Technical Notes

Power Density of a Bare Electrodynamic Tether Generator

Claudio Bombardelli*
Technical University of Madrid,
28040 Madrid, Spain

Nomenclature

A	=	tether cross-sectional area
B_S	=	surface magnetic field intensity at equator
E_t	=	motional electric field projected along the tether
F	=	Lorentz force
i_{av}	=	average dimensionless current
i_c	=	dimensionless current at the load
L	=	tether length
m_e	=	electron mass, $m_e = 9.10938188 \times 10^{-31}$ kg
p	=	perimeter of the tether cross section
q_e	=	electron charge, $q_e = 1.60217646 \times 10^{-19}$ C)
r_p	=	planet radius
r	=	orbit radius
$\mathbf{u}_B$	=	magnetic field unit vector
$\mathbf{u}_t$	=	tether line unit vector
$\boldsymbol{v}_{\mathrm{pl}}$	=	plasmasphere velocity
$\boldsymbol{v}_{\mathrm{sc}}$	=	spacecraft velocity
$\dot{W}_\ell$	=	load power
$\dot{W}_{\mathrm{mag}}$	=	magnetic power
$\dot{w}$	=	dimensionless load power
Z_c	=	tether-plasma contact impedance
Z_p	=	effective plasma contact impedance
Z_t	=	tether ohmic impedance
$\delta_{\max}$	=	maximum power density
ζ	=	load power efficiency
μ_p	=	planet gravitational parameter
ξ_B	=	nondimensional anodic segment length
ρ	=	tether material density
σ	=	tether material conductivity
φ_c	=	dimensionless bias at the load
ω_p	=	angular velocity of planet-corotating plasmasphere

I. Introduction

ONBOARD power generation is an important application of electrodynamic tethers (EDTs). When working in passive mode, that is, when no active onboard power supplies are present, EDTs can be used to efficiently convert part of the orbital energy lost during the deorbit process into useful onboard power. In particular circumstances, power can also be extracted by the energy of a planet-corotating plasmasphere [1,2]. Similar to other EDT applications, power efficiency can be boosted by adopting a bare conductive tether as an anodic contactor [3–5].

The optimum design of bare EDTs for power-generation purposes was first addressed by Sanmartin et al. [3,4], who underlined the existence of a maximum in the generated power function for a given load current, hence suggesting that power generation could be maximized by proper choice of the impedance Z_ℓ of the power load. The optimum value of Z_ℓ for the case of negligible ohmic effects (which holds true for not-too-long tethers in the Jupiter environment, for instance) was recently computed analytically in Bombardelli et al. [2] and Sanmartin et al. [6]. Conversely, the case in which the tether ohmic impedance Z_t greatly exceeds the tether-plasma contact impedance Z_c was addressed by Sanmartin et al. [6] and Sanjurjo-Rivo and Pelaez [7], who showed that the maximum power-generation condition in this case is obtained by matching Z_ℓ with Z_t.

In a recent article by Bombardelli et al. [8], relatively accurate analytical formulas describing the current and potential profile along a passive bare EDT were derived with the help of perturbation theory. Among different aspects of bare EDTs current collection, the results obtained highlighted an important fact known from previous EDT research [4]: once the characteristics of the bare tether are given (length, cross section, and conductivity) as well as the local electric field and plasma density, the optimum value of the power-generation load impedance corresponds to current levels well below the maximum short circuit value achievable for a passive EDT. In other words, when an EDT is used for maximum power generation, it has to be designed in the small-ohmic-effect regime. Because, in that case, the analytical formulas provided were seen to yield negligible error with respect to the numerical solution, they can be advantageously employed to analyze the optimum power-generation capability of a bare EDT for generic environmental conditions (i.e., not only when the tether ohmic impedance is zero or very large when compared with the plasma contact impedance).

Once optimum design conditions for a power-generating EDT are established, some fundamental questions need to be answered: what is the maximum power density of an EDT generator, and how is it affected by the EDT characteristics (density, conductivity, and geometry) and by external environmental conditions? The goal of the present note is to address this and other related questions.

