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SELF-SIMILAR MOTION OF LASER HALF-SPACE PLASMAS. I DEFLAGRATION REGIME.

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SELF-SIMILAR MOTION OF LASER HALF-SPACE PLASMAS.

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ABSTRACT

The one-dimensional self-similar motion of an initially cold, half-space plasma of electron density n_o, produced by the (anamalous) absorption of a laser pulse of irradiation $\phi=\phi_o t/\tau (0<t\leq\tau)$ at the critical density $n_c (n_c/n_o \equiv \varepsilon <<1)$, is condidered; the analysis allows for electron heat conduction and ion-electron energy exchange (viscosities and ion conduction are found neglibible) and retains three dimensionless numbers: ε, Z_i (ion charge number), and a parameter $\alpha \propto (n_o{}^2\tau/\phi_o)^{2/3}$. For $\alpha >> \varepsilon^{-4/3}$, a deflagration wave (where energy absorption occurs) develops, separating an isentropic compression with a shock bounding the undisturbed plasma, and an isentropic expansion flow to the vacuum. The structures of these three regions are completely determined; in particular, the Chapman-Jouguet condition is proved and the density behind the deflagration is found. The deflagration-compression thickness ratio is large (small) for $\alpha << \varepsilon^{-5/3} (\alpha >> \varepsilon^{-5/3})$. There is a thin transition layer ahead of the deflagration, where density has a maximum and temperature a minimum.

I. INTRODUCTION

In a recent paper[1] (called A hereafter), the authors analyzed the self-similar motion generated in a plasma (initially cold and filling the entire space with uniform density n_o) by the deposition, in a given plane, of a pulse of energy per unit area and time, $\phi = \phi_o t/\tau$ ($t \leq \tau$). The results showed in detail the importance of heat conduction and electron-ion temperature relaxation against convection of energy, and their dependence on a single parameter $\alpha \propto (n_o^2 \tau/\phi_o)^{2/3}$. The existence of self-similar motion in a plasma with heat conduction and different ion and electron temperatures was noticed by Anisimov[2]; a broad discussion of conditions for self-similar plasma motion has been given recently[3].

Laser fusion requires compressing a *DT* pellet to very high densities, heating only its core[4,5]; the radiation is usually absorbed at electron densities close to the critical value, n_c (much less than n_o) inside a hot, rarefied corona ablated by the laser. Hydrodynamics is a essential part of the phenomenon.

In this work we have considered an extension of A: the plasma occupies initially a half-space and the laser radiation is assumed absorbed uniformly in the plane where the electron density equals n_c. Clearly, there are now two parameters, n_c/n_o and α; moreover, *there is also a new effect, the corona expansion to the vacuum*, competing with ion-electron relaxation and energy conduction and convection. The analysis includes, therefore, the essential hydrodynamics of laser fusion, and the results should have qualitative (and in some parts quantitative) validity for the spherical geometry contemplated in laser fusion. Non-hydrodynamic effects, such as radiation pressure and emission, nuclear fusion and electron degeneracy are not considered in this work.

In this paper we limit ourselves to the case $\alpha >> (n_c/n_o)^{-4/3}$; the complementary range for α is considered in a companion paper. For such large α, a deflagration wave is found to develop. Deflagration waves in plasmas were first approximately discussed by Fauquignon and Floux[6] and Bobin[7]; related qualitative analysis have been carried out by a number of authors[8-13].

In Sec. II. we establish the self-similar variables and equations. Sections III-V deal with the three main regions which appear in the flow. A discussion of the results is given in Sec.VI. Finally, a thin, transition layer is analyzed in the Appendix.

II. STATEMENT OF THE PROBLEM

We consider an unmagnetized, fully ionized plasma, which at $t = 0$ is at rest in the half-space $x \geq 0$, having zero temperature and uniform electron density n_o and ion density n_o/Z_i (Z_i is the ion charge number). At that time, energy per unit time and area, $\phi(t)$, starts being absorbed by the electrons in the plane where the electron density n_e equals the critical density n_c. This process represents anomalous absorption[5] of laser radiation inciding fron the vacuum on the left; n_c, assumed to be much less than n_o, is given by

$$\omega_{pe}(n_c) \equiv (4\pi n_c e^2/m_e)^{1/2} = \omega \quad ,$$

where ω and ω_{pe} are the laser and electron plasma frequencies respectively. We shall assume both a) quasineutrality and b) short enough ion and electron self-collision times, so that either species has a near-maxwellian distribution function; these assumptions, and the approximation of zero initial temperature, will be justified, for the conditions considered in this paper, in Sec.VI. We allow, however, different temperatures T_e and T_i.

Since the resulting motion is onedimensional and the electric current must vanish in the undisturbed plasma far to the right, quasineutrality ($n_e \simeq Z_i n_i \equiv n$) leads to equal velocities, $v_e \simeq v_i \equiv v$. The evolution of the plasma is then governed, as in A, by the equation of continuity for either species

$$\frac{Dn}{Dt} = -n\frac{\partial v}{\partial x} \quad , \quad \left(\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right) \quad , \tag{1}$$

the momentum equation for the ion-electron fluid

$$m_i n \frac{Dv}{Dt} = -\frac{\partial}{\partial x}\left[nk(Z_i T_e + T_i)\right] + Z_i \frac{4}{3}\frac{\partial}{\partial x}\left[(\mu_e + \mu_i)\frac{\partial v}{\partial x}\right] \quad , \tag{2}$$

and the entropy equation for each species[14]

$$nT_e \frac{D}{Dt}\left(k \ln \frac{T_e^{3/2}}{n}\right) = \frac{\partial}{\partial x}\left(K_e \frac{\partial T_e}{\partial x}\right) + \frac{4}{3}\mu_e\left(\frac{\partial v}{\partial x}\right)^2 - \frac{3}{2}kn\frac{T_e - T_i}{t_{ei}}$$

$$+ \phi(t)\delta(x - x_c) \quad , \tag{3a}$$

$$\frac{n}{Z_i} T_i \frac{D}{Dt}\left(k \ln \frac{T_i^{3/2}}{n}\right) = \frac{\partial}{\partial x}\left(K_i \frac{\partial T_i}{\partial x}\right) + \frac{4}{3}\mu_i\left(\frac{\partial v}{\partial x}\right)^2 + \frac{3}{2} kn \frac{T_e - T_i}{t_{ei}} \quad , \tag{3b}$$

x_c being such that

$$n(x_c) = n_c \quad . \tag{4}$$

Because of quasineutrality, there is no electric field term in Eq. (2), where electron inertia has also being neglected ($m_e \ll m_i$). The classical coefficients of viscosity and thermal conductivity, and the ion-electron energy relaxation time, μ_j, K_j and t_{ei}, may be written as [15,16]

$$\mu_j = \bar{\mu}_j T_j^{5/2} \quad , \quad K_j = \bar{K}_j T_j^{5/2} \quad , \quad t_{ei} = \bar{t}_{ei} T_e^{3/2} n^{-1} \quad ;$$

$\bar{\mu}_j$, $\bar{K}_j$ and $\bar{t}_{ei}$ depend weakly on T_j and n trough Coulomb logarithms.

We have initial conditions

$$T_e(x,0) = T_i(x,0) = v(x,0) = 0 \quad , \quad n(x,0) = n_o \cdot \quad (x \geq 0) , \tag{5}$$

and boundary conditions

$$T_e(\infty,t) = T_i(\infty,t) = v(\infty,t) = 0 \quad , \quad n(\infty,t) = n_o \quad , \tag{6a}$$

$$n(x_v,t) = 0 \quad , \qquad v(x_v,t) = x_v \quad , \tag{6b}$$

where $x_v(t)$ is the plasma-vacuum boundary on the left. In addition, conditions on T_j should be given at x_v: for a collision dominated plasma we should have

$$T_e(x_v,t) = T_i(x_v,t) = 0 \quad , \tag{6c}$$

since the mean free paths are proportional to $T_j^{\,2}/n$.[15]

For the case

$$\phi = \phi_o t/\tau \quad , \qquad 0 < t \leq \tau \quad ,$$

the motion of the plasma is self-similar, as in A, if the Coulomb logarithms are taken to be constant. Defining

$$\xi = x/[w\tau(t/\tau)^{4/3}] \ , \qquad u(\xi) = v/[v_o(t/\tau)^{1/3}] \ ,$$
$$\bar{n}(\xi)=n/n_o \ , \quad \theta_j(\xi)=T_j/[T_o(t/\tau)^{2/3}] \ , \quad \alpha = 3kT_o/m_i v_o w \ , \tag{7a}$$

and choosing the free parameters w, v_o and T_o so as to satisfy

$$3v_o = 4w \ , \quad \phi_o = wkn_oT_o \ , \quad \bar{K}_e T_o^{\,5/2} = w^2\tau k n_o \quad , \tag{7b}$$

Eqs. (1)-(4) become

$$\frac{d\bar{n}}{d\xi} = \frac{\bar{n}}{\xi-u}\,\frac{du}{d\xi} \quad , \tag{8}$$

$$u-4(\xi-u)\frac{du}{d\xi} = -\frac{\alpha}{\bar{n}}\,\frac{d}{d\xi}[\bar{n}(Z_i\theta_e+\theta_i)]+\frac{Z_i}{\bar{n}}\,\frac{d}{d\xi}[(a_1\theta_e^{\,5/2}+a_2\theta_i^{\,5/2})\frac{du}{d\xi}] \ , \tag{9}$$

$$\bar{n}[\theta_e(1+\frac{4}{3}\frac{du}{d\xi})-2(\xi-u)\frac{d\theta_e}{d\xi}]=\frac{d}{d\xi}(\theta_e^{5/2}\frac{d\theta_e}{d\xi})+\frac{4a_1}{3\alpha}\theta_e^{5/2}(\frac{du}{d\xi})^2$$

$$-a_3\alpha\bar{n}^2\frac{\theta_e-\theta_i}{\theta_e^{3/2}}+\delta(\xi-\xi_c)\quad, \tag{10a}$$

$$\frac{\bar{n}}{Z_i}[\theta_i(1+\frac{4}{3}\frac{du}{d\xi})-2(\xi-u)\frac{d\theta_i}{d\xi}]=a_4\frac{d}{d\xi}(\theta_i^{5/2}\frac{d\theta_i}{d\xi})$$

$$+\frac{4a_2}{3\alpha}\theta_i^{5/2}(\frac{du}{d\xi})^2+a_3\alpha\bar{n}^2\frac{\theta_e-\theta_i}{\theta_e^{3/2}}\quad, \tag{10b}$$

$$\bar{n}(\xi_c)=n_c/n_o\equiv\varepsilon<<1\quad, \tag{11}$$

with boundary conditions

$$\begin{aligned}&\theta_e(\infty)=\theta_i(\infty)=u(\infty)=0\quad,\quad \bar{n}(\infty)=1\\ &\bar{n}(\xi_v)=\theta_e(\xi_v)=\theta_i(\xi_v)=0\quad,\quad u(\xi_v)=\xi_v\quad.\end{aligned} \tag{12}$$

