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We discuss the transient effects of mass and energy accumulation processes and of an applied acoustic field interaction on the droplet ignition phenomenon. In order to observe simultaneously the influence of those processes on the ignition, we assume that the chemical reaction occurs in the droplet unsteady far field, where the transient accumulation processes and the acoustic field are as important as the other processes (i.e., the conduction process and the chemical reaction). We also consider the Lewis number as being constant and nonunity. The results show that the ignition time is modified by the acoustic field.

INTRODUCTION

In this work, we analyse the unsteady effects of the mass and energy accumulation processes in the gas phase and of an applied acoustic field on the droplet ignition phenomenon, for any constant Lewis number. We include acoustic perturbations in this analysis because they are found in all kinds of combustion chambers.

The droplet ignition problems analysed so far consider the ratio of the gas-liquid densities very small, so that the gas phase can be divided into two regions: one of them near the droplet surface, where its processes, which are quasi-steady in relation to the processes inside of the droplet, have the main control on the vaporization rate [1]; the other region is far from the droplet, where mass and energy accumulation unsteady processes are as important as mass diffusion and conduction [2-4]. The convection process is very small compared to these processes. As the temperature in this region is high, the chemical reaction accelerates to trigger ignition. Thus, these unsteady processes are important to describe the ignition process [5, 6].

Nearly all analyses of the droplet ignition problem do not take into account the influence of transient processes [7-18]. Instead, they find the ignition conditions for the quasi-steady model. Williams' [5] and Chao et al.'s [6] works,

however, considered the transient processes, but even then, for unity Lewis numbers only.

We limit our study to the case where the acoustic oscillation amplitude is of the same order of the changes of temperature in the far region produced by the transient processes mentioned above. To guarantee that the chemical reaction takes place in the far region where unsteady processes become important, we assume that the gas thermal activation energies ratio is of the same order as the change of temperature in relation to the ambient temperature in the far region. This analysis shows, besides the influence of the acoustic field on the ignition process, the limited variation of the temperature, which is caused by the chemical reaction in the case of the ambient temperature being larger than the adiabatic value.

MODEL AND MATHEMATICAL FORMULATION

We assume that the liquid and gas phases have spherical symmetry. The droplet radius at instant t is $a(t)$ and the liquid density is ρ_l . The ambient gas properties, such as the density ρ_∞ , the ambient oxidant mass fraction Y_{O_∞} , the thermal conductivity k_∞ , and the constant pressure specific heat c_p , are constant; the subscript ∞ represents the ambient condition. However, the ambient temperature T_∞ is taken to vary sinusoidally with time with a wavelength larger than the droplet radius. In addition, we consider that the gas thermal conduc-

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tivity and the diffusion coefficients of the fuel and of the oxidant, k , D_F , and D_O , respectively, are functions of the temperature T only, in the form

$$\frac{\rho D_F}{\rho_\infty D_{F\infty}} = \frac{\rho D_O}{\rho_\infty D_{O\infty}} = \frac{k}{k_\infty} = \theta^n,$$

where $\theta = T/T_\infty$ is the dimensionless temperature.

In order to study only the transient effects of the gas phase in the droplet ignition, we eliminate the droplet heating period, assuming that the droplet temperature is already at the boiling temperature, $T = T_B$. Then,

$$\theta = \theta_B, \quad x \leq a(\tau), \quad (1)$$

where $x = r/a(0)$ is the nondimensionalized radial coordinate, $a(0)$ is the initial droplet radius, $a = a/a(0)$ is the instantaneous dimensionless droplet radius, and $\tau = t/t_v$ is the time nondimensionalized with the order of magnitude of the droplet life time, $t_v = a^2(0)/\alpha_\infty \varepsilon$, where $\alpha_\infty = k_\infty/c_p \rho_\infty$ and $\varepsilon = \rho_\infty/\rho_l \ll 1$.

This simplified model for representing the energy conservation (Eq. 1) in the liquid phase is reasonable for describing the droplet vaporization period for fuels such that $\bar{L} = L \cdot M_w/RT_B \gg 1$ (where L is the latent heat of vaporization, M_w is the fuel molecular weight, and R is the universal gas constant). Then the fuel vapor concentration at the droplet surface is very small unless the surface temperature θ_s is so close to boiling that $\theta_B - \theta_s \sim 1/\bar{L}$, as can be seen from the Clausius-Clapeyron relation. Therefore, if $\bar{L} \gg 1$, the heating and the vaporization processes can be treated separately [20].

Due to the constant value of the droplet temperature, the ratio between the vaporization rate and the droplet radius is a constant, say β_n (the value of β_n depends on the exponent n). Then, the droplet mass conservation equation takes the simple form

$$a^2 = 1 - 2\beta_n \tau, \quad (2)$$

where $\beta_n = \lambda/a$ ($\lambda = \dot{m}c_p/4\pi k_\infty a(0)$) is the dimensionless droplet vaporization rate with \dot{m} being the vaporization rate).

The gas phase is described by the conservation equations, which are written in terms of the following variables,

$$\begin{aligned} \tau &= \frac{t\alpha_\infty \varepsilon}{a^2(0)}, & x &= \frac{r}{a(0)}, & \varrho &= \frac{\rho}{\rho_\infty}, \\ v &= \frac{Va(0)}{\alpha_\infty}, & y_O &= \frac{Y_O}{Y_{O\infty}}, & y_F &= Y_F, \\ \theta &= \frac{T}{T_\infty}, & p &= \frac{P}{R_g \rho_\infty T_\infty}, \end{aligned}$$

where V is the gas velocity, Y is the mass fraction, R_g is the specific gas constant, and the subscripts O and F represent the oxidant and the fuel, respectively. Then, those equations become

$$\varepsilon \varrho \frac{\partial \varrho}{\partial \tau} + \frac{1}{x^2} \frac{\partial}{\partial x} (x^2 \varrho v) = 0, \quad x > a(\tau), \quad (3)$$

