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Coronal fluid-dynamics in laser fusion

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The fluid-dynamics of the corona ejected by laser-fusion targets in the direct-drive approach (thermal radiation and atomic physics unimportant) is discussed. A two-fluid model involves inverse bremsstrahlung absorption, refraction, different ion and electron temperatures with energy exchange, different ion and electron velocities and magnetic field generation, and their effect on ion-electron friction and heat flux. Four dimensionless parameters determine coronal regimes for one-dimensional flows under uniform irradiation. One additional parameter is involved in two-dimensional problems, including the stability of one-dimensional flows, and the smoothing of non-uniform driving.

1. Introduction

We discuss here the fluid dynamics of the corona of fully ionized plasma ejected by a laser-irradiated target. The motivation of such a discussion is that, first, coronal flow is a special field in fluid dynamics, and, second, the variety of existing flow regimes needs to be explored prior to a full understanding of laser fusion, particularly of the compression of the imploding (part of the) target. We restrict our study to the direct-drive approach, for which thermal radiation and atomic physics play no dominant role.

The special features of the corona are elaborated in Sec. 2. The equations involved in its analysis and the dimensionless parameters characterizing the coronal regimes are considered in Secs. 3 and 4 respectively. In the following section we review the limiting regimes and time behaviours for uniform laser irradiation, leading to one-dimensional problems described by systems of ordinary differential equations; their solutions involve non-linear eigenvalue determination and provide basic universal laws. In Sec. 6 we consider weakly two-dimensional problems that include the stability of 1-D flows, and the coronal smoothing of weakly non-uniform irradiation of targets. Some effects not included in the model of Sec. 3 are discussed in Sec. 7.

2. Coronal features

In the simplest case, the plasma is characterized by an ion charge number Z_i and mass $Z_i\tilde{m}$, and the laser pulse by its peak power W_m, half-width τ_L, and frequency ω (with wavelength λ_L and critical density n_c then known in terms of universal constants m_e, e, and c). There is, in addition, a characteristic width R: if the target is a foil, R is the focal-spot radius; for a pellet, R is its radius.

The basic features are as follows:

a) There are two widely disparate time-scales, ω^{-1} and $\tau_L \gg \omega^{-1}$. The slow-scale flow, which is of dominant interest here, may be affected by the fast, oscillatory electron motion (light refraction, ponderomotive force).

b) The plasma expands into a vacuum.

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c) Energy deposition takes place at electron densities $n_e < n_c$.

d) The mass critical density, $\rho_c \simeq \bar{m} n_c$, is small compared to that of the imploding target, which will be larger than solid density ($770\rho_c$ for fully stripped aluminum and $1{\cdot}06\ \mu$m light).

Consequently, the additional features are

1) The flow speed reaches sonic values, $v^* \sim (T_e^*/\bar{m})^{\frac{1}{2}} \equiv c_s^*$, somewhere in the expanding plasma (asterisk superscripts mark unknown characteristic values, say in the critical surface). The ion inertia enters the sound speed because the slow-scale motion is quasineutral

$$\lambda_D^* \ll \min(c_s^* \tau_L, R)$$

i.e.

$$(\bar{m}/m_e)^{\frac{1}{2}} \ll \omega\tau_L \quad \text{or} \quad \lambda_L \ll Rc/c_e^*$$

where $c_e \equiv (T_e/m_e)^{\frac{1}{2}}$.

2) Since energy must be taken from critical surface to target densities, and energy convection has an outward direction, thermal conduction must play a dominant role in the overdense region.

3) Formally letting $\rho_{\text{target}}/\rho_c \to \infty$, and because of i) pressure and mass flow rate remain finite and ii) plasma heat conduction is non-linear, one gets T/T^* and $v/v^* \to 0$ at the target, exhibiting a well defined (ablation) surface lying at finite distance from the critical density. The slow recession of that surface may be neglected in the analysis of the plasma outside it—the corona—, which is thus uncoupled from the implosion process. (Results from that analysis, say the light absorption, or the pressure and flow rate at the ablation surface, might be used later to study the implosion).

4) The analysis of the corona requires a collisional two-fluid model. Indeed, if electron conduction and energy convection are to be comparable in the overdense region, its length must be

$$L_c \sim \lambda_{ei}^* (\bar{m}/m_e)^{\frac{1}{2}} \gg \lambda_{ei}^*, \tag{1}$$

λ_{ei} being the ion-electron scattering mean-free-path. From the ion energy equation one then gets

$$\frac{T_e^* - T_i^*}{T_i^*} \sim \frac{\lambda_{ei}^*}{L_c} \left(\frac{\bar{m}}{m_e}\right)^{\frac{1}{2}} \sim 1.$$

From the electron momentum equation the same result is found for $|\bar{v}_e^* - \bar{v}_i^*|/c_s^*$.