In Eqs. (8)-(10)

$$\begin{aligned}&a_1\equiv\frac{4k}{m_i}\frac{\bar{\mu}_e}{\bar{K}_e}\simeq 4.9\times10^{-4}\qquad (\text{for } Z_i=1)\\ &a_2\equiv\frac{4k}{m_i}\frac{\bar{\mu}_i}{\bar{K}_e}\simeq 3.9\times10^{-2}A_i^{-1/2}\qquad (\text{for } Z_i=1)\\ &a_3\equiv\frac{2m_i}{3k^2}\frac{\bar{K}_e}{\bar{\tau}_{ei}}\simeq 4.3\,b(Z_i)\quad [b\equiv Z_i\frac{\bar{K}_e(Z_i)}{\bar{K}_e(1)}]\\ &a_4\equiv\frac{\bar{K}_i}{\bar{K}_e}\simeq 4.1\times10^{-2}A_i^{-1/2}\qquad (\text{for } Z_i=1)\end{aligned} \tag{13}$$

where A_i is the ion mass number, and $b(Z_i)$, ranging from 1 at $Z_i = 1$ to 4.2 at $Z_i = \infty$, may be found from Ref. 15. To obtain the numerical coefficients, all Coulomb logarithms have been assigned a common, arbitrary, value. Clearly, electron heat conduction is the dominant transport mechanism; in fact, we shall hereafter neglect ion conduction and ion and electron viscosities. We notice that, then, α and Z_i are the only parameters involved in Eqs.(8)-(10); one must solve for $\bar{n}$, u and θ_j as functions of ξ and of the dimensionless numbers ε, α and Z_i.

It will prove convenient to derive conservation laws, from Eqs. (8)-(10), for mass, momentum and energy

$$\bar{n} = \frac{d}{d\xi}\left[\bar{n}(\xi-u)\right] \qquad , \tag{14a}$$

$$5\bar{n}u = \frac{d}{d\xi}\left[4\bar{n}u\,(\xi-u) - \alpha\bar{n}\,(Z_i\theta_e+\theta_i)\right] \qquad , \tag{14b}$$

$$3\bar{n}(Z_i\theta_e+\theta_i+\frac{4u^2}{3\alpha}) = \frac{d}{d\xi}\{2\bar{n}(\xi-u)(Z_i\theta_e+\theta_i+\frac{4u^2}{3\alpha})$$

$$-\frac{4}{3}\bar{n}u(Z_i\theta_e+\theta_i)+Z_i\theta_e^{5/2}\frac{d\theta_e}{d\xi}\}+Z_i\delta(\xi-\xi_c) \qquad . \tag{14c}$$

In addition, with appropriate reference values for laser fusion

$$n_r = 4.7 \times 10^{22}\,cm^{-3}, \quad \phi_r = 10^{14}\,W/cm^2, \quad \tau_r = 10^{-9}\,sec. \quad ,$$

Eqs. (7) lead to useful expressions for v_o, T_o and α

$$v_o = \frac{4}{3} w = \frac{4}{3}\left(\frac{\bar{K}_e^{\,2}\phi_o^{\,5}}{k^7 n_o^{\,7}\tau^2}\right)^{1/9} \simeq 1.73 \times 10^7 \left(\frac{b^2}{Z_i^{\,2}}\,\frac{\phi_o^{\,5}}{\phi_r^{\,5}}\,\frac{\tau_r^{\,2}}{\tau^2}\,\frac{n_r^{\,7}}{n_o^{\,7}}\right)^{1/9} \frac{\text{cm}}{\text{sec}} ,$$

$$T_o = \left(\frac{\phi_o^{\,2}\tau}{k n_o \bar{K}_e}\right)^{2/9} \simeq 1.05 \left(\frac{Z_i}{b}\,\frac{\phi_o^{\,2}}{\phi_r^{\,2}}\,\frac{\tau}{\tau_r}\,\frac{n_r}{n_o}\right)^{2/9} \text{keV} \tag{15}$$

$$\alpha = \frac{9k}{4m_i}\left(\frac{\tau k^2 n_o^{\,2}}{\phi_o \bar{K}_e}\right)^{2/3} \simeq \frac{12.9}{A_i}\left(\frac{Z_i}{b}\,\frac{\phi_r}{\phi_o}\,\frac{\tau}{\tau_r}\,\frac{n_o^{\,2}}{n_r^{\,2}}\right)^{2/3} ,$$

also

$$x = \xi\left(\frac{t}{\tau}\right)^{4/3}\left(\frac{\bar{K}_e^{\,2}\phi_o^{\,5}\tau^7}{k^7 n_o^{\,7}}\right)^{1/9} \simeq 1.30 \times 10^{-2}\,\xi\left(\frac{t}{\tau}\right)^{4/3}\left(\frac{b^2}{Z_i^{\,2}}\,\frac{\phi_o^{\,5}}{\phi_r^{\,5}}\,\frac{\tau^7}{\tau_r^{\,7}}\,\frac{n_r^{\,7}}{n_o^{\,7}}\right)^{1/9} \text{cm}. \tag{16}$$

We have now taken the Coulomb logarithms arbitrarily equal to 8; this approximation will be discussed in Sec. VI. In the next three sections we shall solve Eqs. (8)-(12) for, Z_i arbitrary, ε small and $\alpha \gg \varepsilon^{-4/3}$.

III. ISENTROPIC COMPRESSION REGION

We expect the results of A for the dense plasma flow on the right side, to remain valid here. Then, for $\alpha \gg 1$, energy convection gives rise to a shock wave bounding the disturbed plasma; the flow behind the shock is isentropic, ion and electron temperatures being equal, and viscosity and heat conduction negligible. Actually, there is a very thin precursor[17] ahead of the shock (see Sec. VI).

As in A the jump conditions across the shock, which may be directly obtained from system (14), give

$$\bar{n}_f = 4, \quad u_f = 3\xi_f/4, \quad \theta_f = 3\xi_f{}^2/4(Z_i + 1)\alpha \quad , \tag{17}$$

where subscript f labels conditions just behind the shock. Then, integrating Eqs. (14) between ξ_v and ξ_f leads to overall conservation laws

$$\int_{\xi_v}^{\xi_f} \bar{n}d\xi = \xi_f \quad , \tag{18a}$$

$$\int_{\xi_v}^{\xi_f} 5\bar{n}ud\xi = 0 \quad , \tag{18b}$$

$$\int_{\xi_v}^{\xi_f} 3\bar{n}(Z_i\theta_e+\theta_i+\frac{4u^2}{3\alpha})\, d\xi = Z_i \quad . \tag{18c}$$

The last equation, together with Eqs. (17), leads to $\xi_f = \gamma\alpha^{1/3}$, where γ is an unknown constant, determining the shock position, which must be of the order of unity or smaller.

Introducing into Eqs. (8)-(10) new variables

$$\eta = \xi/\xi_f, \quad \nu = \bar{n}/\bar{n}_f, \quad y = u/u_f, \quad z = \theta_j/\theta_f \quad , \tag{19}$$

one can verify inmediately that the heat conduction and viscosity terms, being of order $\gamma^3\alpha^{-3/2}$, may indeed be neglected, while the ion-electron energy relaxation term is of order $\alpha^{3/2}\gamma^{-3}$, so that we do have $\theta_e = \theta_i$. Then, adding Eqs. (10) the relaxation terms drop out and we get

$$2z(1 + dy/d\eta) - (4\eta - 3y)\,dz/d\eta = 0 \quad , \tag{20}$$

while Eqs. (8) and (9) become

$$\frac{d\nu}{d\eta} = \frac{3\nu}{4\eta-3y}\,\frac{dy}{d\eta} \quad , \tag{21}$$

$$\nu\left[y - (4\eta-3y)\,\frac{dy}{d\eta}\right] = -\frac{d}{d\eta}(\nu z) \quad ; \tag{22}$$

Eqs. (20)-(22) must be solved subject to boundary conditions

$$\nu(1) = y(1) = z(1) = 1 \quad . \tag{23}$$

The electron absorption term has not been included in (20) since the critical density plane lies outside the region of validity of these equations.

A first integral of Eqs. (20)-(22) is

$$\nu^{7/3}(4\eta-3y)\,z^{-2} = 1 \quad , \tag{24}$$

which is the usual adiabatic integral of self-similar, isentropic flow[17]. Next, eliminating ν between (21) and (22) leads to

$$y - (4\eta-3y)\,\frac{dy}{d\eta} = -\frac{dz}{d\eta} - \frac{3z}{4\eta-3y}\,\frac{dy}{d\eta} \quad , \tag{25}$$

which must be solved together with Eq. (20). Defining

$$Y = y/\eta \quad , \quad Z = z/\eta^2 \quad ,$$

one is lead to a phase-plane (Z,Y) equation. A detailed discussion of the integration of this equation is given in A; final numerical results, obtained there for ν, y and z versus η are represented in Fig. 1. In the neighborhood of a value $\bar{\eta} \simeq 0.82$, the solution behaves as

$$y \simeq \frac{4}{3}\bar{\eta} - \frac{2}{5}(\eta-\bar{\eta}), \quad z \simeq B_1(\eta-\bar{\eta})^{3/13}, \quad \nu \simeq B_2(\eta-\bar{\eta})^{-3/13} \quad , \qquad (26)$$

where $B_1 \simeq 1.70$, $B_2 \simeq 0.78$. We notice that the expression $\nu(4\eta-3y)$ (mass flow relative to the moving plane η) goes to zero as η approaches $\bar{\eta}$ from the right. However, since $\bar{n}$ must be positive everywhere, Eq. (14a) shows that $\bar{n}(\xi-u)$ must be monotonic throughout the plasma, increasing righwards from a zero value at the plasma-vacuum boundary; thus, we should have $\xi-u>0$, that is, $4\eta-3y>0$, everywhere. Clearly, then, the isentropic compression solution ceases to be valid in the neighbourhood of $\bar{\eta}$, where the plasma becomes highly dense and cold.

