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A general formulation for fault detection in stochastic continuous-time dynamical systems

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Abstract

In this work, a general formulation for fault detection in stochastic continuous-time dynamical systems is presented. This formulation is based on the definition of a pre-Hilbert space so that orthogonal projection techniques, based on the statistics of the involved stochastic processes can be applied. The general setting gathers different existing schemes within a unifying framework.

Key words: fault diagnosis, continuous-time dynamical systems, stochastic processes, pre-Hilbert space

1 Introduction

In this paper, a general framework for fault diagnosis in stochastic continuous-time dynamical systems is presented. Few schemes have been proposed in the literature to address this problem from different perspectives [2, 4]. In this work we propose a unifying framework based in the definition of a pre-Hilbert space. Within this space, the knowledge about the fault translates into different projections.

The paper is organized as follows. In Section 2, the general problem of system faults is presented. The detection scheme starting in the basic residual generation is shown on Section 3; the different projections based on the available knowledge on the fault are elaborated in Section 4. Section 5 comments some basic issues to consider when building the estimators and applying the test to the projection quantities. Concluding remarks are summarized in Section 6.

2 Problem statement

Let us consider the following nonlinear time-variant dynamical system

$$\dot{x}(t) = f(x(t), u(t), \theta_0, t) + \eta(t) + \mathcal{B}(t - T_0)\phi(t), \qquad (1)$$

$$\begin{aligned} y(t) &= h(x(t), u(t), t), \\ x(0) &= x_0 \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the system state, which has known initial value $x_0 \in \mathbb{R}^n$; $u(t) \in \mathbb{R}^m$ is the control input; the known function $f \in C^1(\mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^+, \mathbb{R}^n)$ represents the dynamics of the nominal model; the random vector $\eta : \mathbb{R}^+ \to \mathbb{R}^n$, which gathers external disturbances and modelling errors, corresponds to an n-dimensional stochastic process whose components are Gaussian generalized processes, given by a linear combination of an MS-continuous Gaussian random process and white Gaussian noise. The derivatives of stochastic system (1) are interpreted as MS derivatives.

$y(t) \in \mathbb{R}^l$ is the measurable output, and the nonlinear mapping $h : \mathbb{R}^n \times \mathbb{R}^p \times \mathbb{R} \to \mathbb{R}^l$ can represent different output availability situations. If x can be computed from y, u and t via an implicit function theorem reasoning, this is equivalent to a full-state measurement, a common assumption in most nonlinear deterministic system diagnosis schemes [5, 9]. Alternatively, when the full state is not directly computable, on one hand fault detectability has been addressed provided the pair f, h satisfies specific observability conditions [3, 10]. On the other hand, high gain observers have been employed to build up robust (insensitive to faults) estimators of the state x for systems with specific structures [6, 4]. Then, full state based diagnosis procedures are proposed, making use of such state estimators. In this work, we will assume full state availability.

Finally, different types of faults are gathered in this model. On one hand, we can consider any additive faults due to some fault process $\phi : \mathbb{R}^+ \to \mathbb{R}^n$ representing the changes in the system dynamics, which is assumed to be an n-dimensional stochastic process continuous in mean square (MS-continuous) and with Gaussian marginal densities. $\mathcal{B}(t - T_0)$ is a diagonal matrix representing the time profile of the fault, such that in the case of an abrupt (sudden) fault, those functions will take the form of a step function and in the case of an incipient (slowly developing) fault they will be ramp-type functions.

Alternatively, parametric faults are modelled by a change in the parameter vector from θ_0 to θ_1 at the unknown instant T_0. The parametric changes can also be modelled in additive form with $\mathcal{B}(t - T_0)\phi(t) = s(t - T_0)$, step function, and

$$\phi(t) = \phi(x(t), u(t), \theta_1, \theta_0, t) = f(x(t), u(t), \theta_1, t) - f(x(t), u(t), \theta_0, t). \tag{2}$$

3 Detection scheme

3.1 Residual generation

Taking into account the structure of system (1), a convenient parity-checker may be constructed based on a Luenberger observer type structure ([2, 4, 8]), which is determined by the nominal part of the model plus a stabilizing term:

$$\dot{\hat{x}}(t) = -\Lambda(\hat{x}(t) - x(t)) + f(x(t), u(t), t), \quad \hat{x}(0) = x_0, \tag{3}$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the state estimation and the matrix $\Lambda = \mathrm{diag}(\lambda_1, \ldots, \lambda_n)$, with $\lambda_i > 0$, $i = 1, \ldots, n$.

