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On the practical uselessness of structural designs derived from some theoretical optimal solutions

Mariano Vázquez Espí · Jaime Cervera Bravo

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Abstract A recent study by Pichugin et al. recall the Hemp's solution for uniform load of 1974, showing that if allowable tensile and compressive stresses are unequal then the Hemp's arch is optimal provided the ratio of stresses falls within a certain interval. This work is undoubtedly an important pass forward to find an optimal solution for the mathematical problem stated by Hemp. Furthermore, the Authors suggest that their optimal solutions are potentially reasonable from a practical perspective for materials with more allowable compressive stress than tensile one, as this kind of materials used to be not too much expensive.

In this paper we profoundly analyse the solutions of the Authors from this practical perspective finding that the original Hemp's solution —albeit sub-optimal for the mathematical problem— leads to real designs that are more efficient than the theoretic optimal solutions of the Authors. We show that the reasons for this shocking fact has to do with the class of problems considered by Hemp and the Authors.

Keywords Support cost · Michell trusses · fixed boundary problems · free loading problems

1 Introduction

Hemp (1974) proposed a parametric layout for the problem of uniform load w between two pinned support, see Fig. 1. It consists in an arch, **OAB**, vertical hangers (as **DE**) between the curve **ODB** and the line **OM**, and a family of orthogonal arches and curved hangers (as **OC** and **AD**) into the region **OABDO**. The main parameter is the slope of the arch at **O**, ϕ_1, that is determined with the condition that a displacement field exits such that the absolute deformation of all the members are equal and its sign equal to the corresponding internal force, i.e., a fully-stressed design. Hemp determined its value when the allowable tensile and compressive stresses are equal. He concluded that his solution, albeit better than the traditional solution with parabolic arch and vertical hangers, is suboptimal because it does not fulfil his own optimality criterion (Hemp, 1973), since the maximum absolute strain somewhere in the field is greater than this value in the members. Pichugin et al. (2012) generalised the Hemp's equations for any

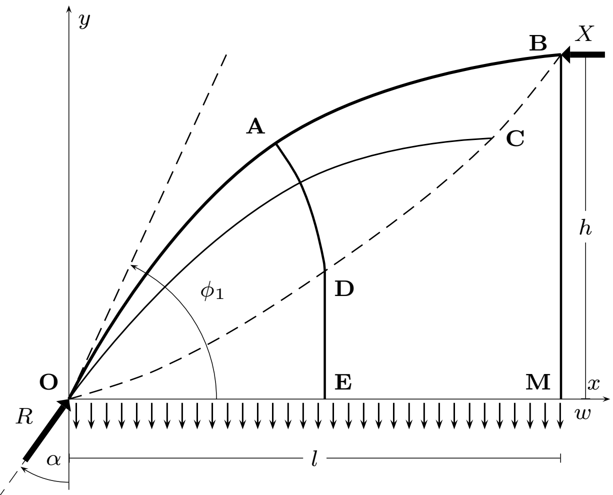

Fig. 1 Half-span of the parametric layout proposed by Hemp (1974)

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ratio of the allowable stresses, σ_T/σ_C, showing that for $\sigma_T/\sigma_C \leq 0.4177$ the resultant layouts from the Hemp's family are absolute optima.

The method of Pichugin et al. (2012) can be summarised as follows: for each value of $\sigma_T/\sigma_C \in [0,1]$, the value of ϕ_1 is the numerical solution of their Eq. (17). Then, the height h of the solution is determined with their (6) and finally the volume of material is computed with (39). Remember that these values are furthermore optimal if $\sigma_T/\sigma_C \leq 0.4177$.

The pinned supports are at **O** and its symmetric point respect to the **MB** line. For each *feasible* solution ϕ_1 we have a vertical reaction $Y = wl$ being l the half-span. And the horizontal reaction X can be determined by simple equilibrium equations of the half-solution:

$$X = \frac{wl^2}{2} \cdot \frac{1}{h(\phi_1)} = Y \cdot \frac{l}{2h(\phi_1)} \qquad (1)$$

Therefore, the angle α and magnitude R of the oblique reaction at **O** will be:

$$\alpha(\phi_1) = \arctan \frac{l}{2h(\phi_1)} \qquad R(\phi_1) = \frac{wl}{\sqrt{1+\tan^2\alpha}} \qquad (2)$$

2 The design problem

As the reaction in the pinned supports depends of ϕ_1, the optimal design problem coupled with the mathematical problem of Hemp can be stated as "to find a solution ϕ_1 with minimal volume, i.e., minimal sum of the foundation volume and the arch volume." (Let us outline that herein "arch's volume" stands for the whole pinned structure, that is the arc, the fan and hangers families and the bottom tie in the cases of hangers not being vertical.)