IV. ISENTROPIC EXPANSION REGION

We shall now make the ansatz that the plasma expands (moving leftwards) all through the interval between η_v and $\bar{\eta}$, the density being there of the order of the critical density

$$\nu \sim \bar{n}_c/\bar{n}_f = O(\varepsilon) \quad . \tag{27}$$

To match the isentropic compression solution, the density should then increase from zero at η_v to infinity at $\bar{\eta}$, while the temperature should vanish at both η_v and $\bar{\eta}$, having a maximum in between. As in A, a very thin transition layer of dense, cold plasma, where ν peaks, z bottoms up and y changes sign, should exist between the expansion and compression regions; we defer to the Appendix a discussion of this layer, which affects the solution outside it negligibly.

Writing Eq. (14b) in the variables defined in (19) and integrating across the transition layer, show that in the expansion region, we have

$$z = O(\varepsilon^{-1}) \quad . \tag{28}$$

Further, we notice that the compression solution satisfies

$$\int_{\bar{\eta}}^{1} 5\nu y d\eta = B_1 B_2 \quad ,$$

and thus Eq. (16b) leads to $\int_{\eta_v}^{\bar{\eta}} 5\nu y d\eta = O(1)$; since the last condition in (12) shows that $y/\eta = O(1)$, we arrive at $\nu y^2 = O(1)$ or

$$y = O(\varepsilon^{-1/2}) \quad , \quad \eta = O(\varepsilon^{-1/2}) \quad . \tag{29}$$

Thus, using appropiately scaled variables we find that η is negative throughout the expanding plasma, the interval $0<\eta<\bar{\eta}$ being of negligible extent. Similarly, one may set $y(0) \simeq y(\bar{\eta}^-) \simeq 0$, since $y(\bar{\eta}^+) = O(1)$; moreover, $y < 0$ in the entire expansion, as assumed, because $(4\eta-3y)$ must be positive throughout the plasma (see end of last section). Finally, the energy in the compressed region comes out to be a small fraction of the total energy in the plasma; that is, $\gamma \ll 1$, Eq. (18c) leading directly to $\gamma = \bar{\gamma}\varepsilon^{1/6}$, where $\bar{\gamma}$ is an unknown constant of order of unity.

Introducing variables scaled according to (27)-(29), into Eqs. (8)-(10), we *find that the conduction and viscosity terms are* $O(\alpha^{-3/2}\varepsilon^{-2})$ as compared with convection terms of order of unity, while the ion-electron energy relaxation term is $O(\alpha^{3/2}\varepsilon^{2})$. Thus, if $\alpha \gg \varepsilon^{-4/3}$, *the expansion region would result to be isentropic* (conduction and viscosity being negligible and ion and electron temperatures being equal). However, since electron heat conduction must be important near the critical density plane, a conduction layer, neccesarily thin compared with the size given in (29), should exist within the expansion region, lying between the isentropic expansion flow on the left and $\bar{\eta}$. The structure of this layer, which may be considered as a deflagration wave because temperature and velocity increase, and density and pressure decrease across it, will be determined in the next section.

Clearly, if $\alpha = O(\varepsilon^{-4/3})$, the isentropic expansion and the deflagration layer merge into each other, and the transport and relaxation terms become important throughout the expansion flow. Thus, there is no deflagration unless $\alpha >> \varepsilon^{-4/3}$.

Now, let ν_d, z_d, and y_d be the unknown values of ν, z and y at the base of the isentropic expansion (just behind the deflagration), assumed of order ε, ε^{-1} and $\varepsilon^{-1/2}$ respectively; then defining

$$\hat{\eta} = \eta/(-y_d), \quad \hat{y} = y/(-y_d), \quad \hat{\nu} = \nu/\nu_d, \quad \hat{z} = z/z_d \ , \tag{30}$$

Eqs. (20)-(22) directly yield

$$\frac{d\hat{\nu}}{d\hat{\eta}} = \frac{3\hat{\nu}}{4\hat{\eta}-3\hat{y}} \frac{d\hat{y}}{d\hat{\eta}} \quad , \tag{31}$$

$$\hat{\nu}[\hat{y} - (4\hat{\eta}-3\hat{y})\, d\hat{y}/d\hat{\eta}\,] = -\,Hd(\hat{\nu}\hat{z})/d\hat{\eta} \quad , \tag{32}$$

$$2\hat{z}(1 + d\hat{y}/d\hat{\eta}) - (4\hat{\eta}-3\hat{y})d\hat{z}/d\hat{\eta} = 0 \quad , \tag{33}$$

where $H = z_d/y_d^2$. As boundary conditions we have

$$\begin{aligned} &\hat{\nu} = \hat{z} = -\hat{y} = 1 \qquad \text{at} \qquad \hat{\eta} = 0 \\ &\hat{\nu} = \hat{z} = 0 \ , \quad \hat{y}_v = 4\hat{\eta}_v/3 \ , \quad \text{at} \quad \hat{\eta} = \hat{\eta}_v \ . \end{aligned} \tag{34}$$

As in Sec. III, a first integral of system (31)-(33) is

$$\hat{\nu}^{7/3}(4\hat{\eta}-3\hat{y})\ \hat{z}^{-2} = 3 \qquad . \tag{35}$$

It is easy to see that if $\hat{\nu}(\hat{\eta}_v) = 0$, condition $\hat{z}(\hat{\eta}_v) = 0$ follows from (35) and needs not be included as a boundary condition, that is, the isentropic expansion is indeed collision dominated [see comment preceding Eq. (6c)]. Then, we must solve Eq. (33) and a combination of (31) and (32)

$$\hat{y} - (4\hat{\eta}-3\hat{y})\ \frac{d\hat{y}}{d\hat{\eta}} = -H\left[\frac{d\hat{z}}{d\hat{\eta}} + \frac{3\hat{z}}{4\hat{\eta}-3\hat{y}}\ \frac{d\hat{y}}{d\hat{\eta}}\right] \quad , \tag{36}$$

for $\hat{y}$ and $\hat{z}$. Now defining

$$\hat{Y} = \hat{y}/\hat{\eta} \qquad , \qquad \hat{Z} = H\hat{z}/\hat{\eta}^2 \qquad , \tag{37}$$

we arrive at the same phase-space equation of the compression region (see A)

$$\frac{d\hat{Z}}{d\hat{Y}} = \frac{\hat{Y}-17/15}{\hat{Y}+2/5}\ \frac{2\hat{Z}}{\hat{Y}-4/3}\ \frac{\hat{Z}-\hat{Z}_N(\hat{Y})}{\hat{Z}-\hat{Z}_D(\hat{Y})} \qquad , \tag{38}$$

together with

$$\hat{\eta}\ \frac{d\hat{Y}}{d\hat{\eta}} = -\left(\hat{Y} + \frac{2}{5}\right)\ \frac{\hat{Z}-\hat{Z}_D(\hat{Y})}{\hat{Z}-\hat{Z}_P(\hat{Y})} \qquad , \tag{39}$$

where

$$\hat{Z}_P = \frac{9}{5}\ (\hat{Y} - \frac{4}{3})^2 \quad , \tag{40}$$

$$\hat{Z}_N = \frac{9}{5}\ \frac{(\hat{Y}-4/3)(\hat{Y}-1)-\hat{Y}/9}{\hat{Y}-17/15}\ (Y - \frac{4}{3}) \quad , \tag{41}$$

$$Z_D = \frac{9}{5}\ \frac{\hat{Y}(\hat{Y}-1)}{\hat{Y}+2/5}\ (\hat{Y} - \frac{4}{3}) \quad . \tag{42}$$

Since $4\hat{\eta}-3\hat{y} \geq 0$, the solution $\hat{Z}(\hat{Y})$ to Eq. (38) must stay to the right of $\hat{Y} = 4/3$; for $\hat{Y} \geq 4/3$ all three curves $\hat{Z}_P(\hat{Y})$, $\hat{Z}_N(\hat{Y})$, and $\hat{Z}_D(\hat{Y})$ cross each other at P and N [which are singular points of (38)] and behave as $(9/5)\hat{Y}^2$ as $Y \to \infty$ (see Fig. 2). We also notice that in dimensional variables, $\hat{Z} = \hat{Z}_P(\hat{Y})$ reads

$$(v - \frac{dx}{dt})^2 = \frac{5}{3}\ \frac{(Z_i+1)nkT}{m_i n} \quad ,$$

where $x = w\tau(t/\tau)^{4/3}\xi$ is the moving plane corresponding to a given ξ; thus, the plasma velocity relative to this plane would be sonic on the curve $\hat{Z}_P(\hat{Y})$. For $\hat{Z} \gtrless \hat{Z}_P(\hat{Y})$ the velocity would be subsonic and supersonic respectively; for large $\hat{Y}$, these conditions correspond to $H \gtrless 9/5$.

Equations (34), (35), (37), and (38) lead to the conditions

$$\hat{Z} \sim H\hat{Y}^2 \qquad \text{as} \qquad \hat{Y} \to \infty \tag{43}$$

$$\hat{Z} = 0, \qquad \hat{\nu}^{7/3} = \frac{\hat{\nu}_v^{\,3}}{H^2}\ \frac{\hat{Z}^2}{\hat{Y}-4/3} = 0, \quad \text{at} \quad \hat{Y} = 4/3 \quad . \tag{44}$$

Now, all solutions to Eq. (38) behave as $\hat{Z} \sim \hat{Y}^2$ for large $\hat{Y}$. On the other hand solutions to Eq. (38), crossing point P, may have any of the three following behaviours

$$\hat{Z} = C(\hat{Y}-4/3)^{3/13}, \quad \hat{Z} = 0 \ , \quad \hat{Z} = \hat{Y}-4/3 \quad ,$$

C being an arbitrary constant. However, the first two behaviours may be ruled out since they lead to $\hat{\nu}^{7/3} \neq 0$ at $\hat{Y} = 4/3$, and $\hat{Z} = 0$ everywhere, respectively. Thus, one can integrate numerically Eq. (38), starting at point P, along the curve $\hat{Z} = \hat{Y}-4/3$. The solution is then found to cross the nodal point N and go to infinity as $\hat{Z} = 1.66\hat{Y}^2$.

