Simple analytical approach for the calculation of winding resistance in gapped magnetic components

Fermín A. Holguín, Rafael Asensi, Roberto Prieto, José A. Cobos

Abstract— **The Dowell expression is the most commonly used method for the analytic calculation of the equivalent resistance in windings of magnetic components. Although this method represents a fast and useful tool to calculate the equivalent resistance of windings, it cannot be applied to components that do not fit with classical 1D assumption, which is the case of gapped magnetic components. These structures can be accurately analyzed using finite-element analysis (FEA) with the time cost that this represents.**

Modifying the Dowell´s equation, and taking advantage of the orthogonality between skin and proximity effects, a simple solution that allows its application in gapped magnetic components is proposed in this work, which results shows a very good accuracy compared with experimental measurements.

Index Terms— **Winding resistance, magnetic components, air gap.**

I. INTRODUCTION

Accurate estimation of losses in windings of magnetic components used in power electronics applications is a very important task. An imprecise calculation of the equivalent resistances of the windings can lead to under or overestimating the losses in the component, resulting in unexpected temperature rise or a costly component oversize. Several methods have been developed to accurately predict the losses in windingsmade with round conductors [1-6]. The most extended one, often called the Dowell method [1], consists on dividing the windings into portions, considering every portion as an equivalent foil conductor with equal total sectional area and then multiplying the dc resistance of each layer by a corresponding factor to obtain the ac resistance of the winding. This method, however, may have considerable errors at high frequencies [8] and many authors have proposed new models based on modifications of this method [3, 4 and 7]. Another choice to accurately predict the resistance in windings of magnetic components is to use finite element analysis (FEA) tools to model and simulate the component [9-11]. These tools are considered to provide reliable results with the disadvantage of relatively large simulation time.

When it comes to gapped magnetic components, Dowell based methods cannot be properly applied because they are only valid when the windings of the magnetic component are arranged in a specific way and the magnetic field along the window breath can be considered as one-dimensional (1D), which clearly is not the case of gapped components. In [12] a solution to calculate the power losses in round conductors in gapped magnetic components is proposed but it can require a big computation time as it uses the mirror-image method [13] to obtain the magnetic field over the considered conductor and the winding must be considered turn by turn.

When a fast computation of the equivalent resistance of the windings is needed, the simplicity of the Dowell's method represents a huge advantage because it can be easily integrated in magnetic modeling and design tools without, practically, penalize the computation time. This simplicity, however, cannot be exploited when gapped magnetic components are being analyzed forcing the designer to use a time consuming calculation method in order to predict the winding resistance in this kind of components. In this paper, a correction factor, based on geometrical parameters of the windings and the local magnetic field in the component, which improves the calculation of the equivalent resistance of the windings in gapped magnetic components (using the foundations of Dowell's method) is proposed. The results of the proposed method have small errors compared with measurements in many cases.

II. ESTIMATING PROXIMITY EFFECT LOSSES

A. The Dowell's equation

In the Dowell method, a layer of round conductors with diameter D is replaced by a single foil conductor with thickness d equal to $\sqrt{\pi}\,D/2$ (see figure 1). Given that the equivalent foil conductor has been extended to become a conductor with equal height of the window bread b, it must be corrected by a parameter η, called the porosity factor, equal to $d/(d+s)$, where s is the spacing between turns, in order to match its dc resistance with the original winding [1, 2]. Then the ac resistance of the m_{th} layer can be calculated with expression (1), where *Rac* and *Rdc* are the ac and dc resistance of a given layer in the winding, m is the number of the layer in consideration and X is equal to d/δ, being δ the skin depth defined as $1/\sqrt{\pi\mu\sigma f}$ where μ and σ are, respectively, the permeability and conductivity of the conductor and f is the frequency of the sinusoidal current.

$$R_{ac} = R_{dc} \frac{X}{2} \left[\frac{\sinh X + \sin X}{\cosh X - \cos X} + \left(\frac{m^2 - 1}{3} \right) \frac{\sinh X - \sin X}{\cosh X + \cos X} \right] \quad (1)$$

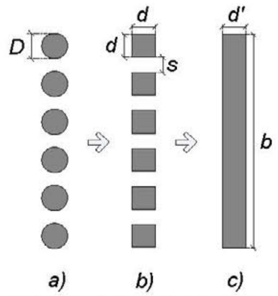


Figure 1. Obtaining the equivalent foil conductor from round conductors: a) layer of round conductors with diameter *D*, b) spaced square conductors of width *d* and c) the equivalent foil conductor of width *d'* and height *b*.