To this end, the shortcoming of the theory of Hemp (1973) is simply that the volume of the foundations is not accounted in anyway, so we generally can find a solution of minimal structure volume but of sub-optimal overall volume. This shortcoming was just noted in 1965 by Cox (1965:95,97) and Owen (1965:64). The former named this approach "fixed boundary"—for the use of theoretic models, as pinned support. According with these authors, the functional to be minimized in this problem from a practical perspective is:

$$V_{\text{total}} = V_{\text{abutments}}(R, \alpha) + V_{\text{arch}}(\phi_1) \qquad (3)$$

We can distinguish two special cases: (i) For problems of very small size —like bridges for children parks— the design of abutments will be restricted to minimal dimensions for practical reasons and $V_{\text{abutments}}$ will be constant. (ii) For Maxwell's problems, i.e., problems fulfilling the first condition of the theory of Michell (1904:Eq. (1)), R and α are constant and so $V_{\text{abutments}}$. This approach was named "free loading" by Cox.

In the case (i) we can count with a constant abutment so we can apply the Hemp's theory—providing that the minimal abutment will have enough bear capacity— and the optimal shape will vary with σ_T/σ_C (Cox, 1965:116). In this case the solutions of Pichugin et al. (2012) has practical interest because we will use a kind of "existing structure", as Cox named the given abutments.

In the case (ii), we also can count with a constant abutment but we cannot apply the Hemp's theory, as the condition of constant reaction is incompatible with kinematic support conditions. But fortunately we can recall the original Michell's theory, and the optimal shape will be independent of σ_T/σ_C, only depending of R and α. In this case we cannot use the equations of Pichugin et al. (2012) to determine the optimal value for ϕ_1. Notice also that the designer has to select a pair (R, α) before the Michell's theory can be useful.

For any other case we must account the two volumes and for the time being there are not sound, optimality-criterion theory for this target—albeit some advance in this direction exists, see the extension of the Prager-Rozvany theory proposed by Rozvany and Sokół (2012), that perhaps can tackle the problem in the illustrative example below. This is the case of normal bridge design, where the design of the abutments is part of the whole design problem. We can anyway refer to the design theory of Cox (1965:115), with include as useful tools both the Michell's theory and the Hemp's one —albeit being different.

3 An illustrative example

As the distinction between "free loading" and "fixed boundary" established by Cox (1965) is nowadays under discussion (see Vázquez Espí and Cervera Bravo, 2011; Rozvany, 2011; Sokół and Lewiński, 2011) we will show the key meanings of our statements in §2 by means of a concrete example. Let us stress that we only want to show that —for the *design problem*, $\min V_{\text{total}}$— the "optimal" solution since Pichugin et al. (2012) is feasible but worse that other feasible solutions, such as the Hemp's one. We only use in the sequel solutions belong to the Hemp's parametric layout, but we do not know what the optimal solution is and it could have a very different layout.

Let be σ_T/σ_C equal to 0.2. If ϕ_1 is the optimal value for the given σ_T/σ_C we use the Eq. (39) from Pichugin et al. to calculate $V_{\text{arch}}(\phi_1)$.

For any case else, we use the two well-known equations of Michell (1904) (see also Hemp, 1958; Cox, 1965;

Table 1 Feasible solutions for the design problem with $\sigma_T/\sigma_C = 0.2$ and prismatic foundations orthogonal to reactions (soil cohesion equal $0.002\sigma_C$, internal friction angle of 28°, $w/(\sigma_C l) = 0.0001$).