5) Ion conduction and viscosity, and electron viscosity, represent small corrections of order $(m_e/\bar{m})^{\frac{1}{2}} Z_i^{-\frac{1}{2}}$ and $Z_i m_e/\bar{m}$, respectively.

3. Model equations

The two-fluid model consists of Maxwell equations

$$\nabla \cdot \bar{E} = 4\pi e(Z_i n_i - n_e), \quad \nabla \wedge \bar{E} = -\frac{1}{c}\frac{\partial \bar{B}}{\partial t},$$

$$\nabla \cdot \bar{B} = 0, \quad \nabla \wedge \bar{B} = \frac{1}{c}\frac{\partial \bar{E}}{\partial t} + \frac{4\pi}{c}\bar{J},$$

and continuity, momentum, and energy (entropy) equations for either species ($\alpha = e,i$)

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot n_\alpha \bar{v}_\alpha = 0$$

$$m_\alpha n_\alpha\left(\frac{\partial}{\partial t} + \bar{v}_\alpha \cdot \nabla\right)\bar{v}_\alpha = -\nabla(n_\alpha T_\alpha) + n_\alpha q_\alpha\left(\bar{E} + \frac{v_\alpha}{c} \wedge \bar{B}\right) + \bar{R}_{\beta\alpha} + \text{viscous term}$$

$$n_\alpha T_\alpha\left(\frac{\partial}{\partial t} + \bar{v}_\alpha \cdot \nabla\right) \ln \frac{T_\alpha^{\frac{3}{2}}}{n_\alpha} = -\nabla \cdot \bar{q}_\alpha + Q_{\beta\alpha} + \text{laser heating} + \text{viscous heating}$$

together with a description of light propagation and absorption. Note that $\bar{R}_{\beta\alpha} + \bar{R}_{\alpha\beta} = 0$, $Q_{\beta\alpha} + Q_{\alpha\beta} + (\bar{v}_\alpha - \bar{v}_\beta) \cdot \bar{R}_{\beta\alpha} = 0$.

Ion conduction and viscous terms are negligible as previously indicated. Quasi-neutrality requires $Z_i n_i \simeq n_e$ and allows to neglect the displacement current and the electron inertia (see also Pert 1987). It also leads to $\nabla \cdot \bar{J} = 0$ where $\bar{J} = eZ_i n_i \bar{v}_i - en_e\bar{v}_e \simeq -en_e\bar{u}(\bar{u} \equiv \bar{v}_e - \bar{v}_i)$. One then gets the following system of equations

$$\frac{\partial n_e}{\partial t} + \nabla \cdot n_e \bar{v}_i = 0 \tag{2}$$

$$\bar{m} n_e\left(\frac{\partial}{\partial t} + \bar{v}_i \cdot \nabla\right)\bar{v}_i = -\nabla\left(n_e T_e + \frac{n_e}{Z_i} T_i\right) - n_e \bar{u} \wedge \frac{e\bar{B}}{c} \tag{3}$$

$$\frac{n_e}{Z_i} T_i\left(\frac{\partial}{\partial t} + \bar{v}_i \cdot \nabla\right) \ln \frac{T_i^{\frac{3}{2}}}{n_i} = Q_{ei} \tag{4}$$

$$n_e T_e\left(\frac{\partial}{\partial t} + \bar{v}_i \cdot \nabla\right) \ln \frac{T_e^{\frac{3}{2}}}{n_e} = -n_e T_e \bar{u} \cdot \nabla \ln \frac{T_e^{\frac{3}{2}}}{n_e} - \nabla \cdot \bar{q}_e - \bar{u} \cdot \bar{R}_{ie} - Q_{ei} - \nabla \cdot \bar{S}_L \tag{5}$$

$$\nabla \wedge \left(\frac{e\bar{B}}{c}\right) = -\frac{k_L^2 m_e}{n_c} n_e \bar{u}, \quad (k_L \equiv \omega/c). \tag{6}$$