One should notice, nonetheless, that this solution needs not be valid beyond point N, since there is an infinite number of solutions to Eq. (38) crossing N with the same first few derivatives. In fact, the analysis of the deflagration structure in Sec. V proves that H in (43) cannot be less than 9/5, that is, the flow velocity relative to the deflagration wave, just behind it, cannot be supersonic. In addition, it can be easily shown that solutions with $H > 9/5$ approach P in the form $\hat{Z} = C(\hat{Y}-4/3)^{3/13}$, and may thus be excluded. Consequently, we must have $H = 9/5$; this proves that the sonic or Chapman-Jouguet condition is satisfied here.

We find that Eq. (38) admits two solutions with $H = 9/5$ in Eq. (43), lying to the left and right of $\hat{Z}_P$ respectively. The first one, however, does not reach point N, and approaches P as $\hat{Z} \sim (\hat{Y}-4/3)^{3/13}$; it may also be ruled out. On the other hand, the curve to the right of $\hat{Z}_P$, first moves away from it and then turns and reaches point N; it is, then, clear that this curve, together with the solution which was previously found to go from P to N, make up the function $\hat{Z}(\hat{Y})$ looked for. The entire function is shown in Fig.2. Once $\hat{Z}(\hat{Y})$ is known, one may find inmediately $\hat{y}$, $\hat{z}$ and $\hat{\nu}$ versus $\hat{y}$, numerically, from Eqs. (39), (37) and (35); they are shown in Fig.3.

V. THE STRUCTURE OF THE DEFLAGRATION WAVE

The deflagration layer which separates the isentropic expansion and compression regions is governed by dissipative terms, the dominant one being electron heat conduction. Hence, the thickness of this layer can be determined by requiring that energy convection and conduction be comparable. Writing the conservation equations (14) in variables $\hat{\eta}$, $\hat{y}$, $\hat{\nu}$ and $\hat{z}$ (allowing for different temperatures $\hat{z}_e$ and $\hat{z}_i$, since, as we shall see, the layer is too thin for temperature relaxation to be effective),we obtain

$$4\hat{\nu} = \frac{d}{d\hat{\eta}}\left[\hat{\nu}(4\hat{\eta}-3\hat{y})\right] \quad , \tag{45a}$$

$$5\hat{\nu}\hat{y} = \frac{d}{d\hat{\eta}}\left[\hat{\nu}\hat{y}(4\hat{\eta}-3\hat{y}) - H\hat{\nu}\frac{Z_i\hat{z}_e+\hat{z}_i}{Z_i+1}\right] \tag{45b}$$

$$3\hat{\nu}\left(\hat{y}^2+H\frac{Z_i\hat{z}_e+\hat{z}_i}{Z_i+1}\right) = \frac{d}{d\hat{\eta}}\left[\frac{\hat{\nu}}{2}(4\hat{\eta}-3\hat{y})\left(\hat{y}^2+H\frac{Z_i\hat{z}_e+\hat{z}_i}{Z_i+1}\right) - \hat{\nu}\hat{y}H\frac{Z_i\hat{z}_e+\hat{z}_i}{Z_i+1} + \right.$$

$$\left. + \frac{c_1\hat{z}_e^{5/2}}{\varepsilon^2\alpha^{3/2}}\frac{d\hat{z}_e}{d\hat{\eta}}\right] + c_2\delta(\hat{\eta}-\hat{\eta}_c) \quad , \tag{45c}$$

where both c_1 and c_2

$$c_1 \equiv (\frac{3}{4})^{5/2} \frac{Z_i H^{2-3}_{\gamma} z_d^{3/2} \varepsilon^{5/2}}{4\nu_d(Z_i+1)^{7/2}} \quad , \quad c_2 \equiv \frac{Z_i \varepsilon^{-1/2}}{3\nu_d(-y_d)^{3-3}_{\gamma}} \quad , \tag{46}$$

are of order of unity. Since all four $\hat{y}$, $\hat{\nu}$, $\hat{z}_e$ and $\hat{z}_i$ must also be of order of unity, there then results a deflagration wave thickness $\hat{\eta} = O(\varepsilon^{-2}\alpha^{-3/2})$; it also follows that such a wave (a conduction layer, thin compared with the isentropic expansion region) will exist as long as $\alpha >> \varepsilon^{-4/3}$, as found before. We notice, on the other hand, that in the compression region $\hat{\eta} = O(\varepsilon^{1/2})$, and therefore the deflagration will be thinner or thicker than this region, for $\alpha >> \varepsilon^{-5/3}$ and $\alpha << \varepsilon^{-5/3}$ respectively.

We now introduce a new scaled variable

$$\hat{\hat{\eta}} = \frac{\alpha^{3/2}\varepsilon^2}{c_1} [\hat{\eta} - \frac{\bar{\eta}}{(-y_d)}] \quad , \tag{47}$$

into Eqs. (45) and integrate them once, neglecting their left-hand sides. Making use of the conditions behind the deflagration $(\hat{\hat{\eta}} \to -\infty)$

$$\hat{\nu} = \hat{z}_e = \hat{z}_i = 1 \quad , \qquad \hat{y} = -1 \quad ,$$

and dropping $\hat{\hat{\eta}}$ everywhere, we arrive at

$$\hat{\nu}\hat{y} = -1 \quad , \tag{48a}$$

$$3\hat{y} + H(Z_i\hat{z}_e + \hat{z}_i)/\hat{y}(Z_i + 1) = -3-H \quad , \tag{48b}$$

$$\frac{3}{2}\hat{y}^2 + \frac{5}{2}H\,(Z_i\hat{z}_e+\hat{z}_i)/(Z_i+1) + \hat{z}_e^{\,5/2}d\hat{z}_e/d\hat{\hat{\eta}} + c_2\sigma = \frac{1}{2}(3+5H) \quad , \tag{48c}$$

σ being zero if $\hat{\hat{\eta}} < \hat{\hat{\eta}}_c$, and unity if $\hat{\hat{\eta}} > \hat{\hat{\eta}}_c$. A fourth equation, to determine ion temperature, is provided by (10b)

$$\hat{z}_i\,\frac{d\hat{y}}{d\hat{\eta}} + \frac{3}{2}\,\hat{y}\,\frac{d\hat{z}_i}{d\hat{\eta}} = \frac{3a_3 Z_i^{\,2}}{4(Z_i+1)^2}\,H^2\hat{\nu}\,\frac{\hat{z}_e-\hat{z}_i}{\hat{z}_e^{\,5/2}} \quad . \tag{48d}$$

Evaluating Eqs. (48b) and (48c) ahead of the wave ($\hat{\hat{\eta}} = 0$), where $\hat{z}_e/\hat{z}_i = 1$, $\hat{z}_e$ and $\hat{y}$ vanish, and $\hat{\nu}\hat{z}_e = B_1B_2/\nu_d z_d$, we find the jump conditions across the wave

$$3 + H = HB_1B_2/\nu_d z_d \quad , \qquad c_2 = \frac{1}{2}(3 + 5H) \quad . \tag{49}$$

As in A, the continuity equation yields no condition, the mass flow at $\hat{\eta}$ being unknown.

System (48) yields

$$\hat{\nu} = -1/\hat{y} \quad , \qquad \hat{z}_i = -Z_i\hat{z}_e - H^{-1}(Z_i+1)\hat{y}(3+H+3\hat{y}) \quad , \tag{50}$$

together with coupled equations for $\hat{z}_e$ and $\hat{y}$

$$\frac{d\hat{z}_e}{d\hat{\eta}} = \frac{6F(\hat{y})}{\hat{z}_e^{5/2}} , \quad \frac{d\hat{y}}{d\hat{\eta}} = \frac{3a_3H^2Z_i}{4(Z_i+1)} \frac{[\hat{z}_e-\hat{z}_1(\hat{y})][\hat{z}_e-\hat{z}_2(\hat{y})]}{\hat{y}\hat{z}_e^{5/2}[\hat{z}_e-\hat{z}_p(\hat{y})]} \quad , \tag{51}$$

where

$$\begin{aligned} F(\hat{y}) &= (\hat{y}+1)\ [\hat{y}+(3+5H)/12] \quad , \quad \sigma = 0 \\ &= \hat{y}\ [\hat{y}+5(3+H)/12] \quad , \quad \sigma = 1 \end{aligned} \tag{52}$$

$$\hat{z}_p(\hat{y}) = -12\ \frac{Z_i+1}{Z_i}\ \frac{\hat{y}}{H}\ [\hat{y}+5(3+H)/24] \quad , \tag{53}$$

$$\hat{z}_{1,2}(\hat{y}) = -\frac{\hat{y}}{2H}\ [H+3+3\hat{y}\pm\{(H+3+3\hat{y})^2+\frac{48}{a_3}\ \frac{Z_i+1}{Z_i}\ F(\hat{y})\}^{1/2}] \ . \tag{54}$$

Equations (51) lead to a phase-space equation for $\hat{z}_e$ and $\hat{y}$

$$\frac{d\hat{z}_e}{d\hat{y}} = \frac{8(Z_i+1)}{a_3Z_iH^2} \quad \frac{\hat{y}F(\hat{y})[\hat{z}_e-\hat{z}_p(\hat{y})]}{[\hat{z}_e-\hat{z}_1(\hat{y})][\hat{z}_e-\hat{z}_2(\hat{y})]} \quad , \tag{55}$$

which must be solved subject to two boundary conditions

$$\hat{z}_e(\hat{y} = -1) = 1, \qquad \hat{z}_e(\hat{y} = 0) = 0 \quad .$$

We notice, however, that both H and the value of $\hat{y}$ at the critical density plane $\hat{y}_c$ (such $\sigma = 0$ for $\hat{y} < \hat{y}_c$, and $\sigma = 1$ for $\hat{y} > \hat{y}_c$), must be found as part of the problem.

