Bifurcation diagrams for polymer blends with diffuse interfaces in confined and adaptive geometries

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The dynamics of binary mixtures such as polymer blends, and of fluids near the critical point, is described by model-H, a phenomenological mean field model which couples momentum transport and diffusion of the components [1]. We present model-H with added boundary conditions allowing us to study the combined effect of phase separation within a film of polymer blend and structuring of the surface of the film itself [2]. We apply it to analyze the stability of vertically stratified steady extended films and show that convective transport leads to new mechanisms of instability as compared to the simpler purely diffusive case described by the Cahn-Hilliard model [3]. We carry out this analysis for realistic parameter values corresponding to polymer blends used in experiments with, e.g. PS/PVME mixtures [4, 5].

However, geometrically more complicated states are possible that involve lateral structuring, strong deflections of the free surface, oblique diffuse interfaces, checkerboard modes, or droplets of one component above the other one, at critical composition studied by solving the Cahn-Hilliard equation in the static limit for quadratic [6] and rectangular [7] fixed domains or with deformable free surfaces, i.e., surfaces that can adapt to the concentration field inside the domain [7]. Here, we extend these results towards off-critical compositions, since a balanced overall composition is not very common in experiments. In particular, we study steady nonlinear solutions of the Cahn-Hilliard equation for two-dimensional layers of fixed rectangular geometry and with deformable free surface. Furthermore we distinguish the cases with and without energetic bias at the free surface.

We numerically analyse the steady films of off-critical polymer blends with arbitrary surface deflections based on minimizing the underlying free energy functional at given composition and volume constraints using a variational approach based on the Cahn-Hilliard equation. The problem is solved numerically using the finite element method (FEM). Results are presented in the form of bifurcation diagrams that give the free energy, and the L_2-norms of surface deflection and the concentration field, as functions of lateral domain size and mean composition.

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