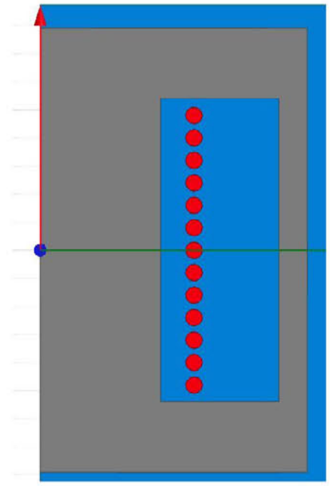
Expression (1) assumes constant field along the window breath and the factor $(m^2 - 1)/3$, from now on *F(m)*, represents the increment of the field due to the contribution of the corresponding layer to the next one respect to the existing field. As skin and proximity effects are orthogonal [2, 3], it is possible to calculate both effects separately and (1) can be split as follows:

$$F_{Skin} = \frac{X}{2} \left[\frac{\sinh X + \sin X}{\cosh X - \cos X} \right] \quad (2)$$

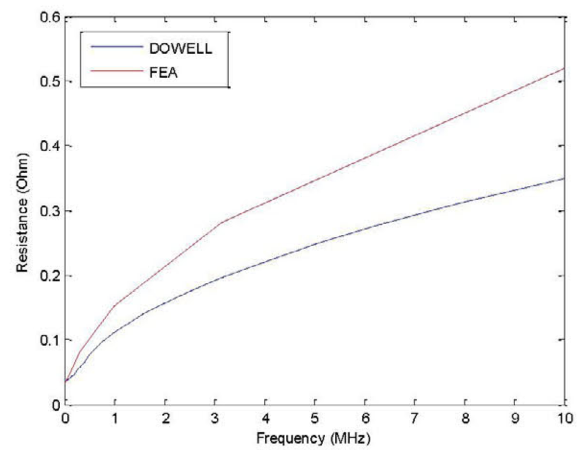
$$F_{Prox} = \frac{X}{2} \left[F(m) . \frac{\sinh X - \sin X}{\cosh X + \cos X} \right] \quad (3)$$

As Dowell's approach only considers the proximity effect between layers, its influence over the equivalent resistance of a winding with only one equivalent layer would be zero, neglecting the actual proximity effect between turns "inside" the equivalent layer. As a consequence, the relative error between the calculated resistances, by means of Dowell method and FEA tools, goes bigger as the frequency increases. This situation can be seen in figure 2, where the calculated resistance of a given component is compared with FEA results.

Moreover, if there is a gap in the magnetic core, the windings, besides the proximity effect between turns, will be influenced by the fringing field of the gap which will be reflected as an increment of the equivalent resistance (see figure 3). In this situation, although the windings could easily be converted into equivalent layers, the magnetic field along the window breath cannot be considered as one-dimensional and, therefore, Dowell's method cannot be properly applied.



(a)



(b)

Figure 2. (a) Axis-symmetric representation of a magnetic component with 13 turns (D = 0.6 mm and s = 0.1 mm) and (b) calculated and FEA simulated equivalent resistance vs. frequency.

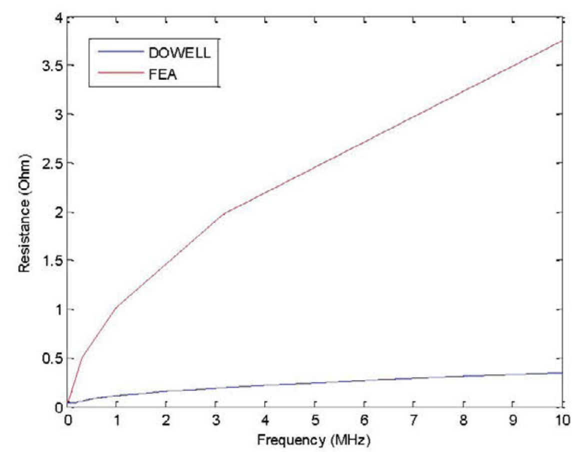


Figure 3. Calculated and FEA simulated equivalent resistance vs. frequency in the component 2a with a central gap of 0.4 mm.