$$\begin{aligned} \varepsilon \varrho \frac{\partial y_O}{\partial \tau} + \varrho v \frac{\partial y_O}{\partial x} - \frac{1}{x^2} \frac{\partial}{\partial x} \left(\frac{x^2 \theta^n}{L_O} \frac{\partial y_O}{\partial x} \right) \\ = -S \varrho \bar{D} a y_O^{\nu_1} y_F^{\nu_2} e^{-\Sigma/\theta}, \quad x > a(\tau), \end{aligned} \quad (4)$$

$$\begin{aligned} \varepsilon \varrho \frac{\partial y_F}{\partial \tau} + \varrho v \frac{\partial y_F}{\partial x} - \frac{1}{x^2} \frac{\partial}{\partial x} \left(\frac{x^2 \theta^n}{L_F} \frac{\partial y_F}{\partial x} \right) \\ = -\varrho \bar{D} a y_O^{\nu_1} y_F^{\nu_2} e^{-\Sigma/\theta}, \quad x > a(\tau), \end{aligned} \quad (5)$$

$$\begin{aligned} \varepsilon \varrho \frac{\partial \theta}{\partial \tau} + \varrho v \frac{\partial \theta}{\partial x} - \frac{1}{x^2} \frac{\partial}{\partial x} \left(x^2 \theta^n \frac{\partial \theta}{\partial x} \right) \\ = \varepsilon \frac{\gamma - 1}{\gamma} \frac{dp}{d\tau} + Q \varrho \bar{D} a y_O^{\nu_1} y_F^{\nu_2} e^{-\Sigma/\theta}, \\ x > a(\tau), \end{aligned} \quad (6)$$

where $\bar{D} a = Ba^2(0)Y_{O\infty}^{\nu_1}/\alpha_\infty$ is the Damköhler number (B is the frequency factor), ν_1 and ν_2 are the reaction global orders, $S = s/Y_{O\infty}$ (s is the mass of oxidant burnt under stoichiometric conditions per unit fuel mass), $Q = q/c_p T_\infty$ (where q is the heat of combustion, i.e., the amount of heat released per unit mass of fuel consumed), $\Sigma = E/RT_\infty$ (where E is the activation energy), $\gamma = c_p/c_v$ (where c_v is the gas constant volume specific heat), and $L_O =$

$\alpha_\infty/D_{O\infty}$ and $L_F = \alpha_\infty/D_{F\infty}$ are the oxidant and the fuel Lewis numbers, respectively.

The system of Eqs. 3 to 6 is closed with the gas equation of state, on which is superimposed an acoustic perturbation:

$$p = 1 + \Lambda p_1 \cos(\omega\tau - \delta) = \rho\theta, \quad x > a(\tau), \quad (7)$$

where $\Lambda \ll 1$ and p_1 are constants, and $\omega = kt_v$, k being the perturbation frequency and δ being the phase angle, all of them known quantities.

Equations 3 to 6 are integrated with the following boundary and initial conditions [19]:

$$y_{O_s} \beta_n a(\tau) - \left(\frac{x^2 \theta^n}{L_O} \frac{dy_O}{dx} \right) = 0, \quad y_{F_s} \beta_n a(\tau) - \left(\frac{x^2 \theta^n}{L_F} \frac{dy_F}{dx} \right) = \beta_n a(\tau), \quad x = a(\tau), \quad (8)$$

$$\left(x^2 \theta^n \frac{d\theta}{dx} \right) = l \beta_n a(\tau), \quad x = a(\tau), \quad (9)$$

$$\theta = \theta_B, \quad y_F = y_{F_s}, \quad y_O = y_{O_s}, \quad x = a(\tau), \quad (10)$$

$$\theta = \theta_\infty, \quad \rho = \rho_\infty, \quad y_F = Y_{F\infty}, \quad y_O = 1, \quad x \rightarrow \infty, \quad (11)$$

$$\theta = \theta_\infty, \quad \rho = \rho_\infty, \quad y_F = Y_{F\infty}, \quad y_O = 1, \quad x > a(\tau), \quad \tau = 0, \quad (12)$$

where $l = L/c_p T_\infty$, and the subscript s represents the condition at the droplet surface. Owing to the pressure oscillatory behavior, conditions θ_∞ and ρ_∞ are

$$\theta_\infty = 1 + \Lambda \theta_{\infty 1} \cos(\omega\tau - \delta)$$

and

$$\rho_\infty = 1 + \Lambda \rho_{\infty 1} \cos(\omega\tau - \delta), \quad (13)$$

where $\theta_{\infty 1}$ and $\rho_{\infty 1}$, according to Eqs. 6 and 7, are equal to $p_1(\gamma - 1)/\gamma$ and p_1/γ , respectively.

THE STEADY DROPLET VAPORIZATION PROCESS

When the chemical reaction rate is small enough in the zone near the droplet surface to cause significant changes in the temperature prior to ignition, then we can consider the reaction to be frozen. As $\varepsilon \ll 1$, then the gas phase has a quasi-steady behavior in this zone, so that the droplet vaporization problem under these conditions can be formulated as

$$x^2 \varrho v = \beta_n a(\tau), \quad x \geq a(\tau), \quad (14)$$

$$\frac{\beta_n a(\tau)}{x^2 \theta^n} = \frac{1}{\theta + l - \theta_B} \frac{d\theta}{dx}, \quad x \geq a(\tau), \quad (15)$$

$$\frac{\beta_n a(\tau)}{x^2 \theta^n} = \frac{1}{L_F(1 - y_F)} \frac{dy_F}{dx}, \quad x \geq a(\tau), \quad (16)$$

$$\frac{\beta_n a(\tau)}{x^2 \theta^n} = \frac{1}{l_O y_O} \frac{dy_O}{dx}, \quad x \geq a(\tau), \quad (17)$$

The integration of Eq. 15 with condition 10 yields the temperature distribution θ :

$$\beta_n \left(1 - \frac{a}{x} \right) = \int_{\theta_B}^{\theta(x)} \frac{z^n}{z - \theta_B + l} dz, \quad x \geq a(\tau), \quad (18)$$

and by imposing condition 11, we determine the value of β_n :

$$\beta_n = \int_{\theta_B}^{\theta_\infty} \frac{z^n}{z - \theta_B + l} dz. \quad (19)$$

