Spin magnetotransport in a two-dimensional electron system confined in a quantum well

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Abstract

We show a simple model that computes the magnetoconduction in a two-dimensional electron system (2DES) where the spin is another degree of freedom in the system. The 2DES is confined in a quantum well (QW) immersed in a heterostructure, where the Rashba spin–orbit interaction is present. When an external magnetic field is applied to the system, the competition between the spin–orbit interaction and the Zeeman effect on the magnetoconduction of the 2DES is analysed, in the cases where one or two sub-bands are occupied in the QW. In the model different spin-oriented 2DES can be treated independently, with a spin current associated with each system. The model has been tested with experimental results obtained from a 2DES formed in an InGaAs layer.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The advances in the fabrication of mesoscopic systems with few impurities and defects lead to the macroscopic observation of microscopic quantum effects, such as the quantum Hall effect (QHE) and Subnikov–de Haas (SdH) oscillations [1]. These phenomena are related to the magnetoconductance of charged particles in a two-dimensional electron system (2DES), where the spin of the charge carriers plays a relevant role, and is responsible for the appearance of even and odd plateaux in the integer QHE. It is also possible to manipulate not only the charge current in the devices, but also the spin of the carriers by means of magnetic and/or electric fields. In fact, in 1990, Datta and Das [2] proposed a spin-polarized field effect transistor (FET). The gate electrode on the top of the FET device is used to control, by means of an electric field, the spin of the electrons. This electric field induces a spin–orbit interaction (SOI) that breaks the spin degeneration of the energy states in the 2DES. Even without any external magnetic/electric field, the carriers of the 2DES are also spin polarized by the internal built-in electric field due the structure inversion asymmetry (SIA) of the semiconductor heterostructure. The first theoretical study of this effect was made by Rashba [3] in 1960 (the SOI due to SIA is called the Rashba effect). In 1989 Das et al obtained evidence of spin splitting carrier populations at zero magnetic fields in InGaAs/InAlAs heterostructures [4]. Additionally, zinc-blende-type semiconductors have bulk inversion asymmetry (BIA). Due to this asymmetry the local electric field varies along the crystal directions and therefore the SOI [5] (Dresselhaus effect). More recent devices are proposed by Schliemann \textit{et al} [6] and Nitta \textit{et al} [7], both based on the spin manipulation by means of an electric field. Schielemann has proposed a spin-field-effect transistor based on SOI of both SIA and BIA types, where the spin-independent scattering processes have no influence on the spin transport, and also showed how the interplay between SIA and BIA can lead to $k$-independent spin wave functions. Nitta has proposed a device based on the interference of spinning currents guided in narrow wire rings.

This work analyses the electrical magnetoconductance (magnetoresistance) behaviour of a 2DES confined in a heterostructure quantum well (QW), under QHE conditions, and with Rashba SOI effect (at low temperature) using a simple model based on semiconsidereations and taking into
account the spin orientation degree of freedom. From the theoretical point of view several attempts to understand SDH magnetoconductance oscillations and the integer QHE have been published. The most accepted one is based on the ‘gendanken’ experiment thought up by Laughlin [8], where the 2DES-localized states due to ionized impurities and defects play a crucial role to explain the plateaux of the Hall affect and the SDH oscillations of the magnetoconductivity, with minima values close to zero. However, experimental evidence shows that the measures made on the 2DES with higher electron mobility (materials with few defects and impurities) provide better plateaux precision. The model that we proposed does not use localized states to explain the QHE and SDH effects, but a simple one-electron theory with two assumptions: first, the existence of a flow of carriers from/to the QW to/from the heterostructure where it is immersed (the heterostructure of the 2DES carrier concentration occur with negligible variations in the 3D carriers density of the environment; second, external magnetic fields and/or SOI lifts the spin degeneration, splitting the 2DES into two independent 2DESs, one with parallel spin and the other with antiparallel spin. If the 2DES is confined in a QW with subbands’ energy levels \(E_i(\alpha = 1, 2, \ldots, n)\), the eigenvalues of (1), assuming only the Rashba effect, are given by the expression [3, 11]