In addition, the electron momentum equation is used in Faraday's law to obtain

$$\frac{\partial}{\partial t}\left(\frac{e\bar{B}}{c}\right) + \nabla \wedge \left(\frac{e\bar{B}}{c} \wedge (\bar{v}_i + \bar{u}) + \frac{\bar{R}_{ie}}{n_e}\right) = \nabla T_e \wedge \nabla \ln n_e. \tag{7}$$

The collisional results for electron heat flux, and ion-electron friction and energy exchange rate are (Braginskii 1965)

$$\bar{q}_e = -n_e T_e\left(\frac{\tau_e}{m_e} \bar{\bar{\gamma}} \cdot \nabla T_e - \bar{\bar{\beta}} \cdot \bar{u}\right)$$

$$\bar{R}_{ie} = -n_e\left(\frac{m_e}{\tau_e} \bar{\bar{\alpha}} \cdot \bar{u} + \bar{\bar{\beta}} \cdot \nabla T_e\right)$$

$$Q_{ei} = \frac{3 m_e n_e}{Z_i \bar{m} \tau_e}(T_e - T_i),$$

where τ_e is some characteristic electron collision time

$$\tau_e = \frac{3}{4(2\pi)^{\frac{1}{2}}} \frac{m_e^{\frac{1}{2}} T_e^{\frac{3}{2}}}{e^4 Z_i n_e \ln \Lambda}$$

and $\ln \Lambda$ is a Coulomb logarithm.

The tensors $\bar{\bar{\alpha}}$, $\bar{\bar{\beta}}$, and $\bar{\bar{\gamma}}$ are dimensionless functions of Z_i and $\frac{eB}{c}\frac{\tau_e}{m_e} \equiv \omega_{ce}\tau_e$, ω_{ce} being the electron cyclotron frequency. Their symmetric and antisymmetric parts are even and odd in B respectively, and represent laws or effects well established in other fields of physics

$$\bar{\bar{\alpha}} = \bar{\bar{\alpha}}_s(\text{Ohm}) + \bar{\bar{\alpha}}_a(\text{Hall})$$

$$\bar{\bar{\gamma}} = \bar{\bar{\gamma}}_s\ (\text{Fourier}) + \bar{\bar{\gamma}}_a\ (\text{Righi-Leduc})$$

$$\bar{\bar{\beta}} = \bar{\bar{\beta}}_s\begin{pmatrix}\text{Seebeck}\\ \text{Peltier}\end{pmatrix} + \bar{\bar{\beta}}_a\begin{pmatrix}\text{Ettinghausen}\\ \text{Nernst}\end{pmatrix} \text{ for } \begin{pmatrix}\bar{q}_e\\ \bar{R}_{ie}\end{pmatrix}.$$

If ray tracing is not required (negligible ray crossing) the light energy flux is

$$\bar{S}_L = \bar{s}_i I_i + \bar{s}_r I_r$$

where the subscripts refer to rays incident and reflected (at the critical surface). One then has (Landau and Lifshitz 1960)

$$\nabla \cdot \bar{s}_{i,r} I_{i,r} = -k_{br} I_{i,r} \tag{8}$$

$$\bar{s}_{i,r} \cdot \nabla \bar{s}_{i,r} = [\nabla - \bar{s}_{i,r}(\bar{s}_{i,r} \cdot \nabla)] \ln (1 - n_e/n_c)^{\frac{1}{2}} \tag{9}$$

where k_{br} is the absorption coefficients for inverse bremsstrahlung (Johnston & Dawson 1973)

$$k_{br} = \frac{n_e/n_c}{c\tau_e(1 - n_e/n_c)^{\frac{1}{2}}}.$$

4. Dimensionless parameters

Equations (2)–(9), together with the auxiliary equations for $\bar{q}_e$, $\bar{R}_{ie}$, Q_{ei}, and k_{br}, and the expressions for τ_e and $\bar{S}_L$, is a set of ten equations for the ten variables

$$n_e,\ \bar{v}_i,\ T_e,\ T_i,\ \bar{u},\ e\bar{B}/c,\ \bar{s}_{i,r},\ I_{i,r}.$$

The system involves a number of dimensional parameters and a dimensionless one (Z_i).