Solution	ϕ_1 (°)	h/l	$V_{\text{arch}}/(wl^2/\sigma_C)$	X/wl	R/wl	α (°)	$V_{\text{abutments}}/(wl^2/\sigma_C)$	$V_{\text{total}}/(wl^2/\sigma_C)$
PTG	38.872	0.36648	5.5887	1.3643	1.9650	53.760	16.302	21.891
Hemp	63.126	0.67687	6.5117	0.7387	1.5955	36.452	5.7580	12.270

Owen, 1965; Parkes, 1965):

$$\mathcal{Q}_T - \mathcal{Q}_C = C \qquad \frac{\mathcal{Q}_T}{\sigma_T} + \frac{\mathcal{Q}_C}{\sigma_C} = V(\sigma_T, \sigma_C) \tag{4}$$

being $\mathcal{Q}$ the functional Michell (1904:Eq (3)) used (the "quantity" of the truss) that is defined as the sum of the absolute internal force of each member times its length, and C the negative of the virtual work of the external forces in equilibrium when the space undergoes a contraction that reduces it to a point (Maxwell, 1870). In our case $C = -2Xl$, and we can compute the volume for any ratio σ_T/σ_C knowing $\mathcal{Q}_T$ and $\mathcal{Q}_C$ with the last equation. These equations give:

$$\mathcal{Q}_T = \frac{\sigma_C \sigma_T V(\sigma_T, \sigma_C) + \sigma_T C}{\sigma_T + \sigma_C}$$
$$\mathcal{Q}_C = \frac{\sigma_C \sigma_T V(\sigma_T, \sigma_C) - \sigma_C C}{\sigma_T + \sigma_C} \tag{5}$$

and we can compute $\mathcal{Q}_T$ and $\mathcal{Q}_C$ knowing the volume for a given ratio σ_T/σ_C and the value of C.

Therefore we will proceed as follow: for each ϕ_1 and with the ratio $(\sigma_T/\sigma_C)_{\text{opt}}$ for which ϕ_1 is optimal for the original problem of Pichugin et al. (2012), we compute V_{opt} with their Eq. (39), X, R and α with our Eqs. (1) and (2), and $\mathcal{Q}_T$ and $\mathcal{Q}_C$ with Eq. (5). Finally, with the ratio $\sigma_T/\sigma_C = 0.2$ we compute the corresponding volume V_{arch} with the second of (4).

To calculate V_{total} with (3) we need a model for the foundation. We use a standard, simple model with the same material of the structure, and with a standard cohesive soil. The abutments will have a square contact surface $A = a \times a$ orthogonal to the direction of R, i.e., this surface has a slope $\tan\alpha$ with the horizontal plane. We consider a prismatic body of base A and depth d —notice that this is not an optimal shape for the foundation. Let us outline that we cannot use others kind of foundations (like a horizontal foundation, $\alpha = 0$, with horizontal friction force, see Vázquez Espí 2012) because the solutions of Pichugin et al. (2012) would be unfeasible for normal soil conditions (e.g., normal friction coefficient between foundation and soil).

We consider a standard soil with an effective cohesion of $0.002\sigma_C$ and an effective stress angle of internal friction of 28°. The allowable stress of the soil under foundation is (Caltrans, 2003: Art. 4.4.7.1.1.8):

$$\sigma_S \approx \sigma_C(0.0212\alpha^2 - 0.0798\alpha + 0.0693) = \sigma_C F_S(\alpha) \tag{6}$$

being α expressed in radians. We have accounted only the cohesion term to keep the model simple. The term due to specific weight of soil depends of the value of R and to add it leads to the same conclusion that we will get later, but increasing the difference of volumes between the solutions we will analyse. The term due to overburden pressure is zero as we consider superficial foundations.

The side of the square base is $a = \sqrt{R/\sigma_S}$, and the depth must be $d \geq \lambda a$, being λ a constant for each shape dependant of the ratios σ_S/σ_C —given by (6) for each $\alpha(\phi_1)$— and σ_T/σ_C:

$$\lambda = \sqrt{\varPhi \frac{\sigma_S/\sigma_C}{\sigma_T/\sigma_C}} \quad \text{with} \quad \sigma_T/\sigma_C \leq 1 \tag{7}$$

being $\varPhi$ a shape factor, equal to 3 for a prismatic foundation. The volume of the two foundations is:

$$V_{\text{abutments}}(R, \alpha) = 2da^2 = 2\lambda(\alpha)\left(\frac{R}{\sigma_S}\right)^{\frac{3}{2}} \tag{8}$$

Let us expand this equation to show clearly the dependence on the main parameters using (6) and (7):

$$V_{\text{abutments}}(R, \alpha) = 2R^{\frac{3}{2}} \frac{\sqrt{\varPhi}}{\sigma_C F_S(\alpha)\sqrt{\sigma_T}} \tag{9}$$

Notice that although the prismatic body is a sub-optimal shape, to use an optimal shape will change only $\varPhi$ and being this factor independent of R or α, this change would decrease the cost of foundations by the same factor, $\sqrt{\varPhi}$, for all feasible solutions. Hence the use of the prismatic body has no influence in our argument and keeps the example simple.