We can find the profile of y_F by combining Eqs. 15 and 16 to eliminate the term $\beta_n a/x^2 \theta^n$ and then integrating the resultant equation, which gives

$$\frac{(1 - y_F)^{1/L_F}}{(1 - y_{F\infty})^{1/L_F}} = \frac{\theta - \theta_B + l}{\theta_\infty - \theta_B + l}, \quad x \geq a(\tau). \quad (20)$$

Following the same procedure with Eqs. 15 and 17, one can write

$$y_O^{1/L_O} = \frac{\theta - \theta_B + l}{\theta_\infty - \theta_B + l}, \quad x \geq a(\tau). \quad (21)$$

Corrections in the temperature and in the fuel and oxidant mass fractions are of the order of $\sqrt{\varepsilon}$, due to transient processes of energy and mass accumulation in the gas phase located at $x \sim \varepsilon^{-1/2}$ from the droplet surface in the outer zone [2–4]. In this zone, conditions are such that the chemical reaction rate can increase quickly, triggering the ignition. The temperature and the fuel and oxidant mass fraction profiles are

$$\theta = 1 + \sqrt{\varepsilon} \theta_1 + o(\sqrt{\varepsilon}),$$

$$y_F = \sqrt{\varepsilon} y_{F1} + o(\sqrt{\varepsilon}),$$

$$y_O = 1 + \sqrt{\varepsilon} y_{O1} + o(\sqrt{\varepsilon}),$$

and

$$\rho = 1 + \sqrt{\varepsilon} \rho_1 + o(\sqrt{\varepsilon}).$$

Functions θ_1 , y_{F1} , y_{O1} , and ρ_1 are determined in the region $x \sim \varepsilon^{-1/2}$, as shown in the next section.

MATHEMATICAL FORMULATION OF THE IGNITION PROCESS

Due to the high value of the activation energy Σ , the chemical reaction rate is important, i.e., it is capable of triggering the ignition process, where the temperature of the fuel–oxidant mixture is $1 - \theta \sim \Sigma^{-1}$. According to Eq. 18, this condition occurs in the gas-phase zone at a distance $x \sim \Sigma$ from the droplet surface. If $\Sigma \gg \varepsilon^{-1/2}$, ignition takes place in a more distant zone where the molecular transport processes are not important, so that the ignition process proceeds as in the thermal-explosion regime. However, if $\Sigma \ll \varepsilon^{-1/2}$, the chemical reaction occurs in the quasi-steady zone and the accumulation of mass and energy processes can be neglected compared to the other processes [6]. We are interested in the limit $\Sigma = \varepsilon^{-1/2} = \Lambda^{-1}$, which allows for the simultaneous observation of the transient process influence of the mass and energy accumulation and of an applied acoustic field on the ignition process. Chao et al. [6] studied the case $\Sigma = \varepsilon^{-1/2}$, but they did not include the effect of the acoustic field nor did they admit a nonunity Lewis number.

In order to analyse the ignition process, it is convenient to change the coordinate variable:

$$X = x\sqrt{\varepsilon}.$$

For $X \sim 1$, the dependent variables, according to Eqs. 14, 18, 20, and 21, are

$$\varrho = 1 + \sqrt{\varepsilon} \Phi(X, \tau) + \dots,$$

$$\theta = 1 + \sqrt{\varepsilon} \Theta(X, \tau) + \dots,$$

$$y_F = \sqrt{\varepsilon} \Psi_F(X, \tau) + \dots,$$

$$y_O = 1 + \sqrt{\varepsilon} \Psi_O(X, \tau) + \dots,$$

and

$$v = \varepsilon \Pi(X, \tau) + \dots. \quad (22)$$

Substituting the solutions in Eqs. 22 into the system of Eqs. 3 to 6, we find

$$\frac{\partial \Phi}{\partial \tau} + \frac{1}{X^2} \frac{\partial}{\partial X} (X^2 \Pi) = 0, \quad (23)$$

$$\frac{\partial \Psi_O}{\partial \tau} - \frac{1}{X^2} \frac{\partial}{\partial X} \left(\frac{X^2}{L_O} \frac{\partial \Psi_O}{\partial X} \right) = -S \text{Da} \Psi_F^{\nu_2} e^\theta, \quad (24)$$

$$\frac{\partial \Psi_F}{\partial \tau} - \frac{1}{X^2} \frac{\partial}{\partial X} \left(\frac{X^2}{L_F} \frac{\partial \Psi_F}{\partial X} \right) = -\text{Da} \Psi_F^{\nu_2} e^\theta, \quad (25)$$

$$\begin{aligned} \frac{\partial \Theta}{\partial \tau} - \frac{1}{X^2} \frac{\partial}{\partial X} \left(X^2 \frac{\partial \Theta}{\partial X} \right) \\ = \frac{d}{d\tau} [\Theta_\infty \cos(\omega\tau - \delta)] \\ + Q \text{Da} \Psi_F^{\nu_2} e^\theta, \end{aligned} \quad (26)$$

where $\text{Da} = \tilde{\text{Da}} \varepsilon^{(\nu_2-3)/2} \exp(\varepsilon^{-1/2})$ is a modified Damköhler number. Recall also that $\theta_{\infty 1} = \Theta_\infty = p_1(\gamma - 1)/\gamma$.

As we can see from the system of Eqs. 23 to 26, it is not necessary to integrate Eqs. 23 and 24 in order to determine the ignition conditions.

