Fault Detection and Isolation on a Noisy Nonlinear Circuit

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Abstract:
In this paper fault detection and isolation (FDI) schemes are applied in the context of the surveillance of emerging faults in an electrical circuit. The FDI problem is studied on a noisy nonlinear circuit, where both abrupt and incipient faults in the voltage source are considered. A rigorous analysis of fault detectability precedes the application of the fault detection (FD) scheme; then, the fault isolation (FI) phase is accomplished with two alternative FI approaches, proposed as new extensions of that FD approach. Numerical simulations illustrate the applicability of the mentioned schemes.

Keywords: Fault detection, fault isolation, analytical redundancy, continuous-time statistics, hypothesis testing.

1. INTRODUCTION

Fault diagnosis techniques in system design and control have received much attention in the last two decades Basseville and Nikiforov (1993); Iserman (2006). System modeling based paradigms implement analytical redundancy as the fundamental framework for Fault Detection and Isolation (FDI); for doing so, they employ both linear Gertler (1998) as well as nonlinear deterministic models Polycarpou and Vemuri (1995). Recently, the Fault Detection (FD) problem in stochastic continuous-time dynamical systems was analytically characterized in Castillo and Zufiria (2009). There, the proposed FD approaches were based on the application of hypothesis testing schemes on continuous-time estimators. In addition, those FD schemes were widely analyzed in the framework of their characteristics, such as *fault detectability*, *false alarms* and *missed detection*. In particular, a collection of *sufficient detectability conditions* were given for a class of faults, characterizing those faults which can be detected with certain formalized guarantee by the given FD schemes, and providing also an upper bound for the detection time in a probabilistic sense.

Circuit theory emerges as an important field where fault detection schemes can be naturally applied Caro et al. (2001). In fact, circuit reliability and fault tolerant design have been gaining much interest due to large scale integration requirements Dubrova (2008). Precisely, component wearout Li et al. (2008), electromigration phenomenae Pierce and Brusius (1997) and radiation effects Schwank et al. (2008) have become key reliability issues in circuit design. So far, these studies have only addressed fault predictability in the design phase, but no on-line fault detection and isolation schemes have been considered yet.

In this paper a FD procedure given in Castillo and Zufiria (2009) and two FI schemes, rooted in the former, are applied to the treatment of a faulty voltage source in a noisy nonlinear circuit. Section 2 formally states the FDI problem in such context. An initial analysis on the fault fulfilling the detectability conditions helps to select the appropriate FD scheme (Section 4). Besides that, two alternative FI approaches are proposed in Section 5: the first approach employs, in parallel, different hypothesis tests on a statistic of the residual signal, one test for each possible fault; the second scheme is based on the application of a discrimination rule based on Bayes decision theory.

2. PROBLEM STATEMENT

Let us consider the RLC circuit with a noisy nonlinear resistor given in Fig. 1; it can be modeled by a bidimensional nonlinear dynamical system.

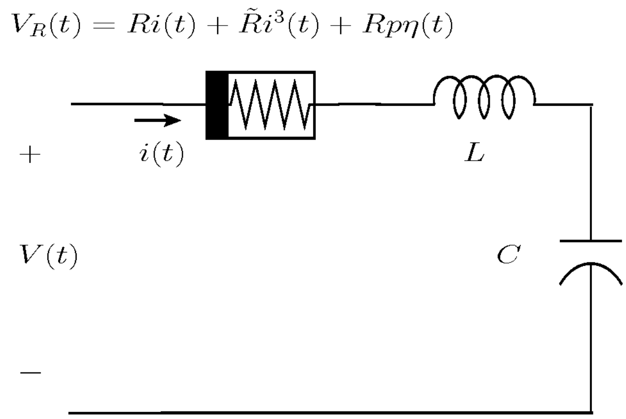

Fig. 1. RLC circuit with a noisy nonlinear resistor

Thermal noise is referred here, which is due to the random thermal motion of electrons and is present in all passive resistive elements. It is customary to represent the noisy resistor by an ideal resistor in parallel with a current noise source. The noise source generates a stream of electrons, but they are assumed to be generated at a rate so high that the noise current appears continuous. In that case, and assuming that the resistor is held at constant temperature, the actual noise current can be modelled by a constant p multiplying a white Gaussian noise process (WGN):

$$i_n(t) \triangleq p\eta(t).$$

More information about thermal noise can be found in Franco (2002); and about modeling the noise current by WGN the reader is referred to the book Larson and Shubert (1979).