Writing $c\tau_e = m_e c \times \tau_e/m_e$ in k_{br} and defining $\bar{K}(z)$ by setting

$$\frac{\tau_e}{m_e} \equiv \frac{\bar{K}(Z_i)}{\gamma_0(Z_i)} \frac{T_e^{\frac{3}{2}}}{n_e}$$

we have eight parameters

$$W_m,\ \tau_L,\ R,\ n_c,\ \bar{m},\ k_L^2 m_e,\ \bar{K}, \text{ and } m_e c;$$

$\gamma_0 \bar{\bar{I}}$ is the limiting form of $\bar{\bar{\gamma}}$ in $\bar{q}_e$ when $\omega_{ce}\tau_e \to 0$. Note that $\bar{K}$ is then the coefficient in Spitzer's thermal conductivity

$$K(\text{Spitzer}) \equiv \bar{K} T_e^{\frac{5}{2}};$$

we neglect its weak dependence on n_e and T_e due to the Coulomb logarithm.

Five dimensionless combinations of the above parameters, can be obtained. However, if a dimensional analysis of equations (2)–(9) is carried out only four combinations are found. A fifth one, $n_c R^3$, the number of electrons in a macroscopic volume, involves microscopic information for which our equations have no use.

Thus, finally, dimensionless results from the present formulation will depend on the dimensionless numbers

$$\sigma_1 \equiv \left(\frac{W_m}{R^2 \bar{m} n_c}\right)^{\frac{1}{3}} \frac{\tau_L}{R}, \quad \sigma_2 \equiv \frac{\bar{K} \bar{m}^{\frac{5}{2}} R^3}{n_c \tau_L^4},$$

$$\sigma_3 \equiv \frac{\bar{m} R}{m_e c \tau_L}, \quad \sigma_4 \equiv \frac{m_e}{\bar{m}} (k_L R)^2,$$

and Z_i.

5. One-dimensional solutions

For atomic number Z not too high, atoms in the corona are fully stripped ($Z_i = Z$) and the dependence of the flow on Z_i shows no particular features. The main effect is that as Z_i increases, ion pressure and entropy per unit volume become negligible compared to the corresponding electron values. There is a slight simplification: T_i remains in equation (4) only, and may be ignored when determining all other variables.

Consider now target and irradiation that are both spherically symmetric. Current and magnetic field then vanish identically. As a result σ_4 drops off the analysis, leaving σ_1–σ_3 as remaining parameters.

For σ_1 small the corona is a thin layer ($L \sim c_s^* \tau_L \ll R$) that may be considered planar (one-dimensional, straight flows). Introducing convenient, reference values of velocity and intensity

$$U \equiv (n_c \tau_L / \bar{m}^{\frac{5}{2}} \bar{K})^{\frac{1}{3}}, \quad I_0 \equiv W_m / 4\pi R^2$$

we then use parameters $\bar{I}_0$ and $\bar{U}$ instead of σ_2 and σ_3:

$$\bar{I}_0 \equiv \frac{I_0}{\rho_c U^3} = \frac{\sigma_1^3 \sigma_2}{4\pi} = \frac{W_m \bar{K} \bar{m}^{\frac{3}{2}}}{4\pi R^2 n_c^2 \tau_L}$$

$$\bar{U} \equiv \frac{\bar{m} U}{m_e c} = \frac{\sigma_3}{\sigma_2^{\frac{1}{3}}} = \frac{n_c^{\frac{1}{3}} \tau_L^{\frac{1}{3}} \bar{m}^{\frac{1}{6}}}{m_e c \bar{K}^{\frac{1}{3}}}.$$

One easily verifies that $L_c / c_s^* \tau_L \sim \bar{I}_0$.

For $\bar{I}_0$ small we have $L_c \ll L$: conduction is restricted to a thin sublayer, having both overdense and underdense regions and lying next to the target. This sublayer may be considered quasi-steady. If $W(t) \propto t^{\frac{3}{2}}$, the larger region outside the layer has a self-similar behaviour; as a consequence the entire problem is reduced to the analysis of systems of ordinary differential equations. For $\bar{U}/\bar{I}_0^{\frac{2}{3}}$ small, inverse bremsstrahlung is negligible and surface absorption at n_c must be considered (Sec. 7). If then $W(t) \propto t$, the entire corona has a self-similar behaviour, leading again to ordinary differential equations.