Starting from the asymptotic solution for the current and local bias longitudinal profiles developed in prior work [8] and under the validity of orbital motion limited (OML) theory, the optimum value of the load impedance allowing maximum power generation is computed as a function of tether geometry, material, and environmental properties. The corresponding maximum power and power density are then evaluated together with the efficiency of the transfer of Lorentz force power to onboard energy (called load power efficiency). Finally, a numerical evaluation of the bare EDT power-generation performance in circular equatorial Earth and Jupiter orbits is carried out and discussed.

II. Optimum Load Impedance

Let us consider a bare EDT of length L with a uniform cross section of area A, mass density ρ, and electrical conductivity σ. Assuming the cathodic part is insulated, a design favoring maximum power generation, and falls within the low-ohmic-effect regime, the dimensionless current, local bias, and generated power are, respectively, as follows:

$$
\begin{aligned}
i_c &= i(1) = \xi_B^{3/2} - \frac{2}{5}\epsilon\xi_B^3 + \frac{29}{400}\epsilon^2\xi_B^{9/2},\\
\varphi_c &= \varphi(1) = (1-\xi_B)(-1+\epsilon i_c),\\
\dot{w} &= -i_c\varphi_c = \xi_B^{3/2}(1-\xi_B) \times \left[1 - \frac{7}{5}\epsilon\xi_B^{3/2} + \frac{349}{400}\epsilon^2\xi_B^3\right.\\
&\quad \left. - \frac{61}{200}\epsilon^3\xi_B^{9/2} + \frac{29}{500}\epsilon^4\xi_B^6 - \frac{841}{160000}\epsilon^5\xi_B^{15/2}\right] + o(\epsilon^5) \quad (1)
\end{aligned}
$$

where ξ_B is the nondimensional zero-bias abscissa along the tether, and ϵ is the ratio between the tether ohmic impedance Z_t and an equivalent tether-plasma contact impedance Z_c under OML conditions, such that

$$\epsilon = \frac{Z_t}{Z_c} \quad (2)$$

$$Z_t = \frac{L}{\sigma A} \quad (3)$$

$$Z_c = \frac{3\pi}{2p\sqrt{L}} \frac{\sqrt{E_t}}{N_e} \sqrt{\frac{m_e}{2q_e^3}} \quad (4)$$

with p, E_t, m_e, and q_e indicating, respectively, the perimeter of the tether cross section, the motional electric field projected along the tether line, the electron mass, and the electron charge [8].

It is important to highlight that, once the tether geometry, the tether conductivity, and the environmental properties are set, the parameter ξ_B is completely determined by the load impedance Z_ℓ and by the potential difference ΔV_{HC} between the cathodic plasma contactor and the surrounding plasma. When efficient plasma contactors (e.g., hollow cathodes) are employed and sufficiently long tethers are considered, the small effect of ΔV_{HC} on the bias and current profile can be safely neglected [8].

The optimum value of ξ_B can be computed by solving

$$\frac{d\dot{w}}{d\xi_B} = 0 \quad (5)$$

which yields

$$
\begin{aligned}
&8410s^{17}\epsilon^5 - 7569\epsilon^5 s^{15} - 78880s^{14}\epsilon^4 + 69600s^{12}\epsilon^4\\
&\quad + 341600s^{11}\epsilon^3 - 292800\epsilon^3 s^9 - 767800s^8\epsilon\\
&628200\epsilon s^6 + 896000\epsilon s^5 - 672000\epsilon s^3\\
&\quad - 400000s^2 + 240000 = 0 \quad (6)
\end{aligned}
$$

$$\xi_B^{\mathrm{opt}} = s^2 \quad (7)$$

Equations (6) and (7) can be solved numerically for $0 \le s \le 1$.

Once an optimum design value for ξ_B is established, the corresponding value of the load impedance has to be computed. For this purpose, the circuit equation at the cathodic end is considered (see Bombardelli et al. [8]). Neglecting the plasma contactor potential drop, the circuit equation provides the optimum load impedance as follows:

$$Z_\ell^{\mathrm{opt}} = Z_c \times \left(-\frac{\varphi_c}{i_c}\right)_{\xi_B = \xi_B^{\mathrm{opt}}} \quad (8)$$

In the limit case of $\epsilon = 0$, one has

$$Z_\ell^{\mathrm{opt}}|_{\epsilon=0} = \frac{2\sqrt{15}}{9} Z_c = Z_p$$

The term Z_p can be defined as the effective plasma contact impedance of the bare EDT measured at the cathodic end. Matching the load impedance Z_ℓ with Z_p provides maximum power transfer to the load when $\epsilon = 0$.