In order to obtain the circuit model, the relationship between current and voltage in the resistor must be first considered:

$$V_R(t) = Ri(t) + \tilde{R}i^3(t) + Rp\eta(t), \quad \eta(t) \triangleq WGN$$

where the term $p\eta(t)$ represents the effect of thermal noise, briefly explained earlier.

Regarding the inductor and the capacitor, the well-known relations are:

$$V_L(t) = L\frac{di(t)}{dt},$$
$$V_C(t) = \frac{1}{C}\int^t i(\tau)d\tau,$$

so that, based on the Kirchhoff's voltage law

$$V(t) = V_L(t) + V_R(t) + V_C(t),$$

and assuming the current can be measured, the circuit model is given by the system

$$\begin{aligned}\frac{dV_C(t)}{dt} &= \frac{1}{C}i(t),\\ \frac{di(t)}{dt} &= -\frac{1}{L}V_C(t) - \frac{R}{L}i(t) - \frac{\tilde{R}}{L}i^3(t) - \frac{R}{L}p\eta(t) + \frac{1}{L}V(t),\\ y(t) &= i(t).\end{aligned}$$

Under healthy conditions the voltage source is generating a constant voltage

$$V(t) = V_0, \quad 0 \le t < T_0,$$

where T_0 is the time instant when a fault occurs in the circuit.

So, the model can be rewritten as

$$\begin{aligned}\frac{dV_C(t)}{dt} =& \frac{1}{C}i(t)\\ \frac{di(t)}{dt} =& -\frac{1}{L}V_C(t) - \frac{R}{L}i(t) - \frac{\tilde{R}}{L}i^3(t) - \frac{R}{L}p\eta(t)\\ &+ \frac{1}{L}V_0 + \beta(t-T_0)\phi(t)\\ y(t) =& i(t), \quad 0 \le t < T_0.\end{aligned}$$

where $\phi(t)$ represents the fault function and $\beta(t-T_0)$ its time profile.

Concerning the possible faults, it is also assumed that the circuit suffers a fault whose consequences are entirely reflected as a change in the voltage function $V(t)$, more specifically it can only suffer one of the three different types of faults explained below:

1. **The source suddenly short-circuits**. The voltage function is then

$$V(t) = \begin{cases} V_0, & t < T_0,\\ 0, & t \ge T_0.\end{cases}$$

In this case, the system model after the fault occurs is given by

$$\begin{aligned}\frac{dV_C(t)}{dt} &= \frac{1}{C}i(t),\\ \frac{di(t)}{dt} &= -\frac{1}{L}V_C(t) - \frac{R}{L}i(t) - \frac{\tilde{R}}{L}i^3(t) - \frac{R}{L}p\eta(t) + 0,\\ y(t) &= i(t), \quad t \ge T_0.\end{aligned}$$

Hence, the fault and its time profile function result in [1]

$$\phi_1(t) = -\frac{V_0}{L},$$
$$\beta_1(t-T_0) = \begin{cases}0, & t < T_0,\\ 1, & t \ge T_0,\end{cases}$$

being this fault an abrupt fault.

2. **The source voltage decreases slowly from V_0 to** 0, that is the voltage function takes the form:

$$V(t) = \begin{cases} V_0, & t < T_0,\\ V_0 e^{-\alpha(t-T_0)}, & t \ge T_0,\end{cases} \quad (1)$$

with $\alpha > 0$.

When the fault takes place the system becomes:

$$\begin{aligned}\frac{dV_C(t)}{dt} =& \frac{1}{C}i(t),\\ \frac{di(t)}{dt} =& -\frac{1}{L}V_C(t) - \frac{R}{L}i(t) - \frac{\tilde{R}}{L}i^3(t)\\ &- \frac{R}{L}p\eta(t) + \frac{V_0}{L}(e^{-\alpha(t-T_0)}),\\ y(t) =& i(t), \quad t \ge T_0,\end{aligned}$$

corresponding to the incipient fault:

$$\phi_2(t) = -\frac{V_0}{L}, \quad (2)$$
$$\beta_2(t-T_0) = \begin{cases}0, & t < T_0,\\ 1 - e^{-\alpha(t-T_0)}, & t \ge T_0,\end{cases}$$

where $\alpha > 0$.

