Comment on "Steady-state planar ablative flow"[Phys. Fluids 25, 1644 (1982)]

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In a recent paper Manheimer *et al.*[1] attempted to proceed beyond older analyses[2] on the ablative corona ejected by planar laser targets. Here we comment on some aspects of the analysis of Ref. 1, common to most previous papers on the subject.

(i) Manheimer *et al.* assumed that the layer (A) between the ablation and the critical surfaces is steady, and that it links onto an unsteady isothermal rarefaction (B). We agree with (B) being unsteady but claim that it will be isentropic if the ablation layer (A) is steady. The type of rarefaction affects its density, velocity, and temperature profiles.

The irradiance $I(t)$ considered rises to a value I_0 and remains constant threreafter. At some stage the temperature at density ρ_c stops growing and the ablation layer becomes steady with a constant thickness $\Delta x \propto KT_c^2/\rho_c \propto KI_0^{4/3}/\rho_c^{7/3}$, given in Eqs. (12) and (37) of Ref. 1. Obviously, however, the overall length of plasma continues growing so that as (A) becomes steady at "large" t, it also becomes shorter than (B) [put it this way, to neglect the time derivative $\partial/\partial t$ against the convective one, $v\,\partial/\partial x$, in (A) but not in (B), this region must be longer than that one]. Now, the conduction term in the energy equation [Eq. (7a)][1] determines the structure and thickness of (A) and is thus "of order unity" there; it involves a second-order derivative $\partial^2/\partial x^2$, against the convective terms ($v\,\partial/\partial x$). It is then clear that, densities, velocities, and temperatures being comparable in both regions, conduction will be negligible in the rarefaction (which would be isentropic) when it becomes longer than (A), at the time this layer reaches steady state.

Manheimer *et al.* did notice, in their Eq. (10), that the energy equation violates the isothermal rarefaction model. They explained this difficulty by making the model correspond to the limit $K\to\infty$, when the temperature gradient vanishes in Eq. (10). However, they also have $\Delta x \propto K$ so that as $K\to\infty$ both Δx and the time Δt ($\propto \Delta x/T_c^{1/2} \propto KI_0/\rho_c^2$) for (A) to reach steady state blow up to infinity. One cannot have it both ways—steady ablation and isothermal rarefaction. Notice that the isothermal model is based on dropping the temperature gradient in the pressure term of the momentum equation. This requires $\rho|\partial T/\partial x| \ll T|\partial\rho/\partial x|$. Using Eqs. (4a), (5), (10), and (12) of Ref. 1 this condition becomes $t \ll KI_0/4\rho_c^2$(*). Hence, the model itself shows that the rarefaction (a) cannot be isothermal for large t, and (b) ceases being isothermal at about the time ablation reaches a steady state.

(ii) Condition (*) can be made more definite for an irradiance $I(t) \simeq I_0 t/\tau$.[3] If $\bar{I}_0 \equiv (4M_i/9Z)^{3/2}(\bar{K}/k_B^{7/2})I_0/n_c^2\tau$ is small there exist both an ablation layer and a longer isentropic rarefaction; the layer is quasisteady, that is, $\partial/\partial t$ is negligible, so that steady equations, like Eqs. (1), (2), and (7) of Ref. 1, may be used (deflagration regime). If $\bar{I}_0 > 1$ [a condition equivalent to (*) because $\bar{K} \propto K, n_c \propto \rho_c$], the rarefaction is isothermal, but the overdense flow is not quasisteady. [This flow regime is called intermediate in Ref. 3; if $\bar{I}_0 > (n_0/n_c)^2$, n_0 = solid density, a thermal wave enters the target, the rarefaction being thin and isothermal.] To discuss an irradiance that rises to a constant value I_0, we use $I(t)/t$ for I_0/τ in $\bar{I}_0$ [$I(t)/t \equiv I_0/\tau$ for the linear pulse]. If the irradiance rise is sharp, the flow will be initially in the intermediate (or even the thermal wave) regime. As time grows, however, we get $\bar{I}_0 \propto I_0/t$, and a transition to the deflagration regime will occur; for $I_0 = 10^{13}$ W/cm^2, $\lambda = 1.06$ μm, $Z_i = 3.5$, $Z_{\rm eff} = \ln\Lambda = 5$, we have $\bar{I}_0 \simeq 8.5/t$ (nsec). The information that $I(t)$ has reached a constant value travels from the ablative layer outward so that *far* away ($\rho \ll \rho_c$) the flow would be roughly isothermal for some time after ablation became steady.