Clearly the solution curve S will comprise two pieces: a solution to Eq. (55) with $\sigma = 1$, passing through the origin, $S_1(H)$, and a solution to Eq. (55) with $\sigma = 0$, passing through point (-1,1), $S_2(H)$, both pieces meeting at the critical point $(\hat{y}_c, \hat{z}_c)$. We shall prove in what follows that S does not exist for H less than 9/5, while it does for $H = 9/5$; since, on the other hand, the range $H > 9/5$ was ruled out by the analysis of Sec. IV, we would then conclude that $H = 9/5$.

We, first, find out that for $H \leq (9/5)(Z_i+1)/(1+3Z_i/5)$, point (-1,1) lies outside the parabola $\hat{z}_e = \hat{z}_p(\hat{y})$, while $S_1(H)$ lies inside it for $\hat{z}_e$ small and positive, for all H. Thus, for $H \leq 9/5$, $\hat{z}_e - \hat{z}_p(\hat{y})$ must vanish somewhere on S. And it also follows that either $\hat{z}_p - \hat{z}_1$ or $\hat{z}_p - \hat{z}_2$ should vanish at that point too, since otherwise we would have $d\hat{\hat{\eta}}/d\hat{y} = 0$ there, and $\hat{y}$ would not be a single valued function of $\hat{\hat{\eta}}$.

We next find out that $S_1(H)$ should reach the critical point inside the parabola. Figure 4 shows $\hat{z}_p$, $\hat{z}_1$ and $\hat{z}_2$ in the range $-1 \leq \hat{y} \leq 0$, for both $\sigma = 1$ and $\sigma = 0$, $Z_i = 1$ and $H = 9/5$; the parabola and the loop formed by $\hat{z}_1(\sigma = 1)$ and $\hat{z}_2(\sigma = 1)$, have no common point, except the origin. As H decreases to zero, both the loop and the parabola get narrower and taller, but remain separate. On the other hand, as Z_i increases, the loop becomes broader while $\hat{z}_p(\hat{y})$ is reduced (by as much as one half), and they finally cross each other for large enough Z_i. Nonetheless, $d\hat{z}_1/d\hat{y}$ is positive at the cross-point; since $S_1(H)$ leaves the origin between $\hat{z}_1(\hat{\eta})$ and $\hat{z}_p(\hat{y})$, it cannot, clearly, reach that point.

Now, for $H < 9/5$, point $(-1,1)$ is a saddle having a positive and a negative slope, and $\hat{z}_1(\sigma=0)$ lies in between. It is easy to show that $\hat{y}$ must be increasing at $(-1,1)$ and that the branch with positive slope, $S_{21}(H)$, cannot be part of S, since it is incompatible with Eqs. (51). To prove that the other branch, $S_{22}(H)$, can neither, we consider the relative positions of the line $\hat{y} = -(3 + 5H)/12$ where $F(\hat{y}, \sigma = 0)$ vanishes, and point M where $\hat{z}_p(\hat{y})$ and $\hat{z}_1(\hat{y}, \sigma = 0)$ meet. Let $H^*(Z_i)$ be the value of H for which M lies on that line (H^* ranges from 27/19 at $Z_i = 1$, to unity for Z_i infinite). For $H<H^*$, M lies to the left of the line, so that the slope of S_{22} must remain negative until crossing either $\hat{z}_p$ or $\hat{z}_1$; since it can be shown that $\hat{z}_p>1$ at M, $S_{22}(H)$ can not be part of S. Actually, for H small enough $\hat{z}_1(\sigma = 0)$ and $\hat{z}_2(\sigma = 0)$ form two loops, one starting at the origin, the other crossing both $(-1,1)$ and $(-1,0)$; then, $S_{22}(H)$ is trapped inside this second loop and finally crosses $\hat{z}_2(\sigma = 0)$ toward $\hat{y}$ decreasing. For $9/5>H>H^*$, M lies to the right of $\hat{y} = -(3+5H)/12$, which meets $\hat{z}_1(\sigma = 0)$ at a point which is a node of Eq. (55), while M is a saddle. We find that $S_{22}(H)$ does not reach M. Thus, S does not exist for $H < 9/5$. We notice that, actually, a negative slope at $(-1,1)$ would imply that heat would flow toward the plane where energy is absorbed (far to the left).

We find numerically that for Z_i not too large, $S_{21}(H<9/5)$ crosses the node from above $\hat{z}_1(\sigma = 0)$ and reaches M from below $\hat{z}_1(\sigma = 0)$. As H goes to 9/5, the node moves to point $(-1,1)$; then, both S_{21} and S_{22} leave this point below $\hat{z}_1(\sigma = 0)$ and are thus compatible with Eqs. (55). Since clearly S_{21} still reaches M, H must be 9/5, and the solution S will be composed of $S_1(9/5)$ and $S_{21}(9/5)$, which should meet inside the parabola, determining $\hat{y}_c$. We notice, however, that as Z_i increases, the critical point approaches the parabola; we let to a future note the discussion of the deflagration for large Z_i.

The entire solution is shown in Fig. 4 for $Z_i = 1$. Once H and $\hat{y}_c$ are known, Eq. (48a) leads to ν_d:

$$-\frac{1}{\hat{y}_c} = \hat{\nu}_c = \frac{\nu_c}{\nu_d} = \frac{\varepsilon}{4\nu_d} \qquad .$$

Then, z_d and c_2 can be obtained from (49). Finally, using $y_d = (5z_d/9)^{1/2}$, both $\overline{\gamma}$ and the scale length c_1 are given by Eqs. (46). All $\hat{y}$, $\hat{z}_e$, $\hat{z}_i$ and $\hat{\nu}$ are shown in Fig. 5, as functions of $\hat{\eta}$, for $Z_i = 1$. To complete the solution in the isentropic expansion, we give ν_d, z_d, y_d, $\overline{\gamma}$ and c_1 in Table I for $Z_i = 1$, 2, and 4. Table I also shows the minimum of z and the maximum of ν and its location η_1.

VI. DISCUSSION OF RESULTS

We have studied the one-dimensional self-similar motion of an initially cold, half-space plasma of electron density n_o, when a laser pulse of irradiation $\phi = \phi_o t/\tau$ $(0 < t \leq \tau)$ is (anomalously) absorbed at the critical density, n_c (its location being part of the solution). We allowed electron heat conduction and different temperatures T_e and T_i, and showed that viscosities and ion conduction, which could be included in the analysis, were numerically negligible. Then, the solution is governed by the ion charge number Z_i, the ratio $\varepsilon \equiv n_c/n_o$ (assumed small), and a parameter $\alpha \propto (n_o^2 \tau/\phi_o)^{2/3}$. We have here considered the case $\alpha >> \varepsilon^{-4/3}$, that is, $(n_c^2 \tau/\phi_o)^2$ large.

We found that a deflagration wave (inside which energy absorption occurs, ion and electron temperatures are unequal, and conduction is important) separates an isentropic compression flow on the right, bounded by a shock from the undisturbed plasma, and an isentropic expansion flow on the left, bounded by the vacuum. We determined the motion entirely throughout these three regions. A very thin transition layer, lying between the deflagration and the isentropic compression is studied in the Appendix. The order of magnitude, throughout the plasma, of the self-similar variables defined id Sec. II, is

Isentropic Compression

$$\bar{n}=O(1),\ u=O(\alpha^{1/3}\varepsilon^{1/6}),\ \Delta\xi=O(\alpha^{1/3}\varepsilon^{1/6}),\ \theta_j=O(\alpha^{-1/3}\varepsilon^{1/3}) \tag{56}$$

Transition Layer

$$\bar{n}=O(\varepsilon^{-3/20}),\ u=O(\alpha^{1/3}\varepsilon^{1/6}),\ \Delta\xi=O(\alpha^{-7/6}\varepsilon^{13/24}),\ \theta_j=O(\alpha^{-1/3}\varepsilon^{29/60}) \tag{57}$$

Deflagration

$$\bar{n}=O(\varepsilon),\ u=O(\alpha^{1/3}\varepsilon^{-1/3}),\ \Delta\xi=O(\alpha^{-7/6}\varepsilon^{-7/3}),\ \theta_j=O(\alpha^{-1/3}\varepsilon^{-2/3}) \tag{58}$$

Isentropic Expansion

$$\bar{n}=0(\varepsilon),\ u=0(\alpha^{1/3}\varepsilon^{-1/3}),\ \Delta\xi=0(\alpha^{1/3}\varepsilon^{-1/3}),\ \theta_j=0(\alpha^{-1/3}\varepsilon^{-2/3})\ . \quad (59)$$

Also there is a thin precursor[17] ahead of the shock, of width $\Delta\xi = 0(\alpha^{-7/6}\varepsilon^{2/3})$.[18]

We notice the following results: 1) A deflagration exists if, and only if, $\alpha >> \varepsilon^{-4/3}$, 2) When a deflagration exists, the expansion flow is isentropic (not isothermal), 3) The deflagration layer is thin compared to the expansion flow; however, it will be thinner (thicker) than the compression flow for $\alpha >> \varepsilon^{-5/3}$ ($\alpha << \varepsilon^{-3/5}$). 4) We completely determined the structure of the deflagration layer; in particular, we proved the Chapman-Jouguet condition and found the value of the density behind the deflagration (not n_c). 5) There is a density maximum and a temperature minimum in a thin transition layer ahead of the deflagration.

In the analysis, we assumed near-Maxwellian distributions functions; this implies $\lambda_j << \Delta x$ and $t_j << t$, where λ_j and t_j are the mean free path and collision time for species j. In self-similar variables, the first condition reads

$$\theta_j^2/\bar{n}\,\Delta\xi << \alpha^{1/2}(m_i/m_e)^{1/2} \quad ; \quad (60)$$

using Eqs. (56)-(59), the left-hand side is $0(\alpha^{1/2})$ in the deflagration and much less elsewhere, so that (60) is well satisfied. Condition $t_j << t$ is more restrictive for ions, and reads

$$\theta_i^{3/2}/\bar{n} << \alpha\,(m_i/m_e)^{1/2} \quad ; \tag{61}$$

for the expansion and the deflagration (where temperature is larger and density smaller) condition (61) becomes $(\alpha\varepsilon^{4/3})^{3/2}(m_i/m_e)^{1/2} >> 1$, which is clearly satisfied.

