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Technical Briefs

Cavitation in Short Bearings

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In this note we obtain the boundaries of the cavitation bubbles when the Reynolds conditions are used to treat the problem of the cavitation, in short journal bearings and in mechanical face seals. A universal solution to this problem is presented in some detail, in the sense that the solution of an ordinary nonlinear first order differential equation, that is necessary to describe the downstream boundary of the cavity, only depends on the eccentricity ratio, but not on the cavitation incipience eccentricity.

Nomenclature

a = defined as $a=\beta\psi(\beta_0)$
C = radial clearance in journal bearings
e = eccentricity
h_a = mean film thickness in face seals
L = bearing length
p = pressure
p_v = cavitation pressure
R = bearing radius
R_e = exterior face seal radius
R_i = interior face seal radius
r = radial coordinate in face seals
W = defined as $3\mu\omega L^2/\{p(L) - p(0)\}C^2$, in journal bearings, and $3\mu\omega(R_e - R_i)^2/\{p(R_i) - p(R_e)\}h_a^2$, in face seals
z = axial coordinate in journal bearings
β = eccentricity ratio defined as e/C in journal bearings, and $\gamma R_e/h_a$ in face seals
β_0 = cavitation incipience eccentricity
ζ = defined as $(\xi_i - \xi_{b2})(\psi\phi_v/W)^{-1/2}$ and $(\xi_{b1} - \xi_i)\{(1 + \phi_v)\psi/W\}^{-1/2}$
θ = angular coordinate in journal bearings and face seals
θ_b = value of θ for the upstream boundaries of the cavity
θ_i = value of θ where the bubble is initiated upstream, equation (3)
θ_M = value of θ where the width of the cavity is maximum
θ_r = value of θ for the downstream boundaries of the cavity
μ = viscosity
ξ = defined as z/L in journal bearings and $(R_e - r)/(R_e - R_i)$ in face seals
ξ_b = value of ξ for the boundaries of the cavity
ξ_{b1} = value of $\xi_b > \xi_i$
ξ_{b2} = value of $\xi_b < \xi_i$
ξ_i = value of ξ where the cavity is initiated upstream (given by equation (5) with $\theta_b = \theta_i$)
ϕ = nondimensional pressure; $\{p - p(0)\}/\{p(L) - p(0)\}$ for journal bearings and $\{p - p(R_e)\}/\{p(R_i) - p(R_e)\}$ for face seals
ϕ_v = defined as $\{p(0) - p_v\}/\{p(L) - p(0)\}$ for journal bearings and $\{p(R_e) - p_v\}/\{p(R_i) - p(R_e)\}$ for face seals
ψ = defined by equation (4)
ω = angular velocity of journal bearing and face seal
γ = angle of tilt of the face seal

1 Introduction

The treatment of the cavitation problem for short journal bearings and mechanical face seals is given in references [1-3]. The solution is obtained upon the hypothesis that the ratios L/R, in journal bearings, and $(R_e - R_i)/R_e$, in face seals, are small.

The geometrical pattern of the cavity boundary at the upstream side of the cavity is given by algebraic relations but, downstream, the boundaries are given by the solution of an ordinary nonlinear first order differential equation. In reference [3] it was shown that the geometrical pattern of the cavity boundaries depends only on the cavitation incipience eccentricity and on the operating eccentricity.

In this note we show that the solution of the nonlinear differential equation depends only on the eccentricity, but not on the cavitation incipience eccentricity that, of course, enter to determine the actual cavity boundary, as shown in [3].

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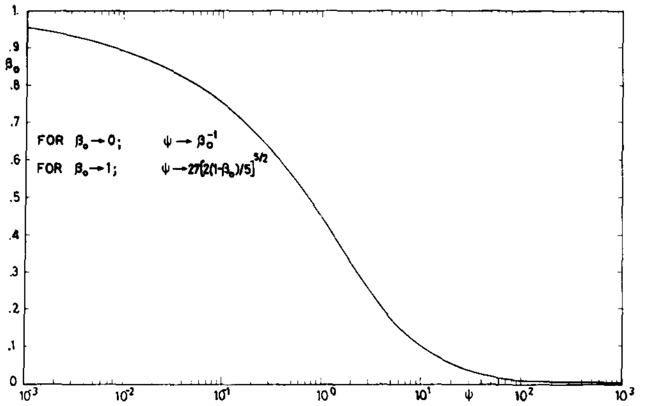