Let us express $V_{\text{abutments}}$ in terms of wl^2/σ_C for a prismatic foundation ($\varPhi = 3$):

$$V_{\text{abutments}} = \frac{wl^2}{\sigma_C} \cdot \left\{ \frac{2\sqrt{3}}{F_S(\alpha)} \left(\frac{R}{wl}\right)^{\frac{3}{2}} \sqrt{\frac{w}{\sigma_T l}} \right\} \tag{10}$$

As the volume of foundations depends on the ratio $w/(\sigma_T l) = w/(\sigma_C l)/(\sigma_T/\sigma_C)$ we must fix the ratio $w/(\sigma_C l)$ to compare different solutions. For example, with $w = 100$kN/m, $l = 100$m and $\sigma_C = 8800$kPa this ratio is about 0.0001. With this value we get finally:

$$V_{\text{abutments}}(R, \alpha, \sigma_T/\sigma_C) = \frac{wl^2}{\sigma_C} \cdot \left\{ \frac{\sqrt{3}}{50 F_S(\alpha)} \left(\frac{R}{wl}\right)^{\frac{3}{2}} \frac{1}{\sqrt{\sigma_T/\sigma_C}} \right\} \tag{11}$$

According with Pichugin et al. (2012) we have the “optimal” solution for the structure volume, labelled “PTG” in table 1, obtained with their method in respect to the volume of the arch, and completed with the volume of the foundations calculated with (11). The latter is almost three times the former, i.e., the cost of the foundations cannot be ignored.

We compute also the design for $\sigma_T/\sigma_C = 0.2$ that results using the Hemp's original layout, named “Hemp” in table 1. As expected, the volume of the arch is greater than the one of PTG solution, but being the reaction R and the angle α lesser the foundation volume results lesser too. As a result, the total volume of the “Hemp” design is 56.1% of the PTG one.

Let us outline that the Hemp solution will be better than the PTG one if $V^{\text{PTG}}_{\text{abutments}} \geq V^{\text{Hemp}}_{\text{arch}} - V^{\text{PTG}}_{\text{arch}}$, because $V_{\text{abutments}}$ will be greater for PTG layout than for Hemp one with any sound foundation model and selected parameters since the difference on reactions, i.e, when $V^{\text{PTG}}_{\text{abutments}}$ is greater than 16.5% of $V^{\text{PTG}}_{\text{arch}}$ in the case $\sigma_T/\sigma_C = 0.2$.

4 Discussion

The structural design theory has to account the whole cost of materials that are necessary for a given target, frequently defined by given useful load, a structure size and a set of surfaces where the structure can lay, see Hemp (1958:1–2). Of course, this is more important for an *optimal structural design theory*. This rule has its origin in thermodynamical accounting rules (Clausius, 1885) and was strongly suggested by Maxwell (1870). Following him, Michell (1904) shown a method fully respectful with this rule that can help to find an optimal structure when a set of external forces in equilibrium is given. Although the work of Michell was ignored during decades, Cox (1965) recalled it and was able to formulate a sound structural design theory with some extensions, in particular the distinction between the “free loading” and “fixed boundary” approaches, pointing out the context where each of them can be useful.

After Cox's work, some others methods for solving mathematical optimization problems derived from structural analysis problems was established—see the classic books of Hemp (1973) or Rozvany (1976). Albeit very interesting from a mathematical perspective, many of them cannot tackle easily the problem of considering the whole cost of the resultant structure from the optimal mathematical layout. As our example shows, these mathematical solutions can be useless from a practical perspective.

It should be noted that although our example is simple for the sake of brevity, more complex model of the soil strength or other model of foundations will lead to similar conclusions, as the shortcoming of using the Hemp's approach to design normal bridges lies in that the whole cost is not considered, and has nothing to do with the cost model adopted.