3. **Voltage source faults periodically**. The voltage function becomes a piecewise constant function, as for example

$$V(t) = \begin{cases} V_0, & t < T_0,\\ 0, & T_0 < t \le T_0 + \triangle t,\\ V_0, & T_0 + \triangle t < t \le T_0 + 2\triangle t,\\ 0, & T_0 + 2\triangle t < t \le T_0 + 3\triangle t,\\ \dots \end{cases}$$

with $\triangle t > 0$.

The system with this fault is

$$\begin{aligned}\frac{dV_C(t)}{dt} &= \frac{1}{C}i(t),\\ \frac{di(t)}{dt} &= -\frac{1}{L}V_C(t) - \frac{R}{L}i(t) - \frac{\tilde{R}}{L}i^3(t) - \frac{R}{L}p\eta(t) + \frac{V(t)}{L},\\ y(t) &= i(t), \quad t \ge T_0,\end{aligned}$$

where $V(t)$ is the piecewise constant function given above. In this case, the fault and profile functions are:

$$\phi_3(t) = \frac{V(t) - V_0}{L},$$

[1] See the general model in reference Castillo and Zufiria (2009).

$$\beta_3(t-T_0) = \begin{cases} 0, & t < T_0, \\ 1, & t \geq T_0. \end{cases}$$

3. RESIDUAL GENERATION

The observer equations, required by any of the here proposed FDI schemes, are given in this problem by

$$\frac{d\hat{V}_C(t)}{dt} = \frac{1}{C}i(t) + \lambda_1\big(\hat{V}_C(t) - V_C(t)\big)$$
$$\frac{d\hat{i}(t)}{dt} = -\frac{R}{L}i(t) - \frac{\tilde{R}}{L}i^3(t) - \frac{V_C(t)}{L} + \frac{V_0}{L} + \lambda_2(\hat{i}(t) - i(t))$$
$$\hat{i}(0) = i(0),$$

with $\lambda_k < 0, \;\; k = 1, 2$. See Castillo and Zufiria (2009) for a more detailed explanation.

Hence, the **residual** components will be given by

$$\epsilon_1(t) = V_C(t) - \hat{V}_C(t),$$
$$\epsilon_2(t) = i(t) - \hat{i}(t),$$

and its evolution is determined by the system

$$\frac{d\epsilon_1(t)}{dt} = \lambda_1\epsilon_1(t),$$
$$\frac{d\epsilon_2(t)}{dt} = \lambda_2\epsilon_2(t) - \frac{R}{L}p\eta(t) + \beta_k(t-T_0)\phi_k(t),$$
$$\epsilon_1(0) = \epsilon_2(0) = 0,$$

where $t \geq 0$ and $k \in \{1, 2, 3\}$. Solving that residual equations leads to

$$\epsilon_1(t) = 0,$$
$$\epsilon_2(t) = \int_0^t e^{\lambda_2(t-\tau)}p^*\eta(\tau)d\tau + \int_0^t e^{\lambda_2(t-\tau)}\beta_k(\tau - T_0)\phi_k(\tau)d\tau,$$

where $p^* = -\frac{R}{L}p$, and $\eta^*(t) = p^*\eta(t)$ is a WGN process with zero mean and autocorrelation function

$$R_{\eta^*}(t_1, t_2) = (p^*)^2\delta(t_1 - t_2).$$

Since $\epsilon_1(t) = 0, \;\; \forall t \geq 0$ the FDI process is uniquely based on the residual component $\epsilon_2(t)$. For the sake of simplicity $\epsilon_2(t)$ is renamed as $\epsilon(t)$ in the following.

The next step is the construction of the corresponding FDI scheme, based on the analysis of the **residual** $\epsilon(t) = i(t) - \hat{i}(t)$.