Fig. 1 Cavitation incipience eccentricity, β_0, as function of ψ

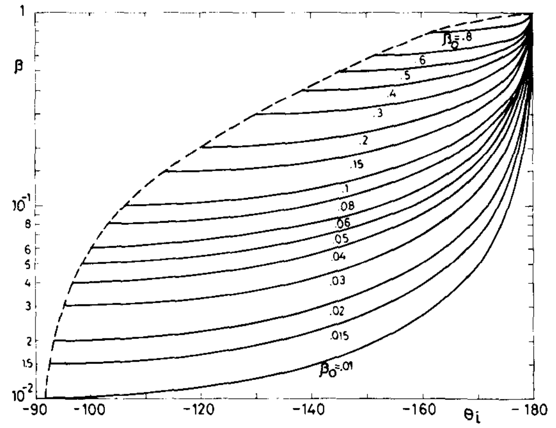

Fig. 2 Angular coordinate where the cavity is initiated upstream, θ_i, as function of β and β_0

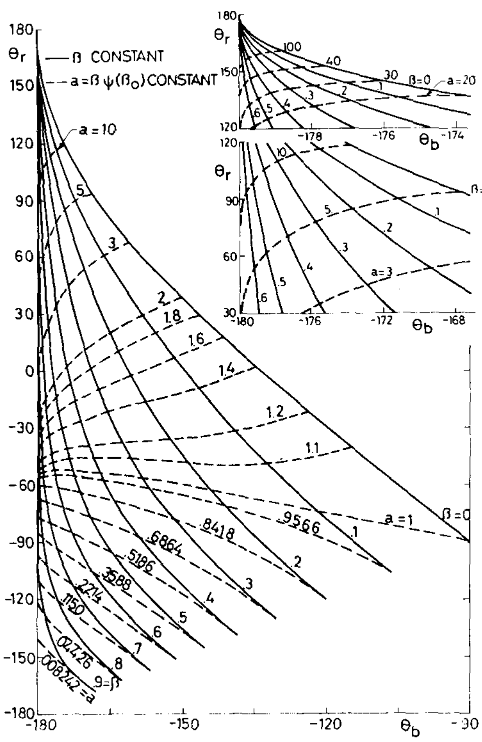

Fig. 3 Angular coordinate of the downstream boundaries of the cavity, θ_r, as function of the angular coordinate of the upstream boundaries, θ_b, and the eccentricity β

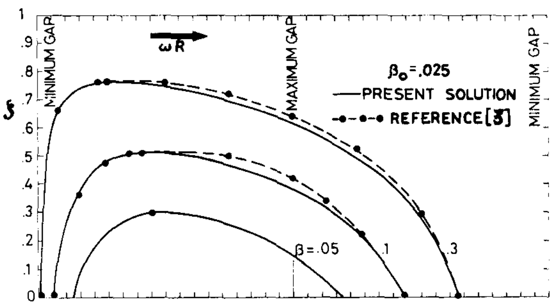

Fig. 4(a)

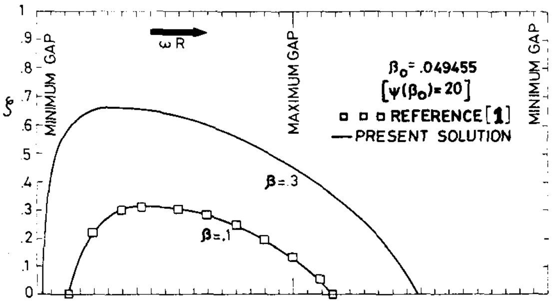

Fig. 4(b)

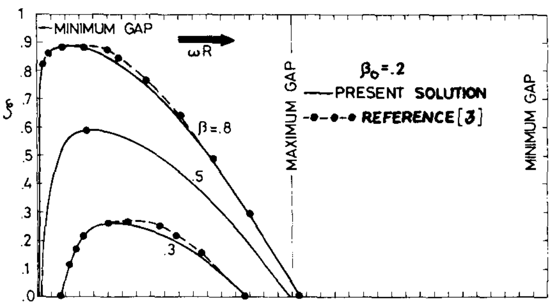

Fig. 4(c)

Fig. 4(a,b,c) Dimensionless universal cavity shapes for different values of β and β_0

2 Analysis

The detailed analysis of the cavitation problem can be seen in references [1, 2]. The nonlinear differential equation that must be solved to obtain the downstream boundaries of the cavity is,

$$\frac{d\theta_r}{d\theta_b} = \frac{(1+\beta\cos\theta_b)^2}{\sin\theta_b} \cdot \frac{(\cos\theta_b - \cos\theta_r)(3\beta + \cos\theta_b - 2\beta\cos^2\theta_b)}{\sin\theta_r(1+\beta\cos\theta_b)^3 - \sin\theta_b(1+\beta\cos\theta_r)^3}, \quad (1)$$

that must be integrated with the initial condition,

$$\theta_r = \theta_b = \theta_M(\beta), \quad (2)$$

Using the dimensionless variables defined in the Nomenclature, equation (1) coincides, after some algebraic manipulations, with equation (15) of reference [1] and with equation (24) of reference [2].