Acknowledgements We thank Jose María Rodríguez Ortiz (Madrid) for his help in geotechnical matters.

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Structural and Multidisciplinary Optimization - Decision on Manuscript ID SMO-12-0182

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Para: Mariano.Vazquez.Espi@upm.es

26 de noviembre de 2012 06:21

26-Nov-2012

Dear Dr. Espí: (Mariano, this is a standard computer letter, George)

I write you in regards to manuscript # SMO-12-0182 entitled "On the practical uselessness of structural designs derived from some theoretical optimal solutions" which you submitted to the Structural and Multidisciplinary Optimization.

In view of the criticisms of the reviewer(s), your manuscript is unsuitable for publication in this journal. However, you could consider submitting it to another journal with different scope and acceptance requirements.

Thank you for considering the Structural and Multidisciplinary Optimization for the publication of your research. I hope the outcome of this specific submission will not discourage you from the submission of future manuscripts.

Sincerely,
Prof. George Rozvany
Editor in Chief, Structural and Multidisciplinary Optimization
smo.rozvany@t-online.hu

Reviewer(s)' Comments to Author:
Reviewer: 1
Comments to the Author
Please see the attached report.

Reviewer: 2
Comments to the Author

The considered text is classed as a Brief Note, but it is really a Discussion on a Paper by Pichugin et al. (1912). My main objections to this text are as follows.

(1) The main claim of the Discussion is, that Pichugin et. al. should have taken the foundation costs in their 'bridge' problem into consideration. In mathematical studies of the Michell truss literature, nobody has taken foundation costs into considerations, with the exception of a recent paper by Rozvany and Sokol (2012). Such truss topology studies are mostly of theoretical interest (although can be used as benchmarks), and it is the authors' prerogative to decide which cost factors are included in their optimization procedure. By the same token, one could insist on including the cost of connections (welds, bolts), maintenance (painting), and scaffolding during construction and also transport costs.

(2) The authors of this Discussion have already made the same point in another Discussion of a paper by Sokol and Lewinski (2010), making this repetition redundant.

(3) This reviewer had been working in bridge design for many years and is therefore thoroughly familiar with the foundation costs of bridges. The cost model used in the Discussion is not particularly representative, because the foundation costs can be rather small (e. g. on hard rock), or huge (e.g. in soft clay or mud with deep bedrock, requiring piles, caissons and/or anchoring cables). How could a rather theoretical mathematical study take care of all those possibilities? Moreover, the considered truss structure is not necessarily a bridge.

(4) It was pointed out by Hemp (1974) himself that his solution is not a global optimum, and shown by Chan (1975) that it becomes optimal for non-uniform loading. In a very elegant mathematical study, Pichugin et al. (1912) have shown that Hemp's layout is also optimal for unequal permissible stresses in tension and compression. In investigating the range of validity of Hemp's solution, one cannot bring an entirely different problem into the picture.

For the above reasons, this reviewer believes that the above Discussion is unsuitable for SMO.

Review Editor's Comments to Author:
Review Editor
Comments to the Author:
I am sending you some suggestions by email.

report.pdf
58K

Mariano Vazquez Espi and Jaime Cervera Bravo

"On the practical uselessness of structural designs derived from some theoretical optimal solutions"

The presented note provides a commentary on the recent paper by Pichugin et al (2012), which demonstrated that Hemp's Michel structure for a uniformly loaded arch between two pinned supports is optimal for sufficiently small ratio of tensile to compressive yield stresses. The original paper by Hemp only looked at the case of equal yield stresses and concluded that Hemp's structure must be suboptimal in that case. The present authors make a point that in realistic bridge design the material cost of foundations must be taken into account and demonstrate that, for some arbitrarily chosen foundation model, the material cost of the Hemp's structure is actually lower than the material cost of the optimal structure by Pichugin et al. The authors of the note believe that this conclusion proves that "Hemp's approach" to the design of optimal structures is not particularly relevant for practical applications.

It is the strong belief of this referee that the authors of the note do not properly appreciate either the thought processes involved in setting up a mathematical model for an optimal structure or the nature of results obtained by either Hemp or Pichugin et al. Let me attempt to argue these points.