For σ_1 large ($c_s^* \tau_L \gg R$), quasi-steady conditions are obtained during most of the pulse, and the characteristic length of the corona is R itself. We are led once more to ordinary differential equations. Introducing a velocity

$$V \equiv (n_c R / \bar{m}^{\frac{5}{2}} \bar{K})^{\frac{1}{4}}$$

we then use parameters $\bar{W}$ and $\bar{V}$ instead of σ_2 and σ_3:

$$\bar{W} \equiv \frac{I_0}{\rho_c V^3} = \frac{\sigma_1^3 \sigma_2^{\frac{3}{4}}}{4\pi} = \frac{W_m \bar{K}^{\frac{3}{4}} \bar{m}^{\frac{7}{8}}}{4\pi R^{\frac{11}{4}} n_c^{\frac{7}{4}}},$$

$$\bar{V} \equiv \frac{\bar{m} V}{m_e c} = \frac{\sigma_3}{\sigma_2^{\frac{1}{4}}} = \frac{\bar{m}^{\frac{3}{8}} R^{\frac{1}{4}} n_c^{\frac{1}{4}}}{m_e c \bar{K}^{\frac{1}{4}}}.$$

One easily verifies that $L_c/R \sim \bar{W}^{\frac{4}{3}}$. For $\bar{W}^{\frac{4}{3}}$ small we have $L_c \ll L$: conduction is restricted to a thin layer having both overdense and underdense regions and lying next to the target. For $\bar{V}/\bar{W}$ small, inverse bremsstrahlung becomes negligible.

Consider next a foil target. For σ_1 large, the flow will be quasi-steady and divergent, but only in a crude sense may it be taken as (hemispherically) symmetric; $\bar{B}$ and $\bar{u}$ effects will be essential to the analysis. For σ_1 small, however, we again have a thin corona ($L \sim c_s^* \tau_L \ll R$), which may be considered planar if, as usual, transverse variations across the laser spot have its radius R as characteristic length. One may then repeat the previous discussion on $\bar{I}_0$ and $\bar{U}$, just omitting the factor $\frac{1}{4}$ in I_0 and $\bar{I}_0$.

In all above cases, universal laws for quantities such as peak electron and ion temperatures, mass ablation rate, ablation pressure, and fractional absorption can be obtained. The laws are analytical in a sense usual in fluid mechanics; they represent dimensionless functions of a few dimensionless parameters, result from ordinary differential equations as nonlinear boundary value problems, and usually involve eigenvalues determination by requiring the crossing of some singular points.

There is clearly a richness of behaviour depending on coronal length relative to pellet or spot radius, conduction-to-coronal length ratio, fractional inverse bremsstrahlung absorption, and even ion-to-electron energy storage ratio. That richness is reflected in target implosion. For example, the hydrodynamic efficiency η_H for the acceleration of a thin foil is a function of the ratio of ablated mass ΔM at the end of the pulse to the initial mass M_0; $\eta_H(M/M_0)$ was calculated for different coronal regimes and found to depend substantially on the particular regime considered (Sanmartín *et al.* 1985b; figure 2).

Different regimes have been analyzed for both $\sigma_1 \to 0$ (Sanmartín & Barrero 1978a,b; Barrero & Sanmartín 1980; Sanmartín *et al.* 1983; Ramis & Sanmartín 1983; Nicolás 1986) and $\sigma_1 \to \infty$ (Montañés & Sanmartín 1980; Sanz *et al.* 1981; Sanz & Sanmartín 1983; Sanmartín *et al.* 1985a; Nicolás & Sanmartín 1985).

6. Weakly two-dimensional flows

When target or irradiation conditions are not perfectly symmetric or uniform, coronal flows become two-dimensional and the parameter σ_4, characterizing current and magnetic effects, enters the picture. The analysis of such flows may be quite complex.

Particularly simple are problems that may be considered as only *weakly* two-dimensional. The main examples are the analysis of a) the stability of the 1-D flows of Sec. 5 and b) the *smoothing* of impressed, weak, *nonuniformities* in irradiation or surface finish: i.e. perturbations of the Sec. 5 flows. The ratio of perturbation wavelength to coronal length is an additional parameter characterizing these problems (Sanmartín *et al.* 1987; Sanz *et al.* 1988a,b; Sanz 1988; Pérez-Saborid *et al.* 1988).

