

AUTOMATIC GENERATION OF SIGNALS AND SYSTEMS EXERCISES

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Abstract

A computer based training tool has been developed for Signals and Systems undergraduate course students. It consists in an automatic exercise generator and an automatic self assessment system that can be used by the students as many times as they want. The students use the tool to generate numerical variation of the problem, they solve it and afterward use again the tool to answer a series of questions that are automatically assessed. The proposed problem comprised a continuous time domain characterization of a linear and time invariant system and its Fourier domain and Laplace domain analysis. The automatic exercise generator is based on a simple algorithm that varies the parameters that defined the exercise in a controlled way, so the students can be trained to analyze different kind of first and second order filters. The proposed algorithm requires only an arbitrary input number to generate a new exercise. More than 2000 numerically different exercises are automatically generated and assessed. This tool can also be used as a computer based assessment system when used in an environment that guarantees the identification of the student. When used as assessment system, it has the advantage that the teacher can select the difficulty of the exercise (all the student are required to analyze the same order and type filter but with different parameters, so the cheating is quite difficult). The algorithm can be easily transferred to almost any kind of problem in Signal and Systems and other engineering disciplines.

Keywords: Computer based assessment; automatic item generation; engineering education.

1 INTRODUCTION

Computer based assessment (CBA) and web-based assessment have become common tools in all educational levels. Many platforms and systems have been developed in recent years to easily generate quizzes intended for instructional or assessment purposes [1-5]. The core of many CBA systems is a large and high quality item bank for any topic to be assessed. A challenge of these systems is to generate the item bank. During the last decade a great effort has been devoted to develop automatic item generators and exercises generators [6-11]. However, some of these generators use a database where the answer to each numerical variation of a given question is stored [8, 10], so it is a hard task to create numerical variation of the same exercise, solve them and store the numerical input data and its corresponding solutions.

Here a method to automatically generate a larger set of numerical variations of the same exercise is proposed. A quite general class of problem in Signal and System undergraduate course has been selected: the analysis of a continuous time linear and time invariant (LTI) system. An application has been developed to generate the numerical values of the parameters that define a LTI system from any input number. A guide to easily assess and grade the works is also proposed. The application can be used by the students in a repetitive way to solve as many numerically different exercises as they want: they just have to introduce a new input number to obtain a new exercise.

The method is based on the usage of modulo operation (which gives the remainder of the division of two numbers) applied to any input number. This modulo operation is applied with different divisors in order to get some integer values that will be used to generate the constant coefficients of the differential equation. If the application input number is any identification number of the student (i.e. his/her passport number, national identity card number, etc.), the application becomes a personalized assessment tool. The idea can be easily applied to any kind of parameter dependent problems. The analysis of LTI systems described by linear differential equation with constant coefficients (LDECC) example has been chosen because LDECC model most of systems that can be found in the engineering and scientific applications.

Following this introduction, the description of the problem is made, afterward the algorithm to automatically generate numerically different exercises is described, a set of self assessment questions is proposed, the obtained results and applicability of the algorithm are discussed, and finally some concluding remarks are pointed out.

2 DESCRIPTION OF THE PROBLEM

Signal and Systems appears as a main topic in many engineering disciplines. An important question in such a topic is to analyze and classify (if it is possible) the frequency response behavior of a LTI system. Here the attention is focused on first and second order continuous time LTI system described by a LDECC. This algorithm can be extended to any engineering or science problem described by a first or second order LDECC.

A typical problem in Signal and Systems is to derive the transfer function (and to draw the pole-zero diagram), the impulse response, the frequency response function, and a block diagram representation for a system described by a LDECC. Once the magnitude of frequency response is sketched, its classification as an approximation of an ideal filter is usually required. These are analysis exercises: the system is defined and the student should analyze the behavior of the system in the time and frequency domain, and relate this behavior to different representations of the system: the LDECC given as a description of the system, the system function, the pole-zero and block diagrams of the system. For example, an exercise statement as the one below (exercise 1 in the following) can be found in many Signals and Systems textbooks (see for example [12-14]):

“The input $x(t)$ and the output $y(t)$ of a causal LTI system are related by the differential equation:

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_2 \frac{d^2 x(t)}{dt^2} + b_1 \frac{dx(t)}{dt} + b_0 x(t). \quad (1)$$