\[
E_i^{\alpha} = E_i + \hbar \omega \left[ N_L + s \frac{1}{2} \left( 1 - |g^*| \frac{m^*}{2m_0} \right)^2 + \frac{\gamma}{B} N_L \right]
\]

with \(s = \pm\) for \(N_L = 1, 2, 3, \ldots, n = +\) for \(N_L = 0\), \(\gamma = 8a^2m^2/\hbar^2e\), \(m_0\) is the free electron mass and \(\omega = eB/m^*\). The states given by (1) are highly degenerate [11], with a degeneracy of \((2\pi l^2)^{-1}\), where \(l = \hbar/(\sqrt{cB})\) is the magnetic length. In the limit of large magnetic fields the Zeeman term dominates the spin splitting, obtaining \(\Delta E_{\text{spin}} = g^*\mu_B B\). In the opposite limit when \(B \to 0\), \(\Delta E_{\text{spin}} = 2akF\) at the Fermi energy, where \(kF = \sqrt{2\pi n_0}\) is the Fermi wave vector and \(n_0\) the total 2DES carrier concentration. If we compare equation (2) with the spin-up and spin-down energy states associated with a conventional Landau level \(N_L\), this corresponds to the \(E_{N_L}^{\alpha}\) and \(E_{N_L+1}^{\alpha}\) states, i.e. \(\Delta E_{\text{spin}} = |E_{N_L}^{\alpha} - E_{N_L+1}^{\alpha}|\).

At zero magnetic field, the SIA spin-split energy of a 2DES is \(E_{\pm}(k) = \mu_s k^2 \pm \alpha k\) where \(\mu_s = \hbar^2/2m\), and the density of states (DOS) of the spin-split branches at zero temperature has the form [12]

\[
D_{\pm}(E) = \int_{-\infty}^{\infty} \frac{1}{2\pi} \frac{\alpha}{\sqrt{4\mu_s E + \alpha^2}} \, dE, \quad E \geq 0
\]

\[
D_{-}(E) = \int_{-\infty}^{0} \frac{1}{2\pi} \frac{\alpha}{\sqrt{4\mu_s E + \alpha^2}} \, dE, \quad E < 0
\]

The DOS converges to the constant value \(D_0 = m^*/\pi\hbar^2\) (no spin degeneration is considered) when \(\alpha\) is zero. In a QW with two filled subbands with energies \(E_1\) and \(E_2\), the 2DES can be considered as the sum of four 2DESs, everyone related to the \(E_{1\uparrow}, E_{1\downarrow}, E_{2\uparrow}\) and \(E_{2\downarrow}\) states. Hence the whole DOS of the four subsystems is computed by the expression

\[
D(E) = \sum_{s} \sum_{i} D_s(E_i) \quad (4)
\]
The Rashba effect. To model the DOS we have used a Rashba
in figure 1 when an external magnetic field is applied and exists

\[ D(E) = (eB/h) \sum_s \sum_{N_i} \left\{ \frac{\pi}{2} \Gamma_{N_i}^2 \right\}^{-1/2} \times \exp \left\{ -2 \frac{(E - E_{N_s})^2}{\Gamma_{N_i}^2} \right\} \]

Figure 1 shows the DOS of the 2DES confined in a QW with two subbands (with energy levels \( E_1 \) and \( E_2 \)). The whole
density of states is considered as the sum of the four DOS
independent related to the four 2DESs.