Note that since the state availability is assumed, in equation (3) the function $f(x(\cdot),\cdot,\cdot)$ is used instead of $f(\hat{x}(\cdot),\cdot,\cdot)$ as it would correspond to a proper Luenberger observer. Such substitution does facilitate the residual generation.

Subtracting observer (3) from system (1), and since the MS derivative is a linear operator, we can obtain the differential equations system which explains the evolution of the state estimation error or residual, $\epsilon(t) = x(t) - \hat{x}(t)$, namely,

$$\dot{\epsilon}(t) = -\Lambda\epsilon(t) + \eta(t) + \mathcal{B}(t - T_0)\phi(t), \qquad \epsilon(0) = 0. \tag{4}$$

The analysis of this residual vector is the key to conclude if a fault has occurred in the system or not. We can express the solution of such differential equations system (4) as

$$\begin{aligned} \epsilon(t) &= \int_0^t e^{-\Lambda(t-\tau)}\eta(\tau)\, d\tau + \int_0^t e^{-\Lambda(t-\tau)}\mathcal{B}(\tau - T_0)\phi(\tau)\, d\tau, \\ &= \epsilon_{H_0}(t) + \epsilon_\phi(t) \end{aligned}$$

where $\epsilon_{H_0}(t)$ stands for the residual under hypothesis H_0 (no fault) and it is an Ornstein-Uhlenbech process with $E[\epsilon_{H_0}(t)] = 0$; on the other hand, $\epsilon_\phi(t)$ gathers all the information concerning the fault, its properties depending on $\phi(t)$.

3.2 Residual evaluation

Most fault diagnosis schemes are based on the study of the properties of the residual $\epsilon(t)$ in order to detect the existence of some $\epsilon_\phi(t)$ added to the "background signal" $\epsilon_{H_0}(t)$. Hence, the implementation of detection schemes is determined by the a priori available information about $\epsilon_{H_0}(t)$ and $\epsilon_\phi(t)$.

Classical signal detection schemes rely on the knowledge of the profile of $\epsilon_\phi(t)$ over a period of time. If $\epsilon_\phi(t)$ (or the set of possible signals) is known, the detection (and classification of the signal) can be implemented via an appropriate projection on a detection space which, in the context of white noise background noise, leads to a matched filter detector structure [7]. Unfortunately, these ideas cannot be directly applied to the residual $\epsilon(t)$ due to the nature and limited knowledge of the stochastic processes $\epsilon_{H_0}(t)$ and $\epsilon_\phi(t)$.

In this work we propose the use of some pre-Hilbert function spaces where $\epsilon(t)$ can be framed, and where $\epsilon_{H_0}(t)$ and $\epsilon_\phi(t)$ tend to be orthogonal elements belonging to known subspaces. Hence, the existence of $\epsilon_\phi(t)$ can be detected via a orthogonal decomposition of $\epsilon(t)$ using projection techniques.

Definition: Let us consider $T, \lambda_i > 0$ and the Gaussian white noise $\eta(t)$ with autocorrelation $R_\eta(t_1, t_2) = \delta(t_1 - t_2)$. For each $t > T$, the vector space V is the set of all stochastic processes $\gamma(t)$ defined in the interval $[t-T, t]$ such that $\forall\gamma(t) \in V$ it is $\gamma(t) = \overline{\gamma}(t) + \tilde{\gamma}(t)$ where $\overline{\gamma}(t) = E[\gamma(t)]$ is continuous and $\tilde{\gamma}(t) = \int_0^t e^{-\Lambda(t-\tau)}\sigma_\gamma\eta(\tau)\, d\tau$, $\sigma_\gamma > 0$.

In such vector space, we define the (moving) inner product $< \cdot,\cdot >_T (t)$ as follows (see [4] for a detailed formulation in scalar stochastic processes):

Definition: Given $\alpha(t), \beta(t) \in V$, the moving inner product $< \alpha, \beta >_T (t)$ is defined as

$$< \alpha, \beta >_T (t) = \frac{1}{T}\int_{t-T}^t E[\alpha(\tau) \bullet \beta(\tau)]\, d\tau,$$

where $\bullet$ stands for the standard inner product in $\mathbb{R}^n$. Note that in a rigorous manner, the expectation must be taken conditional to the available measurements. This fact is not explicitly indicated to simplify the notation. It is easy to prove that $< \cdot, \cdot >$ is an inner product and $(V, < \cdot, \cdot >)$ a pre-Hilbert space.

