Numerical simulation of problems of great deformation in rock masses and its application in mining

Simulation starker Gebirgsverformungen mittels numerischer Verfahren – Anwendung im Bergbau

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The design of a salt working requires detailed studies of the chambers and galleries which will remain opened during the operation. The excavation of the cavity changes the natural distribution of tensions, and the resulting derived cause the flow of salt towards the cavity. The relation between the diverting tensions and the velocity of the associated deformation is defined by a constitutive law depending on temperature. There are a series of computer programs which solve the equations of the thermic and mechanical phenomena, which are used to produce results to compare and combine with the measurements gathered in the mine. So it is a study of the rheologic behavior of a salt material in a visco-elasto-plastic performance, representing such performances by means of programs of numerical simulation based on procedures of finite differences with integration in time.

Der Abbauplan zur Salzgewinnung erfordert eine detaillierte Untersuchung der Kammern und Strecken, die während des Betriebs geöffnet bleiben. Der Abbau des Hohlraums verändert die natürliche Verteilung von Spannungen, und als Folge davon wird der Salzfluß in Richtung des Hohlraums verursacht. Die Beziehung zwischen der Umlenkung der Spannungen und der Geschwindigkeit der damit verbundenen Deformation wird durch ein Grundgesetz in Abhängigkeit von der Temperatur definiert. Es gibt eine Reihe von Computerprogrammen zur Lösung der Gleichungen für die thermischen und mechanischen Phänomene; ihre Ergebnisse werden mit den in der Grube gesammelten Meßresultaten verglichen und kombiniert. Es handelt sich also um eine Studie des rheologischen Verhaltens von Salz in einer visko-elasto-plastischen Darstellung; diese Darstellungen erfolgen mit Hilfe von Programmen zur numerischen Simulation auf der Grundlage von Methoden der Finiten Differenzen mit Integration über die Zeit.

One of the most fundamental and difficult problems rock engineers have to face is determining loads and supports for mining cavities. Theoretical solutions to these problems can be quickly sorted out by applying mathematical principles to the statics theory of mining cavities stability. Using point-by-point data (i. e. probes), we can find out the behaviour of a certain rock mass. There can be, however, large differences between laboratory predictions carried out by means of various testings on point elements, and what occurs in situ in rock mass itself. Today tendency is to elaborate more or less complicated models, either real ones (such as models built with equivalent materials: scale models) or abstract (mathematically based), from which to proceed to a real operation at the mine site.

Fundamental relationships in rheology and rheological modelling

Generally speaking, rheology is the science studying the deformations certain types of solids and materials experience along a time period. There is, obviously, no relationship between that concept and the stationary states.

Special regard is given to those time dependent stresses and changes of strains (deviating strains), as well as the relationships between different strains.

Rheology deals with facts such as: independent procedures, problems in the elastic linearity of materials, applications related to the theory of plasticity, etc.

In general, rheological properties can be expressed using a number of rheological diagrams. Rheological models can be established to solve 2 or 3-dimensional problems. Since our paper has been done on a plane basis, we will limit it to 2 dimensions.

In the case of 2 dimensions, the two diagrams most commonly used are those dealing with strain/deformation problems, that is:

$\sigma = f(\delta, t)$ $\quad \delta$ = constant value

$\delta = g(\varepsilon, t)$ $\quad \varepsilon$ = constant value

$\delta \rightarrow$ strain

$\varepsilon \rightarrow$ deformation
t $\rightarrow$ time
or, if expressed in more general terms: $\varnothing = (\varepsilon, \delta, t) =$ constant value.
Whenever a strain field is applied to a certain material, as in underground mining, it experiences significant deformations. These deformations are given by complicated theoretical functions and, in most of the cases, are unsolvable when an attempt is made to put them into practice.
That's the reason we tried to simplify the problem by using modelling, even though the mistakes being made are to be considered and quantified in order to check the reliability of the refered models.
First of all, we should distinguish between *logical models* and *mathematical models.*
As a logical model, the substantial implementation of the deformation process is understood to be a more or less complicated mechanism; while the mathematical model attempts to find an exact mathematical formula for the deformation that is being applied in this case in the model.
In the deformation process of a solid material, the rheological substitutes are models we refer to as structurals, hence a process such as the aforementioned is divided in elementary processes that are being idealized by elementary substances.

