INSTITUTO NACIONAL DE TECNICA AEROESPACIAL
"ESTEBAN TERRADAS"

THE INFLUENCE OF LAUNCHING ERRORS ON THE TRAJECTORY OF SPACE PROBES

INTA REPORT I. C. 1

APPENDIX 7.III

by

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INTA REPORT I.C.1 (Appendix 7.III)

THE INFLUENCE OF LAUNCHING ERRORS ON THE TRAJECTORY OF SPACE PROBES

1. Introduction.- In the following a study of launching errors is presented more complete than the one given in Chapter 7 of INTA REPORT I.C. 1 "The Influence of Launching Errors on the Trajectory of Space Probes".

The matched conics approximation is also used here; however, "patching" between the geocentric and heliocentric orbits is assumed to take place at the Earth's sphere of influence (radius $r_{tr} \simeq 900000$ Km).

We acknowledge the help of Dr. J. Vandenkerckhove who suggested this extension of our previous work, and kindly provided us the first two matrices given below.

The nomenclature is as given in the figures.

2. Relationships between the injection conditions and the orbital parameters.

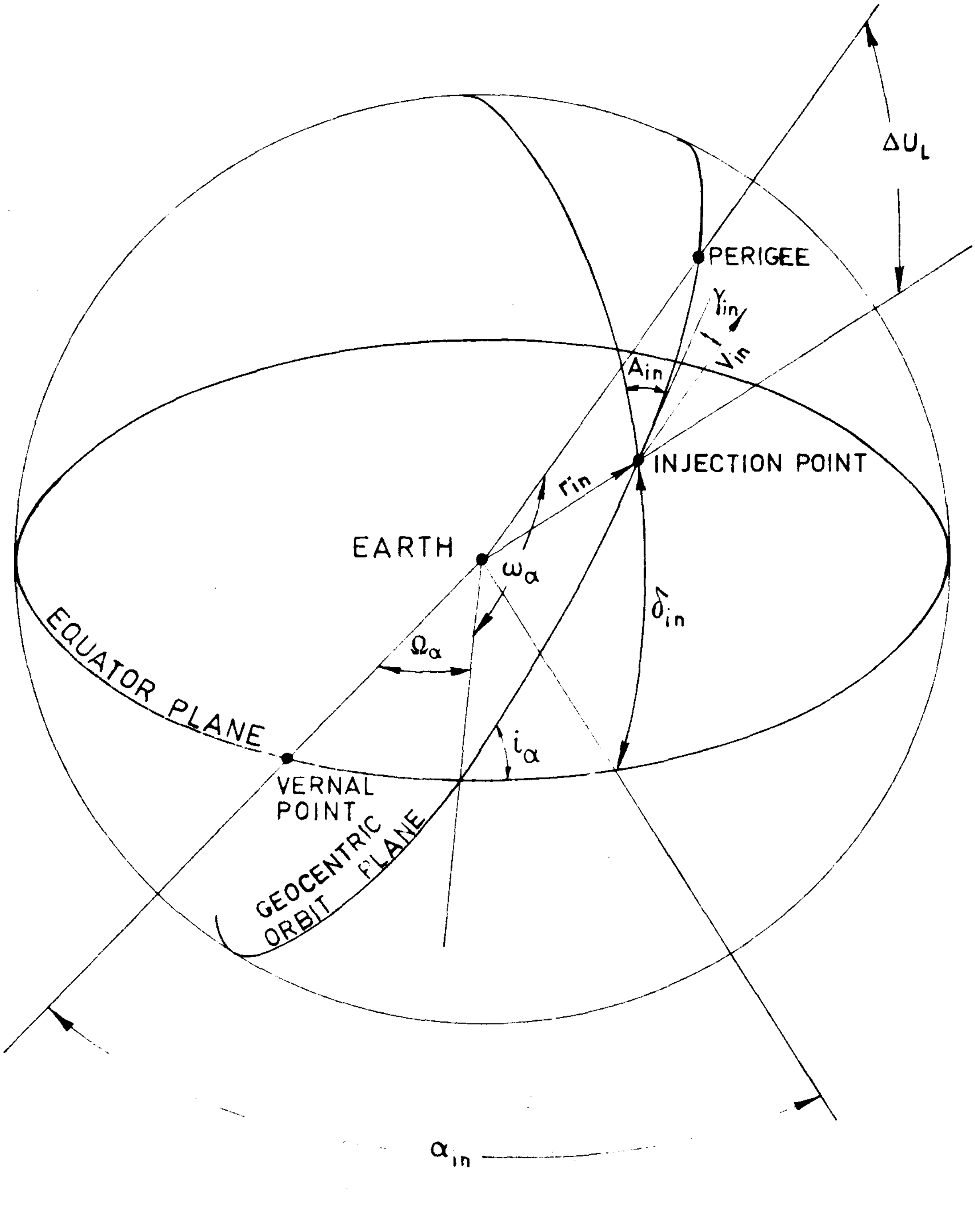
We give below in terms of the seven injection conditions :

$$r_{in}\ ,\ V_{in},\ \gamma_{in},\ t_{in}\ ,\ \alpha_{in},\ \delta_{in},\ A'_{in}$$

the six orbital parameters in a geocentric equatorial inertial system :

$$a_\alpha\ ,\ e_\alpha\ ,\ t_{p_\alpha},\ i_\alpha\ ,\ \Omega_\alpha\ ,\ \omega_\alpha$$

INJECTION AND GEOCENTRIC EQUATORIAL ORBITAL PARAMETERS



The first three orbital parameters are independent of the selected coordinate system; they are given by the following relations:

A, Elliptic orbit.

$$a_\alpha = r \ /(2 - r\, V^2/\mu_\oplus)$$

$$e_\alpha = \sqrt{1 - \left(\frac{2}{r} - \frac{V^2}{\mu_\oplus}\right)\left(\frac{r^2\ V^2\ \cos^2\gamma}{\mu_\oplus}\right)}$$

$$t - t_{p_\alpha} = \frac{1}{2E_\oplus}\left[\left(2\,E_\oplus\ r^2 + 2\mu_\oplus\, r - r^2V^2\cos^2\gamma\right)^{1/2} + \right.$$

$$\left. + \frac{\mu_\oplus}{\sqrt{-2E_\oplus}}\ \text{arc sin}\ \frac{\mu_\oplus + 2\,E_\oplus r}{\left(\mu_\oplus^2 + 2r^2V^2\ \cos^2\gamma\ \ E_\oplus\right)^{1/2}}\right]$$

with $E_\oplus = V^2/2 - \mu_\oplus/r$ (total energy)

B. Hyperbolic orbit

$$a_\alpha = r/\left(rV^2/\mu_\oplus - 2\right)$$

$$e_\alpha = \sqrt{1 - \left(\frac{2}{r} - \frac{V^2}{\mu_\oplus}\right)\left(\frac{r^2V^2\ \cos^2\gamma}{\mu_\oplus}\right)}$$

$$t_p - t_{p_\alpha} = \frac{1}{2E_\oplus}\left[\left(2E_\oplus\ r^2 + 2\mu_\oplus\cdot r - r^2V^2\cos^2\gamma\right)^{1/2} - \right.$$

$$\left. - \frac{\mu_\oplus}{\sqrt{2E_\oplus}}\ l_n\left\{\frac{\mu_\oplus + 2E_\oplus r}{\sqrt{2E_\oplus}} + \left(2E_\oplus\ r^2 + 2\mu_\oplus\, r - r^2V^2\cos^2\gamma\right)^{1/2}\right\}\right]$$

The second three orbital parameters are dependent on the coordinate system, they are given by :

$$i_{\alpha} = \text{arc cos}\,(\sin A'_{in} \cos \delta_{in})$$

$$\Omega_{\alpha} = \alpha_{in} - \text{arc tan}\left[\tan A'_{in} \sin \delta_{in}\right]$$

$$\omega_{\alpha} = \text{arc cot}\left[\cos A'_{in} \cot \delta_{in}\right] + \Delta U_L$$

Effect of injection errors on orbital parameters

Differentiation of the preceding equations permits to express the errors of the orbital parameters, in terms of the errors in the injection parameters; symbolically we can write:

$$|\Delta_{\alpha}| = |\mathcal{M}_{in,\alpha}||\Delta_{in}|$$

where the matrix $\mathcal{M}_{in,\alpha}$ is given below as 2-1

Where the expressions for the partial derivatives are

$$\frac{\partial a_{\alpha}}{\partial r} = \pm \frac{2\,\mu_{\oplus}^2}{(r\,V^2 - 2\,\mu_{\oplus})^2}$$ (+ for ellipse, - for hyperbola)

$$\frac{\partial a_{\alpha}}{\partial V} = \pm \frac{2\,r^2 V\,\mu_{\oplus}}{(rV^2 - 2\,\mu_{\oplus})^2}$$

$$\frac{\partial e_{\alpha}}{\partial r} = \frac{V^2 \cos^2 \gamma}{\mu_{\oplus}\, e_{\alpha}} \left(\frac{r\,V^2}{\mu_{\oplus}} - 2\right)$$

$$\frac{\partial e_\alpha}{\partial V} = \frac{2\, r\, V \cos^2\gamma}{\mu_\oplus e_\alpha}\left(\frac{r\, V^2}{\mu_\oplus} - 1\right)$$

$$\frac{\partial e_\alpha}{\partial \gamma} = - \frac{r\, V^2 \sin\gamma \cos\gamma}{\mu_\oplus e_\alpha}\left(\frac{r\, V^2}{\mu_\oplus} - 2\right)$$

For elliptic orbits

$$\frac{\partial t_{p\alpha}}{\partial r} = 3\,\frac{\mu_\oplus}{r}\,\frac{t - t_{p\alpha}}{r\, V^2 - 2\mu_\oplus} -$$

$$- \frac{r\, V \sin\gamma \left[r\, V^2 \cos^2\gamma + \mu_\oplus\right]\left[rV^2 - \mu_\oplus\right]}{\left(rV^2 - 2\mu_\oplus\right) e_\alpha^2\, \mu_\oplus^2} +$$

$$+ \frac{r\, V \cos\gamma}{\sqrt{\mu_\oplus^2 e_\alpha^2 - \left(rV^2\cos^2\gamma - \mu_\oplus\right)^2}}\left\{1 + \frac{1}{e_\alpha^2}\left[\frac{rV^2}{\mu_\oplus} - 1\right]\left[1 - \frac{rV^2\cos^2\gamma}{\mu_\oplus}\right]\right\}$$

$$\frac{\partial t_{p\alpha}}{\partial V} = \frac{3\, rV(t - t_{p\alpha})}{rV^2 - 2\mu_\oplus} - \frac{2r^2 \sin\gamma\, (rV^2\cos^2\gamma + \mu_\oplus)(rV^2 - \mu_\oplus)}{(rV^2 - 2\mu_\oplus)(\mu_\oplus^2 e_\alpha^2)} +$$

$$+ \frac{2\, r^2 \cos\gamma}{\sqrt{\mu_\oplus^2 e_\alpha^2 - (rV^2\cos^2\gamma - \mu_\oplus)^2}}\left\{1 + \frac{1}{e_\alpha^2}\left(\frac{rV^2}{\mu_\oplus} - 1\right)\left(1 - \frac{rV^2\cos^2\gamma}{\mu_\oplus}\right)\right\}$$

$$\frac{\partial t_{p\alpha}}{\partial \gamma} = \frac{1}{\mu_\oplus^2 e_\alpha^2}\, r^2 \sin^2\gamma \left[rV^2\cos^2\gamma + \mu_\oplus\right] -$$

$$- \frac{2\, r^2 \sin\gamma}{\sqrt{\mu_\oplus^2 e_\alpha^2 - (rV^2\cos^2\gamma - \mu_\oplus)^2}} \left\{ 1 + \frac{1}{2e_\alpha^2} \left[\frac{rV^2}{\mu_\oplus} - 2 \right] \left[1 - 1 \frac{rV^2\cos^2\gamma}{\mu_\oplus} \right] \right\}$$