(iii) The assumptions by Manheimer *et al.* that the unsteady rarefaction begins at ρ_c, and with isothermal sound speed, are invalid, like the isothermal model. In Ref. 3 we studied ablation layer and rarefaction as inner and outer problems of a singular asymptotic expansion in the small ratio of their characteristic lengths. There is no definite surface separating both regions; the characteristic length of the underdense part of the ablation layer is several times the distance between ablation and critical surfaces.

Since our ablation was quasisteady, its structure is universal and thus valid for the case of Ref. 1. Different ion and electron temperatures were allowed for. The ablation layer ends at isentropic sound speed, with density below ρ_c (0.62 ρ_c for $Z = 1$, 0.80 ρ_c for $Z\to\infty$). The underdense fall in temperature is 28% for $Z=1$, 7% for $Z\rightarrow\infty$. The isothermal Mach number $\mathscr{M}$ at ρ_c is 0.75 for $Z=1$, 1 for $Z\rightarrow\infty$. For $Z\rightarrow\infty$, the structure is reasonably simple.[4]

Our results for P_a improve the agreement at high I in Fig. 4 of Ref. 1: for $Z_i=3.5$, our P_a is 25% higher than that given in Ref. 1. For low I, inverse bremsstrahlung should count. We also improve the agreement in Fig. 5 of Ref. 1. The speed measured is about twice v_c (the only characteristic velocity of the isothermal model); for the isentropic rarefaction, the speed begins at about 1.4 v_c and remains finite throughout, independently of geometry and time effects. Table I of Ref. 1 confirms that $0.75<\mathscr{M}<1$.

[1]W. M. Manheimer, D. G. Colombant, and J. H. Gardner, Phys. Fluids **25**, 1644 (1982).

[2]R. E. Kidder, Nucl. Fusion **8**, 3 (1968); A. Caruso and R. Gratton, Plasma Phys. **10**, 867 (1968); J. L. Bobin, Phys. Fluids **14**, 2341 (1971); F. S. Felber, Phys. Rev. Lett. **39**, 84 (1977).

[3]J. R. Sanmartin and A. Barrero, Phys. Fluids **21**, 1957 (1978). Also, J. R. Sanmartin and A. Barrero, Phys. Fluids **21**, 1967 (1978); A. Barrero and J. R. Sanmartín, Plasma Phys. **22**, 617 (1980).

[4]J. Sanz, A. Liñán, M. Rodriguez, and J. R. Sanmartin, Phys. Fluids **24**, 2098 (1981); see Sec. IV.

Reply to the comment by Sanmartín, Montañés, and Barrero

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In our previous work "Steady-state Planar Ablative Flow,"[1] we used a steady-state model to calculate planar ablative flow under both uniform and nonuniform illumination. The model assumed an underdense isothermal rarefaction linking up to a steady-state overdense plasma. Also, Ref. 1 stated that the isothermal rarefaction must ultimately be powered by outward heat flow from the critical surface, so some temperature drop in the underdense plasma must occur.

In their comment, Sanmartín, Montañés, and Barrero maintain that the underdense plasma cannot be in steady state if the rarefaction is isothermal. To reach this conclusion, they cite their own work in which they calculate similarity solutions for the case where the absorbed irradiance increases linearly in time.[2] Needless to say, both this and our steady-state model are approximations to the actual fluid behavior.