1) When a mathematical model for an optimal structure is formulated, a number of simplifying assumptions are made to make the associated mathematical problem solvable. In the case of Michell structures a number of strong assumptions are made from the very start: for instance, one has to accept the somewhat unrealistic concept of structural continuum, one has to accept that the stability of compression members is not necessarily ensured, one has to accept that optimization is only ensured for a specific single load case. Assuming that these limitations are accepted, if one is interested to learn what would be the Michell structure for a particular problem, one has to choose a combination of static and kinematic conditions that best describe the problem of interest. The choice of these conditions constitutes an absolutely critical step in the problem formulation. If the supports available in the engineering problem of interest do not provide significant horizontal reaction forces, one has to either attempt seeking the optimal structure for e.g. roller supports or to factor the foundation costs into the cost of structure, just as suggested by the authors.
2) Both papers mentioned by the present authors assume pinned boundary conditions at supports, i.e. assume, essentially, "free" horizontal reaction forces at the supports. Clearly, the resulting structures do not represent efficient solutions in situations when such reaction forces are unavailable. Nether they should. At the same time, the authors of the note appear to make an implicit assumption that such situation is unrealistic. This referee is prepared to accept that such situations are less common, yet, one can easily come up with situations, not necessarily in bridge design, where Hemp's (or Pichugin et al.'s) structures would be more relevant. For example, in the case when the material of the structure is much softer than the material of the base, the stiffness contrast can easily make the cost of the necessary horizontal reaction force negligible. The point is, both Hemp and Pichugin et al. solved different optimisation problem from the one that seems to preoccupy the authors of the present note. The solutions obtained by Hemp and Pichugin et al. are, quite obviously, not relevant to the problem discussed by the authors of the note. Any conclusions that authors appear to draw from their comparisons are simply superficial.
3) The last point is best illustrated by the following simple illustration. The choice of angle ϕ_1 in Hemp's paper is made, effectively, to ensure vanishing of horizontal displacements at the support (Hemp phrases it differently, but his results can easily be reformulated appropriately). Since the

authors of the present note chose foundation model that is very sensitive to the horizontal reaction forces at the supports, they have absolutely no reason to pick the value of ϕ_1 chosen by Hemp. In fact, by choosing larger values of ϕ_1 they will find new Hemp-type structures whose resulting total volume (considered together with the value of foundation suggested by the authors) would be even lower than the values reported in Table 1!

Overall then, it seems that the authors of the present note took optimal solutions for one class of problems and showed that their optimality is questionable when a completely different class of problems is concerned. This referee does not see much value or novelty in this message, and given also a relatively low quality of English, it does not seem possible to recommend publishing this note in the Structural and Multidisciplinary Optimization.

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Decision lwetter on your Brief Note 12-0182

2 mensajes

George Rozvany <smo.rozvany@t-online.hu> 26 de noviembre de 2012 09:24
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Dear Mariano,

The reviewers were unanimously against the publication of the above text in its present form, and since it is basically a Discussion on a paper by other authors, I have not been in a position to reverse this opinion.

Basically, you are saying, that - looking at this class of problems from a 'practical' viewpoint -a least-volume truss design is 'useless', if it does not take the foundation costs into consideration. You pointed out the same earlier in a published Discussion on a paper by Lewinski and Sokol.

The problem is, that the solutions by Michell (whom you regard very highly), are also entirely unrealistic, because e. g. they do not take buckling into consideration, and have an infinite number of truss members. Nonetheless Michell trusses constitute a popular classical field, which receives a lot of attention by leading researchers, and also useful in providing benchmarks for numerical solutions.

Your proposed design of footingss is also fairly academic, because there are many types of bridge foundations in practice, depending on the soil conditions, and the considered truss may not be a bridge at all, but part of a building, for example.

In summary, you could possibly write a paper comparing the cost of bridges with or without horizontal reactions, taking the cost of foundations into consideration, but in my opinion you should not call all studies of Michell trusses 'useless' if they do not take reaction costs into consideration.

You could alternatively submit such a paper to a more practically oriented, e. g. architectural, journal.

Your previous two Discussions are published online, and will soon appear in print.

Best wishes,

George

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Dear George.