In the other limit case of $\epsilon \to \infty$, it can be seen (by numerical analysis) that

$$\lim_{\epsilon\to\infty} Z_\ell^{\mathrm{opt}} = \epsilon Z_c = Z_t$$

meaning that, when the tether-plasma contact impedance is negligible, the optimum load impedance for maximum power generation has to match the ohmic impedance of the tether in agreement with the literature [6].

Figure 1 shows the variation of the optimum load impedance as a function of the parameter ϵ. Note that, for $\epsilon > 5$, the optimum load impedance is very close to the tether ohmic impedance Z_t and practically independent of local environmental conditions (N_e, E_t). Conversely, the lower the parameter ϵ, the more Z_ℓ^{opt} depart from Z_t and the more it becomes dependent on the local environment.

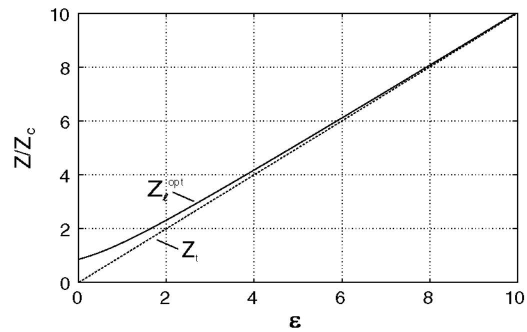

Fig. 1 Comparison between optimum load impedance and tether ohmic impedance as a function of the parameter ϵ. The quantity Z_c is the tether-plasma contact impedance [Eq. (4)].

III. Maximum Power Density

The dimensional power generated at the load can be obtained as

$$\dot{W}_\ell = \dot{w} I_{\mathrm{ch}} E_t L \quad (9)$$

where I_{ch} is the tether characteristic current [8]:

$$I_{\mathrm{ch}} = \epsilon\sigma A E_t \quad (10)$$

When large tethers are considered, the tether mass $m_t = \rho A L$ becomes the dominant term in the total mass of an EDT-based power plant so that the power density yields

$$\delta = \frac{\dot{W}_\ell}{m_t} = \epsilon\dot{w}\frac{\sigma E_t^2}{\rho} \quad (11)$$

The function $\epsilon\dot{w}$ under the optimum design condition ($\xi_B = \xi_B^{\mathrm{opt}}$) grows monotonically with ϵ, reaching the maximum asymptotic value of 0.25 (Fig. 2). The maximum power density achievable with a bare EDT is, then, as follows:

$$\delta_{\max} = \lim_{\epsilon\to\infty} \delta_{\mathrm{opt}} \simeq 0.25 \times \frac{\sigma E_t^2}{\rho} \quad (12)$$

which is, again, in agreement with previously published results [6]. Equation (11), together with Fig. 2, highlights the fact that, to maximize power generation, the best use of tether mass is done by increasing tether length and decreasing cross-sectional area and that the improvement obtained by increasing ϵ beyond, say, 10 or 20, is very modest.

The power density for the case of zero ohmic effects is derived by substituting Eq. (2) into Eq. (11):

$$\delta_{\mathrm{opt}}(\epsilon = 0) = \frac{4\sqrt{15}}{125\pi}\frac{p}{A}\frac{N_e}{\rho}\sqrt{\frac{2E_t^3}{m_e}q_e^3 L^3} \quad (13)$$

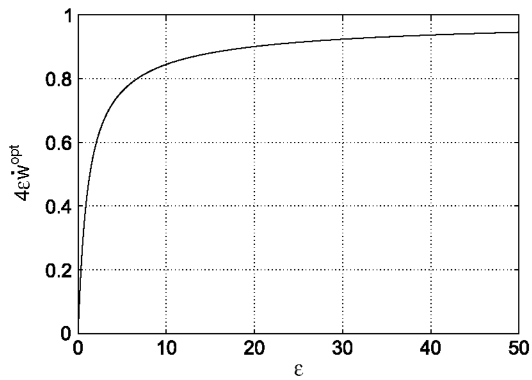

Fig. 2 Variation of the function $\epsilon\dot{w}$ under optimum design condition as a function of the parameter ϵ.

showing that, for the zero-ohmic-effect case, the tether length, cross section geometry, and plasma density all have a strong influence on the achievable power density.