The first question is whether steady flow profiles do in fact occur if the absorbed laser irradiance is uniform in time. Reference 1 gave sketchy details of five fluid simulations. Here we give additional details concerning the achievement of steady flow.

To proceed we consider the fourth row in Table I of Ref. 1, that of a plastic target accelerated by an absorbed irradiance of 10^{13} W/cm^2 deposited at $n_e=10^{21}$ cm^{-3}. This corresponds most closely to the Naval Research Laboratory experiments. In the simulations, the absorbed irradiance increased linearly from 0 to 10^{13} W/cm^2 over a time of 2 nsec. For longer times, the absorbed irradiance was held constant. The time dependence of the separation between critical and ablation surface is shown in Fig. 1. Clearly, for times longer than about 4 nsec, a steady state has been achieved. At steady state, the time for the fluid to flow through the ablation layer is about 1.2 nsec, so once the irradiance is held constant, the steady state forms in just under two transit times. This result is characteristic of all of the simulations reported in Ref. 1.

The next question is whether the rarefaction is isothermal. To get the condition for this, we perform the calculation alluded to in Ref. 1, namely we calculate the drop in temperature necessary to power the assumed isothermal rarefaction. As discussed in Ref. 1, the outward thermal flux at the critical surface is $\rho_c v_c T_c$ (temperature has dimension of ve-

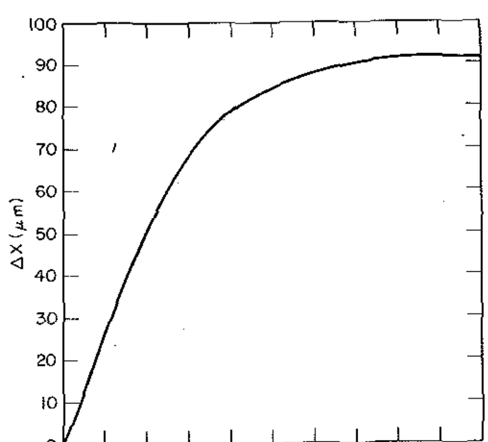

FIG. 1. The separation between critical and ablation surface as a function of time for $\lambda=1\,\mu$m, $I=10^{13}$ W/cm. Note the unmistakable achievement of steady state.

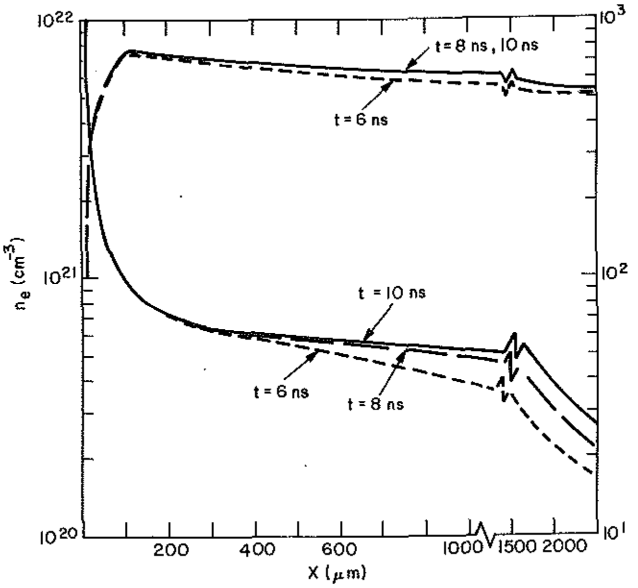

FIG. 2. Steady-state plots of density and temperature at three different times (observe the scale change at $X = 1000\,\mu$m).

locity squared) which is $\rho_c T_c^{3/2}/M^{1/2}$ if the isothermal Mach number is unity.