For hyperbolic orbits

$$\frac{\partial t_{p\alpha}}{\partial r} = \frac{3\mu_\oplus}{2Er^2}(t - t_{p\alpha}) - \frac{\mu_\oplus^3 + B\,\mu_\oplus^2 - 4E^2r^2\mu_\oplus + B(2Er + \mu_\oplus) - 2Er\,\mu_\oplus^2}{(2E)^{5/2}\,(\mu_\oplus + 2Er + B)}$$

$$\frac{\partial t_{p\alpha}}{\partial V} = \frac{3V(t - t_{p\alpha})}{2E} - \frac{V\left[\mu_\oplus^2 B + \mu_\oplus B^2 + (2Er + B)(B^2 + 4E^2r^2\sin^2\gamma) - 2Er\,\mu_\oplus\right]}{(2E)^{5/2}\left[\mu_\oplus + 2Er + B\right] B}$$

$$\frac{\partial t_{p\alpha}}{\partial \gamma} = \frac{(2Er + B)\, r^2 V \cos\gamma \sin\gamma}{(2E)^{1/2}\,(\mu_\oplus + 2Er + B)\, B}$$

Where $E = \frac{V^2}{2} - \frac{\mu_\oplus}{r}$ and $B = \sqrt{4E^2r^2 + 4E\mu_\oplus r - 2Er^2V^2\cos^2\gamma}$

$$\frac{\partial i_\alpha}{\partial \delta_{in}} = \frac{\sin A'_{in} \sin \delta_{in}}{\sqrt{1 - \cos^2 \delta_{in} \sin^2 A'_{in}}} = \frac{\sin A'_{in} \sin \delta_{in}}{\sin i_\alpha}$$

$$\frac{\partial i_\alpha}{\partial A'_{in}} = - \frac{\cos A'_{in} \cos \delta_{in}}{\sin i_\alpha}$$

$$\frac{\partial \Omega_\alpha}{\partial \delta_{in}} = - \frac{\cos \delta_{in} \sin A'_{in} \cos A'_{in}}{\sin^2 i_\alpha}$$

$$\frac{\partial \Omega_\alpha}{\partial A'_{in}} = - \frac{\sin \delta_{in}}{\sin^2 i_\alpha}$$

$$\frac{\partial \omega_\alpha}{\partial \delta_{in}} = \frac{\cos A'_{in}}{\sin^2 i_\alpha}$$

$$\frac{\partial \omega_\alpha}{\partial A'_{in}} = \frac{\cos \delta_{in} \sin \delta_{in} \sin A'_{in}}{\sin^2 i_\alpha}$$

$$\frac{\partial \omega_\alpha}{\partial r} = \frac{V^2 \cos^2 \gamma}{\sqrt{\mu_\oplus^2 e_\alpha^2 - (rV^2 \cos^2 \gamma - \mu_\oplus)^2}} \left\{ 1 + \frac{1}{e_\alpha^2} \left(\frac{rV^2}{\mu_\oplus} - 1 \right) \left(1 - \frac{rV^2 \cos^2 \gamma}{\mu_\oplus} \right) \right\}$$

$$\frac{\partial \omega_\alpha}{\partial V} = \frac{2\, rV \cos^2 \gamma}{\sqrt{\mu_\oplus^2 e_\alpha^2 - (rV^2 \cos^2 \gamma - \mu_\oplus)^2}} \left\{ 1 + \frac{1}{e_\alpha^2} \left(\frac{rV^2}{\mu_\oplus} - 1 \right) \left(1 - \frac{rV^2 \cos^2 \gamma}{\mu_\oplus} \right) \right\}$$

$$\frac{\partial \omega_\alpha}{\partial \gamma} = - \frac{r\, V^2 \sin 2\gamma}{\sqrt{\mu_\oplus^2 e^2 - (rV^2 \cos^2 \gamma - \mu_\oplus)^2}} \left\{ 1 + \frac{1}{2 e_\alpha^2} \left(\frac{rV^2}{\mu_\oplus} - 2 \right) \left(1 - \frac{rV^2 \cos 2\gamma}{\mu_\oplus} \right) \right\}$$

(2, II)

$$
\begin{vmatrix} \Delta a_\alpha \\ \Delta e_\alpha \\ \Delta t_{p_\alpha} \\ \Delta i_\alpha \\ \Delta \Omega_\alpha \\ \Delta \omega_\alpha \end{vmatrix}
=
\begin{vmatrix}
\frac{\partial a_\alpha}{\partial r_{in}} & \frac{\partial a_\alpha}{\partial v_{in}} & \frac{\partial a_\alpha}{\partial \gamma_{in}} & 0 & 0 & 0 & 0 \\
\frac{\partial e_\alpha}{\partial r_{in}} & \frac{\partial e_\alpha}{\partial v_{in}} & \frac{\partial e_\alpha}{\partial \gamma_{in}} & 0 & 0 & 0 & 0 \\
\frac{\partial t_{p_\alpha}}{\partial r_{in}} & \frac{\partial t_{p_\alpha}}{\partial v_{in}} & \frac{\partial t_{p_\alpha}}{\partial \gamma_{in}} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{\partial i_\alpha}{\partial A'_{in}} & \frac{\partial i_\alpha}{\partial \delta_{in}} \\
0 & 0 & 0 & 0 & 1 & \frac{\partial \Omega_\alpha}{\partial A'_{in}} & \frac{\partial \Omega_\alpha}{\partial \delta_{in}} \\
\frac{\partial \omega_\alpha}{\partial r_{in}} & \frac{\partial \omega_\alpha}{\partial v_{in}} & \frac{\partial \omega_\alpha}{\partial \gamma_{in}} & 0 & 0 & \frac{\partial \omega_\alpha}{\partial A'_{in}} & \frac{\partial \omega_\alpha}{\partial \delta_{in}}
\end{vmatrix}
\begin{vmatrix} \Delta r_{in} \\ \Delta v_{in} \\ \Delta \gamma_{in} \\ \Delta t_{in} \\ \Delta \alpha_{in} \\ \Delta A'_{in} \\ \Delta \delta_{in} \end{vmatrix}
$$

3. Transformation from an equatorial into an ecliptical geocentric and inertial coordinate system.

The orbital parameters a_ε , e_ε , t_{p_ε} , i_ε , Ω_ε, and ω_ε measured in an ecliptical geocentric and inertial coordinate system are related to the orbital parameters a_α , e_α , t_{p_α} , i_α , Ω_α and ω_α measured in an equatorial geocentric and inertial coordinate system, by the following relationships.

$$a_\varepsilon = a_\alpha$$

$$e_\varepsilon = e_\alpha$$

$$t_{p_\varepsilon} = t_{p_\alpha}$$

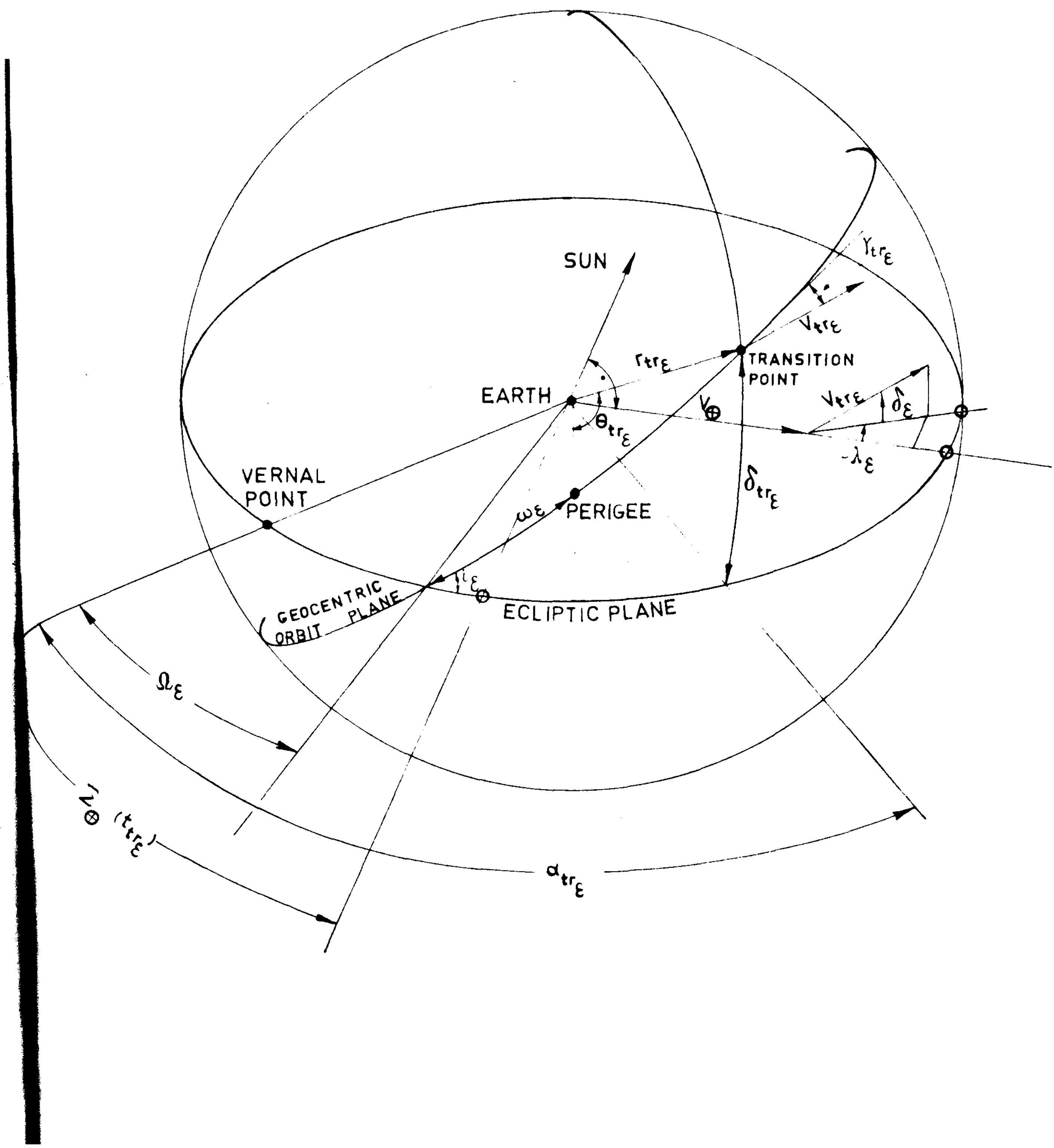
$$\cos i_\varepsilon = \cos \varepsilon \cdot \cos i_\alpha + \sin \varepsilon \cdot \sin i_\alpha \cdot \cos \Omega_\alpha$$

$$\cos \Omega_\varepsilon = \frac{\cos \varepsilon \cdot \sin i_\alpha \cdot \cos \Omega_\alpha - \sin \varepsilon \cdot \cos i_\alpha}{\sin i_\varepsilon}$$

$$\cos\left(\omega_\alpha - \omega_\varepsilon\right) = \frac{\cos \varepsilon \cdot \sin i_\alpha - \sin \varepsilon \cdot \cos i_\alpha \cdot \cos \Omega_\alpha}{\sin i_\varepsilon}$$