Since both $\bar{B}$ and $\bar{u}$ are then small quantities, one finds in general that

$$\bar{R}_{ie} \simeq -n_e\left(\frac{m_e}{\tau_e}\alpha_0\bar{u} + \frac{\beta_0''}{\delta_0}\frac{\tau_e e\bar{B}}{m_e c}\wedge\nabla T_e + \beta_0\nabla T_e\right),$$

$$\bar{q}_e \simeq -n_e T_e\left[\frac{\tau_e}{m_e}\left(\gamma_0\nabla T_e + \frac{\gamma_0''}{\delta_0}\frac{\tau_e e\bar{B}}{m_e c}\wedge\nabla T_e\right) - \beta_0\bar{u}\right].$$

Then, Faraday's law becomes

$$\frac{\partial}{\partial t}\left(\frac{e\bar{B}}{c}\right) + \nabla\wedge\left[\frac{e\bar{B}}{c}\times\left(\bar{v}_i - \frac{\beta_0''}{\delta_0}\frac{\tau_e}{m_e}\nabla T_e\right) - \frac{m_e}{\tau_e}\alpha_0\bar{u}\right] = \nabla T_e\wedge\nabla\ln n_e \tag{10}$$

and the first three terms on the right-hand side of equation (5) become

$$-n_eT_e\bar{u}\cdot\nabla\ln\frac{T_e^{\frac{3}{2}}}{n_e}-\nabla\cdot\bar{q}_e-\bar{u}\cdot\bar{R}_{ie}$$
$$\rightarrow -n_eT_e\bar{u}\cdot\nabla\ln\frac{T_e^{\frac{3}{2}}}{n_e}+\nabla\cdot(\bar{K}T_e^{\frac{5}{2}}\nabla T_e)+\nabla\cdot\left(\bar{K}T_e^{\frac{5}{2}}\frac{\gamma_0''}{\gamma_0\delta_0}\frac{\tau_e e\bar{B}}{m_e c}\wedge\nabla T_e\right). \quad (11)$$

Note that only Ohm and Nernst terms enter (10), while the $\bar{u}$-convection of electron entropy, and Righi-Leduc conduction, enter (11). The thermoelectric effects (β_0-terms) and the term $\bar{J}\wedge\bar{B}/c$ in equation (3) drop out. On the contrary, the refraction equations (8), (9) are an essential part of weakly 2-D flows. The Braginskii coefficients (α_0, β_0'', etc.) only depend on Z_i.

Full 2-D coronal flows can be simplified in special limits. Consider first σ_1 large. For $\sigma_4\rightarrow 0$, Faraday's law generates a current through Ohm's term, the heating in (5) is due to $\bar{u}$ convection of entropy, and Ampere's law gives $\bar{B}$, which is small ($\omega_{ce}\tau_e\ll 1$). For $\sigma_4\rightarrow\infty$, the $\partial\bar{B}/\partial t$ and $\bar{v}_i\wedge\bar{B}$ terms and the Nernst effect enter Faraday's law, yielding $\bar{B}$ (now we have $\omega_{ce}\tau_e\sim 1$), Ampere's law gives a small $\bar{u}$, and Righi-Leduc heating occurs. In both the small and large σ_4 limits, the force $\bar{J}\wedge\bar{B}/c$ is negligible. For σ_1 small the same results apply, if $\sigma_1^2\sigma_4$ is small and large respectively.

7. Corrections to the model

7.1. *Thermal radiation*

For atomic number Z not too large, and thus ions fully stripped, the transport of thermal radiation has a simple description if polarization and refraction are neglected: only bremsstrahlung processes and Thomson scattering are involved. Letting $I_\nu(\bar{\Omega})$ be the specific intensity for frequency ν and directional unit vector $\bar{\Omega}$, we have (Pomraning 1973)

$$\bar{\Omega}\cdot\nabla I_\nu(\bar{\Omega})=\kappa_{a\nu}'[I_{p\nu}-I_\nu(\bar{\Omega})]+\kappa_{s\nu}\int d\bar{\Omega}'\frac{3}{4\pi}[1+(\bar{\Omega}\cdot\bar{\Omega}')^2][I_\nu(\bar{\Omega})-I_\nu(\bar{\Omega}')], \quad (10)$$

where the time derivative was neglected ($c\tau_L\gg R$) and

$$I_{p\nu}=2h\nu^3/c^2(e^{h\nu/T_e}-1),$$
$$\kappa_{a\nu}'=\frac{\pi g}{3^{\frac{1}{2}}\ln\Lambda}\left(\frac{\hbar\omega}{T_e}\right)^2\frac{n_e/n_c}{c\tau_e}\frac{1-e^{-h\nu/T_e}}{(h\nu/T_e)^3},$$
$$\kappa_{s\nu}=\frac{(2\pi)^{\frac{1}{2}}}{Z_i\ln\Lambda}\left(\frac{T_e}{m_ec^2}\right)^{\frac{3}{2}}\frac{1}{c\tau_e}.$$