B. Modifying the Dowell's model

If we neglect the edge effect of the core over the conductors and consider that the skin effect contribution in the equivalent resistance of the component only depends on the frequency and the characteristics of the conductive material, it can be assumed that gap and proximity effects are the main contributors of errors in the calculated resistance. In this order, we can divide the total proximity effect (including gap effect) in two components: intra-layer proximity and inter-layer proximity which are, respectively, the proximity effect between turns within an equivalent layer and the proximity effect between layers (the gap effect has been included in this component). Since *F(m)*, in expression (3), represents the increment of the *H* field from one layer to another, we can modify this factor in order to take into account the influence of both components of the proximity effect and correct the calculations and its contribution in the equivalent resistance of the winding.

To do that, many 2-D and 3-D FEA simulations have been run in order to establish a relationship between the current *H* field along the layers and winding area and the influence of geometrical characteristics of the winding in its equivalent resistance. For this, a simple procedure was followed:

1- Three groups of magnetic components, divided in coreless, gapped and non-gapped, are modeled and simulated.

2- The results are processed in order to clear the proximity factor from expression (1) and the resulting information is used to make a mathematical regression and fit the data into a new chosen proximity factor, $F_X(m)$, which will replace *F(m)* in (3) of the form:

$$F_X = a(m)X^{b(m)} \quad (4)$$

Where

$$a(m) = a_1 m + a_0 \quad (5)$$

$$b(m) = b_2 m^2 + b_1 m + b_0 \quad (6)$$

With *m* equal to the corresponding layer.

3- Finally, the coefficients of the new factor are expressed in terms of geometrical and magnetic characteristics of the winding.

C. Defining the coefficients

The coefficients a_1, a_0, b_2, b_1, and b_0 in (5) and (6) have been expressed, through a data fitting process from FEA results, as functions of geometrical parameters (diameter of the conductors, distance between conductors and layers) and magnetic field *H* around the equivalent layer and its distance from the air gap.

First, the first component of the proximity factor, the intra-layer factor, was evaluated. For this case, since there should not be any core influence over the windings, a group of coreless components (with one layer of conductors) was simulated in 2D and 3D FEA tools and the coefficients were related with the winding structure (diameter of the conductors and distance between turns). Then, in order to evaluate the second component of the proximity factor, the magnetic core was included in the analysis and the group of non-gapped and multilayered components was simulated to expand the correction factor to multilayered windings in such a way that a_1 and b_1 could be expressed as a function of the wire diameter, distance between turns and layers, d_t and d_l respectively, and the number of the corresponding layer, *m*, and b_2 as a constant according to expressions (7), (8) and (9), respectively. Coefficients a_0 and b_0, as the gap influence is not yet taken into account, can be considered as constants equal to -1.171 and 0.12 respectively. At this point, as the proximity factor has been corrected with the intra-layer proximity effect, the modified model can be used to calculate equivalent resistance in both coreless and non-gapped magnetic components.

$$a_1 = 1.045(1 + \frac{d_t}{mD}) \quad (7)$$

$$b_1 = \frac{d_l d_t}{D^2} + 0.13 \quad (8)$$

$$b_2 = -0.037 \quad (9)$$

Coefficients a_0 and b_0 could be modified according to the information of the fringing field and the distance from the corresponding layer to the air gap in order to include its influence in the calculations. If we take, as a reference, the magnetic field, H_a, produced by an air cored component with one layer of N turns carrying a current I, and having in consideration that the magnetic field intensity due to the air gap over a given conductor is inversely proportional to its distance from the gap, *x*, a_0 and b_0 can be expressed as functions of the magnetic field factor, *H*, the number of layers, *n*, the corresponding layer, *m*, the length of the reference air cored layer, l_a and the length of the air gap, l_g, as follows:

$$H_g = {}^{NI}/_{l_g} \quad ; \quad H_a = {}^{NI}/_{l_a} \quad (10)$$

$$H = {}^{H_g}/_{H_a} = {}^{l_a}/_{l_g} \quad (11)$$

$$a_0 = [(13.31 + 1.4m) + 0.69n - 70.9K_d]\left[\frac{n-(m-1)}{1.5}\right] \quad (12)$$

$$b_0 = K_d \ln(H) \quad (13)$$

$$K_d = 0.0048\left(\frac{D}{x}\right)^{-2.586} + 0.195 \quad (14)$$

D. Model validity

Since the study was focused on the windings structure and the effect of the air gap over the equivalent resistance, and considering that the model has been developed from the basis of Dowell's method, it is considered that the characteristics of the magnetic core (permeability, shape) have no influence over the calculated resistance of the windings. The study, therefore, was carried out based on the results of FEA simulations of many components in which no especial care has been taken in the shape of the core or the magnetic material used for the simulations. Actually, all FEA simulations were run with POT cores, of different sizes, and a linear generic ferrite with a relative permeability equal to 1000. The windings of the simulated components were modeled with copper round conductors with diameters from 0.1 mm to 1 mm, distance between turns from 0.09 mm to 3 mm and distance between layers from 0.1 mm to 2 mm. Outside these conditions, the validity of the described method has not been tested.