The errors Δ_ε in system ε are, therefore, related to the errors in system α by a set of equations, which can be written conveniently, in matrix form :

GEOCENTRIC ECLIPTIC ORBITAL AND TRANSITION PARAMETERS



$$
\begin{vmatrix} \Delta a_\varepsilon \\ \Delta e_\varepsilon \\ \Delta t_{p_\varepsilon} \\ \Delta i_\varepsilon \\ \Delta \Omega_\varepsilon \\ \Delta \omega_\varepsilon \end{vmatrix}
\begin{vmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\partial i_\varepsilon}{\partial i_\alpha} & \frac{\partial i_\varepsilon}{\partial \Omega_\alpha} & 0 \\
0 & 0 & 0 & \frac{\partial \Omega_\varepsilon}{\partial i_\alpha} & \frac{\partial \Omega_\varepsilon}{\partial \Omega_\alpha} & 0 \\
0 & 0 & 0 & \frac{\partial \omega_\varepsilon}{\partial i_\alpha} & \frac{\partial \omega_\varepsilon}{\partial \Omega_\alpha} & 1
\end{vmatrix}
\begin{vmatrix} \Delta a_\alpha \\ \Delta e_\alpha \\ \Delta t_{p_\alpha} \\ \Delta i_\alpha \\ \Delta \Omega_\alpha \\ \Delta \omega_\alpha \end{vmatrix}
$$

or, symbolically.

$$\left| \Delta_\varepsilon \right| = \left| \mathfrak{M}_{\alpha,\varepsilon} \right| \left| \Delta_\alpha \right|$$

with the following expressions for the partial derivatives:

$$\frac{\partial i_\varepsilon}{\partial i_\alpha} = \cos\left(\omega_\alpha - \omega_\varepsilon\right)$$

$$\frac{\partial i_\varepsilon}{\partial \Omega_\alpha} = \frac{\sin \varepsilon . \sin i_\alpha . \sin \Omega_\alpha}{\sin i_\varepsilon}$$

$$\frac{\partial \Omega_\varepsilon}{\partial i_\alpha} = - \frac{\cos \varepsilon . \cos i_\alpha . \cos \Omega_\alpha + \sin \varepsilon . \sin i_\alpha - \cos \Omega_\varepsilon . \cos(\omega_\alpha - \omega_\varepsilon) \cos i_\varepsilon}{\sin i_\varepsilon \; . \; \sin \Omega_\varepsilon}$$

$$\frac{\partial \Omega_\varepsilon}{\partial \Omega_\alpha} = \frac{\sin i_\alpha \cdot \sin \Omega_\alpha}{\sin i_\varepsilon \cdot \sin \Omega_\varepsilon} \cdot \left[\cos \varepsilon + \cos \Omega_\varepsilon \cdot \sin \varepsilon \cdot \operatorname{cotg} i_\varepsilon\right]$$

$$\frac{\partial \omega_\varepsilon}{\partial i_\alpha} = \frac{\left[\cos \varepsilon \cdot \cos i_\alpha + \sin \varepsilon \cdot \sin i_\alpha \cdot \cos \Omega_\alpha\right] - \cos^2(\omega_\alpha - \omega_\varepsilon) \cos i}{\sin i_\varepsilon \cdot \sin(\omega_\alpha - \omega_\varepsilon)}$$

$$\frac{\partial \omega_\varepsilon}{\partial \Omega_\alpha} = \frac{\sin \varepsilon \cdot \cos i_\alpha \cdot \sin \Omega_\alpha - \cos(\omega_\alpha - \omega_\varepsilon) \operatorname{cotg} i_\varepsilon \cdot \sin \varepsilon \cdot \sin i_\alpha \cdot \sin \Omega_\alpha}{\sin i_\varepsilon \cdot \sin(\omega_\alpha - \omega_\varepsilon)}$$

4. Relationships between the orbital parameters and transition point conditions.

The following parameters determine the transition point in the Eart's sphere of influence and the velocity of the probe there, as well as the transit time :

$$r_{tr_\varepsilon} \text{ " } \delta_{tr_\varepsilon} \text{ " } \alpha_{tr_\varepsilon} \text{ " } V_{tr} \text{ " } \lambda_\varepsilon \text{ " } \delta_\varepsilon \text{ " } t_{tr_\varepsilon}$$

These are given in terms of the orbital parameters :

$$a_\varepsilon \text{ " } e_\varepsilon \text{ " } t_{p_\varepsilon} \text{ " } i_\varepsilon \text{ " } \Omega_\varepsilon \text{ " } \omega_\varepsilon$$

by the following relations for hyperbolic orbits.

$$\sin \delta_{tr_\varepsilon} = \sin i_\varepsilon \cdot \sin (\omega_\varepsilon + \theta_{tr_\varepsilon})$$

$$\tan (\alpha_{tr_\varepsilon} - \Omega_\varepsilon) = \cos i_\varepsilon \cdot \tan (\omega_\varepsilon + \theta_{tr_\varepsilon})$$

$$V_{tr} = \sqrt{\frac{\mu_\oplus (2a_\varepsilon + r_{tr_\varepsilon})}{a_\varepsilon \, r_{tr_\varepsilon}}}$$

$$r_{tr_\varepsilon} = \text{constant } (\sim 900000 \text{ Km})$$

$$\cos \theta_{tr_\varepsilon} = \frac{a_\varepsilon (e_\varepsilon^2 - 1)}{e_\varepsilon \, r_{tr_\varepsilon}} - \frac{1}{e_\varepsilon}$$

$$\sin \delta_\varepsilon = \sin i_\varepsilon \cdot \cos (\Upsilon_{tr_\varepsilon} - \omega_\varepsilon - \theta_{tr_\varepsilon})$$

$$\tan\left(\sum_{\oplus}(t_{tr_\varepsilon}) - \lambda_\varepsilon - \Omega_\varepsilon\right) = \frac{\tan(\Upsilon_{tr_\varepsilon} - \omega_\varepsilon - \theta_{tr_\varepsilon})}{\cos i_\varepsilon}$$

$$\cos \Upsilon_{tr_\varepsilon} = \sqrt{\frac{\mu_\oplus a_\varepsilon(e_\varepsilon^2 - 1)}{r_{tr_\varepsilon} V_{tr_\varepsilon}}}$$

$$t_{tr_\varepsilon} - t_{p_\varepsilon} = \frac{1}{2E}\Bigg[\sqrt{2Er_{tr_\varepsilon}^2 + 2\mu_\oplus r_{tr_\varepsilon} - J^2} -$$

$$- \frac{\mu_\oplus}{\sqrt{2E}} L\left(\mu_\oplus + 2E\, r_{tr_\varepsilon} + \sqrt{2E}\sqrt{2E\, r_{tr_\varepsilon}^2 + 2\mu_\oplus r_{tr_\varepsilon} - J^2}\right)\Bigg] +$$

$$+ \frac{a_\varepsilon^{3/2}}{\mu_\oplus^{1/2}} L(\mu_\oplus e_\varepsilon)$$

Where $E = \dfrac{\mu_\oplus}{2 a_\varepsilon}$ " $J = \sqrt{\mu_\oplus a_\varepsilon (e_\varepsilon^2 - 1)}$

Effect of orbital parameters errors on transition conditions.

Differentation of preceding equations permits to express the errors in the transition parameters in terms of the errors in the orbital parameters; symbolically:

$$\left|\Delta_{tr_\varepsilon}\right| = \left|\mathcal{M}_{\varepsilon, tr_\varepsilon}\right| \left|\Delta_\varepsilon\right|$$

where $\left|\mathcal{M}_{tr_\varepsilon, \varepsilon}\right|$ is given by 4.1.

$$
\begin{vmatrix}
\Delta r_{tr_\varepsilon} \\
\Delta \alpha_{tr_\varepsilon} \\
\Delta \delta_{tr_\varepsilon} \\
\Delta v_{tr} \\
\Delta \lambda_\varepsilon \\
\Delta \delta_\varepsilon \\
\Delta t_{tr_\varepsilon}
\end{vmatrix}
=
\begin{vmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\partial \alpha_{tr_\varepsilon}}{\partial a_\varepsilon} & \frac{\partial \alpha_{tr_\varepsilon}}{\partial e_\varepsilon} & 0 & \frac{\partial \alpha_{tr_\varepsilon}}{\partial i_\varepsilon} & 1 & \frac{\partial \alpha_{tr_\varepsilon}}{\partial \omega_\varepsilon} \\
\frac{\partial \delta_{tr_\varepsilon}}{\partial a_\varepsilon} & \frac{\partial \delta_{tr_\varepsilon}}{\partial e_\varepsilon} & 0 & \frac{\partial \delta_{tr_\varepsilon}}{\partial i_\varepsilon} & 0 & \frac{\partial \delta_{tr_\varepsilon}}{\partial \omega_\varepsilon} \\
\frac{\partial v_{tr}}{\partial a_\varepsilon} & 0 & 0 & 0 & 0 & 0 \\
\frac{\partial \lambda_\varepsilon}{\partial a_\varepsilon} & \frac{\partial \lambda_\varepsilon}{\partial e_\varepsilon} & \frac{\partial \lambda_\varepsilon}{\partial t_{p\varepsilon}} & \frac{\partial \lambda_\varepsilon}{\partial i_\varepsilon} & 1 & \frac{\partial \lambda_\varepsilon}{\partial \omega_\varepsilon} \\
\frac{\partial \delta_\varepsilon}{\partial a_\varepsilon} & \frac{\partial \delta_\varepsilon}{\partial e_\varepsilon} & 0 & \frac{\partial \delta_\varepsilon}{\partial i_\varepsilon} & 0 & \frac{\partial \delta_\varepsilon}{\partial \omega_\varepsilon} \\
\frac{\partial t_{tr_\varepsilon}}{\partial a_\varepsilon} & \frac{\partial t_{tr_\varepsilon}}{\partial e_\varepsilon} & 1 & 0 & 0 & 0
\end{vmatrix}
\begin{vmatrix}
\Delta a_\varepsilon \\
\Delta e_\varepsilon \\
\Delta t_{r_\varepsilon} \\
\Delta i_\varepsilon \\
\Delta \Omega_\varepsilon \\
\Delta \omega_\varepsilon
\end{vmatrix}
\qquad \boxed{4.1}
$$