In the initial condition (2), $\theta_M(\beta)$ is the value of θ for which the width of the cavity is maximum: $\theta_M(\beta) = \cos^{-1}\{(1 - \sqrt{1+24\beta^2})/4\beta\}$. Note that the initial point of integration is a saddle, and must be left following the direction $\theta_r = \theta_M - 2(\theta_b - \theta_M)$.

The final point of integration of equation (1) is given by $\theta_b = \theta_i\ (\beta, \psi)$. Here θ_i is the value of θ where the cavity is initiated upstream and is given by,

$$(1+\beta\cos\theta_i)^3+\beta\psi\sin\theta_i=0, \quad (3)$$

where

$$\psi=W/\{1+2\phi_v\pm2[\phi_v(1+\phi_v)]^{1/2}\}, \quad (4)$$

(the minus sign must be used only when $\phi_v < 0$).

For each β and ψ values, equation (3) provides two solutions for θ_i. The appropriate solution is the one that gives the origin of the cavity in the divering part of the gap, closer to the minimum gap.

The cavitation incipience eccentricity, β_0, is the value of β for which the cavity reduces to a point and is given by the condition: θ_i $(\beta_0,\ \psi)$ = $\theta_M(\beta_0)$. In Fig. 1 β_0 is given as a function of ψ, and in Fig. 2 θ_i is given as a function of β and β_0.

The advantage of our formulation compared with those of the references [1, 2], lies in the fact that only one parameter, the eccentricity β, appears in the differential equation and in the initial condition; the second parameter, the cavitation incipience eccentricity β_0, determines the end point of integration. Thus a one-parameter family of curves, shown in Fig. 3, is necessary to describe the downstream boundary of the cavity. The dashed lines in Fig. 3 characterize the end point of integration in terms of the parameter $a = \beta\psi(\beta_0)$. The intersection of the lines a = constant with β = constant provides the value $\theta_b = \theta_i$, and therefore $\theta_r = \theta_r(\theta_i)$, associated with given β and β_0; although the value of θ_i can be calculated directly using equation (3) or Fig. 2.

Once we have θ_r as function of θ_b, from equation (1), we can obtain the cavity boundary in the following way:

(i) First we have algebraic relations for the cavity boundaries at the upstream side [1-3] that can be written in the universal form [3],

$$\zeta=1-\frac{(1+\beta\cos\theta_b)^{3/2}}{\{-\beta\psi(\beta_0)\sin\theta_b\}^{1/2}}, \quad (5)$$

depending only on the parameters β and β_0.

(ii) Second, since inside the cavity the circumferential Couette flux remains constant along the lines ζ = constant, one of the conditions used to obtain equation (1), the value of ζ at the upstream boundaries of the cavity, given by equation (5), remains the same for the downstream boundaries, but the value of θ is now $\theta_r(\theta_b)$, given by Fig. 3, instead of θ_b. See notation to translate ζ to the physical variable z, for journal bearings, or r, for face seals.

3 Results

The boundaries of the cavity, according to equations (1) and (5), are given in Figs. 4(*a*, *b*, and *c*) for several values of β and β_0. In these figures the range of the angular scale corresponds to the extent of the film between two consecutive minimum gaps, when there are more than one circumferential wave as is the case in reference [1], and it covers the entire developed film in our case. The values obtained in reference [3] have also been plotted in Figs. 4(*a* and *c*). As a consequence of the approximation used in [3] to solve equation (1), there are slight differences in the downstream boundaries between our results and those of reference [3]; differences in the order of five percent.

The cavity boundaries calculated in reference [1] are reproduced by our calculations as the curve β = 0.1 in Fig. 4(*b*). Note that in the $\zeta-\theta$ variables the two branches of the cavity boundaries collapse in one.

References

1 Findlay, J. A., "Cavitation in Mechanical Face Seals," ASME Journal of Lubrication Technology, Vol. 90, 1968, pp. 356-364.

2 Pan, C. H. T., "An Improved Short Bearing Analysis for the Submerged Operation of Plain Journal Bearings and Squeeze-Film Dampers," ASME Journal of Lubrication Technology, Vol. 102, 1980, pp. 320-332.

3 Pan, C. H. T., and Ibrahim, R. A., "Cavitation in Short Bearing with Pressurized Lubricant Supply," ASME Journal of Lubrication Technology, Vol. 103, 1981, pp. 337-349.

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