Following a list of some of them:
- Elastic material (EM)
  Its typical relationships are:
  $\sigma = E.\ \varepsilon$
  $Z = G.\ \gamma$
  $\sigma \rightarrow$ Normal strain
  z $\rightarrow$ Tangential strain
  $\varepsilon \rightarrow$ Relative deformation
  $\gamma \rightarrow$ Relative distortion
  E $\rightarrow$ Strain Elasticity Module
  G $\rightarrow$ Slip Elasticity Module
  The model used for this material is a spring.
- Viscous material (VM).
  Between the strain and the moving speed there is a directly proportional relationship, so called flow relationship.
  Typical relationships of this behaviour are:
  $\sigma = \lambda d\varepsilon/dt$
  $z = n d\gamma/dt$
  $\lambda \rightarrow$ Normal viscosity coefficent
  n $\rightarrow$ Tangential viscosity coefficent

The rheological model used is a hydraulic piston.
- Plastic material (PM)
  Transition is made from a solid to a liquid, as a behaviour conforming to solid/liquid deformation limits. 2 sliding discs are used as the rheological model.

More complicated matter behaviours, like elastic-viscous or elastic-plastic, can be modelled as a combination of the simple behaviours described above.
In our present case, the Bingham rheological model has been taken as the one most suitable, due to the viscous/elastic/plastic behaviour of salt, a polycrystalline type of material. This model is represented by a series connection of a spring, a hydraulic piston and a slider.
Its general rheological equation is:
$K1 + K2\varepsilon + K3\ d\varepsilon/dt = K4.\sigma + K5\ d\sigma/dt$

Characteristics and behaviour of saline deposits

The degree of difficulty in engineering, regarding the numerical treatment of the different problems to be solved, depends on the actual knowledge available on that specific problem. In mining, that knowledge is purely relative, depending also on the type of deposit that is being studied.
When we start to consider the exploitation of a saline deposit (potassium salts) and we want to treat numerically the stability of its mining boards (chambers and pillars), we are going to face the difficulty of not being able to use a precisely determined or known constituent equation of the said material. On the other hand, when using any kind of computerized numerical representation, the very same limitation of this type of program may lead to an inadequate treatment of the information so obtained and to the establishment of false and unreliable solutions.
Constituent equations for most materials can be determined before starting any project through lab testing. In our case, as existing literature on the subject makes it clear, salt behaviour in situ as opposed to salt behaviour predicted by use of lab models originates considerable deviations and mistakes.
Our present paper is designed to determine those final equations for the material that can justify the deposit's real behaviour. To reach that goal, we'll begin from previously estimated constituent equations, and deducted empirical equations from different mining tests. Later on, these will be introduced into a mathematical model designed to solve the problem. The process will obviously have to be done with the help of a computer. The shifts and deformations we obtain from the model have to be similar to those measured in the mine.
Mine measurements were carried out horizontally as well as vertically at mine drills with attached anchorings, using mechanical extensometers to take several readings. Fig. 1 shows type curves at these anchorings.
In general, saline rocks are considered to be solids of a viscous/elastic/plastic behaviour.

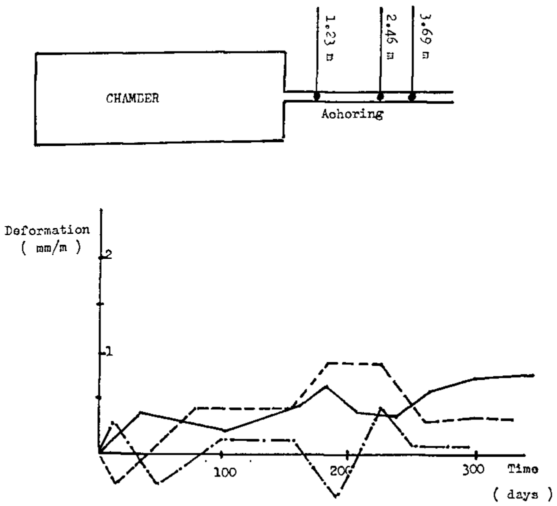

Fig. 1: Type curves of extensometers.

Medium and long term response of this kind of material is given by equations such as:

$\varepsilon = A.\sigma^n.\ e^{-H/KT}$

where

$\sigma = [3/2\ Sij.\ Sij]^{1/2}$ → Effective strain

$\varepsilon = [2/3\ dij.\ dij]^{1/2}$ → Effective deformation speed

Sij → Deviating stretcher of Cauchy strains

dij → Deformation speed stretcher

A, n, H → material's constant values

K → Boltzmann constant

T → Absolute temperature

Isotropic conditions lead to:

$dij = 2/3\ A\sigma^{(n-1)} \cdot e^{-H/KT} \cdot Sij$

The possibility to represent behaviour types as those previously mentioned is a common feature in computer programs. We will later on indicate the one we have used.