IV. Load Power Efficiency

In general, a passive EDT generator can extract power from its own orbital energy (and the one of the spacecraft to which it is attached) and from the energy of the plasmasphere with which it interacts. In this regard, it becomes important to establish what fraction of the available power is actually converted into useful power for onboard use. The load power efficiency, introduced by Sanmartin et al. [4], can be written as

$$\zeta = \frac{\dot{W}_\ell}{|\dot{W}_{\text{mag}}|} \tag{14}$$

where $\dot{W}_\ell$ is the power generated at the load, and $\dot{W}_{\text{mag}}$ is the power associated with the Lorentz force $\boldsymbol{F}$, acting both on the tether and on the ambient plasma

$$\dot{W}_{\text{mag}} = \boldsymbol{F} \cdot (\boldsymbol{v}_{\text{sc}} - \boldsymbol{v}_{\text{pl}}) \tag{15}$$

with $\boldsymbol{v}_{\text{sc}}$ and $\boldsymbol{v}_{\text{pl}}$ indicating the spacecraft and plasma velocity vectors with respect to an inertial reference system.

The Lorentz force can be written as

$$\mathbf{F} = I_{\text{ch}} i_{\text{av}} BL(\mathbf{u}_t \wedge \mathbf{u}_B) \tag{16}$$

where B and $\mathbf{u}_B$ are the magnetic field intensity and unit vector, $\mathbf{u}_t$ is the tether unit vector, and i_{av} is the dimensionless current averaged along the tether line [8], which can be determined as follows:

$$i_{\text{av}} = \frac{\xi_B^{3/2}}{5}(5 - 2\xi_B) + \frac{\epsilon\xi_B^3}{40}(7\xi_B - 16) + \frac{\epsilon^2\xi_B^{9/2}}{4400}(319 - 130\xi_B) \tag{17}$$

From Eqs. (15) and (16) one has

$$\dot{W}_{\text{mag}} = -LI_{\text{ch}} i_{\text{av}} E_t \tag{18}$$

which substituted into Eq. (14), together with Eq. (9), yields

$$\zeta = \frac{\dot{w}}{i_{\text{av}}} \tag{19}$$

Finally, the power load efficiency ζ^{opt} for the maximum power density can be derived by substituting Eqs. (1) and (17) into Eq. (19) and evaluating for $\xi_B = \xi_B^{\text{opt}}$. For the limit cases of $\epsilon = 0$ and $\epsilon \to \infty$, one has

$$\zeta^{\text{opt}}(\epsilon = 0) = \frac{10}{19} \simeq 0.526, \qquad \zeta^{\text{opt}}(\epsilon \to \infty) = 0.5$$

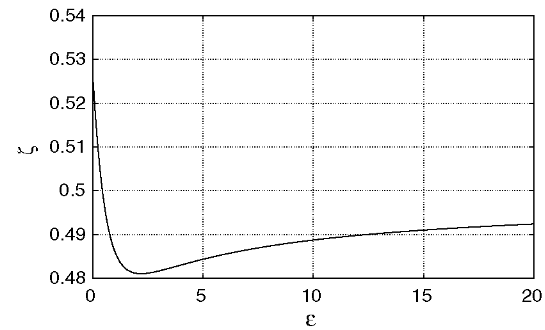

Fig. 3 Optimum load power efficiency as a function of the parameter ϵ.

The function $\zeta^{\text{opt}}(\epsilon)$ is plotted in Fig. 3, showing that the efficiency is close to 50% throughout all the range of ϵ in line with results published in Sanmartin et al. [4].

V. Applications

A. Earth Orbit

Assuming the tether is in a circular equatorial orbit, perfectly aligned with the local vertical and neglecting the small magnetic field variations along the orbit due to tilt, offset and higher harmonics E_t become solely a function of the orbit radius r so that, for a prograde ($-$) and retrograde ($+$) orbit, one has

$$E_t \cong \left(\sqrt{\frac{\mu_p}{r}} \pm \omega_p r\right) \times \frac{B_S r_p^3}{r^3} \tag{20}$$

where μ_p, r_p, ω_p, and B_S are, respectively, the planet gravitational constant, the planet radius, the angular velocity of the planet-corotating plasmasphere, and the planet surface magnetic field measured at the magnetic equator.