- a) Find the system function and draw its pole-zero diagram.*
- b) Depict a block diagram representation of the system.*
- c) Based on the pole-zero diagram, sketch the impulse response indicating its qualitative behavior.*
- d) Find the impulse response of the system.*
- e) Find the frequency response of the system.*
- f) Sketch the magnitude of the frequency response and classify it (if it is possible) as an approximation to an ideal filter. Indicate some characteristic parameters as cutoff frequency(s), maximum value of the magnitude, frequency for which that maximum is reached...”*

Note that usually only one or two of the questions are asked at once (depending on the chapter of the book we are consulting). Then the proposed class of exercise can be understood as a long exercise that covers many aspects of the methods to analyze the LTI systems behavior (in the time and frequency domain). The exercise solution will be quantitatively and qualitatively different depending on the particular set of coefficients. In the following, a method to generate the $a=[a_2, a_1, a_0]$ and $b=[b_2, b_1, b_0]$ coefficients from a given integer number is proposed. Each selection of a and b yields to a numerically different problem. The coefficients will be selected in such a way they satisfy the following the criteria:

The system should be stable (in order to have frequency response) and causal (in order to the system has physical meaning, as a mechanical damper or a passive electric circuit). This is satisfied if the coefficients a_i are real positive numbers (a_2 could be zero) [2].

The system should represent an approximation to one of the following ideal filters: low-pass, high-pass, band-pass, band-stop or all-pass filter.

All the coefficients should be not too high integers (up to 3 digits). Then the required calculations will be quite easy. The purpose is that the student will learn the mechanical steps he/she must follow to obtain the particular solution.

A large enough set of different problems should be generated. The purpose of this condition is that each student that attends a particular course will have enough numerically different exercises to be solved.

An integer number is enough as input to the algorithm in order to generate a particular set of coefficients (a, b).

By solving this kind of exercises the student should learn to relate the behavior of the impulse response and the frequency response amplitude to the pole-zero diagram, to the block diagram or to the system function.

3 DESCRIPTION OF THE ALGORITHM

3.1 Generating the Coefficients

The starting point of the algorithm will be an input integer number (N_0 in the following). The teacher can indicate the same set of input numbers for all the students, so they solve the same set of exercises, or the teacher could assign different sets of input numbers to each student, so each student solves a different set of numerically different exercises. Additionally the student can use the algorithm and the self assessment test to practice as many times as he/she wants.

The second step is to calculate the N_0 modulo n values for $n=2$ (m_2), $n=3$ (m_3), $n=5$ (m_5), $n=7$ (m_7), and $n=11$ (m_{11}). More mn being n the following prime numbers can be added if we want to generate a larger set of a and b vectors. The reason to use prime numbers is to generate uncorrelated mn values (for example: the knowledge of m_4 implies the knowledge of m_2 , and knowing m_2 and m_3 , m_6 is known).

The third step is to decide which kind of filter (low-pass, high-pass...) will be generated. The m_5 , that can take 0, 1, 2, 3, 4 value, will be used for this purpose. In the case of all-pass (AP), low-pass (LP) and high-pass (HP) filters, m_2 will be used to select first or second order filter. For the case of band-pass (BP) and band-stop (BS) filters, that will be 2nd order filters for exercise 1, different values of m_2 will modify the amplitude of the frequency response. Table 1 shows the proposed correspondence between filter type and order and the values of these two numbers.

TABLE 1: Type and order of the generated filters for each combination of m_2 and m_5 values. This is an arbitrary selection proposed by the author

m_5	FILTER TYPE	m_2	FILTER ORDER
0	All-pass	0	First order
		1	Second order
1	Low-pass	0	First order
		1	Second order
2	High-pass	0	First order
		1	Second order
3	Band-pass	0/1	Second order
4	Band-stop	0/1	Second order

The fourth step is to build first and second order Butterworth low-pass approximations to generate a low-pass prototype: $a_{p1}=[0, 1, 1]$ (first order case) or $a_{p2}=[1, \sqrt{2}, 1]$ (second order case) and $b_p=[0, 0, 1]$. These prototypes have 1 rad/s cutoff frequency. Finally, a frequency transformation is applied to the prototype to obtain the a and b vectors that correspond to the proposed filter [13].

Following these steps, conditions C.1 and C.2 are automatically fulfilled. The trick of the proposed algorithm is to appropriately select the new cutoff frequency (or central frequency and bandwidth) in order to satisfy C.3 and C.4 conditions.