Expressions of the partial derivatives

$$\frac{\partial \alpha_{tr_\varepsilon}}{\partial a_\varepsilon} = \frac{\cos i_\varepsilon \cdot \cos^2(\alpha_{tr_\varepsilon} - \Omega_\varepsilon)}{\cos^2 (\omega_\varepsilon + \theta_{tr_\varepsilon})} \cdot \frac{1 - e_\varepsilon^2}{e_\varepsilon \, r_{tr_\varepsilon} \sin \theta_{tr_\varepsilon}}$$

$$\frac{\partial \alpha_{tr_\varepsilon}}{\partial e_\varepsilon} = \frac{\cos i_\varepsilon \cdot \cos^2(\alpha_{tr_\varepsilon} - \Omega_\varepsilon)}{\cos^2 (\omega_\varepsilon + \theta_{tr_\varepsilon})} \cdot \frac{a_\varepsilon (e_\varepsilon^2 - 1) + r_{tr_\varepsilon}}{r_{tr_\varepsilon} e_\varepsilon^2 \sin \theta_{tr_\varepsilon}}$$

$$\frac{\partial \alpha_{tr_\varepsilon}}{\partial i_\varepsilon} = - \cos^2 (\alpha_{tr_\varepsilon} - \Omega_\varepsilon) \cdot \sin i_\varepsilon \cdot \tan (\omega_\varepsilon + \theta_{tr_\varepsilon})$$

$$\frac{\partial \alpha_{tr_\varepsilon}}{\partial \omega_\varepsilon} = \frac{\cos i_\varepsilon \cdot \cos^2 (\alpha_{tr_\varepsilon} - \Omega_\varepsilon)}{\cos^2 (\omega_\varepsilon - \theta_{tr_\varepsilon})}$$

$$\frac{\partial \delta_{tr_\varepsilon}}{\partial a_\varepsilon} = \frac{\sin i_\varepsilon \cdot \cos (\omega_\varepsilon + \theta_{tr_\varepsilon})}{\cos \delta_{tr_\varepsilon}} \cdot \frac{1 - e_\varepsilon^2}{r_{tr_\varepsilon} e_\varepsilon \sin \theta_{tr_\varepsilon}}$$

$$\frac{\partial \delta_{tr_\varepsilon}}{\partial e_\varepsilon} = \frac{\sin i_\varepsilon \cdot \cos (\omega_\varepsilon + \theta_{tr_\varepsilon})}{\cos \delta_{tr_\varepsilon}} \cdot \frac{a_\varepsilon (e_\varepsilon^2 - 1) + r_{tr_\varepsilon}}{r_{tr_\varepsilon} e_\varepsilon^2 \sin \theta_{tr_\varepsilon}}$$

$$\frac{\partial \delta_{tr_\varepsilon}}{\partial i_\varepsilon} = \frac{\cos i_\varepsilon \cdot \sin (\omega_\varepsilon + \theta_{tr_\varepsilon})}{\cos \delta_{tr_\varepsilon}}$$

$$\frac{\partial \delta_{tr_\varepsilon}}{\partial \omega_\varepsilon} = \frac{\sin i_\varepsilon \cdot \cos(\omega_\varepsilon + \theta_{tr_\varepsilon})}{\cos \delta_{tr_\varepsilon}}$$

$$\frac{\partial V_{tr}}{\partial a_\varepsilon} = - \frac{\mu_\oplus}{2\, V_{tr}\, a_\varepsilon^2}$$

$$\frac{\partial \lambda_\varepsilon}{\partial a_\varepsilon} = - \frac{\cos^2\left[\Sigma_\oplus(t_{tr_\varepsilon}) - \lambda_\varepsilon - \Omega_\varepsilon\right]}{\cos i_\varepsilon \cdot \cos^2(\Upsilon_{tr_\varepsilon} - \omega_\varepsilon - \theta_{tr_\varepsilon})}\left[\frac{e_\varepsilon^2 - 1}{e_\varepsilon \Upsilon r_{tr_\varepsilon}} \quad \frac{1}{\sin \theta_{tr_\varepsilon}} - \frac{3\, a_\varepsilon + r_{tr_\varepsilon} - a_\varepsilon e_\varepsilon^2}{\sin \Upsilon_{tr_\varepsilon}} \sqrt{\frac{r_{tr_\varepsilon}(e_\varepsilon^2 - 1)}{2a_\varepsilon + r_{tr_\varepsilon}}}\right] + \frac{2\pi}{365'25} \frac{\partial t_{tr_\varepsilon}}{\partial a_\varepsilon}$$

$$\frac{\partial \lambda_\varepsilon}{\partial e_\varepsilon} = - \frac{\cos^2\left[\Sigma_\oplus(t_{tr_\varepsilon}) - \lambda_\varepsilon - \Omega_\varepsilon\right]}{\cos i_\varepsilon \cdot \cos^2(\Upsilon_{tr_\varepsilon} - \omega_\varepsilon - \theta_{tr_\varepsilon})}\left[\frac{a_\varepsilon(e_\varepsilon^2 + 1) + r_{tr_\varepsilon}}{e_\varepsilon^2\, r_{tr_\varepsilon}} \cdot \frac{1}{\sin \theta_{tr_\varepsilon}} - \frac{a_\varepsilon}{\sin \Upsilon_{tr_\varepsilon}} \sqrt{\frac{e_\varepsilon}{r_{tr_\varepsilon}(2a_\varepsilon + r_{tr_\varepsilon})(e_\varepsilon^2 - 1)}}\right] + \frac{2\pi}{365'25} \frac{\partial t_{tr_\varepsilon}}{\partial e_\varepsilon}$$

$$\frac{\partial \lambda_\varepsilon}{\partial t_{p_\varepsilon}} = \frac{2\pi}{365{,}25}$$

$$\frac{\partial \lambda_{\varepsilon}}{\partial i_{\varepsilon}} = - \frac{\cos^2\left[\Sigma_{\oplus}(t_{tr_{\varepsilon}}) - \lambda_{\varepsilon} - \Omega_{\varepsilon}\right] \cdot \tan(\gamma_{tr_{\varepsilon}} - \omega_{\varepsilon} - \theta_{tr_{\varepsilon}}) \cdot \sin i_{\varepsilon}}{\cos^2 i_{\varepsilon}}$$

$$\frac{\partial \lambda_{\varepsilon}}{\partial \omega_{\varepsilon}} = \frac{\cos^2\left[\Sigma_{\oplus}(t_{tr_{\varepsilon}}) - \lambda_{\varepsilon} - \Omega_{\varepsilon}\right]}{\cos i_{\varepsilon} \cdot \cos^2(\gamma_{tr_{\varepsilon}} - \omega_{\varepsilon} - \theta_{tr_{\varepsilon}})}$$

$$\frac{\partial \delta_{\varepsilon}}{\partial a_{\varepsilon}} = \frac{\sin i_{\varepsilon} \cdot \sin(\gamma_{tr_{\varepsilon}} - \omega_{\varepsilon} - \theta_{tr_{\varepsilon}})}{\cos \delta_{\varepsilon}}$$

$$\frac{3 a_{\varepsilon} - r_{tr_{\varepsilon}} - a_{\varepsilon} e_{\varepsilon}^2}{\sin \gamma_{tr_{\varepsilon}}} \sqrt{\frac{r_{tr_{\varepsilon}}(e_{\varepsilon}^2 - 1)}{2a_{\varepsilon} - r_{tr_{\varepsilon}}}} - \frac{e_{\varepsilon}^2 - 1}{e_{\varepsilon}\, r_{tr_{\varepsilon}}} \quad \frac{1}{\sin \theta_{tr_{\varepsilon}}} \Bigg]$$

$$\frac{\partial \delta_{\varepsilon}}{\partial e_{\varepsilon}} = \frac{\sin i_{\varepsilon} \cdot \sin(\gamma_{tr_{\varepsilon}} - \omega_{\varepsilon} - \theta_{tr_{\varepsilon}})}{\cos \delta_{\varepsilon}}$$

$$\frac{a_{\varepsilon}}{\sin \gamma_{tr_{\varepsilon}}} \sqrt{\frac{e_{\varepsilon}}{r_{tr_{\varepsilon}}(2a_{\varepsilon} + r_{tr_{\varepsilon}})(e_{\varepsilon}^2 - 1)}} - \frac{1}{\sin \theta_{tr_{\varepsilon}}} \frac{a_{\varepsilon}(e_{\varepsilon}^2 + 1) + r_{tr_{\varepsilon}}}{e_{\varepsilon}^2\, r_{tr_{\varepsilon}}} \Bigg]$$

$$\frac{\partial \delta_{\varepsilon}}{\partial i_{\varepsilon}} = \frac{\cos i_{\varepsilon} \cdot \cos(\gamma_{tr_{\varepsilon}} - \omega_{\varepsilon} - \theta_{tr_{\varepsilon}})}{\cos \delta_{\varepsilon}}$$

$$\frac{\partial \delta_{\varepsilon}}{\partial \omega_{\varepsilon}} = \frac{\sin i_{\varepsilon} \cdot \sin(\gamma_{tr_{\varepsilon}} - \omega_{\varepsilon} - \theta_{tr_{\varepsilon}})}{\cos \delta_{\varepsilon}}$$

$$\frac{\partial t_{tr_{\varepsilon}}}{\partial a_{\varepsilon}} = \frac{1}{\sqrt{\mu_{\oplus}}} \left\{ \frac{r_{tr_{\varepsilon}}^{2} + 4\, a_{\varepsilon}\, r_{tr_{\varepsilon}} - 3a_{\varepsilon}^{2}\,(e_{\varepsilon}^{2} - 1)}{2\sqrt{a_{\varepsilon}\, r_{tr_{\varepsilon}}^{2} + 2r_{tr_{\varepsilon}} a_{\varepsilon}^{2} - a_{\varepsilon}^{2}\,(e_{\varepsilon}^{2} - 1)}} \right. +$$

$$+ \frac{3\, a_{\varepsilon}^{1/2}}{2}\, L \left(\frac{a_{\varepsilon} + r_{tr_{\varepsilon}} + \sqrt{r_{tr_{\varepsilon}}^{2} + 2r_{tr_{\varepsilon}}\, a_{\varepsilon} - a_{\varepsilon}^{2}\,(e_{\varepsilon}^{2} - 1)}}{a_{\varepsilon}\, e_{\varepsilon}} \right) -$$

$$\left. - \frac{r_{tr_{\varepsilon}}\, a_{\varepsilon}^{1/2}}{\sqrt{r_{tr_{\varepsilon}}^{2} + 2\, r_{tr_{\varepsilon}}\, a_{\varepsilon} - a_{\varepsilon}^{2}\,(e_{\varepsilon}^{2} - 1)}} \right\}$$

$$\frac{\partial t_{tr_{\varepsilon}}}{\partial e_{\varepsilon}} = \frac{-1}{\sqrt{\mu_{\oplus}}} \left[\frac{e_{\varepsilon}\, a_{\varepsilon}^{3}}{\sqrt{a_{\varepsilon}\, r_{tr_{\varepsilon}}^{2} + 2\, r_{tr_{\varepsilon}}\, a_{\varepsilon}^{2} - a_{\varepsilon}^{2}\,(e_{\varepsilon}^{2} - 1)}} \right. +$$

$$\left. + a_{\varepsilon}^{3/2} \frac{r_{tr_{\varepsilon}} + a_{\varepsilon}}{\sqrt{r_{tr_{\varepsilon}}^{2} + 2\, r_{tr_{\varepsilon}}\, a_{\varepsilon} - a_{\varepsilon}^{2}\,(e_{\varepsilon}^{2} - 1)}} \right]$$

5. Relationships between the transition conditions with respect to the Earth and the injection conditions in the heliocentric orbit.

