted symbol block $m$ assuming different QAM constellation sizes per activated MIMO layer results in

$$P_{e,k}^{(m)} = \frac{1}{\sum_{\nu=1}^{L} \log_2(M_k \nu)} \sum_{\ell=1}^{L} \log_2(M_k \ell) P_{e,k}^{(\ell,m)}.$$  \hspace{1cm} (35)

When considering time-variant channel conditions, rather than an AWGN channel, the BER can be derived by considering the different transmission block SNRs.

### 4 Adaptive MIMO-layer Power Allocation

In systems, where channel state information is available at the transmitter side, the knowledge about how the symbols are attenuated by the channel can be used to adapt the transmit parameters. Power allocation (PA) can be used to balance the bit-error probabilities in the activated MIMO layers and has been widely investigated in the literature [KRJ00, AL07, JL03].

In order to suppress the DL MUI efficiently, the DL preprocessing matrix has to be designed according to equation (20).

However, the user-specific BER of the uncoded MIMO system is dominated by the specific layers having the lowest SNR’s. As a remedy, a MIMO-layer transmit PA scheme is required for minimizing the overall BER under the constraint of a limited total MIMO transmit power. The proposed PA scheme scales the half-level transmit amplitude $U_{s,k}^{(\ell)}$ of the $\ell$th MIMO layer by the factor $\sqrt{\tilde{p}_{k,\ell}^{(m)}}$. This results in a MIMO layer-specific transmit amplitude of $U_{s,k}^{(\ell)} \sqrt{p_{k,\ell}^{(m)}}$ for the QAM symbol of the transmit data vector transmitted at the time $m$ over the MIMO layer $\ell$. Together with the DL preprocessing design, the layer-specific power allocation parameter at the time $m$ results in:

$$\sqrt{p_{k,\ell}^{(m)}} = \sqrt{\beta^{(m)}} \sqrt{\tilde{p}_{k,\ell}^{(m)}}.$$  \hspace{1cm} (36)

Applying MIMO-layer PA, the information about how the symbols are attenuated by the channel, i.e., the singular-values, has to be sent via a feedback channel to the transmitter side and leads to a high signalling overhead that is contradictory to the fixed transmission modes that require no signalling overhead. However, as shown in [AL09] a vector quantizer (VQ) can be used to keep the signalling overhead moderate. Here, a VQ for the power allocation parameters instead of the singular values guarantees a better adaption at a given codebook size, since the power level vectors has less or equal dimensions than the singular-value vectors [AL09]. Moreover, its elements are much smaller digits ranged from 0 to 1, rather than from 0 to $+\infty$ in the singular-value vector case. Hence, the entropy of the power level vectors is smaller, which benefits the quantization accuracy and the feedback overhead.

The aim of the forthcoming discussions is now the determination of the values $\sqrt{p_{k,\ell}^{(m)}}$ for the activated MIMO layers. Unfortunately, the Lagrange multiplier method often leads
to excessive-complexity optimization problems. Therefore, suboptimal power allocation strategies having a lower complexity are of common interest [AL07]. A natural choice is to opt for a PA scheme, which results in an identical signal-to-noise ratio

$$\rho_{PA}^{(\ell,m)} = \frac{(U^{(\ell,m)}_{PA})^2}{U_R^2} = \rho_{k,\ell}^{(m)} = \frac{3 \xi_{k,\ell}^{(m)} \rho_{k,\ell}^{(m)} E_s}{L (M_k \ell - 1) N_0}$$

(37)

for all activated MIMO layers at the time $m$, i.e., in

$$\rho_{PA}^{(\ell,m)} = \text{constant} \quad \ell = 1, 2, \cdots, L.$$  

(38)

The power to be allocated to each activated MIMO layer at the time $m$ can be shown to be calculated as follows [AL07]:

$$\tilde{p}_{k,\ell}^{(m)} = \frac{(M_k \ell - 1)}{\xi_{k,\ell}^{(m)}} \cdot \frac{L}{\sum_{\nu=1}^{L} \frac{(M_k \nu - 1)}{\xi_{k,\nu}^{(m)}}}.$$  

(39)

5 Results

In this contribution fixed transmission modes are used regardless of the channel quality. Assuming predefined transmission modes, a fixed data rate can be guaranteed.

5.1 Single-User System

Considering a non-frequency selective SDM (spatial division multiplexing) single-user MIMO link ($K = 1$) composed of $n_T = 4$ transmit and $n_R = 4$ receive antennas, the obtained BER curves are depicted in Fig. 2 for the different QAM constellation sizes and MIMO configurations of Tab. 1, when transmitting at a bandwidth efficiency of 8 bit/s/Hz. Assuming a uniform distribution of the transmit power over the number of activated MIMO layers, it turns out that not all MIMO layers have to be activated in order to achieve the best BERs.

PA can be used to balance the bit-error probabilities in the different number of activated MIMO layers. As shown in Fig. 2, unequal PA is only effective in conjunction with the optimum number of MIMO layers and at high SNR. Using all MIMO layers, our PA scheme would assign much of the total transmit power to the specific symbol positions per data block having the smallest singular values and hence the overall performance would deteriorate.