For the cases of low-pass and high-pass filters, the proposed selection of the cutoff angular frequency is:

$$\omega_c = \frac{(1+m11)}{2} (\sqrt{2})^{m2}. \quad (2)$$

In such a way we obtain 11 different semi-integer or integer values of the cutoff frequency for the case of first order filter ($m2=0$) that span from $1/2$ to $11/2$. For the case of second order filters ($m2=1$) we obtain the same 21 values multiplied by $\sqrt{2}$ for the angular cutoff frequency. After application of the frequency transformation, the coefficients that define the filter (a_t, b_t), are integers or semi-integers. Finally the proposed coefficients are calculated as:

$$\begin{aligned} a &= 2^k (2+3m3)a_t \\ b &= 2^k (1+m7)b_t \end{aligned} \quad (3)$$

The factor 2^k with $k=0$ (when all elements of a_t and b_t are integers) or $k=1$ (if there is some semi-integer element in a_t or b_t) assures that all coefficients will be integers and the factors $(2+3m3)$ and $(1+m7)$, modify the amplitude of the frequency response of the system. The largest integer that could appear as an element of a or b is $7 \times 2 \times \left(\frac{11}{2}\sqrt{2}\right)^2 = 847$. It must be noted that the factor $(1+m7)/(2+3m3)=1$ when $m7=1$ and $m3=0$ and when $m7=4$ and $m3=1$, then $2 \times 3 \times 7 \times 11 \times 2 - 2 \times 2 \times 11 = 880$ numerically different exercises are generated.

For the cases of band-pass and band-stop filters, the proposed selection of the central angular frequency and bandwidth is:

$$\begin{aligned} \omega_0 &= \frac{1+m11}{2} \\ B &= \omega_0 \left(\frac{m3+1}{4} \right) \end{aligned} \quad (4)$$

After the transformation of the prototype filter into a band-pass or band-stop filter, the obtained (a_t, b_t) vectors are modify as:

$$\begin{aligned} a &= 2^k (1+3m2)a_t \\ b &= 2^k (1+m7)b_t \end{aligned} \quad (5)$$

Again the factor 2^k with $k=0, 1, 2, 3$, depending on the values of a_t and b_t , is introduced to assure that all the elements in a and b are integers. The largest possible element of the generated a and b vectors is $8 \times 4 \times (11/2)^2 = 968$. It must be noted that the factor $(1+m7)/(1+3m2)=1$ when $m7=m2=0$ and when $m7=3$, and $m2=1$, so the number of numerically different filters obtained following the described method is $2 \times 3 \times 7 \times 11 \times 2 - 2 \times 3 \times 11 = 858$.

Finally the generation of an all pass filter with integer coefficients is direct and quite easy. There are several possibilities to obtain all pass filters satisfying the 5 selected criteria. Among all these possibilities the following choice is proposed:

$$\begin{aligned} a &= [1+m7, 1+m11] \\ b &= (1+m3) \times [1+m7, -1-m11] \end{aligned} \quad (6)$$

for the first order case and:

$$\begin{aligned} a &= [1, 1+m7, 1+m11] \\ b &= (1+m3) \times [1, -1-m7, 1+m11] \end{aligned} \quad (7)$$

for the second order case. With this selection, the pole or poles always have negative real part, and the system can be stable and causal. It is obvious that the selected coefficients for AP filters are integers in the range from -33 to 33. In this way $3 \times 7 \times 11 \times 2 = 462$ numerically different problems are generated.

With the proposed method, 2200 different pairs of a and b vectors are generated. They satisfy the criteria C.1 to C.5, and define numerically different exercises. This number can be easily and largely increased by using $m13$, $m17$, etc. but the condition C.3 will not be necessarily satisfied: quite large integer numbers are required, or rational numbers must be considered.

As previously commented this algorithm can be used by the students as many times they want to generate numerically different exercises as the exercise 1. They can train themselves in solving this kind of exercises. By answering a set of test questions that are automatically assessed they will know if they are solving correctly the exercise. They should solve the exercise (with different input number) till they are sure to be able to solve any other numerically different example.

3.2 Self Assessment Questions

The developed application automatically assess a set of proposed questions related to the exercise 1. Before assessing the student answers, checking that they use the coefficients assigned to the particular exercise they are solving is necessary so they must introduce the N_0 as first input data. The application calculates the corresponding mn values and a and b vectors. The answers to the following questions are easily programmable from these mn values, Table 1 and Eqs. 2-7.