These injection conditions are the following in a heliocentric inertial reference system:

$$r_{tr_\sigma},\ \delta_{tr_\sigma},\ \alpha_{tr_\sigma},\ V_I,\ \gamma_\sigma,\ i_\sigma,\ t_{tr_\sigma}$$

That are given in terms of the transition conditions with respect to a geocentric ecliptic inertial reference system:

$$r_{tr_\varepsilon},\ \delta_{tr_\varepsilon},\ \alpha_{tr_\varepsilon},\ V_{tr},\ \lambda_\varepsilon,\ \delta_\varepsilon,\ t_{tr_\varepsilon}$$

by the following relations:

$$\tan\alpha_{tr_\sigma} = \frac{\cos\delta_{tr_\varepsilon} \cdot \sin\alpha_{tr_\varepsilon} + \frac{r_\oplus}{r_{tr_\varepsilon}} \sin \Sigma_\oplus (t_{tr_\varepsilon})}{\frac{r_\oplus}{r_{tr_\varepsilon}} \cos \Sigma_\oplus (t_{tr_\varepsilon}) + \cos\alpha_{tr_\varepsilon} \cos\delta_{tr_\varepsilon}}$$

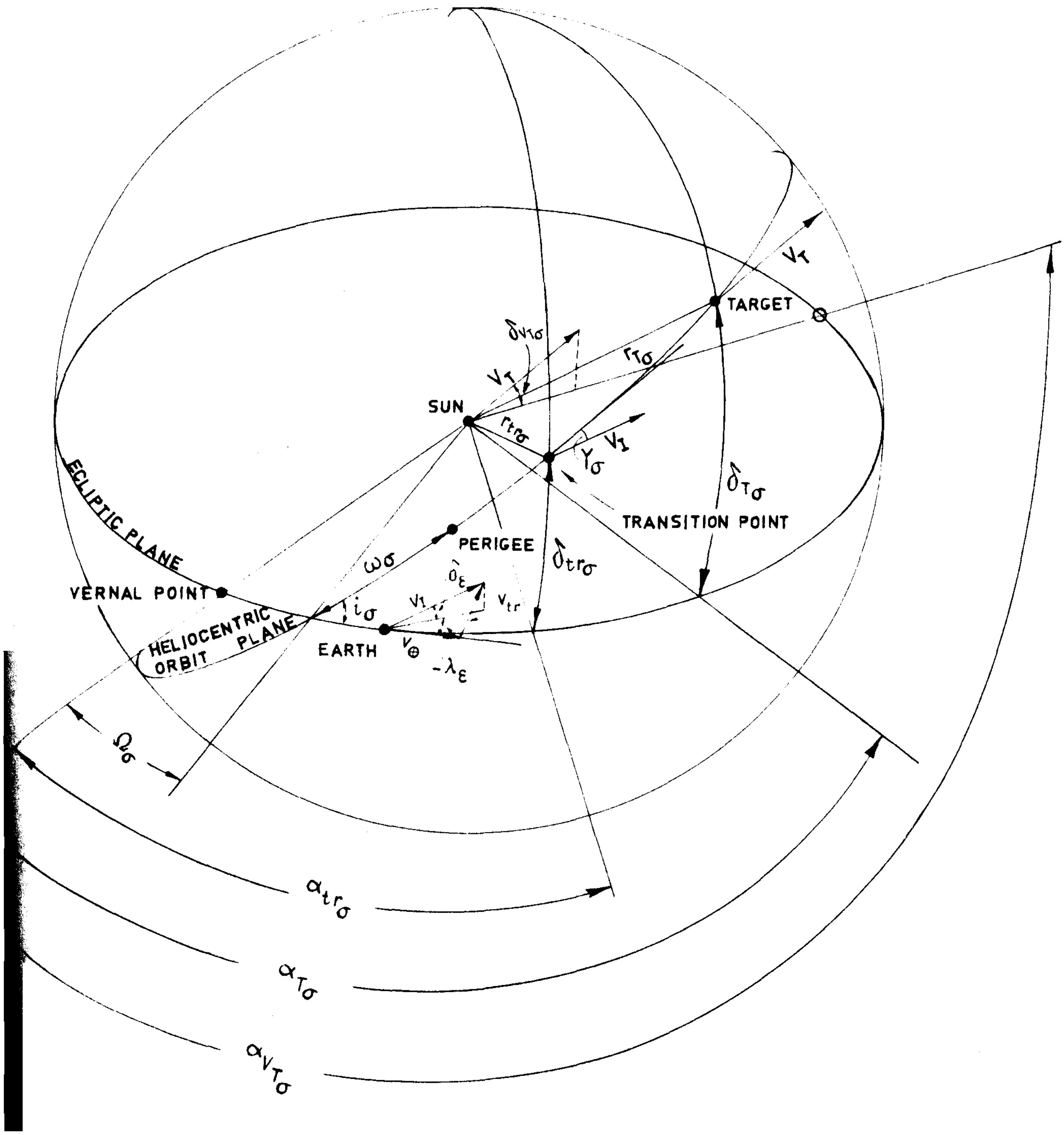
$$\tan^2 \delta_{tr_\sigma} = \frac{r_{tr_\varepsilon}^2 \sin^2 \delta_{tr_\varepsilon}}{r_\oplus^2 + r_{tr_\varepsilon}^2 \cos^2\delta_{tr_\varepsilon} + 2r_\oplus r_{tr_\varepsilon} \cos\delta_{tr_\varepsilon} \cos\left[\alpha_{tr_\varepsilon} - \Sigma_\oplus (t_{tr_\varepsilon})\right]}$$

$$r_{tr_\sigma} = \frac{r_{tr_\varepsilon} \sin\delta_{tr_\varepsilon}}{\sin\delta_{tr_\sigma}}$$

$$V_I^2 = V_\oplus^2 + 2V_{tr} V_\oplus \cos\delta_\varepsilon \cos\lambda_\varepsilon + V_{tr}^2$$

$$t_{tr_\sigma} = t_{tr_\varepsilon}$$

TRANSITION AND HELIOCENTRIC ORBITAL PARAMETERS



$$\mathrm{Sen}\,\gamma_\sigma = (V_I)^{-1}\Big\{V_{tr}\cos\delta_\varepsilon \cos\delta_{tr_\sigma} \sin\left[\lambda_\varepsilon + \alpha_{tr_\sigma} - \Sigma_\oplus(t_{tr_\varepsilon})\right] + V_{tr}\sin\delta_\varepsilon \sin\delta_{tr_\sigma} + V_\oplus \cos\delta_{tr_\sigma} \sin\left[\alpha_{tr_\sigma} - \Sigma_\oplus(t_{tr_\varepsilon})\right]\Big\}$$

$$\cos i_\sigma = (V_I \cos\gamma_\sigma)^{-1}\Big\{V_\oplus \cos\delta_{tr_\sigma} \cos\left[\alpha_{tr_\sigma} - \Sigma_\oplus(t_{tr_\varepsilon})\right] + V_{tr}\cos\delta_\varepsilon \cos\delta_{tr_\sigma} \cos\left[\alpha_{tr_\sigma} + \lambda_\varepsilon - \Sigma_\oplus(t_{tr_\varepsilon})\right]\Big\}$$

However, taking into account that $r_{tr_\varepsilon}/r_\oplus$ is a small number ($\simeq 0.9/150$) we shall use the following appoximate relations in which terms of order $[r_{tr_\varepsilon}/r_\oplus]^2$ have been:

$$\alpha_{tr_\sigma} = \Sigma_\oplus(t_{tr_\varepsilon}) + \frac{r_{tr\varepsilon}}{r_\oplus}\cos\delta_{tr_\varepsilon} \sin\left[\alpha_{tr_\varepsilon} - \Sigma_\oplus(t_{tr_\varepsilon})\right]$$

$$\delta_{tr_\sigma} = \frac{r_{tr\varepsilon}}{r_\oplus}\sin\delta_{tr_\varepsilon}$$

$$r_{tr_\sigma} = r_\oplus + r_{tr_\varepsilon}\cos\delta_{tr_\varepsilon}\sin\left[\alpha_{tr_\varepsilon} - \Sigma_\oplus(t_{tr_\varepsilon})\right]$$

$$V_I^2 = V_\oplus^2 + 2V_{tr}V_\oplus \cos\delta_\varepsilon \cos\lambda_\varepsilon + V_{tr}^2$$

$$t_{tr_\sigma} = t_{tr_\varepsilon}$$

$$\sin\gamma_\sigma = \frac{V_{tr}\cos\delta_\varepsilon \sin\lambda_\varepsilon + \left[\alpha_{tr_\sigma} - \Sigma_\oplus(t_{r_\varepsilon})\right](V_\oplus + V_{tr}\cos\delta_\varepsilon\cos\lambda_\varepsilon) + \delta_{tr_\sigma}\sin\delta_\varepsilon V_{tr}}{V_I}$$

$$\cos i_\sigma = \frac{V_\oplus + V_{tr}\cos\delta_\varepsilon\cos\lambda_\varepsilon - \left[\alpha_{tr\sigma} - \Sigma_\oplus(t_{tr\varepsilon})\right]V_{tr}\cos\delta_\varepsilon\sin\lambda_\varepsilon}{V_I\cos\gamma}$$

Effect of transition errors on injection error in the heliocentric orbit.