For an optically thin corona ($L\ll\kappa_{a\nu}'^{-1}$, $\kappa_{s\nu}^{-1}$), equation (10) becomes

$$\bar{\Omega}\cdot\nabla I_\nu(\bar{\Omega})\simeq\kappa_{a\nu}'I_{p\nu}.$$

The thermal radiation flux, $\bar{S}_{th}\equiv\int d\nu\, d\bar{\Omega} I_\nu(\bar{\Omega})\bar{\Omega}$, is then given by

$$\nabla\cdot\bar{S}_{th}=\frac{4\alpha}{3^{\frac{1}{2}}\ln\Lambda}\frac{n_eT_e^2}{m_ec^2\tau_e}$$

where $\alpha\equiv e^2/hc\simeq 137^{-1}$ is the constant of fine structure, and a Gaunt factor $g=1$ was used. A term $-\nabla\cdot\bar{S}_{th}$, representing plasma-radiation energy exchange, should be added to the left-hand side of equation (5). With the dimensionless analysis of Secs. 4–6, the above exchange term is of order $\alpha\bar{V}/\bar{W}^{\frac{3}{2}}$ for σ_1 large; since $\bar{W}$ is large for usual conditions, rarely need thermal radiation be included in the analysis. Similarly, for σ_1 small we have $-\nabla \cdot \bar{S}_{th} \sim \alpha\sigma_1^{\frac{1}{2}}\bar{U}/\bar{I}_0^{\frac{1}{2}}$. One easily verifies that the corona is indeed optically thin; for σ_1 large, for instance, we have

$$R\kappa'_{av} \sim \frac{\bar{V}^5}{\sigma_4 \bar{W}^{\frac{7}{3}}}\left(\frac{m_e}{\alpha\bar{m}}\right)^2,$$

$$R\kappa_{sv} \sim \frac{\bar{V}^4}{Z_i}\left(\frac{m_e}{\bar{m}}\right)^{\frac{1}{2}}.$$

7.2. *Ionization energy*

This effect was also ignored in the model of Sec. 3. If $j^2\varepsilon_j$ is the energy required to strip an atom off its j^{th} electron, the total ionization energy is $\sum \varepsilon_j j^2$, $j = 1, 2, \ldots, Z_i$; here

$$\varepsilon_j = I_H \times (\text{a factor of order unity})$$

$$I_H \equiv \frac{m_e e^4}{2\hbar^2} = \frac{1}{2}\alpha^2 m_e c^2 \simeq 13{\cdot}6\ \text{eV}.$$

For $Z_i \gg 1$, when the effect is largest, the total plasma energy per electron is $\frac{1}{2}\bar{m}v^2 + \frac{3}{2}T_e + \frac{1}{3}\bar{\varepsilon}Z_i^2$, $\bar{\varepsilon}$ being some average value. To just get an estimate, we set $\bar{m}v^2 = T_e = T_e^*$, $I_0 = n_c(T_e^*/\bar{m})^{\frac{1}{2}} \times (2T_e^* + \frac{1}{3}\bar{\varepsilon}Z_i^2)$ and $\bar{\varepsilon} = 10\ \text{eV}$ (midway between 4 and 25 eV). Then we have $T_e^* \propto 1 + Z_i^2\bar{\varepsilon}/9T_e^*$, the last term being the ionization correction, assumed weak. In dimensionless form, this term reads as

$$\frac{Z_i^2\bar{\varepsilon}}{9T_e^*} \simeq \frac{\alpha^2\bar{m}}{16m_e}\frac{Z_i^2}{\bar{V}^2\bar{W}^{\frac{2}{3}}} \simeq \left(\frac{Z_i}{9\bar{V}\bar{W}^{\frac{1}{3}}}\right)^2.$$

Typically a Z ($\simeq Z_i$) of about 10–15 is the largest atomic number for which the ionization energy may be reasonably neglected.

7.3. *Ionization state*

For Z large enough, the ion charge number Z_i lags behind Z. This fact introduces complex atomic physics into the analysis. First, some appropriate value for a mean Z_i must be determined as part of the problem; the ion populations for different ionization states may in fact be needed. Secondly, line emission and absorption, which are frequently quite large, will need to be accounted for in the radiation transport; the plasma-radiation exchange must be considered in the plasma energy balance. Finally the ionization energy will also be substantial.