Afterwards, the student must introduce the coefficients of a and b vectors. The application checks these values and permits to the student answer a subset of the following questions if the students introduce an a , b pair compatible with the N_0 previously introduced.

The proposed questions and their possible answers are listed below. They refer to the tasks a) to f) in the exercise 1.

a.1) What is the order of the numerator of the system transfer function? Possible answers: 0, 1, 2, other value.

a.2) What is the order of the denominator of the system transfer function? Possible answers: 1, 2, other value.

a.3) Where are the poles of the system transfer function. Possible answers: on the negative real axis, left half complex plane, on the imaginary axis, on the real positive axis, right half complex plane, none of these possibilities.

a.4) Calculate the poles and zeros of the system transfer function. Open answer.

b.1) Which is the minimum number of integrators necessary to obtain a block diagram of the system? Possible answers: 0, 1, 2, other value.

c.1) What is the behavior (for $t > 0$) of the system impulse response? Possible answers: periodic oscillations, monotonically increasing, monotonically decreasing, constant, increasing oscillations, decreasing oscillations, other behavior.

d.1) What is the value of the impulse response for $t \rightarrow \infty$? Open answer.

d.2) What is the value of the impulse response for $t \rightarrow 0$? Open answer.

d.3) How is the expression of the impulse response? Possible answers:

i) $c_1 \exp(-c_2 t) u(t)$

ii) $c_0 \delta(t) - c_1 \exp(-c_2 t) u(t)$

iii) $c_1 \exp(-c_2 t) \sin(-c_3 t) u(t)$

iv) $c_0 \delta(t) - c_1 \exp(-c_2 t) \cos(-c_3 t) u(t)$

v) $c_1 \exp(-c_2 t) \sin(-c_3 t) u(t) + c'_1 \exp(-c'_2 t) \cos(-c'_3 t) u(t)$

$$\text{vi) } c_0 \delta(t) + c_1 \exp(-c_2 t) \sin(-c_3 t) u(t) + c'_1 \exp(-c'_2 t) \cos(-c'_3 t) u(t)$$

$$\text{vii) } c_1 \exp(-c_2 t) u(t) + c_3 \exp(-c_4 t) u(t)$$

$$\text{viii) } c_0 \delta(t) + c_1 \exp(-c_2 t) u(t) + c_3 \exp(-c_4 t) u(t)$$

ix) Other expression.

(When answering this question, it is assumed that the coefficients of the expressions i)-viii) have non zero values. Once a selection is made, the values of the coefficients are also asked for).

e.1) What amplification/reduction gives the system for a constant entry? Open answer.

e.2) What is the maximum value of the frequency response amplitude? Open answer.

e.3) For which frequency does the frequency response amplitude reach its maximum value? Open answer.

f.1) What ideal filter does the frequency response of the system approximate? Possible answers: low-pass, high-pass, band-pass, band-stop, all-pass, none of these.

f.2) What is the 3 dB cutoff angular frequency (low or high-pass cases) or frequencies (band-pass or band-stop filter) of the system frequency response? Open answer.

f.3) From the straight-line approximations to the Bode diagram, what is the slope (in dB/decade) for frequencies well below the (lower) cutoff frequency? Possible answers: 0, 10, 20, 30, 40 dB/decade, other.

f.4) From the straight-line approximations to the Bode diagram, what is the slope (in dB/decade) for frequencies well above the (higher) cutoff frequency? Possible answers: 0, 10, 20, 30, 40 dB/decade, other.

In the case of open answer, a range of possible values (within a $\pm 5\%$ of the real value) are considered as right answers.

4 DISCUSSION

An application has been developed for implementing the described algorithm and the assessment questions. Then up to 2200 numerically different exercises that satisfy the criteria C.1-C.5 are generated with the same questions than exercise 1. A set of test questions are proposed for self assessment. Each student that attend the course could use the application entering several input integer numbers to generate as many exercise as he/she wants. The student should solve enough exercises to be sure that he/she is able to correctly do a similar exercise with different numerical values. Then the teacher could use the application for automatically assess and grade all students that attend the course. This could be done in several ways for example:

a) The teacher decides an arbitrary input number to generate the same new problem to be solved by all students.

b) The teacher assigns a personalized number to each student. There are many possibilities to assign a personalized input number, for example, the student passport number, or its ordinal number in an alphabetically ordered list (L) of the students that attend the course. The input numbers can be selected as a given integer plus $5 \times L$ and then all students solve the same type of filter, or a given integer plus $10 \times L$ in order all students solve same type and order filter, so the expected difficulty is the same for all students. The teacher can change the selected integer to change the type of filter.