The DOS of a 2DES under the application of a magnetic field normal
to the system has a shape like a ‘comb’, where the pinned ‘teeth’ are related to the \( E_{N_i}^s \) values, and can be modeled with an ‘ad hoc’ Gaussian shape function [20]:

\[ D(E)_{N_i} = (eB/h) \sum_s \sum_{N_i} \left\{ \frac{\pi}{2} \Gamma_{N_i}^2 \right\}^{-1/2} \times \exp \left\{ -2 \frac{(E - E_{N_s})^2}{\Gamma_{N_i}^2} \right\} \]

The level broadening \( \Gamma_{N_i} \) is strongly dependent on the range
of the scattering potentials. For short-range scatters \( \Gamma_{N_i} \) depends on the strength of the magnetic field. The broadening due to long-range potentials is proportional to the
fluctuations of the local potential energy \((V(r) - V(r'))^2\), and can be considered negligible in samples where the
impurities are far from the 2DES. Then, we consider only short-range scatters and use the expression [20] \( \Gamma_{N_i} = \Gamma_0 + \kappa \sqrt{\frac{(2\hbar^2 \pi)}{\alpha^2}} \), where \( \Gamma_0 \) and \( \kappa \) are fitting parameters,
and \( \tau \) is the relaxation time that takes into account the transport and
spin relaxation processes. We assume the Dyakonov–Perel
relaxation mechanism [21, 22] which describes the spin
relaxation of free electrons.

Figure 2(a)–(f) show the evolution of the DOS presented in
figure 1 when an external magnetic field is applied and exists
the Rashba effect. To model the DOS we have used a Rashba
parameter of \( \alpha = 0.7 \times 10^{-11} \text{ eVm} \) and an effective g-factor
\( g^* = 4 \). In order to compute the width of the Gaussian function
of energy levels, we use the fitting parameters \( \Gamma_0 = 0.01E_F \),
\( \kappa = 1 \). The relaxation time is \( \tau = 10^{-12} \text{ s} \) given by Burg
et al [23].

Figures 2(a) and (b) show the oscillations and nodes of the
DOS. The maxima and minima values of the oscillations
occur when there is coincidence of the energy levels of the
different spin 2DES, i.e. \( D(E) \) have a maximum value when
\( E = E_{N_i}^s = E_{N_i}^t \), \( (N_i \neq N_j) \) in every subband, and also
when there is coincidence of the energy levels of the maxima
values of the DOS in the two subbands, and at the same time
the coincidence of the minima values in the oscillations. The
nodes occur when there is no coincidence between energy states
[9] in the DOS, i.e. when \( E = E_{N_i}^s = E_{N_j}^t \). The number
of nodes and their position depend on the energy balance
between Rashba and Zeeman terms. The Rashba term grows
with the momentum, and hence with the energy, while the
Zeeman term remains constant. The lower the magnetic field
\( B \), the larger the number of levels \( N_i \), and more nodes can occur.

Figures 2(c) and (d) show with clarity the energy levels in both subbands. In figure 2(c) there is an overlapping
of the \( E_{N_i}^s \) and \( E_{N_j}^s \) levels in the DOS in each subband
and when both subbands are added. In figure 2(d) there is coincidence of energy levels of different spin in each
subband, i.e. \( E_{N_i}^s = E_{N_j}^t \), \( (N_i \neq N_j) \), but there is no
overlapping of the levels of the two subbands. Figures 2(d)
and (f) show the DOS at high magnetic fields (8 T and 12 T
respectively). The height of the DOS levels depends again on
the coincidence of levels intrasubband and the overlapping of
levels intersubbands. As we will see below this DOS behaviour
and its value at Fermi level explain the magnetococonductance of the 2DES.

3. Magnetococonducton

When the applied magnetic field increases, the energy levels
\( E_{N_i}^s \) move to the Fermi level \( (E_F) \), and the conduction occurs
when each level crosses \( E_F \), providing the SdH oscillation
in the magnetococonductivity. The minimal values of the SdH
oscillations occur when there is no coincidence between \( E_{N_i}^s \)
and \( E_F \), and the maximal values occur when \( E_{N_i}^s = E_F \). On
the other hand, when two kinds of carriers are present in the
system the SdH oscillations show a beating pattern behaviour.
In a 2DES confined in semiconductor heterostructures the
SdH beating pattern arises from the existence of two different
2DES, spin-up and spin-down electrons systems respectively
[4].