Differentiation of the preceding equations permits to express the errors in the injection conditions in terms of the errors in - transition conditions.

$$\left| \Delta_{tr,\sigma} \right| = \left| \mathcal{M}_{tr\varepsilon, tr\sigma} \right| \quad \left| \Delta_{tr_\varepsilon} \right|$$

where $\left| \mathcal{M}_{tr\varepsilon, tr\sigma} \right|$ is given by 5.1

with the following expressions for the derivatives

$$\frac{\partial r_{tr\sigma}}{\partial r_{tr_\varepsilon}} = \cos \delta_{tr_\varepsilon} \cdot \sin \left[\alpha_{tr_\varepsilon} - \Sigma_{\oplus}(t_{tr_\varepsilon}) \right]$$

$$\frac{\partial r_{tr\sigma}}{\partial \alpha_{tr_\varepsilon}} = r_{tr_\varepsilon} \cdot \cos \delta_{tr_\varepsilon} \cdot \cos \left[\alpha_{tr_\varepsilon} - \Sigma_{\oplus}(t_{tr_\varepsilon}) \right]$$

$$\frac{\partial r_{tr\sigma}}{\partial \delta_{tr_\varepsilon}} = - r_{tr_\varepsilon} \cdot \sin \left[\alpha_{tr_\varepsilon} - \Sigma_{\oplus}(t_{tr_\varepsilon}) \right] \quad \sin \delta_{tr_\varepsilon}$$

$$\frac{\partial r_{tr\sigma}}{\partial t_{tr_\varepsilon}} = - r_{tr_\varepsilon} \cdot \cos \delta_{tr_\varepsilon} \cdot \cos \left[\alpha_{tr_\varepsilon} - \Sigma_{\oplus}(t_{tr_\varepsilon}) \right] \frac{2\pi}{365.25}$$

5.1

$$
\begin{vmatrix} \Delta r_{tr_\sigma} \\ \Delta \alpha_{tr_\sigma} \\ \Delta \delta_{tr_\sigma} \\ \Delta V_I \\ \Delta i_\sigma \\ \Delta \gamma_\sigma \\ \Delta t_{tr_\sigma} \end{vmatrix}
=
\begin{vmatrix}
\frac{\partial r_{tr_\sigma}}{\partial r_{tr_\varepsilon}} & \frac{\partial r_{tr_\sigma}}{\partial \alpha_{tr_\varepsilon}} & \frac{\partial r_{tr_\sigma}}{\partial \delta_{tr_\varepsilon}} & 0 & 0 & 0 & \frac{\partial r_{tr_\sigma}}{\partial t_{tr_\varepsilon}} \\
\frac{\partial \alpha_{tr_\sigma}}{\partial r_{tr_\varepsilon}} & \frac{\partial \alpha_{tr_\sigma}}{\partial \alpha_{tr_\varepsilon}} & \frac{\partial \alpha_{tr_\sigma}}{\partial \delta_{tr_\varepsilon}} & 0 & 0 & 0 & \frac{\partial \alpha_{tr_\sigma}}{\partial t_{tr_\varepsilon}} \\
\frac{\partial \delta_{tr_\sigma}}{\partial r_{tr_\varepsilon}} & 0 & \frac{\partial \delta_{tr_\sigma}}{\partial \delta_{tr_\varepsilon}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\partial V_1}{\partial V_{tr}} & \frac{\partial V_1}{\partial \lambda_\varepsilon} & \frac{\partial V_1}{\partial \delta_\varepsilon} & 0 \\
\frac{\partial i_\sigma}{\partial r_{tr_\varepsilon}} & \frac{\partial i_\sigma}{\partial \alpha_{tr_\varepsilon}} & \frac{\partial i_\sigma}{\partial \delta_{tr_\varepsilon}} & \frac{\partial i_\sigma}{\partial V_{tr}} & \frac{\partial i_\sigma}{\partial \lambda_\varepsilon} & \frac{\partial i_\sigma}{\partial \delta_\varepsilon} & \frac{\partial i_\sigma}{\partial t_{tr_\varepsilon}} \\
\frac{\partial \gamma_\sigma}{\partial r_{tr_\varepsilon}} & \frac{\partial \gamma_\sigma}{\partial \alpha_{tr_\varepsilon}} & \frac{\partial \gamma_\sigma}{\partial \delta_{tr_\varepsilon}} & \frac{\partial \gamma_\sigma}{\partial V_{tr}} & \frac{\partial \gamma_\sigma}{\partial \lambda_\varepsilon} & \frac{\partial \gamma_\sigma}{\partial \delta_\varepsilon} & \frac{\partial \gamma_\sigma}{\partial t_{tr_\varepsilon}} \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{vmatrix}
\begin{vmatrix} \Delta r_{tr_\varepsilon} = 0 \\ \Delta \alpha_{tr_\varepsilon} \\ \Delta \delta_{tr_\varepsilon} \\ \Delta V_{tr} \\ \Delta \lambda_\varepsilon \\ \Delta \delta_\varepsilon \\ \Delta t_{tr_\varepsilon} \end{vmatrix}
$$

$$\frac{\partial \alpha_{tr_\sigma}}{\partial r_{tr_\varepsilon}} = \frac{\cos \delta_{tr_\varepsilon}}{r_\oplus} \sin \left[\alpha_{tr_\varepsilon} - \Sigma_\oplus \left(t_{tr_\varepsilon} \right) \right]$$

$$\frac{\partial \alpha_{tr_\sigma}}{\partial \alpha_{tr_\varepsilon}} = \frac{r_{tr_\varepsilon}}{r_\oplus} \cdot \cos \delta_{tr_\varepsilon} \cos \left[\alpha_{tr_\varepsilon} - \Sigma_\oplus \left(t_{tr_\varepsilon} \right) \right]$$

$$\frac{\partial \alpha_{tr_\sigma}}{\partial \delta_{tr_\varepsilon}} = - \frac{r_{tr_\varepsilon}}{r_\oplus} \sin \delta_{tr_\varepsilon} \sin \left[\alpha_{tr_\varepsilon} - \Sigma_\oplus \left(t_{tr_\varepsilon} \right) \right]$$

$$\frac{\partial \alpha_{tr_\sigma}}{\partial t_{tr_\varepsilon}} = \frac{2\pi}{365{,}25} \left\{ 1 - \frac{r_{tr_\varepsilon}}{r_\oplus} \cos \delta_{tr_\varepsilon} \cdot \cos \left[\alpha_{tr_\varepsilon} - \Sigma_\oplus \left(t_{tr_\varepsilon} \right) \right] \right\}$$

$$\frac{\partial \delta_{tr_\sigma}}{\partial r_{tr_\varepsilon}} = \frac{\sin \delta_{tr_\varepsilon}}{r_\oplus}$$

$$\frac{\partial \delta_{tr_\sigma}}{\partial \delta_{tr_\varepsilon}} = \frac{r_{tr_\varepsilon}}{r_\oplus} \cos \delta_{tr_\varepsilon}$$

$$\frac{\partial V_I}{\partial V_{tr}} = \frac{V_\oplus \cos \delta_\varepsilon \cos \lambda_\varepsilon + V_{tr}}{V_I}$$

$$\frac{\partial V_I}{\partial \lambda_\varepsilon} = - \frac{V_{tr} V_\oplus \cos \delta_\varepsilon \cos \lambda_\varepsilon}{V_I}$$

$$\frac{\partial V_I}{\partial \delta_\varepsilon} = - \frac{V_{tr} V_\oplus \sin \delta_\varepsilon \cos \lambda_\varepsilon}{V_I}$$

$$\frac{\partial i_\sigma}{\partial r_{tr_\varepsilon}} = \frac{V_{tr}\cos\delta_\varepsilon \cos\lambda_\varepsilon \left(\frac{\partial \alpha_{tr\sigma}}{\partial r_{tr_\varepsilon}}\right) - V_I \cos i_\sigma \cdot \sin\gamma_\sigma \left(\frac{\partial \gamma_\sigma}{\partial r_{tr_\varepsilon}}\right)}{V_I \cos\gamma_\sigma \sin i_\sigma}$$

$$\frac{\partial i_\sigma}{\partial \alpha_{tr_\varepsilon}} = \frac{V_{tr}\cos\delta_\varepsilon \cos\lambda_\varepsilon \left(\frac{\partial \alpha_{tr\sigma}}{\partial \alpha_{tr\varepsilon}}\right) - V_I \cos i_\sigma \sin\gamma_\sigma \left(\frac{\partial \gamma_\sigma}{\partial \alpha_{tr\varepsilon}}\right)}{V_I \cos\gamma_\sigma \sin i_\sigma}$$

$$\frac{\partial i_\sigma}{\partial \delta_{tr_\varepsilon}} = \frac{V_{tr}\cos\delta_\varepsilon \cos\lambda_\varepsilon \left(\frac{\partial \alpha_{tr\sigma}}{\partial \delta_{tr\varepsilon}}\right) - V_I \cos i_\sigma \sin\gamma_\sigma \left(\frac{\partial \gamma_\sigma}{\partial \delta_{tr_\varepsilon}}\right)}{V_I \cos\gamma_\sigma \sin i_\sigma}$$

$$\frac{\partial i_\sigma}{\partial V_{tr}} = -(V_I \cos\gamma_\sigma \sin i_\sigma)^{-1}\Big\{\cos\delta_\varepsilon \cos\lambda_\varepsilon - \cos\delta_\varepsilon \sin\lambda_\varepsilon\left[\alpha_{tr_\sigma} - \Sigma_\oplus (t_{tr_\varepsilon})\right] + V_I \cos i_\sigma \sin\gamma_\sigma \left(\frac{\partial \gamma_\sigma}{\partial V_{tr}}\right)\Big\}$$

$$\frac{\partial i_\sigma}{\partial \lambda_\varepsilon} = (V_I \cos\gamma_\sigma \sin i_\sigma)^{-1}\Big\{\cos i_\sigma \cos\gamma_\sigma \left(\frac{\partial V_I}{\partial \lambda_\varepsilon}\right) - V_I \cos i_\sigma \sin\gamma_\sigma \left(\frac{\partial \gamma_\sigma}{\partial \lambda_\varepsilon}\right) + V_{tr}\cos\delta_\varepsilon \sin\lambda_\varepsilon + V_{tr}\cos\delta_\varepsilon \cos\lambda_\varepsilon\left[\alpha_{tr_\sigma} - \Sigma_\oplus (t_{tr_\varepsilon})\right]\Big\}$$

$$\frac{\partial i_\sigma}{\partial \delta_\varepsilon} = (V_I \cos\gamma_\sigma \sin i_\sigma)^{-1}\Big\{\cos i_\sigma \cos\gamma_\sigma \left(\frac{\partial V_I}{\partial \delta_\varepsilon}\right) - V \cos i_\sigma \sin\gamma_\sigma \left(\frac{\partial \gamma_\sigma}{\partial \delta_\varepsilon}\right) + V_{tr}\sin\delta_\varepsilon \cos\lambda_\varepsilon - V_{tr}\sin\lambda_\varepsilon \sin\delta_\varepsilon\left[\alpha_{tr_\sigma} - \Sigma_\oplus (t_{tr_\varepsilon})\right]\Big\}$$

$$\frac{\partial i_\sigma}{\partial t_{r_\varepsilon}} = (V_I \cos\gamma_\sigma \sin i_\sigma)^{-1}\Big\{V_{tr}\cos\delta_\varepsilon \cos\lambda_\varepsilon \left(\frac{\partial \alpha_{tr\sigma}}{\partial t_{tr\varepsilon}}\right) - \frac{2\pi}{365{,}25} V_{tr}\cos\delta_\varepsilon \sin\lambda_\varepsilon - V_I \cos i_\sigma \sin\gamma_\sigma \left(\frac{\partial \gamma_\sigma}{\partial t_{r_\varepsilon}}\right)\Big\}$$