Boundary conditions as $X \rightarrow 0$ for Eqs. 25 and 26 are determined by the matching of the

solutions of the inner zone with those of the outer zone:

$$X^2 \frac{\partial \Psi_F}{\partial X} = h(\tau) = -L_F \beta_n \sqrt{(1 - 2\beta_n \tau)},$$

$$X \rightarrow 0, \quad (27)$$

$$X^2 \frac{\partial \Theta}{\partial X} = g(\tau) = (1 - \theta_B + l) \beta_n \sqrt{(1 - 2\beta_n \tau)},$$

$$X \rightarrow 0. \quad (28)$$

Other boundary conditions, defined as $X \rightarrow \infty$, are found by matching the outer solutions with the equilibrium solutions. This yields

$$\Theta - \Theta_\infty \cos(\omega\tau - \delta) = \Psi_F = 0, \quad X \rightarrow \infty. \quad (29)$$

We assume that the initial condition is the frozen solution with fluxes $g(0)$ and $h(0)$. Then,

$$\left\{ \begin{array}{c} \Theta - \Theta_\infty \cos(\omega\tau - \delta) \\ \Psi_F \end{array} \right\} = - \left\{ \begin{array}{c} g(0) \\ h(0) \end{array} \right\} \frac{1}{X} \operatorname{erfc}(X/2\sqrt{\tau^*}),$$

$$X > 0, \quad \tau \rightarrow 0, \quad (30)$$

where

$$\tau^* = \begin{cases} \tau, & \Theta - \Theta_\infty \cos(\omega\tau - \delta), \\ \tau/L_F, & \Psi_F \end{cases}$$

as shown in the appendix.

CASE WHERE $L_F = 1$

For $L_F = 1$, the heat and fuel mass transport processes have the same rate, so that they are similar. In this condition, we can combine Eqs. 25 and 26 in the form $Q\Psi_F + \Theta$ to eliminate the chemical reaction term. The function $H = Q\Psi_F + \Theta$ is the total enthalpy, also known as

the Schvab-Zel'dovich's variable. It satisfies the following equations:

$$\frac{\partial H}{\partial \tau} - \frac{1}{X^2} \frac{\partial}{\partial X} \left(X^2 \frac{\partial H}{\partial X} \right) = \frac{d}{d\tau} [\Theta_\infty \cos(\omega\tau - \delta)], \quad (31)$$

$$X^2 \frac{\partial H}{\partial X} = g(\tau) + Qh(\tau) = I\sqrt{(1 - 2\beta_n \tau)},$$

$$X \rightarrow 0, \quad (32)$$

$$H = \Theta_\infty \cos(\omega\tau - \delta), \quad X \rightarrow \infty, \quad (33)$$

$$H = -\frac{I}{X} \operatorname{erfc}(X/\sqrt{4\tau}) + \Theta_\infty \cos(\omega\tau - \delta),$$

$$X > 0 \quad \text{at} \quad \tau \rightarrow 0, \quad (34)$$

where

$$I = (1 + l - \theta_B - Q)\beta_n.$$

The solution to Eqs. 31 to 34 is given by

$$H - \Theta_\infty \cos(\omega\tau - \delta) = -IU(X, \tau), \quad (35)$$

where

$$U(X, \tau) = \frac{1}{2\sqrt{\pi}} \int_0^\tau \sqrt{1 - 2\beta_n \xi} \times \frac{\exp[-X^2/4(\tau - \xi)]}{(\tau - \xi)^{3/2}} d\xi.$$

Equation 35 is used to eliminate the function Ψ_F from Eq. 26 or the function Θ from Eq. 25. We choose to eliminate the former. Then, the evolution of the temperature $\hat{\Theta}$, defined as $\Theta - \Theta_\infty \cos(\omega\tau - \delta)$ (choosing a global reaction order $\nu_2 = 1$) is given by:

$$\frac{\partial \hat{\Theta}}{\partial \tau} - \frac{1}{X^2} \frac{\partial}{\partial X} \left(X^2 \frac{\partial \hat{\Theta}}{\partial X} \right) = -\operatorname{Da} [IU(X, \tau) + \hat{\Theta}] e^{\Theta_\infty \cos(\omega\tau - \delta) + \hat{\Theta}}, \quad (36)$$

with the boundary and initial conditions of Eqs. 28, 29, and 30, respectively, suitably modified in terms of $\hat{\Theta}$.

CASE WHERE $I < 0$

The value of the gas temperature $\hat{\Theta}$ is always negative if the problem is in the frozen state, except for $X \rightarrow \infty$ where $\hat{\Theta} = 0$. In the reactive problem, the chemical reactions are responsible for raising the gas temperature to its

maximum value, $-IU(X, \tau)$, according to Eq. 35, along with the equilibrium condition, $\Psi_F = 0$. Then, if $I < 0$, i.e., if $Q > 1 + l - \theta_B$, $\hat{\Theta}$ reaches positive values, $-IU(X, \tau) - Q\Psi_F > 0$, in a region with a thickness of the order of unity and, inside, there exists a zone where the conditions are such that the temperature increases exponentially to the value $-IU(X, \tau)$. The rapid increase of $\hat{\Theta}$ indicates that the ignition process took place, as we can see in Fig. 1.

The induction time τ_{ig} , for the ignition, is the time period measured from the instant that the droplet enters the oxidizing atmosphere until the chemical reaction begins (ignition process). This time period is an important parameter for designing a combustion chamber; it helps to determine the length of the chamber. Its value as a function of the Damköhler number, Da , is shown in Fig. 2. From this figure, we can see that the induction time does not exist for values of Da smaller than a certain value, say Da_m , which depends on the value of I . Another fact that we observe is that τ_{ig} decreases rapidly from a high value with a small variation of Da near the value Da_m . The high value of τ_{ig} shows that most of the droplet has already vaporized, so that the fuel mass fraction in the gas phase where the chemical reaction takes place is large, and the ignition process is controlled by the temperature of the gases in that region. The reaction temperature sensitivity decreases as Da increases, showing that the control of the ignition process is transferred to the fuel diffusion process, from the droplet to the reaction zone.

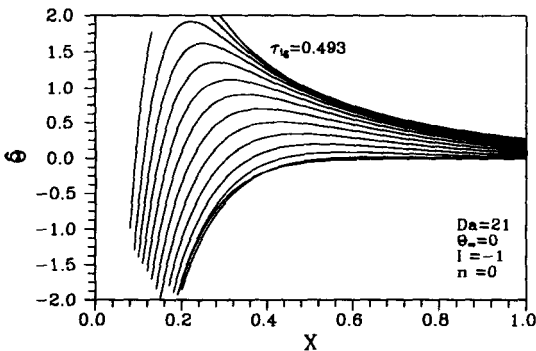


Fig. 1. $\hat{\Theta}$ profile evolution up to ignition. The lowest profile corresponds to $\tau = 0.01$ and the highest one is for $\tau_{ig} = 0.493$.