In order to obtain the magnetococonductivity of the 2DES,
formed in the semiconductor heterostructure, we have to
calculate the density of carriers. Assuming that the 2DES
is confined in a QW with two filled subbands, each subband
energy level can be considered as a pocket that contains two
independent 2DES, with spins parallel and antiparallel to the
magnetic field. Therefore, the whole carrier concentration
confined in the QW is given by the sum of the four 2DES
concentrations:

\[ n = \sum_i \sum_s n_{is} \]

where again \( i = 1, 2 \) refers to each subband \( E_i \), and \( s \) refers
to each spin orientation, and the carrier concentrations are
obtained by the expressions

\[ n_{is} = \int_{-\infty}^{\infty} f_0(E) D_{is}(E) \, dE \].
Figure 2. (a)–(f) show the evolution of the density of states of a 2DES when the magnetic field increases. The electron system is confined in a QW with two filled subbands.
is no coincidence of the levels pattern with a node near to 2.2 T. The nodes occur when there are related to carrier concentration at levels with the magnetic field. The red-dotted line and blue line are related to spin-up and spin-down orientations respectively. The black thin lines are related to carrier concentration at levels $E_1$ and $E_2$, and the black bold line is related to the whole 2DES carrier concentration.

$$N = \sum_{s} \sum_{i} N_{is}$$  \hspace{1cm} (7)

where $N_{is} = \int_{-\infty}^{\infty} \left( -\partial f_0/\partial E \right) D_{is}(E) \, dE$.

If the carrier concentration at zero external magnetic field is known, the Fermi level of the system is determined from equation (6). To compute the magnetoresistance we use experimental data obtained by Can Min Hu et al. [24]. The 2DES is formed in a 20 nm thick In$_{0.53}$Ga$_{0.47}$As layer where the two subbands levels are filled. The electron concentration at zero magnetic field is $n_0 = 3.6 \times 10^{16}$ m$^{-2}$ and the carrier concentrations of the subbands are $n_1 = 2.8 \times 10^{16}$ m$^{-2}$ and $n_2 = 8 \times 10^{15}$ m$^{-2}$. The calculated Fermi level is $E_F = 0.172$ eV and the computed subband levels are $E_1 = 0.038$ eV and $E_2 = 0.134$ eV. The effective mass is 0.05$m_0$. Figure 3(a) shows the evolution of the total carrier concentration in the whole 2DES ($n$) when the external magnetic field increases, the evolution of the two subbands’ carrier concentrations ($n_1$ and $n_2$), and the evolution of the spin up/down 2DES that forms each subband ($n_{1+}$, $n_{1-}$, $n_{2+}$, $n_{2-}$).

Figure 3(b) shows the evolution of the carrier concentration $N$ computed at Fermi Level in the whole 2DES, in the two subbands ($N_1$ and $N_2$), and the evolution of the spin up/down 2DES that forms each subband ($N_{1+}$, $N_{1-}$, $N_{2+}$, $N_{2-}$). The values of $N_1$ and $N$ show a beating pattern with a node near to 2.2 T. The nodes occur when there is no coincidence of the levels $E_{N_1}$ and $E_{N_2}$, at Fermi level. The $N$ value also shows an envelope modulation created by the sum of the $N_2$ value.

Consider a competition between Zeeman and Rashba effects. Both effects cancel each other when $E_{N_2} = E_{N_2+} = E_F$ [10, 17]. In this case there is a coincidence of the value of the spin-up and spin-down carrier concentrations at Fermi level, i.e. the spin split energy is zero. We can see in figure 3(b) that this occurs at values of the magnetic field close to 3 T for the 2DES confined in the $E_2$ subband and 6 T for the 2DES confined in the $E_1$ subband, where $N_{1+} = N_{1-}$ in both subbands.