Quasineutrality has been also assumed throughout the analysis. This condition requires a Debye length small compared with Δx^*, and in self-similar variables reads

$$\omega\tau(t/\tau) >> (A_i\,\varepsilon\,\alpha\,\theta_e/\bar{n})^{1/2}\,(10/\Delta\xi) \quad , \tag{62}$$

(ω and A_i being laser frequency and ion mass number). Clearly, charge separation is not self-similar; we shall consider $0.1\tau < t < \tau$ in discussing (62). This condition is less restrictive for the isentropic regions than for the deflagration layer, for which it becomes

$$\omega\tau >> 10^2 A_i^{1/2}(\alpha\varepsilon^{4/3})^{3/2} \quad ;$$

for $\omega \simeq 1.8\times10^{15}\ \mathrm{sec}^{-1}$ (Nd laser), $\tau = 30$ nsec, $A_i = 2.5$ (DT) and $\alpha\varepsilon^{4/3} = 10^{3/2}$, the condition leads to $10^3 >> 1$. We notice that according to Eqs. (15), $\alpha\varepsilon^{4/3} \propto (Z_i\tau/\phi_o)^{2/3}\omega^{8/3}A_i^{-1}$ cannot be large (unless ϕ_o is very much less than 10^{14} W/cm^2) except for the largest ω, A_i^{-1} and τ; for the values just taken, $\phi_o \simeq 10^{11}$ W/cm^2.

For the transition layer, quasineutrality would require

$$\omega\tau >> 10^2 A_i^{1/2}(\alpha\varepsilon^{4/3})^{3/2}\ \varepsilon^{-69/40} \quad ,$$

and it may thus be easily violated. However, a review of the analysis in the Appendix shows that the only modification in the results given there, is the change $\bar{n} \rightarrow (\bar{n}_e Z_i + \bar{n}_i)/(Z_i + 1)$. Moreover, we stress the point that the structure of the transition layer bears no dependence upon the results outside it.

While we assumed initially a half-space of plasma at zero temperature, this is not essential to the analysis; we could have as well a half-space of solid matter, as long as its initial specific energy could be later neglected. Letting aside the transition layer, the assumption that the perturbed matter is a plasma is most restrictive for the dense, relatively cold compression region; clearly, however, the results would not be modified if we had there a monotomic ideal gas instead of a plasma.

The assumption of constant Coulomb logarithms only affects the deflagration layer (where heat conduction and ion electron energy relaxation was used), but certainly, it can be only considered as an approximation. We notice, however, that in the structure of the layer itself, Coulomb logarithms enter only as a ratio, through the coefficient a_3 defined in Sec. II; furthermore, the expressions for such parameter as α, T_o, and v_o (see Sec. II) depend only weakly on a Coulomb logarithm.

We finally point out that the results of the present analysis should remain qualitatively valid for spherical geometries and other pulse shapes. Furthermore, our results for thin layers (such as deflagration) should in fact remain quantitatively valid.

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APPENDIX

To analyse the deflagration, we assumed $\hat{y}(\hat{\hat{\eta}} = 0) = 0$. Actually, for matching with the isentropic compression solution, we should have $y(\eta = \bar{\eta}) = 4\bar{\eta}/3$, that is, $y(\hat{\hat{\eta}} = 0) = 4\bar{\eta}/3(-y_d) = 0(\varepsilon^{1/2})$. Then, for $\hat{\hat{\eta}} \to 0$, we obtain

$$z_i \simeq z_e \simeq z_d(45/4)^{2/5}(-\hat{\hat{\eta}})^{2/5} \quad ,$$

$$\nu \simeq B_1 B_2/z_e \quad , \qquad y \simeq 4\bar{\eta}/3 + (3y_d/8)z_e/z_d \quad , \tag{A1}$$

which are just the deflagration results, except for the shift in y.

The analysis of the transition layer around $\bar{\eta}$ is now similar to that carried out in A. Defining $J \equiv \nu(4\eta - 3y)$ and using Eqs. (A1), we have

$$J(\eta \to \bar{\eta}^-) \to -9B_1B_2\, y_d/8z_d = 0(\varepsilon^{1/2}) \quad . \tag{A2}$$

On the other hand, the value of J in the isentropic compression is, from Eqs. (26), $(26B_2/5)(\eta-\bar{\eta})^{10/13}$, which is $0(\varepsilon^{1/2})$ at a point

$$\eta_1 = \bar{\eta} + b\,\varepsilon^{13/20} \quad , \qquad b = 0(1) \quad .$$

Equations (26) also give $z(\eta=\eta_1) = 0(\varepsilon^{3/20})$. Then, we introduce a stretched variable

$$\eta^* = (\eta-\eta_1)\,\alpha^{3/2}\,\varepsilon^{-3/8} \quad ,$$

to appropiately retain electron heat conduction near η_1; we verify that electron and ion temperatures are equal (and νz=constant=B_1B_2) throughout the layer. The equation for J is

$$\frac{dJ}{d\eta^*} = \frac{\varepsilon^{3/8}}{\alpha^{3/2}}\,\frac{4B_1B_2}{z} \quad . \tag{A3}$$

Since $z = O(\varepsilon^{3/20})$, *the change in J across the layer is small com-*pared to $\varepsilon^{1/2}$; we shall try as solution

$$J = (26\,B_2/5)\,b^{10/13}\varepsilon^{1/2} + J_1\,, \qquad J_1 = O(\varepsilon^{9/40}\alpha^{-3/2}) \ . \tag{A4}$$

The equation for z then reads

$$B_1B_2 - \frac{13}{3}B_2\,b^{10/13}\alpha^{3/2}\varepsilon^{1/8}\frac{dz}{d\eta^*} = \left(\frac{3}{4}\right)^{5/2}\frac{Z_i\bar{\gamma}^3\alpha^{3/2}}{4(Z_i+1)^{7/2}\varepsilon^{1/4}}\,\frac{d}{d\eta^*}\,z^{5/2}\,\frac{dz}{d\eta^*} . \tag{A5}$$

Once z is known, J_1 follows from (A3).

Let us now assume

$$z = B_1\,b^{3/13}\,\varepsilon^{3/20} + z_1\,, \qquad |z_1| \ll \varepsilon^{3/20} \quad . \tag{A6}$$

Then, linearizing Eq. (A5), and integrating twice we get

$$z_1 = \alpha^{-3/2}\varepsilon^{-1/8}\left\{\frac{3B_1\eta^*}{13\,b^{10/13}} + F + G\exp\left[-\eta^*\,\frac{52B_2b^{5/26}(Z_i+1)^{7/2}}{3Z_iB^{5/2}\,\bar{\gamma}^3}\left(\frac{4}{3}\right)^{5/2}\right]\right\}, \tag{A7}$$

where F and G are unknown constants. For $\eta^* \to \infty$, (A7) gives

$$z \simeq B_1\, b^{3/13}\, \varepsilon^{3/20} + \frac{3\, B_1}{13\, b^{10/13}}\, \varepsilon^{-1/2}(\eta-\eta_1) \quad , \tag{A8}$$

while from (A3) and (A4) we get

$$J = (26\, B_2/5)\, b^{10/13}\, \varepsilon^{1/2} + \frac{4\, B_2}{b^{3/13}}\, \varepsilon^{-3/20}(\eta-\eta_1) \quad ; \tag{A9}$$

Eqs. (A8) and (A9) match to the behaviors of z and J in the isentropic compression, given in (26), near η_1.

For η^* negative, Eq. (A7) shows that (A6) ceases to be valid when $|\eta^*| = O(\ln \varepsilon^{-1})$. Defining then $z = B_1\, b^{3/13}\, \varepsilon^{3/20}\, z^*$, dropping $B_1 B_2$ in (A5), and integrating twice we arrive at

$$-\eta^* = \left(\frac{3}{4}\right)^{5/2} \frac{3\, B_1^{5/2}}{52\, B_2\, b^{5/26}}\, \frac{\bar{\gamma}^3\, Z_i}{(Z_i+1)^{7/2}} \left[\frac{2}{5} z^{*5/2} + \frac{2}{3} z^{*3/2} + 2 z^{*1/2} + \ln \frac{z^{*1/2}-1}{z^{*1/2}+1}\right] ;$$

for $\eta^* \to -\infty$,

$$z^* = \frac{4\, b^{1/13}}{3\, B_1} \left[\frac{130 (Z_i+1)^{7/2}}{3 \bar{\gamma}^3\, Z_i}\right]^{2/5} (-\eta^*)^{2/5} \quad .$$

Then, matching to Eqs. (A1) and (A2), we find b, z_{min} and ν_{max}; they are given in Table I.

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18 In A this width was $O(\alpha^{-7/6})$ [erroneously given as $O(\alpha^{-5/6})$].

TABLE I

Z_i	1	2	4
$\varepsilon^{-1} \nu_d$	0.152	0.165	0.184
εz_d	3.27	3.01	2.70
$\varepsilon^{1/2} y_d$	-1.35	-1.29	-1.22
$\overline{\gamma}$	0.530	0.677	0.869
c_1	0.203	0.166	0.089
$\varepsilon^{-3/20} z_{min}$	0.965	0.975	0.992
$\varepsilon^{3/20} \nu_{max}$	1.37	1.36	1.34
$\varepsilon^{-13/20}(\eta_1-\overline{\eta})$	0.086	0.090	0.097

TABLE I. Values of normalized density, ν_d, temperature, z_d, and velocity, y_d, behind the deflagration; shock location parameter, $\overline{\gamma}$ (Sec. IV), and deflagration scale length, c_1 [Eq. (46)]; minimum temperature, z_{min}, maximum density, ν_{max}, and their location, η_1 (see the Appendix).