Based on these definitions, in case that $< \epsilon_{H_0}, \epsilon_\phi >_T (t) \approx 0$, we have that $\|\epsilon\|_T^2(t) \approx \|\epsilon_{H_0}\|_T^2(t) + \|\epsilon_\phi\|_T^2(t)$. Hence, if we can measure $\|\epsilon\|_T(t)$, the fault will be detected provided the size of $\|\epsilon_\phi\|_T(t)$ is large enough when compared to $\|\epsilon_{H_0}\|_T(t)$.

In case that $\|\epsilon_\phi\|_T(t)$ is not large, but the profile of $\epsilon_\phi(t)$ is known, some projections of $\epsilon(t)$ will be defined below to allow detection. This information will also be useful for isolation purposes.

4 Knowledge on the residual and appropiate projections

In general, the possible forms we expect on $\phi(t)$ (and accordingly on $\epsilon_\phi(t)$) will determine the projection operations via inner products. Hence, some assumptions and approximations are needed to carry out the detection schemes.

4.1 Approximation due to unknown T_0

In all cases, a first limitation comes from the fact that T_0 is unknown. If we define

$$\phi_{LP}(t) = \int_0^t e^{-\Lambda(t-\tau)}\phi(\tau)\ d\tau,$$

then we can write $\epsilon_\phi(t) = \phi_{LP}(t) - e^{-\Lambda(t-T_0)}\phi_{LP}(T_0)$, so that $\epsilon_\phi(t) \approx \phi_{LP}(t)$ for $t \gg T_0$.

Provided $\phi(t)$ is known, $\phi_{LP}(t)$ can be computed as an approximation of ϵ_ϕ. This function can then be employed as a reference signal where to project the residual $\epsilon(t)$ for testing purposes.

Note that the quality of this approximation is guaranteed for t larger than T_0; this fact can affect the scheme detection time.

4.2 Known profile of $\phi(t)$

Sometimes, the possible fault profiles of $\phi(t)$ are known. This happens, for instance, when the possible failure profiles are typified a priori, due to some knowledge on the system. (Note that when $\phi(t)$ is known, and only T_0 is unknown, we are framed in a typical problem of target detection.)

Given a known deterministic function $\phi(t)$, we can compute

$$\begin{aligned} < \epsilon_{H_0}, \phi_{LP} >_T (t) &= \frac{1}{T}\int_{t-T}^{t} E[(\epsilon_{H_0}(\tau) \bullet \phi_{LP}(\tau)]\ d\tau \\ &= \frac{1}{T}\int_{t-T}^{t} \phi_{LP}(\tau) \bullet E[\epsilon_{H_0}(\tau)]\ d\tau \\ &= 0 \end{aligned}$$

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which will be obtained when there is no fault. On the other hand, if we compute

$$\begin{aligned} <\epsilon, \phi_{LP}>_T(t) &= <\epsilon_{H_0}, \phi_{LP}>_T(t) + <\epsilon_\phi, \phi_{LP}>_T(t) \\ &= <\epsilon_\phi, \phi_{LP}>_T(t) \\ &\approx \|\phi_{LP}\|_T^2(t) \end{aligned}$$

Hence, the magnitude of this inner product can be employed as a fault indicator.

4.2.1 Known parameter change

If parameter changes with known θ_1 are considered, and the state space is available (we call the specific sample $x_S(\tau),\ \tau \leq t$), then $\phi(t)$ can be conditionally estimated. Note that in this special case of fault generated by a parameter variation, the randomness of the process $\phi(t)$ is fully characterized by the sigma-algebra $\sigma(\eta(\tau), 0 \leq \tau \leq t)$ and therefore it is also determined by the available state measurement:

$$\begin{aligned} \hat{\phi}(t)(t) &= E[\phi(t)/x_S(\tau),\ \tau \leq t] \\ &= E[f(x(t), u(t), \theta_1, t) - f(x(t), u(t), \theta_0, t)/x_S(\tau),\ \tau \leq t] \\ &= f(x_S(t), u(t), \theta_1, t) - f(x_S(t), u(t), \theta_0, t) \\ &= \phi_S(t). \end{aligned}$$