To simplify the problem we are going to use simplified empirical equations, such as:

$\dot{\varepsilon} = K1.\sigma$ or $\sigma = K\dot{\varepsilon}^m$

where

σ → Material's strain

$\dot{\varepsilon}$ → Deformation speed

In some particular cases, such as the potassium mines in the North of Spain, the equation is:

$\dot{\varepsilon} = 3.5 \times 10^{-\varepsilon}\ \sigma^3$

Finally, we should indicate that the starting equations mentioned above definitively apply to the General Rheological Law described in former paragraph about fundamental relationships in rheology and rheological modelling.

Numerical modelling

If numerical models are used for real problems, as in our present case for salt mining exploits, it will be possible to do mathematical check of the reliability and development of the exploitation design that has been commonly produced by means of empirical methods.

The purpose of using numerical analysis to produce models is:

- To predict the closing speed of chambers against time (in those mining exploitations using chambers and pillars).

To predict the relative displacements occurring at the contact points between 2 or more different materials.

The program that has been used is PR2D, produced by Principio Mechanica Limited, in London, 1984. It is a program of finite differences designed to solve 2-dimension mechanical problems.

Calculus progress by means of explicit integration of momentum conservation equations.

Constituent equations are incrementally formulated, allowing to easily introduce the viscous/elastic/plastic type of treatment required in dealing with the problems we face in our salt exploitations. The program uses as temporary derivative Jaumann's derivative of Cauchy's strain stretcher.

Formulae used is of Lagrangian type. The mesh nodes and elements possess a mass not altered by time. The mesh changes according to the considered medium.

The discrete system used (Goicolea, 1985) is a modification of the mixed discrete system proposed by Marti and Cundall (1982). The mesh is made of triangles requiring the deviating components of the strain and deformation stretchers.

The program proceeds to operate by integration cycles. The following operations are carried out at each integration interval:

- Strains around the nodes are integrated to produce resulting forces that, once divided by the nodal masses, will provide instant accelerations. These accelerations are then explicitly integrated to obtain speeds and displacements.
- Updated speeds and geometrical configurations are used to determine deformation and running speeds. After that, constituent equations allow calculation of strain increments and the start of a new cycle.

Courant's stability condition requires the integration interval to be small enough as to prevent compression valves not to enter any element during that time. This condition decouples momentum equations, enabling them to be independently resolved for each node.

The problem is solved moving forward as many integration intervals as necessary to cover the time period the problem lasts.
Fig. 2 is showing the kind of geometry introduced in the program. The three symmetry axes provided by the model and used to simulate an unlimited number of chambers can be observed here.

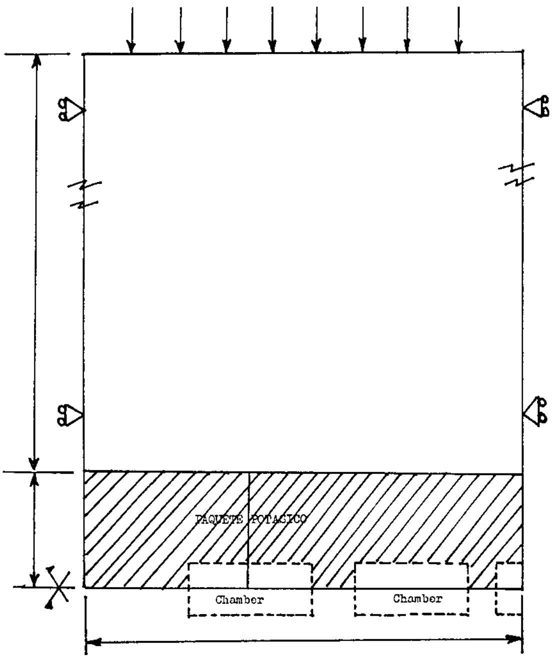

Fig. 2: Kind of geometry introduced in the program.

Summary and conclusions

Field measurements and numerical predictions of closure rates in a potash mine have been compared with satisfactory although imperfect results. This has allowed improving (somewhat) this viscoplastic creep law derived from laboratory experiments. The improved law has been used for design calculations in relation with a future exploitation at greater depths.
The calculations have been carried out by explicit integration in the time domain. A mixed discretisation referred to a Lagrangian frame was used for spatial representation of the equations. Dynamic relaxation procedures were implemented to study the quasi-static problems of interest. The results show the robustness of the algorithm in spite of the constitutive nonlinearities triggered, the large strains developed and the establishment of contacts between roof and pillar associated to their progressive bulging.

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