4. FAULT DETECTION. DETECTABILITY CONDITIONS

The key element in the detection approach is the following hypotheses test on the residual mean

$$H_0 : E[\epsilon(t)] = 0,$$
$$H_1 : E[\epsilon(t)] \neq 0.$$

In order to construct the test, its size γ $(0 < \gamma < 1)$ must be set first. Since γ represents also the false alarm rate at each instant of time t, γ must be selected by considering the consequences of a false alarm and a missing fault in the problem under consideration. Precisely, a false alarm may imply to stop the circuit functioning and to replace the (non-faulty) voltage source, whereas a missing fault will result in a short circuit. If $\gamma \approx 1$ is taken, a fault in the source will be detected quickly, but the number of replacements of right voltage sources (false alarms) may be excessive. Otherwise, if $\gamma \approx 0$ then non-faulty sources would be rarely replaced due to false alarms but the detection of a fault could happen too late. Hence there is a compromise between the two extreme situations, depending on the preferences imposed by the real circuit operating conditions. In this work $\gamma = 0.05$ will be taken.

Concerning the choice of a mean estimator for the test [2], the simplest one is

$$\mu(t) = \epsilon(t),$$

so that its capability for detecting any of the three possible faults which can occur in the voltage source must be evaluated; equivalently one must see if those faults fulfil the sufficient detectability condition associated with that raw estimator.

Since the faults in this problem are represented by non-positive deterministic functions $\phi_k(t)$, the detectability conditions (given in Castillo and Zufiria (2009)) for the proposed estimator consist of the existence of a value T_0^* such that

$$\beta_k(t - T_0)\phi_k(t) \leq -\varepsilon \quad \forall t > T_0^* \geq T_0,$$

where

$$\varepsilon > \varepsilon_k(\mu, \gamma, \vartheta) = -\lambda \lim_{t\to\infty}\left(h_{\frac{\gamma}{2}}\sqrt{Var_{\mu/H_0}(t)} + h_{\frac{\vartheta}{2}}\sqrt{Var_{\mu/\{H_1,\phi_k\}}(t)}\right)$$

for $k \in \{1, 2, 3\}$ (corresponding to each one of the possible faults).

In order to follow the detectability analysis, a new parameter ϑ must be set, since it is required for the definition of *detectability* established in Castillo and Zufiria (2009)). For simplicity, $\vartheta = \gamma = 0.05$ is taken in this work, and so $h_{\frac{\vartheta}{2}} = h_{\frac{\gamma}{2}} = 1.96$. Then, the required variances are

$$Var_{\mu/H_0}(t) = Var_{\epsilon/H_0}(t) = R_{\epsilon/H_0}(t,t) = -\frac{\sigma^2_{\eta^*}}{2\lambda}(1 - e^{2\lambda t}) \underset{t\to\infty}{\longrightarrow} -\frac{\sigma^2_{\eta^*}}{2\lambda} = -\frac{\frac{R^2}{L^2}p^2}{2\lambda}.$$

And, since fault functions are deterministic

$$Var_{\mu/\{H_1,\phi_k\}}(t) = Var_{\mu/H_0}(t),$$

so that

$$\varepsilon_k(\mu, \gamma, \vartheta) = 1.96\frac{R^2}{L^2}p^2$$

for the three faults ($\forall k \in \{1, 2, 3\}$).

It is straightforward to check now if each one of the faults fulfills the corresponding sufficient detectability condition:

1. For the first abrupt fault with $\phi_1(t) = -\frac{V_0}{L}$, the sufficient detectability condition is

   $$V_0 > 1.96\frac{R^2}{L}p^2.$$

   If this condition is fulfilled, there is a guarantee for the first fault to be detected by the FD approach

[2] Some more proposals of mean estimators can be found in Castillo and Zufiria (2009).

determined by $\mu(t) = \epsilon(t)$. Otherwise, alternative mean estimators must be considered, such as

$$\mu(t) = \frac{1}{t}\int_0^t \epsilon(\tau)d\tau,$$

whose associated (detectability condition) value is

$$\varepsilon_k(\mu,\gamma,\vartheta) = 0 \quad \forall k \in \{1,2,3\},$$

so that the sufficient detectability condition would be fulfilled by any $V_0 > 0$.

2. The second (incipient) fault with fault and time profile functions given in (2) leads to an associated detectability condition which results in the existence of a value T_0^* such that

$$V_0\big(1 - e^{-\alpha(t-T_0)}\big) \geq \varepsilon' > 1.96\frac{R^2}{L}p^2 \qquad \forall t > T_0^* \geq T_0$$

which is equivalent to

$$V_0 > 1.96\frac{R^2}{L}p^2.$$

So, if this condition is satisfied in the circuit under study, the choice $\mu(t) = \epsilon(t)$ will be valid; alternatively, as mentioned earlier, another mean estimator should be selected.