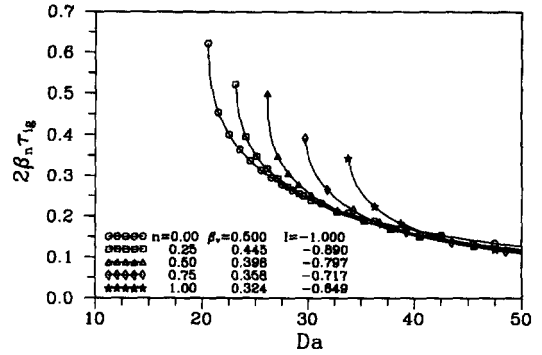


Fig. 2. Induction time τ_{ig} for various values of n .

In these cases, we considered the ambient temperature unperturbed, $\Theta_{\infty} = 0$. Next, we discuss the cases for $\Theta_{\infty} \neq 0$. Then, the perturbation in the ambient temperature leads to variations in the chemical reaction rate, which behave as $\exp[\Theta_{\infty} \cos(\omega\tau - \delta)]$. If Θ_{∞} is small, the influence of the acoustic field on the ignition process is very small, but if $\Theta_{\infty} = O(1)$, the chemical reaction rate suffers large variations, which causes a drastic reduction in the induction time and, in some cases, in a discontinuous fashion, as shown in Fig. 3. This is because the induction time occurs when $\cos(\omega\tau - \delta) > 0$ and, then, with an increase in Θ_{∞} , the conditions are such that the ignition occurs in the previous time period when $\cos(\omega\tau - \delta) > 0$.

Representing the induction time vs. the perturbed Damköhler number, $Da_{osc} = Da \times \exp[\Theta_{\infty} \cos(\omega\tau - \delta)]$, the curve $\tau_{ig} \times Da_{osc}$ has a Z shape, as can be seen in Fig. 4.

Moreover, Figs. 5 and 6 show the influence of the frequency ω and the phase angle δ ,

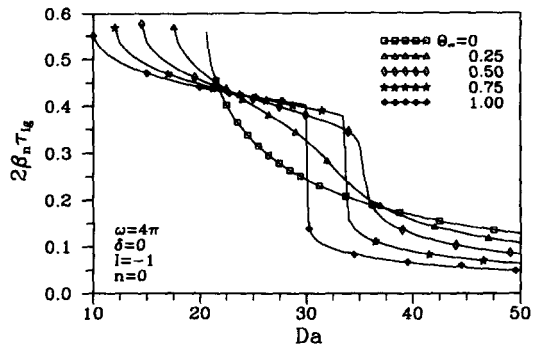


Fig. 3. Induction time τ_{ig} for various values of Θ_{∞} .

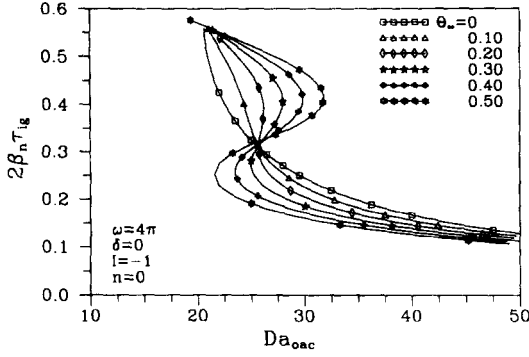


Fig. 4. Induction time $2\beta_n \tau_{ig}$ as a function of Da_{osc} (defined as $Da e^{\Theta_x \cos(\omega\tau - \delta)}$) for various values of Θ_x .

respectively, in the induction time. We observe that, in the case $\omega = \pi$, the induction time is the smallest, showing that the ignition process is anticipated due to the now-existing larger time interval given by the term $\cos(\omega\tau - \delta)$ positive. The effect of the phase angle δ on the induction time is due to the displacement of the time interval for $\cos(\omega\tau - \delta) > 0$, which, together with the fuel concentration and the value of Da , can anticipate the ignition, as shown in Fig. 5.

CASE WHERE $I \geq 0$

For $I \geq 0$, the problem conditions are such that the gas temperature $\hat{\Theta}$, after reaching the equilibrium state, is smaller than the ambient temperature, according to Eq. 35. This temperature behavior means that the maximum of the chemical reaction rate is a constant all the time, a fact that does not occur if $I < 0$, as shown in Fig. 1. Another feature of the problem when $I \geq 0$ is that the chemical reaction

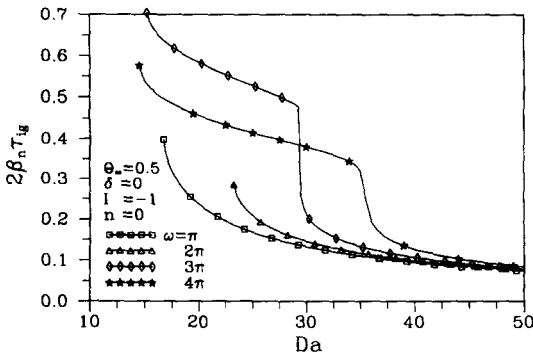


Fig. 5. Induction time τ_{ig} for various values of ω .