Non-frequency selective MIMO links have attracted a lot of research. By contrast, frequency selective MIMO links require substantial further research, where spatio-temporal vector coding (STVC) introduced by RALEIGH seems to be an appropriate candidate for
broadband transmission channels, where multipath propagation is no longer a limiting factor [RC98, RJ99, Ges04].

When considering a frequency selective SDM MIMO link, composed of $n_T$ transmit and $n_R$ receive antennas, the block-oriented system has to take the $(L_c + 1)$ non-zero elements of the resulting symbol rate sampled overall channel impulse response between the given transmit and receive antenna combinations into account [ABP09a]. Throughout this paper, it is assumed that the $(L_c + 1)$ channel coefficients between the given transmit and receive antenna combinations have the same averaged power and undergo a Rayleigh distribution.

The obtained BER curves are depicted in Fig. 3 and 4 for the different QAM constellation sizes and MIMO configurations of Table 1, when transmitting at a bandwidth efficiency of 8 bit/s/Hz within a given bandwidth over a frequency-selective MIMO channel. Comparing the results, depicted in Fig. 3 for the two-path MIMO channel and in Fig. 4 for the five-path MIMO channel, it can still be seen that delay spread is highly beneficial for the overall performance.

5.2 Multi-User System

The parameters of the exemplarily studied two-users MIMO system are chosen as follows\textsuperscript{2}: $P_{s_k} = 1 \text{ V}^2$, $n_{R_k} = 4$ (with $k = 1, 2$), $K = 2$, $n_R = n_T = 8$. The obtained user-specific BER curves are depicted in Fig. 5 for the different QAM constellation sizes and MIMO configurations of Table 1 and confirm the obtained results for the single-user.

\textsuperscript{2}In this contribution a power with the dimension (voltage)$^2$ (in V$^2$) is used. At a real, constant resistor this value is proportional to the physical power (in W).
Figure 3: BER with PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Table 1 and transmitting 8 bit/s/Hz over frequency selective channels (two-path channel model, $L_c = 1$)

Figure 4: BER with PA (dotted line) and without PA (solid line) when using the transmission modes introduced in Table 1 and transmitting 8 bit/s/Hz over frequency selective channels (five-path channel model, $L_c = 4$)
system \((K = 1)\). Assuming a uniform distribution of the transmit power over the number of activated MIMO layers, it still turns out that not all MIMO layers have to be activated in order to achieve the best BERs. However, the lowest BERs can only be achieved by using bit auction procedures leading to a high signalling overhead [WCLM99]. Analyzing the probability of choosing a specific transmission mode by using optimal bitloading, as depicted in Table 2, it turns out that only an appropriate number of MIMO layers has to be activated, e.g., the \((16, 4, 4, 0)\) QAM configuration.

Table 2: Probability of choosing specific transmission modes at a fixed data rate by using optimal bitloading \((10 \cdot \log_{10}(E_s/N_0) = 10\, \text{dB})\)

<table>
<thead>
<tr>
<th>mode</th>
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<tr>
<td>(16, 16, 0, 0)</td>
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<td>(16, 4, 4, 0)</td>
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<tr>
<td>(4, 4, 4, 4)</td>
<td>QAM</td>
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</table>

Besides, as depicted in Fig. 5, the gap between the different transmission modes becomes smaller and the influence of PA, as depicted in Fig. 6, diminishes.

Analyzing frequency selective MIMO links instead of non-frequency selective ones, the obtained BER curves are depicted in Fig. 7 for the investigated frequency-selective MIMO channels and confirm the BER results depicted in Fig. 5. From Fig. 7 it can be seen that a high delay spread is beneficial for minimizing the overall BER.

Figure 5: User-specific BERs without PA when using the transmission modes introduced in Tab. 1 and transmitting 8 bit/s/Hz over non-frequency selective channels
Figure 6: User-specific BER with PA (dashed line) and without PA (solid line) when using the transmission modes introduced in Tab. 1 and transmitting 8 bit/s/Hz over non-frequency selective channels.

Figure 7: User-specific BERs without PA when using the transmission modes introduced in Tab. 1 and transmitting 8 bit/s/Hz over frequency selective channels (two-path channel model (solid line, $L_c = 1$), five-path channel model (dashed line, $L_c = 4$)).
6 Conclusion

In this paper, the DL performance of multiuser MIMO system is investigated theoretically and by software simulation. Both frequency selective and non-frequency selective MIMO channels are considered and conditions to eliminate the multiuser and multi-antenna interferences are established using the SVD of individual user channel matrix. Furthermore, bit and power allocation in multiuser MIMO systems were investigated for constant throughput. Here, it turned out that the choice of the number of bits per symbol as well as the number of activated MIMO layer substantially affects the performance of a MIMO system, suggesting that not all user-specific MIMO layers have to be activated in order to achieve the best BERs. Additionally, unequal PA was found to be effective in conjunction with the optimum number of MIMO layers and delay spread was found to be beneficial for the overall performance.

References