The temperature equation is

$$\frac{3}{2}\rho\frac{\partial}{\partial t}T+\frac{3}{2}\rho v\frac{\partial T}{\partial x}+\rho T\frac{\partial v}{\partial x}=K\frac{\partial}{\partial x}T^{5/2}\frac{\partial T}{\partial x}. \qquad (1)$$

Assuming $T = T + \delta T$ and $\delta T \ll T$ (that is $\delta T \sim K^{-1}$), we find the equation for δT is

$$\rho T_c\frac{\partial v}{\partial x}=K\frac{\partial}{\partial x}T_c^{5/2}\frac{\partial \delta T}{\partial x}. \qquad (2)$$

Using the fact that at $x = 0$, $\delta T = 0$, $-KT_c^{5/2}\partial\delta T/\partial x = \rho T_c^{3/2}/M^{1/2}$, we find

$$\delta T = -(\rho_c t/KT_c^{1/2})\{1-\exp[-x/(T/M)^{1/2}t]\}, \qquad (3)$$

where we have assumed ρ and v on the left-hand side have the space and time dependence of an isothermal rarefaction wave. Thus as $x \to \infty$,

$$\Delta T|_{x=\infty} = -\rho_c L/KT_c^2 \equiv 0.15L/\chi_c, \qquad (4)$$

where L is the scale length of the assumed isothermal rarefaction wave, $L = (T_c/M)^{1/2}t$, and x_c is the spacing between critical and ablation surface. According to Ref. 1, $x_c = 0.15KT_c^2/\rho$. Thus, as Sanmartín *et al.* state, the scaling law shows that the length of the underdense plasma for which an isothermal approximation is valid is proportional to the length of the steady-state plasma. However the coefficient of proportionality is large, particularly since L is the e-folding length of the rarefaction while x_c is the total length of the steady region. Thus, in practice, the isothermal approximation can be extremely good. Fluid simulations show that the underdense plasma is isothermal for lengths in excess of ten times the steady-state region.

Here we present additional details of the fluid simulation mentioned in Fig. 1. In Fig. 2 are shown the spatial plots of density and temperature at times $t = 6,8$, and 10 nsec of the simulation of Fig. 1. The graphs are displaced from one another in space so that the ablation point is fixed. (During the 4 nsec, the ablation surface moves inward about $10\,\mu$m.) Again, it is clear that steady states do form. Furthermore, while there is some falloff in temperature, the underdense expansion is much closer to being isothermal than adiabatic. For instance the temperature falls by less than 30% while the density falls by a factor of 5. This drop in temperature appears to be what is necessary to power the isothermal expansion.

Thus, to summarize, the ablation region is steady state for steady-state absorbed irradiance, and the underdense plasma, in practice, is often well-described as isothermal.

Additional points are

(1) As shown in Fig. 2, the underdense plasma does not link directly onto a rarefaction wave, but there is a density transition, out to about $0.7\rho_c$. This verifies one aspect of the theory of Ref. 2. However there does not appear to be any temperature transition region.

(2) We do not think the explanation for the measured electron velocity has to do with isothermal versus abiabatic rarefaction. Once the electrons move from the surface, three-dimensional effects are a much more likely explanation of the velocity limit.

(3) One great advantage of the steady-state theory is its relative simplicity. Our derivation (in the uniform illumination case) took 4½ journal pages and 18 equations. The derivations of Sanmartín *et al.*, took 15 journal pages and 100 equations. Since accurate solutions are always available via fluid simulation, our principal goal was to get simple analytical solutions which shed light on the basic physics. The simplicity of the uniform illumination solutions also allow us to calculate results for nonuniform illumination. This is a vital consideration in laser fusion, and we feel it is important to get some sort of analytical result somewhere between the simplest "cloudy day" effect and a full fluid simulation. It is not clear whether the similarity solutions of Ref. 2 can be extended to treat nonuniform illumination.

[1] W. M. Manheimer, D. G. Colombant, and J. H. Gardner, Phys. Fluids **25**, 1644 (1982).

[2] J. R. Sanmartín and A. Barrero, Phys. Fluids **21**, 1957 (1978); J. R. Sanmartín and A. Barrero, Phys. Fluids **21**, 1967 (1978).