$$\frac{\partial \gamma_\sigma}{\partial r_{tr_\varepsilon}} = \frac{(V_\oplus + V_{tr}\cos\delta_\varepsilon\cos\lambda_\varepsilon)\left(\frac{\partial\alpha_{tr\sigma}}{\partial r_{tr_\varepsilon}}\right) + \sin\delta_\varepsilon V_{tr}\left(\frac{\partial\delta_{tr\sigma}}{\partial r_{tr_\varepsilon}}\right)}{V_I\cos\gamma_\sigma}$$

$$\frac{\partial \gamma_\sigma}{\partial \alpha_{tr_\varepsilon}} = \frac{V_\oplus + V_{tr}\cos\delta_\varepsilon\cos\lambda_\varepsilon}{V_I\cos\gamma_\sigma}\left(\frac{\partial\alpha_{tr\sigma}}{\partial\alpha_{tr_\varepsilon}}\right)$$

$$\frac{\partial \gamma_\sigma}{\partial \delta_{tr_\varepsilon}} = \frac{(V_\oplus + V_{tr}\cos\delta_\varepsilon\cos\lambda_\varepsilon)\left(\frac{\partial\alpha_{tr\sigma}}{\partial\delta_{tr\varepsilon}}\right) + \sin\delta_\varepsilon V_{tr}\left(\frac{\partial\delta_{tr\sigma}}{\partial\delta_{tr\varepsilon}}\right)}{V_I\cos\gamma_\sigma}$$

$$\frac{\partial \gamma_\sigma}{\partial V_{tr}} = -(V_I\cos\gamma_\sigma)^{-1}\Big\{\sin\gamma_\sigma\left(\frac{\partial V_I}{\partial V_{tr}}\right) - \cos\delta_\varepsilon\sin\lambda_\varepsilon - \cos\lambda_\varepsilon\cdot\cos\delta_\varepsilon\cdot\left[\alpha_{tr_\sigma} - \Sigma_\oplus(t_{tr_\varepsilon})\right] + \delta_{tr_\sigma}\sin\delta_\varepsilon\Big\}$$

$$\frac{\partial \gamma_\sigma}{\partial \lambda_\varepsilon} = -(V_I\cos\gamma_\sigma)^{-1}\Big\{\sin\gamma_\sigma\left(\frac{\partial V_I}{\partial\lambda_\varepsilon}\right) + V_{tr}\cos\delta_\varepsilon\cos\lambda_\varepsilon + V_{tr}\cos\delta_\varepsilon\sin\lambda_\varepsilon\left[\alpha_{tr_\sigma} - \Sigma_\oplus(t_{tr_\varepsilon})\right]\Big\}$$

$$\frac{\partial \gamma_\sigma}{\partial \delta_\varepsilon} = -(V_I\cos\gamma_\sigma)^{-1}\Big\{\sin\gamma_\sigma\left(\frac{\partial V_I}{\partial\delta_\varepsilon}\right) + V_{tr}\sin\delta_\varepsilon\sin\lambda_\varepsilon + V_{tr}\sin\delta_\varepsilon\cos\lambda_\varepsilon\left[\alpha_{tr\sigma} - \Sigma_\oplus(t_{tr_\varepsilon})\right] + V_{tr}\,\delta_{tr}\cos\delta_\varepsilon\Big\}$$

$$\frac{\partial \gamma_\sigma}{\partial t_{tr_\varepsilon}} = \frac{(V_\oplus + V_{tr}\cos\delta_\varepsilon\cos\lambda_\varepsilon)}{V_I\cos\gamma_\sigma}\left[\left(\frac{\partial\alpha_{tr\sigma}}{\partial t_{tr\varepsilon}}\right) - \frac{2\pi}{365{,}25}\right]$$

6. Relationships between in the injection conditions and orbital parameters of the heliocentric orbit.

We give below the relation between the orbital parameters in a heliocentric ecliptic inertial reference system:

$$a_\sigma \quad e_\sigma \quad t_{p\sigma} \quad i_\sigma \quad \Omega_\sigma \quad \omega_\sigma$$

and the injection conditions into the orbit.

$$r_{tr_\sigma}, \quad \delta_{tr_\sigma}, \quad \alpha_{tr_\sigma}, \quad V_I, \quad \gamma_\sigma, \quad i_\sigma, \quad t_{tr_\sigma}$$

These relations are as follows

$$a_\sigma = \frac{r_{tr_\sigma}}{2 - \dfrac{r_{tr_\sigma} V_I^2}{\mu_\odot}}$$

$$e_\sigma = \sqrt{1 - \left(\frac{2}{r_{tr_\sigma}} - \frac{V_I^2}{\mu_\odot}\right)\left(\frac{r_{tr_\sigma}^2 V_I^2 \cos^2 \gamma_\sigma}{\mu_\odot}\right)}$$

$$t_{tr_\sigma} = t_{p_\sigma} = \frac{1}{2E_\odot}\Big[(2E_\odot r_{tr_\sigma}^2 + 2\mu_\odot r_{tr_\sigma} - r_{tr_\sigma}^2 V_I^2 \cos^2 \gamma_\sigma)^{\frac{1}{2}} +$$

$$+ \frac{\mu_\odot}{\sqrt{-2E_\odot}} \sin^{-1} \frac{\mu_\odot + 2E_\odot r_{tr_\sigma}}{(\mu_\odot^2 - 2\, r_{tr_\sigma}^2 V_I^2 \cos^2 \gamma_\sigma E_\odot)^{\frac{1}{2}}}\Big]$$

$$E_\odot = \frac{V_I^2}{2} - \frac{\mu_\odot}{r_{tr_\sigma}}$$

$$i_\sigma = i_\sigma$$

$$\cos\,(\alpha_{tr_\sigma} - \Omega_\sigma) = \operatorname{ctg}\, i_\sigma \tan \delta_{tr_\sigma}$$

$$\sin\,(\omega_\sigma - \Delta\omega_\sigma) = \frac{\sin \delta_{tr_\sigma}}{\sin i_\sigma}$$

The errors in the heliocentric orbital parameters may be expressed symbolically in terms of the errors in the injection conditions by

$$|\Delta_\sigma| = |\mathfrak{M}_{tr\sigma,\sigma}|\,|\Delta_{tr\sigma}|$$

Where the matrix $|\mathfrak{M}_{tr\sigma,\sigma}|$ is given by 6.1 with the following expressions for the derivatives.

$$\frac{\partial a_\sigma}{\partial r_{tr_\sigma}} = \frac{2\mu_\odot^2}{(r_{tr_\sigma} V_I^2 - 2\mu_\odot)^2}$$

$$\frac{\partial a_\sigma}{\partial V_I} = \frac{2\mu_\odot V_I\, r_{tr_\sigma}^2}{(r_{tr_\sigma} V_I^2 - 2\mu_\odot)^2}$$

$$\frac{\partial e_\sigma}{\partial r_{tr_\sigma}} = \frac{V_I^2 \cos^2 \gamma_\sigma}{\mu_\odot\, e_\sigma} \left(\frac{r_{tr_\sigma} V_I^2}{\mu_\odot} - 1 \right)$$

$$\frac{\partial e_\sigma}{\partial V_I} = \frac{2\, r_{tr_\sigma} V_I \cos^2 \gamma_\sigma}{\mu_\odot\, e_\sigma} \left(\frac{r_{tr_\sigma} V_I^2}{\mu_\odot} - 1 \right)$$

$$\begin{Vmatrix} \Delta a_\sigma \\ \Delta e_\sigma \\ \Delta t_{p_\sigma} \\ \Delta i_\sigma \\ \Delta \Omega_\sigma \\ \Delta \omega_\sigma \end{Vmatrix} = \begin{Vmatrix} \frac{\partial a_\sigma}{\partial r_{tr_\sigma}} & 0 & 0 & \frac{\partial a_\sigma}{\partial V_1} & 0 & 0 & 0 \\ \frac{\partial e_\sigma}{\partial r_{tr_\sigma}} & 0 & 0 & \frac{\partial e_\sigma}{\partial V_1} & 0 & \frac{\partial e_\sigma}{\partial \gamma_\sigma} & 0 \\ \frac{\partial t_{p_\sigma}}{\partial r_{tr_\sigma}} & 0 & 0 & \frac{\partial t_{p_\sigma}}{\partial V_1} & 0 & \frac{\partial t_{p_\sigma}}{\partial \gamma_\sigma} & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{\partial \Omega_\sigma}{\partial \alpha_{tr_\sigma}} & \frac{\partial \Omega_\sigma}{\partial \delta_{tr_\sigma}} & 0 & \frac{\partial \Omega_\sigma}{\partial i_\sigma} & 0 & 0 \\ \frac{\partial \omega_\sigma}{\partial r_{tr_\sigma}} & 0 & \frac{\partial \omega_\sigma}{\partial \delta_{tr_\sigma}} & \frac{\partial \omega_\sigma}{\partial V_1} & \frac{\partial \omega_\sigma}{\partial i_\sigma} & \frac{\partial \omega_\sigma}{\partial \gamma_\sigma} & 0 \end{Vmatrix} \begin{Vmatrix} \Delta r_{tr_\sigma} \\ \Delta \alpha_{tr_\sigma} \\ \Delta \delta_{tr_\sigma} \\ \Delta V_1 \\ \Delta i_\sigma \\ \Delta \gamma_\sigma \\ \Delta t_{tr_\sigma} \end{Vmatrix} \qquad (6.1)$$

$$\frac{\partial e_\sigma}{\partial \gamma_\sigma} = - \frac{r_{tr_\sigma} V_I^2 \sin \gamma_\sigma \cos \gamma_\sigma}{\mu_\odot \quad e_\sigma} \left(\frac{r_{tr_\sigma} V_I^2}{\mu_\odot} - 2 \right)$$

$\frac{\partial t_{p\sigma}}{\partial r_{tr_\sigma}}$, $\frac{\partial t_{p\sigma}}{\partial V_I}$ and $\frac{\partial t_{p\sigma}}{\partial r_\sigma}$ are those given in matrix 2.1 when $t_{p\alpha}$, r_{in}, V_{in}, γ_{in} e_α, a_α and $\mu_\oplus$ are substituted by $t_{p\sigma}$, r_{tr_σ}, V_I, γ_σ, e_σ, a_σ, and $\mu_\odot$

$$\frac{\partial \Omega_\sigma}{\partial t_{r_\sigma}} = - \frac{\operatorname{ctg} \; i_\sigma}{\cos^2 \delta_{tr\sigma} \; \sin \left(\alpha_{tr\sigma} - \Omega_\sigma \right)}$$

$$\frac{\partial \Omega_\sigma}{\partial i_\sigma} = \frac{\tan \; \delta_{tr_\sigma}}{\sin^2 i_\sigma \; . \; \sin \left(\alpha_{tr_\sigma} - \Omega_\sigma \right)}$$