After taking $B_S = 30\ \mu\text{T}$, $r_p = 6378$ km, $\omega_p = 7.2722 \times 10^{-5}$ rad/s, $\mu_p = 3.986 \times 10^{14}\ \text{m}^3\,\text{s}^{-2}$, and substituting Eq. (20) into Eq. (12), one obtains the maximum power density in circular orbit, which is plotted in Fig. 4.

The obtained power density values can be higher than current flexible foldout solar arrays (~40–100 W/kg [9]), which, unlike EDTs, require continuous sun tracking to avoid losing performance. On the other hand, when comparing EDTs with conventional power-generation systems, one has to consider that the generated power comes at the expense of orbital energy, which may be an important drawback of this approach, unless the deorbiting operation is included in the mission objectives. Should this not be the case, the EDT orbit would need to be constantly reboosted if continuous power is desired. Because the load power efficiency is about 50% for an optimized system, the amount of orbital energy loss per unit time will be about twice the generated power.

It is important to underline that the power-density values plotted in Fig. 4 refer to the ideal case, in which $\epsilon \to \infty$ and $Z_\ell^{\text{opt}} \to Z_t$. In real applications, the parameter ϵ is limited by the maximum tether length

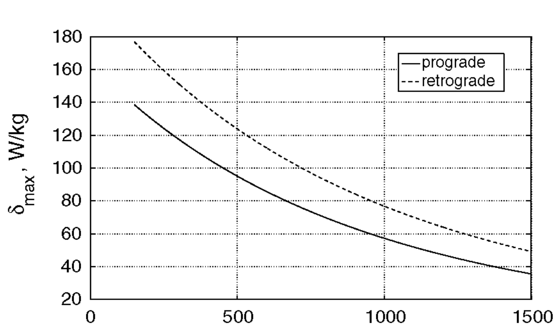

Fig. 4 Maximum instantaneous power density achievable for an aluminium EDT in circular (prograde and retrograde) equatorial Earth orbit of altitude h.

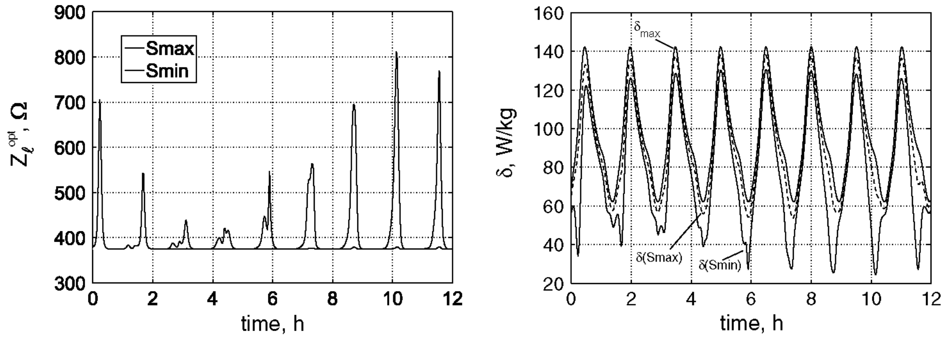

Fig. 5 Optimum load impedance (left) and power density (right) for a 20 km tape aluminium tether of 0.05 mm thickness placed in a 600 km altitude circular equatorial orbit around the Earth and under maximum and minimum solar activity conditions. It is assumed that the EDT electrodynamic drag force is counteracted by a continuous thrust force to keep all orbital elements constant.

L that can be deployed and the minimum tether cross section A, providing reasonable strength, as well as by the available environmental plasma density N_e. The latter can vary along the orbit by one to two orders of magnitude, primarily due to solar illumination conditions (a drop in plasma density is experienced during orbital eclipse transition).