LIST OF FIGURES

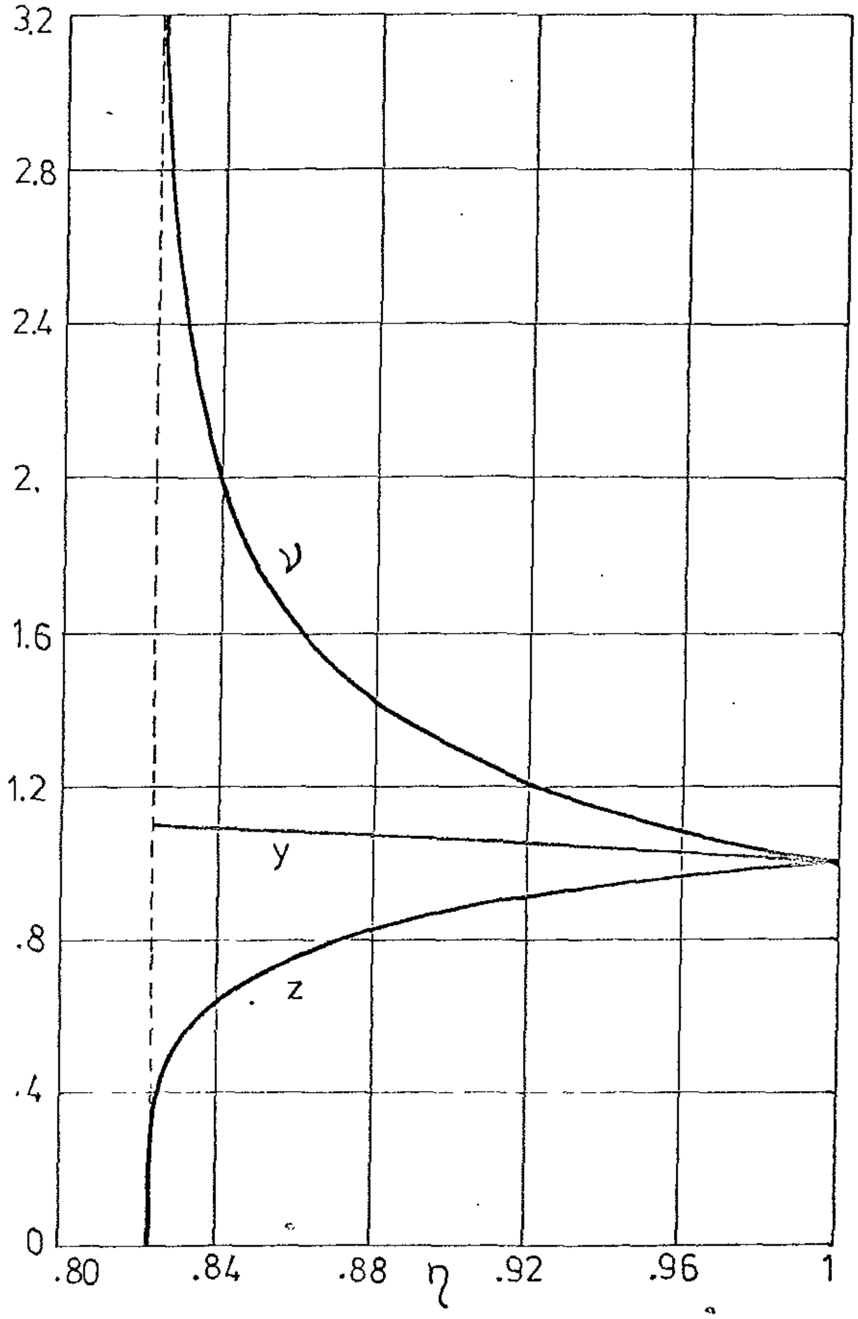
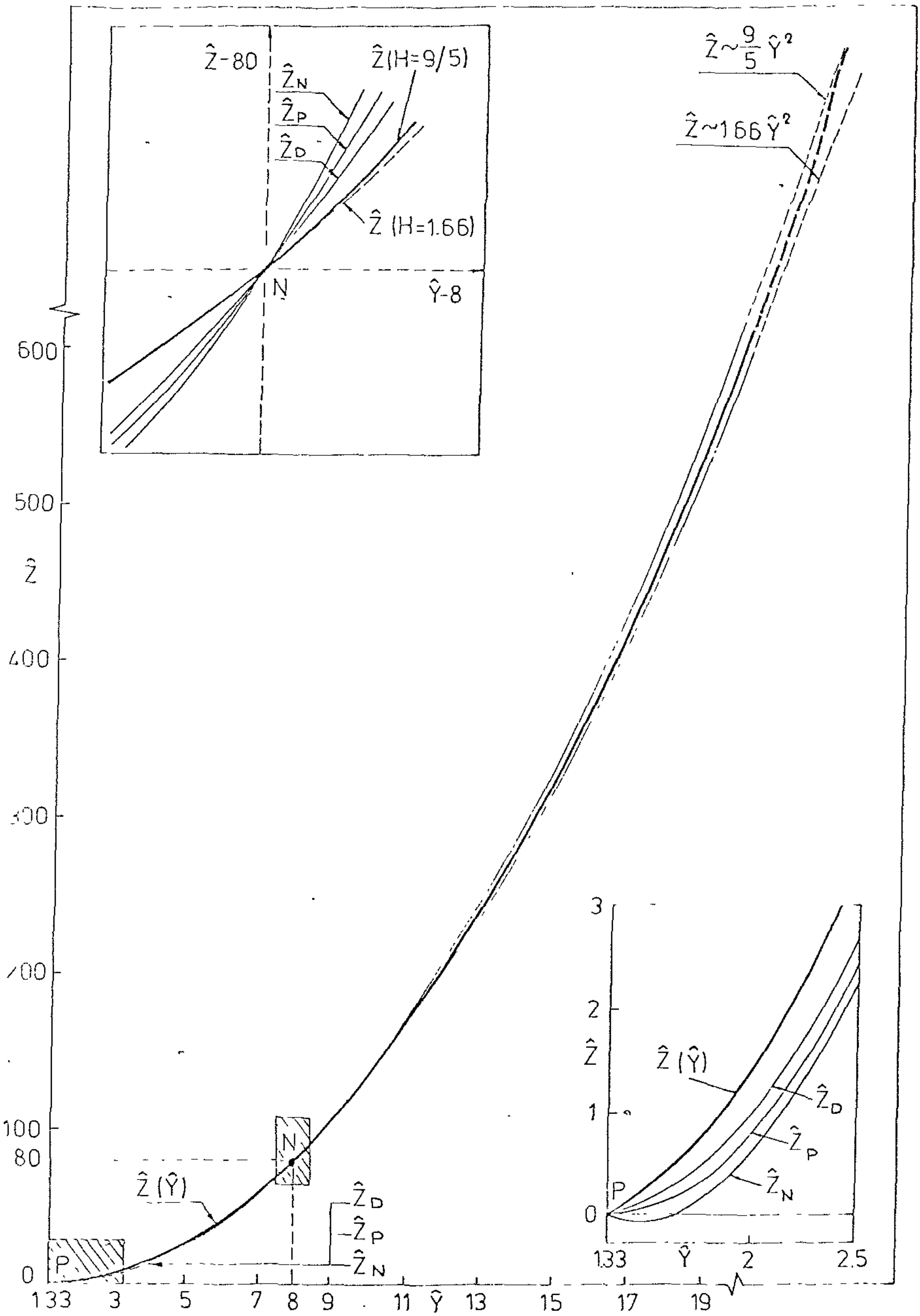


Fig. 1



$\hat{Z}-80$
$\hat{Z}_N$
$\hat{Z}_P$
$\hat{Z}_D$
$\hat{Z}(H=9/5)$
$\hat{Z}(H=1.66)$
N
$\hat{Y}-8$
$\hat{Z}\sim\frac{9}{5}\hat{Y}^2$
$\hat{Z}\sim 1.66\,\hat{Y}^2$
$\hat{Z}$
$\hat{Z}(\hat{Y})$
$\hat{Z}_D$
$\hat{Z}_P$
$\hat{Z}_N$
P
$\hat{Y}$