$\frac{\partial \omega_\sigma}{\partial r_{tr_\sigma}}$, $\frac{\partial \omega_\sigma}{\partial V_I}$ and $\frac{\partial \omega_\sigma}{\partial \gamma_\sigma}$ are those given in matrix 2.1 when ω_α, r_{in}, V_{in}, γ_{in}, e_α, a_α and $\mu_\oplus$ are substituted by ω_σ, $r_{tr\sigma}$, V_I, γ_σ, e_σ, a_σ and $\mu_\odot$

$$\frac{\partial \omega_\sigma}{\partial \delta_{tr_\sigma}} = \frac{\cos \; \delta_{tr_\sigma}}{\sqrt{\sin^2 i_\sigma - \cos^2 \delta_{tr_\sigma}}}$$

$$\frac{\partial \omega_\sigma}{\partial i_\sigma} = - \frac{\cos \delta_{tr_\sigma} \; . \; \cos i_\sigma}{\sin i_\sigma \sqrt{\sin^2 i_\sigma - \cos^2 \delta_{tr_\sigma}}}$$

7.1. Relationships between the errors in the position and velocity of the probe at arrival and the errors in the heliocentric orbit parameters (with $r_{T\sigma}$ fixed)

We give below the relation between the probe's position and velocity at the moment of arrival at the target :

$$r_{T\sigma}\,,\ \alpha_{T\sigma}\,,\ \delta_{T\sigma}\,,\ V_T\,,\ \alpha_{V_{T\sigma}}\,,\ \delta_{V_{T\sigma}}\,,\ t_{T\sigma}$$

and the heliocentric orbital parameters :

$$a_\sigma\,,\ e_\sigma\,,\ t_{p_\sigma}\,,\ i_\sigma\,,\ \Omega_\sigma\,,\ \omega_\sigma$$

These relations are as follows :

$$\sin \delta_{T\sigma} = \sin i_\sigma \sin(\omega_\sigma + \theta_{T\sigma})$$

$$\tan(\alpha_{T\sigma} - \Omega_\sigma) = \cos i_\sigma \tan(\omega_\sigma + \theta_{T\sigma})$$

$$V_T = \sqrt{\frac{\mu_\odot (2a_\sigma + r_{T\sigma})}{a_\sigma\, r_{T\sigma}}}$$

$$r_{T\sigma} = \text{constant}$$

$$\cos \theta_{T\sigma} = \frac{a_\sigma (1 - e_\sigma^2)}{e_\sigma\, r_{T\sigma}} - \frac{1}{e_\sigma}$$

$$\cos \gamma_{T\sigma} = \sqrt{\frac{\mu_\odot a_\sigma (1 - e_\sigma^2)}{r_{T\sigma} V_T}}$$

$$\tan(\alpha_{V_{T\sigma}} - \Omega_\sigma) = \cos i_\sigma \operatorname{ctg}(\Upsilon_{T\sigma} - \omega_\sigma - \theta_{T\sigma})$$

$$\sin \delta_{V_{T\sigma}} = \sin i_\sigma \cdot \cos(\Upsilon_{T\sigma} - \omega_\sigma - \theta_{T\sigma})$$

$$t_{T\sigma} - t_{p_\sigma} = \frac{1}{2E}\left\{\sqrt{2Er^2_{T\sigma} + 2\mu_\odot r_{T\sigma} - J^2} + \frac{\mu_\odot}{\sqrt{-2E}}\left[\sin^{-1}\left(\frac{\mu_\odot + 2E\, r_{T\sigma}}{\sqrt{\mu_\odot^2 + 2EJ^2}}\right) - \frac{\pi}{2}\right]\right\}$$

where

$$E = -\frac{\mu_\odot}{2a_\sigma} \quad J = \sqrt{a_\sigma \mu_\odot (1 - e^2_\sigma)}$$

The errors in the position and velocity of the probe at arrival (with $r_{T\sigma}$ constant) are given in terms of the errors in the heliocentric orbital parameters by

$$|\Delta_{T\sigma}| = |\mathcal{M}_{\sigma, T\sigma}|\,|\Delta_\sigma|$$

where the matrix $|\mathcal{M}_{\sigma, T\sigma}|$ is given by 8.1, with the expressions given below for the derivatives.

The partial derivatives of $r_{T\sigma}$, $\delta_{T\sigma}$, $\alpha_{T\sigma}$, V_T and $\delta_{V_{T\sigma}}$ are the same as those given in matrix 4.1 if we substitute

$$r_{tr_\varepsilon}, \delta_{tr_\varepsilon}, \alpha_{tr_\varepsilon}, V_{tr}, \delta_\varepsilon \text{ and } a_\varepsilon, e_\varepsilon, t_{p_\varepsilon}, i_\varepsilon, \Omega_\varepsilon, \omega_\varepsilon$$

by

$$r_{T\sigma}, \delta_{T\sigma}, \alpha_{T\sigma}, V_T, \delta_{V_{T\sigma}} \text{ and } a_\sigma, e_\sigma, t_{p_\sigma}, i_\sigma, \Omega_\sigma, \omega_\sigma$$

and, the heliocentric orbit being eliptic, the combination $e^2_\varepsilon - 1$ by $1 - e^2_\sigma$.

7.1

$$
\begin{vmatrix} \Delta r_{T\sigma} \\ \Delta\alpha_{T\sigma} \\ \Delta\delta_{T\sigma} \\ \Delta v_T \\ \Delta\alpha_{vT\sigma} \\ \Delta\delta_{vT\sigma} \\ \Delta t_{T\sigma} \end{vmatrix}
=
\begin{vmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
\frac{\partial\alpha_{T\sigma}}{\partial a_\sigma} & \frac{\partial\alpha_{T\sigma}}{\partial e_\sigma} & 0 & \frac{\partial\alpha_{T\sigma}}{\partial i_\sigma} & 1 & \frac{\partial\alpha_{T\sigma}}{\partial\omega_\sigma} \\
\frac{\partial\delta_{T\sigma}}{\partial a_\sigma} & \frac{\partial\delta_{T\sigma}}{\partial e_\sigma} & 0 & \frac{\partial\delta_{T\sigma}}{\partial i_\sigma} & 0 & \frac{\partial\delta_{T\sigma}}{\partial\omega_\sigma} \\
\frac{\partial v_T}{\partial a_\sigma} & 0 & 0 & 0 & 0 & 0 \\
\frac{\partial\alpha_{vT\sigma}}{\partial a_\sigma} & \frac{\partial\alpha_{vT\sigma}}{\partial e_\sigma} & 0 & \frac{\partial\alpha_{vT\sigma}}{\partial i_\sigma} & 1 & \frac{\partial\alpha_{vT\sigma}}{\partial\omega_\sigma} \\
\frac{\partial\delta_{vT\sigma}}{\partial a_\sigma} & \frac{\partial\delta_{vT\sigma}}{\partial e_\sigma} & 0 & \frac{\partial\delta_{vT\sigma}}{\partial i_\sigma} & 0 & \frac{\partial\delta_{vT\sigma}}{\partial\omega_\sigma} \\
\frac{\partial t_{T\sigma}}{\partial a_\sigma} & \frac{\partial t_{T\sigma}}{\partial e_\sigma} & 1 & 0 & 0 & 0
\end{vmatrix}
\begin{vmatrix} \Delta a_\sigma \\ \Delta e_\sigma \\ \Delta t_{p\sigma} \\ \Delta i_\sigma \\ \Delta\Omega_\sigma \\ \Delta\omega_\sigma \end{vmatrix}
$$

The remaining derivatives are as follows :

$$\frac{\partial \alpha_{V_{T_\sigma}}}{\partial a_\sigma} = - \frac{\cos i_\sigma \cos^2(\alpha_{V_{T\sigma}} - \Omega_\sigma)}{\sin^2(\gamma_{T_\sigma} - \omega_\sigma - \theta_{T_\sigma})} \; \frac{\sqrt{1 - e_\sigma^2}}{r_{T_\sigma}} \left[\frac{\sqrt{1 - e_\sigma^2}}{e_\sigma \sin \theta_{T_\sigma}} - \right.$$

$$\left. - \sqrt{\frac{\mu_\odot}{a_\sigma}} \; \frac{1}{2 \, V_T \sin \gamma_{T_\sigma}} \right]$$

$$\frac{\partial \alpha_{V_{T_\sigma}}}{\partial e_\sigma} = - \frac{\cos i_\sigma \cos^2(\alpha_{V_{T_\sigma}} - \Omega_\sigma)}{\sin^2(\gamma_{T_\sigma} - \omega_\sigma - \theta_{T_\sigma})} \left[\frac{e_\sigma}{r_{T_\sigma} V_T \sin \gamma_{T_\sigma}} \sqrt{\frac{\mu_\odot \, a_\sigma}{1 - e_\sigma^2}} + \right.$$

$$\left. + \frac{1}{e_\sigma^2 \sin \theta_{T_\sigma}} \left(1 + a_\sigma - a_\sigma e_\sigma^2 \right) \right]$$

$$\frac{\partial \alpha_{V_{T_\sigma}}}{\partial i_\sigma} = \cos^2 (\alpha_{V_{T_\sigma}} - \Omega_\sigma) \sin i_\sigma \; \tan \; (\gamma_{T_\sigma} - \omega_\sigma - \theta_{T_\sigma})$$

$$\frac{\partial \alpha_{V_{T_\sigma}}}{\partial \omega_\sigma} = + \frac{\cos i_\sigma \cos^2(\alpha_{V_{T_\sigma}} - \Omega_\sigma)}{\sin^2 (\gamma_{T_\sigma} - \omega_\sigma - \theta_{T_\sigma})} \quad , \quad \frac{\partial \alpha_{V_{T_\sigma}}}{\partial \Omega_\sigma} = 1$$

$$\frac{\partial t_{T_\sigma}}{\partial a_\sigma} = - \sqrt{\frac{\mu_\odot^3}{8 a_\sigma^5}} \; \frac{- 8 r_{T_\sigma} a_\sigma + 3 r_{T_\sigma}^2 + 2 a_\sigma^2 (1 - e_\sigma^2)}{\sqrt{4 r_{T_\sigma} a_\sigma - r_{T_\sigma}^2 - 2 a_\sigma^2 (1 - e_\sigma^2)}} +$$

$$+ \frac{1}{2} \left(\frac{\mu_\odot}{a_\sigma} \right)^{3/2} \left[\sin^{-1} \left(\frac{a_\sigma - r_\sigma}{a_\sigma \, e_\sigma} \right) - \frac{\pi}{2} \right] -$$

$$- \left(\frac{\mu_\odot}{a_\sigma}\right)^{3/2} \frac{r_\sigma}{\sqrt{(a_\sigma e_\sigma)^2 - (a_\sigma - r_\sigma)^2}}$$

$$\frac{\partial t_{T\sigma}}{\partial e_\sigma} = - \frac{\mu_\odot^2 \; e_\sigma (2a_\sigma \;)^{1/2}}{\sqrt{4 \, r_{T\sigma} a_\sigma \mu_\odot - \mu_\odot r_{T\sigma}^2 - \mu_\odot a_\sigma (1 - e_\sigma^2)}} +$$

$$+ \left(\mu_\odot a_\sigma\right)^{1/2} \frac{a_\sigma - r_{T\sigma}}{e_\sigma \sqrt{(a_\sigma e_\sigma)^2 - (a_\sigma - r_{T\sigma})^2}}$$