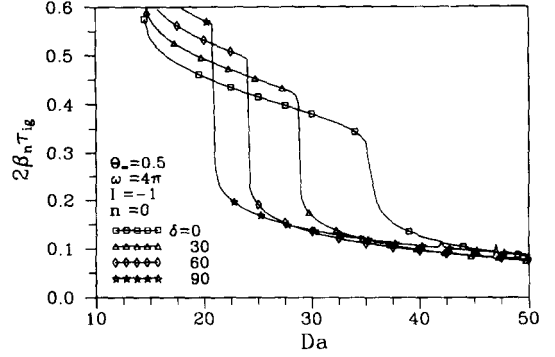


Fig. 6. Induction time τ_{ig} for various values of δ .

zone is very thin, according to Liñán and Crespo [14] and Liñán and Rodrigues [15], and it takes place near the temperature value of $\hat{\Theta} = -IU(X, \tau)$. Liñán [21] defined this combustion regime as a *premixed flame*, because on one side of the chemical reaction zone, the flow is frozen and on the other side, it is in equilibrium. In our case, this occurs from the droplet surface to the flame and from the flame to ambient, respectively.

Because the maximum chemical reaction rate is a constant with a value that does not raise the temperature above the ambient temperature, the reaction does not run away. The combustion of the gases takes place slowly in a thin zone. Therefore, the changes in the gas temperature depend on the Damköhler number Da .

In order to observe the temperature evolution due to the chemical reaction in the far zone, it is convenient to follow the changes in the temperature relative to the frozen temperature. To do this, we employ the transformation $\hat{\Theta} = \hat{\Theta}_q + \hat{\Theta}_f$, where $\hat{\Theta}_f$ is the solution of the frozen problem [Eqs. 26, 28 to 30, where $\hat{\Theta}_f = -g(0)U(X, \tau)$]. $\hat{\Theta}_q$ represents the changes of temperature caused by the chemical reaction and is determined by

$$\begin{aligned} \frac{\partial \hat{\Theta}_q}{\partial \tau} - \frac{1}{X^2} \frac{\partial}{\partial X} \left(X^2 \frac{\partial \hat{\Theta}_q}{\partial X} \right) \\ = -Da \left[h(0)U(X, \tau) + \hat{\Theta}_q \right] \\ \times e^{\Theta_x \cos(\omega\tau - \delta) - g(0)U(X, \tau) + \hat{\Theta}_q}, \end{aligned} \quad (37)$$

with boundary conditions

$$\frac{\partial \hat{\Theta}_q}{\partial X} = 0; \quad X \rightarrow 0$$

and

$$\hat{\Theta}_q = 0; \quad X \rightarrow \infty, \quad \tau > 0, \quad (38)$$

and initial condition

$$\hat{\Theta}_q = 0; \quad \tau \rightarrow 0, \quad X > 0. \quad (39)$$

Figure 7 shows the evolution of the temperature $\hat{\Theta}_q$: Figs. 7a corresponds to $0.05 \leq \tau \leq 0.65$ and Fig. 7b, to $0.7 \leq \tau \leq 1$. We can see that the ignition does not occur even for a high Damköhler number, $Da = 50$. The criterion for identifying the ignition process in this mathematical formulation is the rapid increase in $\hat{\Theta}_q$ as $X \rightarrow 0$. In Fig. 8, we present the values of $\hat{\Theta}_q(0, \tau)$ for some values of Da . Figure 8a shows the behavior of $\hat{\Theta}_q(0, \tau)$ when the gases ignite, for $I = -1$, and Fig. 8b again shows $\hat{\Theta}_q(0, \tau)$ vs τ for $I = 0.5$. Both cases are for unperturbed ambient conditions.

In the problem $I \geq 0$, the droplet does not burn fast and the vaporized fuel is always burned at a slow rate, causing only small modifications in the temperature. Therefore, these changes lead to small modifications in the droplet vaporization rate [22].

Figure 9 presents the temperature behavior $\hat{\Theta}_q(0, \tau)$ during the droplet lifetime for various values of the frequency and phase angles.

CASE WHERE $L_F \neq 1$

Here, we analyse the effect of the difference between the rates of the molecular transport of fuel mass and energy on the ignition process.

If $L_F < 1$, the fuel mass diffusion process is faster than the heat conduction process, so that in the zone where the ignition takes place, the fuel mass fraction increases more rapidly than temperature decreases. Under these conditions, the smaller L_F is, the earlier ignition will be achieved. However, for $L_F > 1$, the ignition zone loses heat to the droplet faster than it receives fuel from the droplet; then the low values of temperature and fuel mass fraction retard the ignition. Following the induction time behavior, τ_{ig} , we can quantify the effects of L_F on the ignition, as shown in Fig. 10.

CASE WHERE $\omega \gg 1$

We now discuss the effect of high-frequency acoustic perturbations on the ignition process. The ignition problem in this condition has two time scales, one of them on the order of the inverse of the frequency, ω^{-1} , the other being the droplet lifetime. Therefore, to follow the evolution of the ignition process, we need to observe the phenomenon, scaling it with the former one, i.e., with ω^{-1} , so that it is convenient to use a new variable σ , $\sigma = \omega\tau$. Then, Eqs. 25 and 26 written in this new time scale σ will have the form

$$\omega \frac{\partial \Psi_F}{\partial \sigma} - \frac{1}{X^2} \frac{\partial}{\partial X} \left(X^2 \frac{\partial \Psi_F}{\partial X} \right) = -Da e^{\Theta_x \cos(\sigma - \delta)} \Psi_F^{\nu_2} e^{\hat{\Theta}}, \quad (40)$$

$$\omega \frac{\partial \hat{\Theta}}{\partial \sigma} - \frac{1}{X^2} \frac{\partial}{\partial X} \left(X^2 \frac{\partial \hat{\Theta}}{\partial X} \right) = Q Da e^{\Theta_x \cos(\sigma - \delta)} \Psi_F^{\nu_2} e^{\hat{\Theta}}. \quad (41)$$

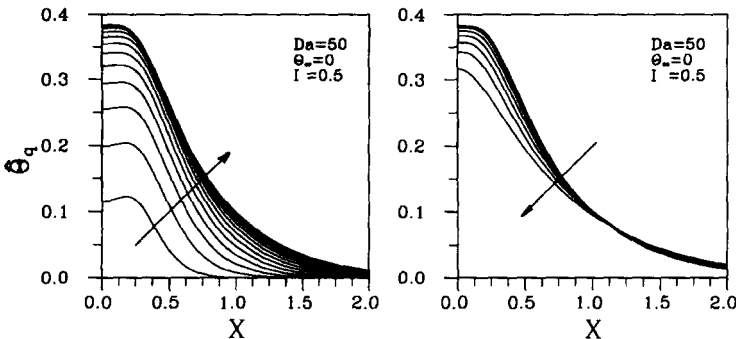


Fig. 7. Evolution of $\hat{\Theta}_q$ for various values of Damköhler number Da .