3. The third case fault function is a piecewise constant function which takes only values V_0 and 0, periodically; hence, this fault function does not fulfil the sufficient detectability condition neither for the proposed estimator (except in some intervals), nor for the rest of mean estimators proposed in Castillo and Zufiria (2009). Hence no guarantee can be given that this fault is to be detected by the present FD approach.

In the following, the study is restricted to RL circuits such that

$$V_0 > 1.96\frac{R^2}{L}p^2. \tag{3}$$

If it is not the case, the mean estimator must be changed to

$$\mu(t) = \frac{1}{t}\int_0^t \epsilon(\tau)d\tau,$$

so that the procedure that follows also applies. (Note that no matter if (3) is satisfied, or even if other estimators are employed, the detection of the third fault will not be guaranteed.)

In addition, it is possible to calculate an approximation for the time the FD scheme takes to detect a given fault, that is $T_d - T_0$. In fact, $\hat{T}_d^\vartheta$ is an upper bound for the detection instant of time T_d with probability $1-\frac{\vartheta}{2}$, so that $\hat{T}_d^\vartheta - T_0$ is also an upper bound for $T_d - T_0$ (see Castillo and Zufiria (2009)).

The test acceptance region (see Castillo and Zufiria (2009)) is

$$(l(t),u(t)] = \Big(E[\mu(t)/H_0] - h_{\frac{\gamma}{2}}\sqrt{Var_{\mu/H_0}(t)},\; E[\mu(t)/H_0] + h_{\frac{\gamma}{2}}\sqrt{Var_{\mu/H_0}(t)}\Big],$$

and the symmetric confidence interval required to define $\hat{T}_d^\vartheta$ (see Castillo and Zufiria (2009)) is

$$(a_k^\vartheta(t), b_k^\vartheta(t)] = \Big(E[\mu(t)/\{H_1,\phi_k\}] - h_{\frac{\vartheta}{2}}\sqrt{Var_{\mu/\{H_1,\phi_k\}}(t)},\; E[\mu(t)/\{H_1,\phi_k\}] + h_{\frac{\vartheta}{2}}\sqrt{Var_{\mu/\{H_1,\phi_k\}}(t)}\Big]$$

varying with each fault ($k = 1,2,3$). In the present case the involved values are

$$E(\mu(t)/H_0) = 0$$

$$E(\mu(t)/\{H_1,\phi_k\}) = \int_{T_0}^t e^{\lambda(t-\tau)}\beta_k(\tau - T_0)\phi_k(\tau)d\tau$$

$$Var_{\mu/\{H_1,\phi_k\}}(t) = Var_{\mu/H_0}(t) \underset{t\to\infty}{\longrightarrow} -\frac{\frac{R^2}{L^2}p^2}{2\lambda}.$$

Since the fault functions satisfy $\phi_k(t) \leq 0$ then

$$\hat{T}_d^\vartheta = T_d^{b,l}(\vartheta) = \inf\{t \geq T_0 \;/\; b_k^\vartheta(t) < l(t)\}.$$

The continuity of the residual mean and variance (see Castillo and Zufiria (2009)) implies that functions $b_k^\vartheta(t)$ and $l(t)$ are also continuous, so that $\hat{T}_d^\vartheta$ fulfils

$$b_k^\vartheta(\hat{T}_d^\vartheta) = l(\hat{T}_d^\vartheta). \tag{4}$$

Making some calculations

$$l(t) \underset{t\to\infty}{\longrightarrow} -h_{\frac{\gamma}{2}}\sqrt{\frac{-\frac{R^2}{L^2}p^2}{2\lambda}}$$

$$b_k^\vartheta(t) \underset{t\to\infty}{\longrightarrow} \int_{T_0}^t e^{\lambda(t-\tau)}\beta_k(\tau - T_0)\phi_k(\tau)d\tau + h_{\frac{\vartheta}{2}}\sqrt{\frac{-\frac{R^2}{L^2}p^2}{2\lambda}},$$

and considering equality (4)

$$\int_{T_0}^{\hat{T}_d^\vartheta} e^{\lambda(\hat{T}_d^\vartheta-\tau)}\beta_k(\tau - T_0)\phi_k(\tau)d\tau \approx -(h_{\frac{\gamma}{2}} + h_{\frac{\vartheta}{2}})\sqrt{\frac{-\frac{R^2}{L^2}p^2}{2\lambda}}. \tag{5}$$

This is an equation on $\hat{T}_d^\vartheta$ and T_0, which, in many cases, can be written on the increment $\hat{T}_d^\vartheta - T_0$. Then, particularizing for each fault, it provides an approximation for the time the FD scheme takes to detect such particular fault.