Once the students have solved their exercise, they use the application to be automatically assessed and graded. In this way the workload of the teacher is reduced to read the grade that the application assigns to each student.

In the following some examples are analyzed. Table 2 shows the values of the a and b vectors generated for 5 consecutive numbers (for example the order number in an alphabetically ordered list of students that attend the course). The corresponding mn values as well as the filter type are also listed. It can be seen that all a and b components are integers with 3 or less digits.

TABLE 2: Examples of generated filters (defined by a pair of a and b vectors) when the first 5 integers are taken as N_0 input values. The corresponding mn values, and filter type are also shown

N_0	m_2	m_3	m_5	m_7	m_{11}	a	b	Type
1	1	1	1	1	1	[5,10,10]	[0,0,4]	LP
2	0	2	2	2	2	[0,8,12]	[0,3,0]	HP
3	1	0	3	3	3	[2,1,8]	[0,1,0]	BP
4	0	1	4	4	4	[8,10,50]	[20,0,125]	BS
5	1	2	0	5	5	[1,6,6]	[3,-18,18]	AP

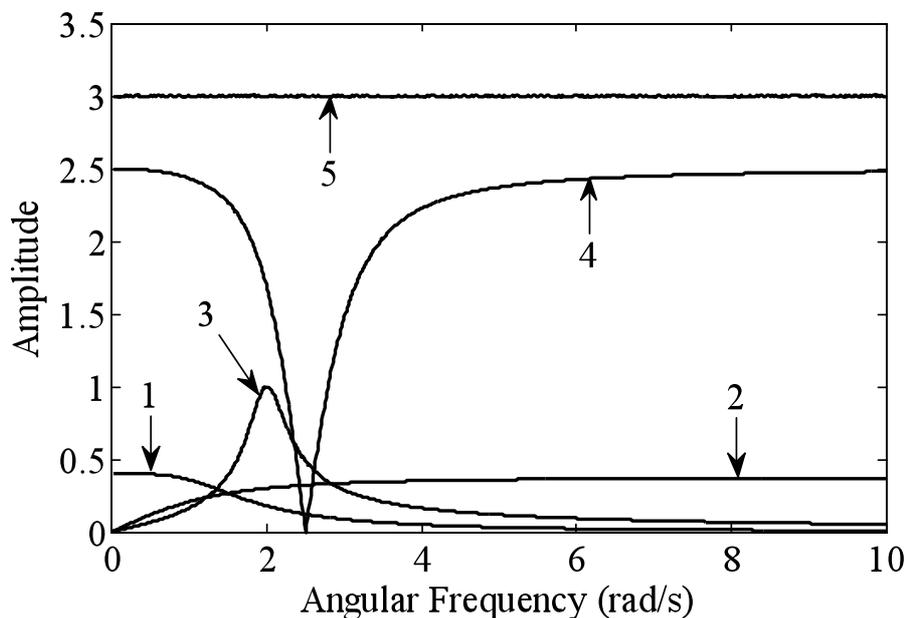


Fig. 1. Frequency response amplitude corresponding to a and b vectors generated from first 5 integers (listed in Table II). The numbers show the correspondence of the selected N_0 with its frequency response

5 CONCLUSION

A simple algorithm to automatically generate many numerically different exercises is proposed. The exercises have a common structure and the same enunciate, but differ in the parameters that define them. With the proposed selection rules, 2200 numerically different exercises are generated. The generated coefficients are such that the exercise refers to a stable and causal LTI system with frequency response that approximates an ideal filter, the coefficients are up to 3 digits integer numbers, and the teacher can check the assignment of a particular set of coefficients to any student just knowing the input number used to generate the exercise. An application to self-assess the answers to these questions can be easily implemented following the algorithm.

This algorithm has been conceived in the context of a Signals and Systems course, but can be applied to any other problem governed by a linear differential equation with constant coefficients that could appear in any engineering or scientific area.

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