With the approximation of the fault profile proposed for unknown T_0, we have

$$\begin{aligned} <\epsilon_{H_0}, \phi_{SLP}>_T(t) &= \frac{1}{T}\int_{t-T}^{t} E[\epsilon_{H_0}(\tau) \bullet \phi_{SLP}(\tau)/x(\tau_1), \tau_1 \leq \tau]\ d\tau \\ &= \frac{1}{T}\int_{t-T}^{t} \phi_{SLP}(\tau) \bullet E[\epsilon_{H_0}(\tau)/x(\tau_1), \tau_1 \leq \tau]\ d\tau \\ &\approx \frac{1}{T}\int_{t-T}^{t} \phi_{SLP}(\tau) \bullet E[\epsilon_{H_0}(\tau)]\ d\tau \\ &= 0 \end{aligned}$$

where we have assumed that $\epsilon_{H_0}(t)$ is uncorrelated to $x(t)$, which happens to be a reasonable hypothesis in nonlinear systems. Therefore, in general, computing

$$\begin{aligned} <\epsilon, \phi_{SLP}>_T(t) &= <\epsilon_{H_0}, \phi_{SLP}>_T(t) + <\epsilon_\phi, \phi_{SLP}>_T(t) \\ &\approx <\epsilon_\phi, \phi_{SLP}>_T(t) \\ &= \frac{1}{T}\int_{t-T}^{t} E[\epsilon_\phi(\tau) \bullet \phi_{SLP}(\tau)/x(\tau_1), \tau_1 \leq \tau]\ d\tau \\ &= \frac{1}{T}\int_{t-T}^{t} E[(\phi_{LP}(\tau) - e^{-\Lambda(\tau - T_0)}\phi_{LP}(T_0)) \bullet \phi_{SLP}(\tau)/x(\tau_1), \tau_1 \leq \tau]\ d\tau \\ &= \frac{1}{T}\int_{t-T}^{t} (\phi_{SLP}(\tau) - e^{-\Lambda(\tau - T_0)}\phi_{SLP}(T_0)) \bullet \phi_{SLP}(\tau)\ d\tau \\ &\approx \frac{1}{T}\int_{t-T}^{t} \phi_{SLP}(\tau) \bullet \phi_{SLP}(\tau)\ d\tau \\ &= \|\phi_{SLP}\|_T^2(t) \end{aligned}$$

we obtain again that the inner product can be employed as a fault indicator.

In case that θ_1 is unknown, we may have a large variety of possible profiles for $\phi(t)$; later we will show that for small parameter perturbations, the set of possible profiles in $\phi(t)$ can be efficiently characterized.

4.3 Known profile of $\phi(t)$ except for a proportional constant

In case the profile of $\phi(t) = k\varphi(t)$ is known, except for the value of K, we can define

$$\varphi_{LP}(t) = \int_0^t e^{-\Lambda(t-\tau)}\varphi(\tau)\ d\tau.$$

Assuming that

$$< \epsilon_{H_0}, \varphi_{LP} >_T (t) \quad \approx \quad 0$$

We can compute

$$\begin{aligned} < \epsilon, \varphi_{LP} >_T (t) &= < \epsilon_{H_0}, \varphi_{LP} >_T (t) + < \epsilon_{\phi}, \varphi_{LP} >_T (t) \\ &\approx < \epsilon_{\phi}, \varphi_{LP} >_T (t) \\ &\approx K\|\varphi_{LP}\|_T^2(t) \end{aligned}$$

so that the quality of the indicator depends on the unknown value of K.