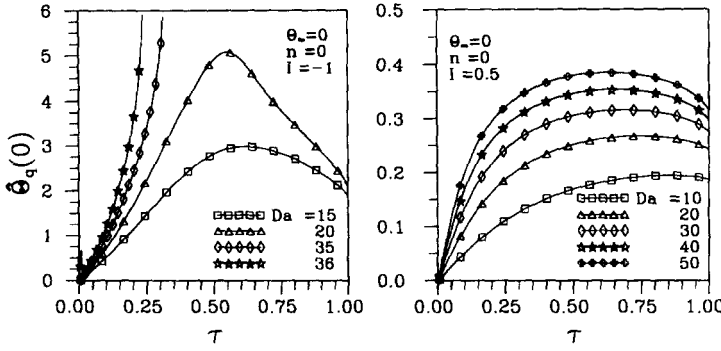


Fig. 8. Evolution of $\hat{\Theta}_q(0, \tau)$ for various values of Damköhler number Da , for an unperturbed ambient for (a) $I = -1$ and for (b) $I = 0.5$.

If $\omega \gg 1$, the diffusive processes of fuel mass and energy do not have an important effect on the ignition process, as they modify the solution on the order of ω^{-1} only. Then, the solution of Eqs. 40 and 41 can be expressed as

$$\hat{\Theta} = \hat{\Theta}_0 + \omega^{-1} \hat{\Theta}_1 + \dots$$

and

$$\Psi_F = \Psi_{F0} + \omega^{-1} \Psi_{F1} + \dots$$

In addition, as the chemical reaction must be of the same order as its largest term, one can write

$$Da = \omega Da_0 + Da_1 + \omega^{-1} Da_2 + \dots$$

Taking into account these solutions, we can find that

$$\frac{\partial \Psi_{F0}}{\partial \sigma} = -Da_0 e^{\theta_\infty \cos(\sigma - \delta)} \Psi_{F0}^{\nu_2} e^{\hat{\Theta}_0}, \quad (42)$$

$$\frac{\partial \hat{\Theta}_0}{\partial \sigma} = Q Da_0 e^{\theta_\infty \cos(\sigma - \delta)} \Psi_{F0}^{\nu_2} e^{\hat{\Theta}_0}. \quad (43)$$

Combining Eqs. 42 and 43 in such a way to obtain the function $\hat{\Theta}_0 + Q\Psi_{F0}$, the chemical reaction term is eliminated. The equation giving

the time evolution for $\hat{\Theta}_0 + Q\Psi_{F0}$ can be integrated, yielding

$$\hat{\Theta}_0 + Q\Psi_{F0} = F(X, \tau_1), \quad (44)$$

where the function $F(X, \tau_1)$ is determined from the unperturbed reacting Eqs. 25 and 26. We consider the ambient acoustic perturbation beginning at time $\tau = \tau_1$.

Equations 42 and 43 decouple when we substitute Eq. 44 into them. Then the ignition problem for a high-frequency perturbed ambient is

$$\frac{\partial \hat{\Theta}_0}{\partial \sigma} = Q^{1-\nu_2} Da_0 e^{\theta_\infty \cos(\sigma - \delta)} \times (F(X, \tau_1) - \hat{\Theta}_0)^{\nu_2} e^{\hat{\Theta}_0}, \quad (45)$$

which can be integrated as

$$\int_{\hat{\Theta}_0(X, \tau_1, 0)}^{\hat{\Theta}_0(X, \tau_1, \sigma)} \frac{\exp(-\chi) d\chi}{(F(X, \tau_1) - \chi)^{\nu_2}} = Q^{1-\nu_2} Da_0 \int_0^\sigma e^{\theta_\infty \cos(\zeta - \delta)} d\zeta. \quad (46)$$

The condition that identifies the onset of the ignition is the rapid temperature increase to very high values, which justifies imposing the

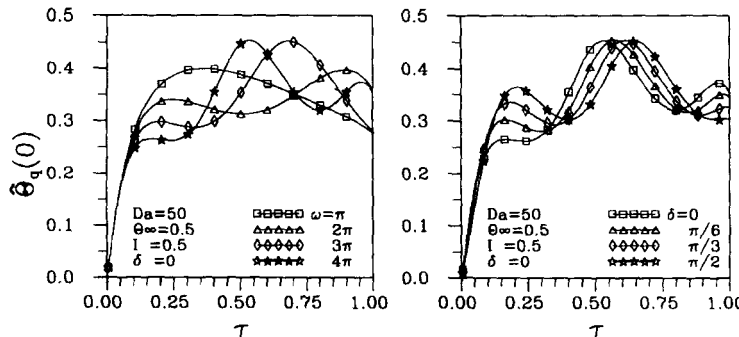


Fig. 9. Evolution of $\hat{\Theta}_q(0, \tau)$ for (a) various values of the frequency ω and for (b) various values of the phase angle δ .