Both high target atomic-number and special target design, result in copious thermal (x-rays) radiation in the corona of hohlraum targets. On the whole, atomic physics overwhelms fluid mechanics in the hohlraum approach to laser fusion. By restricting our study to the direct-drive approach, we explicitly exclude targets with high atomic number.

7.4. *Non-classical heat flux*

At a large ionization state, the mean free paths for ion-electron scattering, λ_{ei}, and electron–electron relaxation $\lambda_{ee} \sim Z_i\lambda_{ei}$ are widely disparate. From equation (1), $L_c \sim \lambda_{ei}(\bar{m}/m_e)^{\frac{1}{2}}$, the ratio

$$\frac{\lambda_{ee}}{L_c} \sim Z_i(m_e/\bar{m})^{\frac{1}{2}}.$$

need not be small. One then expects that the hypothesis of a local near-Maxwellian electron population may break down, invalidating the classical calculation of transport coefficients. Actually, i) the electrons contributing mostly to the heat-flux have energy ε somewhat above T_e, and ii) their motion between electron-electron collisions is a random walk; the condition for a break-down of local thermal equilibrium for electrons is then that $Z_i^{\frac{1}{2}}(\varepsilon/T_e)^2$ gets near $(\bar{m}/m_e)^{\frac{1}{2}}$. The crude expression $\bar{q} = -fn_eT_e(T_e/m_e)^{\frac{1}{2}}\nabla T_e/|\nabla T_e|$, normally used under such conditions, involves an "ad hoc", overall, flux limit factor f, that must change with the regime of the coronal flow. A non-local kinetic formalism should depend on the dimensionless number $Z_i^{\frac{1}{2}}(\varepsilon/T_e)^2 \times (m_e/\bar{m})^{\frac{1}{2}}$ (Luciani *et al.* 1985, Albritton *et al.* 1986).

7.5. *Plasma-light interaction*

The fast (ω^{-1}) time-scale interaction between laser beam and plasma electrons basically yields the dielectric function, $1 - n_e/n_c$, for light propagation, and the absorption coefficient k_{br}. There are however additional interaction phenomena, not included in our model.

First, for Z_i large, the ratio $\lambda_{ee}/\lambda_{ei}$ is of order Z_i, as earlier noticed. This affects the classical result for k_{br} when $Z_iI_0/cn_cT_e^*$ reaches about unity (Langdon 1980).

Secondly, (linear) resonance absorption occurs at the critical density for oblique incidence and p-polarization. For weakly two-dimensional flow, the absorption will depend on the value of $(\lambda\,\nabla \ln n_e)^{\frac{2}{3}}(1 - \bar{s}_i \cdot \nabla n_e/|\nabla n_e|)$ at n_c (Friedberg *et al.* 1972).

Finally, (nonlinear) parametric effects may affect the fate of light propagating inwards, up to the critical surface. Such inconvenient effects may be avoided by the use of broad laser bandwidth.

7.6. *Target thickness*

If the target is a foil, or a spherical shell, so thick that the inward moving perturbation has not reached its backface by the end of the pulse, then an overall momentum balance shows that

$$\text{inward velocities} \sim c_s^* \times (\rho_c/\rho_{\text{target}})^{\frac{1}{2}} \ll c_s^*.$$

This inequality was the basis for the neglect of the receding motion of the ablation surface, an essential point in our model for a coronal analysis. The target thickness ΔR is large enough if

$$\Delta R > \tau_L c_s^*(\rho_c/\rho_{\text{target}})^{\frac{1}{2}} \quad \text{or} \quad \rho_c/\rho_{\text{target}} < (\Delta R/c_s^*\tau_L)^2. \tag{11}$$

If, however, the opposite is true, then we have

$$\text{inward velocities} \sim c_s^* \times \frac{c_s^*\tau_L}{\Delta R} \times \frac{\rho_c}{\rho_{\text{target}}},$$

so that the receding motion may be neglected if

$$\tau_L c_s^*(\rho_c/\rho_{\text{target}}) \ll \Delta R < \tau_L c_s^*(\rho_c/\rho_{\text{target}})^{\frac{1}{2}}.$$

The first inequality, weaker than (11), basically means that the ablated mass fraction be small. For a spherical shell of radius R, the pulse will be on, at shell collapse, if

$$c_s^*\tau_L > (R\Delta R)^{\frac{1}{2}}(\rho_{\text{target}}/\rho_c)^{\frac{1}{2}}$$

or

$$\Delta R < \tau_L c_s^*(\rho_c/\rho_{\text{target}}) \times c_s^*\tau_L/R.$$

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