4.3.1 Moving cosine

In this case, an alternative measure comes from the computation of the following generalized cosine

$$\begin{aligned} \cos(\epsilon, \varphi_{LP})_T(t) &= \frac{< \epsilon, \varphi_{LP} >_T (t)}{\|\epsilon\|_T(t) \cdot \|\varphi_{LP}\|_T(t)} \\ &\approx \frac{K\|\varphi_{LP}\|_T^2(t)}{\sqrt{K^2\|\varphi_{LP}\|_T^2(t) + \|\epsilon_{H_0}\|_T^2(t)} \cdot \|\varphi_{LP}\|_T(t)} \\ &= \frac{K}{\sqrt{K^2 + \frac{\|\epsilon_{H_0}\|_T^2(t)}{\|\varphi_{LP}\|_T^2(t)}}} \end{aligned}$$

which can be close to one provided $\|\epsilon_{H_0}\|_T(t)$ is small enough when compared to K and $\|\varphi_{LP}\|_T(t)$. $\|\epsilon_{H_0}\|_T^2(t)$ can be approximately computed under the assumption of $t > T_{\min} \gg 0$. In general, it is customary to assume that such $T_{\min} > T_0$ so that $\|\epsilon_{H_0}\|_T^2(t)$ does not change by the time the fault begins to show up.

4.3.2 Small parameter change

In case of parametric changes where θ_1 is unknown, if we consider small variations on θ, so that $\theta_1 - \theta_0 = \Delta\theta$, the fault process $\phi(t)$ can be linearly approximated

$$\begin{aligned}\phi(x(t),u(t),\theta_1,\theta_0,t) &\approx \left.\frac{\partial f(x(t),u(t),\theta,t)}{\partial\theta}\right|_{\theta_0} \cdot \Delta\theta \\ &= \left.\frac{\partial f(x(t),u(t),\theta,t)}{\partial\theta}\right|_{\theta_0} \cdot \frac{\Delta\theta}{\|\Delta\theta\|}\|\Delta\theta\| \\ &= \varphi(x(t),u(t),\theta_0,t)\cdot\|\Delta\theta\|\end{aligned}$$

so that if the direction of the parameter change $\Delta\theta$ is known, the profile can be determined, except for a proportionality constant, due to the linearity on the parameter variation. For example, this applies for changes involving a single parameter; in such case $\varphi(x(t),u(t),\theta_0,t)$ would be the corresponding normalized column of the Jacobian matrix of f.

Again, if we consider state space availability, $\varphi_S(t)$ can be computed and we can define the previous reasoning with $\|\Delta\theta\| = K$.

In [4], a detailed analysis is carried out of this cosine-based scheme in the scalar case for the purpose of both detection and isolation of faults.

4.4 Known statistics of $\phi(t)$

Alternatively, in many applications $\phi(t)$ can only be characterized as an stochastic process, and the detection problem becomes even more involved. In this setting, general results exist in discrete time systems for the basic problem of detecting changes in the scalar parameter of an independent sequence (precisely, the distribution P_{θ_0} of the sequence of independent measurements y is assumed to be known [1]). This procedure cannot be directly applied to the residual process $\epsilon(t)$; still some detection schemes can be developed for such residual when the changing parameter can be easily estimated from the sequence.

4.4.1 Case of known non-zero mean of $\phi(t)$

Let us assume that $\phi(t)$ is independent of the system noise $\eta(t)$ and that $E[\phi(t)] = \bar{\phi}(t) \neq 0$. If $\bar{\phi}(t)$ was to be known, we could use

$$\bar{\phi}_{LP}(t) = \int_0^t e^{-\Lambda(t-\tau)}\bar{\phi}(\tau)\, d\tau,$$

as a reference signal, so that

$$\begin{aligned}<\epsilon,\bar{\phi}_{LP}>_T(t) &= <\epsilon_{H_0},\bar{\phi}_{LP}>_T(t) + <\epsilon_{\phi},\bar{\phi}_{LP}>_T(t) \\ &= \frac{1}{T}\int_{t-T}^t E[\epsilon_{H_0}(\tau)\bullet\bar{\phi}_{LP}(\tau)]\, d\tau\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{T}\int_{t-T}^{t} E[(\phi_{LP}(\tau)-e^{-\Lambda(\tau-T_0)}\phi_{LP}(T_0))\bullet\bar{\phi}_{LP}(\tau)]\;d\tau \\
=\;& \frac{1}{T}\int_{t-T}^{t} E[\epsilon_{H_0}(\tau)]\bullet\bar{\phi}_{LP}(\tau)\;d\tau \\
&+\frac{1}{T}\int_{t-T}^{t} E[(\phi_{LP}(\tau)-e^{-\Lambda(\tau-T_0)}\phi_{LP}(T_0))]\bullet\bar{\phi}_{LP}(\tau)\;d\tau \\
=\;& \frac{1}{T}\int_{t-T}^{t} (\bar{\phi}_{LP}(\tau)-e^{-\Lambda(\tau-T_0)}\phi_{LP}(T_0))\bullet\bar{\phi}_{LP}(\tau)\;d\tau \\
\approx\;& \frac{1}{T}\int_{t-T}^{t} \bar{\phi}_{LP}(\tau)\bullet\bar{\phi}_{LP}(\tau)\;d\tau \\
=\;& \|\bar{\phi}_{LP}\|_T^2(t).
\end{aligned}
$$