Fig. 2

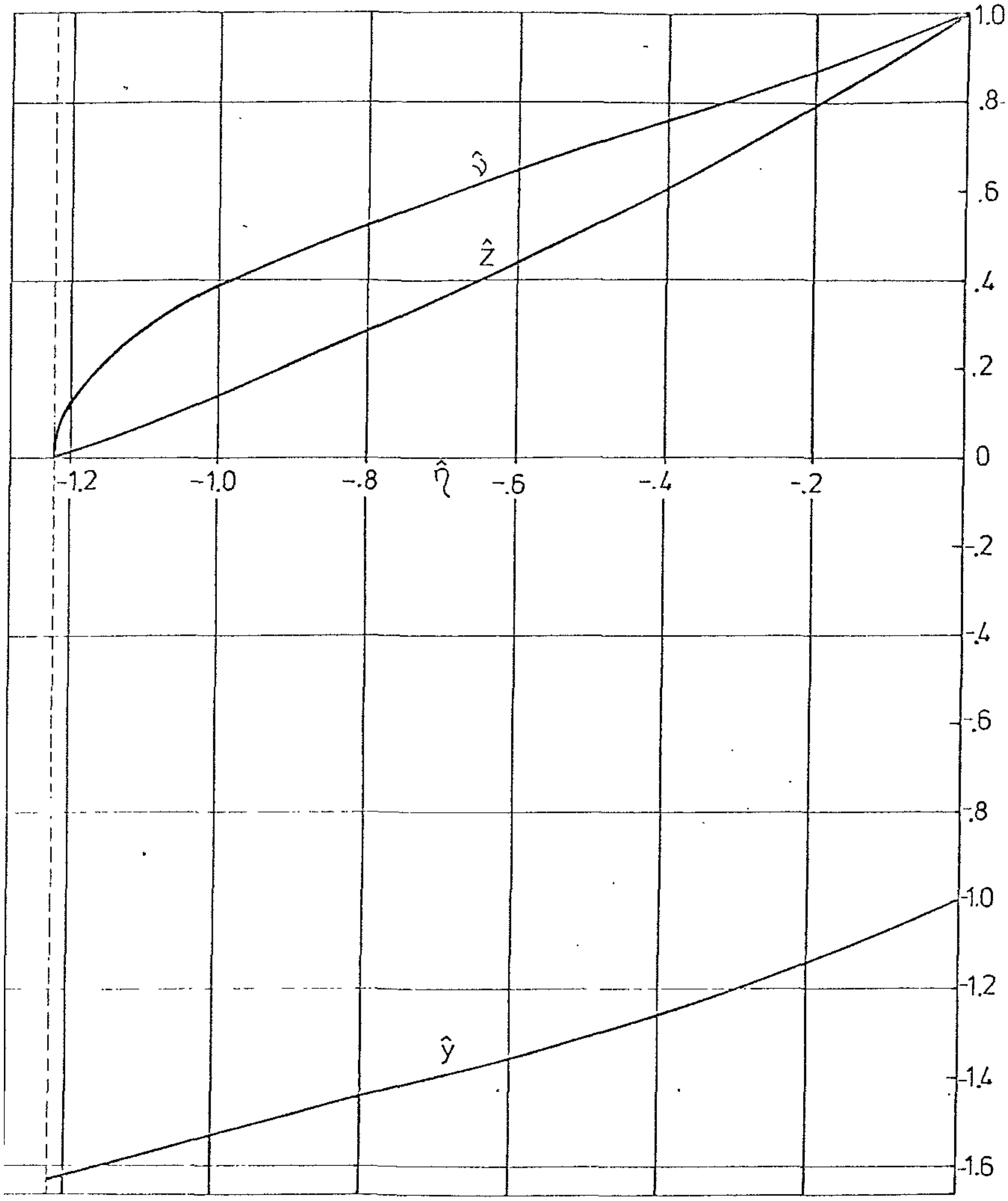
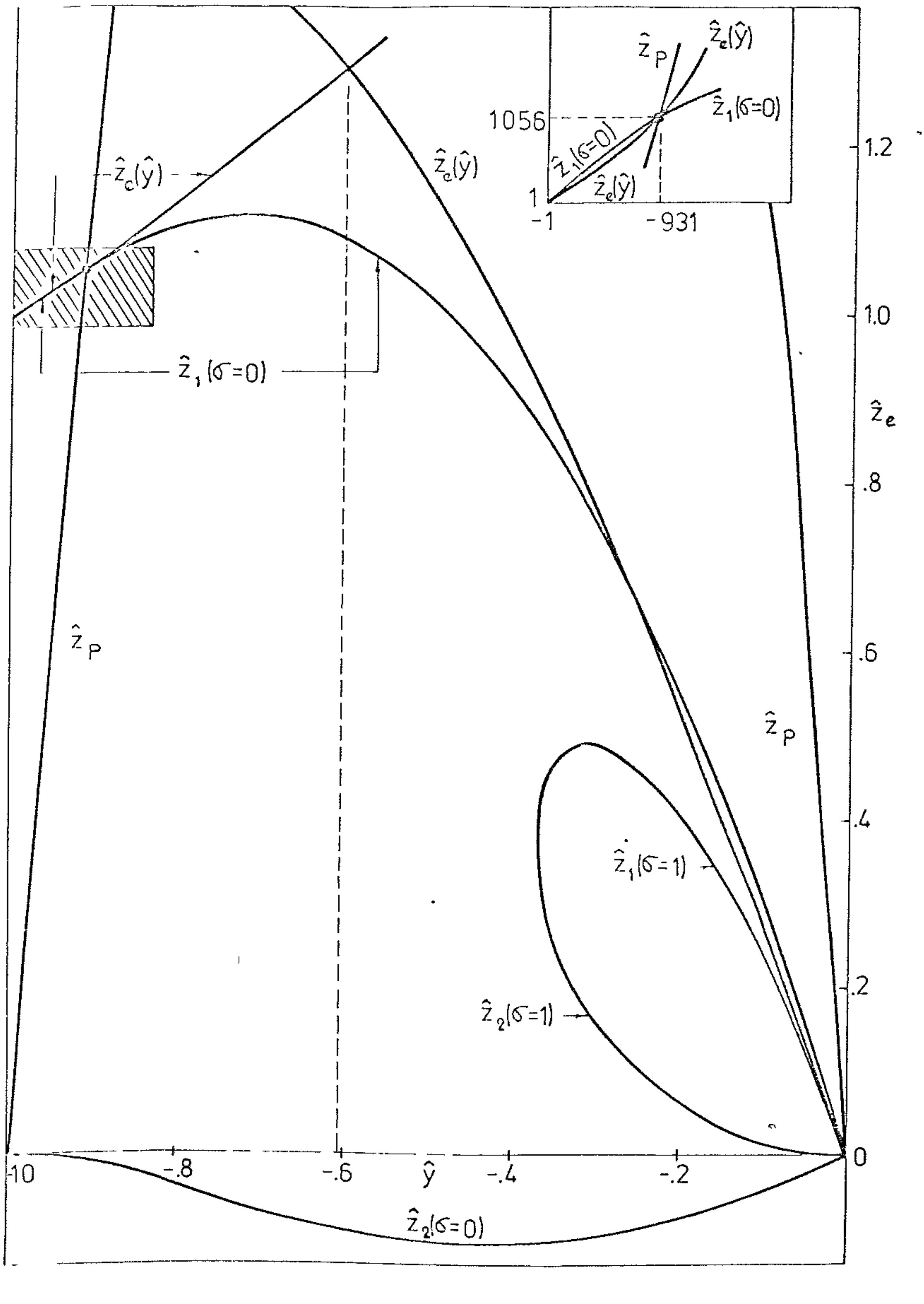
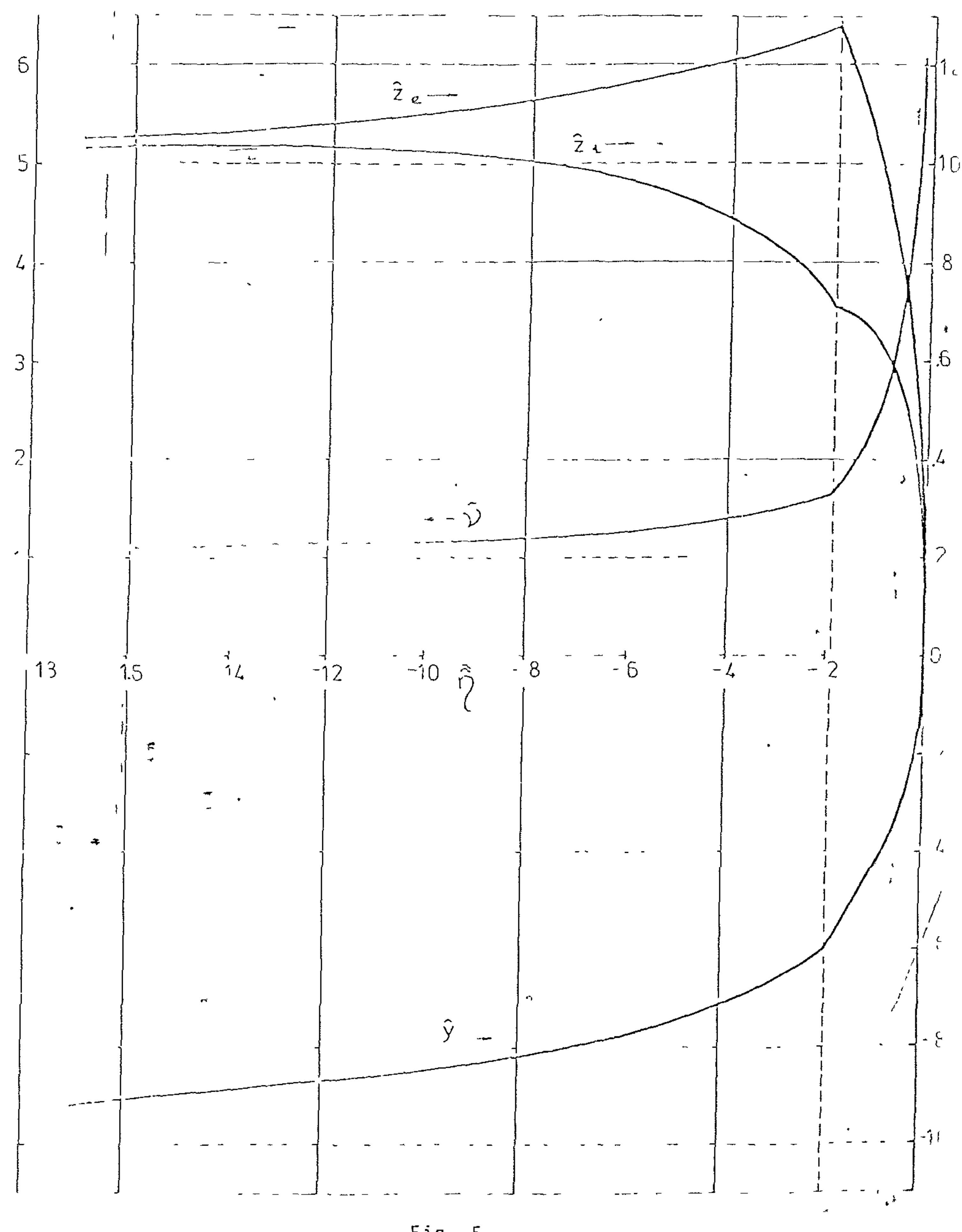


Fig. 3



$\hat{z}_c(\hat{y})$
$\hat{z}_e(\hat{y})$
$\hat{z}_1(\sigma=0)$
$\hat{z}_P$
$\hat{z}_P$
$\hat{z}_e$
$\hat{z}_1(\sigma=1)$
$\hat{z}_2(\sigma=1)$
$\hat{z}_2(\sigma=0)$
$\hat{y}$
-10
-.8
-.6
-.4
-.2
0
.2
.4
.6
.8
1.0
1.2
$\hat{z}_P$
$\hat{z}_e(\hat{y})$
$\hat{z}_1(\sigma=0)$
1056
1
-1
-931
$\hat{z}_1(\sigma=0)$
$\hat{z}_e(\hat{y})$

Fig. 4



$\hat{z}_e$
$\hat{z}_i$
$\hat{\nu}$
$\hat{y}$
$\hat{\eta}$
6
5
4
3
2
-13
-15
-14
-12
-10
-8
-6
-4
-2
0
10
8
.6
.4
2

Fig. 5