For example, calculating the integral in the abrupt fault case $\phi_1(t) = -\frac{V_0}{L}$, *and substituting into (5)*

$$-\frac{V_0}{L}\frac{\big(1 - e^{\lambda(\hat{T}_d^\vartheta - T_0)}\big)}{\lambda} \approx -(h_{\frac{\gamma}{2}} + h_{\frac{\vartheta}{2}})\sqrt{\frac{-\frac{R^2}{L^2}p^2}{2\lambda}}$$

so that

$$\hat{T}_d^\vartheta - T_0 \approx \frac{1}{\lambda}\ln\left(\left(1 - \frac{L\lambda}{V_0}(h_{\frac{\gamma}{2}} + h_{\frac{\vartheta}{2}})\right)\sqrt{\frac{-\frac{R^2}{L^2}p^2}{2\lambda}}\right).$$

Taking for instance $R = L = 1$, $V_0 = 7$, $p = 1$, $\lambda = -0.5$ *and* $\gamma = \vartheta = 0.05$, *the value* $\hat{T}_d^\vartheta - T_0 \approx 0.675$ *is obtained, as an upper bound for the time the detection of this first fault takes, with probability* 0.975.

The same way such value $\hat{T}_d^\vartheta - T_0$ *can be computed for fault* ϕ_2 *and even for fault* ϕ_3, *which despite not fulfilling the sufficient detectability condition is* $\hat{T}_d^\vartheta$*-detectable.*

The theoretical analysis provides the required elements to run the simulations. In the following the simulation results corresponding to faults $\{\phi_k(t),\ k = 1,2,3\}$ at time $T_0 = 50$, in a circuit with

$$R = 1\ \Omega, \quad L = 1\ \text{H}, \quad V_0 = 7\ \text{V}, \quad p = 1,$$

are presented, for the FDI scheme with $\gamma = 0.05$ as the size of all the tests involved in the procedures.

- The first simulation corresponds to the occurrence of the abrupt fault $\phi_1(t) = -\frac{V_0}{L}$. The fault is detected quite quickly, $T_d = 50.31$, so the detection process takes 0.31; as expected, this value is under the upper bound given above ($\hat{T}_d^{\vartheta} - T_0 \approx 0.675$). The process is represented in Fig. 2.

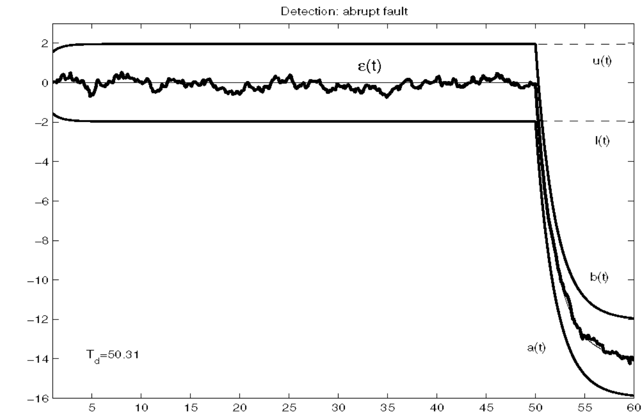

Fig. 2. Detection of the abrupt fault

- The next simulation considers the incipient fault

$$\phi_2(t) = -\frac{V_0}{L},$$
$$\beta_2(t - T_0) = \begin{cases} 0, & t < T_0, \\ 1 - e^{-\alpha(t-T_0)}, & t \geq T_0, \end{cases}$$

with $\alpha = 0.01$. The result can be seen in Fig. 3; obviously, it takes more time to be detected, since the fault effect increases gradually. For this particular realization $T_d = 66.78$.