2012/11/26 George Rozvany <smo.rozvany@t-online.hu>

> The reviewers were unanimously against the publication of the above text in its present form, and since it is basically a Discussion on a paper by other authors, I have not been in a position to reverse this opinion.

Our text is not a discussion at all. The paper of Puchugin and colleagues is a very important step forward the theretical solution of the problem stated by Hemp. We said exactly this in the abstract of the paper. Furthermore, the numerical solution of Fig.2 is of a very high importance in our opinion for searching the absolute optima for the Hemp's problem. As a consequence we have no point of discussion on the main aim of that paper.

Our Brief Note was motivated for a statement of little importance that thoses author included in the last paragraph of the paper:

> Finally, although the form of the resulting structure is
> undeniably complex, it should perhaps be noted that stipulation
> of a limiting tensile stress which is lower than the

> limiting compressive stress is potentially reasonable from a
> practical perspective. This is because materials with good
> mechanical characteristics in compression (only) are generally
> much less expensive than those which also have good
> tensile capacity, and can therefore be used to form compressive
> elements which are efficient provided adequate
> restraint against buckling is available.

The authors suggested that their solutions are "potentially reasonable" out of their theoretical realm. And this suggestion is wrong in our opinion. And it is worth of a brief note that takes into account the distance between theory and practice. We prepared a simple but feasible example. Note that a simpler foundation with friction force and horizontal contact surface is not feasible with the layout of the optimal solution of the paper of Pichugin et al. because of the great horizontal reaction that that layout requires (we outlined this fact in our paper).

> Basically, you are saying, that - looking at this class of problems from a 'practical' viewpoint -a least-volume truss design is 'useless', if it does not take the foundation costs into consideration. You pointed out the same earlier in a published Discussion on a paper by Lewinski and Sokol.
>
> The problem is, that the solutions by Michell (whom you regard very highly), are also entirely unrealistic, because e. g. they do not take buckling into consideration, and have an infinite number of truss members. Nonetheless Michell trusses constitute a popular classical field, which receives a lot of attention by leading researchers, and also useful in providing benchmarks for numerical solutions.

There is no foundamental problem with theoretic solutions if they can suggest to the designer interesting layouts and insights. In our brief note we compare two theoretic layouts and we showed that the Hemp's layout is more interesting from a practical perspective although it is not the optimal theoretic solution for the theoretical problem. With our foundation model the Hemp layout with foundation is better than the PTG one. Furthermore, the Hemp layout is feasible with simple foundation with friction force and normal soil conditions whilst the PTG one is not. A elementary calculation shows that the Hemp solution is better for any foundation model when the foundation cost of the PTG layout would be greater than the 17% of the structure cost of this layout in our example, because for any foundation model the reactions required for the Hemp layout will be more favorable than the one of the PTG layout, and as a consequence the savings in the Hemp foundation will be compensate the increase of the structure cost respect to the optima PTG layout. In summary: from a practical perspective the Hemp layout continues being of most interest for practical designers. This means that the geometric properties of this layout (angle of the arch in the support, slenderness, etc) will be better guide for real designs in the real world.

> Your proposed design of footingss is also fairly academic, because there are many types of bridge foundations in practice, depending on the soil conditions, and the considered truss may not be a bridge at all, but part of a building, for example.

I don't understand this comment: we only speak of bridges in our note: we only try to call attention of a problem that sometimes arises. Any way, if the building you refer is to be designed also, the problem it is the same as with the foundation: this building must bear different reactions with different layouts and this could make a small or great difference of the overal cost. Only when the designer account this overall cost, he or she can take a reasonable decision about the selected desing... So the designer will make well considering several layouts, not only to the theoretic optimum.

> In summary, you could possibly write a paper comparing the cost of bridges with or without horizontal reactions, taking the cost of foundations into consideration, but in my opinion you should not call all studies of Michell trusses 'useless' if they do not take reaction costs into consideration.

As the title of the paper say, we call "useless" to "structural designs derived from some theretical optimal solutions". We do not call "useless" to any study neither the optimal solutions they get. So, once again, I do not understand your comment. I have the sensation that you (and the reviewers) have some prejudices about our aims, and they lead to you to a bad interpretation of our statements and purposes.

> You could alternatively submit such a paper to a more practically oriented, e. g. architectural, journal.
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> Best wishes,
>
> George

Best regards

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Salud
Mariano Vázquez Espí