Figure 5 plots the instantaneous optimum load impedance and power density achievable with a 20-km aluminum tape tether of 0.05 mm thickness flying on a 600 km altitude equatorial orbit around the Earth during maximum and minimum solar activity. (The reference dates for the simulations are 1 January 2000 and 1 January 1996 for maximum and minimum solar activity, respectively). The IRI2007 ionospheric model and the IGRF95 Earth magnetic field models were employed. During maximum solar activity, the optimum load impedance is almost constantly equal to the tether ohmic impedance, meaning that the parameter epsilon [Eq. (2)] for this particular tether design and environmental conditions is practically always greater than $\sim$5 (see Fig. 2). Conversely, during minimum solar activity, the same tether has a much more variable optimum power-generation impedance. More specifically, the peaks in Fig. 5a correspond to eclipse transition periods, during which a decrease in plasma density (corresponding to a decrease in ϵ) is observed. By looking at Fig. 5b, one can notice that power-density values close to the maximum possible are obtained during high solar activity, whereas a relatively small decrease in the average value is experienced during low solar activity. Note that the oscillatory behavior of the power density is due to the different Earth magnetic field harmonics accounted for by the IGRF95 model. Nevertheless, the maximum average power density remains close to the one plotted in Fig. 4.

B. Jupiter Orbit

The lower ionospheric density and stronger magnetic field of Jupiter, compared with the Earth case, make ohmic effects negligible for EDT of reasonable size (say less than 20 km) [8], so that Eq. (13), rather than (12), must be used for computing the achievable power density.

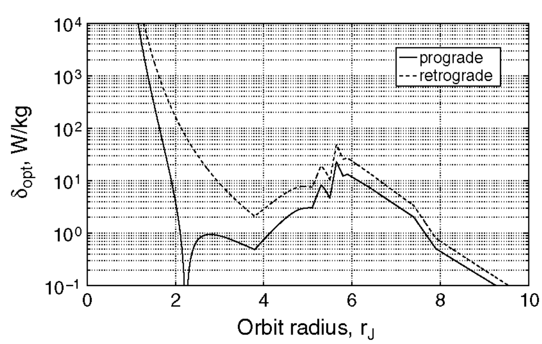

Fig. 6 Optimum instantaneous power density achievable for a 20 km long aluminium tape EDT, with tape thickness of 0.05 mm, in circular (prograde and retrograde) equatorial Jupiter orbit.

The motional electric field for circular equatorial orbits is computed using Eq. (20), as in the Earth case, and taking $B_S = 420\ \mu\mathrm{T}$, $r_p = 71492$ km, $\omega_p = 1.7585 \times 10^{-4}$ rad/s, and $\mu_p = 1.267 \times 10^{17}\ \mathrm{m^3\,s^{-2}}$. By substituting Eq. (20) into (13) and employing a Divine-Garrett ionospheric density model [10], one finally obtains the zero-ohmic-effect power density, which is plotted in Fig. 6, considering an aluminum tape tether of 0.05 mm thickness and 20 km length.

The plot highlights the advantage of using EDTs as power plants (as opposed to low-power-density radioisotope thermoelectric generator (RTG)-based power sources (typical power density levels for RTGs do not exceed 8 W/kg) in the lower part of Jupiter's plasmasphere, as already pointed out in the literature [1,2,11–13].

VI. Conclusions

The performance of bare EDTs as power-generating systems under OML-theory conditions has been analyzed based on a previously published asymptotic model for the tether current and bias. The model allows for the easy determination of the optimum load impedance for maximum power generation at the cathodic end of the tether, and, in turn, the computation of the corresponding optimum power, power density, and load power efficiency. Limit cases of zero and dominant tether ohmic impedance provide closed-form analytical formulas that match previously published results.

The maximum power density theoretically achievable by an EDT is seen to be proportional to the square of the local motional electric field along the tether and to the conductivity/density ratio of the tether material. A numerical evaluation of the maximum performance in circular equatorial Earth and Jupiter orbits show that power density levels comparable to the ones of solar arrays (for the Earth) and superior to RTGs (for Jupiter) are achievable. Simulations with a 20-km-long 0.05-mm-thick tape EDT in low circular equatorial Earth orbit at 600 km altitude show that nearly optimum power-generation conditions are reached during maximum solar activity, with optimum load impedance always very close to tether ohmic impedance and, therefore, virtually insensitive to local plasma density variations. A power density decrease is experienced by the same system during eclipse transitions under minimum solar activity conditions, in which a tuning of the load impedance based on the local ionospheric density becomes necessary.

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