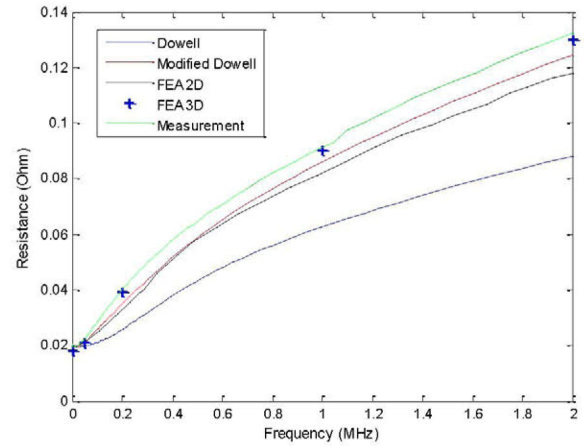
III. RESULTS

To state the accuracy of the proposed model, the equivalent resistance of several components has been calculated using the described model and compared with measurements. The tested devices are coreless (air-cored) inductors of 1 and 2 layers and gapped inductors of 1 and 4 layers (with 15 turns per layer) made with AWG24 magnet wire placed in a RM8/I core (material 3F3 from FERROXCUBE) with two different air gaps: 0.4 mm and 2.2mm. The impedances of all components were measured with a precision impedance analyzer (Agilent 4294A) in a frequency range from 40Hz to 2MHz.

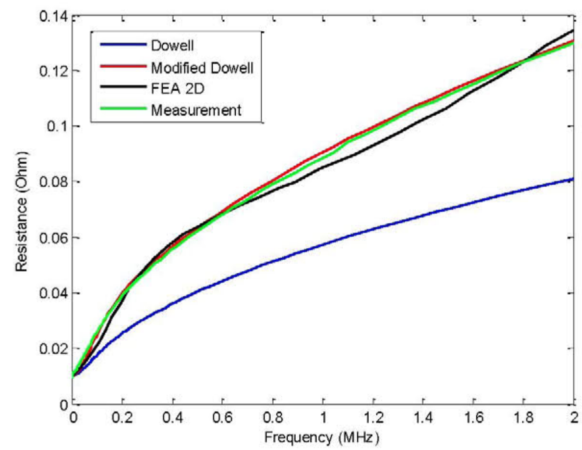
The R_{dc} of each layer of conductors was calculated according to its number of turns, length and section of the wire with a copper resistivity for 25°C (room temperature). The calculated resistance in coreless and gapped components, using the proposed method, is compared with classical Dowell, 2-D FEA simulations and experimental data in figures 4 and 5 respectively, where a good agreement with measurements can be appreciated (for visibility issues, due to the resonance between the inductance and parasitic capacitance, the equivalent resistance in the full frequency range of the measurement is not showed in figure 5).

Although the results are being compared with Dowell, this information is merely informative because, considering the nature of the considered components, the classical method of Dowell cannot be properly applied to these configurations. For similar reasons, the results obtained for these particular components (gapped) cannot be compared with others solutions, but [12] and FEA (here they are compared with FEA only), because they do not include the influence of the fringing field over conductors.

The results in figure 5 do not include the effect of the resonance between the inductance and the parasitic capacitance of the windings. If we include the measured inductance and parasitic capacitance of the component in figure 2d (182.95 µH and 27.03 pF respectively) in the results, the calculated equivalent resistance matches very well with measurements for this component (see figure 6).



(a)



(b)

Figure 4. Equivalent resistance vs. frequency in a coreless component with 9 turns, turn diameter of 8.4 mm, 0.1 mm of distance between turns and wire diameter of 0.6 mm. (a) Single layer and (b) two parallel layers.