In the semiclassical approximation, when an electric field is applied (normal to external applied magnetic field), the carriers move with velocity $v = v_d + v_c$, where $v_d$ is the drift velocity and $v_c$ the cyclotron one. We assume that the mean value $\langle v_c \rangle = 0$ when the carriers move in the system and we use the Boltzmann distribution function for carriers perturbed by an electric and magnetic field [25, 26]. Taking into account the previous assumptions and using the linear relationship $j = [\sigma] E$, where $E$ is the applied electric field and

$$[\sigma] = \sum_{s} \sum_{i} [\sigma]_{i,s} \hspace{1cm} (8)$$

which is the magnetoconductivity tensor, with components

$$\sigma_{xx} = \sigma_{yy} = \sum_{s} \sum_{i} \left[ (e^2 N_{is} \tau_{is}/m^*)/ \left( 1 + (\omega \tau_{is})^2 \right) \right] \hspace{1cm} (9a)$$

$$\sigma_{xy} = -\sigma_{yx} = \sum_{s} \sum_{i} \left[ (e^2 N_{is} \tau_{is}/m^*) \cdot \omega \tau_{is}/ \left( 1 + (\omega \tau_{is})^2 \right) \right] \hspace{1cm} (9b)$$

the current density for the two subband problem can be expressed:

$$j = [\sigma] E = (j_1 + j_2)_x + (j_1 + j_2)_y \hspace{1cm} (10)$$

The magnetoresistivities are obtained by the relationship between tensors $[\rho] = [\sigma]^{-1}$, with components $\rho_{xx} = \rho_{yy} = \sigma_{xx}/(\sigma_{xx}^2 + \sigma_{xy}^2)$ and $\rho_{yx} = -\rho_{xy} = \sigma_{xy}/(\sigma_{xx}^2 + \sigma_{xy}^2)$.

We reproduce the value of the magnetoresistivity given in reference [24] when the whole 2DES carrier concentration...
is $3.6 \times 10^{16} \text{ m}^{-2}$ and two energy subbands in the QW are filled. Figure 4(a) shows the SdH oscillations of the magnetoresistivity with a visible node near to 2.2 T, and figure 4(b) shows a detailed plot in the interval 0.6 T–1.5 T of the magnetic field, where two more nodes at values near 0.75 T and 1.1 T respectively are also shown. As we said before, the nodes occur when there is no coincidence between energy levels at $E_F$, i.e. when $E_{N_L}^+ \neq E_{N'_L}^-$ at Fermi level. The appearance and definition of the nodes depend on the overlapping and the width $\Gamma_{N_L}$ of the DOS energy levels.

Figure 5(a) shows the calculated Hall magnetoconductivity $|\sigma_{xy}|$ of the whole 2DES and the Hall magnetoconductivities $|\sigma_{xy1}|$, $|\sigma_{xy2}|$ related to each filled subband versus the external applied magnetic field. (b) Magnetoresistivity of the 2DES versus the external applied magnetic field.

Figure 4. (a) SdH oscillations beating pattern of the magnetoresistivity, with a visible node in the region between 2 T and 2.5 T. (b) Detailed plot of the SdH oscillations that shows a clean node at 1.1 T, and another in the 0.7 T–0.8 T interval.

Figure 5. (a) Magnetococonductivity $|\sigma_{xy}|$ of the whole 2DES and magnetococonductivities $|\sigma_{xy1}|$, $|\sigma_{xy2}|$ of the subsystems related to each filled subband versus the external applied magnetic field. (b) Magnetoresistivity of the 2DES versus the external applied magnetic field.
4. Conclusions

In conclusion, we have developed a simple semiclassical theory that reproduces the magnetoconduction of a 2DES confined in a QW when two subbands are occupied and when the competition between Rashba and Zeeman effects is significant. Then, in the model that we use, the spin plays an important role in the magnetoconduction. The model starts with the whole carrier concentration at zero external magnetic field, that establishes the Fermi level. When two subbands are occupied, the carrier concentration of each subband is obtained from the value of the subband energy level with respect to Fermi level. Each subband is considered as the sum of two independent 2DESs with different spin polarizations due to the Rahsba effect. Therefore, we consider the whole 2DES confined in a QW as two filled subbands, and hence four independent 2DESs. The evolution of the DOS with the external applied magnetic field explains the SdH oscillations and the integer QHE. The model can be generalized to systems with more than two filled subbands.

References

[19] Yaji Koichiro et al 2010 Nature communications 1 17