4.4.2 Case of known constant lower bound of the mean of $\phi(t)$

In case that we do not know a priori the profile of $\bar{\phi}(t)$, but we know that $\bar{\phi}(t)\geq m > 0, \forall t$ (meaning that the inequalities are satisfied component-wise), we have

$$
\begin{aligned}
\bar{\phi}_{LP}(t) \;&=\; \int_0^t e^{-\Lambda(t-\tau)}\bar{\phi}(\tau)\;d\tau \geq \int_0^t e^{-\Lambda(t-\tau)}m\;d\tau \\
&=\; \Lambda^{-1}\cdot(I-e^{-\Lambda(t-\tau)})\cdot m \longrightarrow_{t\to\infty} \Lambda^{-1}\cdot m.
\end{aligned}
$$

Hence, if we use a constant reference vector function (e.g. 1 in all components) we get

$$
\begin{aligned}
<\epsilon,1>_T(t) \;=\;& <\epsilon_{H_0},1>_T(t)+\;<\epsilon_{\phi},1>_T(t) \\
=\;& \frac{1}{T}\int_{t-T}^{t} E[\epsilon_{H_0}(\tau)\bullet 1]\;d\tau \\
&+\frac{1}{T}\int_{t-T}^{t} E[(\phi_{LP}(\tau)-e^{-\Lambda(\tau-T_0)}\phi_{LP}(T_0))\bullet 1]\;d\tau \\
=\;& \frac{1}{T}\int_{t-T}^{t} E[\epsilon_{H_0}(\tau)]\bullet 1\;d\tau \\
&+\frac{1}{T}\int_{t-T}^{t} E[(\phi_{LP}(\tau)-e^{-\Lambda(\tau-T_0)}\phi_{LP}(T_0))]\bullet 1\;d\tau \\
=\;& \frac{1}{T}\int_{t-T}^{t} (\bar{\phi}_{LP}(\tau)-e^{-\Lambda(\tau-T_0)}\phi_{LP}(T_0))\bullet 1\;d\tau \\
\approx\;& \frac{1}{T}\int_{t-T}^{t} \bar{\phi}_{LP}(\tau)\bullet 1\;d\tau \\
\geq\;& \Lambda^{-1}\cdot m\cdot 1
\end{aligned}
$$

which, for large enough m, can be employed for testing purposes.

Note that an equivalent reasoning can be applied for the case $\bar{\phi}(t)\leq m < 0, \forall t$. Obviously, these analyses include the case of $\phi(t)$ being a stationary process with non-zero constant mean.

In [2], different detection schemes are considered under the framework of mean estimation, which can be considered as variations of the above presented projection:

- $\mu_1 = \lim_{T\to 0} < \epsilon, 1 >_T (t)$,
- $\mu_2 =< \epsilon, 1 >_t (t)$,
- $\mu_3 =< \epsilon, 1 >_T (t)$.

Then the detection capabilities are analyzed through the study of estimators.

5 Testing procedures

The projections proposed in the previous section, serve as appropriate residuals for testing purposes. If no fault has occurred, such quantities are supposed to have zero value, whereas when a fault occurs they take values significantly larger than zero.

In all cases, appropriate estimators for such inner products must be constructed, which rely on the specific realization of ϵ which is measured. The evaluation of such residuals leads to the construction of a hypothesis test, which forms the core of the corresponding detection scheme.

Note that since the inner product depends on t, sequential tests can be performed on-line so that detection times can also be evaluated.

6 Concluding remarks

The unified setting, based on the definition of a pre-Hilbert space, successfully gathers the existing detection schemes for continuous-time stochastic dynamical systems [2, 4]. The quantities evaluated in these schemes can be interpreted as different projection operators which depend on the a priori knowledge about the fault.

This general formulation allows for the definition of new detection schemes and for comparative analyses among them.

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