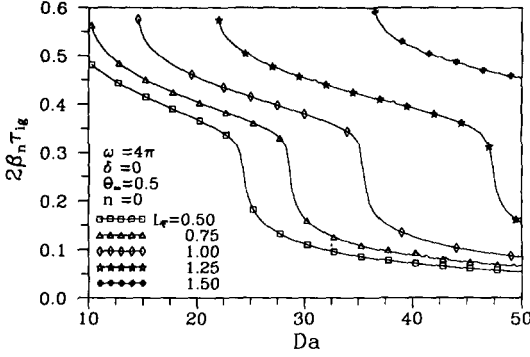


Fig. 10. Induction time $2\beta_n\tau_{ig}$ for various values of the fuel Lewis number, L_F .

condition $\hat{\Theta}_0 \rightarrow \infty$. The place where this condition occurs is determined by the minimum of the function

$$J(X) = \int_{\hat{\Theta}_0(X, \tau_1)}^{\infty} \frac{\exp(-\chi) d\chi}{(F(X, \tau_1) - \chi)^{\nu_2}}, \quad (47)$$

defined as

$$M = \text{Min}[J(X)], \quad (48)$$

so that the induction time, σ_{ig} , is determined by

$$\int_0^{\sigma_{ig}} e^{\Theta_{\infty} \cos(\zeta - \delta)} d\zeta = \frac{M}{Q^{1-\nu_2} \text{Da}_0}. \quad (49)$$

We can see from Eq. 49 that by increasing the modified Damköhler number and decreasing the reaction global order ν_2 , the induction time decreases, as expected. But one result that is not expected is the ignition always occurs independently of the values of Da and ν_2 (for $\text{Da} > 0$, of course). As the conduction process is not fast enough to transport the heat locally generated by the chemical reaction, the temperature increases and the chemical reaction runs away.

For the droplet ignition to occur with the initial condition of Eq. 30, we need to integrate Eq. 47 with the following condition:

$$\hat{\Theta}_0 = \frac{g(0)}{X} \left[\text{erfc}\left(\frac{X}{\sqrt{4\tau_1}}\right) \right] \\ X > 0 \quad \sigma = 0 \quad (\tau_1 \ll 1), \quad (50)$$

with $F(X, \tau_1)$ given as

$$F(X, \tau_1) = \frac{g(0)}{X} \text{erfc}\left(\frac{X}{\sqrt{4\tau_1}}\right) \\ + \frac{Qh(0)}{X} \text{erfc}\left(\frac{X}{\sqrt{4\tau_1/L_F}}\right). \quad (51)$$

For $g(0) = 0.8244$, $h(0) = -0.5$, $\nu_2 = 1$, $L_F = 1.0$, $\tau_1 = 0.01$, and $Q = 3.6488$, we find $M = 1.2645$; thus, for $\text{Da}_0 = 0.1$ and $\Theta_{\infty} = 1.0$, the time induction σ_{ig} is equal to 5.70.

CONCLUSION

The effects of an acoustic field and of the transient mass and energy accumulation processes in the ignition of a vaporizing droplet were investigated showing the following results.

1. Under perturbed conditions, the lowest limit of the Damköhler number, Da_m , for ignition to occur is lower than that limit in the unperturbed conditions; therefore, the acoustic processes anticipate the ignition.
2. For any given ambient conditions, the induction time τ_{ig} decreases fast with a small variation of the Damköhler number near the value Da_m , so that it was not possible to determine the ignition condition for any instant later than half of the droplet lifetime, $2\beta_n\tau_{ig} > 0.6$.
3. The acoustic temperature ($\theta_{\infty} \cos(\omega\tau - \delta)$), when positive, causes an increase in the chemical reaction rate equal to $\exp[\theta_{\infty} \cos(\omega\tau - \delta)]$. Consequently, the temperature increases periodically in the reaction zone until the ignition occurs.
4. As this process occurs when $\theta_{\infty} \cos(\omega\tau - \delta) > 0$ only, then it is possible to anticipate the ignition for the previous period when $\theta_{\infty} \cos(\omega\tau - \delta) > 0$ by increasing the Damköhler number. This leads to the discontinuous behavior of the induction time response, τ_{ig} , with the Damköhler number.
5. The frequency and phase angles, ω and δ respectively, of the acoustic temperature establish the number of periods and the instants of condition $\theta_{\infty} \cos(\omega\tau - \delta) > 0$ in the droplet life. So, by changing these pa-

- rameters, one can obtain better conditions for anticipating, or retarding, the ignition.
6. When the frequency is much higher than the inverse of the droplet lifetime, the heat generated by the excited chemical reaction does not have enough time to be transported to a region of lower temperature. Then the local temperature is an increasing step function with sharp rises between consecutive steps. The temperature increases until ignition sets up, even with very small Damköhler numbers.

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APPENDIX

The corresponding frozen problem (i.e., Eqs. 24 and 25) has a similar solution of the form

$$\Delta = \frac{\Upsilon(\eta)}{X}, \quad (\text{A.1})$$

where $\eta = X/\sqrt{\tau}$ and Δ represents either $\Theta - \Theta_\infty \cos(\omega\tau - \delta)$ or Ψ_F . The function Υ satisfies

$$-\frac{\eta}{2} \frac{d\Upsilon}{d\eta} = \frac{d^2\Upsilon}{d\eta^2}, \quad (\text{A.2})$$

with boundary conditions

$$\Upsilon = \begin{cases} -g(0), & \text{for } \Delta = [\Theta - \Theta_\infty \cos(\omega\tau - \delta)] \\ -h(0), & \text{for } \Delta = \Psi_F \end{cases} \quad \text{as } X \rightarrow 0, \quad (\text{A.3})$$

and

$$\Upsilon = 0, \quad X \rightarrow \infty. \quad (\text{A.4})$$

The solution of the above problem is

$$\Upsilon = - \begin{Bmatrix} g(0) \\ h(0) \end{Bmatrix} \operatorname{erfc} \left(\frac{X}{2\sqrt{\tau^*}} \right), \quad (\text{A.5})$$

where

$$\tau^* = \begin{cases} \tau, & \Delta = \Theta - \Theta_\infty \cos(\omega\tau - \delta) \\ \tau/L_F, & \Delta = \Psi_F. \end{cases}$$

Therefore, the initial conditions for problems 25 to 29 are given by

$$\begin{aligned} & \left\{ \begin{array}{c} \Theta - \Theta_\infty \cos(\omega\tau - \delta) \\ \Psi_F \end{array} \right\} \\ & = - \begin{Bmatrix} g(0) \\ h(0) \end{Bmatrix} \frac{1}{X} \operatorname{erfc} (X/2\sqrt{\tau^*}), \\ & X > 0, \quad \tau \rightarrow 0. \quad (\text{A.6}) \end{aligned}$$