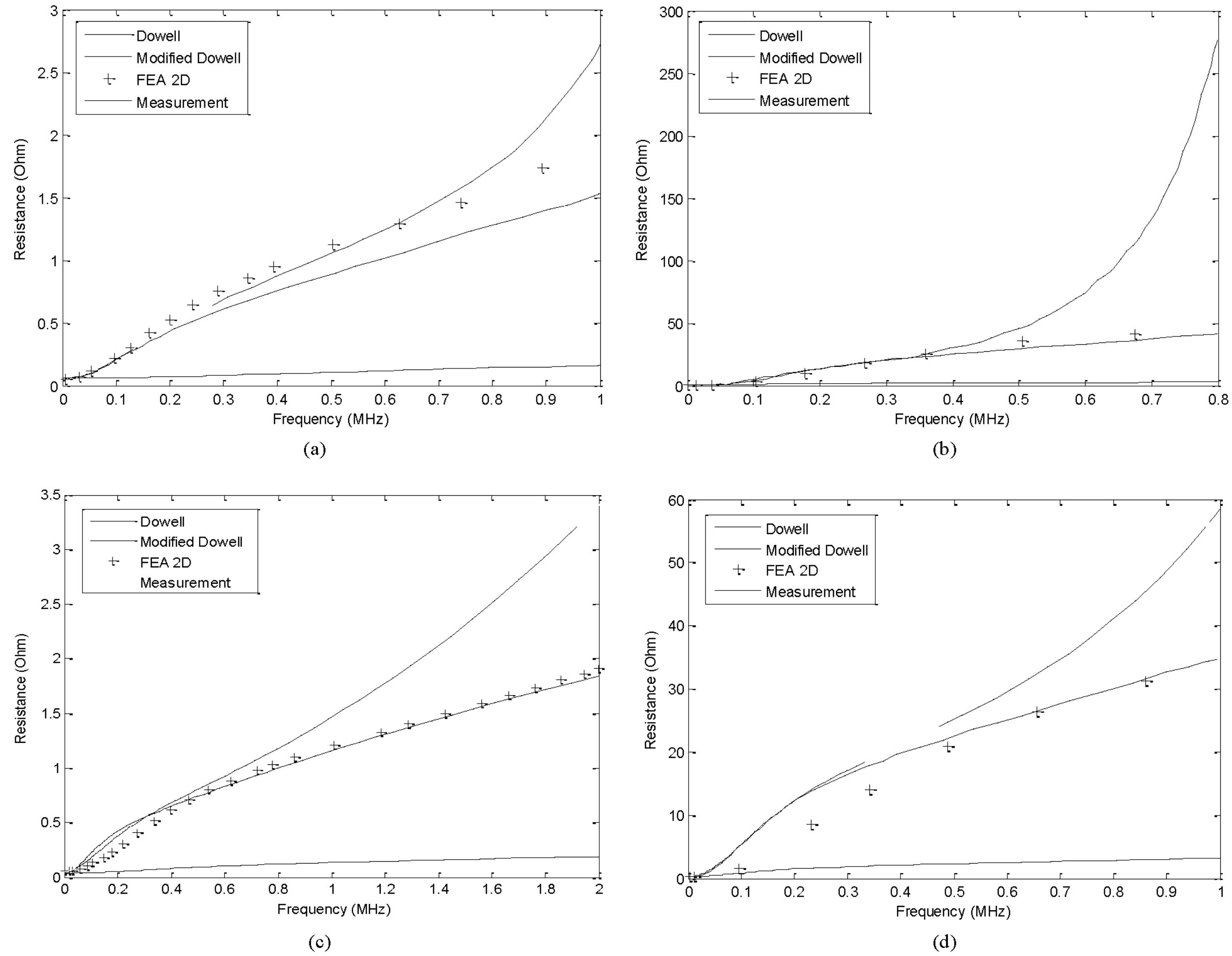


Figure 5. Comparison of equivalent resistance vs. frequency predicted by the proposed model with classic Dowell, 2-D FEA simulations and measurements. (a) And (b) windings of 1 and 4 layers respectively with air gap of 0.4 mm, (c) and (d) windings of 1 and 4 layers respectively with air gap of 2.2 mm.

I. CONCLUSIONS

A correction factor that allows the application of Dowell's method in the estimation of equivalent resistance in windings of gapped magnetic components is proposed. This approach has been studied based in 2D and 3D FEA simulations which results has been used to develop a fast and simple semi-empirical method that shows good agreement with measurements in the considered cases. As the relevant coefficients of the proposed method are related with magnetic and geometrical characteristics of the component it can be used, besides gapped components, in other configuration such non-gapped and core-less components but, as it has been developed based on Dowell's approach, it is restricted to "layered" windings structures.