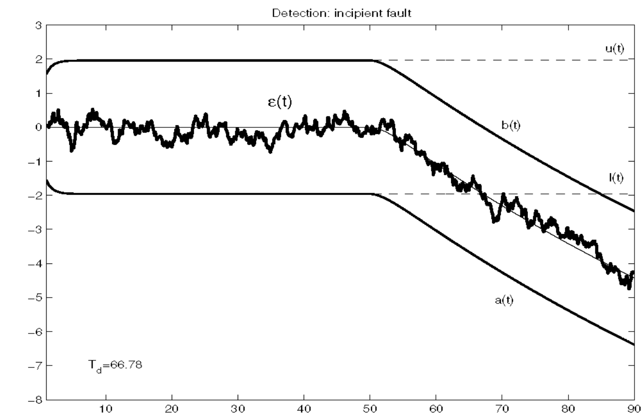

Fig. 3. Detection of the incipient fault

- Finally, the occurrence of the third fault, with piecewise constant function, is simulated. Remember that this fault does not fulfil the sufficient detectability condition. Nevertheless, in this case, since its initial behavior is similar to the abrupt fault, the FD scheme does detect the fault ($T_d = 50.31$). See Fig. 4.

5. FAULT ISOLATION

In this section, the **isolation** phase is addressed. The occurrence of the abrupt fault ϕ_1 is simulated, and two

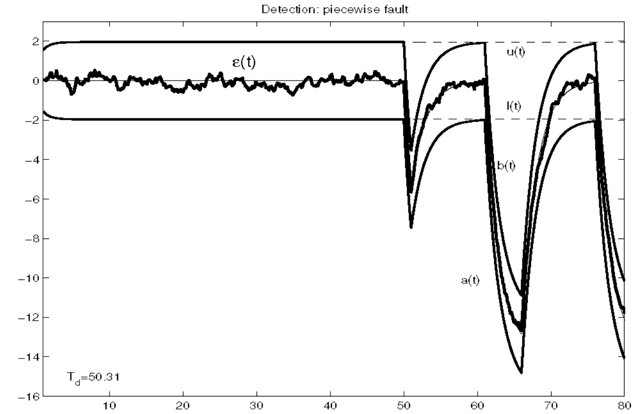

Fig. 4. Detection of the constant piecewise fault

alternative isolation schemes are applied to conclude that this abrupt fault is actually occurring in the system among the three given possibilities.

The fault can be isolated via the collection of **hypotheses tests**

$$\begin{aligned} &H_0 : E[\epsilon(t)] = E[\epsilon(t)/\phi_1] \quad H_0 : E[\epsilon(t)] = E[\epsilon(t)/\phi_2] \\ &H_1 : E[\epsilon(t)] \neq E[\epsilon(t)/\phi_1] \quad H_1 : E[\epsilon(t)] \neq E[\epsilon(t)/\phi_2] \end{aligned}$$

$$\begin{aligned} &H_0 : E[\epsilon(t)] = E[\epsilon(t)/\phi_3] \\ &H_1 : E[\epsilon(t)] \neq E[\epsilon(t)/\phi_3]. \end{aligned}$$

Taking the residual as a raw mean estimator to construct the tests, the following tests acceptance regions are obtained

$$\begin{aligned} (l_k(t), u_k(t)] = \Big(&E[\epsilon(t)/\phi_k] - h_{\frac{\gamma}{2}} \sqrt{Var_{\epsilon/\phi_k}(t)}, \\ &E[\epsilon(t)/\phi_k] + h_{\frac{\gamma}{2}} \sqrt{Var_{\epsilon/\phi_k}(t)} \Big], \end{aligned}$$

with $k = 1, 2, 3$. The region where the residual sample path remains will indicate that the fault corresponding to that region is the actual fault affecting the system.

In order to determine such regions, the following terms must be computed

$$E[\epsilon(t)/\phi_k] = \epsilon(T_d)e^{\lambda(t-T_d)} + \int_{T_d}^{t} e^{\lambda(t-\tau)} \beta_k(\tau - T_0)\phi_k(\tau)d\tau,$$
$$Var_{\epsilon/\phi_k}(t) = Var_{\epsilon/H_0}(t) = -\frac{\frac{R^2}{L^2}p^2}{2\lambda}(1 - e^{2\lambda t}) \underset{t\to\infty}{\nearrow} -\frac{\frac{R^2}{L^2}p^2}{2\lambda}.$$

Applying the proposed isolation method, based on checking the residual mean at each time t, the result represented in Fig. 5 are obtained: the residual realization remains inside the acceptance region corresponding to the abrupt fault ϕ_1 and outside the rest of regions.