Also, as the model was developed from the basis of Dowell's method, it is considered that the characteristics of the magnetic core have no influence over the calculated resistance (edge effect neglected). However, from 2D FEA results, it has been observed that the permeability of the magnetic core does have influence over the equivalent resistance of the windings and, therefore, it should be included in the model. In gapped components the influence of the air gap in the equivalent resistance of the windings is much higher than edge effect and it can be neglected. In non-gapped components, edge effect can be significant as the relative permeability of the magnetic material and frequency increase and the functions to obtain the coefficients related with magnetic properties might not be accurate.

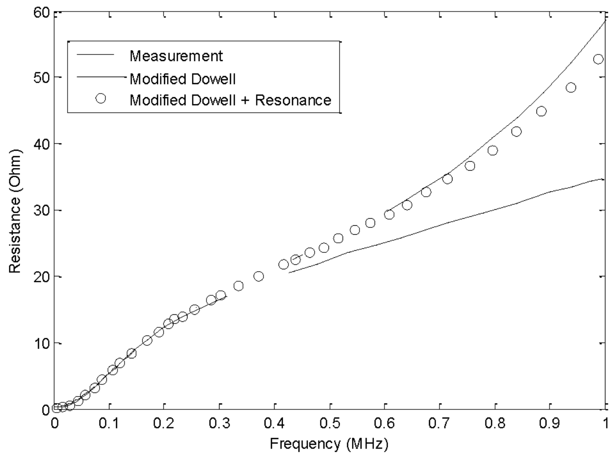


Figure 6. Equivalent resistance vs. frequency predicted by the proposed model including the effect of the resonance in component of 4 layers with air gap of 2.2 mm.

REFERENCES

[1] P.L. Dowell, "Effects of eddy currents in transformer windings", *Proceedings of the IEE*, vol. 113, no. 8, pp. 1387–1394, Aug. 1966.

[2] J. A. Ferreira, "Improved analytical modeling of conductive losses in magnetic components", *IEEE Transactions on Power Electronics*, vol. 9, no. 1, pp. 127–31, Jan. 1994.

[3] Xi Nan and Charles R. Sullivan, "An improved calculation of proximity effect loss in high frequency windings of round conductors", in *34th Annual IEEE Power Electronics Specialists Conference*, 2003, vol. 2, pp. 853–860.

[4] Xi Nan and Charles R. Sullivan, "Simplified high-accuracy calculation of eddy-current loss in round-wire windings", in *35th Annual IEEE Power ElectronicsSpecialists Conference*, 2004, vol. 2, pp. 873 – 879.

[5] M.P. Perry, "Multiple layer series connected winding design for minimum losses", *IEEE Transactions* on *Power Apparatus and Systems,* vol. PAS-98, pp. 116-123, Jan. /Feb. 1979.

[6] M.P. Perry, "Multiple layer parallel connected air-core inductor design", *IEEE Transactions* on *Power Apparatus and Systems,* vol. PAS-98, pp. 1387-1393, Jul./Aug. 1979.

[7] F. Robert, P. Mathys and J. P. Schauwers "A closed-form formula for 2-D ohmic losses calculations in SMPS transformer foils", *IEEE Transactions* on *Power Electronics,* vol. 16, pp. 437-444, May 2001.

[8] R. Prieto, J. A. Oliver, J. A. Cobos, J. Uceda and M. Christini "Errors obtained when 1D magnetic component models are not properly applied", *Annual IEEE Applied Power Electronics and Expositions,* 1999, vol. 1, pp. 206-212.

[9] R. Asensi, J.A. Cobos, O. García, R. Prieto and J. Uceda "A full procedure to model high frequency transformer windings", *25th Annual IEEE Power Electronics Specialists Conference*, 1994, vol. 2, pp. 856-863.

[10] R. Asensi, R. Prieto, J.A. Cobos and J. Uceda "Modeling high-frequency multiwinding magnetic components using finite-element analysis", *IEEE Transactions on Magnetics*, vol. 43, no. 10, pp. 3840-3850, 2007.

[11] R. Prieto, R. Asensi, C. Fernandez, J.J.A. Oliver and J.A. Cobos, "Bridging the gap between FEA field solution and the magnetic component model", *IEEE Transactions on Power Electronics*, vol. 22, no. 3, pp. 943-951, 2007.

[12] Chen Wei, Huang Xiaosheng and Zheng Juanjuan "Improved winding loss theoretical calculation of magnetic component with air gap", *7th International Power Electronics and Motion Control Conference*, 2012, vol. 1, pp. 471-475.

[13] Jiankun Hu and Charles R. Sullivan "Optimization of shapes for round-wire high-frequency gapped-inductor windings", *33th Annual IEEE Industry Applications Conference*, 1998, vol. 2, pp. 907-912.