As an alternative isolation method the **Bayes rule** can also be applied. For doing so the prior probabilities of occurrence of the three faults are first needed; assuming they can occur with the same probability

$$p_1 = p_2 = p_3 = \frac{1}{3},$$

it is straightforward to calculate the posterior probabilities

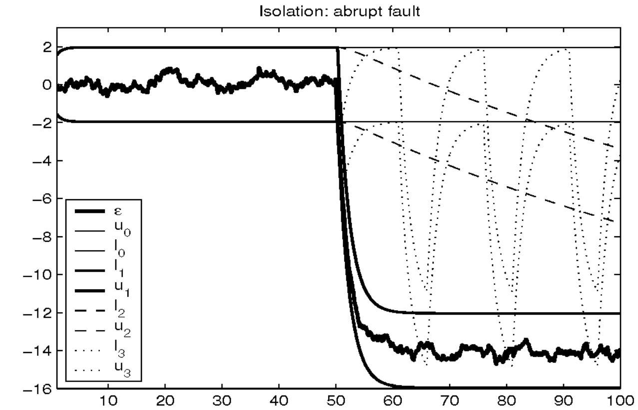

Fig. 5. Isolation via hypotheses tests

$$\pi_k(t) = f(\phi_k/\epsilon(t)) = \frac{p_k\, f(\epsilon(t)/\phi_k)}{f(\epsilon(t))}, \quad k = 1, 2, 3. \tag{6}$$

The (regularly) highest posterior probability will correspond to the fault affecting the system.

Since the fault occurring in the system is unknown, the residual density function $f(\epsilon(t))$ cannot be computed; fortunately, there is no need to obtain such density function to compare the posterior probabilities (since it acts as standardizing term). Hence, it suffices to calculate and compare the quantities

$$f(\epsilon(t))\pi_k(t) = p_k\, f(\epsilon(t)/\phi_k), \quad k = 1, 2, 3.$$

The really important term is the conditioned residual density, which, since the residual is a Gaussian process, it is given by

$$f(\epsilon(t)/\phi_k) = \frac{1}{\sqrt{2\pi\, Var_{\epsilon/\phi_k}(t)}}\, e^{-\frac{1}{2}\frac{\left(\epsilon(t) - E[\epsilon(t)/\phi_k]\right)^2}{Var_{\epsilon/\phi_k}(t)}}.$$

Fig. 6 represents the integral functions

$$\int_{T_d}^{t} p_k\, f(\epsilon(\tau)/\phi_k)d\tau,$$

where it can be observed that the function corresponding to the abrupt fault is clearly over the rest, so one can conclude that fault ϕ_1 is affecting the system, that is the voltage source has suddenly suffered a short-circuit.

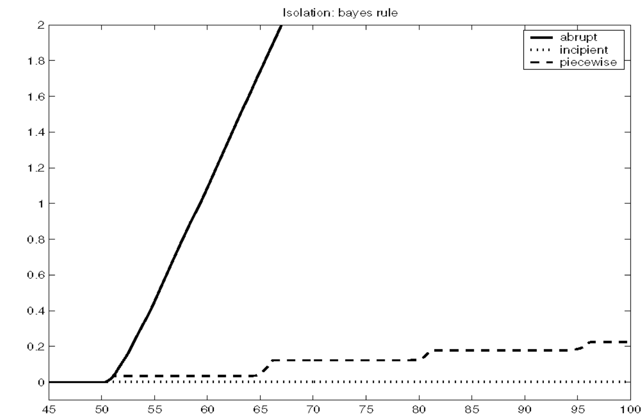

Fig. 6. Isolation via the Bayes rule

6. CONCLUSION

The FDI problem has been studied on a noisy nonlinear circuit, with abrupt and incipient faults affecting the voltage source. The study of the fulfillment of the detectability conditions (given in Castillo and Zufiria (2009)) has been important for the FD scheme selection procedure. The applied isolation approaches are based respectively on hypothesis testing and the Bayes rule. Simulations show the usefulness and effectiveness of the employed FDI schemes (see Castillo and Zufiria (2009)).

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